[Recap] → Q&A

Linear Algebra & co-ordinate Geometry

for ML*

Class starts @ 9:03 PM

(Q1) ORTHOGONAL VECTORS

90°

$x_1 \in \mathbb{R}^d$

$x_1 \perp x_2$ or not

$x_2 \in \mathbb{R}^d$

$x_1 \cdot x_2 = 0$

(Q1a) $\boxed{\text{WAF}}$

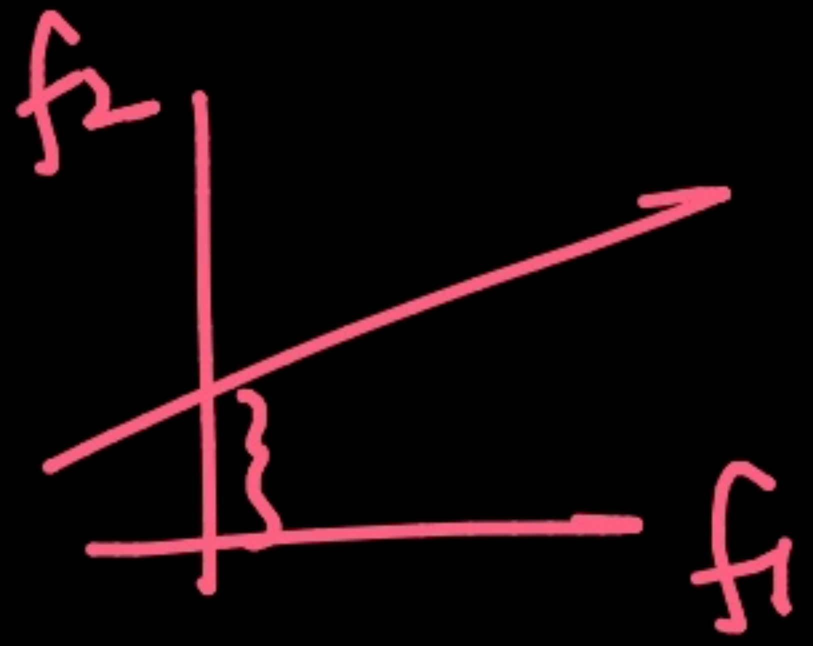
$$\sum_{i=1}^{d} x_{1i}\, x_{2i} = 0$$

$\theta = 90^\circ$

$$x_1 \cdot x_2 = \|x_1\| \, \|x_2\| \cos\theta$$

$$\hookrightarrow \sum_{i=1}^{d} x_{1i} \cdot x_{2i} \rightarrow 0$$

(Q2) Eqn. of π^d : $\vec{w}^T x + w_0 = 0$



$w \in \mathbb{R}^d$ w_0: scaler

$x \in \mathbb{R}^d$

(2a) if $w_0 = 0 \Rightarrow$ what does this mean?

↳ π^d passes through origin ✓

[why?] →

d-dim space

origin $0 = [0, 0, \dots, 0]$

$$w^T x + \underbrace{\cancel{w_0}}_{=0} = 0$$

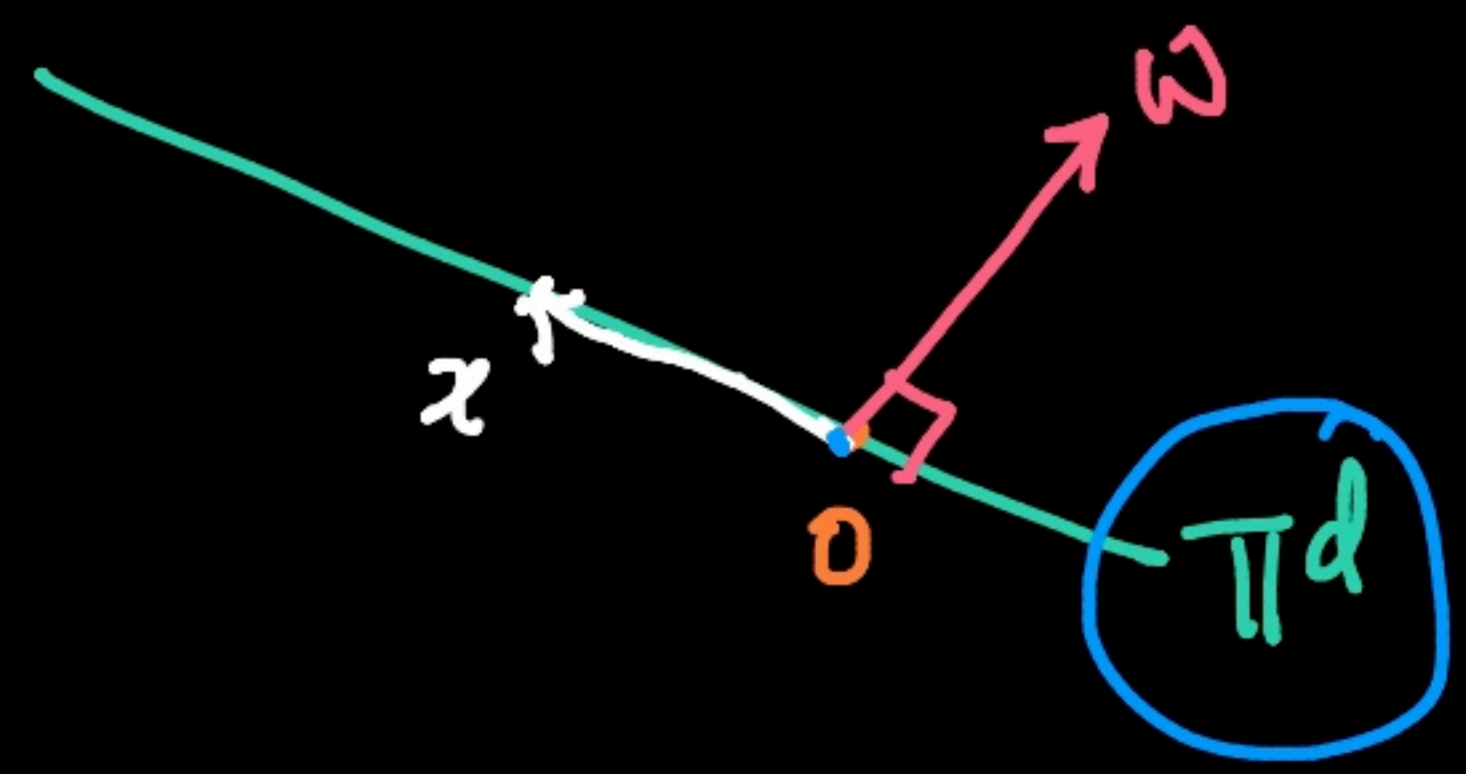
$$w^T x = 0$$

(Q3) what does w represent here?

π_d: $\left[\ \tilde{w}^T \cdot \underline{x} = 0\right.$

$\Rightarrow w \cdot x = 0$

x: any pt on the plane

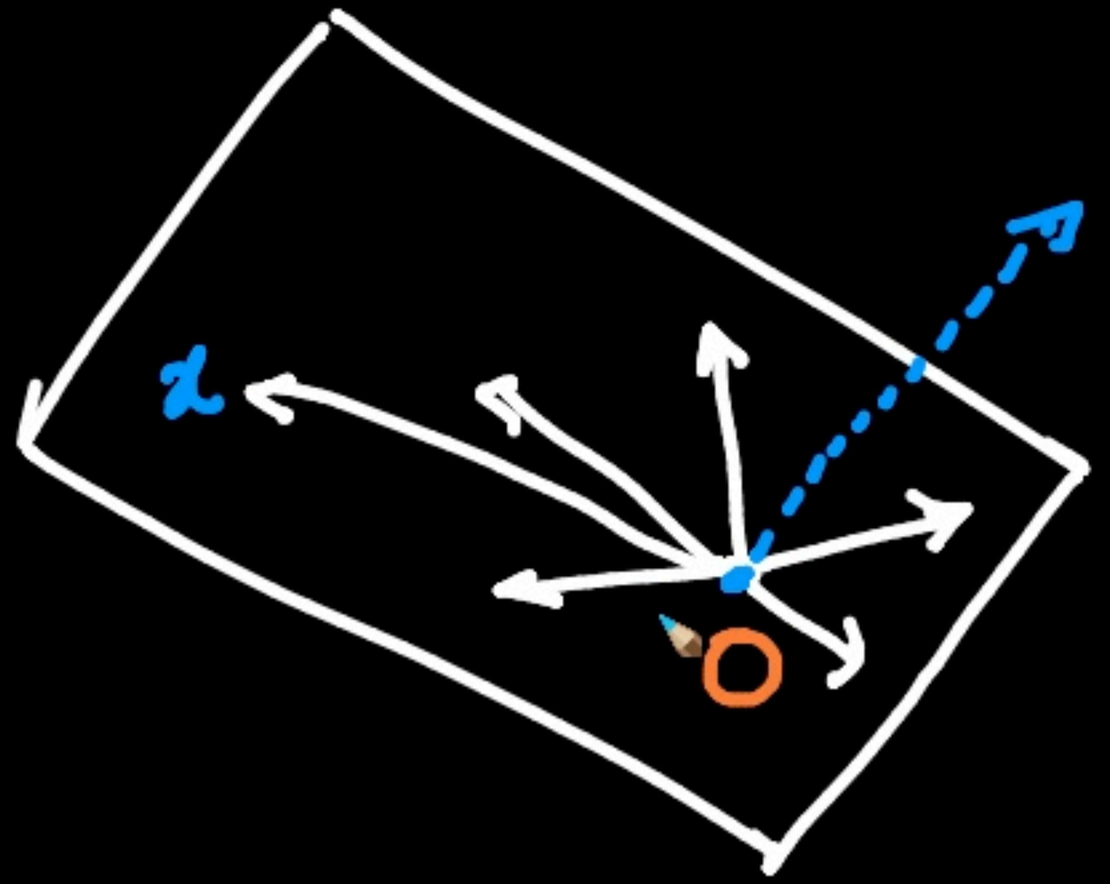


✓ $\left\{ \begin{array}{l} \rightarrow \underbrace{\underline{w} \text{ passes}} \text{ through origin} \\ \rightarrow \underline{w \perp \pi^d} \end{array} \right.$

(why?)

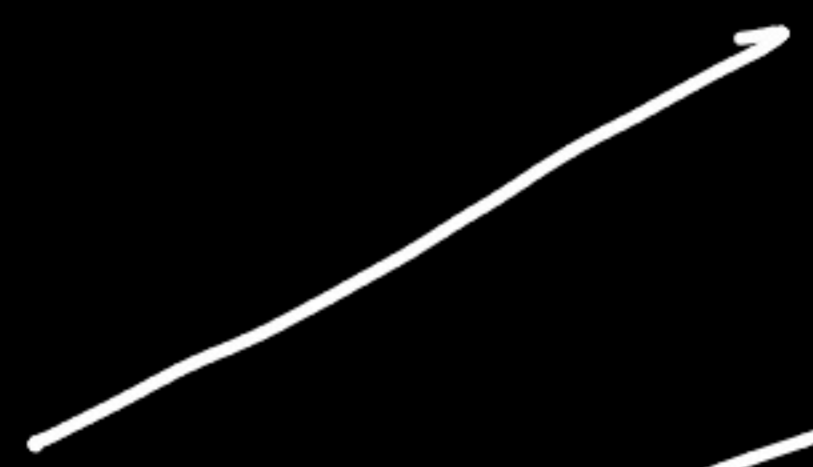
↓

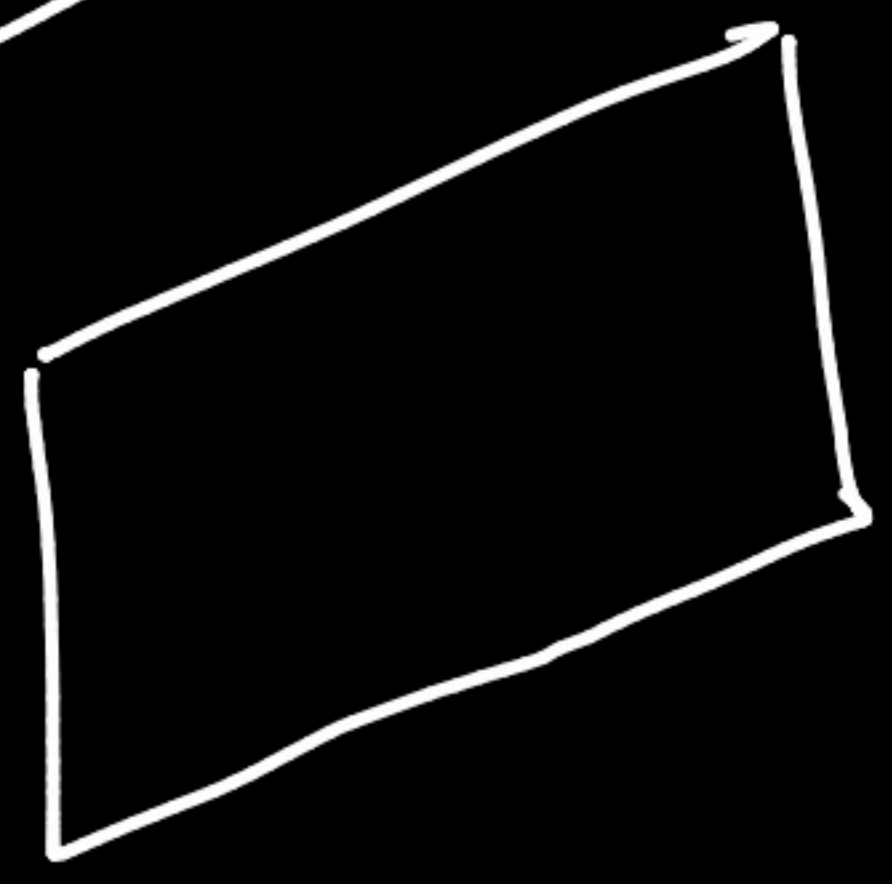
$w \perp$ any point on the π^d

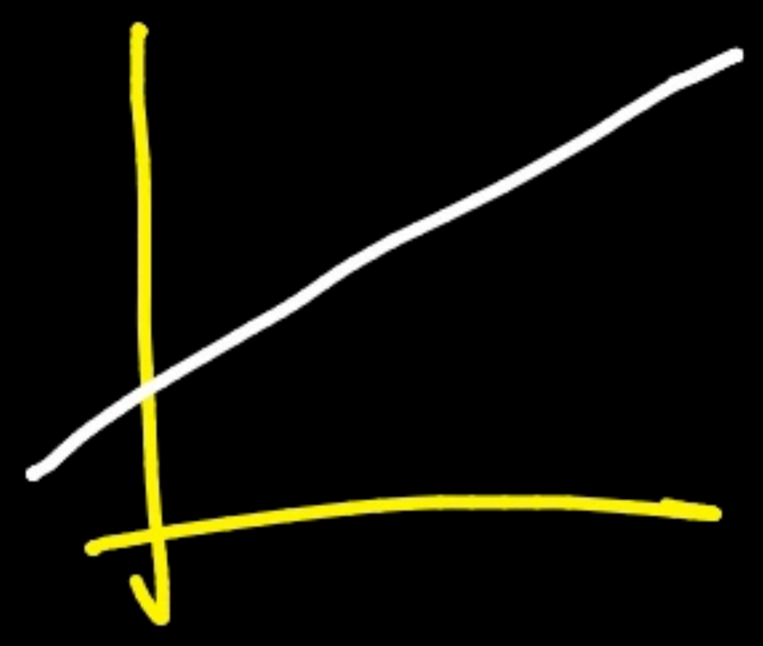


$\overbrace{\text{pt } x} \Rightarrow$ vec from origin to x

Hyper-plane:

2D:  line

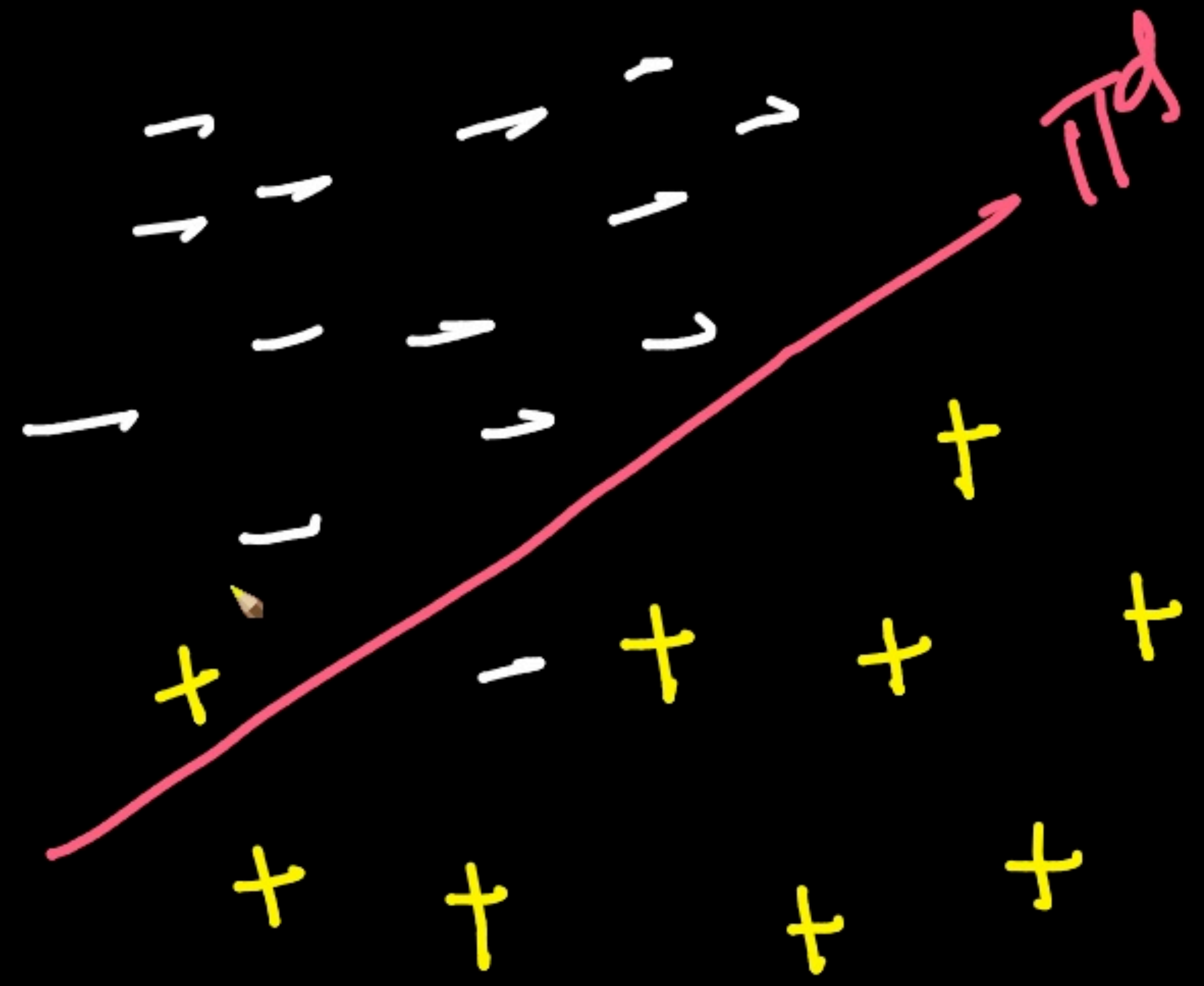
3D:  plane



(Q) 3D-space → can a line seperate the 3D-space

d-dim: hyper-plane

d-dim



(Q)

$w \in \mathbb{R}^d$

w_0 : scalar : $+2, -2, -1.5, +1.5$

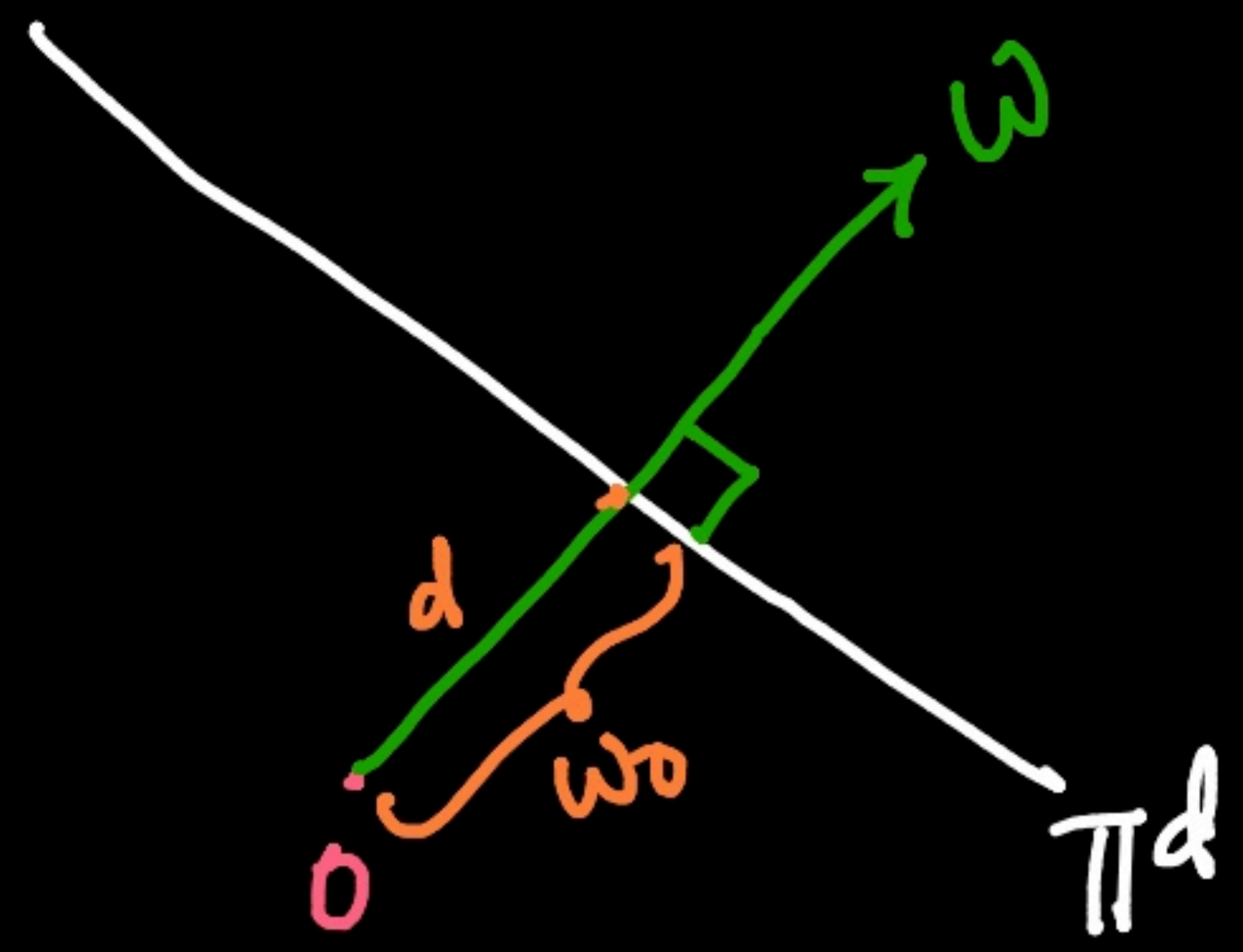
$\pi^d : w^T x + w_0 = 0$

Plane not passing through origin

$\Downarrow$

$w_0 \neq 0$

$d \propto w_0$



w: $w \perp \pi^d$ & w passes through O

w_0: $d = \dfrac{|w_0|}{\|w\|}$

(Q)

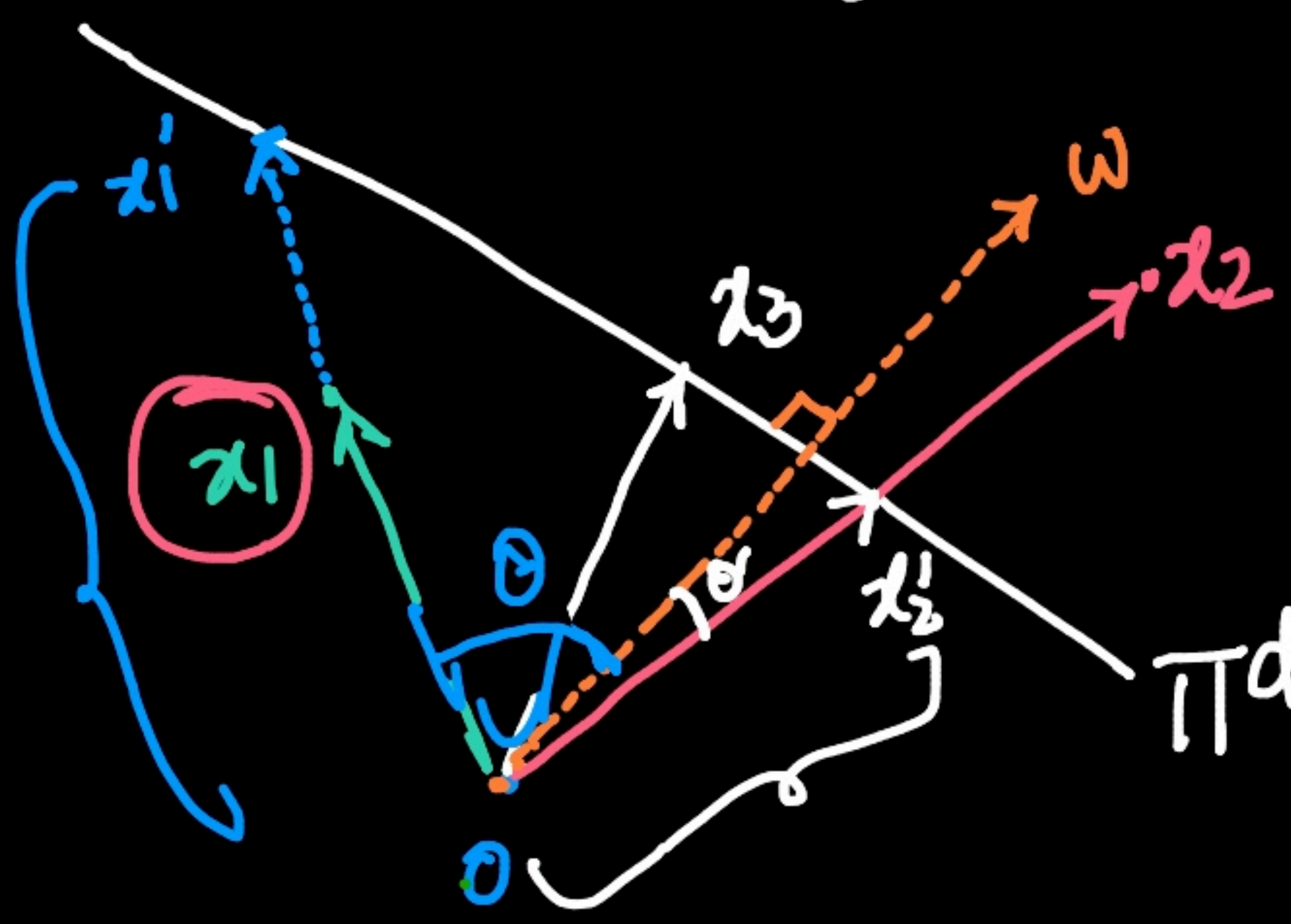
$$\underset{10}{\|x_1'\|} > \underset{6}{\|x_1\|}$$

$$\|w\|\,\underbrace{\|x_2'\|}_{6}\cos\theta' + w_0 = 0$$

$<, >, =$

$$w^T x_1 + w_0 \boxed{<} 0 \ ✓$$

why?



$$\pi^d: \underline{w}^T x + w_0 = 0$$

$\downarrow$

$\neq 0$

$$w^T x_1' + w_0 = 0$$

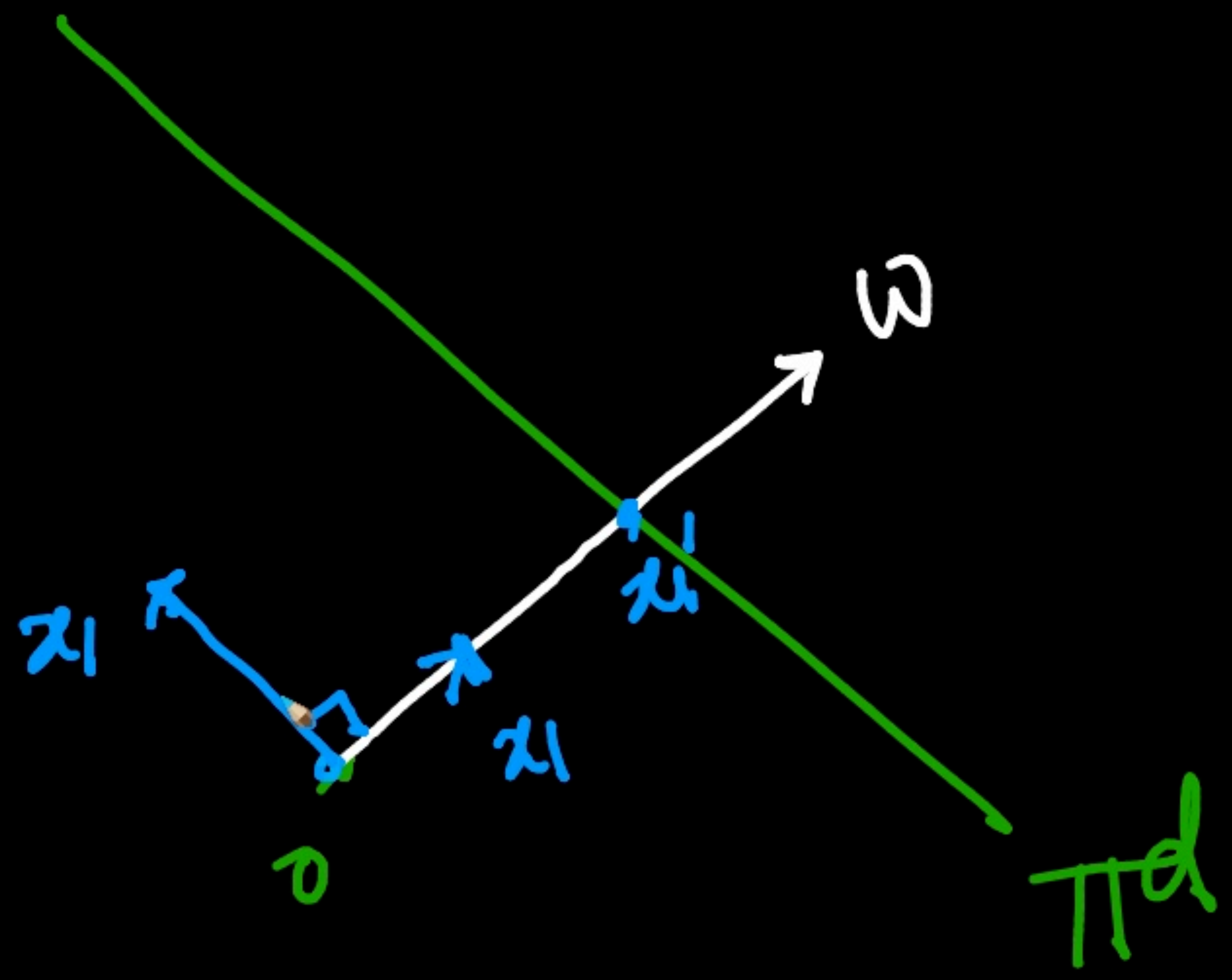
$\downarrow$

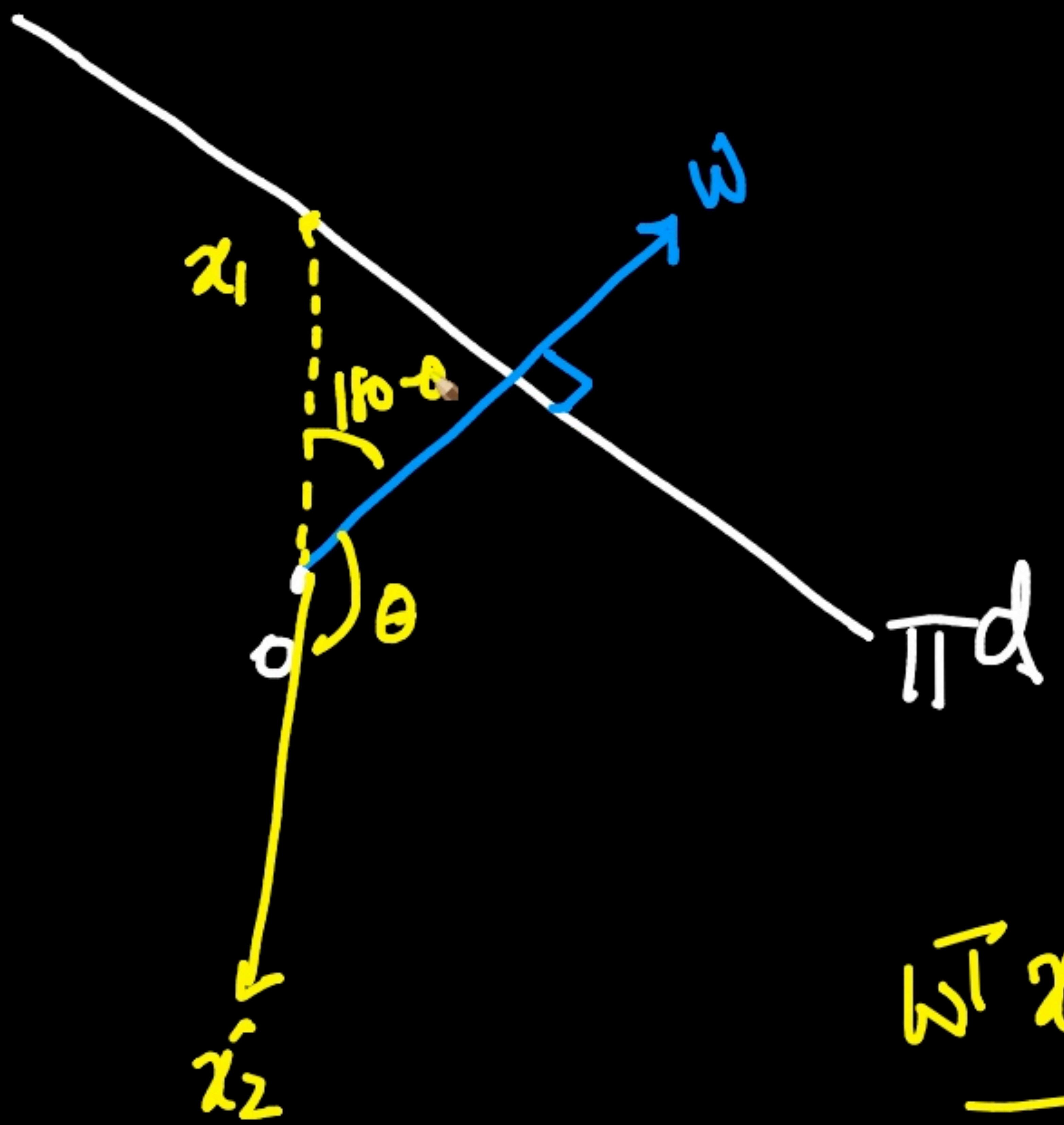
$$\|w\|\,\underbrace{\|x_1'\|}_{6}\cos\theta + w_0 = 0$$

$<, >, =$

$>, <, =$

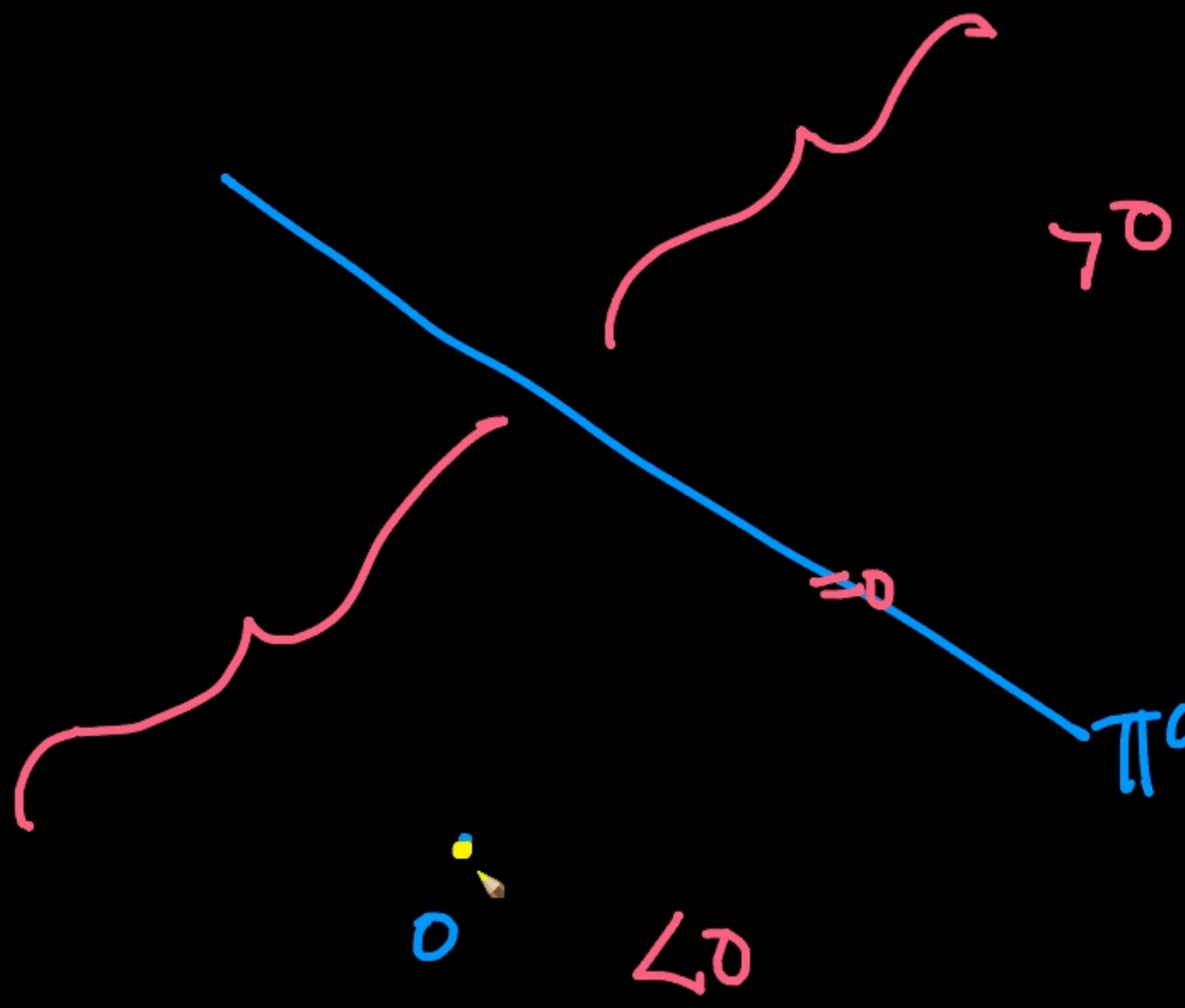
$$w^T x_2 + w_0 \boxed{>} 0$$


w
x1
x1
xi'
0
πd

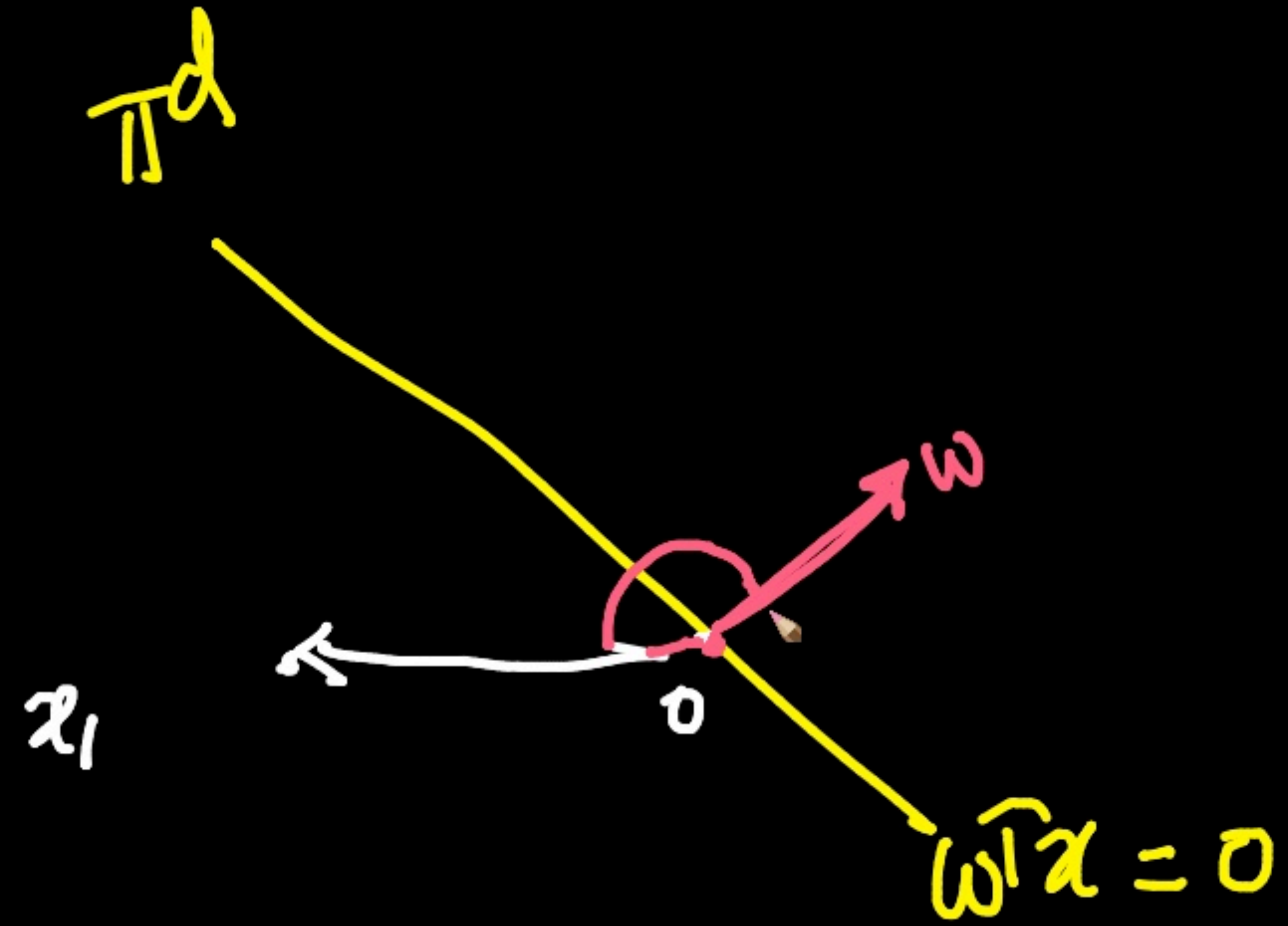


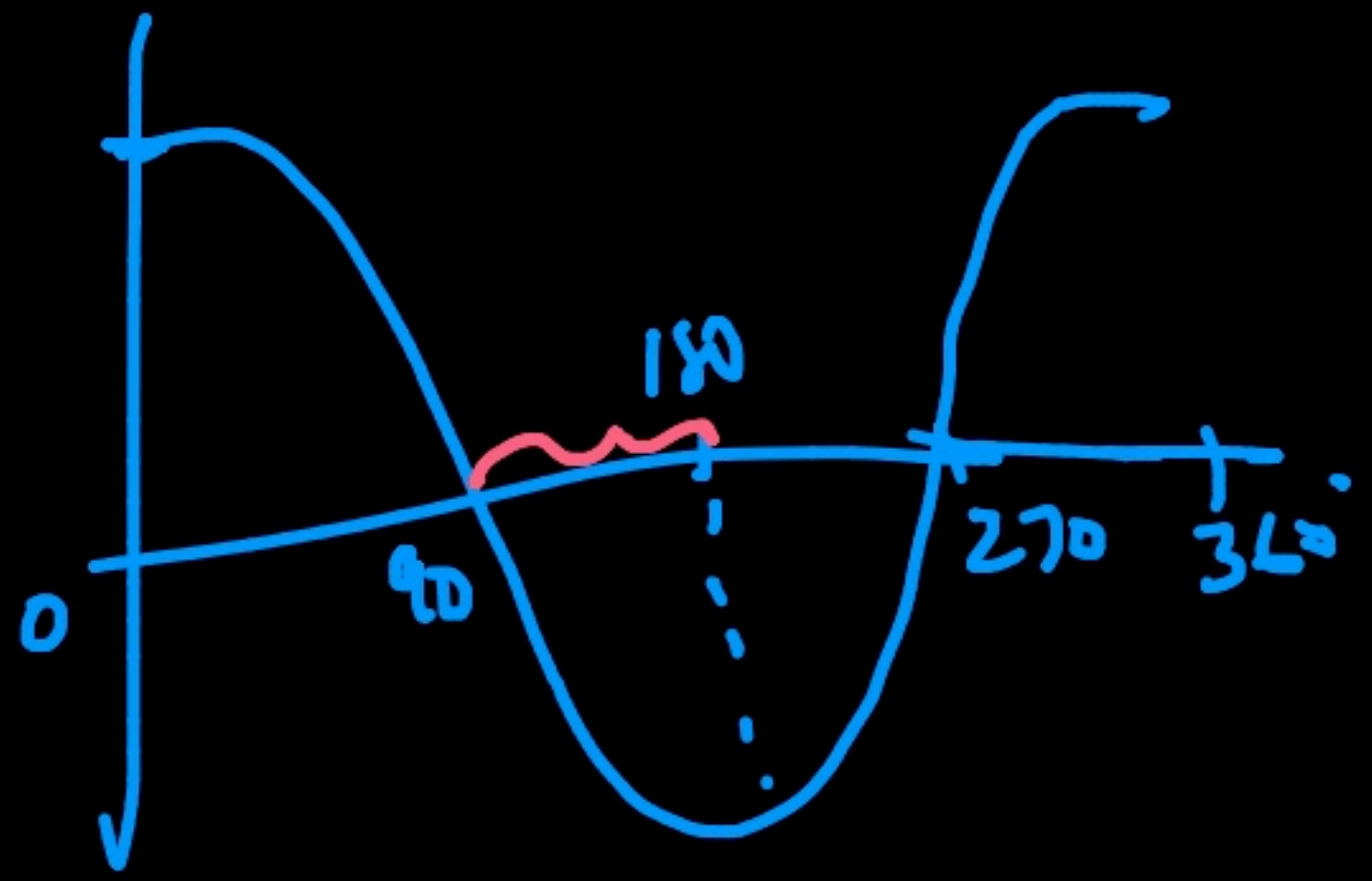
$\underline{w^T x_2} + w_0 < 0 \rightarrow$ Yes

half-spaces



$\pi^d : w^T x + w_0 = 0$





$||x_1|| \; ||w|| \cos\theta + 0$

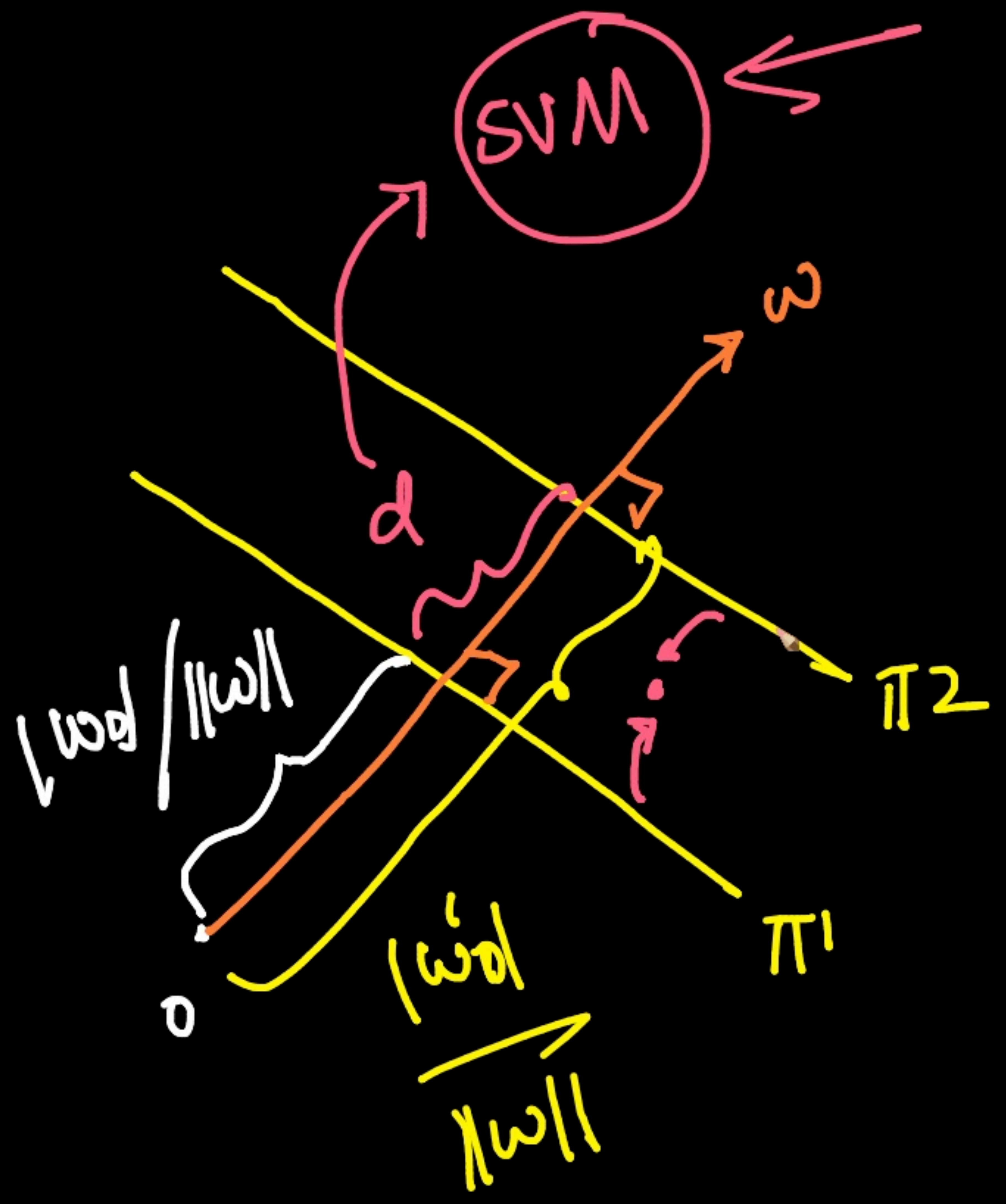
+ve +ve -ve

$\pi_1 : \quad w^T x + w_0 = 0$

$\pi_2 : \quad w^T x + w_0' = 0$

dist b/w π_1 & π_2

$\hookrightarrow \dfrac{|w_0' - w_0|}{||w||}$



(Q)

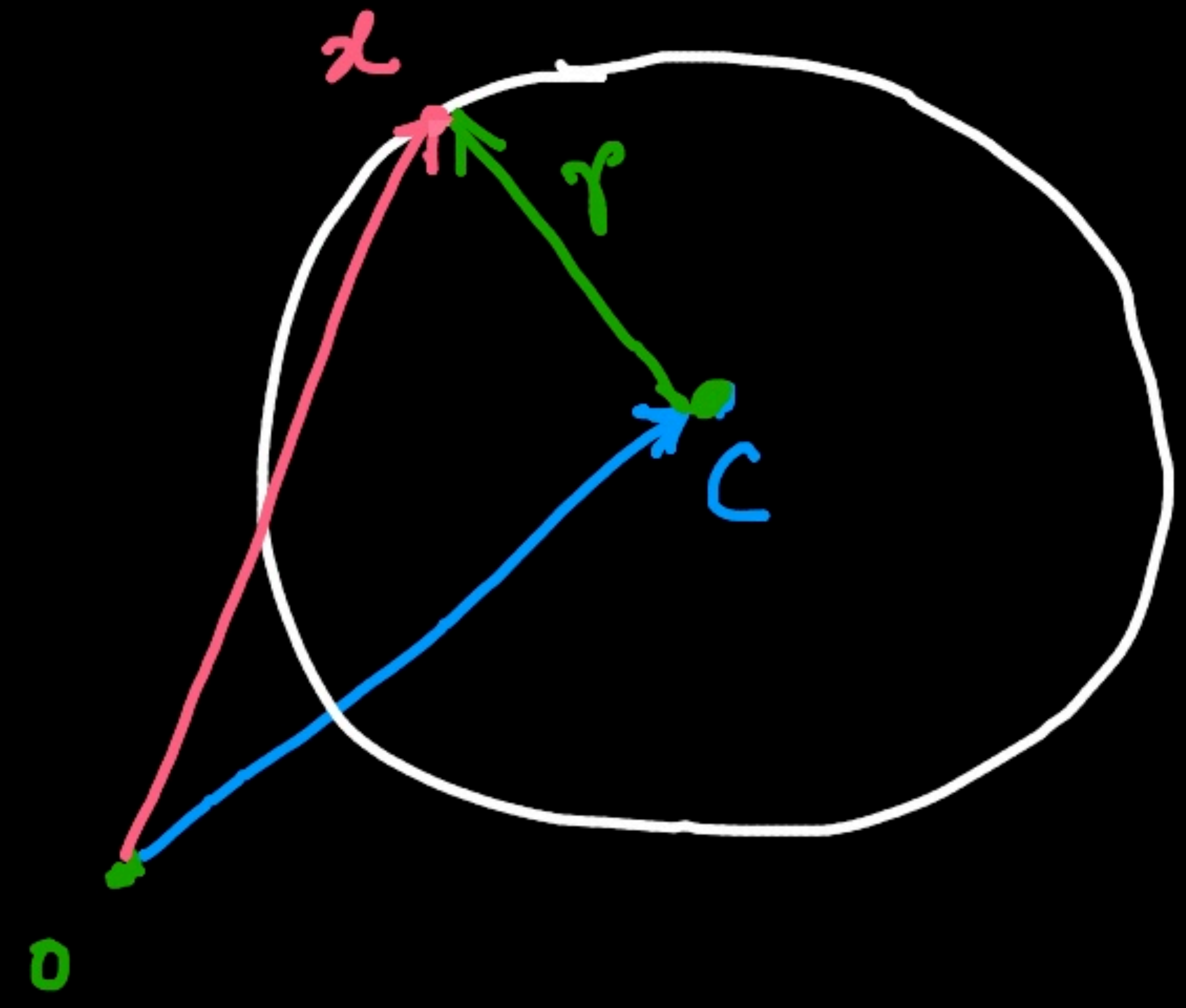
d-dim [hyper sphere]

$\mathbb{R}^d \ni c$: center

scalar ← r: radius.

x ←

$\vec{oc} + \vec{cx} = \vec{ox}$



$r^2 = \|x - c\|^2$

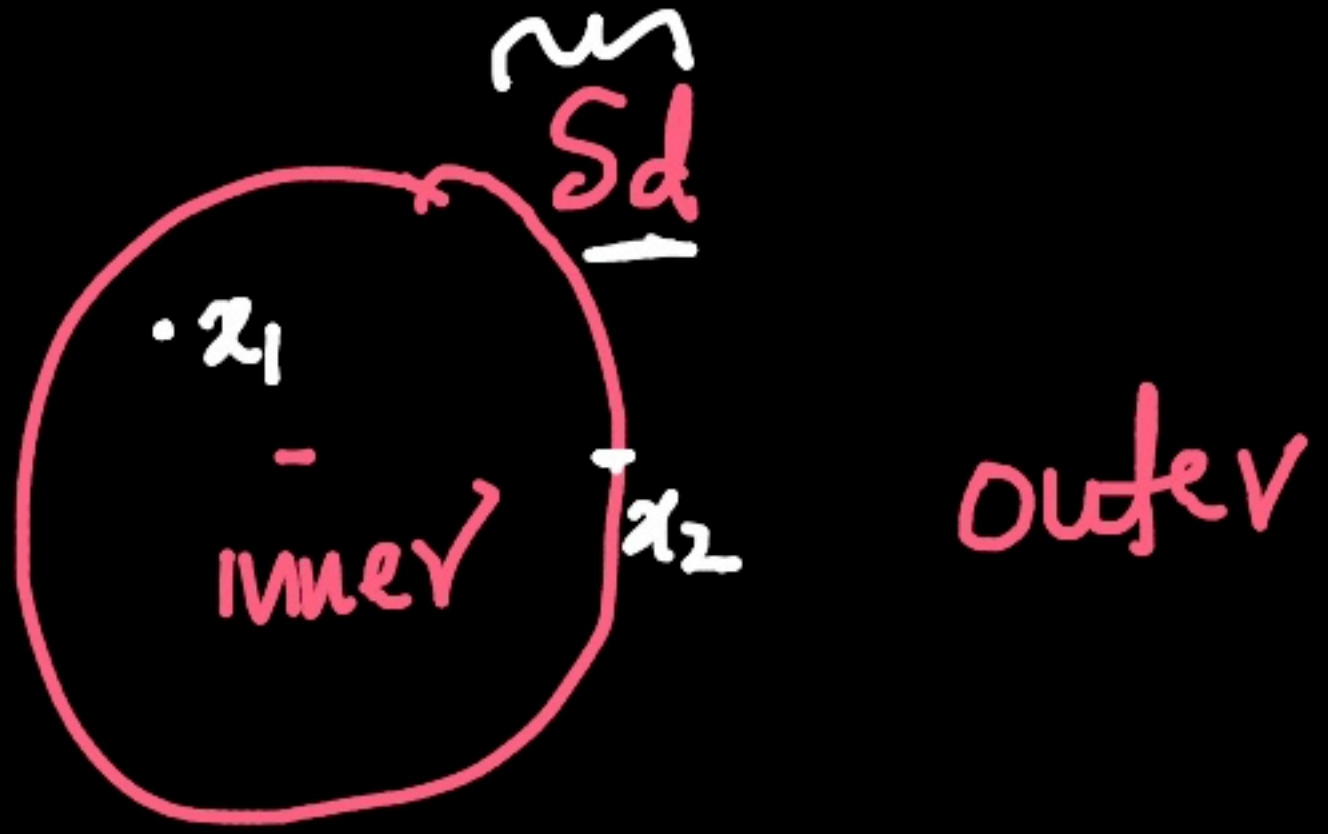
$\|\vec{cx}\| = \|\vec{ox} - \vec{oc}\|$

$r = \|x - c\|$

(Q)

$\pi^d : w, w_0 \rightarrow$ halfspaces

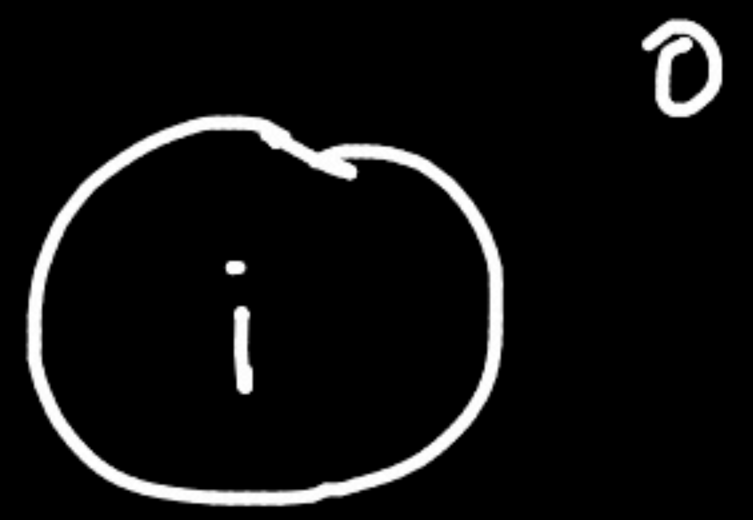
$\cdot x_3$



$$Sd : \|x - c\| = r$$

$$\begin{cases} x_1 \text{ in } Sd \rightarrow \|x_1 - c\| \boxed{<} r \\ x_2 \rightarrow 0 \qquad x_3 > 0 \end{cases}$$

2D:

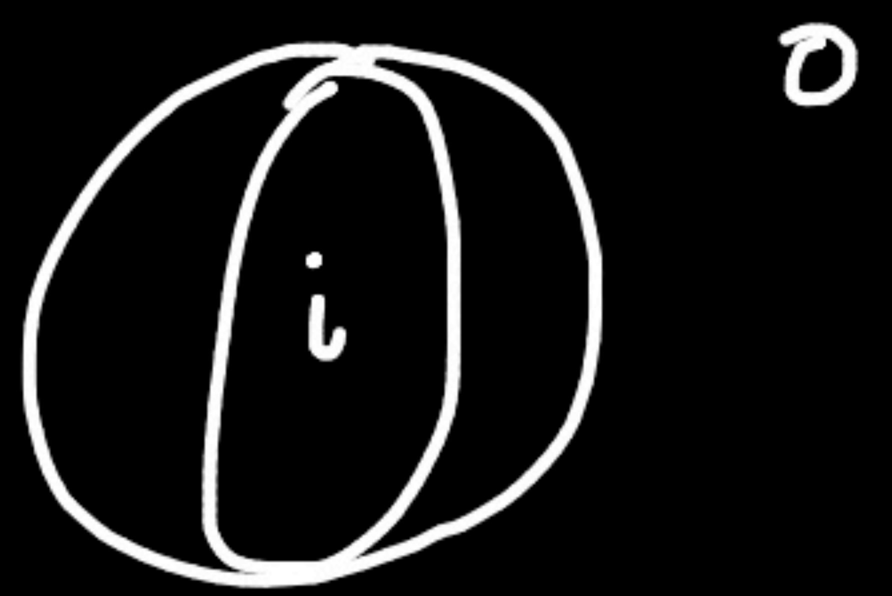


Clustering algo

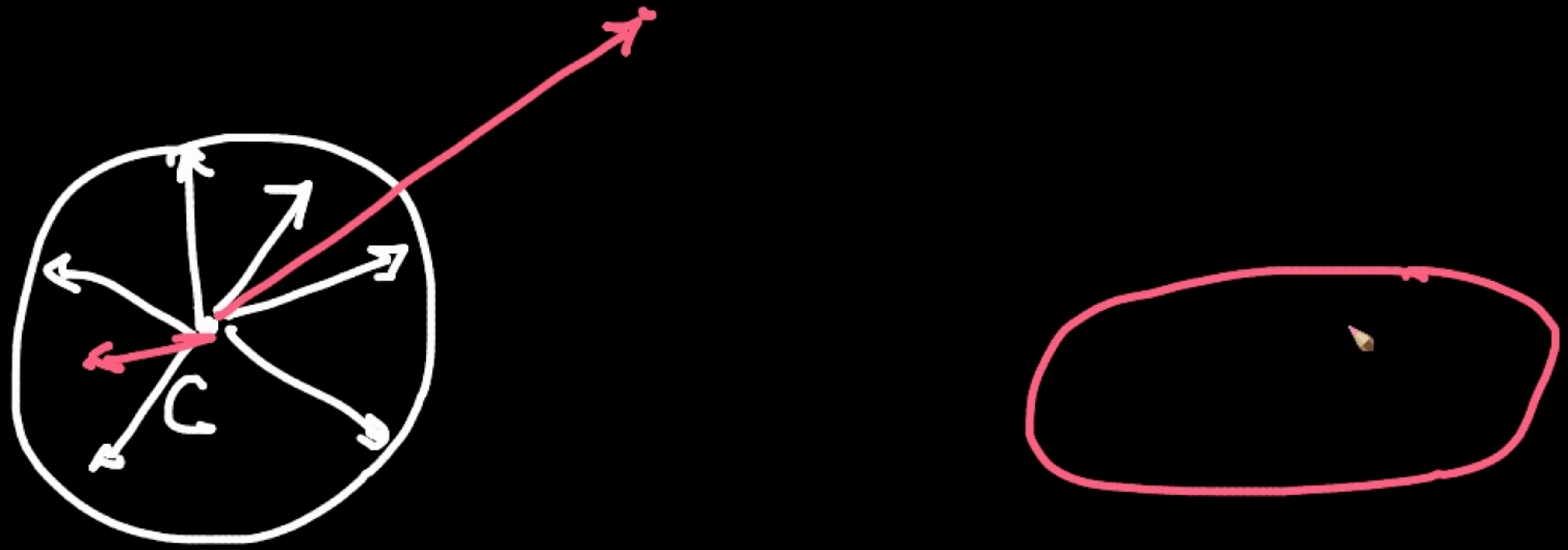
↓

ML2

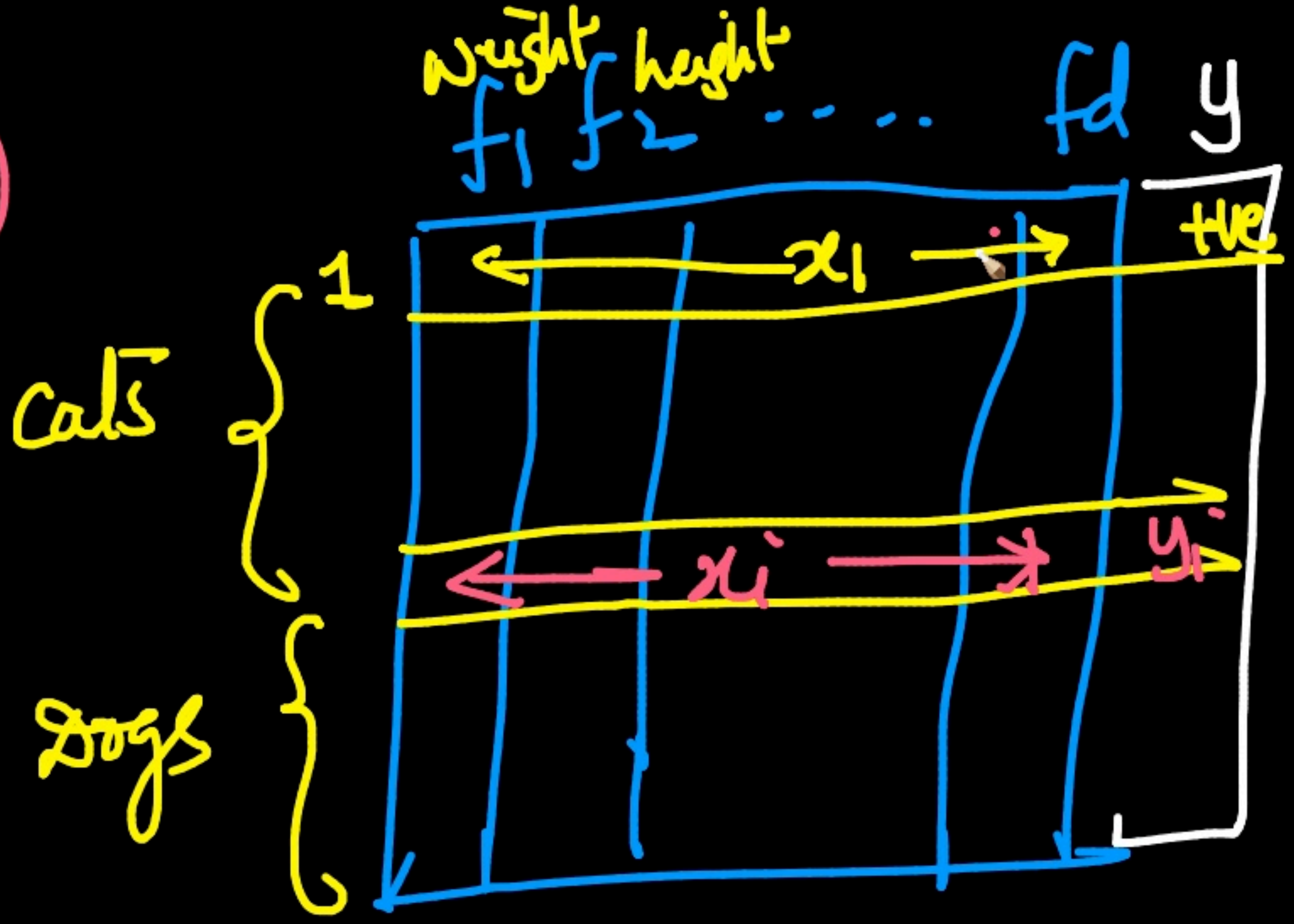
3D:



d-dim → hyper-spheres


C

ML



2-class Classification

100 Cat : +ve

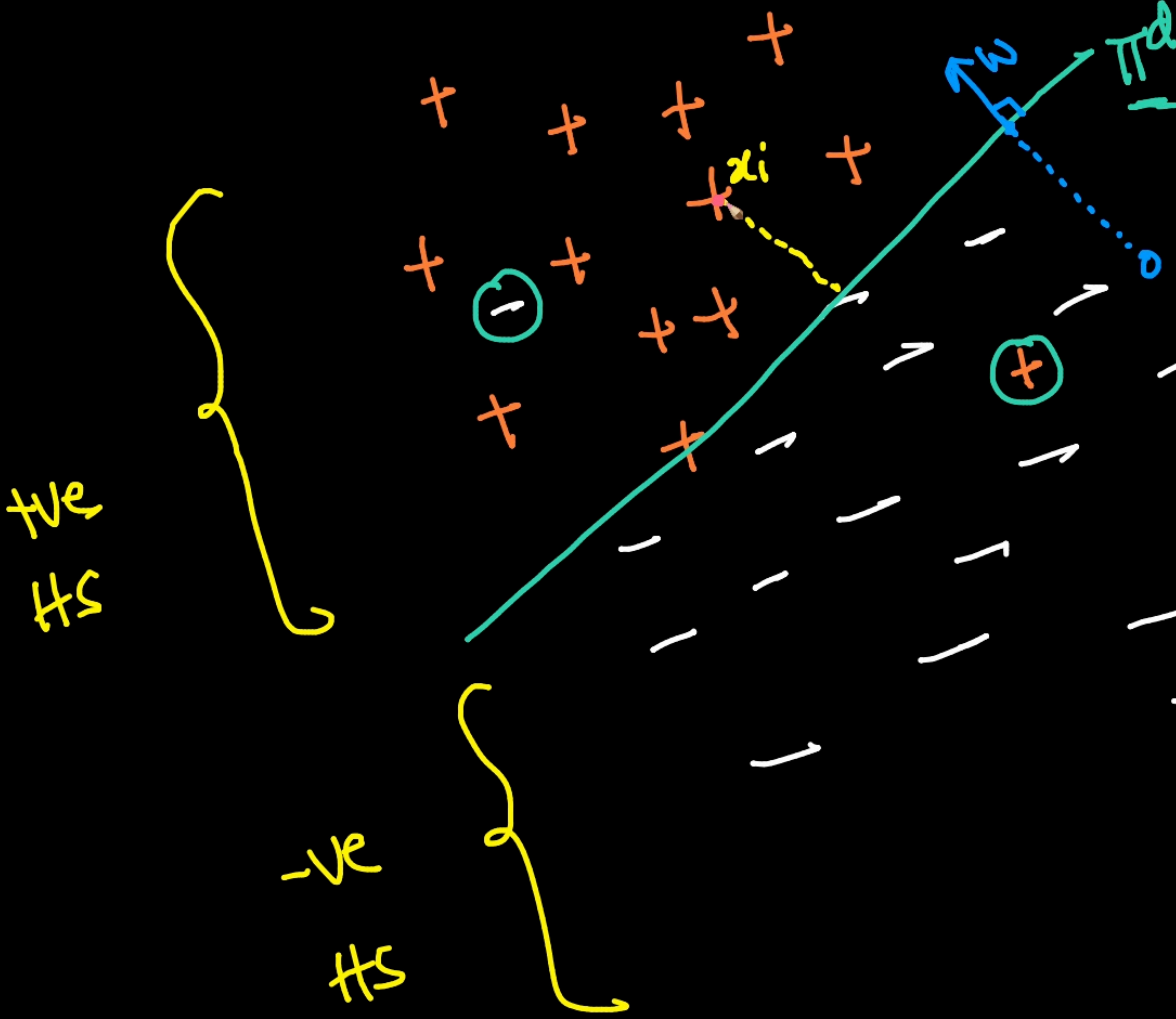
100 Dog : -ve

n=200

$$D = \left\{ (x_i, y_i)_{i=1}^{n} \; ; \; x_i \in \mathbb{R}^d \; ; \; y_i \in \{+1, -1\} \right\}$$

+ve -ve

$\pi^d : w^T x + w_0 = 0$



OBJective

ML: find best π^d that seperates +ve & -ve pts using D_{200}

find w & w_0 s.t

+ve: $w^T x_i + w_0 > 0$ → $d_i > 0$ $= \dfrac{w^T x_i + w_0}{\|w\|}$ as large as possible ⇓ max

-ve: $w^T x_i + w_0 < 0$ → $d_i < 0$: $\dfrac{w^T x_i + w_0}{\|w\|} = d_i$

d_i's as small as possible ⇒ min

+ve: $\frac{\overbrace{w^T x_i + w_0}^{Max}}{\|w\|} \cdot \overset{+1}{y_i} \longrightarrow Max$

-ve: $\underbrace{\frac{w^T x_i + w_0}{\|w\|}}_{min} \cdot \underset{-1}{y_i} \longrightarrow Max$

+ve : $\dfrac{w^T x_i + w_0}{\|w\|} \cdot y_i = z_i$ Max

-ve : $\dfrac{w^T x_i + w_0}{\|w\|} \cdot y_i = z_i$: Max find w & w_0 s.t

for $i = 1$ to n

compute z_i

max all z_i's

Find w & w_0 that Max $z_1, z_2, z_3, \ldots\ldots z_n$

$\Downarrow$

$$\max \sum_{i=1}^{n} z_i = \max \sum_{i=1}^{n} \frac{\overbrace{w^T z_i + w_0}}{\|w\|} \cdot y_i$$

easier

$$\max \prod_{i=1}^{d} z_i$$

(n)

Given $D = \{ (x_i, y_i)_{i=1}^{n} \;.\; x_i \in R^d,\; y_i \in \{+1, -1\} \}$

find the π^d : w, w_0

which maximizes $\sum_{i=1}^{n} \frac{w^T x_i + w_0}{\|w\|} \cdot y_i$

$f(w, w_0)$

$$\boxed{\max \; \underline{\tilde{f}}(w, w_0)}$$

best
w, w_0

✓ { Maxima & minima

↳ f', f''