

UltraRay: Introducing Full-Path Ray Tracing in Physics-Based Ultrasound Simulation

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Code

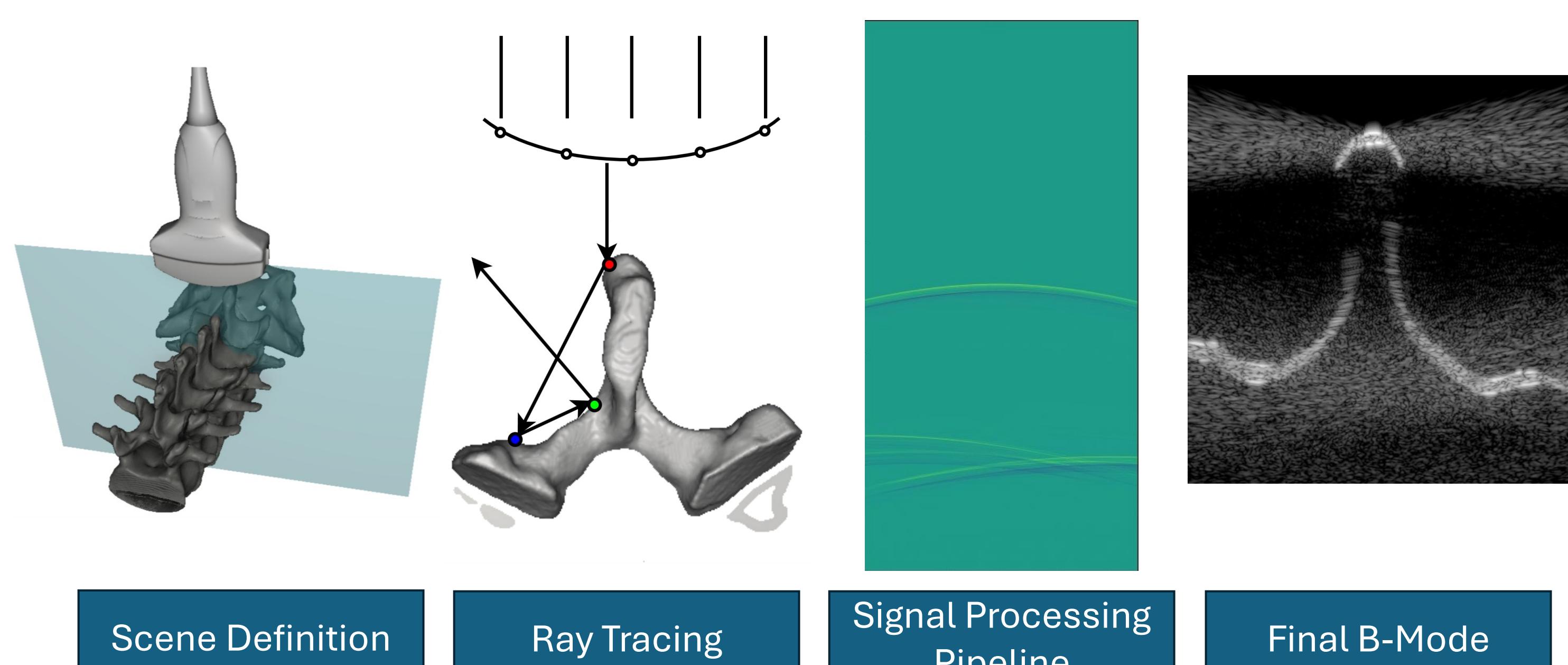


Paper

Motivation

- Ultrasound simulators are used for physician training, transducer design optimization, and the generation of training data for deep learning.
- Existing wave-based simulators such k-Wave [1] are accurate but slow.
- Ray tracing-based ultrasound methods are **faster** than traditional approaches, but prior methods simplify the wave propagation by adding echoes without verifying their return paths.
- In parallel, **physics-based-rendering** (PBR, [2]) has gained prominence, and powerful simulators (e.g. Mitsuba 3 [3]) have been developed to enable quick and convenient simulation of natural images.

Pipeline



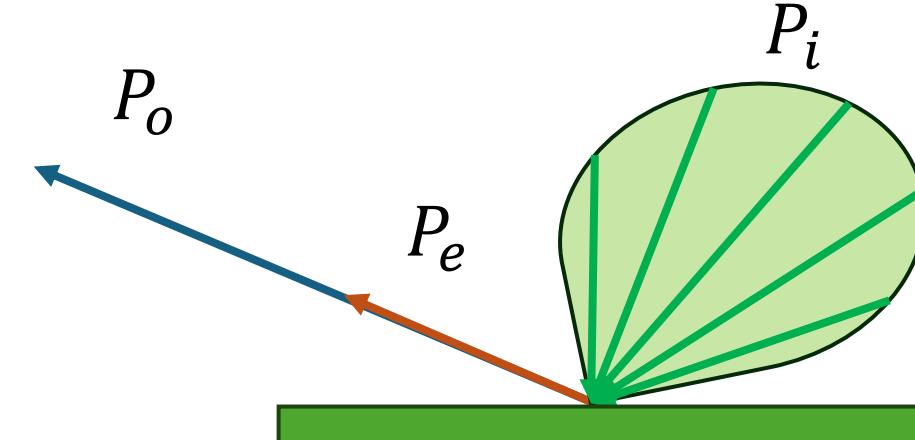
Ray Tracing

We model the received pressure P at one individual element over the hemisphere Ω and received surface A :

$$P(e, t) = \int_{\Omega} \int_A P_i(x, t, \omega_i) f_d(\omega_i) d\omega da,$$

Evaluation of the outgoing (P_o), emitted (P_e) and incoming (P_i) pressure at a surface interaction point x :

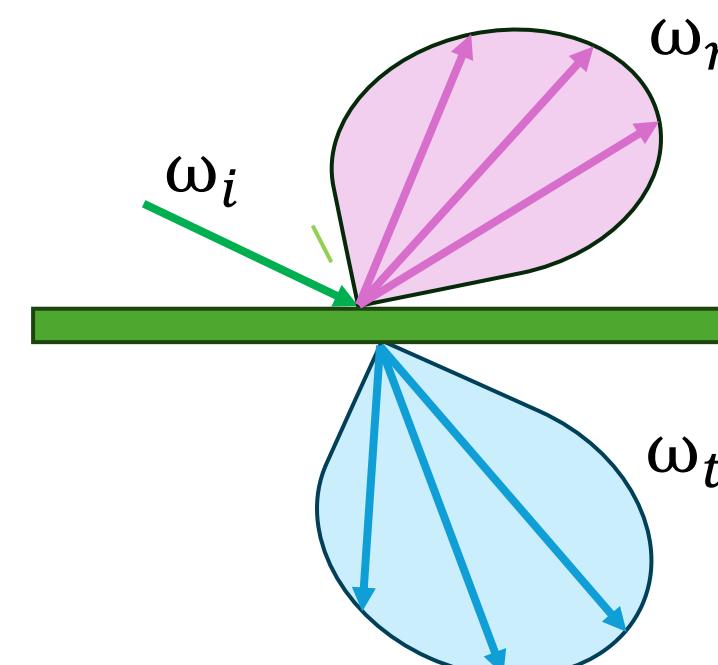
$$P_o(x, t, \omega_o) = P_e(x, t, \omega_o) + \int_{\Omega} f_r(x, \omega_i, \omega_o) P_i(x, t, \omega_i) (n \cdot \omega_i) d\omega,$$



Using Snell's law with $\eta = Z_1/Z_2$ and Z being the acoustic impedance we can derive:

$$\omega_r = \omega_i + 2 \cos \theta_r n,$$

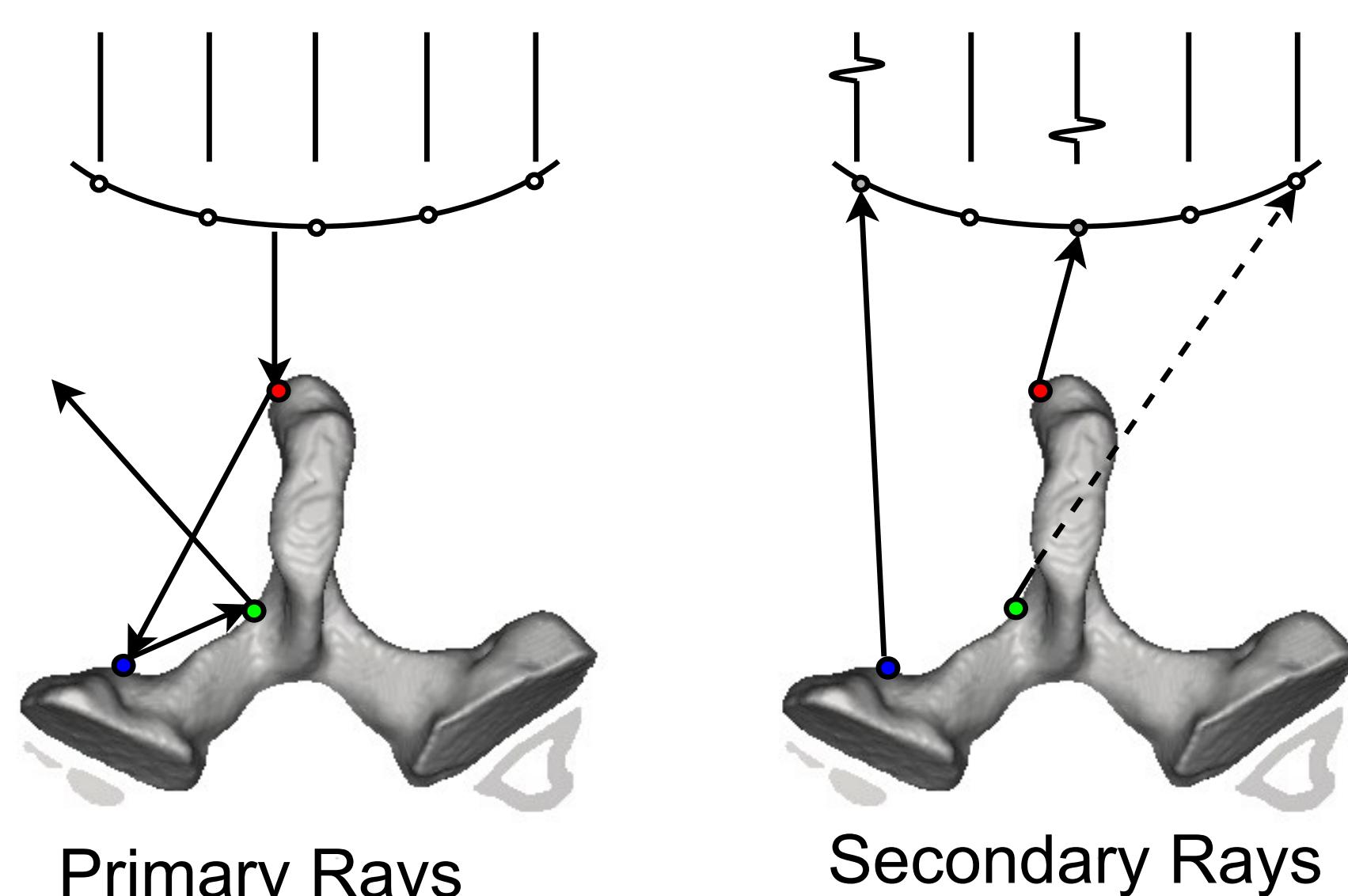
$$\omega_t = \eta \omega_i + (\eta \cos \theta_r - \cos \theta_t) n$$



The intensity I (and amplitude A) for the respective rays are given by the Fresnel equation:

$$A_r = \frac{Z_1 \cos \theta_r - Z_2 \cos \theta_t}{Z_1 \cos \theta_r + Z_2 \cos \theta_t}, \quad I_t = I_i - (A_r)^2$$

When a scene interaction occurs, a randomly sampled emitter is selected, and a secondary ray is cast to that emitter.

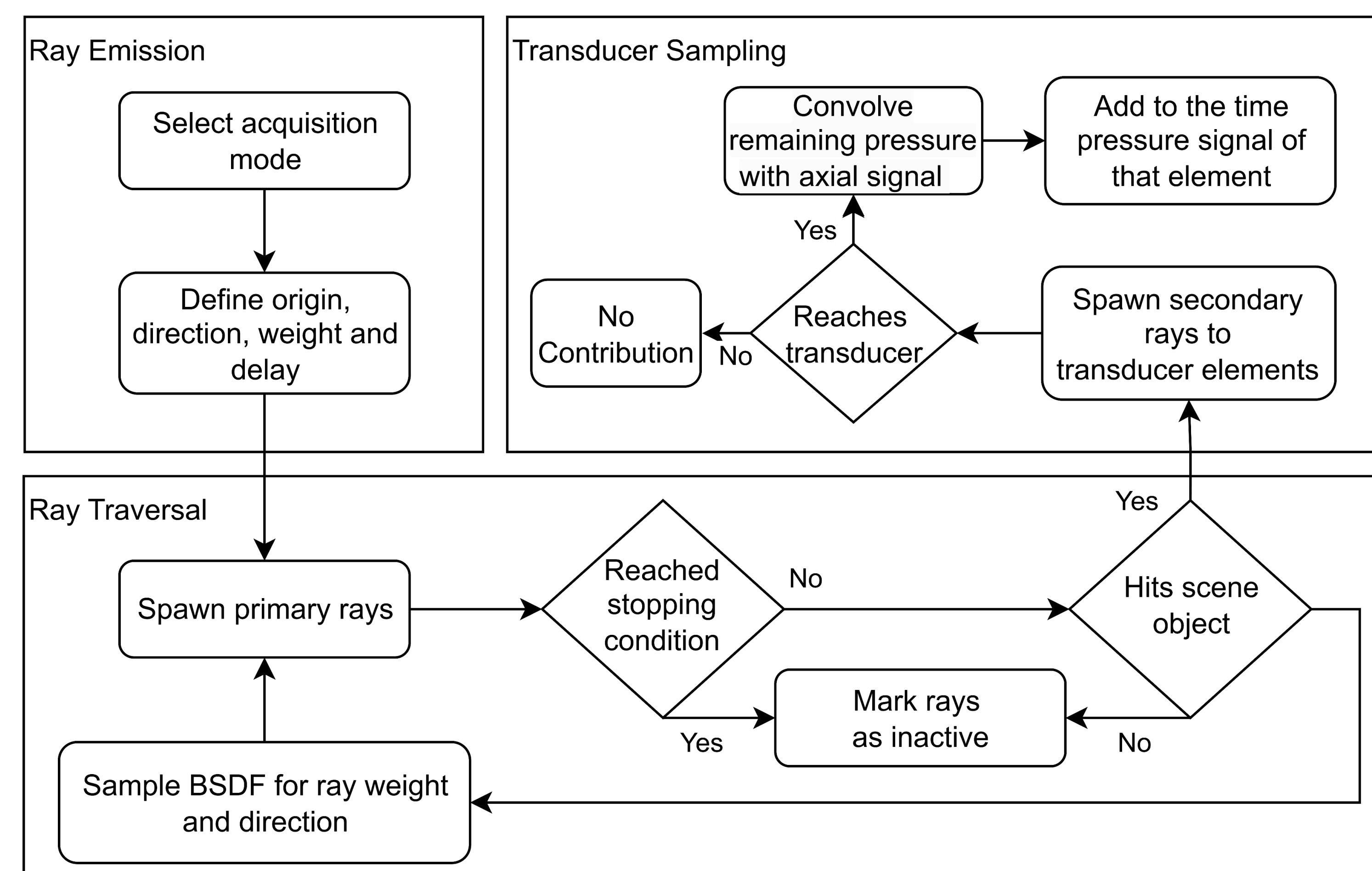


After a secondary ray reaches the transducer element a transmit pulse is written into memory:

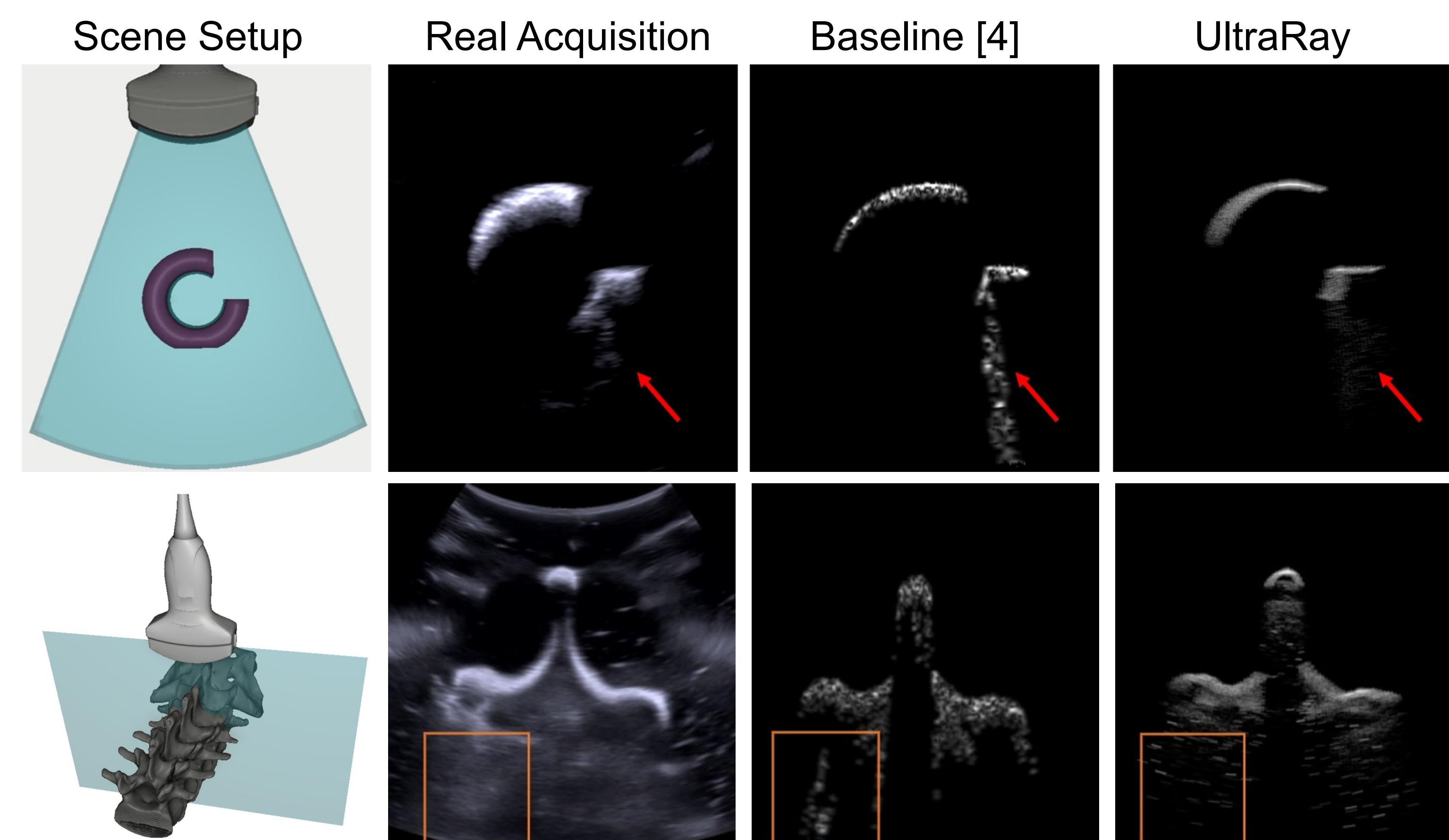
$$s(t) = \sin(2\pi f_c t) \exp\left(-\frac{t^2}{\sigma}\right)$$

References

- [1] Treeby, B.E., Cox, B.T.: k-wave: Matlab toolbox for the simulation and reconstruction of photoacoustic wave fields. *Journal of biomedical optics* 15(2), 021314–021314 (2010)
- [2] Pharr, M., Jakob, W., Humphreys, G.: *Physically based rendering: From theory to implementation*. MIT Press (2023)
- [3] Jakob, W., Speierer, S., Roussel, N., Nimier-David, M., Vicini, D., Zeltner, T., Nicolet, B., Crespo, M., Leroy, V., Zhang, Z.: Mitsuba 3 renderer (2022), <https://mitsuba-renderer.org>
- [4] Mattausch, O., Makhinya, M., Goksel, O.: Realistic ultrasound simulation of complex surface models using interactive monte-carlo path tracing. In: *Computer Graphics Forum*. vol. 37, pp. 202–213. Wiley Online Library (2018)



Results



Summary

We present **UltraRay**, a fast and flexible framework for ultrasound reflection simulation based on ray tracing. Its main features include:

- **Full-path ray tracing** that tracks rays from emission to reception, using a transducer sampling strategy for efficient ray return after interactions.
- **Monte Carlo-based formulation** with a physics-informed model for accurate tissue boundary reflections.
- **End-to-end simulation workflow** integrating plane wave imaging with a standard beamforming pipeline (delay-and-sum, envelope detection, and log compression).