

## Linear Regression - 4

### Agenda

- Finish scratch code for lin Reg ✓
- Adjusted R-square ✓
- Assumption of Linear Regression (by statisticians)

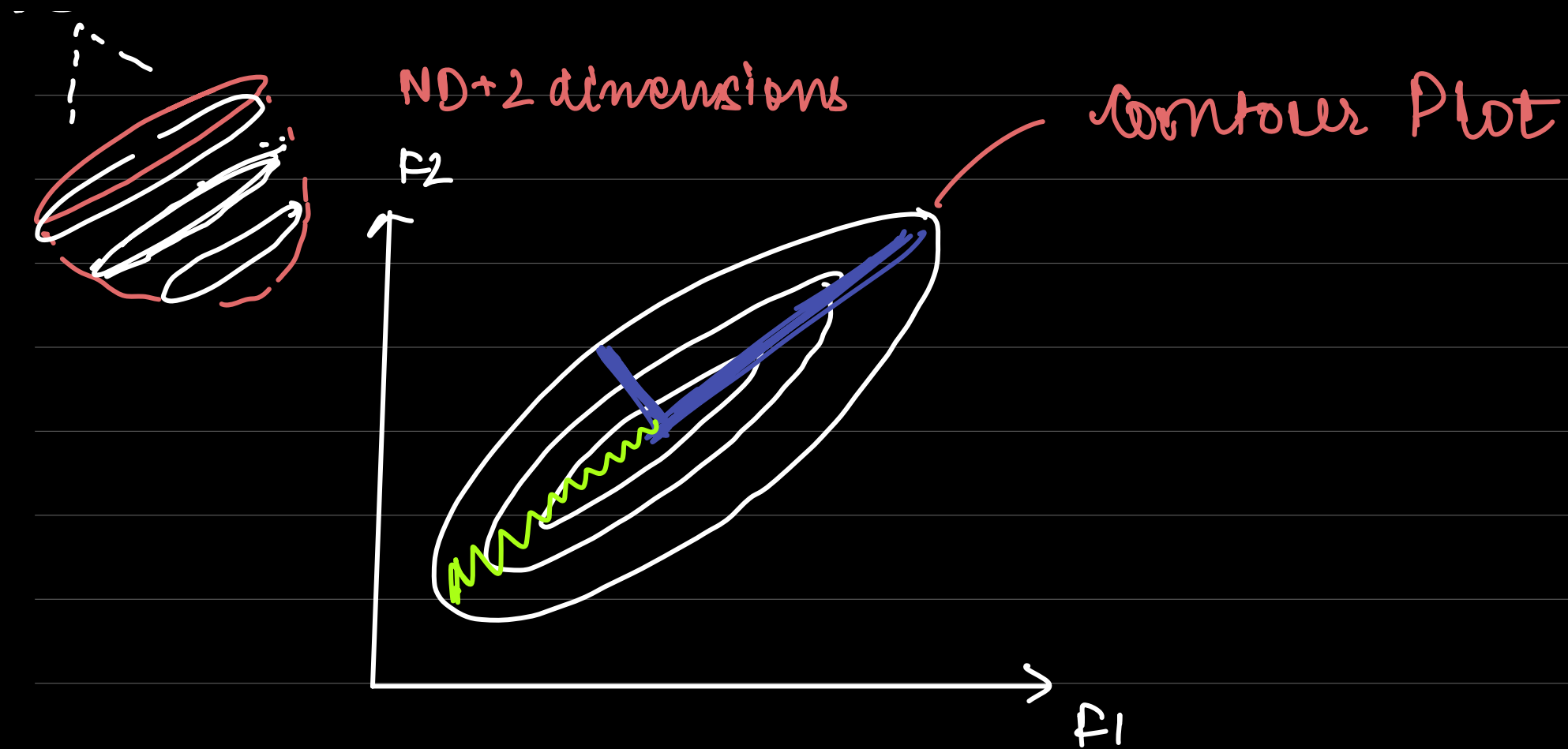
5 assumptions (1, 0.5x2) , 3, 4, 5  
pending

- Meet after break.

### Importance of Feature Scaling in Machine Learning

- Similar units → Interpret the weights / coefficients
- Helps GD converge faster ★★★

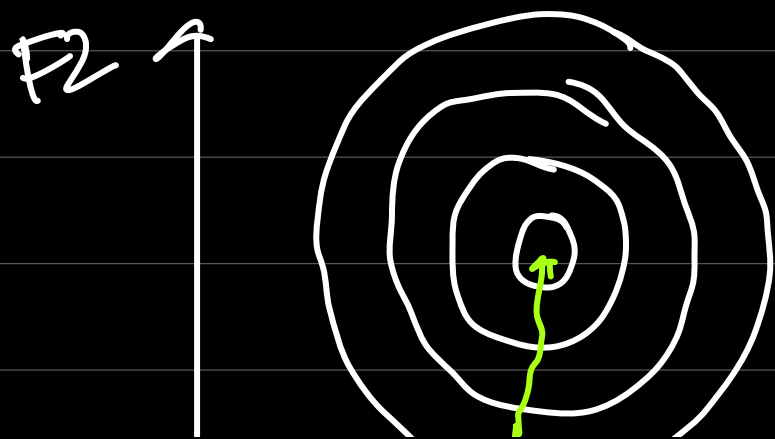




If non feature scaled

- $F_1$   $F_2$  may have diff scale, range, variance

If we scale the feature.



Training would be faster here



# # Problems with R-squared <sup>☆☆</sup>

R-squared ( $R_1$ )

lin Regression - N features  
+  
1 feature

$$w_0 + w_1 x_1 + w_2 x_2 + \dots + w_n x_n \rightarrow w_{n+1} x_{n+1}$$

Re-calculate R-square: ( $R_2$ )

$$R_1 > < = R_2 \quad ???$$



Case 1)  $R_1 > R_2$  ~~X~~

Model always have an option to make  $W_{n+1} = 0$

Case 2)  $R_1 < R_2$  ✓

New feature led to better performance. (Trivial)

Case 3)  $R_1 = R_2$

$d \uparrow \Rightarrow$  Performance —

No added value

Prefer this ✓

$M_1$   
 $d$  features  
 $R$

$M_2$   
 $d+2$  features  
 $R$

Why prefer  $M_1$  over  $M_2$  ??

Ockham's Razor

Preference for simplicity

Goal: Adjust the metric  $R^2$  in such a way that it penalises calculation if with  $\uparrow$  in  $d$ , there is NO increase in performance

$$\text{Adj. } R\text{-squared} = 1 - \left[ \frac{(1 - R^2)(m-1)}{(m-d-1)} \right] \quad A$$

If  $d \uparrow \rightarrow$  denom  $\downarrow$ ,  $R^2$  - no change,  $A \uparrow$ ,  $\text{Adj. } R^2 \downarrow$

if  $dT \rightarrow$  denom  $\downarrow$ ,  $\frac{K}{T}$ ,  $\frac{A}{T} \downarrow$  depends on  $n$  more extreme.

Performance increased

## # Intro to stats model

statistics.

Corab

ML

statsmodel

sklearn

- p-value
- confidence interval
- coefficients / std error

- Residual plot
- QQ plot

- GD
- Feature Scaling
- Cross validation - NEXT CLASS
- Evaluation Metrics

Linear, Classification, Models

Accuracy - computation errors

## Performance

$$R^2 = 0.75$$

75% of variance in  $y$  can be explained by the model.

## # Assumptions of Linear Regression.

1. Linearity ✓

2. NO MULTI-COLLINEARITY 0.5

3. Normality of Residuals (Errors)

4. No Heteroskedasticity

5. No Autocorrelation.

} next class.

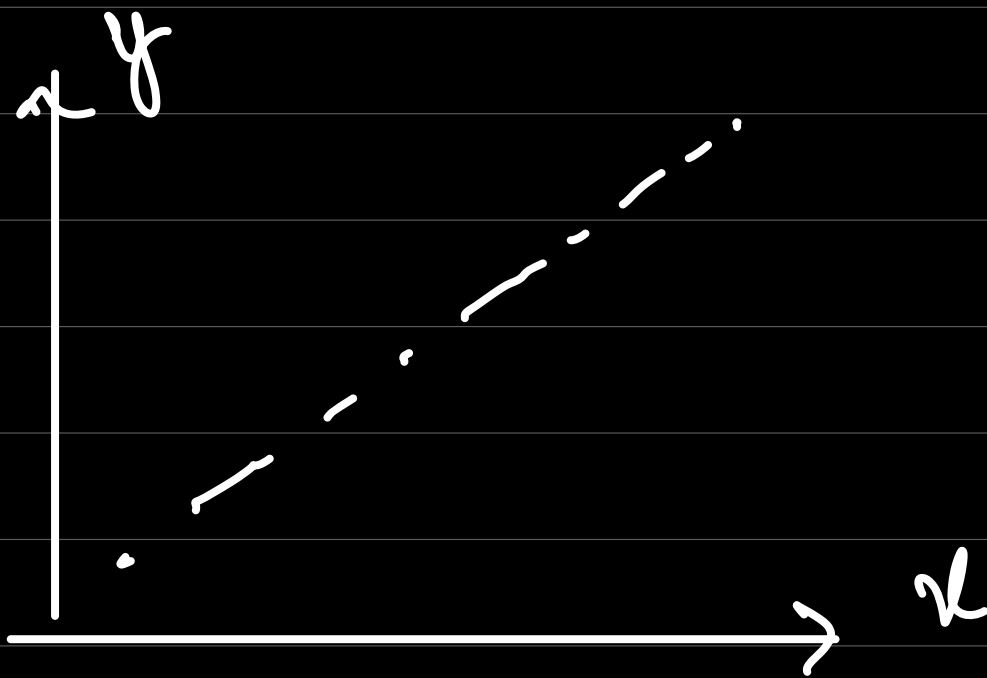
① Assumption of linearity

Your model will perform well if

$$X \xrightarrow{f} y$$

Trivial.

$f$ : linear



② No multi-collinearity



⇒ collinearity

$f_1$  and  $f_2$

$$f_1 = \alpha + \beta f_2$$

$f_1$  and  $f_2$   
are collinear.

$f_1$  - linear Transformation of  $f_2$

collinear variables

- Age of car, model Year
- Reg in miles and F

⇒ Multi-collinearity

$f_1, f_2, f_3, f_4$

$$f_1 = \alpha_1 + \alpha_2 f_2 + \alpha_3 f_3 + \alpha_4 f_4$$

$f_1, f_2, f_3, f_4$  are multicollinear.



They can be written as lin. comb. of each other

NO MULTI-COLLINEARITY. WHY?

$f_1, f_2, f_3$ .

$$M_1 = [w_1, w_2, w_3] = \langle 1, 2, 3 \rangle$$

$f_2$  and  $f_1$  are collinear  $\Rightarrow f_2 = 1.5 f_1$

$$\hat{y} = w_0 + w_1 x_1 + w_2 x_2 + w_3 x_3$$

$$\hat{y} = w_0 + x_1 + 2x_2 + 3x_3$$

$$\hat{y} = w_0 + x_1 + 3x_1 + 3x_3$$

$$\hat{y} = w_0 + 4x_1 + 0 \cdot x_2 + 3x_3$$

$$\langle 4, 0, 3 \rangle \quad \checkmark$$

$$\langle 1, 2, 3 \rangle \quad \checkmark$$

} Feature Imp. ??

# Unstable Training

Goal: To remove redundant features



TO HAVE NO MULTI-COLLINEARITY

VIF - Variance Inflation Factor

~ Next class.