AMRITA VISHWA VIDYAPEETHAM

DEEMED TO BE UNIVERSITY UNDER SECTION 3 OF UGC ACT, 1956

Unit 1 - Sem 4 - 22MAT230

Mathematics for Computing 4

Dr Sunil Kumar S and Prof K P Soman

School of Artificial Intelligence

Amrita Vishwa Vidyapeetham

If you find any mistakes or have any comments to share,

I would be grateful to receive them at s_sunilkumar@cb.amrita.edu

https://github.com/mfcpjt/MFC4_22MAT230

Signal Denoising using ADMM

```
clearvars
cd("/media/user/DATA4LINUX/new1/Repos/Mine/DATAmat/")
load("NoisyECG.mat")
```

$$\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{x}-\mathbf{y}\|_2^2 + \frac{\sigma}{2}\|D\mathbf{x}\|_2^2$$

Two competing terms

$\mathbf{y} \rightarrow$ noisy signal

$\mathbf{x} \rightarrow$reconstructed smooth signal

$$f(\mathbf{x}) = \frac{1}{2}\|\mathbf{x}-\mathbf{y}\|_2^2$$

$$g(\mathbf{x}) = \frac{\sigma}{2}\|D\mathbf{x}\|_2^2$$

ADMM formulation

$$L_\rho(\mathbf{x}, \mathbf{z}, \mathbf{u}) = f(\mathbf{x}) + g(\mathbf{z}) + \frac{\rho}{2}\|\mathbf{x} - \mathbf{z} + \mathbf{u}\|_2^2$$

Update equations

$\mathbf{x}^{(k+1)} = \arg\min_{\mathbf{x}} f(\mathbf{x}) + \frac{\rho}{2}\|\mathbf{x} - \mathbf{z}^{(k)} + \mathbf{u}^{(k)}\|_2^2$, taking derivative and equating to zero, we get

$$\mathbf{x} - \mathbf{y} + \rho(\mathbf{x} - \mathbf{z}^{(k)} + \mathbf{u}^{(k)}) = \mathbf{0}$$

$$\Rightarrow (1 + \rho)\mathbf{x}^{(k+1)} = \mathbf{y} + \rho(\mathbf{z}^{(k)} - \mathbf{u}^{(k)})$$

$$\mathbf{x}^{(k+1)} = \frac{1}{(1 + \rho)}(\mathbf{y} + \rho(\mathbf{z}^{(k)} - \mathbf{u}^{(k)}))$$

$\mathbf{z}^{(k+1)} = \arg\min_{\mathbf{z}} g(\mathbf{z}) + \frac{\rho}{2}\|\mathbf{x} - \mathbf{z}^{(k)} + \mathbf{u}^{(k)}\|_2^2$, taking derivative and equating to zero, we get

$$\sigma D^T D\mathbf{z} - \rho(\mathbf{x}^{(k+1)} - \mathbf{z} + \mathbf{u}^{(k)}) = \mathbf{0}$$

$$\Rightarrow \sigma D^T D\mathbf{z} + \rho\mathbf{z} = \rho(\mathbf{x}^{(k+1)} + \mathbf{u}^{(k)})$$

$$(\sigma D^T D + I\rho)\mathbf{z} = \rho(\mathbf{x}^{(k+1)} + \mathbf{u}^{(k)})$$

$$\mathbf{z}^{(k+1)} = \rho(\sigma D^T D + I\rho)^{-1}(\mathbf{x}^{(k+1)} + \mathbf{u}^{(k)})$$

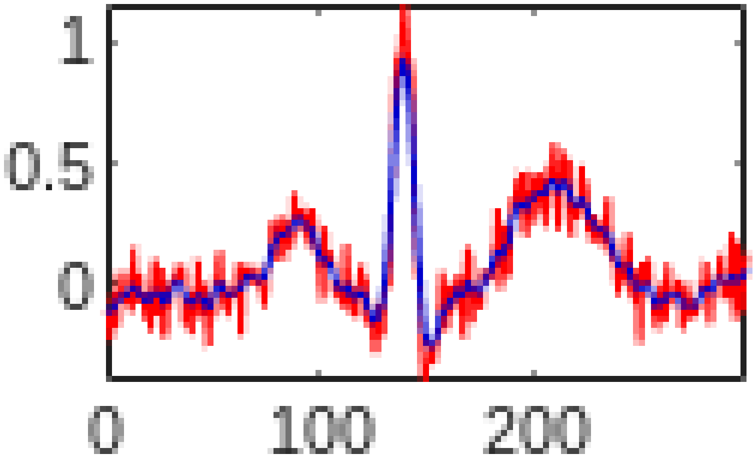
$$\mathbf{u}^{(k+1)} = \mathbf{u}^{(k)} + (\mathbf{x}^{(k+1)} - \mathbf{z}^{(k+1)})$$

```
rho = 1.1;
sigma = 4;

max_Iter = 25;
n = length(y);

x = rand(n,1);
z = rand(n,1);
u = rand(n,1);
e = ones(n, 1); % vector of ones
D = spdiags([e -e], 0:1, n-1, n); % sparse format - 1st order difference matrix
%  full(D)  % uncomment this line to see the full matrix format
B1 = pinv(sigma*(D'*D)/rho+eye(n));
for i=1:max_Iter
    x1 = (y + rho*(z-u))/(1+rho);
    z1 = B1*(x1+u);
    u1 = u + (x1 - z1);

    u = u1;
    z = z1;
end
plot(x1,Color=[0 0 1]); hold on
plot(y,Color=[1 0 0]);
plot(z1,Color=[0 0 0.8]); hold off
```



References

https://www.math.mcgill.ca/yyang/soft.html

https://www.diva-portal.org/smash/get/diva2:560977/FULLTEXT01.pdf

https://arxiv.org/pdf/2309.05925

https://dl.acm.org/doi/pdf/10.1145/3409073.3409077

https://web.stanford.edu/~boyd/papers/admm/

```
path = "/media/user/DATA4LINUX/new1/Repos/Mine/MFC4_22MAT230/";
```

```
path = 
"/media/user/DATA4LINUX/new1/Repos/Mine/MFC4_22MAT230/"
```

```
cd(path)
mlxfile = matlab.desktop.editor.getActive().Filename;
outfile = mlxfile + ".pdf";
export(matlab.desktop.editor.getActive().Filename, outfile);
```

```
if ispc
    winopen(outfile);
elseif ismac
    system(['open ' char(outfile)]);
else
    system("env -u LD_LIBRARY_PATH xdg-open '" + outfile + "' &");
end
```