

$$z = x \cdot w + b$$

$$\frac{\partial J}{\partial w}, \frac{\partial J}{\partial b}$$

$$\frac{\partial J}{\partial w} = \frac{\partial J}{\partial a} \times \frac{\partial a}{\partial z} \times \frac{\partial z}{\partial w}$$

Chain Rule (Backbone of NN)

Forward pass

$$z = \vec{w} \cdot \vec{x} + b$$

$$\text{Act}(z) = \text{Sigmoid}(z) \\ \cong \text{Softmax}(z)$$

Multiclass

J → Categorical cross entropy loss

Backprop

$$\underline{J} = -\frac{1}{2} \left(y_i \log(a_i) + (1-y_i) \log(1-a_i) \right)$$

$$\frac{\partial J}{\partial a} = - \left(y \log(a) + (1-y) \log(1-a) \right)$$

$$= - \left(y \frac{\partial \log(a)}{\partial a} + (1-y) \frac{\partial \log(1-a)}{\partial a} \right)$$

$$\frac{\log a}{\frac{1}{a}} = \frac{1}{a} \left(\frac{1}{a} \right) + (1-y) \left(\frac{-1}{1-a} \right)$$

$$= -\frac{y}{a} + (1-y)\frac{1}{(1-a)} = \frac{(1-a)(-y) + (1-y)a}{a(1-a)}$$

$$= \frac{-y + ya + a - ya}{a(1-a)}$$

$$\frac{1}{x} \Rightarrow x^{-1} \Rightarrow -\frac{1}{x^2}$$

$$\frac{\partial \mathcal{J}}{\partial a} \Rightarrow \frac{a-y}{a(1-a)} \quad \textcircled{1}$$

$$\frac{\partial a}{\partial z} = \frac{\partial}{\partial z} \left(\frac{1}{1+e^{-z}} \right)$$

$$\Rightarrow \frac{-1}{(1+e^{-z})^2} (e^{-z})$$

$$\Rightarrow \frac{1 \times e^{-z} \textcircled{+1} \textcircled{-1}}{(1+e^{-z})(1+e^{-z})}$$

$$= \frac{1+e^{-z}-1}{(1+e^{-z})^2} = \frac{\cancel{1+e^{-z}}}{(1+e^{-z})^2} \cdot \frac{-1}{(1+e^{-z})^2} \Rightarrow \left(\frac{1}{1+e^{-z}} \right) \cdot \frac{-1}{(1+e^{-z})^2}$$

$$\Rightarrow \left(\frac{1}{1+e^{-z}} \right) \left(1 - \frac{1}{1+e^{-z}} \right)$$

$$\Rightarrow \boxed{a(1-a)}$$

$$\boxed{\frac{\partial a}{\partial z} = a(1-a)} \quad \textcircled{2}$$

$$\frac{\partial z}{\partial w} = \frac{\partial (xw + b)}{\partial w} = x \quad \text{--- (3)}$$

$$\frac{\partial J}{\partial w} = \frac{\partial J}{\partial a} \times \frac{\partial a}{\partial z} \times \frac{\partial z}{\partial w} = \frac{(0-y)}{a(1-a)} \times \frac{(a)(1-a)}{a(1-a)} \times x$$

①
②
③

$$\frac{\partial J}{\partial w} = \underbrace{(a-y)}_{\frac{\partial J}{\partial z}} \times x$$

$$\frac{\partial J}{\partial w} = \underbrace{(\text{error})}_{(a-y)} \times \underline{x}$$

error = pred - actual

$$\frac{\partial J}{\partial w} = \text{error} \times \frac{\partial z}{\partial w}$$

$$\frac{\partial J}{\partial b} = \text{error} \times \frac{\partial z}{\partial b}$$

$$\frac{\partial J}{\partial b} = \text{error} \cdot 1$$

$$z = xw + b$$

$$\frac{\partial z}{\partial w} = x$$

$$\frac{\partial z}{\partial b} = (xw + b) \Rightarrow 1$$

Neural n/w from Scratch

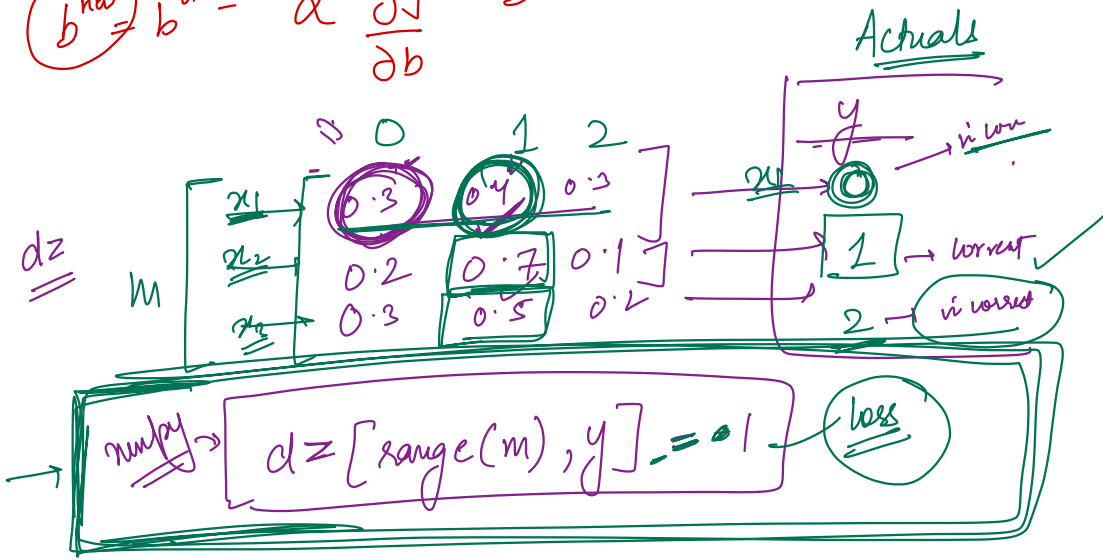
$$\frac{\partial J}{\partial w}$$

$$w^{new} = w^{old} - \alpha \sum_{i=0}^N \frac{\partial J}{\partial w}$$

learning rate

Stochastic GD
Batch GD

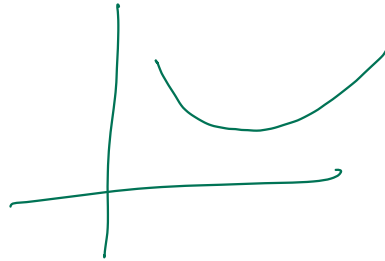
$$b^{new} = b^{old} - \alpha \frac{\partial J}{\partial b}$$



$$Z = 2x+1 \rightarrow \underline{\text{linear}}$$

$$\frac{1}{1+e^{(2x+1)}}$$

\Rightarrow linear
function



$$x^r \rightarrow \underline{x^2}, \underline{x^3}, x^4$$

~~8~~ (SQ)

$$(2x+1)^2 \Rightarrow$$

$$4x^2 + 4x + 2$$

