Deep Learning - Examen Octubre 2022 - Final

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Ej 4. Se tiene la función $y = a * \log(b) + b * c^2$
calcular las derivadas de y respecto a los parámetros a, b, c evaluados en $a = 2, b = 10, c = 0{,}5$
Se deben calcular las derivadas por los siguientes métodos.

* Nota: Se entiende que $\log(b)$ es el logaritmo en base e

a) Utilizando derivadas analíticas (symbolic differentiation)

$$y = a * \log(b) + b * c^2$$

$$\frac{\partial y}{\partial a} = \frac{\partial (a \cdot \log(b))}{\partial a} + \frac{\partial (b \cdot c^2)}{\partial a} = \log(b) + 0$$

$$= \log(10) = 2.302 \Rightarrow \boxed{\frac{\partial y}{\partial a} = 2{,}302}$$

$$\frac{\partial y}{\partial b} = \frac{\partial (a \cdot \log(b))}{\partial b} + \frac{\partial (b \cdot c^2)}{\partial b} = a \cdot \frac{\partial \log(b)}{\partial b} + c^2 \frac{\partial b}{\partial b}$$

$$= a \cdot \frac{1}{b} + c^2 = 2 \cdot \frac{1}{10} + 0{,}25 \Rightarrow \boxed{\frac{\partial y}{\partial b} = 0{,}45}$$

$$\frac{\partial y}{\partial c} = \frac{\partial (a \cdot \log(b))}{\partial c} + \frac{\partial\, b \cdot c^2}{\partial c} = 0 + b \cdot 2c$$

$$= 10 \cdot 2 \cdot 0{,}5 = 10 \Rightarrow \boxed{\frac{\partial y}{\partial c} = 10}$$

b) utilizando límite con un delta numérico pequeño (numerical differentiation)

$$y = a \times \log(b) + b \cdot c^2$$

$a = 2$

$b = 10$

$c = 0,5$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

* para calcular $\frac{\partial y}{\partial a}$ se toma $\Delta a = 0,1$

$$f(a+\Delta a) = (a+\Delta a) \cdot \log(b) + b \cdot c^2$$
$$= (2+0,1) \cdot 2.302 + 10 \cdot 0,25$$
$$= 7.3342$$
$$f(a) = a \times \log(b) + b \cdot c^2$$
$$= 2 \times 2,302 + 10 \cdot 0,25 =$$
$$= 7.104$$

$$f'(a) = \frac{7.3342 - 7.104}{0,1} = 2.302 \Rightarrow \boxed{f'(a) = 2,302}$$

* para calcular $\frac{\partial y}{\partial b}$ se toma $\Delta b = 0,1$

$$f(b+\Delta b) = a \times \log(b+\Delta b) + (b+\Delta b) \cdot c^2$$
$$= 2 \cdot \log(10+0,1) + 10.1 \cdot 0,25$$
$$= 7.149$$
$$f(b) = a \cdot \log(b) + b \cdot c^2$$

$$f(b) = 2 . \log(10) + 10 . 0,25$$
$$= 7.104$$

$$f'(b) = \frac{7.149 - 7.104}{0,1} = 0.450 \Rightarrow F'(b) = 0,45$$

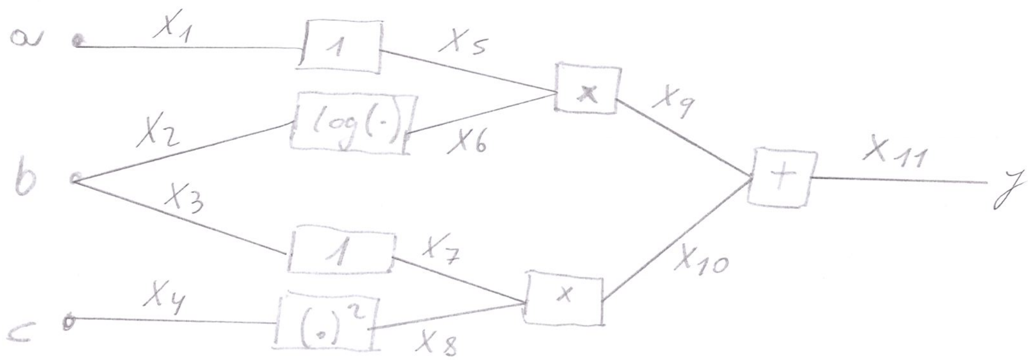
* para calcular $\frac{\partial y}{\partial c}$ se toma $\Delta c = 0,001$

$$f(c+\Delta c) = a . \log(b) + b . (c+\Delta c)^2$$
$$= 2 \log(10) + 10 . 0,251$$
$$= 7.114$$
$$f(c) = a . \log(b) + b . c^2$$
$$= 2 \times \log(10) + 10 \times 0.25$$
$$= 7.104$$

$$f'(c) = \frac{7.114 - 7.104}{0,001} = 10 \Rightarrow F'(c) = 10$$

c) utilizando grafos y la regla de la cadena (chain rule differentiation)

$$y = a \cdot \log(b) + b \cdot c^2$$



$X_1 = a = 2$

$X_2 = b = 10$

$X_3 = b = 10$

$X_4 = c = 0,5$

$X_5 = X_1$

$X_6 = \log(X_2)$

$X_7 = X_3$

$X_8 = X_4^2$

$X_9 = X_5 \times X_6$

$X_{10} = X_7 \times X_8$

$X_{11} = X_9 + X_{10} = y$

$$\frac{\partial X_5}{\partial X_1} = \frac{\partial X_5}{\partial X_5} = \frac{\partial X_1}{\partial X_1} = 1$$

$$\frac{\partial X_6}{\partial X_2} = \frac{\partial \log(X_2)}{\partial X_2} = \frac{1}{X_2}$$

$$\frac{\partial X_7}{\partial X_3} = \frac{\partial X_3}{\partial X_3} = \frac{\partial X_7}{\partial X_7} = 1$$

$$\frac{\partial X_8}{\partial X_4} = \frac{\partial X_4^2}{\partial X_4} = 2X_4$$

$$\frac{\partial X_9}{\partial X_5} = \frac{\partial X_5 \cdot X_6}{\partial X_5} = X_6$$

$$\frac{\partial X_9}{\partial X_6} = \frac{\partial X_5 \cdot X_6}{\partial X_6} = X_5$$

$$\frac{\partial X_{10}}{\partial X_7} = \frac{\partial X_7 \cdot X_8}{\partial X_7} = X_8$$

$$\frac{\partial X_{10}}{\partial X_8} = \frac{d X_7 \cdot X_8}{d X_8} = X_7$$

$$\frac{\partial X_{11}}{\partial X_9} = \frac{\partial (X_9 + X_{10})}{\partial X_9} = 1$$

$$\frac{\partial X_{11}}{\partial X_{10}} = \frac{\partial (X_9 + X_{10})}{\partial X_{10}} = 1$$

* $$\frac{\partial y}{\partial a} = \frac{d X_{11}}{d X_9} \cdot \frac{d X_9}{d X_5} \cdot \frac{d X_5}{d X_1} =$$

$$= 1 \cdot X_6 \cdot 1 = X_6 = \log(X_2)$$

$$= \log(b) = \log(10) = 2.302$$

$$\Rightarrow \boxed{\frac{\partial y}{\partial a} = 2.302}$$

* $$\frac{\partial y}{\partial b} = \frac{d X_{11}}{d X_9} \cdot \frac{d X_9}{d X_6} \frac{d X_6}{d X_2} + \frac{d X_{11}}{d X_{10}} \cdot \frac{d X_{10}}{d X_7} \cdot \frac{d X_7}{d X_3}$$

$$= 1 \cdot X_5 \cdot \frac{1}{X_2} + 1 \cdot X_8 \cdot 1$$

$$\frac{\partial y}{\partial b} = x_5 \cdot \frac{1}{x_2} + x_8 = x_1 \frac{1}{x_2} + x_4^2 =$$

$$= a \cdot \frac{1}{b} + c^2 = 2 \cdot \frac{1}{10} + 0{,}25 = 0{,}45$$

$$\Rightarrow \boxed{\frac{\partial y}{\partial b} = 0.45}$$

$$* \quad \frac{\partial y}{\partial c} = \frac{d x_{11}}{d x_{10}} \cdot \frac{d x_{10}}{d x_8} \frac{d x_8}{d x_4} =$$

$$= 1 \cdot x_7 \cdot 2 x_4 = b \cdot 2c = 10 \times 1$$

$$\Rightarrow \boxed{\frac{\partial y}{\partial c} = 10}$$