Computational Experiment

To demonstrate that the no: zero eigen values of the (unnormalized) graph Laplacian is equal to the no: disconnected components or disconnected subgraphs (or the clusters)

Toy examples

These are toy examples.

Understand the concept using toy examples !

Demonstrate your understanding using toy examples !

Example 1

2 zero eigen values $\Rightarrow 2$ components

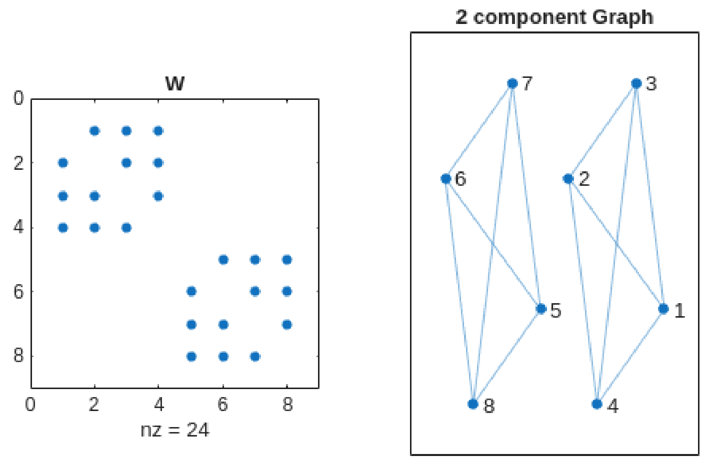
```
clearvars
rng(100)
N = 4;  %no: nodes
```

Make a random adjacency matrix(square symmetric)

```
A = rand(N);
A = A'*A;
Z =  zeros(size(A));
```

Build the Laplacian matrix of a graph with 2 disconnected components.

```
W = [A Z ;Z A];
W = W-diag(diag(W));

n = size(W,1);

D = diag(sum(W));

L_u = D - W;
L_n = eye(n) - pinv(D)*W;
L_ns = eye(n) - D^(-0.5)*W*D^(-0.5);

subplot(1,2,1)
spy(W)
title("W")

subplot(1,2,2)
plot(graph(W))
title("2 component Graph")
```




```
[eig(L_u) eig(L_n) eig(L_ns)]
```

```
ans = 8×3
   -0.0000         0   -0.0000
   -0.0000    1.4338    1.4338
    2.7175    1.3618    1.3618
    2.7175    1.2043    1.2043
    3.7223         0   -0.0000
    3.7223    1.4338    1.4338
    3.8766    1.3618    1.3618
    3.8766    1.2043    1.2043
```

Example 2

3 zero eigen values $\Rightarrow 3$ components

```
clearvars
rng(100)
N = 4;  %no: nodes
```

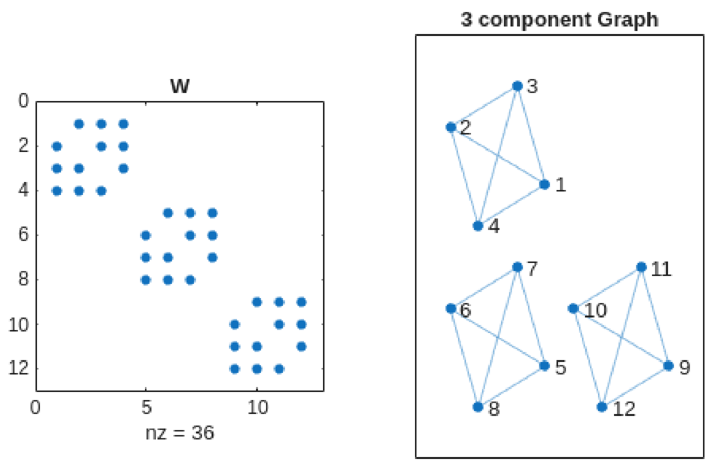
Make a random adjacency matrix(square symmetric)

```
A = rand(N);
A = A'*A;
Z =  zeros(size(A));
```

Build the Laplacian matrix of a graph with 3 disconnected components.

```
W = [A Z Z;Z A Z;Z Z A];
W = W-diag(diag(W));

n = size(W,1);

D = diag(sum(W));
L_u = D - W;
```

```
L_n = eye(n) - pinv(D)*W;
L_ns = eye(n) - D^(-0.5)*W*D^(-0.5);

subplot(1,2,1)
spy(W)
title("W")
subplot(1,2,2)
plot(graph(W))
title("3 component Graph")
```




```
[eig(L_u) eig(L_n) eig(L_ns)]
```

```
ans = 12×3
   -0.0000         0   -0.0000
   -0.0000    1.4338    1.4338
   -0.0000    1.3618    1.3618
    2.7175    1.2043    1.2043
    2.7175         0   -0.0000
    2.7175    1.4338    1.4338
    3.7223    1.3618    1.3618
    3.7223    1.2043    1.2043
    3.7223         0   -0.0000
    3.8766    1.4338    1.4338
    3.8766    1.3618    1.3618
    3.8766    1.2043    1.2043
      ⋮
```

Example 3 **Using 'blkdiag' function**

3 zero eigen values $\Rightarrow 3$ components

```
clearvars
rng(100)
```

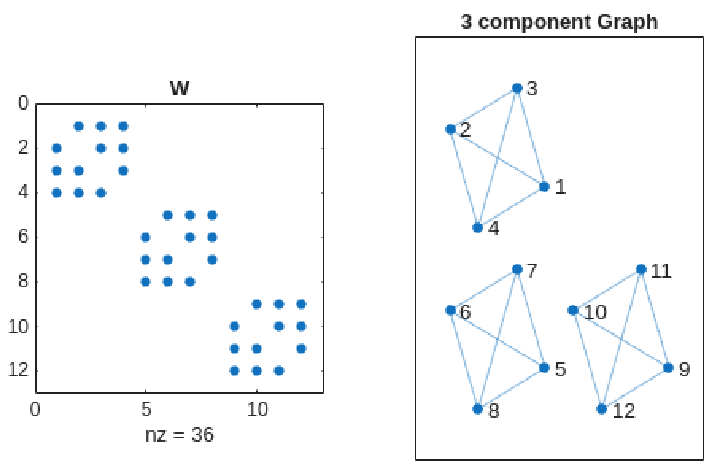
Make a random adjacency matrix(square symmetric)

```
A = rand(4);
```

```
A = A'*A;
```

Build the Laplacian matrix of a graph with 3 disconnected components using blkdiag function.

```
W = blkdiag(A,A,A);
W = W-diag(diag(W));

n = size(W,1);

D = diag(sum(W));

L_u = D - W;
L_n = eye(n) - pinv(D)*W;
L_ns = eye(n) - D^(-0.5)*W*D^(-0.5);

subplot(1,2,1)
spy(W)
title("W")

subplot(1,2,2)
plot(graph(W))
title("3 component Graph")
```




```
[eig(L_u) eig(L_n) eig(L_ns)]
```

```
ans = 12×3
   -0.0000         0   -0.0000
   -0.0000    1.4338    1.4338
   -0.0000    1.3618    1.3618
    2.7175    1.2043    1.2043
    2.7175         0   -0.0000
    2.7175    1.4338    1.4338
    3.7223    1.3618    1.3618
    3.7223    1.2043    1.2043
    3.7223         0   -0.0000
    3.8766    1.4338    1.4338
    3.8766    1.3618    1.3618
```

```
    3.8766    1.2043    1.2043
      ⋮
```

```

```

```
cd("/media/user/DATA4LINUX/new1/Repos/Mine/MFC4_22MAT230/")
mlxfile = matlab.desktop.editor.getActive().Filename;
outfile = mlxfile + ".pdf"
```

```
outfile = 
"/media/user/DATA4LINUX/new1/Repos/Mine/MFC4_22MAT230/U1_EigValues_GraphLaplacian.mlx.pdf"
```

```
% matlab.internal.liveeditor.executeAndSave(mlxfile);
export(matlab.desktop.editor.getActive().Filename, outfile);
```