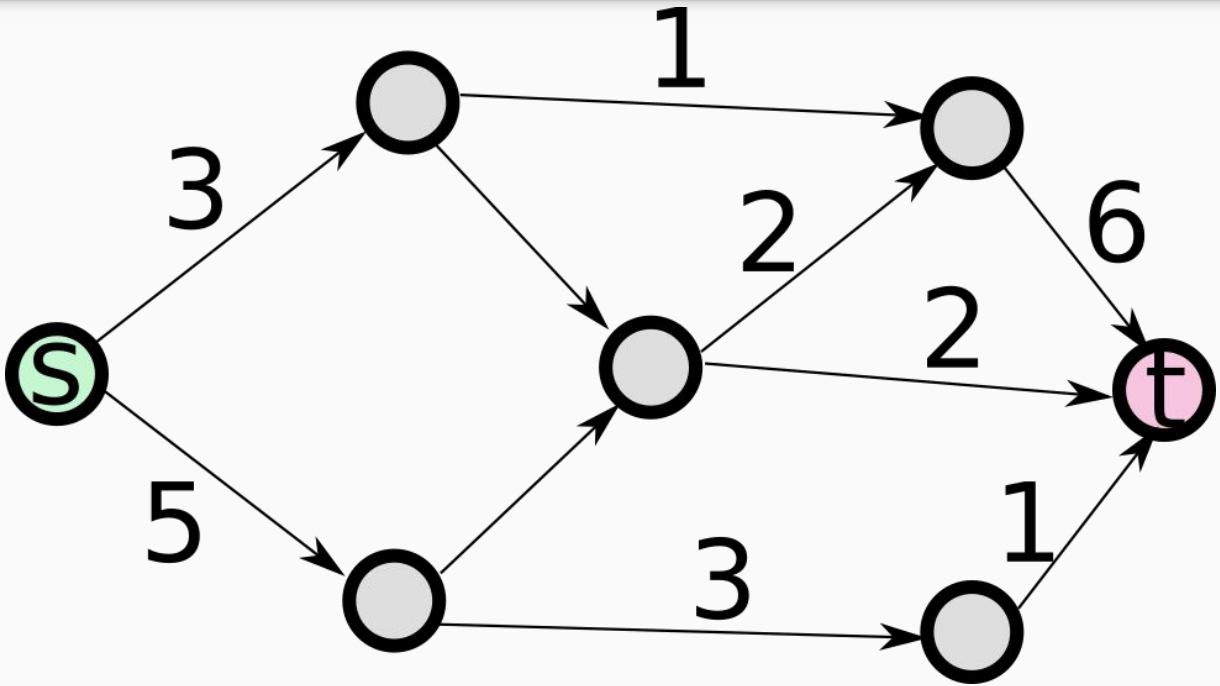
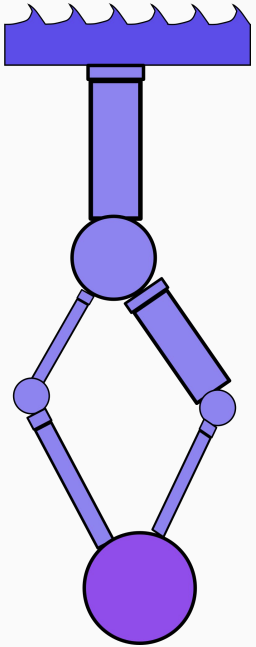
Max Flow

http://bit.ly/VTProgMaxFlow

Network flow problem



Network flow problem



Network flow problem

Network flow problem

- Source produces infinite “flow”
- Edges are directed from A -> B, can only hold a certain amount of “flow”
  - Called the capacity
- Amount of flow into a node is equal to the amount out
- How much flow can we get to the sink?

Maximum flow problem

- The maximum flow problem asks what is the maximum amount of flow we can get from the source to the sink?

Minimum cut problem

- An equivalent question asks what is the “minimum capacity cut”
- We want to disconnect the source from the sink by removing edges that sum to the minimum capacity
- Size of the min-cut is the same as the max flow
- Can easily find the edges of a min-cut after running a max flow algorithm

Applications of network flow

- Bipartite matching
- Counting edge disjoint paths
- Minimum cut
- Baseball elimination
- Project selection

Towards a max flow algorithm

- Bipartite matching from last week
- How can we state this as a flow problem?

Bipartite matching

Max flow

- What did we use to solve this last week?

Ford Fulkerson

- Simply stated:
- If there is an “augmenting path”, augment flow along it
- Repeat until there are no augmenting paths

Augmenting paths

- Path from the source to the sink
- Can push some amount of flow along it
- Also consider backwards flow chains
- http://weierstrass.is.tokushima-u.ac.jp/ikeda/suuri/maxflow/MaxflowApp.shtml?demo1

Augmenting paths

- How can we find augmenting paths?
- Just need a path-finding algorithm from source to sink

Augmenting paths

- How do we deal with backwards (or “dual”) edges?
- We essentially “borrowed” flow

Example

Modeling problems as network flow

Code for Ford Fulkerson

https://spruett.me/blog/static/code/maxflow.java.html

Problems for today

- https://pcs.spruett.me/problems?query=maxflow
- https://pcs.spruett.me/problems?query=matching
  - https://pcs.spruett.me/problems/34