Flight overbooking

= Class starts @ 9:03 PM =

(*) Excel & some tableau lectures added as recorded content for those cohorts for whom they are not covered

Flight overbooking

Flight : 100 seats

overbook : 102 (>100) tickets

Cost per ticket = 5,000 Rs

Some times : ≤ 100 passengers ✓

101, 102 ⟵ reject from boarding

2 additional passengers

↓

Rs 10K inconvinience renumeration per passenger

↓

penalty → loss to the airline

Problem: How many tickets should you sell to maximize your expected profit

↓

100 : capacity

5,000 : Ticket price

10,000 : penalty

Real-world: Lufthansa

2005 → 570,000 additional tickets

↓

$105 million

Additional data :

SIMPLER VERSION

(1) Probability of not showing up

↳ 10% (historically)

Idea 1: Overbook upto 100 + 10%
= 110 (?)

Extreme Cases:

if Penalty = 0 ; overbook $\rightarrow \infty$

if penalty = 5000 × 100 ; overbook $\rightarrow 0$
(V. high)

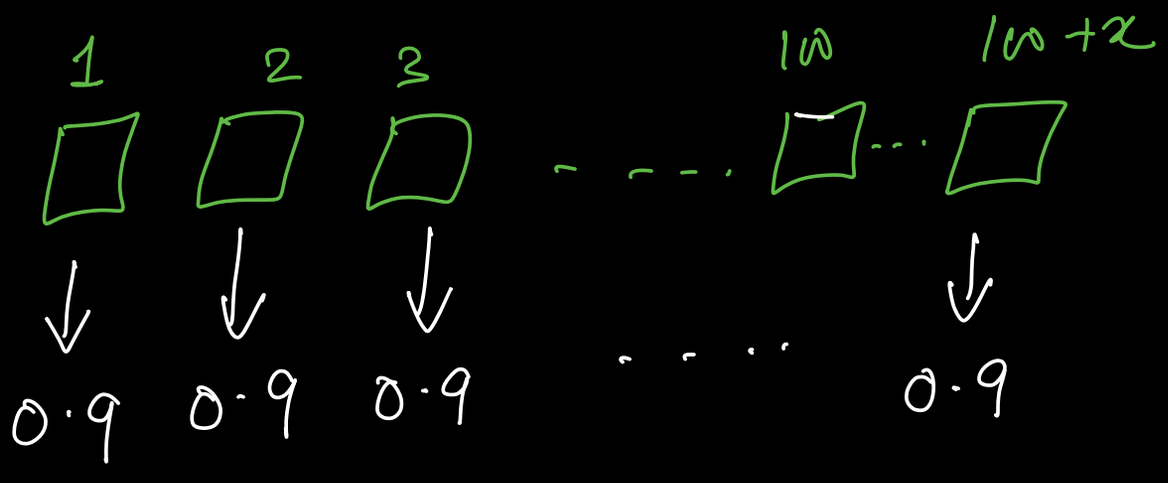
as penalty ↑ # overbookings ($\infty \rightarrow 0$)

100 → seats

$P(\text{one passenger not showing up}) = 0.1$

Let's assume we sold $(100 + x)$ tickets

$$P(100 \text{ people show up}) = {}^{100+x}C_{100}\, 0.9^{100}\, 0.1^{x}$$



Let Y = # people who show up

① $P(Y = 100) = {}_{100+x}C_{100} \; 0.9^{100} \; 0.1^{x} \rightarrow 0$

P_0

② $P(Y = 101) = {}_{100+x}C_{101} \; 0.9^{101} \; 0.1^{100+x-101} \rightarrow 10k$

P_1

$\vdots$

∘ $P(Y = 100+x) = {}_{100+x}C_{100+x} \; 0.9^{100+x} \; 0.1^{0} \rightarrow 10k$ $\ast x$

P_x

Expected Penalty

$$= \sum_{i=0}^{x} P_i \times (10K * i)$$

Revenue

$$(100 + x) * 500$$

Expected Profit $= (100 + x)\,500$

$$- \sum_{i=0}^{x} P_i\,(10K * i)$$

because of overbooking

dice

dice

$1 \rightarrow 1/6$

$2 \rightarrow 1/6$

$\vdots$

$6 \rightarrow 1/6$

$$\frac{1}{6} \times 1 + \frac{1}{6} \times 2 + \frac{1}{6} \times 3 + \cdots \frac{1}{6} \times 6$$

Find x that maximizes

$$f(x) = 5000\,(100 + x) - \sum_{i=0}^{x} P_i \quad (i \times 10K)$$

x : integer ≥ 0

SIMPLEST: $x = 0$

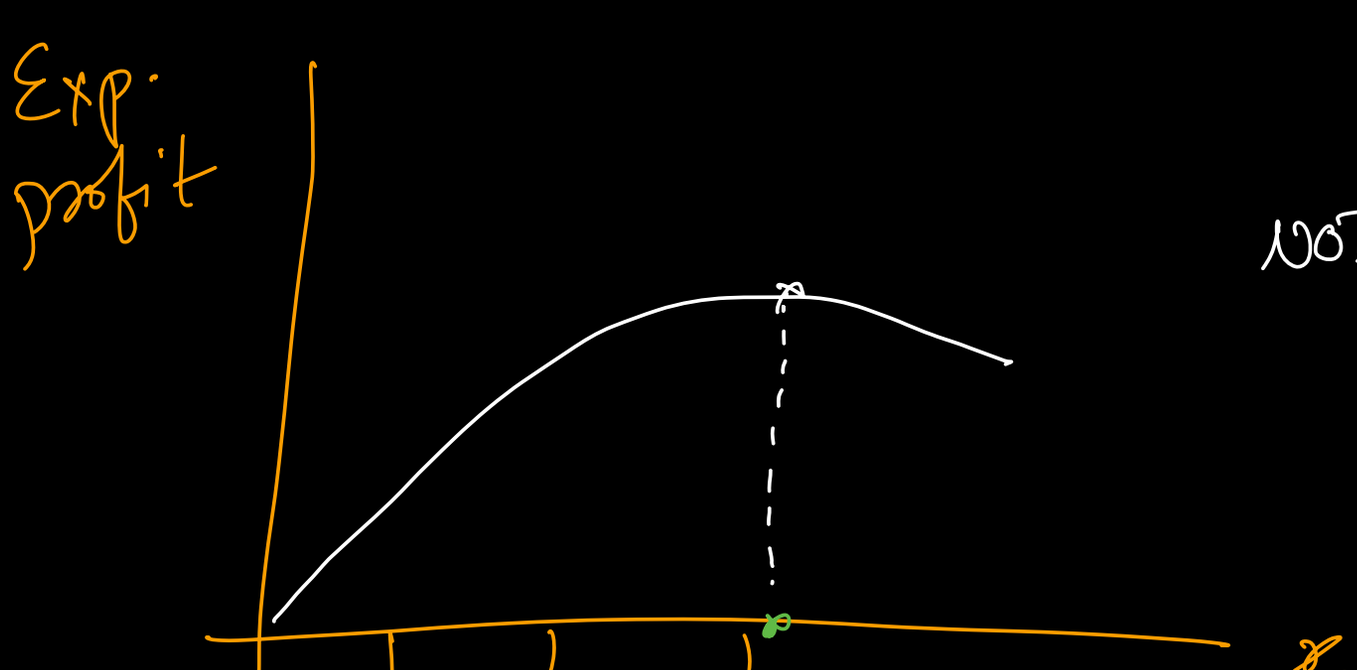
$x = 1$

$x = 2$

$\vdots$

Homework - 1

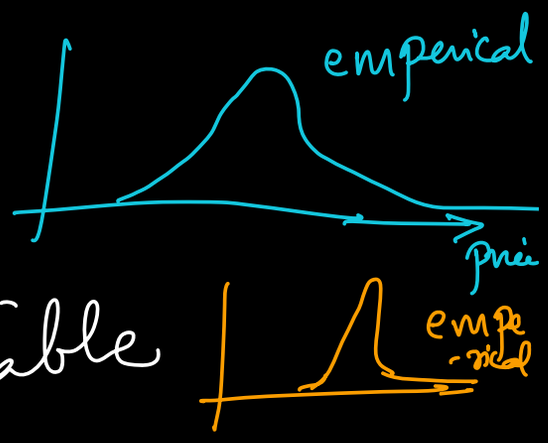
Python - Code



NOT a bell - curve

home work - 2: play with SDV & 10K Chages

What is wrong in the above analysis
(Short-Comings)

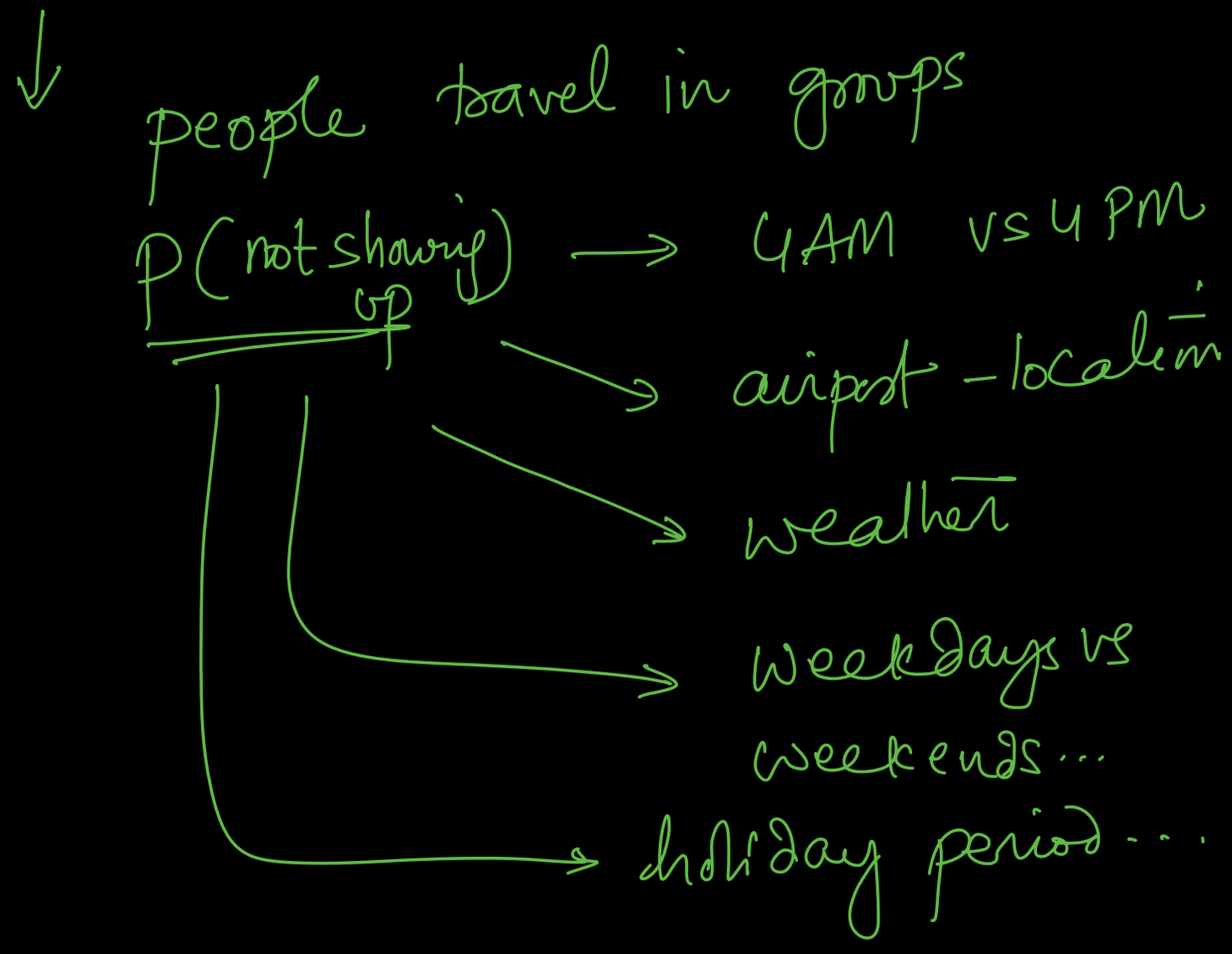


(1) Price is not fixed →

(2) Penalty is also a variable

(3) Probability of x additional passengers showing up is not binomial [why?]

↳ each passenger has to follow bernoulli dist with same 'p' & each have to be indep


people travel in groups
P(not showing up)
4AM vs 4PM
airport - location
weather
weekdays vs weekends...
holiday period...

P(single passenger not showing up)

Segment users → ..., business,

Segment time of day → ..., 8AM,

segment day of week → ..., Mon,

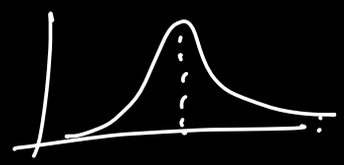
segment by weather → ..., Clear,

Segment by occasion → NO

segment by destination → NY

USING historical data

Now,



① Price: empirical dist (do not know the exact dist)

② Penalty: empirical dist

③ P(not showing up | passenger) = prob case

Expected profit due to overbooking:

Simulate: 1000's of times (Pick an x)

100 seats $+x$ overbook

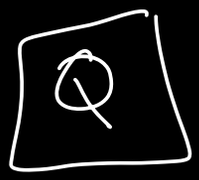
① pick a price from the dist → total ticket sales

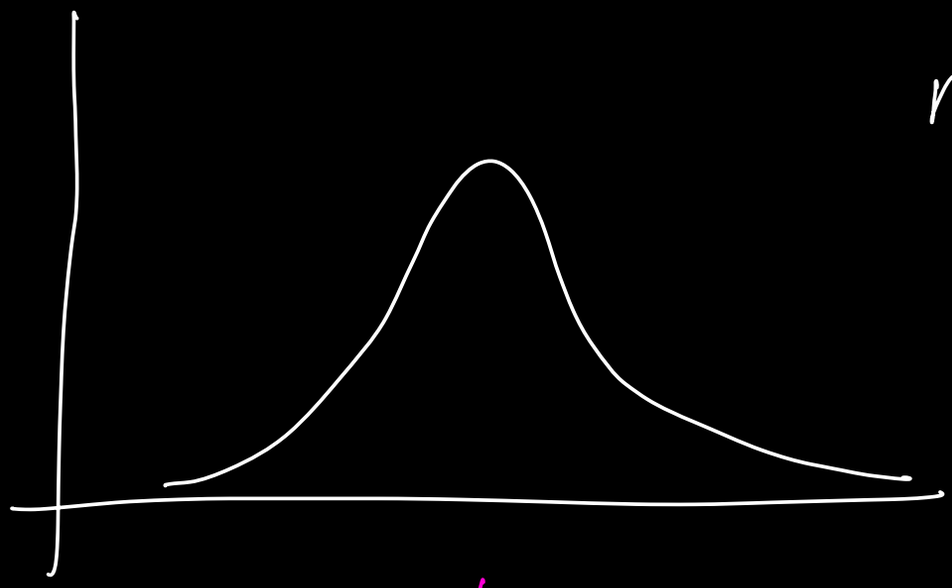
② pick $100+x$ passengers

↓

prob of not showing up

③ Expected # people who show up : 102

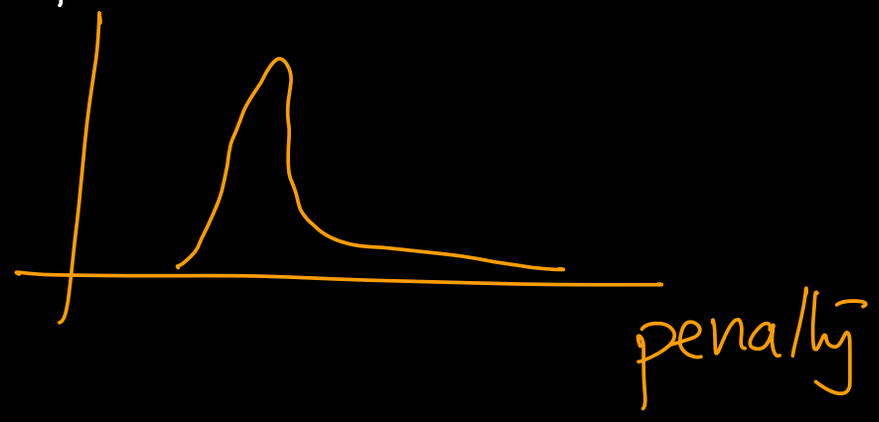




observed/empirical dist → historical

↳ home work: pick/sample random values from this distribution...

④ penalty : empirical dist



⑤ profit due to overbooking

Repeat steps 1 to 5 1000's of times → average

for x in 1 to 50:

for i in 1 to 1000:

→ pick (100+x) prices from price_empirical dist. Calculate total ticket sales

→ pick 100+x passengers. Compute individual P(not showing up using segments)

→ Compute probability of 101, 102 --- 100+x

passengers showing up

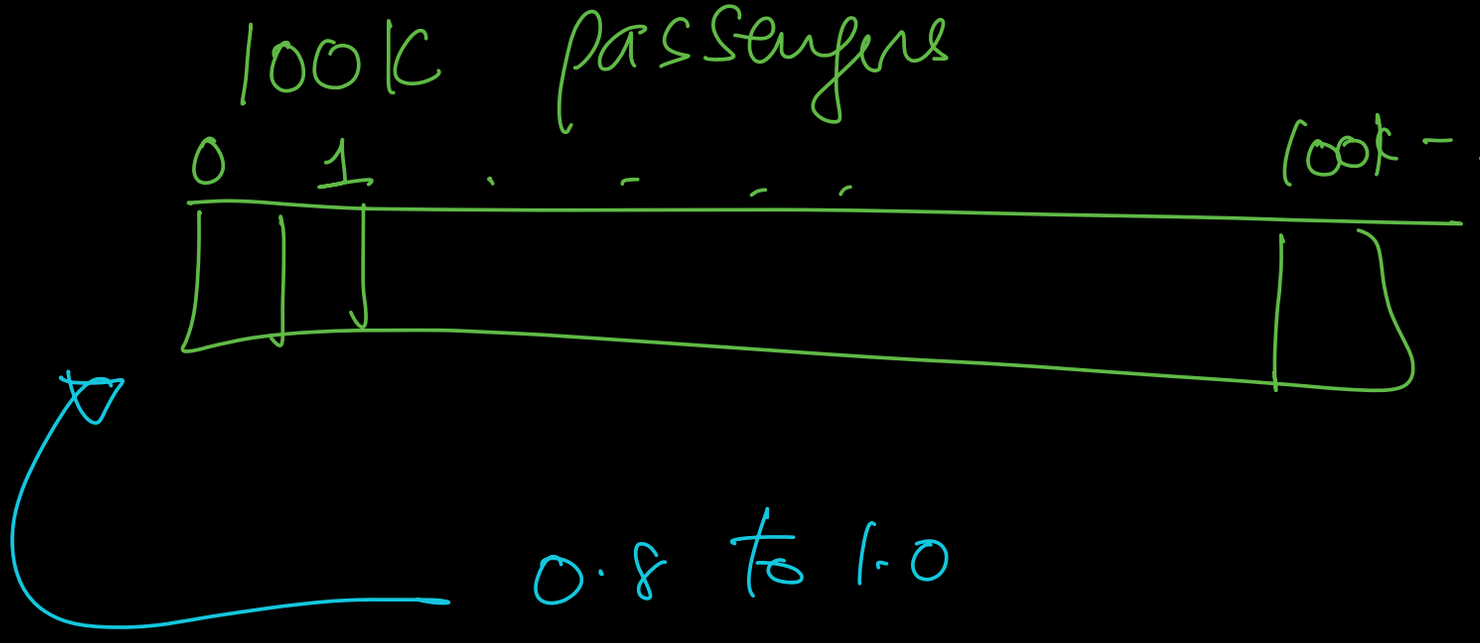
- pick from penalty empirical dist
- Expected penalty: compute
- Compute profit
- Take average of 1000 profits
  = Profit @ x over boolies

Pick x that has maximum (expected) profit

(e.g)

P1	P2	P3
↓	↓	↓
0.9	0.98	0.81

$$P(P1 \ \& \ P2 \ \& \ P3) = 0.9 \times 0.98 \times 0.81$$


100K passengers
0
1
100K - 1
0.8 to 1.0

Price dist:

(Let) Normal (5000, 1000)

Penalty dist

Uniform (8000, 12,000)