



Unit 1 - Sem 4 - 22MAT230

Mathematics for Computing

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If you find any mistakes or have any comments to share,

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Computational Experiment

To demonstrate that the no: zero eigen values of the (unnormalized) graph Laplacian is equal to the no: disconnected components or disconnected subgraphs (or the clusters)

Toy examples

These are toy examples.

Understand the concept using toy examples !

Demonstrate your understanding using toy examples !

Example 1

2 zero eigen values \Rightarrow 2 components

```
clearvars
rng(100)
N = 4; %no: nodes
```

Make a random adjacency matrix(square symmetric)

```
A = rand(N);
A = A'*A;
Z = zeros(size(A));
```

Build the Laplacian matrix of a graph with 2 disconnected components.

```
W = [A Z ; Z A];
```

```

W = W-diag(diag(W));

n = size(W,1);

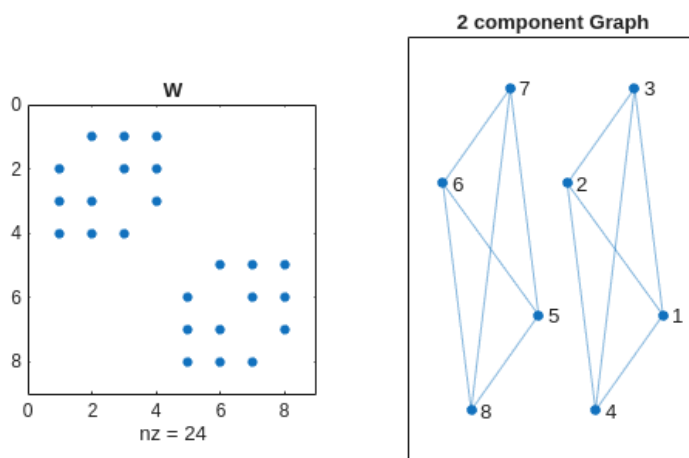
D = diag(sum(W));

L_u = D - W;
L_n = eye(n) - pinv(D)*W;
L_ns = eye(n) - D^(-0.5)*W*D^(-0.5);

subplot(1,2,1)
spy(W)
title("W")

subplot(1,2,2)
plot(graph(W))
title("2 component Graph")

```



```
[eig(L_u) eig(L_n) eig(L_ns)]
```

```

ans = 8×3
-0.0000    0 -0.0000
-0.0000    1.4338    1.4338
 2.7175    1.3618    1.3618
 2.7175    1.2043    1.2043
 3.7223    0 -0.0000
 3.7223    1.4338    1.4338
 3.8766    1.3618    1.3618
 3.8766    1.2043    1.2043

```

Example 2

3 zero eigen values \Rightarrow 3 components

```

clearvars
rng(100)
N = 4; %no: nodes

```

Make a random adjacency matrix(square symmetric)

```
A = rand(N);
A = A'*A;
Z = zeros(size(A));
```

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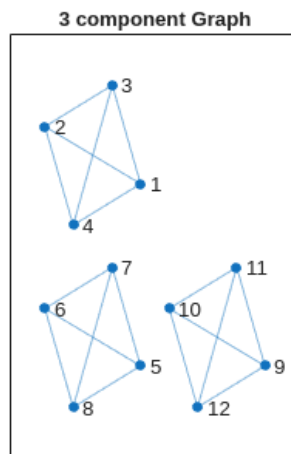
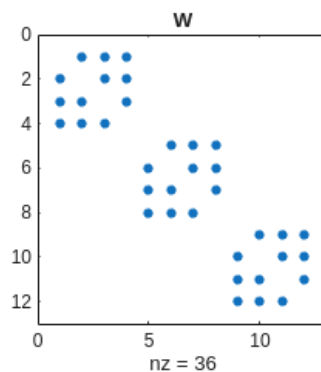
Build the Laplacian matrix of a graph with 3 disconnected components.

```
W = [A Z Z; Z A Z; Z Z A];
W = W-diag(diag(W));

n = size(W,1);

D = diag(sum(W));
L_u = D - W;
L_n = eye(n) - pinv(D)*W;
L_ns = eye(n) - D^(-0.5)*W*D^(-0.5);

subplot(1,2,1)
spy(W)
title("W")
subplot(1,2,2)
plot(graph(W))
title("3 component Graph")
```



```
[eig(L_u) eig(L_n) eig(L_ns)]
```

```
ans = 12×3
-0.0000    0    -0.0000
-0.0000    1.4338    1.4338
-0.0000    1.3618    1.3618
 2.7175    1.2043    1.2043
 2.7175     0    -0.0000
 2.7175    1.4338    1.4338
 3.7223    1.3618    1.3618
 3.7223    1.2043    1.2043
 3.7223     0    -0.0000
 3.8766    1.4338    1.4338
 3.8766    1.3618    1.3618
 3.8766    1.2043    1.2043
⋮
⋮
```

Example 3 Using 'blkdiag' function

3 zero eigen values \Rightarrow 3 components

```
clearvars
rng(100)
```

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Make a random adjacency matrix(square symmetric)

```
A = rand(4);
A = A'*A;
```

Build the Laplacian matrix of a graph with 3 disconnected components using blkdiag function.

```
W = blkdiag(A,A,A);
W = W-diag(diag(W));

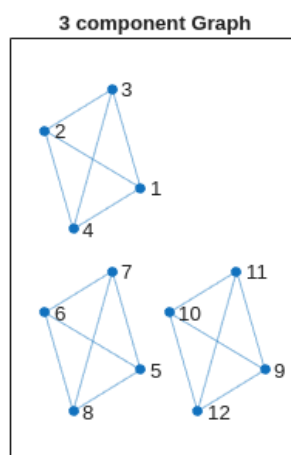
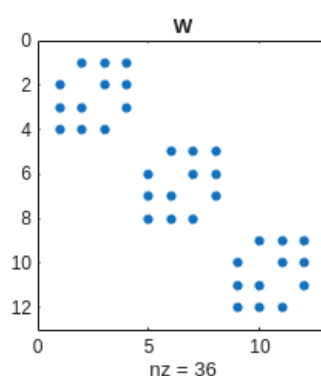
n = size(W,1);

D = diag(sum(W));

L_u = D - W;
L_n = eye(n) - pinv(D)*W;
L_ns = eye(n) - D^(-0.5)*W*D^(-0.5);

subplot(1,2,1)
spy(W)
title("W")

subplot(1,2,2)
plot(graph(W))
title("3 component Graph")
```



```
[eig(L_u) eig(L_n) eig(L_ns)]
```

```
ans = 12x3
-0.0000         0 -0.0000
-0.0000    1.4338    1.4338
-0.0000    1.3618    1.3618
```

2.7175	1.2043	1.2043
2.7175	0	-0.0000
2.7175	1.4338	1.4338
3.7223	1.3618	1.3618
3.7223	1.2043	1.2043
3.7223	0	-0.0000
3.8766	1.4338	1.4338
3.8766	1.3618	1.3618
3.8766	1.2043	1.2043
:		
:		

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```
cd("/media/user/DATA4LINUX/new1/Repos/Mine/MFC4_22MAT230/")
mlxfile = matlab.desktop.editor.getActive().Filename;
outfile = mlxfile + ".pdf";
export(matlab.desktop.editor.getActive().Filename, outfile, PageSize="A4");
```