Unit 1 - Sem 4 - 22MAT230

Mathematics for Computing

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If you find any mistakes or have any comments to share,

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Definition of Norm to be added

L_0 L0 norm

Number of non zero components in $\mathbf{x}$

```
n = 10;
x = rand(n,1);
x(x<0.5) = 0;
lo_x = sum(x~=0)
```



```
lo_x =
4
```

(not strictly a norm function)

Understanding the L_0 Norm: Counting the Non-Zero

The L_0 norm of a vector, denoted as $\|x\|_0$ is defined as the number of non-zero elements in that vector.

For instance, for the vector $\mathbf{x} = \begin{bmatrix} 0 \\ 5 \\ 0 \\ -2 \\ 3 \end{bmatrix}$, the L_0 norm $\|x\|_0$ is 3, as there are three non-zero elements $5, -2$ and 3.

Mathematical Definition:

For a vector $\mathbf{x} \in R^n$, the L_0 norm is given by:

$$\|x\|_0 = \sum_{i=1}^{n} |x_i|^0$$

where $|x_i|^0 = 1$ if $x_i \neq 0$ and 0 if $x_i = 0$.

L_1 L1 norm

Sum of absolute value of the components of $\mathbf{x}$

```
n = 5;
x = randi([-1,4],n,1)
```

```
x = 5×1
     2
    -1
     1
    -1
     4
```

```
l1_x = sum(abs(x))
```

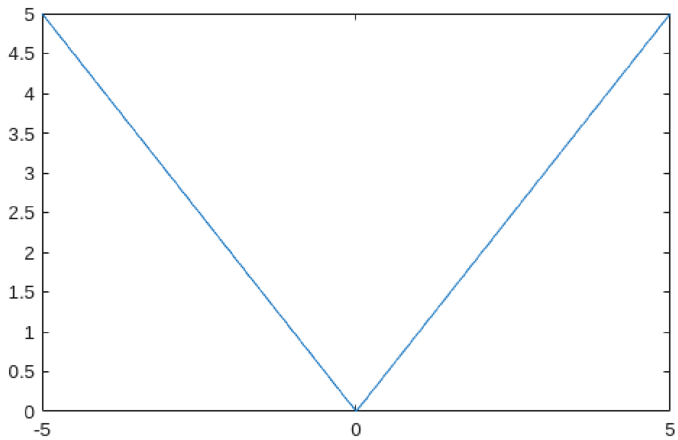
```
l1_x =
9
```

is the sum of the absolute values of each elements of $\mathbf{x}$.

$$\|\mathbf{x}\|_1 = \sum_{i=1}^{n} |x_i|$$

Visualizing $|x|$

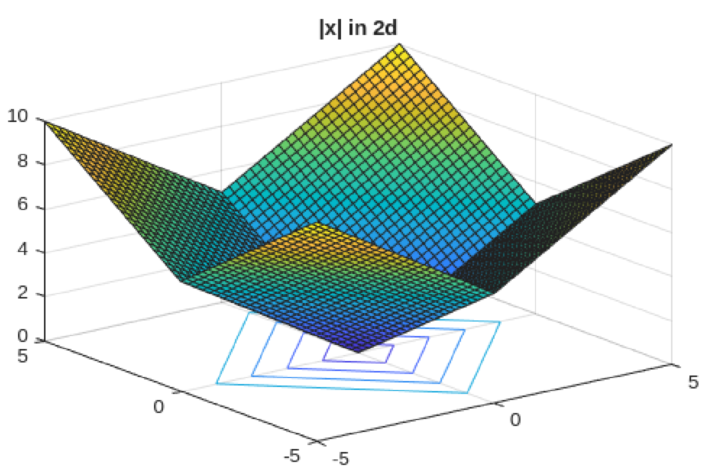
```
x1 = single(-5:0.2:5);
y = abs(x1);
plot(x1,y)
```




```
clearvars
```

Visualizing $|\mathbf{x}|, \mathbf{x} \in R^2$

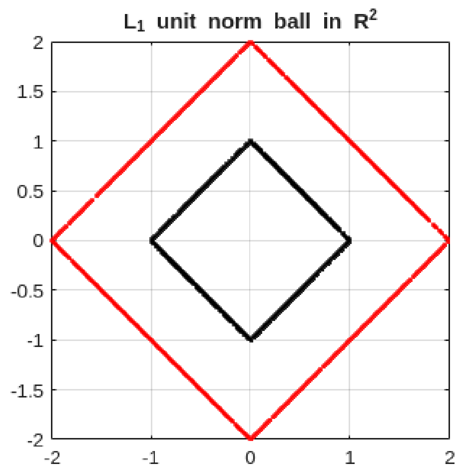
```
x1 = single(-5:0.2:5);
x2 = x1;
[X1,X2] = meshgrid(x1,x2);
Z1 = abs(X1)+abs(X2);
surf(X1,X2,Z1);hold on
contour(X1,X2,Z1,1:4); hold off
title("|x| in 2d")
```



L_1 **Unit norm ball in 2 dimension**

$\mathbf{x} \in R^2$

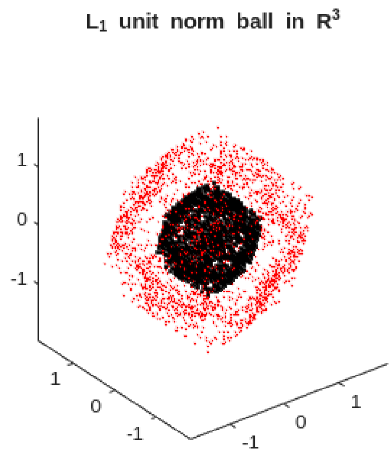

```
clearvars
x = rand(2,2500)-0.5;
x = x./sum(abs(x));
x2 = 2*x;
plot(x(1,:),x(2,:),'k.');hold on
plot(x2(1,:),x2(2,:),'r.');hold off
axis equal
title("L_1 unit norm ball in R^2")
grid on
xlim([-2 2])
ylim([-2 2])
```



L_1 **Unit norm ball in 3 dimension**

$\mathbf{x} \in R^3$

```
clearvars
x = rand(3,2500)-0.5;
x = x./sum(abs(x));
plot3(x(1,:),x(2,:),x(3,:),'k.');hold on
axis equal
title("L_1 unit norm ball in R^3")

x2 = 2*x;
plot3(x2(1,:),x2(2,:),x2(3,:),'r.',MarkerSize=0.5);
hold off
```



Weighted L_1 L1 norm

```
n = 5;
x = randi([-1,4],n,1)
```

```
x = 5×1
     0
    -1
     3
     1
     2
```

```
w = rand(n,1);
w = w/sum(w)
```

```
w = 5×1
    0.3359
    0.0987
    0.2185
    0.2706
    0.0763
```

```
wl1_x = w'*abs(x)
```

```
wl1_x =
1.1774
```

L_2 L2 norm

Sqrt of sum of squares of the components of $\mathbf{x}$

```
n = 5;
x = randi([-1,4],n,1)
```

```
x = 5×1
```

```
3
-1
1
0
3
```

```
l2_x = sqrt(sum(x.*x))
```

```
l2_x =
4.4721
```

```
l2_x = sqrt(sum(x.^2))
```

```
l2_x =
4.4721
```

```
l2_x = sqrt(dot(x,x))
```

```
l2_x =
4.4721
```

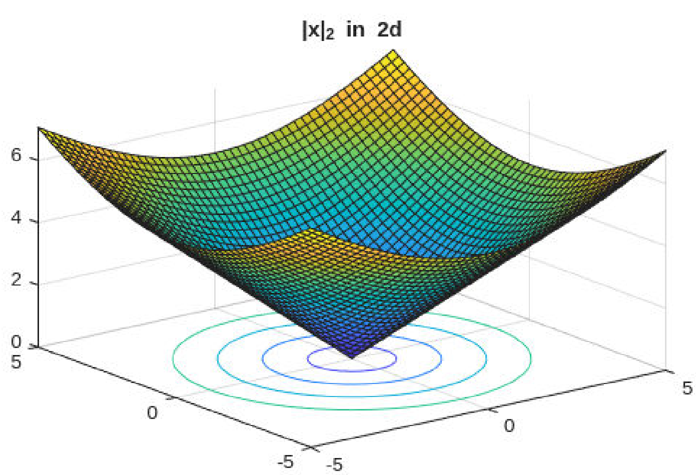
is the square root of the sum of the squares of each values of $\mathbf{x}$.

$\|\mathbf{x}\|_2^2 = \sum_{i=1}^{n} x_i^2$ square of the L_2 norm

L_2 norm $= \sqrt{\sum_{i=1}^{n} x_i^2}$

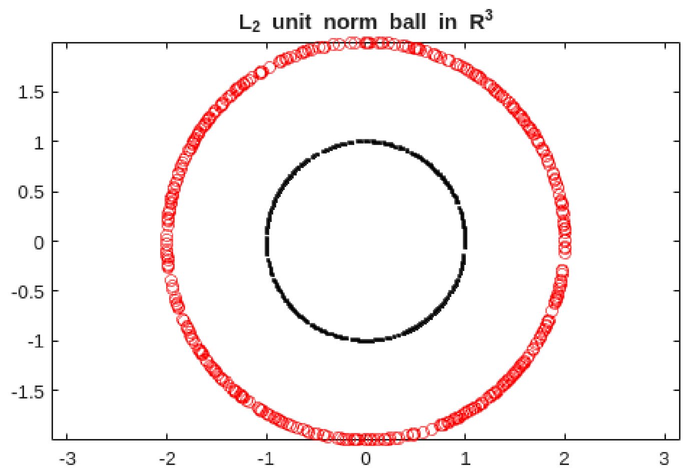
Visualizing $|\mathbf{x}|_2, \mathbf{x} \in R^2$

```
x1 = single(-5:0.2:5);
x2 = x1;
[X1,X2] = meshgrid(x1,x2);
Z1 = sqrt(X1.^2 + X2.^2);
surf(X1,X2,Z1);hold on
contour(X1,X2,Z1,1:4); hold off
title("|x|_2 in 2d")
```



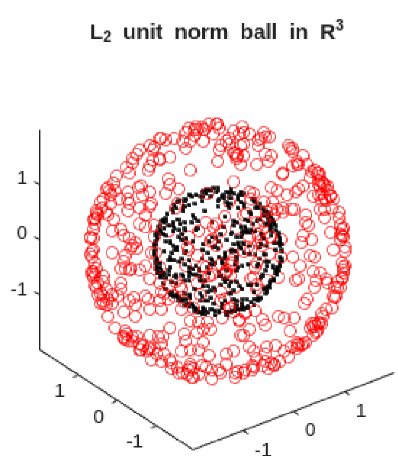
L_2 Unit norm ball in 2 dimension

```
x = rand(2,500)-0.5;
x = x./(sqrt(sum(x.^2)));

plot(x(1,:),x(2,:),'k.');hold on
axis equal
title("L_2 unit norm ball in R^3")
x = 2*x./(sqrt(sum(x.^2)));
plot(x(1,:),x(2,:),'ro');hold off
```



L_2 Unit norm ball in 3 dimension

```
x = rand(3,500)-0.5;
x = x./(sqrt(sum(x.^2)));

plot3(x(1,:),x(2,:),x(3,:),'k.');hold on
axis equal
title("L_2 unit norm ball in R^3")
x = 2*x./(sqrt(sum(x.^2)));
plot3(x(1,:),x(2,:),x(3,:),'ro');hold off
```



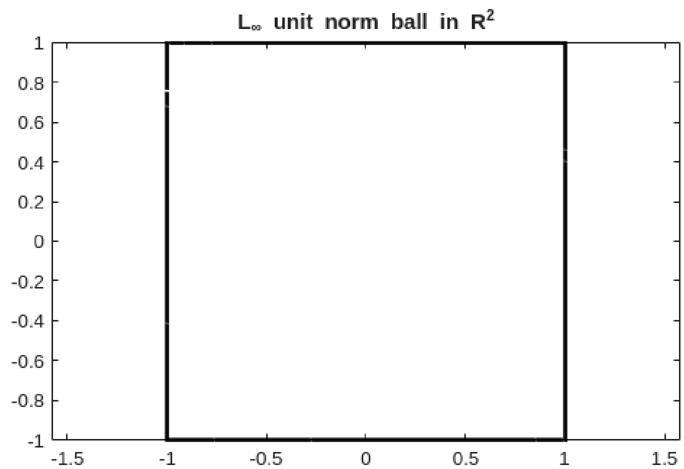
L_∞ norm

is the maximum of the absolute values of $\mathbf{x}$.

$\|\mathbf{x}\|_\infty = \max\{|x_i|\}$

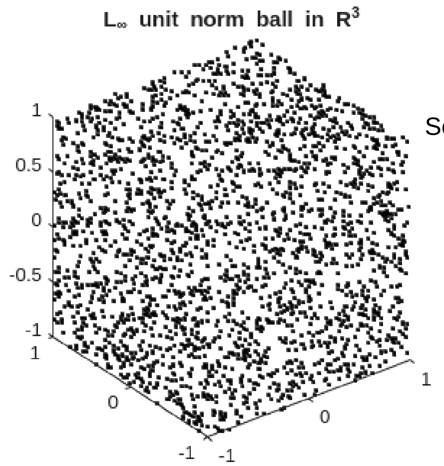
$\mathbf{x} \in R^2$

```
x = rand(2,2500)-0.5;
x = x./(max(abs(x)));
plot(x(1,:),x(2,:),'k.')
axis equal
title("L_\infty unit norm ball in R^2")
```



$\mathbf{x} \in R^3$

```
x = rand(3,2500)-0.5;
x = x./(max(abs(x)));
plot3(x(1,:),x(2,:),x(3,:),'k.')
axis equal
title("L_\infty unit norm ball in R^3")
```



L_p norm

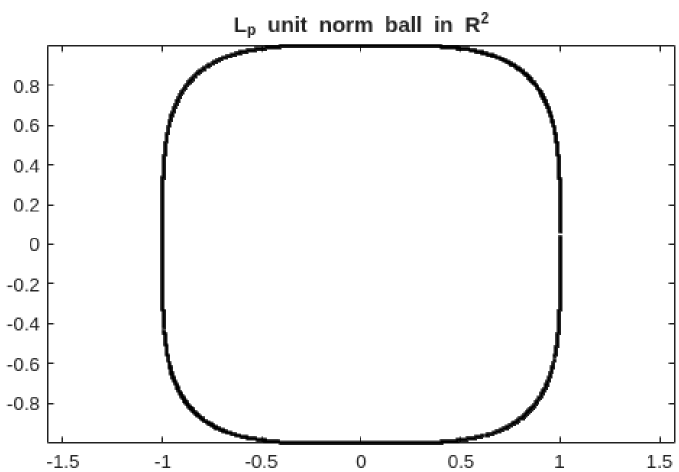
is defined as

$$\|\mathbf{x}\|_p = \left(\sum_{i=1}^{n} |x_i|^p\right)^{\frac{1}{p}}$$

with $1 \leq p < \infty$

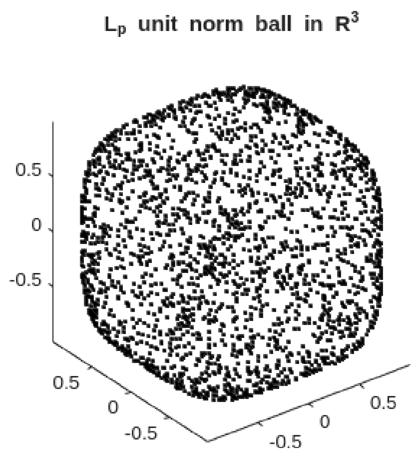
$\mathbf{x} \in R^2$

```
x = rand(2,2500)-0.5;
p = 4;
x = x./power(sum(abs(x).^p),1/p);
plot(x(1,:),x(2,:),'k.')
axis equal
title("L_p unit norm ball in R^2")
```



$\mathbf{x} \in R^3$


```
x = rand(3,2500)-0.5;
p = 4;
x = x./power(sum(abs(x).^p),1/p);
plot3(x(1,:),x(2,:),x(3,:),'k.')
axis equal
title("L_p unit norm ball in R^3")
```



L_* Trace norm, Schatten 1-norm, Nuclear norm

ℓ_1 norm of the vector of singular values of a matrix A

sum of singular values

```
rng(100)
X = randi([-1,4],3,4);

Ln_X = sum(svd(x))
```

```
Ln_X =
102.7801
```

Frobenius norm

ℓ_2 norm of the vector of singular values of a matrix A

```
rng(100)

A = randi([-1,3],3,5)
```

```
A = 3×5
     1     3     2     1    -1
     0    -1     3     3    -1
     1    -1    -1     0     0
```

```
norm(A,"fro")
```

```
ans =
6.2450
```



```
sqrt(sum(A.^2,"all"))
```

```
ans =
6.2450
```

```
sqrt(dot(A(:),A(:)))
```

```
ans =
6.2450
```

```
sqrt(sum(svd(A).^2))
```

```
ans =
6.2450
```

```
sqrt(trace(A'*A))
```

```
ans =
6.2450
```

```
sqrt(trace(A*A'))
```

```
ans =
6.2450
```

$L_{2,1}$ L21norm

Sum of the L_2 norm of the column vectors of A

```
rng(100)

A = randi([-1,3],3,5)
```

```
A = 3×5
     1     3     2     1    -1
     0    -1     3     3    -1
     1    -1    -1     0     0
```

```
sum(sqrt(sum(A.^2)))
```

```
ans =
13.0490
```

Matrix inner product

The inner product of 2 $m \times n$ matrices A and B is defined as trace($A^T B$). It is equal to the dot product of the vectorized form of A and B.

```
rng(100)

A = randi([-1,3],4,2)
```



```
A = 4×2
     1    -1
     0    -1
     1     2
     3     3
```

```
B = randi([-1,3],4,2)
```

```
B = 4×2
    -1    -1
     1    -1
     3     0
     0     3
```

```
trace(A'*B)
```

```
ans =
13
```

```
trace(B'*A)
```

```
ans =
13
```

```
trace(A*B')
```

```
ans =
13
```

```
trace(B*A')
```

```
ans =
13
```

```
dot(A(:),B(:))
```

```
ans =
13
```

Operator Norm

Maximum value of the singular value of a matrix A

Weighted $L_{2,1}$ norm

```
A = repmat(1:5,3,1)
```

```
A = 3×5
     1     2     3     4     5
     1     2     3     4     5
     1     2     3     4     5
```



```
w = rand(5,1)
```

```
w = 5×1
    0.8117
    0.1719
    0.8162
    0.2741
    0.4317
```

```
L21 = sum(sqrt(sum(A.^2)))
```

```
L21 =
25.9808
```

```
x2 = sqrt(sum(A.^2))'
```

```
x2 = 5×1
    1.7321
    3.4641
    5.1962
    6.9282
    8.6603
```

```
WL21 = w'*x2
```

```
WL21 =
11.8802
```

```
sum(w.*x2)
```

```
ans =
11.8802
```

Definition 1 (Weighted L_1-Norm): For matrix $X \in \mathbb{R}^{m \times n}$, the weighted L_1-norm of X is defined as follows:

$$\|X\|_{w_n,1} = \sum_{i=1}^{m} \sum_{j=1}^{n} w_{n_{i,j}} |X_{i,j}|, \tag{1}$$

where $w_n \in \mathbb{R}^{m \times n}$ denotes the norm weight. The weighted L_1-norm is calculated by multiplying the absolute value of X by its corresponding norm weight and then summing all the results.

Definition 2 (Weighted $L_{2,1}$-Norm): For matrix $X \in \mathbb{R}^{m \times n}$, the weighted $L_{2,1}$-norm of X is defined as follows:

$$\|X\|_{w_g,1} = \sum_{j=1}^{n} w_{g_j} \left\| X^{[j]} \right\|_2 , \tag{2}$$

where $w_n \in \mathbb{R}^{m \times n}$ is the group norm weight, and g_j denotes the group corresponding to the jth column. The weighted $L_{2,1}$-norm is calculated by multiplying the L_2-norm of each group by its corresponding group norm weight, and then summing these values.


```
cd("/media/user/DATA4LINUX/new1/Repos/Mine/MFC4_22MAT230/")
mlxfile = matlab.desktop.editor.getActive().Filename;
outfile = mlxfile + ".pdf";
export(matlab.desktop.editor.getActive().Filename, outfile);
```