

>>> Feature Extraction

>>> GRSS Summer School

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Date: [2017-04-26 Wed 10:30]–[2017-04-26 Wed 12:00]

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2. Physical Indices

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    Unsupervised

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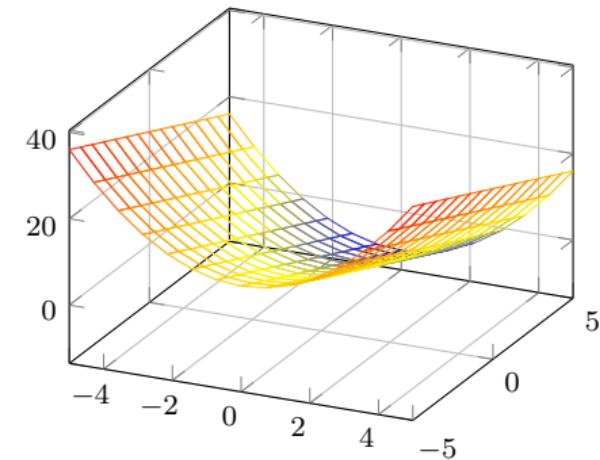
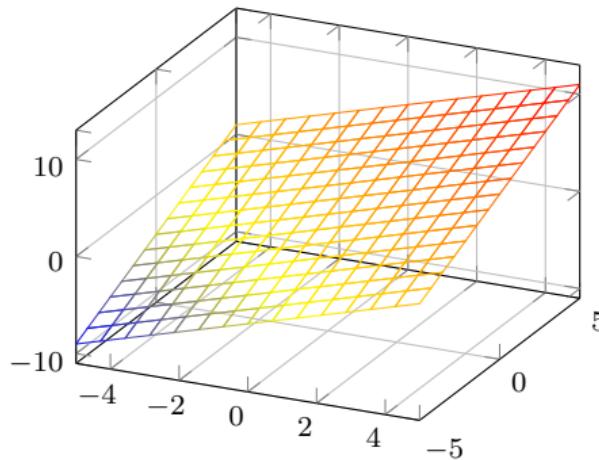
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- \* **Curse of dimensionality:** it is not possible to get enough data to cover all the observation space.  
*High dimensional spaces are mostly empty !*

- \* **Curse of dimensionality:** it is not possible to get enough data to cover all the observation space.  
*High dimensional spaces are mostly empty !*
- \* Multivariate data live in a lower dimensional space, but which one ?



- ★ Feature extraction is important in remote sensing because:
  - ★ It reduces the size of the data,
  - ★ It limits the spatial and the spectral redundancy,
  - ★ It permits visualization of the data,
  - ★ It mitigates the *curse of dimensionality*.
- ★ Extraction techniques:
  - ★ Spectral
    - ★ Physically based method,
    - ★ Statistical methods.
  - ★ Spatial:
    - ★ Linear filters,
    - ★ Non linear techniques (Mathematical Morphology)

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- ★ Spectral indices are a linear/non-linear combination of two (or more) spectral bands.
- ★ They provides information as a *single number* about:
  - ★ Plant structure,
  - ★ Biochemistry,
  - ★ Humidity,
  - ★ Stress.
- ★ Four main types [TLH11]:

Name	Formulae
Difference vegetation index	$R_{\lambda_1} - R_{\lambda_2}$
Ratio vegetation index	$\frac{R_{\lambda_1}}{R_{\lambda_2}}$
Normalized difference vegetation index	$\frac{R_{\lambda_1} - R_{\lambda_2}}{R_{\lambda_1} + R_{\lambda_2}}$
Soil-adjusted vegetation index	$(1 + L) \times \frac{R_{\lambda_1} - R_{\lambda_2}}{R_{\lambda_1} - R_{\lambda_2} + L}$

- ★ The three last indexes are invariant to a multiplicative factor

Index database : <http://www.indexdatabase.de/>

Name	Formulae ( $\lambda$ nm)
Normalized Difference Vegetation index	$\frac{R_{\lambda 800} - R_{\lambda 670}}{R_{\lambda 800} + R_{\lambda 670}}$
Modified Soil-Adjusted Vegetation Index	$\frac{1}{2} \left[ 2R_{\lambda 800} + 1 - \sqrt{(2R_{\lambda 800} + 1)^2 - 8(R_{\lambda 800} - R_{\lambda 670})} \right]$
Modified Chlorophyll Absorption Ratio Index	$[(R_{\lambda 700} - R_{\lambda 670}) - 0.2(R_{\lambda 700} - R_{\lambda 550})] \times \frac{R_{\lambda 700}}{R_{\lambda 670}}$
Normalized Difference Water Index	$\frac{R_{\lambda 858} - R_{\lambda 1240}}{R_{\lambda 858} + R_{\lambda 1240}}$
Datt Reflectance Index	$\frac{R_{\lambda 816} - R_{\lambda 2218}}{R_{\lambda 816} + R_{\lambda 2218}}$
Normalized Difference Redness Index	$\frac{R_{\lambda 540} - R_{\lambda 700}}{R_{\lambda 540} + R_{\lambda 700}}$
Soil Brightness Index	$0.406R_{\lambda 550} + 0.600R_{\lambda 650} + 0.645R_{\lambda 750} + 0.243R_{\lambda 950}$

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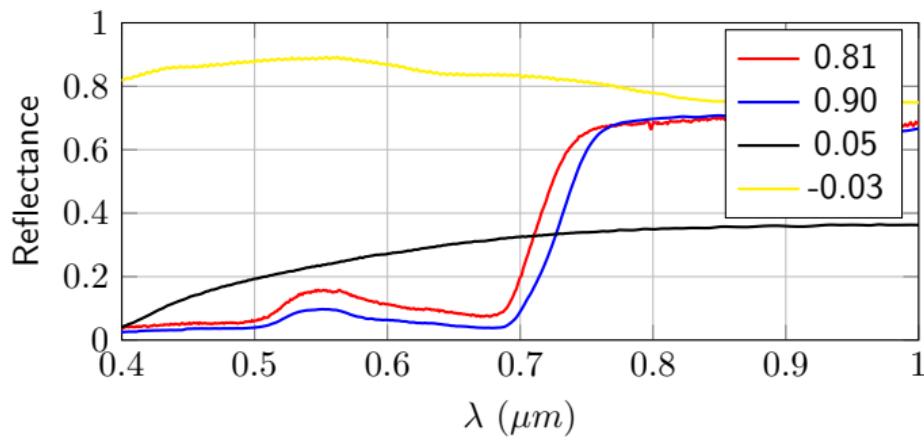
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>>> Normalized difference vegetation index

$$\text{NDVI} = \frac{R_{\lambda_{800}} - R_{\lambda_{670}}}{R_{\lambda_{800}} + R_{\lambda_{670}}}$$

- ★  $-1 \leq \text{NDVI} \leq 1$
- ★  $\text{NDVI} < 0$ : surfaces other than plant cover
- ★  $\text{NDVI} \approx 0$ : bare soil
- ★  $\text{NDVI} \geq 0.1$ : vegetation cover (higher values correspond to more dense covers)



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- ★ Peri-urban area
- ★ Rosis-3 sensor
- ★ 103 Spectral bands (400nm-900nm)
- ★ 1.5 meter per pixel spatial resolution
- ★  $610 \times 340$  pixels

- ★ OTB is a C++ library for remote sensing images processing.
- ★ It is free, open-source and available for most OS (window, apple, linux)
- ★ OTB-Applications are set of tools appropriated for big/large images
- ★ They are available from QGIS, Python and Bash
- ★ To compute the NDVI

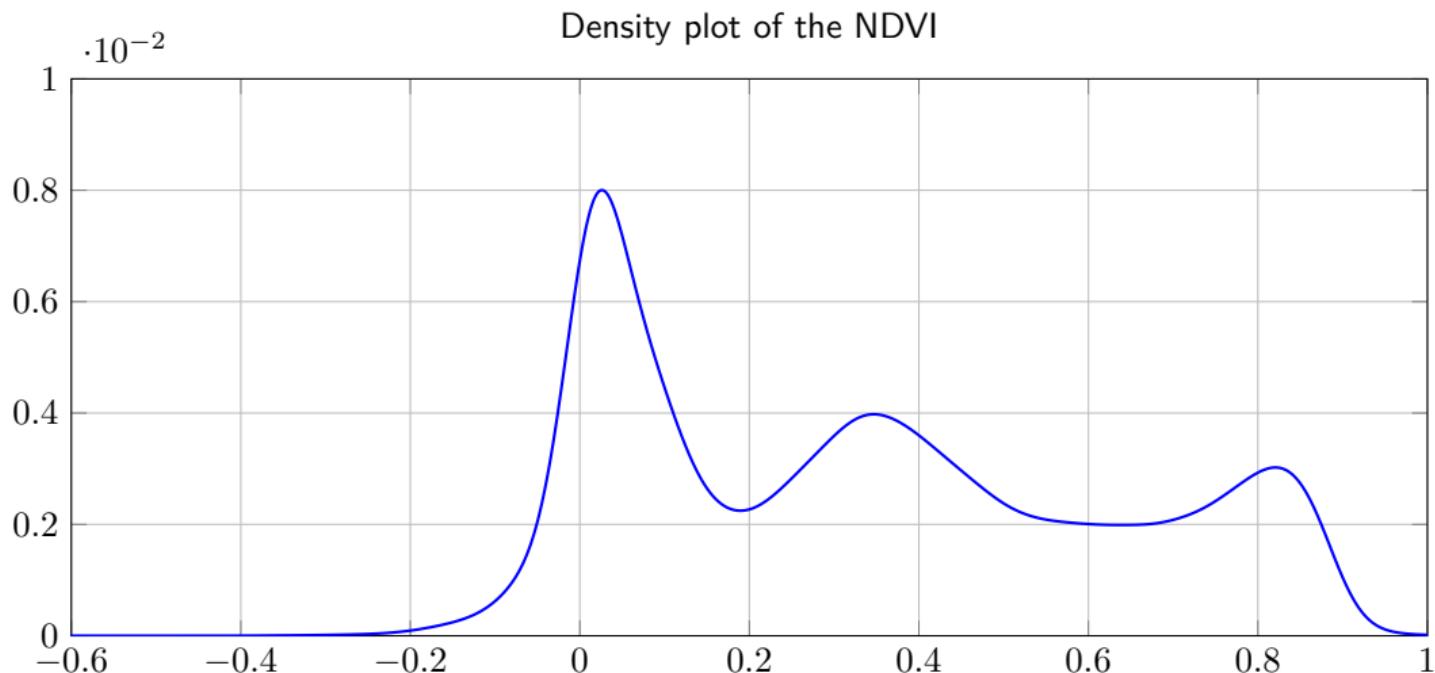
```
# Computation of the NDVI
otbcli_BandMath -il ../Data/university.tif -out ../Data/university_ndvi.tif \
-exp "(im1b83-im1b56)/(im1b83+im1b56)"
```

```
# Computation of the SBI
otbcli_BandMath -il ../Data/university.tif -out ../Data/university_sbi.tif \
-exp "0.406*im1b31 + 0.6*im1b52 + 0.645*im1b73"
```

>>> University of Pavia - Spectral Indices



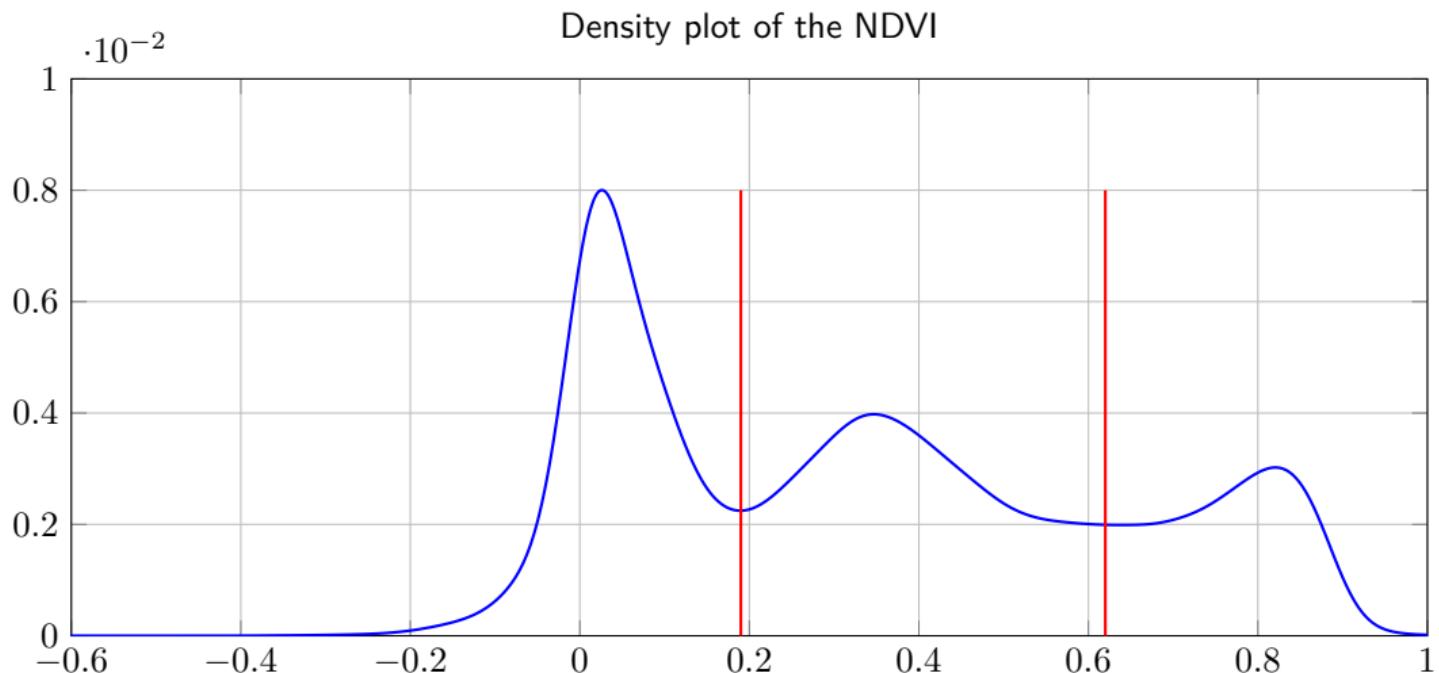
>>> Where is the vegetation 1/2 ?



# Segmentation of the NDVI in three classes

```
otbcli_BandMath -il ../Data/university_ndvi.tif -out ../Data/university_ndvi_segmented.tif \
-exp "(im1b1<0.19?1:(im1b1<0.62?2:3))"
```

>>> Where is the vegetation 1/2 ?



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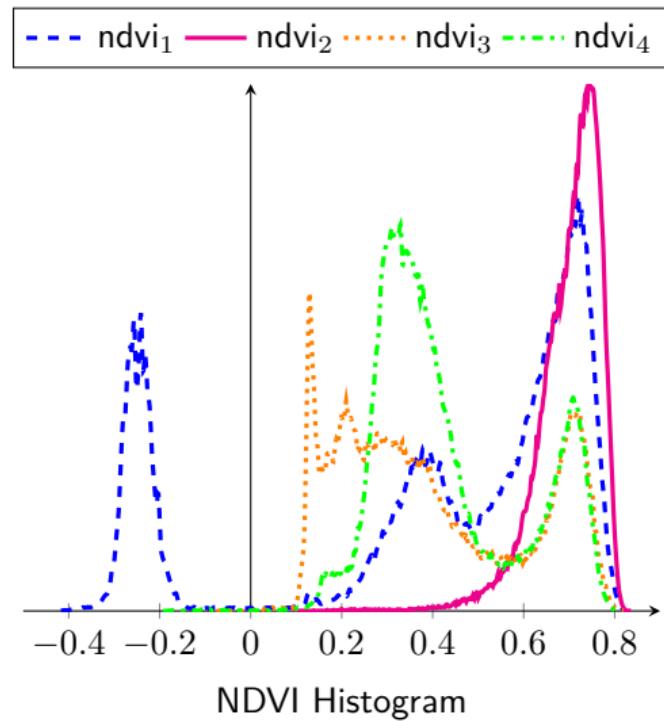
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>>> Could you find the good one ?



Image



From the histogram, which one does correspond to the NDVI of the image ?

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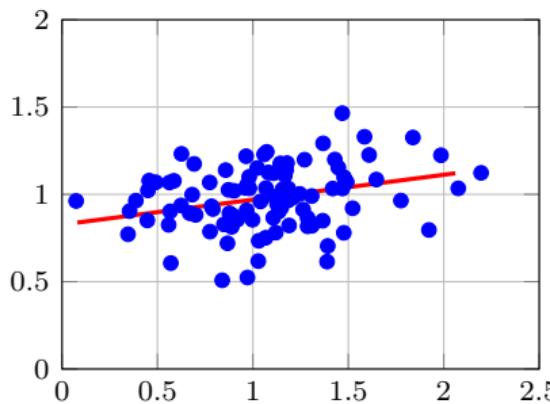
## >>> Principal Components Analysis

- ★ Linear transformation used to reduce the dimensionality of the data [Jol02].

$$z_i = \langle \mathbf{v}_i, \mathbf{x} \rangle$$

- ★ Find features  $\mathbf{z}$  that account for most of the variability of the data:

- ★  $z_1, z_2, z_3, \dots$  are mutually uncorrelated,
- ★  $\text{var}(z_i)$  is as large as possible,
- ★  $\text{var}(z_1) > \text{var}(z_2) > \text{var}(z_3) > \dots$



>>> Maximization of the variance 1/2

- ★ Search  $\mathbf{v}_1$  such as  $\max \text{var}(z_1)$ :

$$\begin{aligned}\text{var}(z_1) &= \text{var}(\langle \mathbf{v}_1, \mathbf{x} \rangle) \\ &= \mathbf{v}_1^\top \Sigma \mathbf{v}_1\end{aligned}$$

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- ★ Indetermined: if  $\hat{\mathbf{v}}_1$  maximizes the variance,  $\alpha \hat{\mathbf{v}}_1$  too! Add a constraint:  $\langle \mathbf{v}_1, \mathbf{v}_1 \rangle = 1$

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- ★ Lagrangian:

$$\mathcal{L}(\mathbf{v}_1, \lambda_1) = \mathbf{v}_1^\top \Sigma \mathbf{v}_1 + \lambda_1(1 - \mathbf{v}_1^\top \mathbf{v}_1)$$

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- ★  $\mathbf{v}_1$  is an eigenvector of the covariance matrix of  $\mathbf{x}$ :

$$\Sigma \mathbf{v}_1 = \lambda_1 \mathbf{v}_1$$

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$$\Sigma \mathbf{v}_1 = \lambda_1 \mathbf{v}_1$$

- ★  $\mathbf{v}_1$  is the eigenvector corresponding to the largest eigenvalues !

$$\text{var}(z_1) = \mathbf{v}_1^\top \Sigma \mathbf{v}_1 = \lambda_1 \mathbf{v}_1^\top \mathbf{v}_1 = \lambda_1$$

>>> Maximization of the variance 2/2

- ★ Search  $\mathbf{v}_2$  such as  $\max \text{var}(z_2)$  and  $\langle \mathbf{v}_2, \mathbf{v}_2 \rangle = 1$  and  $\langle \mathbf{v}_1, \mathbf{v}_2 \rangle = 0$

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- ★ At optimality,  $\langle \mathbf{v}_1, \mathbf{v}_2 \rangle = 0$ . Left-multiplying by  $\mathbf{v}_1^\top$  the above equation:

$$\begin{aligned}\mathbf{v}_1^\top \Sigma \mathbf{v}_2 &= 2\beta_1 \\ \lambda_1 \mathbf{v}_1^\top \mathbf{v}_2 &= 2\beta_1 \\ 0 &= 2\beta_1\end{aligned}$$

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$$\Sigma \mathbf{v}_2 = \lambda_2 \mathbf{v}_2$$

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$$\Sigma \mathbf{v}_2 = \lambda_2 \mathbf{v}_2$$

- ★  $\mathbf{v}_2$  is the eigenvector corresponding the *second largest* eigenvalues

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- ★ Hence, we have

$$\Sigma \mathbf{v}_2 = \lambda_2 \mathbf{v}_2$$

- ★  $\mathbf{v}_2$  is the eigenvector corresponding the *second largest* eigenvalues
- ★  $\mathbf{v}_k$  is the eigenvector corresponding the  $k^{\text{th}}$  *largest* eigenvalues

1. Empirical estimation the mean value:

$$\boldsymbol{\mu} = \frac{1}{n} \sum_{i=1}^n \mathbf{x}_i$$

2. Empirical estimation the covariance matrix:

$$\boldsymbol{\Sigma} = \frac{1}{n-1} \sum_{i=1}^n (\mathbf{x}_i - \boldsymbol{\mu})(\mathbf{x}_i - \boldsymbol{\mu})^\top$$

3. Compute  $p$  first eigenvalues/eigenvectors... How to choose  $p$ ? Explained variance:

$$\frac{\sum_{i=1}^p \lambda_i}{\sum_{i=1}^d \lambda_i}$$

4. Tips for high dimensional data set: if  $n < d$  see [MLC16] page 420

>>> PCA case study 1/3

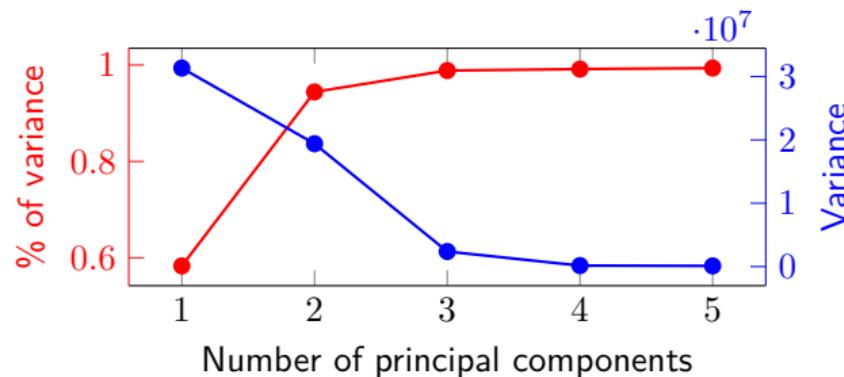
```
import rasterTools as rt
import scipy as sp
from sklearn.decomposition import PCA
import matplotlib.pyplot as plt

# Load data set
im,GeoT,Proj = rt.open_data('../Data/university.tif')
[h,w,b]=im.shape
im.shape=(h*w,b)
wave = sp.loadtxt('../Data/waves.csv',delimiter=',,')

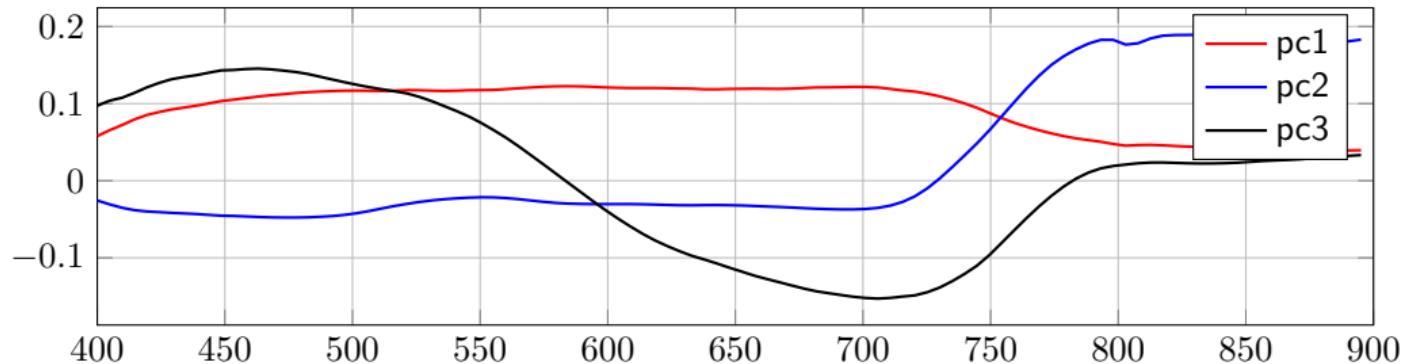
# Do PCA
pca = PCA()
pca.fit(im)

# Save Eigenvectors
D = sp.concatenate((wave[:,sp.newaxis],pca.components_[:,3:,:].T),axis=1)
sp.savetxt('../FeatureExtraction/figures/pca_pcs.csv',D,delimiter=',,')
```

\* Explained variance



\* Principal components



>>> PCA case study 3/3

# Projection of the first PCs

```
imp = sp.dot(im,pca.components_[:3,:].T)  
imp.shape = (h,w,3)
```

# Save image

```
rt.write_data('..../Data/pca_university.tif',imp,GeoT,Proj)
```



>>> Kernel PCA

- ★ PCA is limited to second order information
- ★ To capture higher-order statistics, it is possible to map the data onto another space  $\mathcal{H}$

$$\begin{aligned}\phi : \mathbb{R}^d &\rightarrow \mathcal{H} \\ \mathbf{x} &\mapsto \phi(\mathbf{x}).\end{aligned}$$

- ★ In  $\mathcal{H}$ , conventional PCA can be applied.
- ★ Using the *kernel trick* it is possible to directly work on the *kernel matrix* in  $\mathbb{R}^d$

$$\mathbf{K} = \begin{pmatrix} k(\mathbf{x}_1, \mathbf{x}_1) & k(\mathbf{x}_1, \mathbf{x}_2) & \dots & k(\mathbf{x}_1, \mathbf{x}_n) \\ k(\mathbf{x}_2, \mathbf{x}_1) & k(\mathbf{x}_2, \mathbf{x}_2) & \dots & k(\mathbf{x}_2, \mathbf{x}_n) \\ \vdots & \vdots & \ddots & \vdots \\ k(\mathbf{x}_n, \mathbf{x}_1) & k(\mathbf{x}_n, \mathbf{x}_2) & \dots & k(\mathbf{x}_n, \mathbf{x}_n) \end{pmatrix}.$$

### >>> Kernel PCA

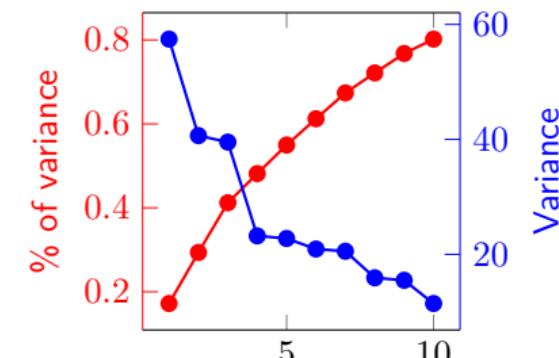
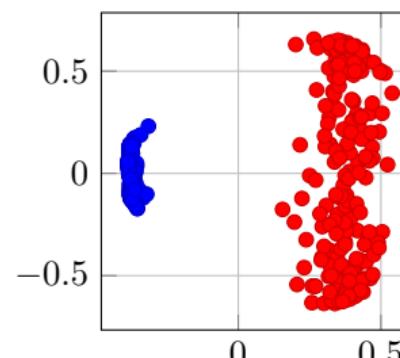
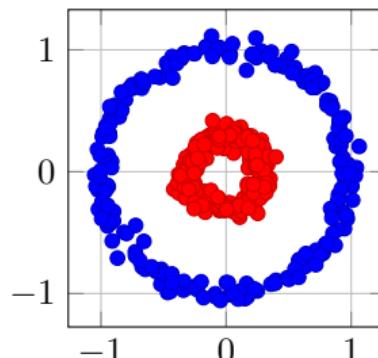
- \* PCA is limited to second order information
- \* To capture higher-order statistics, it is possible to map the data onto another space  $\mathcal{H}$

$$\begin{aligned}\phi : \mathbb{R}^d &\rightarrow \mathcal{H} \\ \mathbf{x} &\mapsto \phi(\mathbf{x}).\end{aligned}$$

- \* In  $\mathcal{H}$ , conventional PCA can be applied.
- \* Using the *kernel trick* it is possible to directly work on the *kernel matrix* in  $\mathbb{R}^d$

$$\mathbf{K} = \begin{pmatrix} k(\mathbf{x}_1, \mathbf{x}_1) & k(\mathbf{x}_1, \mathbf{x}_2) & \dots & k(\mathbf{x}_1, \mathbf{x}_n) \\ k(\mathbf{x}_2, \mathbf{x}_1) & k(\mathbf{x}_2, \mathbf{x}_2) & \dots & k(\mathbf{x}_2, \mathbf{x}_n) \\ \vdots & \vdots & \ddots & \vdots \\ k(\mathbf{x}_n, \mathbf{x}_1) & k(\mathbf{x}_n, \mathbf{x}_2) & \dots & k(\mathbf{x}_n, \mathbf{x}_n) \end{pmatrix}.$$

- \* KPCA versus PCA:



- ★ Choose the kernel and its parameters
- ★ Compute the kernel matrix  $\mathbf{K}$  for all the pixels (or a subset)
- ★ Center the matrix

$$\mathbf{K}_c = \mathbf{K} - \mathbf{1}_n \mathbf{K} - \mathbf{K} \mathbf{1}_n + \mathbf{1}_n \mathbf{K} \mathbf{1}_n$$

- ★ Solve the eigenproblems

$$\lambda \boldsymbol{\alpha} = \mathbf{K} \boldsymbol{\alpha} \text{ subject to } \|\boldsymbol{\alpha}\|_2 = \frac{1}{\lambda}$$

- ★ Project on the  $p$  first *kernel principal components*:  $\phi^{kpc}(\mathbf{x}) = [\phi_1^{kpc}(\mathbf{x}) \quad \dots \quad \phi_p^{kpc}(\mathbf{x})]^t$

$$\phi_j^{kpc}(\mathbf{x}) = \sum_{i=1}^n \alpha_{ki} k(\mathbf{x}_i, \mathbf{x})$$

From [FCB09].

```
import rasterTools as rt
import scipy as sp
from sklearn.decomposition import KernelPCA
import matplotlib.pyplot as plt
from sklearn.preprocessing import StandardScaler

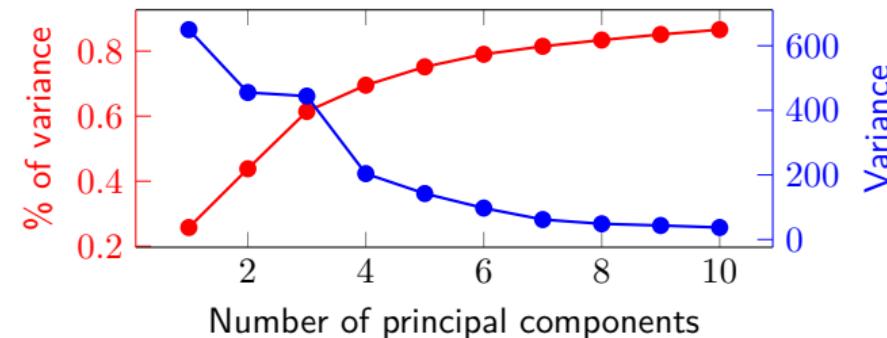
# Load data set
im,GeoT,Proj = rt.open_data('../Data/university.tif')
[h,w,b]=im.shape
im.shape=(h*w,b)
wave = sp.loadtxt('../Data/waves.csv',delimiter=',',)

# Scale data
sc = StandardScaler()
im = sc.fit_transform(im)

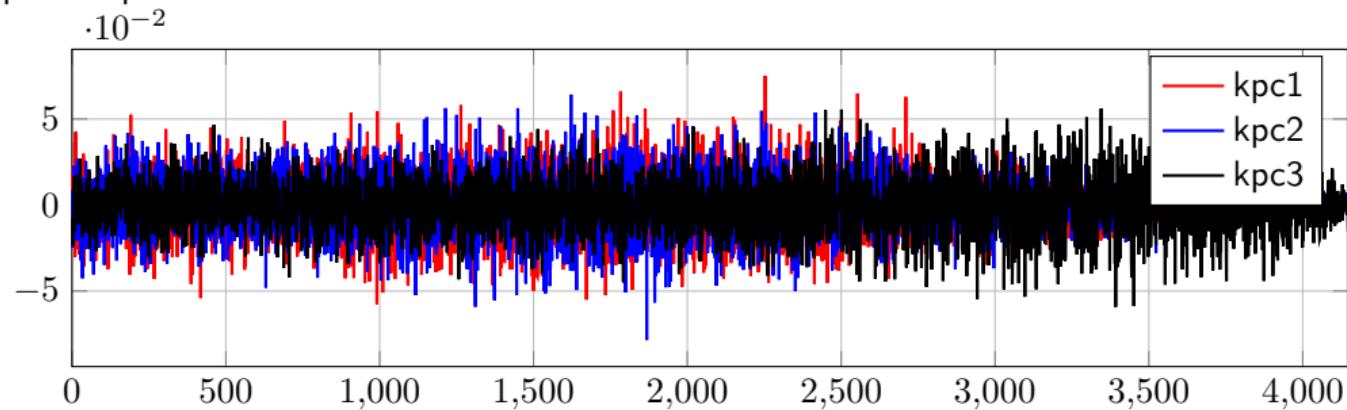
# Do KPCA
kpca = KernelPCA(kernel='rbf',gamma=1.0/b,n_jobs=-1)
kpca.fit(im[:,::50]) # Use a subset of the total number of pixels
```

>>> KPCA case study 2/3

\* Explained variance



\* Principal components



>>> KPCA case study 3/3

```
imp = kpca.transform(im)[:, :, 3]
imp.shape = (h, w, 3)

# Save image
rt.write_data('../Data/kpca_university.tif', imp, GeoT, Proj)
```



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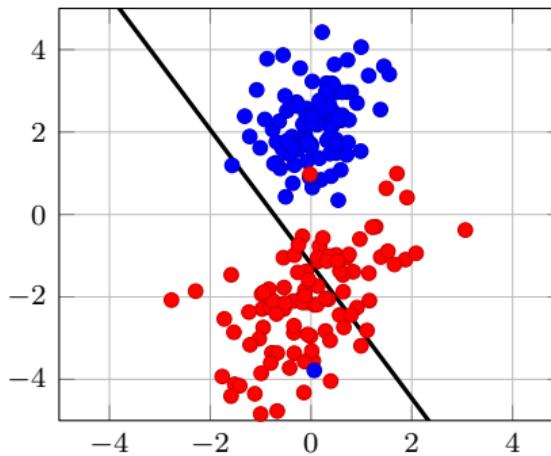
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- ★ We observe some  $\{\mathbf{x}_i, y_i\}_{i=1}^n$
- ★ Use the label information to find the linear features that highlight differences among classes



- ★ FDA: find  $\mathbf{a}$  such as the ratio between the *between projected variance* and the *sample projected variance* is maximal [MLC16] Chap. 8.8

- ★ Between-class covariance matrix:

$$\mathbf{B} = \frac{1}{n} \sum_{c=1}^C n_c (\boldsymbol{\mu}_c - \boldsymbol{\mu})(\boldsymbol{\mu}_c - \boldsymbol{\mu})^\top$$

- ★ Class covariance matrix

$$\boldsymbol{\Sigma}_c = \frac{1}{n_c - 1} \sum_{i=1, i \in c}^{n_c} (\mathbf{x}_i - \boldsymbol{\mu}_c)(\mathbf{x}_i - \boldsymbol{\mu}_c)^\top$$

- ★ Within-class covariance matrix

$$\mathbf{W} = \sum_{c=1}^C \boldsymbol{\Sigma}_c$$

- ★ The Fisher discriminant subspace is given by the eigenvectors of  $\mathbf{W}^{(-1)}\mathbf{B}$
- ★ Remark: there are at most  $C - 1$  eigenvectors because  $\text{Rank}(\mathbf{B}) \leq C - 1$ .

```
>>> FDA case study 1/3
```

```
import rasterTools as rt
import scipy as sp
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis

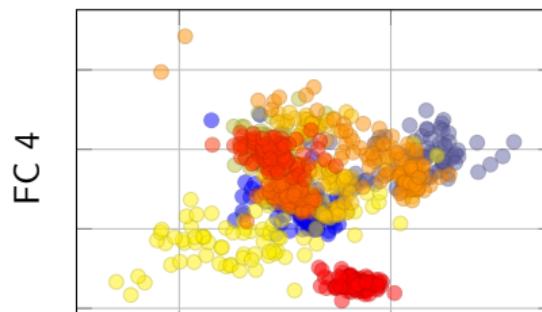
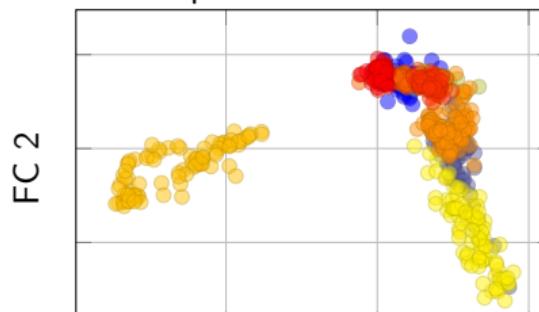
# Load data set
X,y=rt.get_samples_from_roi('../Data/university.tif','../Data/university_gt.tif')
wave = sp.loadtxt('../Data/waves.csv',delimiter=',',)

# Select the same number of samples
nt = 900
xt,yt=[], []
for i in sp.unique(y):
    t = sp.where(y==i)[0]
    nc = t.size
    rp = sp.random.permutation(nc)
    xt.extend(X[t[rp[0:nt]],:])
    yt.extend(y[t[rp[0:nt]]])

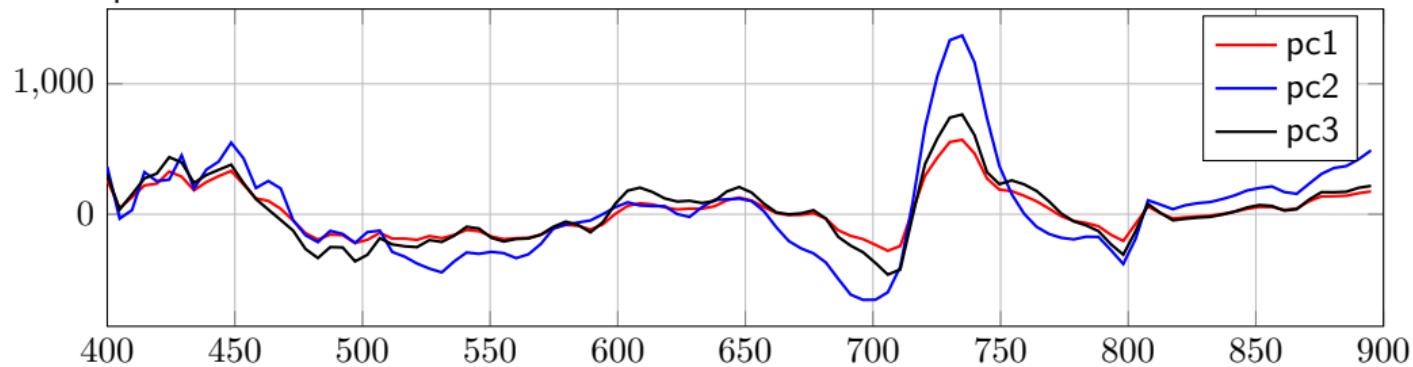
xt = sp.asarray(xt)
yt = sp.asarray(yt)

# Do LDA
lda = LinearDiscriminantAnalysis(solver='eigen', shrinkage='auto')
lda.fit(xt,yt.ravel())
```

\* Projection on Fisher components



\* Fisher components



>>> FDA case study 3/3

```
im,GeoT,Proj = rt.open_data('..../Data/university.tif')
[h,w,b]=im.shape
im.shape=(h*w,b)
imp = lda.transform(im)[:,3]
imp.shape = (h,w,3)
# Save image
rt.write_data('..../Data/lda_university.tif',imp,GeoT,Proj)
```



- ★ Feature selection: pick few features *from* the original ones (no combination)
- ★ In general, for feature selection, we need:
  - ★ *Criterion* to evaluate how perform the model with a given subset
  - ★ *Optimization procedure* to find the subset that minimizes/maximizes the criterion
- ★ For instance:

Criterion	Optimization	Ref.
Entropy	Genetic algorithm	[Cha07]
Jeffries Matusita	Exhaustive Search	[SM07]
Classification error	Forward search/GA	[CBN14; LFG17]
$\ell_1$ norm	Linear-SVM	[Tui+14]

- ★ Fast forward strategy based on a nonlinear model driven by an estimate of the classification error or a measure of separability:

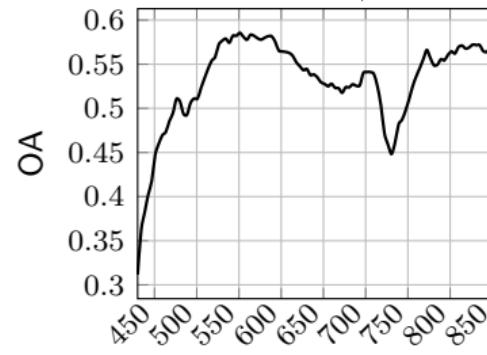
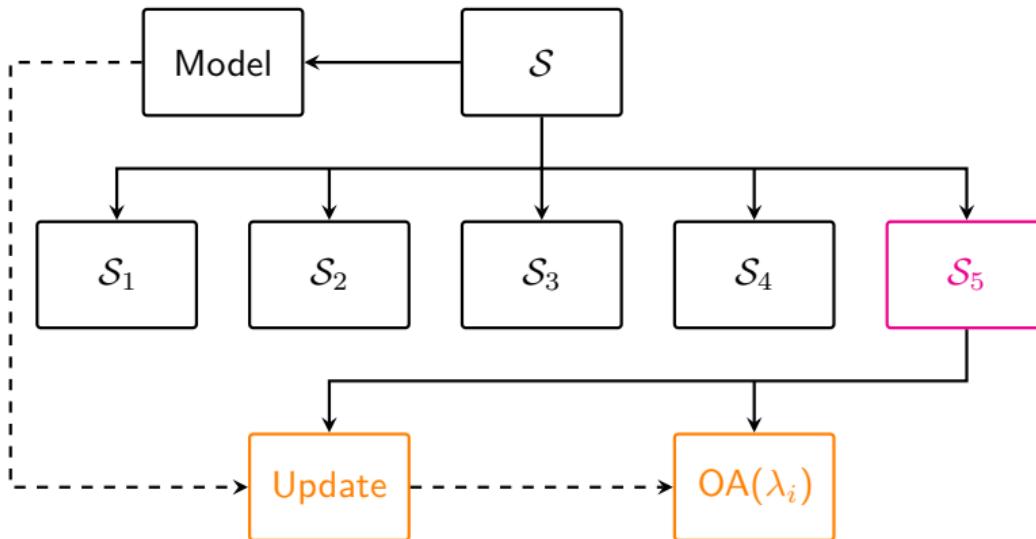
Criterion	Type	Complexity
Overall accuracy	Accuracy	High
Cohen's kappa	Accuracy	High
F1 mean	Accuracy	High
Kullback-Leibler divergences	Divergence	Low
Jeffries-Matusita distance	Divergence	Low

- ★ Use *Gaussian Mixture Models* (natural extension for multiclass problem)
- ★ Fast update and fast forward search [LFG17]: based on linear algebra of semi-definite positive matrices

The forward feature selection works as follow:

1. Starts with an empty pool  $F$  of selected features,
2. Select the feature  $f_1$  that provides the best value for the selected criterion and add it to  $F$ .
3. Select the feature  $f_2$  such that the couple of features  $(f_1, f_2)$  provides the best value for the selected criterion and add it to  $F$ .
4. Select the feature  $f_3$  such that the triplet of features  $(f_1, f_2, f_3)$  ...
5. ...
6. The algorithm stops either if the increase of the criterion is too low or if the maximum number of features is reached.

>>> Algorithm 2/2



>>> FFFS case study 1/3

```
import rasterTools as rt
import scipy as sp
import npfs as npfs

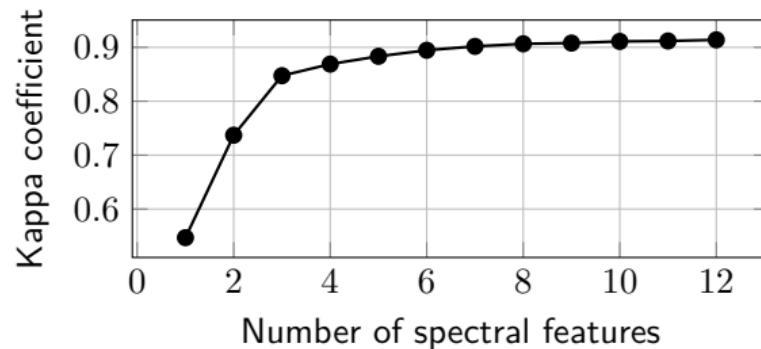
# Load data set
X,y=rt.get_samples_from_roi('..../Data/university.tif','..../Data/university_gt.tif')
wave = sp.loadtxt('..../Data/waves.csv',delimiter=',',)

# Select the same number of samples
nt = 900
xt,yt=[], []
for i in sp.unique(y):
    t = sp.where(y==i)[0]
    nc = t.size
    rp = sp.random.permutation(nc)
    xt.extend(X[t[rp[0:nt]],:])
    yt.extend(y[t[rp[0:nt]]])

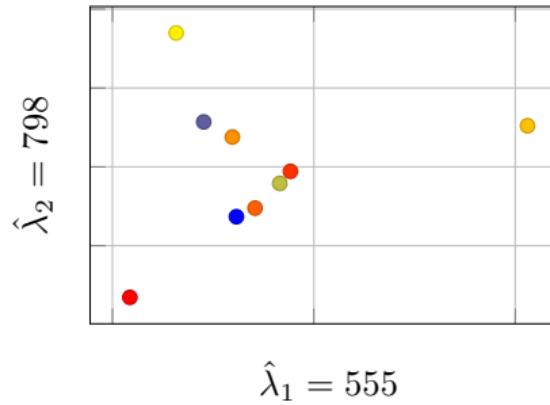
xt = sp.asarray(xt)
yt = sp.asarray(yt)

# Do FFFS
maxVar = 12
model = npfs.GMMFeaturesSelection()
model.learn_gmm(xt,yt)
idx, crit, [] = model.selection('forward',xt, yt,criterion='kappa', varNb=maxVar, nfold=5)
```

\* Criterion



\* Mean projection on best bands



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>>> Number of features

Given a set of observed pixels  $\mathcal{S} = \{(\mathbf{x}_i, y_i)\}_{i=1}^n$  and  $\mathbf{x} \in \mathbb{R}^d$ . The number of classes  $C$  is 3. What is the *maximum* number of features that can be extracted with

- \* PCA, when  $d = 200$  and  $n = 400$ : a) 100, b) 200, c) 400 and d) 3.

Given a set of observed pixels  $\mathcal{S} = \{(\mathbf{x}_i, y_i)\}_{i=1}^n$  and  $\mathbf{x} \in \mathbb{R}^d$ . The number of classes  $C$  is 3. What is the *maximum* number of features that can be extracted with

- \* PCA, when  $d = 200$  and  $n = 400$ : a) 100, b) 200, c) 400 and d) 3.
- \* PCA, when  $d = 200$  and  $n = 100$ : a) 100, b) 200, c) 400 and d) 3.

Given a set of observed pixels  $\mathcal{S} = \{(\mathbf{x}_i, y_i)\}_{i=1}^n$  and  $\mathbf{x} \in \mathbb{R}^d$ . The number of classes  $C$  is 3. What is the *maximum* number of features that can be extracted with

- \* PCA, when  $d = 200$  and  $n = 400$ : a) 100, b) 200, c) 400 and d) 3.
- \* PCA, when  $d = 200$  and  $n = 100$ : a) 100, b) 200, c) 400 and d) 3.
- \* FDA, when  $d = 200$  and  $n = 400$ : a) 100, b) 200, c) 400 and d) 3.

Given a set of observed pixels  $\mathcal{S} = \{(\mathbf{x}_i, y_i)\}_{i=1}^n$  and  $\mathbf{x} \in \mathbb{R}^d$ . The number of classes  $C$  is 3. What is the *maximum* number of features that can be extracted with

- \* PCA, when  $d = 200$  and  $n = 400$ : a) 100, b) 200, c) 400 and d) 3.
- \* PCA, when  $d = 200$  and  $n = 100$ : a) 100, b) 200, c) 400 and d) 3.
- \* FDA, when  $d = 200$  and  $n = 400$ : a) 100, b) 200, c) 400 and d) 3.
- \* KPCA, when  $d = 200$  and  $n = 400$ : a) 100, b) 200, c) 400 and d) 3.

Given a set of observed pixels  $\mathcal{S} = \{(\mathbf{x}_i, y_i)\}_{i=1}^n$  and  $\mathbf{x} \in \mathbb{R}^d$ . The number of classes  $C$  is 3. What is the *maximum* number of features that can be extracted with

- \* PCA, when  $d = 200$  and  $n = 400$ : a) 100, b) 200, c) 400 and d) 3.
- \* PCA, when  $d = 200$  and  $n = 100$ : a) 100, b) 200, c) 400 and d) 3.
- \* FDA, when  $d = 200$  and  $n = 400$ : a) 100, b) 200, c) 400 and d) 3.
- \* KPCA, when  $d = 200$  and  $n = 400$ : a) 100, b) 200, c) 400 and d) 3.
- \* KPCA, when  $d = 200$  and  $n = 100$ : a) 100, b) 200, c) 400 and d) 3.

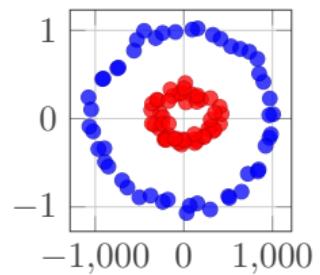
Given a set of observed pixels  $\mathcal{S} = \{(\mathbf{x}_i, y_i)\}_{i=1}^n$  and  $\mathbf{x} \in \mathbb{R}^d$ . The number of classes  $C$  is 3. What is the *maximum* number of features that can be extracted with

- \* PCA, when  $d = 200$  and  $n = 400$ : a) 100, b) 200, c) 400 and d) 3.
- \* PCA, when  $d = 200$  and  $n = 100$ : a) 100, b) 200, c) 400 and d) 3.
- \* FDA, when  $d = 200$  and  $n = 400$ : a) 100, b) 200, c) 400 and d) 3.
- \* KPCA, when  $d = 200$  and  $n = 400$ : a) 100, b) 200, c) 400 and d) 3.
- \* KPCA, when  $d = 200$  and  $n = 100$ : a) 100, b) 200, c) 400 and d) 3.
- \* FFFS, when  $d = 200$  and  $n = 100$ : a) 2, b) 7, c) 15 and d) 200.

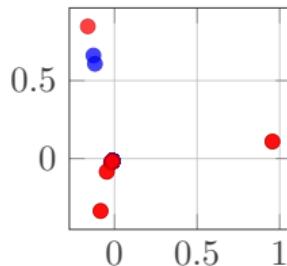
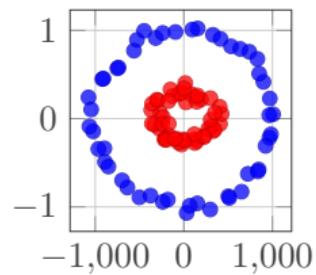
Given a set of observed pixels  $\mathcal{S} = \{(\mathbf{x}_i, y_i)\}_{i=1}^n$  and  $\mathbf{x} \in \mathbb{R}^d$ . The number of classes  $C$  is 3. What is the *maximum* number of features that can be extracted with

- \* PCA, when  $d = 200$  and  $n = 400$ : a) 100, b) 200, c) 400 and d) 3.
- \* PCA, when  $d = 200$  and  $n = 100$ : a) 100, b) 200, c) 400 and d) 3.
- \* FDA, when  $d = 200$  and  $n = 400$ : a) 100, b) 200, c) 400 and d) 3.
- \* KPCA, when  $d = 200$  and  $n = 400$ : a) 100, b) 200, c) 400 and d) 3.
- \* KPCA, when  $d = 200$  and  $n = 100$ : a) 100, b) 200, c) 400 and d) 3.
- \* FFFS, when  $d = 200$  and  $n = 100$ : a) 2, b) 7, c) 15 and d) 200.
- \* FFFS, when  $d = 200$  and  $n = 400$ : a) 2, b) 7, c) 15 and d) 200.

- ★ Given the following data set, using the Gaussian kernel, how do you expect KPCA will behave:
  - as usual,
  - poorly

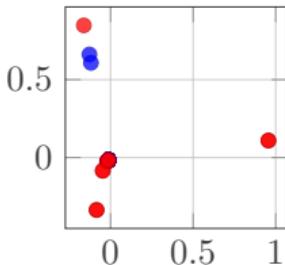
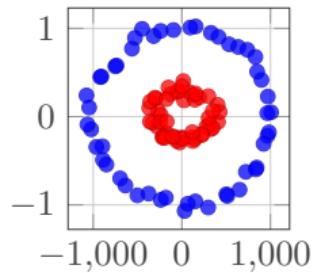


- Given the following data set, using the Gaussian kernel, how do you expect KPCA will behave:
  - as usual,
  - poorly



- ★ Given the following data set, using the Gaussian kernel, how do you expect KPCA will behave:
  - as usual,
  - poorly
- ★ Gaussian kernel:

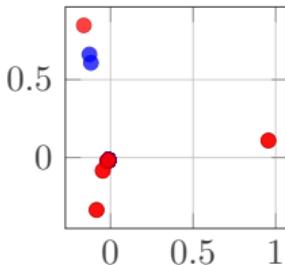
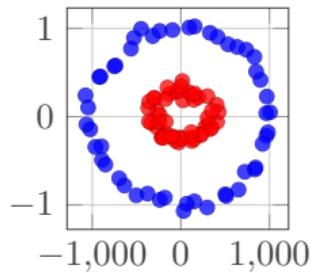
$$k(\mathbf{x}_i, \mathbf{x}_j) = \exp\left[-\frac{\|\mathbf{x}_i - \mathbf{x}_j\|^2}{2\sigma^2}\right] = \exp\left[-\frac{\sum_{l=1}^d (\mathbf{x}_{il} - \mathbf{x}_{jl})^2}{2\sigma^2}\right]$$



- Given the following data set, using the Gaussian kernel, how do you expect KPCA will behave:
  - as usual,
  - poorly
- Gaussian kernel:

$$k(\mathbf{x}_i, \mathbf{x}_j) = \exp\left[-\frac{\|\mathbf{x}_i - \mathbf{x}_j\|^2}{2\sigma^2}\right] = \exp\left[-\frac{\sum_{l=1}^d (\mathbf{x}_{il} - \mathbf{x}_{jl})^2}{2\sigma^2}\right]$$

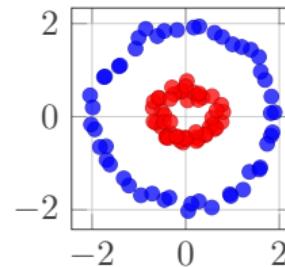
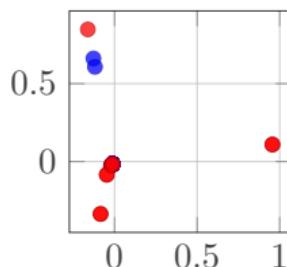
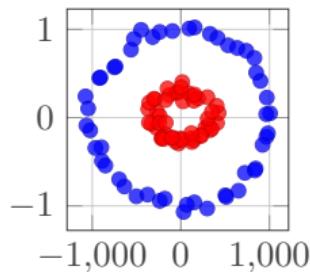
- Solution: scale the features (e.g., zero mean and unit variance)



- Given the following data set, using the Gaussian kernel, how do you expect KPCA will behave:
  - as usual,
  - poorly
- Gaussian kernel:

$$k(\mathbf{x}_i, \mathbf{x}_j) = \exp\left[-\frac{\|\mathbf{x}_i - \mathbf{x}_j\|^2}{2\sigma^2}\right] = \exp\left[-\frac{\sum_{l=1}^d (\mathbf{x}_{il} - \mathbf{x}_{jl})^2}{2\sigma^2}\right]$$

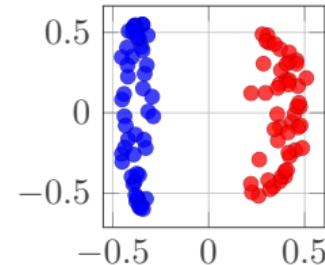
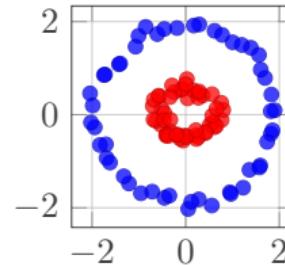
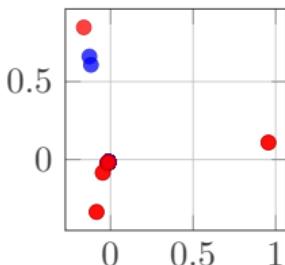
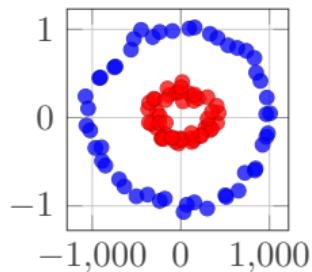
- Solution: scale the features (e.g., zero mean and unit variance)



- Given the following data set, using the Gaussian kernel, how do you expect KPCA will behave:
  - as usual,
  - poorly
- Gaussian kernel:

$$k(\mathbf{x}_i, \mathbf{x}_j) = \exp\left[-\frac{\|\mathbf{x}_i - \mathbf{x}_j\|^2}{2\sigma^2}\right] = \exp\left[-\frac{\sum_{l=1}^d (\mathbf{x}_{il} - \mathbf{x}_{jl})^2}{2\sigma^2}\right]$$

- Solution: scale the features (e.g., zero mean and unit variance)



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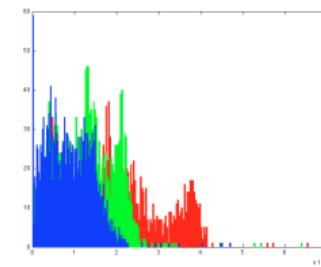
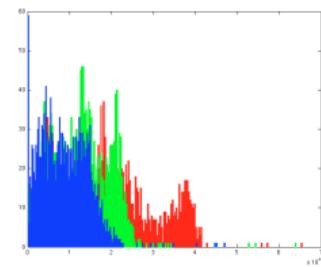
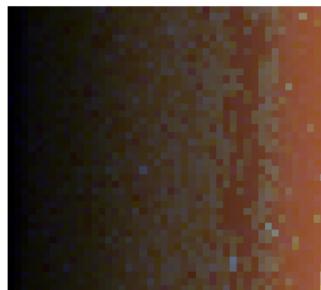
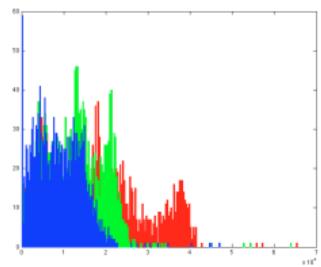
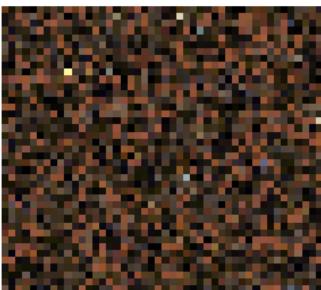
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>>> Why spatial feature extraction?



>>> More on this topics

## **Image analysis of hyperspectral data using mathematical morphology**

Tutorial WHISPERS 2014: Lesson, Labwork and full matlab implementation !

*Dalla Mura, Mauro, & Fauvel, Mathieu. (2014, June).*

*Image analysis of hyperspectral data using mathematical morphology.*

*Zenodo. <http://doi.org/10.5281/zenodo.437195>*

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- ★ The neighborhood of a given pixel is the set of pixels that are connected to it.
- ★ For a flat (grayscale) image :

$\mathbf{x}_{-1,-1}$	$\mathbf{x}_{0,-1}$	$\mathbf{x}_{1,-1}$
$\mathbf{x}_{-1,0}$	$\mathbf{x}_{0,0}$	$\mathbf{x}_{1,0}$
$\mathbf{x}_{-1,1}$	$\mathbf{x}_{0,1}$	$\mathbf{x}_{1,1}$

4-connected

$\mathbf{x}_{-1,-1}$	$\mathbf{x}_{0,-1}$	$\mathbf{x}_{1,-1}$
$\mathbf{x}_{-1,0}$	$\mathbf{x}_{0,0}$	$\mathbf{x}_{1,0}$
$\mathbf{x}_{-1,1}$	$\mathbf{x}_{0,1}$	$\mathbf{x}_{1,1}$

8-connected

- ★ Wide range of processing are based on pixel neighborhood

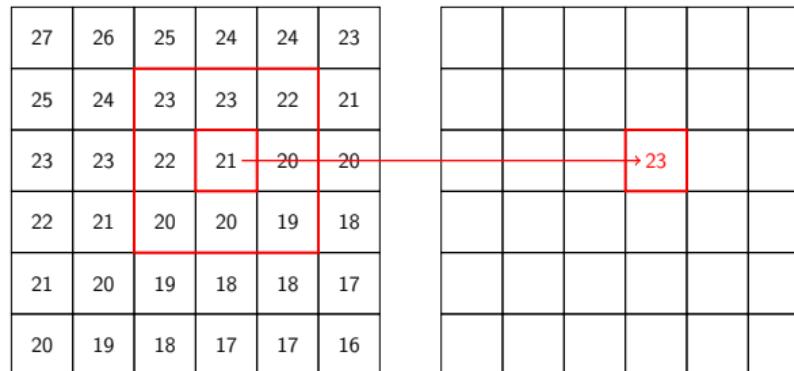
- ★ De noising,
- ★ Texture analysis,
- ★ Edges detection,
- ★ Pattern recognition,
- ★ ...

Steps:

1. Define the template  $G$ : 4/8-connected and size
2. Define the processing  $f$  on the neighborhood. If  $f$  is linear  $\leftrightarrow$  convolution.
3. Scan all the pixels:

$$\mathbf{x}_{ij}^f = f(\mathbf{x}_1, \dots, \mathbf{x}_N), \quad \mathbf{x}_n \in G(i, j)$$

Max Filter



- \*  $G = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ , for a  $3 \times 3$  neighborhood.

- \* Mean filter

$$\mathbf{x}^m(x, y) = \frac{1}{9} \sum_{i,j=-1}^1 \mathbf{x}(x+i, y+j)$$

- \* Variance filter:

$$\mathbf{x}^v(x, y) = \frac{1}{9} \sum_{i,j=-1}^1 (\mathbf{x}(x+i, y+j) - \mathbf{x}^m(x, y))^2$$

- \* Range filter:

$$\mathbf{x}^r(x, y) = \max_{i,j \in G} [\mathbf{x}(x+i, y+j)] - \min_{i,j \in G} [\mathbf{x}(x+i, y+j)]$$

- \* Median filter:

$$\mathbf{x}^m(x, y) = \text{median}_{i,j \in G} [\mathbf{x}(x+i, y+j)]$$

>>> Template filters in action 1/3

For multidimensional images: Use spectral feature extraction to get flat images! See ??

```
# Compute the different filters with a template of size 3x3 and 11x11
for i in 3 11
do
    # Mean filter
    otbcli_BandMathX -il ../Data/pca_university.tif -out ../Data/pca_mean_${i}_${i}_university.tif \
        -exp "mean(im1b1N${i}x${i}); mean(im1b2N${i}x${i}); mean(im1b3N${i}x${i})"

    # Var filter
    otbcli_BandMathX -il ../Data/pca_university.tif -out ../Data/pca_std_${i}_${i}_university.tif \
        -exp "var(im1b1N${i}x${i}); var(im1b2N${i}x${i}); var(im1b3N${i}x${i})"

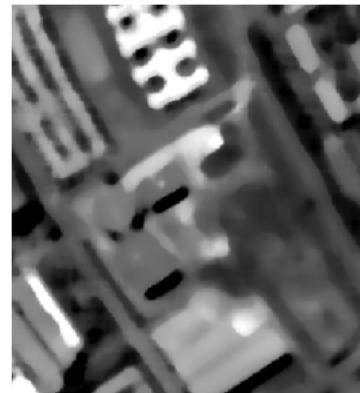
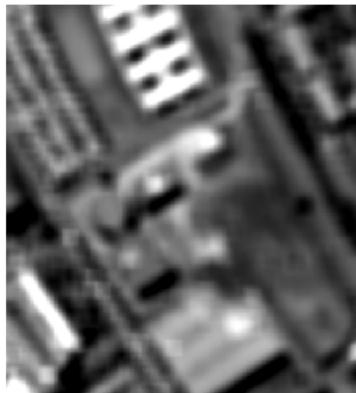
    # Range filter
    otbcli_BandMathX -il ../Data/pca_university.tif -out ../Data/pca_range_${i}_${i}_university.tif \
        -exp "vmax(im1b1N${i}x${i})-vmin(im1b1N${i}x${i}); vmax(im1b2N${i}x${i})-vmin(im1b2N${i}x${i}); \
            vmax(im1b3N${i}x${i})-vmin(im1b3N${i}x${i})"

    # Median filter
    otbcli_BandMathX -il ../Data/pca_university.tif -out ../Data/pca_median_${i}_${i}_university.tif \
        -exp "median(im1b1N${i}x${i}); median(im1b2N${i}x${i}); median(im1b3N${i}x${i})"
done
```

>>> Template filters in action 2/3



>>> Template filters in action 3/3



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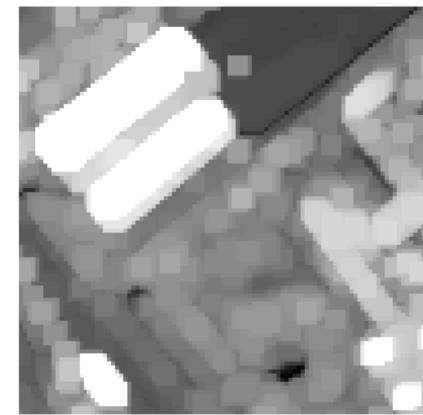
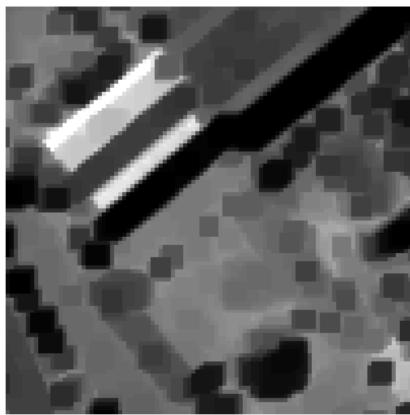
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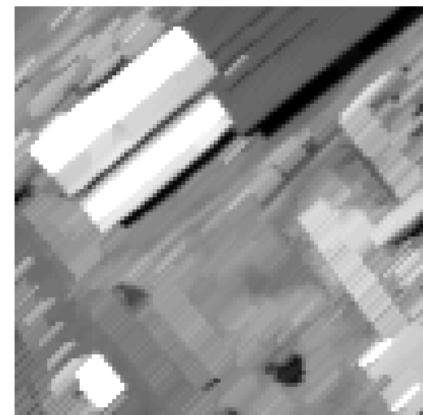
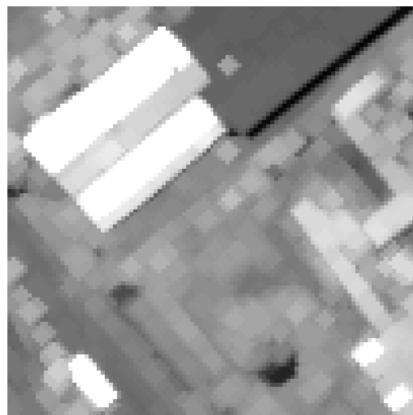
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- ★ Mathematical morphology: non-linear image processing.
- ★ A lot of applications in geoscience and remote sensing, see [SP02]
- ★ Erosion: template filter with a min operation in  $G$  (called *structuring element*)
- ★ Dilation: template filter with a max operation in  $G$



>>> Effects of structuring elements



0	0	1	0	0
0	1	1	1	0
1	1	1	1	1
0	1	1	1	0
0	0	1	0	0

0	0	0	0	0
0	1	1	1	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	1	0	0	0
1	0	0	0	0

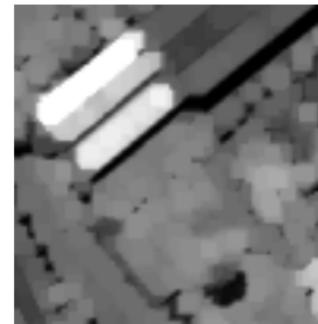
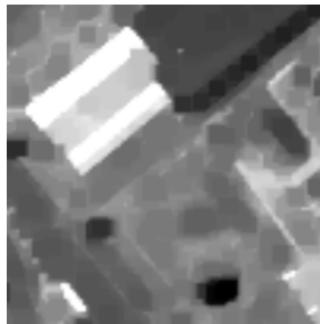
>>> Opening and closing

\* Opening:

- \* *Erosion* followed by a *dilation*
- \* Remove bright objects that are smaller than the SE

\* Closing:

- \* *Dilation* followed by an *erosion*
- \* Remove dark objects that are smaller than the SE



>>> Opening and closing

\* Opening:

- \* *Erosion* followed by a *dilation*
- \* Remove bright objects that are smaller than the SE

\* Opening by reconstruction:

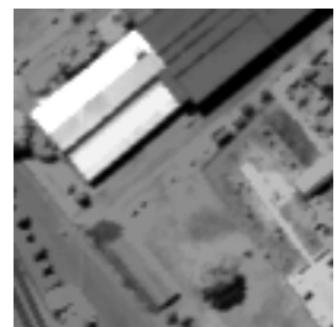
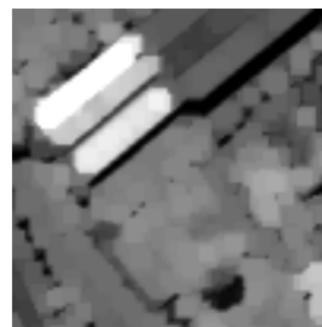
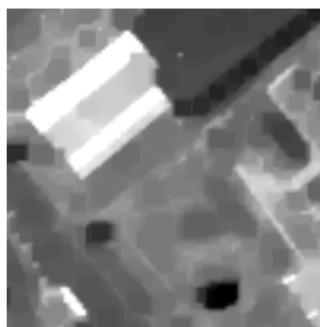
- \* *Erosion* followed by a *reconstruction*
- \* Completely removes bright objects that are smaller than the SE, otherwise preserve it

\* Closing:

- \* *Dilation* followed by an *erosion*
- \* Remove dark objects that are smaller than the SE

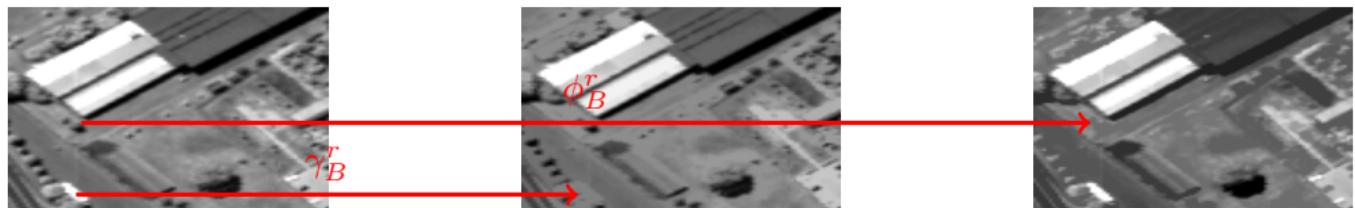
\* Closing by reconstruction:

- \* *Dilation* followed by an *erosion*
- \* Completely removes dark objects that are smaller than the SE, otherwise preserve it



>>> Opening and closing profile

- ★ For a given  $B$ ,  $\gamma_B^r$  (resp.  $\phi_B^r$ ) indicates which clear (dark) objects fit  $B$ .



- ★ Applying  $\gamma_{B_i}$  with a set of  $\{B_i | B_i \subset B_{i+1}, i \in [1, \dots, n]\}$  : **Opening Profile**
- ★ Applying  $\phi_{B_i}$  with a set of  $\{B_i | B_i \subset B_{i+1}, i \in [1, \dots, n]\}$  : **Closing Profile**

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- ★ MM is based on inf and sup operators
- ★ No unambiguous inf / sup for pixel/vector:

$$\begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix} \stackrel{?}{\leqslant} \begin{bmatrix} 0 \\ 6 \\ 1 \end{bmatrix}$$

- ★ Marginal ordering  $\Rightarrow$  by band filtering
- ★ Reduced ordering  $\Rightarrow h : \mathbb{R}^d \rightarrow \mathbb{R}$   
$$\mathbf{x} \mapsto h(x)$$
- ★ Use spectral feature extraction *then* spatial feature extraction.

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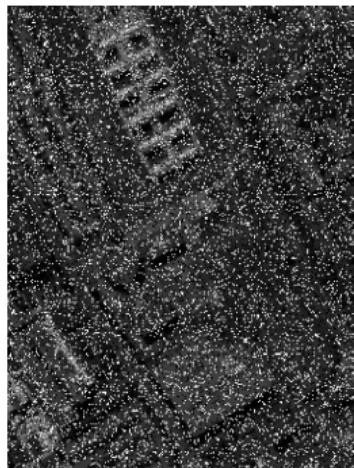
- ★ The image has been corrupted by a *salt and pepper* noise, which filter should we use to filter it ?
  - a) mean filter, b) opening, c) median filter and d) closing.
- ★ To remove the small cars on the road, which filter should we use to filter it ?
  - a) mean filter, b) opening, c) median filter and d) closing.



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