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Department of

Electrical & Electronics Engineering

Abdullah Gül University

Project Report for Paradox of Double Cone Rolling Up an Incline

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Project Report

EE1100 Computation and Analysis (COMA) Capsule

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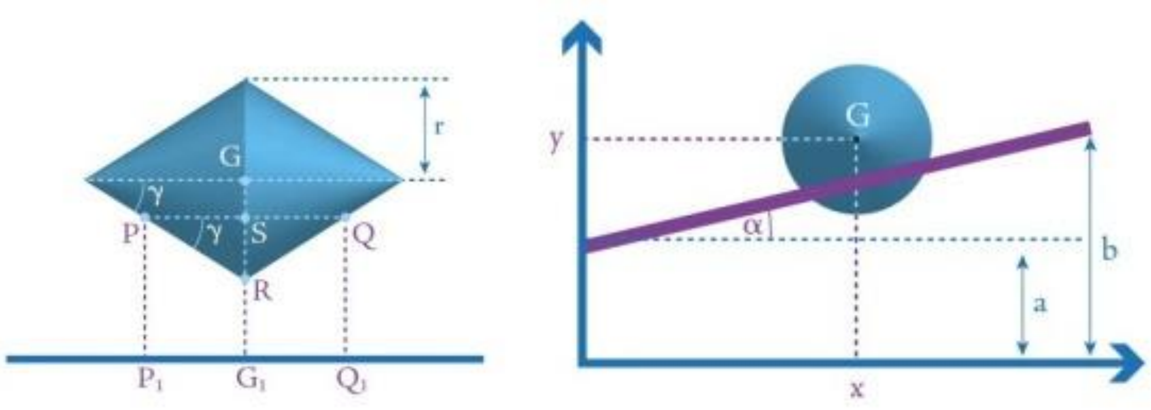
OBJECTIVE

The double cone experiment aims to investigate the motion of a double cone's center of mass along inclined rails, to ascertain the effect of the bases' angle of inclination on the rolling time and the circumstances under which the body will be motionless, and to explain the "anti-gravity" paradox of the double cone motion.



DOUBLE CONE DIAGRAM

FIGURE 1

In this experiment, by connecting two right circular cones at their bases, a double cone is created. When positioned close to the bottom of an inclining triangle track, the double cone rises. The double cone moves upward in an intriguing and unexpected manner. It seems to be moving counter to gravity. As a result, it is also known as an antigravity double cone. We shall investigate the double cone's counter-intuitive motion.

BACKGROUND

For the project, hollow a double cone made of wood with a radius of 4 centimeters and a height of 13 centimeters and the upward movement of the double cone despite the inclined plane in the setup made using 3 proportional boards were examined.

ANALYTICAL AND SIMULATION PROCEDURES

a. Steps of Experiment

In this project

- Hollow double cone made of wood (made using two cones of the same size and shape, glued together at their bases),
- Triangular arrangement made of three proportional boards,
- Cylindrical object

are used.

1. 2 proportional wooden sticks are combined to change the angle between the rails. The other wooden bar, which is proportional to these bars, is connected to the flat ground to form an angle of inclination which is 3°. Thus, the construction of the triangular assembly on which the double cone will move has been completed.

2. After making sure that there are no other objects that could affect the direction of movement of the double cone, the setup is ready for experimentation. (The apparatus in the video was taken as a reference for the experiment. [1*])

3. As a control experiment, a cylindrical object was released at the widest part of the rail (a metal beverage can was used in the experiment). The downhill motion of the cylindrical object was observed as expected.

4. The double cone at the narrow end of the track was released, observing how it irrationally rolled uphill.

5. We recorded a video during the movement of the double cone and cylinder (YouTube video link is attached.)

6. The result of this experiment is that the double cone rounds the slope in an unexpected direction. Instead of rolling downhill as expected due to gravity (like the movement of a cylindrical shape), the double cone rolled uphill and against the direction of gravity. This paradoxical result was inferred from the physical and algebraic calculations determined to be due to the shape and properties of the double cone that allow it to take advantage of the friction and rotational forces in a way that defies the laws of gravity.

Abstract Stages of Experiment

- As seen in the pictures (**Figure 2, Figure 3, Figure 4**), the mechanism is set up to perform the experiment, and the double cone and triangular track are appropriate and measured. Ready to do the experiment.

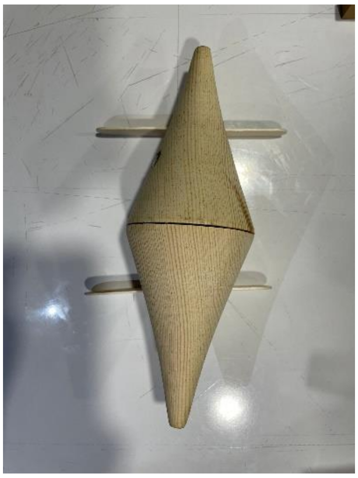


FIGURE 2

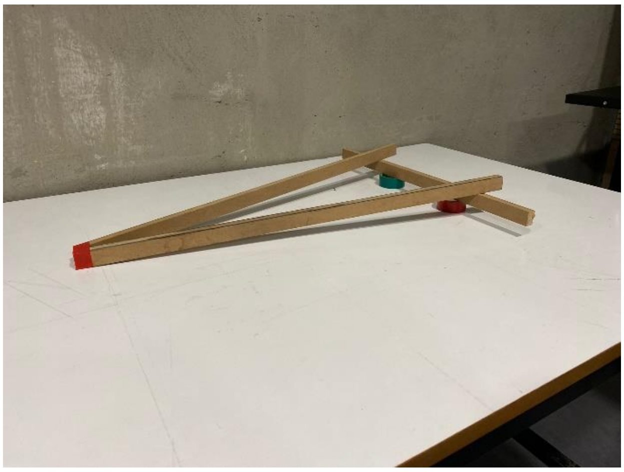


FIGURE 3

- With the help of the materials, experiment of the “anti-gravity” paradox of the double cone motion was performed. By measuring, values such as critical angle of inclination where the double cone starts to move upwards, maximum angular velocity, are taken.
- r is the maximum radius of the double cone, Δz is the maximum vertical displacement, etc. It is appropriate to take other values. These values will be used in the interpretation of physical and algebraic operations below.

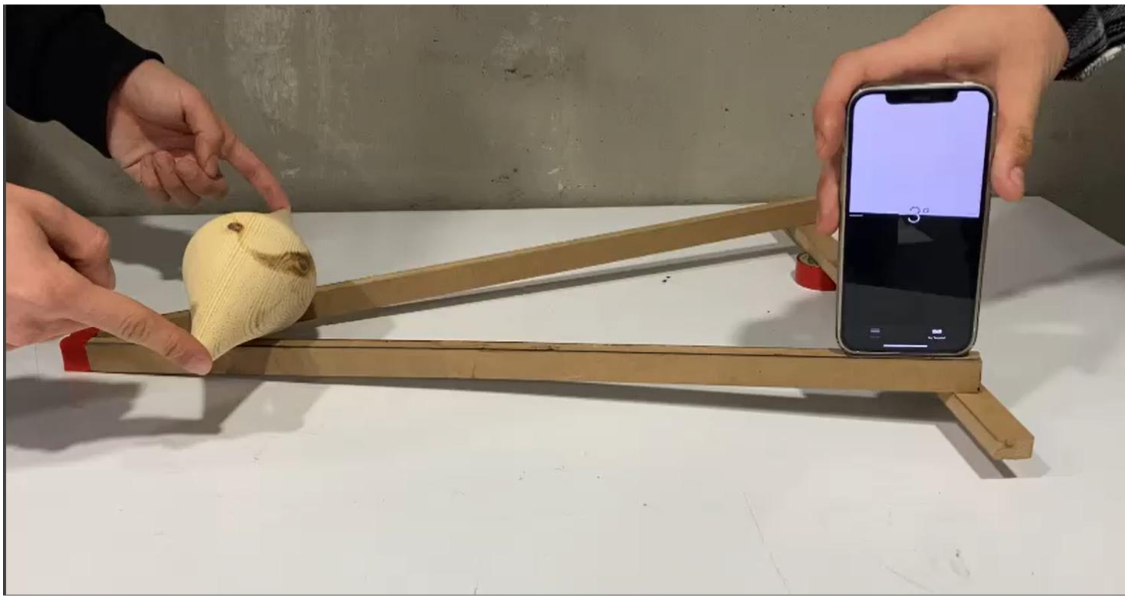


FIGURE 4

b. Evaluation of Problems

The double cone setup consists of two identical cones with mass m, radius R and height H placed in a horizontal plane. The cones are glued together from their base. It has been observed that the cone moves from the lower position to the upper position when the cone is placed on an inclined rail assembly. This unexpected movement is explained by a paradox.

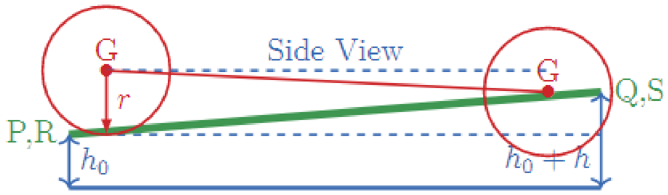


FIGURE 5

h_0= the height of the mechanism itself

h_i= the initial height of the center of mass at lower point

h_f= the final height of the center of mass at upper point

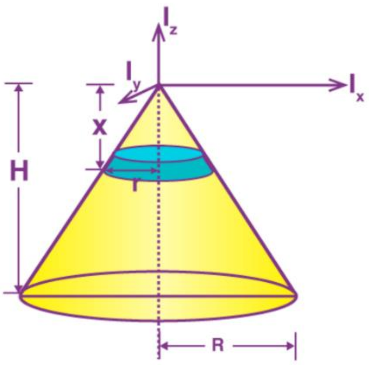
r= radius of double cone

It is known that objects move from a higher level to a lower level due to the force of gravity. To explain the paradox, the concept of center of mass must be considered. The center of mass G of the double cone is located at the point where the axis of symmetry intersects the plane of symmetry. The height of the center of mass G when the cone is at the lower end is $h_i=h_0+r$[2*]. As the cone moves from its lower end to its upper end, the center of mass begins to descend. At the upper end, center of mass G is at a height $h_i=h_0+h$. Since r>h, h_i is greater than h_f. Therefore, h_i is greater than the final height h_f. The object's center of mass moves downward. It is seemed that the double cone moves from the lower part to the upper part of the system. In fact, the center of mass moves down in the system.

The moment of inertia of the double cone changes according to whether the double cone is solid or hollow. The following steps have been done to calculate moment of inertia of solid double cone which is used in the experiment. Since the double cone rotates around the z-axis, the operations are done accordingly:

$$\frac{dm}{M} = \frac{\pi r^2 dx}{\frac{1}{3}\pi R^2 H} \quad \Longrightarrow \quad dm = \frac{3Mr^2 x dx}{R^2 H}$$

If we apply the similarity theorem for the cone, the following results are obtained:



$$\frac{x}{r} = \frac{H}{R} \qquad r = \frac{x.R}{H} \qquad r^2 = \frac{x^2 R^2}{H^2}$$

If we substitute r^2 in the dm equation, outcome will be like below;

$$dm = \frac{3M\frac{x^2 R^2}{H^2} x dx}{R^2 H} = \frac{3Mx^2 dx}{H^3}$$

$$dI = \frac{1}{2} dmr^2 = \frac{1}{2} \frac{3Mx^2 dx}{H^3} \frac{x^2 R^2}{H^2}$$

$$dI = \frac{3MR^2}{2H^5} x^4 dx$$

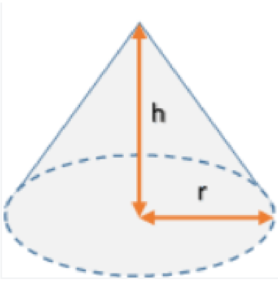
$$\int dI = \int_0^H \frac{3MR^2}{2H^5} x^4 dx = \frac{3}{10} MR^2$$

$$I = \frac{3}{10} MR^2$$

When it is asked to experimentally find the critical angle of inclination at which the double cone starts to move upward, the following solutions are attained:

The critical angle is between the ground and the mechanism. The critical angle can be called as α. This angle α must be less than the product of the tangent of half the angle between the two rails and the arctangent of the r/h ratio of the cone. It can be called the half angle between two rails as β, arctangent of r/h as θ. So, the equation will be this:

$$\tan \boldsymbol{\alpha} < \tan \boldsymbol{\beta} \,.\, \tan \boldsymbol{\theta}$$



According to the results obtained from the experiment, the angle α between the mechanism and the ground is 3°. The h (height) of the cone is 13 centimeters and the r (radius) of cone is 4 centimeters. When their ratio is written , r/h=4/13 is obtained. The $\tan \theta$ is found as follows: $\tan^{-1} \frac{4}{13} = \tan \theta$, $\tan \theta = \tan 12{,}09°$. The half angle between two rails is 30°. So, $\tan \beta$ is 15°. If these are substituted in the equation;

$$\tan 3° < \tan 15° \,.\, \tan 12{,}09°$$

$$\tan 3° \cong 0{,}0524$$

$$\tan 15° \cong 0{,}2141$$

$$\tan 12{,}09° \cong 0{,}2679$$

$$0{,}0524 < 0{,}05735739$$

When making some theoretical predictions about how this critical angle depends on the height and radius of the double cone, the following can be said:

For example, if the height h is increased and the radius is kept constant, the ratio r/h will decrease. As the r/h ratio decreases, $\tan \theta$ value decreases. As the $\tan \theta$ value decreases, the product of $\tan \theta$ and $\tan \beta$ will approach the $\tan \alpha$ value. When this happens, it will be difficult for the double cone to go from the bottom to the top. So, the paradox of double cone will not work.

If the radius was increased and the height kept constant, the r/h ratio would increase. As the r/h ratio increased, the $\tan\theta$ value would also increase. As the $\tan\theta$ value increases, the product of $\tan\theta$ and $\tan\beta$ will be bigger the $\tan\alpha$ value. So, double cone will move easily.

To find maximum angular velocity, the following equation [3*] is used:

$$\boldsymbol{\omega} = \sqrt{\frac{\mathbf{20}.\boldsymbol{g}.\boldsymbol{\Delta z}}{\mathbf{3}\boldsymbol{r}^{\mathbf{2}}}}$$

$\boldsymbol{\omega}$=angular velocity $\boldsymbol{g}$=gravity acceleration

r=radius of double cone $\Delta\boldsymbol{z}$=maximum vertical displacement

$\boldsymbol{g}$ =**9,8 N/kg** **r=**$\frac{4}{100}$**m** $\Delta\boldsymbol{z}$=$\frac{5}{100}$**m**

$$\boldsymbol{\omega} = \sqrt{\frac{20.9{,}8.\frac{5}{100}}{3.(\frac{4}{100})^2}} = \mathbf{45{,}1848} \ldots \boldsymbol{rad/s}$$

Energy conservation equation can be used to check experimentally the equation for the maximum angular velocity:

$$\boldsymbol{m}.\boldsymbol{g}.\Delta\boldsymbol{z} = \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{I}.\boldsymbol{\omega}^{\mathbf{2}}$$

$\boldsymbol{m}$**=**mass of the double cone $\Delta\boldsymbol{z}$=maximum vertical displacement

$\boldsymbol{g}$**=** gravity acceleration $\boldsymbol{I}$=moment of inertia of double cone

The moment of inertia was found as $\frac{3}{10}\boldsymbol{MR}^{\mathbf{2}}$. When the values are substituted in the equation:

$$\boldsymbol{m}.\mathbf{9{,}8}.\frac{\mathbf{5}}{\mathbf{100}} = \frac{\mathbf{1}}{\mathbf{2}}.\frac{\mathbf{3}}{\mathbf{10}}\boldsymbol{m}\left(\frac{\mathbf{4}}{\mathbf{100}}\right)^{\mathbf{2}}.\boldsymbol{\omega}^{\mathbf{2}}$$

When the masses are simplified, it is seen that the value of ω is 45,1848... rad/s. So, it can be said that the $\boldsymbol{\omega} = \sqrt{\frac{\mathbf{20}.\boldsymbol{g}.\Delta\boldsymbol{z}}{\mathbf{3}\boldsymbol{r}^{\mathbf{2}}}}$ formula works.

Using angular velocity, the linear velocity (ϑ) can be found with the following formula:

$$\boldsymbol{\vartheta = \omega . r}$$

$$\boldsymbol{\vartheta \cong 45,1848 \; x \; \frac{4}{100} = 1,807392 \; m/s}$$

In the case where it is desired to create a linear system of equations in the form of Ax=b with any equation algebraically and to explain the results of the changing variables in this system:

$$Ax = B$$

$$\Delta U + \Delta K = 0$$

$$\begin{bmatrix} -7 & 7 \end{bmatrix} \begin{bmatrix} \Delta U \\ \Delta K \end{bmatrix} = [0]$$

The link between rolling kinetic energy and potential energy is demonstrated by the Ax=b matrix system in energy conservation equation (In the article it is 6th equation). The magnitudes of the potential and kinetic energy are represented by the coefficient in the matrix system (A). Potential and rolling kinetic energy are represented by the unknowns in the matrix system (x). The system's result (b) shows that the sum of the potential and kinetic energy in a closed system is equal to zero. The Ax=b matrix system demonstrates that if we modify one unknown's coefficient, changes to the other unknown will have the opposite effect. For instance, if we increase the amount of rolling kinetic energy, the object's potential energy will decrease since, in a closed system, energy is conserved because it is converted from one form of energy to another.

When a vector a is constructed with ω, Δz and r and a vector orthogonal to this vector is found;

$$\boldsymbol{\omega = 45,1848 \; rad/s} \qquad \boldsymbol{\Delta z = \frac{5}{100} m} \qquad \boldsymbol{r = \frac{4}{100} m}$$

$$\begin{bmatrix} \boldsymbol{\omega} \\ \boldsymbol{\Delta z} \\ \boldsymbol{r} \end{bmatrix} \leftrightarrow \begin{bmatrix} \boldsymbol{45,1848} \\ \boldsymbol{5/100} \\ \boldsymbol{4/100} \end{bmatrix}$$

If the result is 0 when the dot product of two vectors is made, these two vectors are said to be orthogonal to each other: $\underset{v}{\rightarrow} \times \underset{y}{\rightarrow} = \boldsymbol{0}$.

$$\begin{bmatrix} \boldsymbol{x} & \boldsymbol{y} & \boldsymbol{z} \end{bmatrix} \begin{bmatrix} \mathbf{45,1848} \\ \mathbf{5/100} \\ \mathbf{4/100} \end{bmatrix} = [\mathbf{0}]$$

$$\mathbf{45,1848}(\boldsymbol{x}) + \frac{\mathbf{5}}{\mathbf{100}}(\boldsymbol{y}) + \frac{\mathbf{4}}{\mathbf{100}}(\boldsymbol{z}) = \mathbf{0}$$

Suppose to find orthogonal matrix → x=0 and y=1;

$$\mathbf{0} + \frac{\mathbf{5}}{\mathbf{100}} + \frac{\mathbf{4}}{\mathbf{100}}(\boldsymbol{z}) = \mathbf{0}$$

Therefore, z is obtained as $\frac{-5}{4}$.

$$\text{Orthogonal Vector} \rightarrow \begin{bmatrix} 0 \\ 1 \\ -5/4 \end{bmatrix}$$

To find the projection p of $b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ onto vector a, the following steps are done:

Formula to obtain projection of b onto a = $\frac{a^T.b}{a^T.a}a$

$$p = a.x = P.b$$

$$x = \frac{a^T.b}{a^T.a}$$

$$a = \begin{bmatrix} 45.1848 \\ 0.05 \\ 0.04 \end{bmatrix} \qquad a^T = [45.1848 \quad 0.05 \quad 0.04]$$

Step One:

$a^T.b$ = (1 · 45.1848) + (2 · 0.05) + (3 · 0.04)

$a^T.b$ = 45.185 + 0.1 + 0.12

$a^T.b$ = 45.405

Step Two:

$a^T \times a = a.a = a^2$= $\sqrt{45{,}1848^2\ +\ 0{,}05^2\ +\ 0{,}04^2}$

a^2= $\sqrt{2.041{,}7\ +\ 0{,}0025\ +\ 0{,}0016}$

a^2= $\sqrt{2{,}041.7}$

$a^T \times a$ =$a.a$ $= 2{,}041.7$

Step Three:

$x = \frac{a^T.b}{a^T.a}$ = $\frac{45.405}{2{,}041.7}$

$x = \frac{a^T.b}{a^T.a}$= $0{,}0222$

Step Four: Multiply Vector a by the Projection Factor

$$p = a.x$$

$$a = \begin{bmatrix} 45.1848 \\ 0.05 \\ 0.04 \end{bmatrix} \qquad x = 0.0222$$

Projection of b onto a = (45.1848 x 0.0222, 0.05 x 0.0222, 0.04 x 0.0222)

Projection of b onto a = $\begin{bmatrix} \mathbf{1,005} \\ \mathbf{0,001112} \\ \mathbf{0,00089} \end{bmatrix}$

To find error vector following formula should be used;

$$\boldsymbol{e = b - p}$$

b - p = ({$x_b - x_p$}, {$y_b - y_p$}, {$z_b - z_p$})

When it is substituted and solved:

b - p = ({1 - 1.005}, {2 - 0.001112}, {3 - 0.00089})

$$\boldsymbol{e} = \begin{bmatrix} \mathbf{-0,005} \\ \mathbf{1,998888} \\ \mathbf{2.99911} \end{bmatrix}$$

To evaluate the projection matrix (P) following formula should be applied:

$$p = a.x = a.\frac{a^T.b}{a^T.a} \rightarrow p = P.b$$

$$P = \frac{a \times a^T}{a^T \times a}$$

Solution:

a=$\begin{bmatrix} 45.1848 \\ 0.05 \\ 0.04 \end{bmatrix}$ a^T=$[45.1848 \quad 0.05 \quad 0.04]$

In order to find $a \times a^T$, following steps are applied:

Step 1:

$$\begin{bmatrix} (45.1848 \times 45.1848) & (45.1848 \times 0.05) & (45.1848 \times 0.04) \\ (0.05 \times 45.1848) & (0.05 \times 0.05) & (0.05 \times 0.04) \\ (0.04 \times 45.1848) & (0.04 \times 0.05) & (0.04 \times 0.04) \end{bmatrix}$$

Step 2:

$$\begin{bmatrix} 2041.66615104 & 2.25924 & 1.807392 \\ 2.25924 & 0.0025 & 0.002 \\ 1.807392 & 0.002 & 0.0016 \end{bmatrix}$$

Result:

$$\begin{bmatrix} \mathbf{2041.66615104} & \mathbf{2.25924} & \mathbf{1.807392} \\ \mathbf{2.25924} & \mathbf{0.0025} & \mathbf{0.002} \\ \mathbf{1.807392} & \mathbf{0.002} & \mathbf{0.0016} \end{bmatrix}$$

The following operations were performed to evaluate $a^T \times a$ for the denominator part.

a^T=$[45.1848 \quad 0.05 \quad 0.04]$ a=$\begin{bmatrix} 45.1848 \\ 0.05 \\ 0.04 \end{bmatrix}$

Step 1:

$$[(45.1848 \times 45.1848 + 0.05 \times 0.05 + 0.04 \times 0.04)]$$

Step 2:

[(2041.66615104+0.0025+0.0016)]

Step 3:

[2041.67025104]

$$P = \frac{a \times a^T}{a^T \times a} = \frac{1}{2041.67025104} \times \begin{bmatrix} 2041.66615104 & 2.25924 & 1.807392 \\ 2.25924 & 0.0025 & 0.002 \\ 1.807392 & 0.002 & 0.0016 \end{bmatrix}$$

RESULTS AND DISCUSSION

In this experiment, we examined the Paradox of Double Cone Rolling Up an Incline. We set up an inclined ramp at an angle of 3 degrees and placed a double cone on the ramp. We then recorded the inter-rail angle and critical tilt angle required for the double cone to round the ramp. We repeated this process at different angles between rails and collected data on the angular velocity at each angle.

The results of this experiment are consistent with the Paradox of Double Cone Rolling Up an Incline. As the angle between the rails increases, the object rolls more easily. This is implausible, as a narrower angle width can be expected to make rolling more difficult and therefore increase rolling time. However, this can be explained by considering the rotational motion of the double cone. The greater the angle between the rails, the more pronounced the rotational action of the double cone, making it easier to round the slope.

In addition, it has been determined that the 15-degree angle is the most efficient angle for rolling. This experiment has some limitations. For example, we only tested one tilt angle, so there may be other angles where the double cone can rotate more efficiently. In summary, this experiment provided evidence for the Double Cone Rolling on a Slope Paradox and showed that the object rolls more easily as the angle between the rails increases. The most productive angle was found to be 15 degrees.

CONCLUSIONS

The result of this experiment is that the double cone rounds the slope in an unexpected direction. Instead of rolling downhill as expected due to gravity (like the movement of a cylindrical shape), the double cone rolled uphill and against the direction of gravity. This paradoxical result was inferred from the physical and algebraic calculations determined to be due to the shape and properties of the double cone that allow it to take advantage of the friction and rotational forces in a way that defies the laws of gravity. From this project, we learnt that exactly same results cannot be obtained due to environmental conditions. The most important thing that we learnt organizing a teamwork, researching a topic which we don't know anything about and sharing information. We are pleased to gain these achievements.

THE VIDEO WE RECORDED

https://youtu.be/3CkZteROANY

REFERENCES

1. https://www.youtube.com/watch?v=hwA-5oCkC_Q
2. https://www.concepts-of-physics.com/anveshika/naest-2018-double-cone-rolling-up.php
3. https://core.ac.uk/download/pdf/53273197.pdf