

Computational Experiment

To demonstrate that the no: zero eigen values of the (unnormalized) graph Laplacian is equal to the no: disconnected components or disconnected subgraphs (or the clusters)

Toy examples

These are toy examples.

Understand the concept using toy examples !

Demonstrate your understanding using toy examples !

Example 1

2 zero eigen values \Rightarrow 2 components

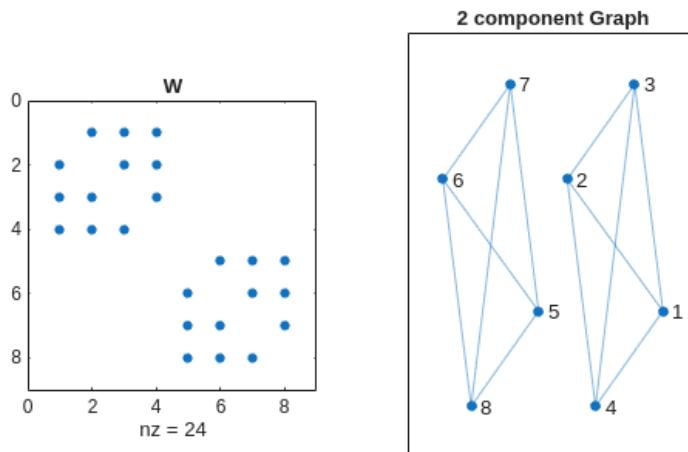
```
clearvars  
rng(100)  
N = 4; %no: nodes
```

Make a random adjacency matrix(square symmetric)

```
A = rand(N);  
A = A' *A;  
Z = zeros(size(A));
```

Build the Laplacian matrix of a graph with 2 disconnected components.

```
W = [A Z ;Z A];  
W = W-diag(diag(W));  
  
n = size(W,1);  
  
D = diag(sum(W));  
  
L_u = D - W;  
L_n = eye(n) - pinv(D)*W;  
L_ns = eye(n) - D^(-0.5)*W*D^(-0.5);  
  
subplot(1,2,1)  
spy(W)  
title("W")  
  
subplot(1,2,2)  
plot(graph(W))  
title("2 component Graph")
```



```
[eig(L_u) eig(L_n) eig(L_ns)]
```

```
ans = 8×3
-0.0000      0    -0.0000
-0.0000  1.4338   1.4338
2.7175  1.3618   1.3618
2.7175  1.2043   1.2043
3.7223      0    -0.0000
3.7223  1.4338   1.4338
3.8766  1.3618   1.3618
3.8766  1.2043   1.2043
```

Example 2

3 zero eigen values \Rightarrow 3 components

```
clearvars
rng(100)
N = 4; %no: nodes
```

Make a random adjacency matrix(square symmetric)

```
A = rand(N);
A = A'*A;
Z = zeros(size(A));
```

Build the Laplacian matrix of a graph with 3 disconnected components.

```
W = [A Z Z; Z A Z; Z Z A];
W = W-diag(diag(W));

n = size(W,1);

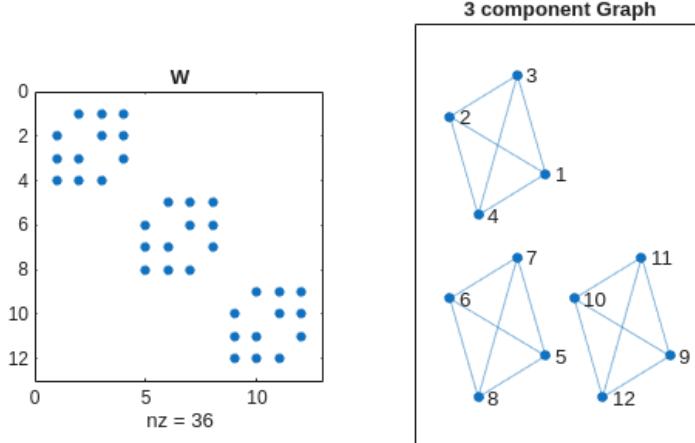
D = diag(sum(W));
L_u = D - W;
L_n = eye(n) - pinv(D)*W;
```

```

L_ns = eye(n) - D^(-0.5)*W*D^(-0.5);

subplot(1,2,1)
spy(W)
title("W")
subplot(1,2,2)
plot(graph(W))
title("3 component Graph")

```



```
[eig(L_u) eig(L_n) eig(L_ns)]
```

```

ans = 12x3
-0.0000      0    -0.0000
-0.0000   1.4338   1.4338
-0.0000   1.3618   1.3618
2.7175   1.2043   1.2043
2.7175      0    -0.0000
2.7175   1.4338   1.4338
3.7223   1.3618   1.3618
3.7223   1.2043   1.2043
3.7223      0    -0.0000
3.8766   1.4338   1.4338
3.8766   1.3618   1.3618
3.8766   1.2043   1.2043
:
:
```

Example 3 Using 'blkdiag' function

3 zero eigen values \Rightarrow 3 components

```

clearvars
rng(100)

```

Make a random adjacency matrix(square symmetric)

```
A = rand(4);
```

```
A = A'*A;
```

Build the Laplacian matrix of a graph with 3 disconnected components using blkdiag function.

```
W = blkdiag(A,A,A);
W = W-diag(diag(W));

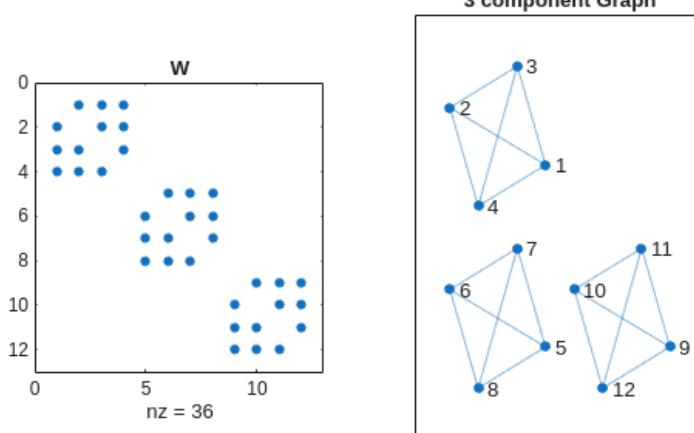
n = size(W,1);

D = diag(sum(W));

L_u = D - W;
L_n = eye(n) - pinv(D)*W;
L_ns = eye(n) - D^(-0.5)*W*D^(-0.5);

subplot(1,2,1)
spy(W)
title("W")

subplot(1,2,2)
plot(graph(W))
title("3 component Graph")
```



```
[eig(L_u) eig(L_n) eig(L_ns)]
```

```
ans = 12x3
-0.0000      0   -0.0000
-0.0000  1.4338  1.4338
-0.0000  1.3618  1.3618
2.7175  1.2043  1.2043
2.7175      0   -0.0000
2.7175  1.4338  1.4338
3.7223  1.3618  1.3618
3.7223  1.2043  1.2043
3.7223      0   -0.0000
3.8766  1.4338  1.4338
3.8766  1.3618  1.3618
```

```
3.8766    1.2043    1.2043
```

```
.
```

```
cd( "/media/user/DATA4LINUX/new1/Repos/Mine/MFC4_22MAT230/" )
mlxfile = matlab.desktop.editor.getActive().Filename;
outfile = mlxfile + ".pdf";

outfile =
"/media/user/DATA4LINUX/new1/Repos/Mine/MFC4_22MAT230/U1_EigValues_GraphLaplacian.mlx.pdf"

export(matlab.desktop.editor.getActive().Filename, outfile);
```