

Neuron



① Sigmoid — Log Reg Unit
② Softmax

1 hidden

→

52%

Linear decision boundary

Activation

$$\frac{\partial a}{\partial z} = \left(\frac{1}{1+e^{-z}} \right)$$

- ① Differentiable (through some hacks)
- ② Compute inexpensive $\rightarrow \underline{w, b}$
- ③ non-linear

Sigmoid

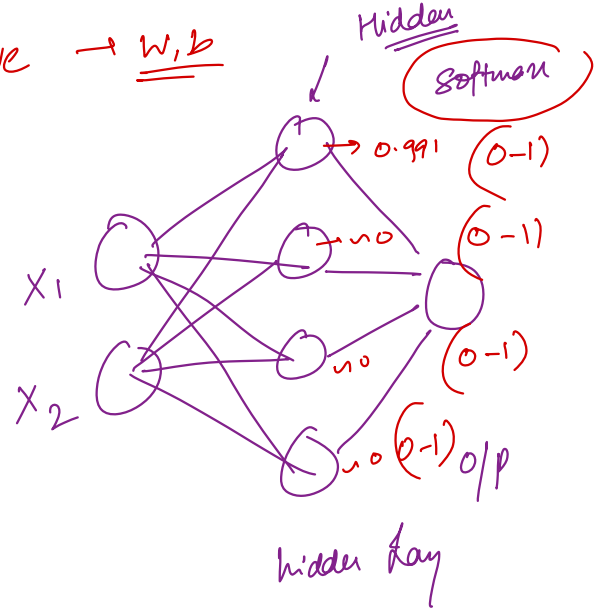
Activation

- ① Regression
 \hookrightarrow linear

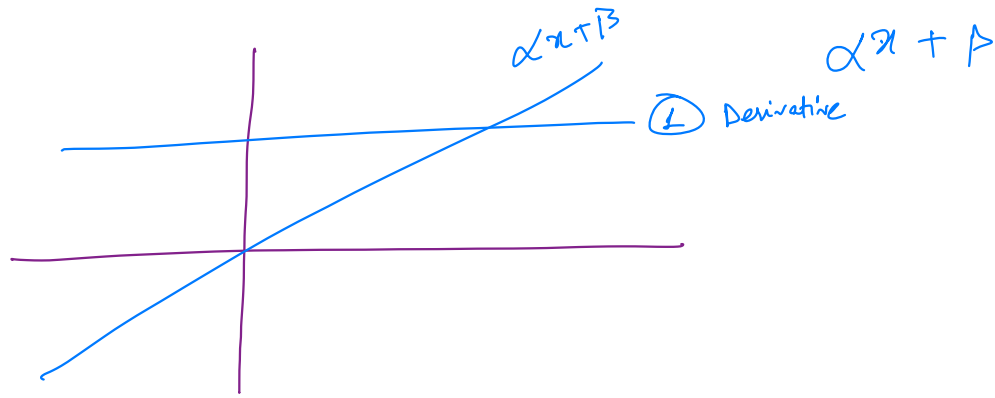
$$\text{act}(z) = \frac{1}{1 + e^{-z}}$$

Trained

$$z \rightarrow (-\infty, \infty)$$

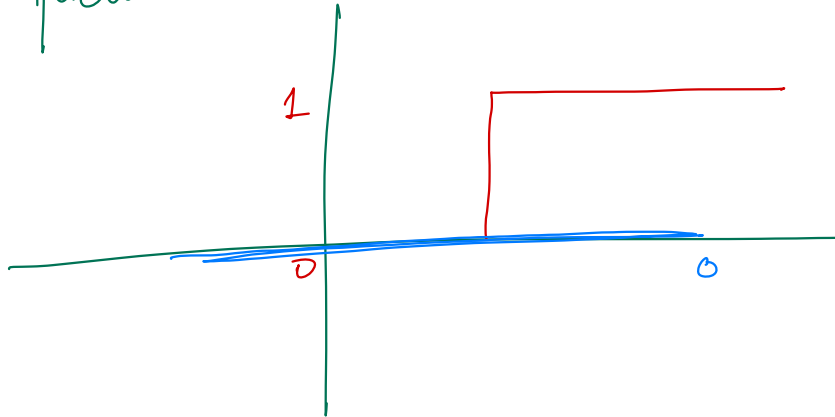


$$\hat{y} = (10, \infty)$$



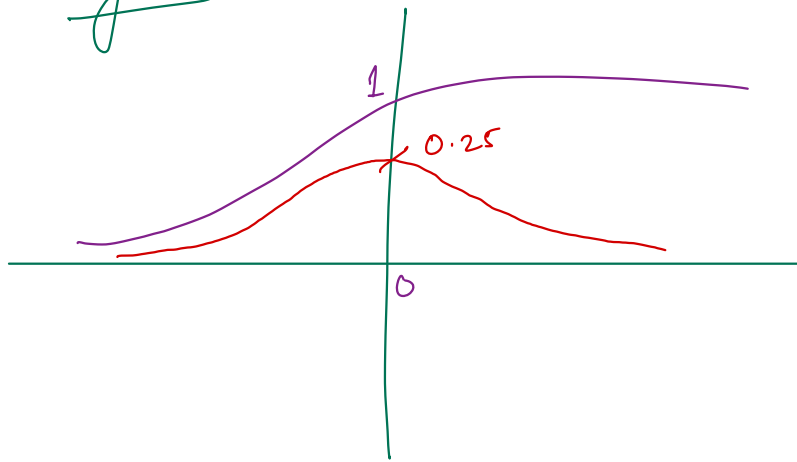
$$\begin{aligned} z > 0 &= 1 \\ z < 0 &= 0 \end{aligned}$$

$\textcircled{2}$ Step function



③

Sigmoid



$$\hat{y} = [0, 1]$$

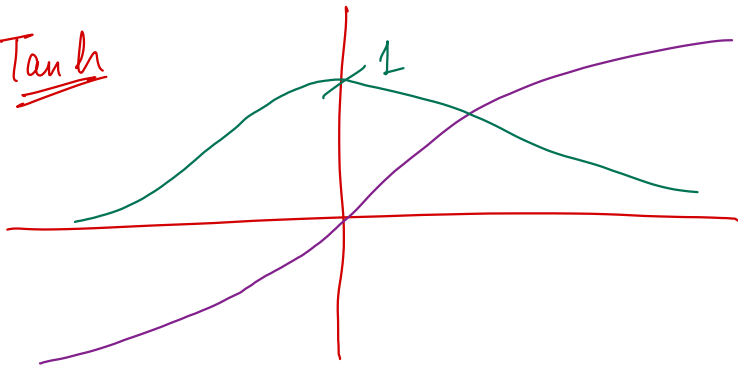
$$\text{Act}(z) = \frac{1}{1 + e^{-z}}$$

④

Softmax

⑤

Tanh



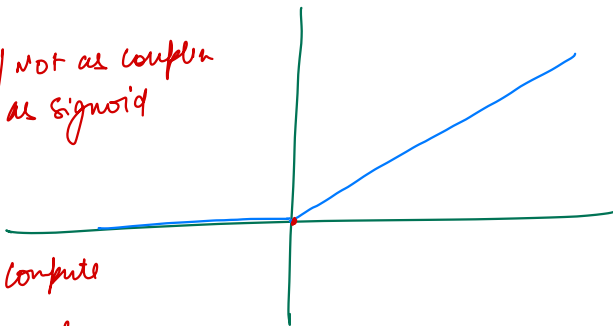
$$\hat{y} = [-1, 1]$$

$$\text{Act}(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

⑥ Relu → Rectified Linear Unit

① Simple / not as complex as sigmoid

$$\text{Act}(z) = \begin{cases} z > 0, z \\ z \leq 0, 0 \end{cases}$$



② Faster compute

③ Differentiable

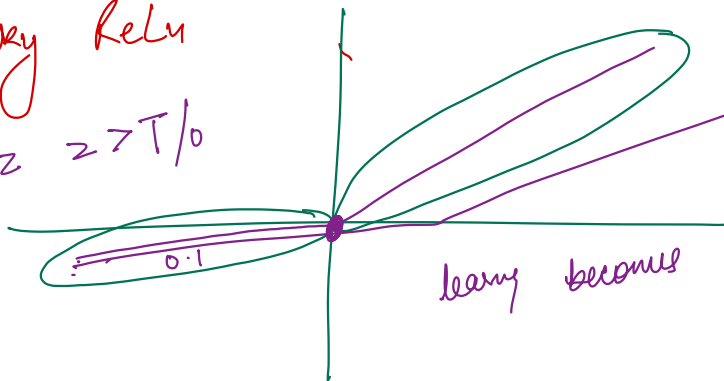
④ Simplest + non-linear function

$$\rightarrow \max(0, z)$$

⑦ Leaky Relu

$$\text{Act}(z) = z \quad z \geq T/0$$

$$0.1z \quad z < 0$$



leaky becomes

In hidden

$$\alpha x + \beta$$

Linear Act. func.

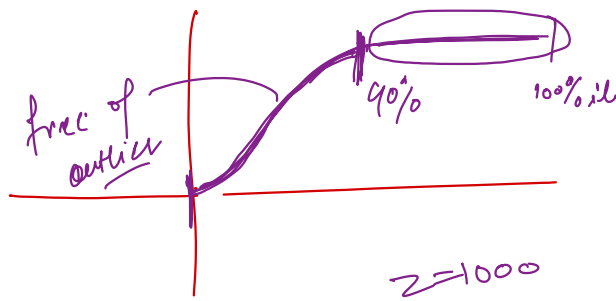
↳ non-linearity X

Step func.

→ kills a lot of info.

Sigmoid

→ Squashing effect



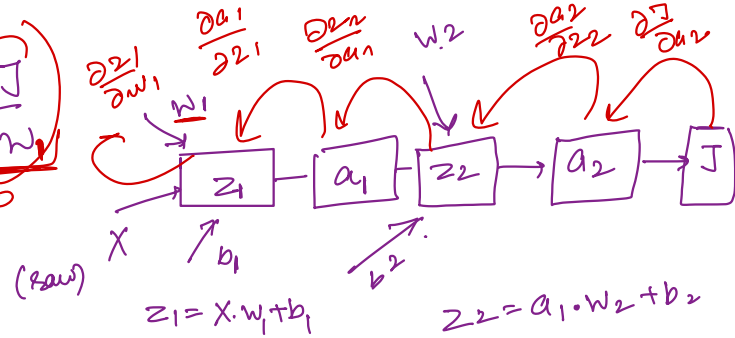
Tanh \rightarrow Similar to sigmoid.
 \rightarrow VSC - case dependent.

Relu \rightarrow hidden layers.

$$w^{new} = w^{old} -$$

$$\frac{\partial J}{\partial w} \approx 0$$

$$\alpha \frac{\partial J}{\partial w}$$



Sigmoid \rightarrow 0.25

50 layers

$$0.25 \times 0.25 \times 0.25 \dots$$

$$\approx 0$$

Vanishing gradients

$$w^{new} = w^{old}$$

Tanh \Rightarrow 1

$\frac{\partial a}{\partial z} = 1$

$z_1 \rightarrow \underline{2 \times 2 \times 2 \times 2}$

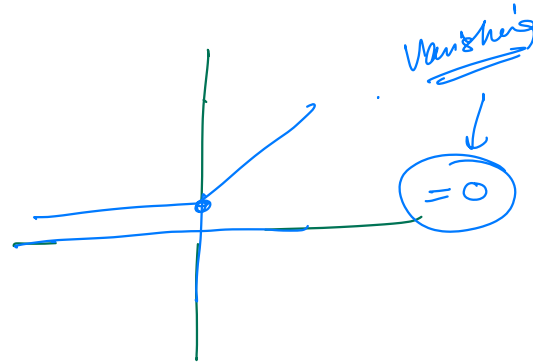
Relu \rightarrow $\max(0, z)$

exploding gradients

$\text{Relu} = \max(0, z)$

$\frac{\partial a}{\partial z} = 1$

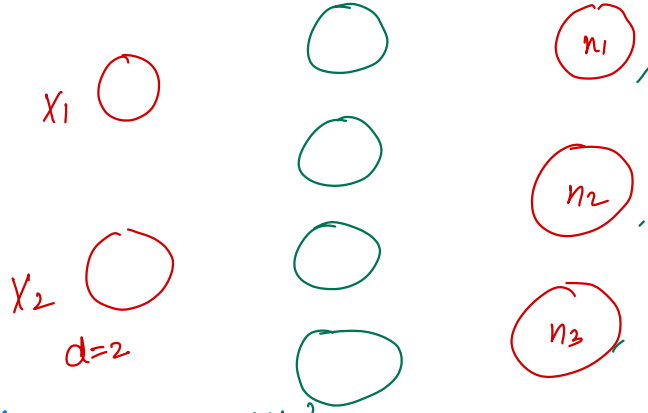
$\frac{\partial a}{\partial z} \neq 1, 0$



$$\text{leaky}(z) = z \quad z > 0 \text{ or Threshold}$$

$$= \underline{0.12} \quad z \leq 0$$

300
data points



$d=2$

Shape

$$A^0 = X \quad (300, 2)$$

$$z^1 = (300, 4)$$

$$w^1 = (2, 4)$$

$$b^1 = (1, 4)$$

$$A^1 = (300, 4)$$

$$w^2 = (4, 3)$$

$$b^2 = (1, 3)$$

$$z^2 = (300, 3)$$

$$A^2 = \hat{y} = (300, 3)$$

$$z^1 = X \cdot w + b$$

$$= (300 \times 2) \cdot (2 \times 4) + (1, 4)$$

$$= (600, 4) + (1, 4)$$

$$z^2 = A^1 \cdot w^2 + b^2$$

$$= (300 \times 4) \cdot (4 \times 3) + (1, 3)$$

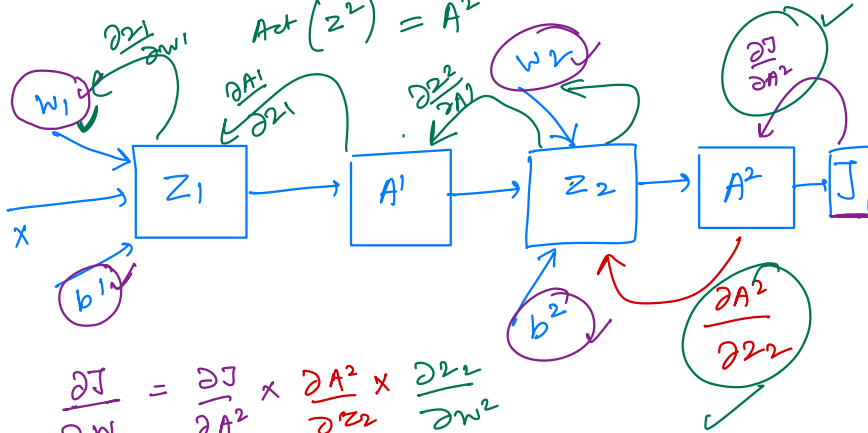
$$\Rightarrow (300, 3) + (1, 3)$$

$$z^1 = X \cdot w_1 + b_1$$

$$A^1 = \text{Act}(z^1)$$

$$z^2 = A^1 \cdot w_2 + b_2$$

$$A^2 = \text{Act}(z^2)$$



$$\frac{\partial J}{\partial w_2} = \frac{\partial J}{\partial A^2} \times \frac{\partial A^2}{\partial z_2} \times \frac{\partial z_2}{\partial w_2}$$

$$\frac{\partial J}{\partial w_1} = \frac{\partial J}{\partial A^2} \times \frac{\partial A^2}{\partial z_2} \times \frac{\partial z_2}{\partial A^1} \times \frac{\partial A^1}{\partial z_1} \times \frac{\partial z_1}{\partial w_1}$$

$$\text{loss} = -\frac{1}{1} y \log(a) + (1-y) \log(1-a)$$

$$w_1, w_2$$

$$\frac{\partial J}{\partial w^2} = \underbrace{\frac{\partial J}{\partial A^2} \times \frac{\partial A^2}{\partial z^2}}_{a-y} \times \frac{\partial z^2}{\partial w^2}$$

$$z^2 = A' \cdot \underline{w^2} + b^2$$

$$\frac{\partial z^2}{\partial w^2} = A'$$

$$\frac{\partial J}{\partial w^2} = (A^2 - y) \cdot A'$$

$$\frac{\partial J}{\partial b^2} = (A^2 - y)$$

$$z^2 = \underline{A' \cdot w^2} + \underline{b^2}$$

$$\frac{\partial z^2}{\partial b^2} = 1$$

$$\frac{\partial J}{\partial w^1} = \frac{\partial J}{\partial A^2} \times \frac{\partial A^2}{\partial z^2} \times \frac{\partial z^2}{\partial A'} \times \frac{\partial A'}{\partial z^1} \times \frac{\partial z^1}{\partial w^1}$$

$$\frac{\partial J}{\partial w^1} = (A^2 - y) \cdot w^2 \cdot 0 \cdot x = 0$$

$$\frac{\partial J}{\partial w^1} = (A^2 - y) \cdot \underline{w^2} \cdot x \quad \checkmark \text{ Reduced form}$$

$$\frac{\partial J}{\partial b^1} = (A^2 - y) \cdot w^2 \quad \checkmark \text{ Reduced form}$$

$$z^2 = \underline{A'} \cdot w^2 + b^2$$

$$\frac{\partial z^2}{\partial A'} = w^2$$

$$A' = \text{ReLU}(z^1)$$

$$\max(0, z^1)$$

$$A' = 0 \quad z^1 \leq 0$$

$$A' = z^1 \quad z^1 > 0$$

$$\frac{\partial A'}{\partial z^1} = 0, 1$$

$$z^1 = x \cdot w^1 + b^1$$

$$\frac{\partial z^1}{\partial w^1} = x$$

