Unit 1 - Sem 4 - 22MAT230

Mathematics for Computing 4

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If you find any mistakes or have any comments to share,

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https://github.com/mfcpjt/MFC4_22MAT230

Signal Denoising using ADMM

```
clearvars
ready = true;
PUBLISH = ready;
```

$$\min_{\mathbf{x}} \underbrace{\frac{1}{2}\|\mathbf{x}-\mathbf{y}\|_2^2}_{\text{fidelity term}} + \underbrace{\frac{\sigma}{2}\|D_1\mathbf{x}\|_2^2}_{\text{regularization term}}$$

$\mathbf{x}$ →denoised signal

$\mathbf{y}$ →noisy signal

D_1 first difference matrix

$$\min_{\mathbf{x}} \underbrace{\frac{1}{2}\|\mathbf{x}-\mathbf{y}\|_2^2}_{\text{fidelity term}} + \underbrace{\frac{\sigma}{2}\|D_2\mathbf{x}\|_2^2}_{\text{regularization term}}$$

$\mathbf{x}$ →denoised signal

$\mathbf{y}$ →noisy signal

D_2 second difference matrix

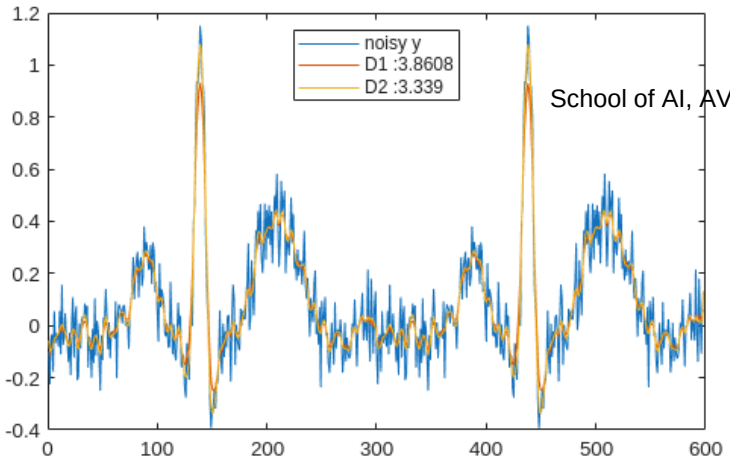
ADMM update equations

$$\mathbf{x}^{(k+1)} = \frac{1}{(1+\rho)}(\mathbf{y} + \rho(\mathbf{z}^{(k)} - \mathbf{u}^{(k)}))$$

$$\mathbf{z}^{(k+1)} = \rho(\sigma D^T D + I\rho)^{-1}(\mathbf{x}^{(k+1)} + \mathbf{u}^{(k)})$$

$$\mathbf{u}^{(k+1)} = \mathbf{u}^{(k)} + (\mathbf{x}^{(k+1)} - \mathbf{z}^{(k+1)})$$

```
cd("/media/user/DATA4LINUX/new1/Repos/Mine/DATAmat/")
load("NoisyECG.mat")
[ye1,y1] = ADMMdenoise([y; y],1.5,4,1);
[ye2,y2] = ADMMdenoise([y; y],1.5,4,2);
plot([y; y]); hold on
plot(y1);
plot(y2); hold off
legend("noisy y","D1 :"+num2str(ye1),"D2 :"+num2str(ye2),'Location','north')
```



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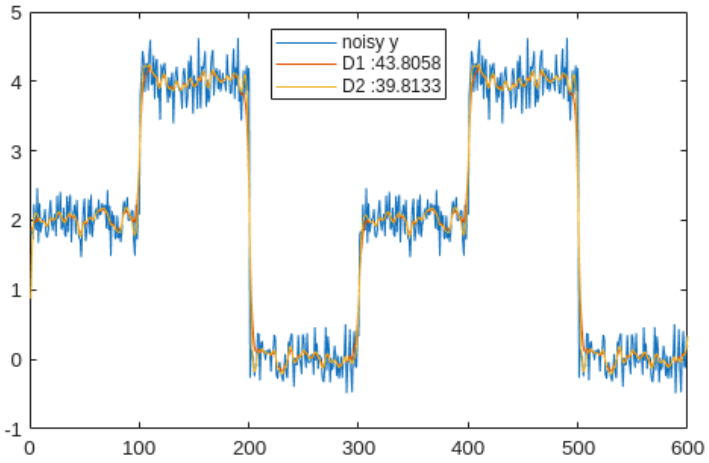
```
n = 100;

y0 = 0.25*randn(1,n);
y4 = 2 + 0.25*randn(1,n);
y5 = 4 + 0.25*randn(1,n);

y = [y4 y5 y0 y4 y5 y0];
y = y(:);

[ye1,y1] = ADMMdenoise(y,1.5,4,1);
[ye2,y2] = ADMMdenoise(y,1.5,4,2);
plot(y); hold on
plot(y1);
plot(y2); hold off
legend("noisy y","D1 :"+num2str(ye1),"D2 :"+num2str(ye2),'location','north')
```



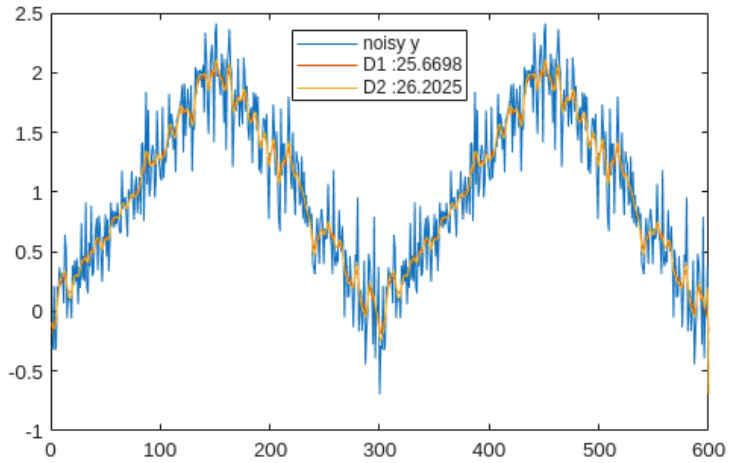

```
n = 150;
y1 = linspace(0,2,n) + 0.25*randn(1,n);
```

```
y2 = linspace(2,0,n) + 0.25*randn(1,n);

y = [y1 y2 y1 y2];
y = y(:);
```

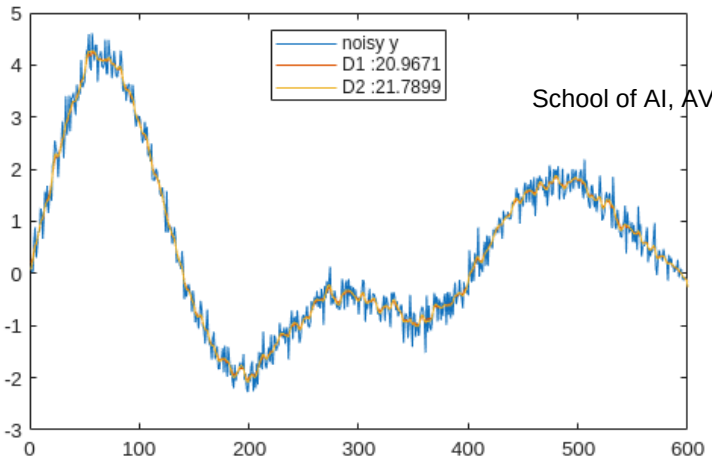


```
[ye1,y1] = ADMMdenoise(y,1.5,4,1);
[ye2,y2] = ADMMdenoise(y,1.5,4,2);
plot(y); hold on
plot(y1);
plot(y2); hold off
legend("noisy y","D1 :"+num2str(ye1),"D2 :"+num2str(ye2),'Location','north')
```




```
n = 600;
t = linspace(0,2,n);
A = [2 1.5 1];
f = [0.8 1.2 1.5];
y = sum(A.*sin(2*pi*t'*f),2) + 0.25*randn(n,1);

[ye1,y1] = ADMMdenoise(y,1.5,4,1);
[ye2,y2] = ADMMdenoise(y,1.5,4,2);
plot(y); hold on
plot(y1);
plot(y2); hold off
legend("noisy y","D1 :"+num2str(ye1),"D2 :"+num2str(ye2),'Location','north')
```



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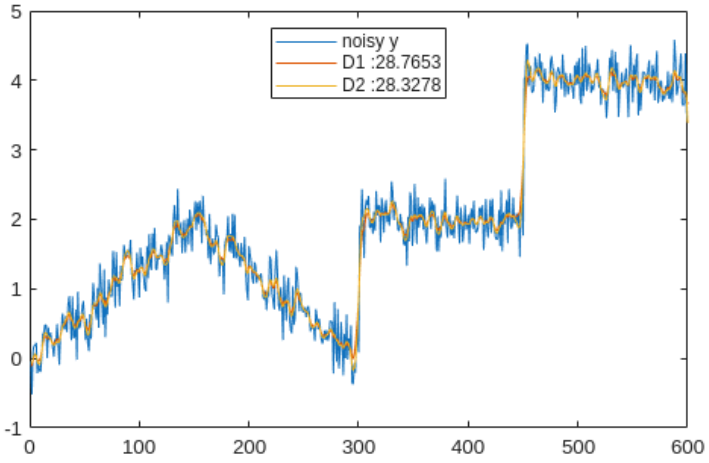
```
n = 150;
y1 = linspace(0,2,n) + 0.25*randn(1,n);
y2 = linspace(2,0,n) + 0.25*randn(1,n);
y4 = 2 + 0.25*randn(1,n);
y5 = 4 + 0.25*randn(1,n);
y = [y1 y2 y4 y5];
y = y(:);

[ye1,y1] = ADMMdenoise(y,1.5,4,1);
[ye2,y2] = ADMMdenoise(y,1.5,4,2);
plot(y); hold on
plot(y1);
plot(y2); hold off
legend("noisy y","D1 :"+num2str(ye1),"D2 :"+num2str(ye2),'Location','north')
```




```
function [Yerr, Ysmooth] = ADMMdenoise(ynoisy,r,s,D)
    rho = r;
    sigma = s;
```

```
    y = ynoisy;
    max_Iter = 25;
    n = length(y);
```



```
    z = rand(n,1);
    u = rand(n,1);
    e = ones(n,1); % vector of ones
    if(D == 1)
        D = spdiags([e -e], 0:1, n-1, n); % sparse format
    elseif(D == 2)
        D = spdiags([e -2*e e], -1:1, n-2, n); % sparse format
    end

    B1 = pinv(sigma*(D'*D)/rho+eye(n));

    for i=1:max_Iter
        x1 = (y + rho*(z-u))/(1+rho);
        z1 = B1*(x1+u);
        u1 = u + (x1 - z1);

        u = u1;
        z = z1;
    end
    Ysmooth = x1;
    Yerr = sum((y-x1).^2);
end
```

```
if(PUBLISH == ready)
    path = '/media/user/DATA4LINUX/new1/Repos/Mine/MFC4_22MAT230/';
    mlxfile = matlab.desktop.editor.getActive().Filename;
    [~, name, ext] = fileparts(mlxfile);
    outfile = [path, name, ext,'.pdf']
    export(matlab.desktop.editor.getActive().Filename, outfile);
    if ispc
        winopen(outfile);
    elseif ismac
        system(['open ' char(outfile)]);
    else
        system("env -u LD_LIBRARY_PATH xdg-open '" + outfile + "' &");
    end
end
```

```
outfile =
'/media/user/DATA4LINUX/new1/Repos/Mine/MFC4_22MAT230/U2_Signal_Denoising_ADMM_D1_D2_function.mlx.pdf'
```