



Unit 1 - Sem 4 - 22MAT230

Mathematics for Computing 4

Dr Sunil Kumar S and Prof K P Soman

School of Artificial Intelligence

Amrita Vishwa Vidyapeetham

If you find any mistakes or have any comments to share,

I would be grateful to receive them at s_sunilkumar@cb.amrita.edu

https://github.com/mfcpj/MFC4_22MAT230

Signal Denoising using ADMM

```
clearvars  
ready = true;  
PUBLISH = ready;
```

$$\min_{\mathbf{x}} \underbrace{\frac{1}{2} \|\mathbf{x} - \mathbf{y}\|_2^2}_{\text{fidelity term}} + \underbrace{\frac{\sigma}{2} \|D_1 \mathbf{x}\|_2^2}_{\text{regularization term}}$$

\mathbf{x} → denoised signal

\mathbf{y} → noisy signal

D_1 first difference matrix

$$\min_{\mathbf{x}} \underbrace{\frac{1}{2} \|\mathbf{x} - \mathbf{y}\|_2^2}_{\text{fidelity term}} + \underbrace{\frac{\sigma}{2} \|D_2 \mathbf{x}\|_2^2}_{\text{regularization term}}$$

\mathbf{x} → denoised signal

\mathbf{y} → noisy signal

D_2 second difference matrix

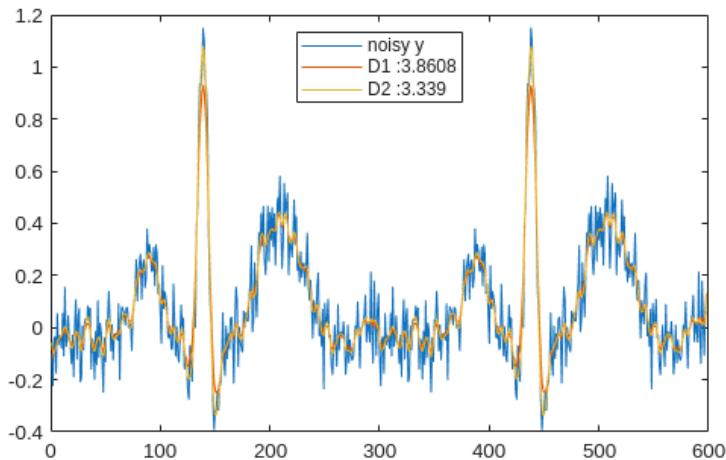
ADMM update equations

$$\mathbf{x}^{(k+1)} = \frac{1}{(1 + \rho)} (\mathbf{y} + \rho(\mathbf{z}^{(k)} - \mathbf{u}^{(k)}))$$

$$\mathbf{z}^{(k+1)} = \rho(\sigma D^T D + I\rho)^{-1}(\mathbf{x}^{(k+1)} + \mathbf{u}^{(k)})$$

$$\mathbf{u}^{(k+1)} = \mathbf{u}^{(k)} + (\mathbf{x}^{(k+1)} - \mathbf{z}^{(k+1)})$$

```
cd( "/media/user/DATA4LINUX/new1/Repos/Mine/DATAmat/" )
load( "NoisyECG.mat" )
[yel,y1] = ADMMdenoise([y; y],1.5,4,1);
[ye2,y2] = ADMMdenoise([y; y],1.5,4,2);
plot([y; y]); hold on
plot(y1);
plot(y2); hold off
legend("noisy y", "D1 :" +num2str(yel), "D2 :" +num2str(ye2), 'Location', 'north')
```



```

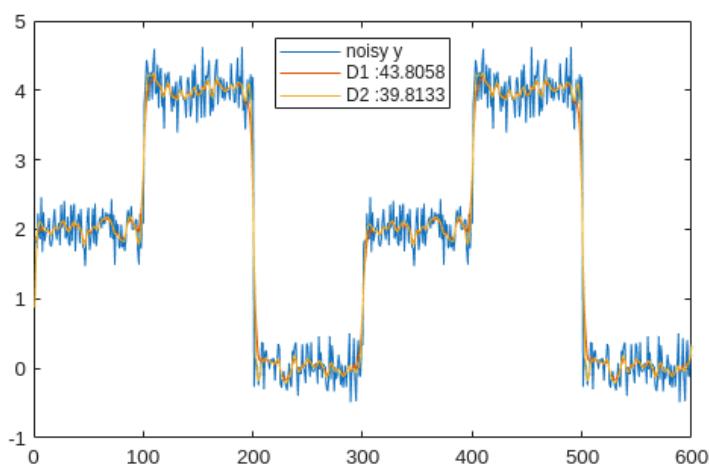
n = 100;

y0 = 0.25*randn(1,n);
y4 = 2 + 0.25*randn(1,n);
y5 = 4 + 0.25*randn(1,n);

y = [y4 y5 y0 y4 y5 y0];
y = y(:);

[ye1,y1] = ADMMdenoise(y,1.5,4,1);
[ye2,y2] = ADMMdenoise(y,1.5,4,2);
plot(y); hold on
plot(y1);
plot(y2); hold off
legend("noisy y", "D1 :" +num2str(ye1), "D2 :" +num2str(ye2), 'location', 'north')

```



```

n = 150;
y1 = linspace(0,2,n) + 0.25*randn(1,n);

```

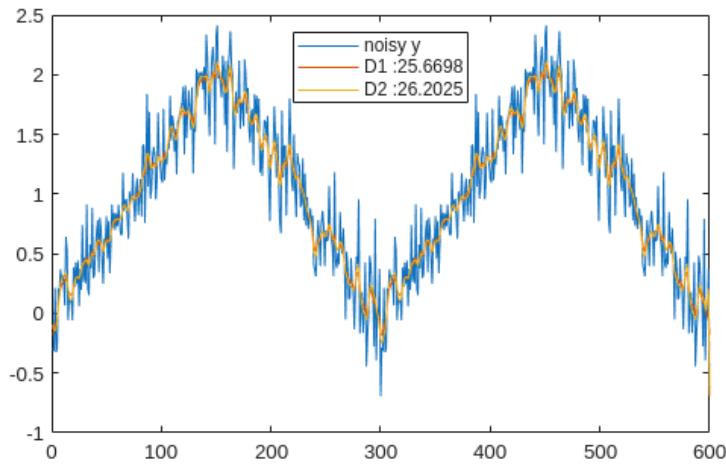
```

y2 = linspace(2,0,n) + 0.25*randn(1,n);

y = [y1 y2 y1 y2];
y = y(:);

[yel,y1] = ADMMdenoise(y,1.5,4,1);
[ye2,y2] = ADMMdenoise(y,1.5,4,2);
plot(y); hold on
plot(y1);
plot(y2); hold off
legend("noisy y", "D1 :" + num2str(yel), "D2 :" + num2str(ye2), 'Location', 'north')

```

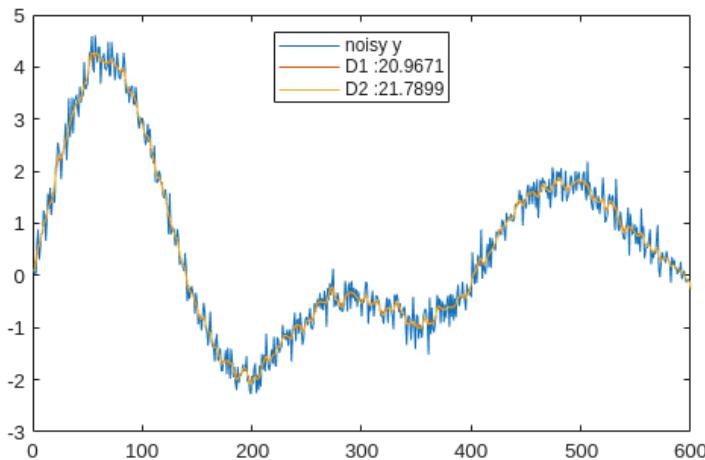


```

n = 600;
t = linspace(0,2,n);
A = [2 1.5 1];
f = [0.8 1.2 1.5];
y = sum(A.*sin(2*pi*t'*f),2) + 0.25*randn(n,1);

[yel,y1] = ADMMdenoise(y,1.5,4,1);
[ye2,y2] = ADMMdenoise(y,1.5,4,2);
plot(y); hold on
plot(y1);
plot(y2); hold off
legend("noisy y", "D1 :" + num2str(yel), "D2 :" + num2str(ye2), 'Location', 'north')

```

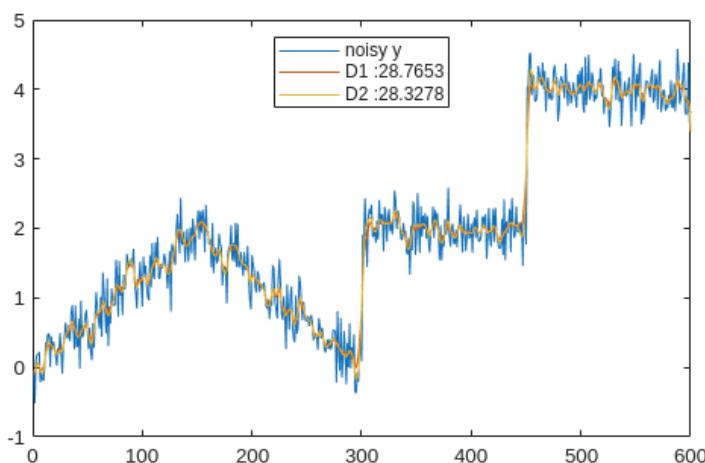


```

n = 150;
y1 = linspace(0,2,n) + 0.25*randn(1,n);
y2 = linspace(2,0,n) + 0.25*randn(1,n);
y4 = 2 + 0.25*randn(1,n);
y5 = 4 + 0.25*randn(1,n);
y = [y1 y2 y4 y5];
y = y(:);

[ye1,y1] = ADMMdenoise(y,1.5,4,1);
[ye2,y2] = ADMMdenoise(y,1.5,4,2);
plot(y); hold on
plot(y1);
plot(y2); hold off
legend("noisy y", "D1 :" + num2str(ye1), "D2 :" + num2str(ye2), 'Location', 'north')

```



```

function [Yerr, Ysmooth] = ADMMdenoise(ynoisy,r,s,D)
rho = r;
sigma = s;

```

```

y = ynoisy;
max_Iter = 25;
n = length(y);

z = rand(n,1);
u = rand(n,1);
e = ones(n,1); % vector of ones
if(D == 1)
    D = spdiags([e -e], 0:1, n-1, n); % sparse format
elseif(D == 2)
    D = spdiags([e -2*e e], -1:1, n-2, n); % sparse format
end

B1 = pinv(sigma*(D'*D)/rho+eye(n));

for i=1:max_Iter
    x1 = (y + rho*(z-u))/(1+rho);
    z1 = B1*(x1+u);
    u1 = u + (x1 - z1);

    u = u1;
    z = z1;
end
Ysmooth = x1;
Yerr = sum((y-x1).^2);
end

```

```

if(PUBLISH == ready)
    path = '/media/user/DATA4LINUX/new1/Repos/Mine/MFC4_22MAT230/';
    mlxfile = matlab.desktop.editor.getActive().Filename;
    [~, name, ext] = fileparts(mlxfile);
    outfile = [path, name, ext, '.pdf']
    export(matlab.desktop.editor.getActive().Filename, outfile);
    if ispc
        winopen(outfile);
    elseif ismac
        system(['open ' char(outfile)]);
    else
        system("env -u LD_LIBRARY_PATH xdg-open '" + outfile + "' &");
    end
end

outfile =
'/media/user/DATA4LINUX/new1/Repos/Mine/MFC4_22MAT230/U2_Signal_Denoising_ADMM_D1_D2_function mlx.pdf'

```