Linear Algebra & Co-ordinate Geometry – 3

① Eqn of line/plane in depth

$y = mx + c$

$ax + by + c = 0$ — 2D

$ax + by + cz + d = 0$ — 3D

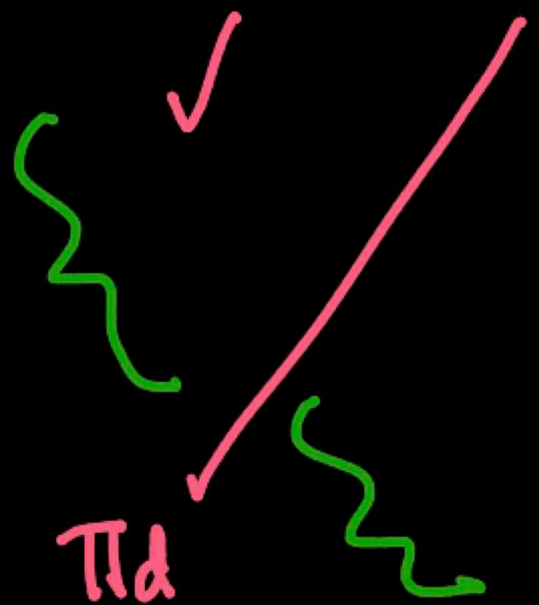
} extrapolate

$w^T x + w_0 = 0$ — π_d

d-dim

d-dim

(2)



ML

on which side of the plane

↳ simplest seperator in d-dim

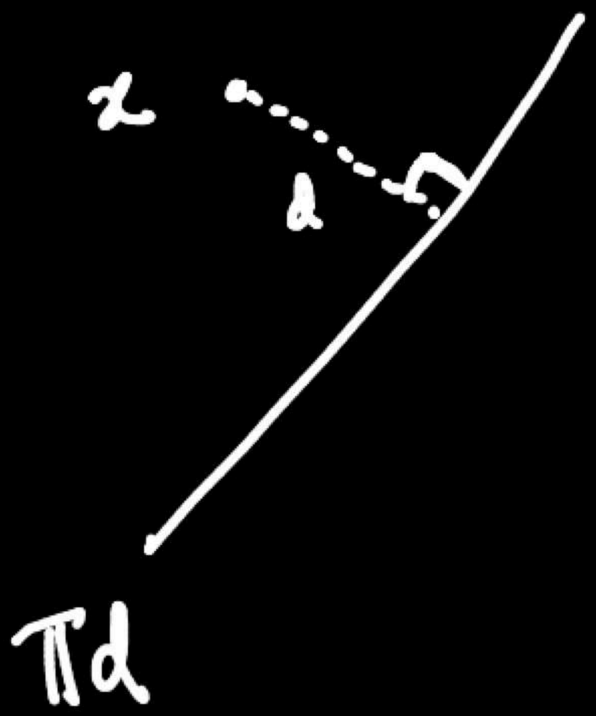
d-dim

↓

2-sides (half-spaces)

(3) How far are we from πd

confidence



④ l, w, non-numeric - - - - Type1 or 2

l → numeric

w → numeric

non-numeric → geometric intuition

(1)

Π_d: $w^T x + w_0 = 0$

w^T → geometrically

x → d-dim

w_0 → scalar

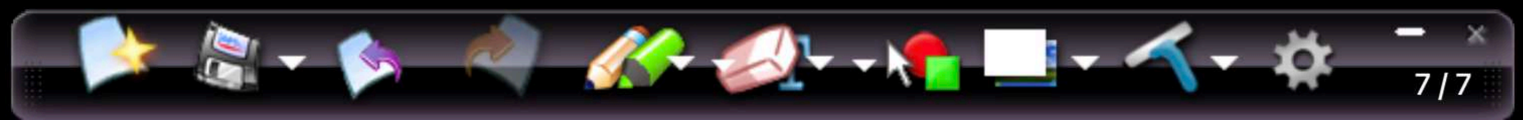
$$w^T x + b = 0$$

↳ angle between vectors

↓

dot-product

→ $w^T x$; w_0 mean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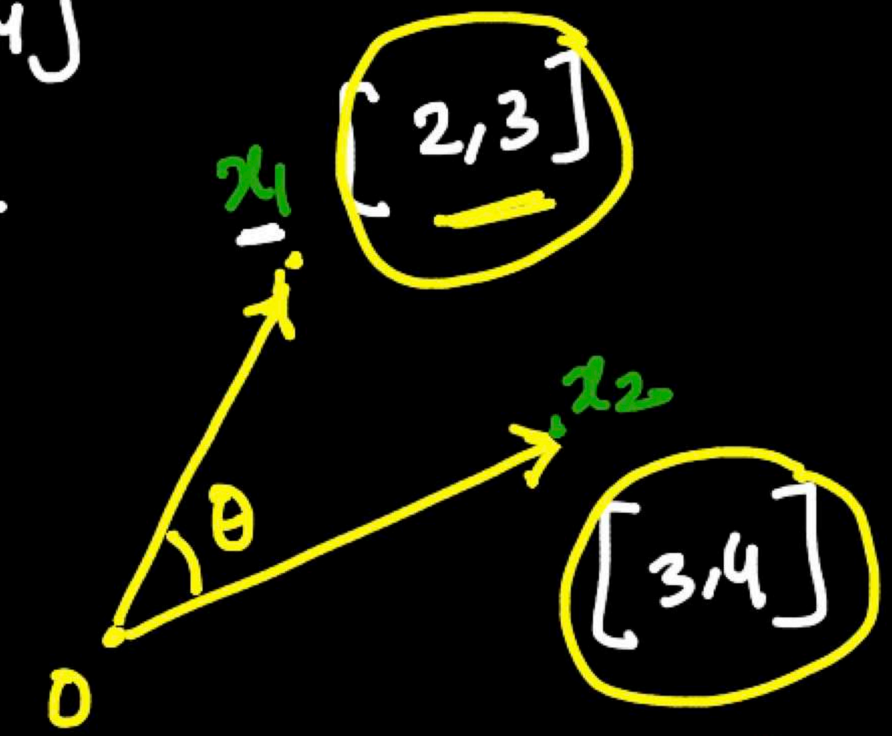
$[2,3]\ \begin{bmatrix}3\\4\end{bmatrix} = 6+12=18$

x_1^T x_2



$x_1, x_2 \cdots \rightarrow$ d-dim data points

$f_1, f_2 \rightarrow$ features

✓ $x_1 \cdot x_2 = x_1^T x_2$ — (1)

$= \|x_1\| \cdot \|x_2\| \cdot \cos\theta$ — (2)

Equivalence proof ✓

$= \sqrt{4+9} = \sqrt{13}$

x_1 [2,3]

$\|x_1\|$

θ

x_2 [3,4]

origin (0,0)

$\|x_2\|$

$\sqrt{3^2+4^2} = 5$

$x_1 \cdot x_2 = x_1^T x_2 = \|x_1\| \|x_2\| \cos\theta$

$\theta = \cos^{-1}\left(\frac{x_1^T x_2}{\|x_1\| \|x_2\|} \right)$ ✓

$\|x_1\| = |x_1|$

↑ confusion

(Q)

$\sqrt{13}$

$\sqrt{2^2+3^2}$

$x_1 [2,3]$

$(0,0)_O$

θ

$s = \sqrt{3^2+4^2}$

$x_2 [3,4]$

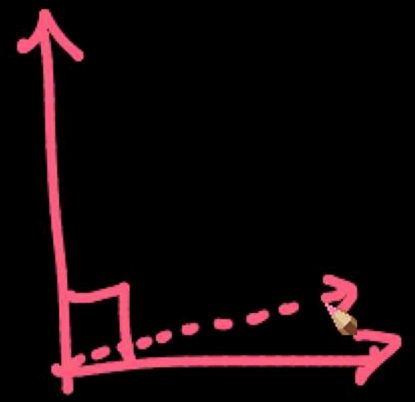
degrees

$\theta = ?$

$$\cos\theta = \frac{x_1^T x_2}{\|x_1\| \, \|x_2\|} = \frac{[2 \;\; 3]\begin{bmatrix}3\\4\end{bmatrix}}{\sqrt{13}\cdot 5}$$

$$= (6+12)/\sqrt{13}\cdot 5$$

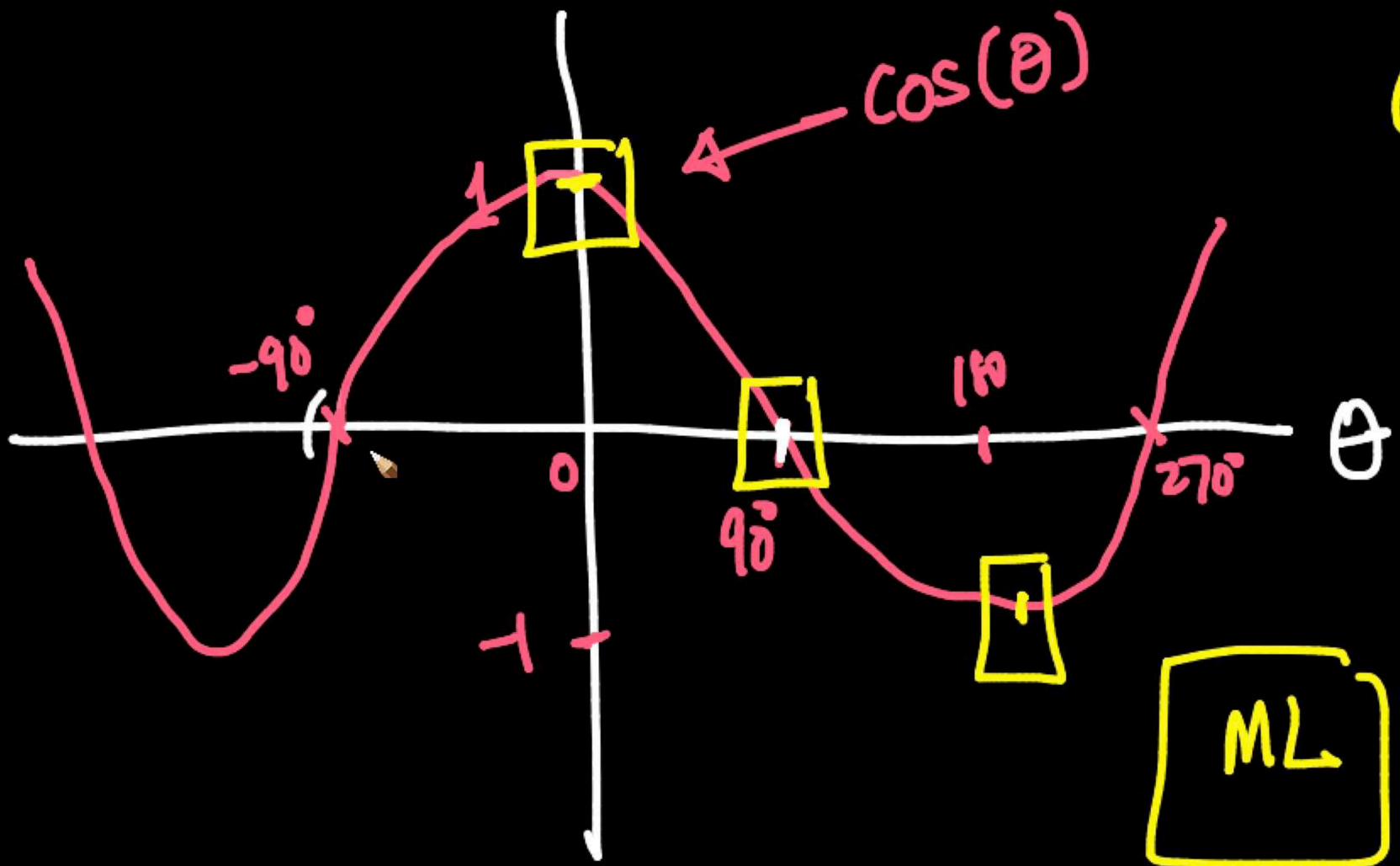
$$\frac{18}{5\cdot\sqrt{13}} = \cos\theta$$



$$\cos^{-1}\left(\frac{18}{5\sqrt{13}}\right)$$

$$\boxed{\theta = 3.179^{\circ}}$$

$$\sim 0.05 \text{ rad}$$



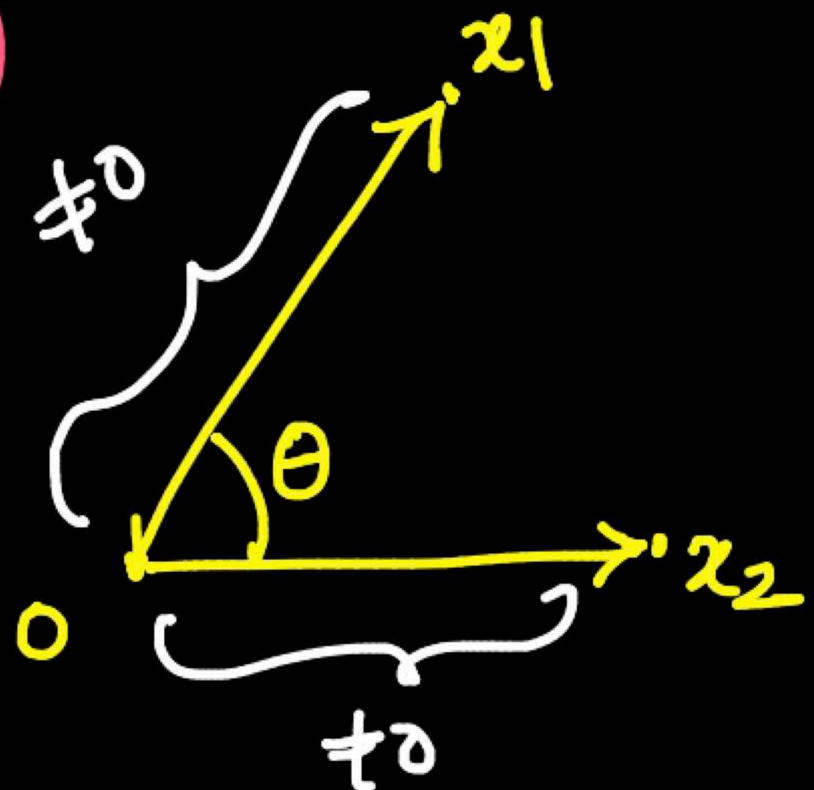
$\cos 0 = 1$

$\cos 90^\circ = 0$

$\cos 180^\circ = -1$

ML

Q



d-dim vec

$x_1 \perp x_2$

Let's assume

$x_1 \cdot x_2 = 0$

$\theta = ?$

$x_1 \cdot x_2 = \|x_1\| \|x_2\| \cos\theta = 0$

$\|x_1\| \neq 0$ $\|x_2\| \neq 0$ $\cos\theta = 0$

$x_1 \perp x_2$ ← perpendicular (ML)

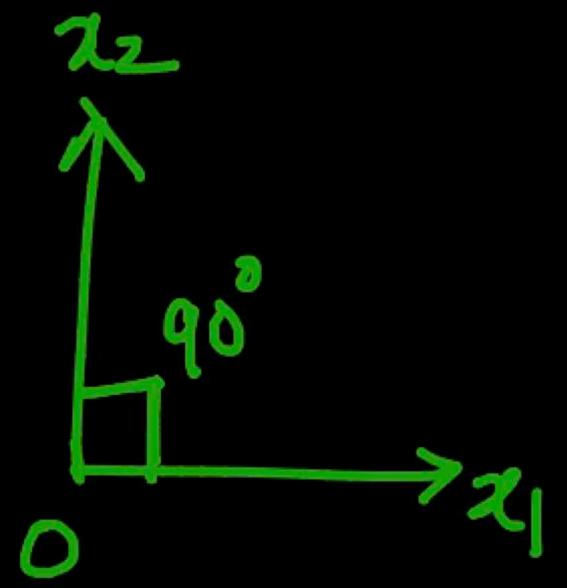
$x_3 \parallel x_4$ ← parallel

Orthogonal vectors

$x_1 \perp x_2$ (perpendicular)

$\overbrace{x_1 \cdot x_2 = 0}$ & $\overbrace{\|x_1\| \neq 0}$

$\underbrace{\|x_2\| \neq 0}$



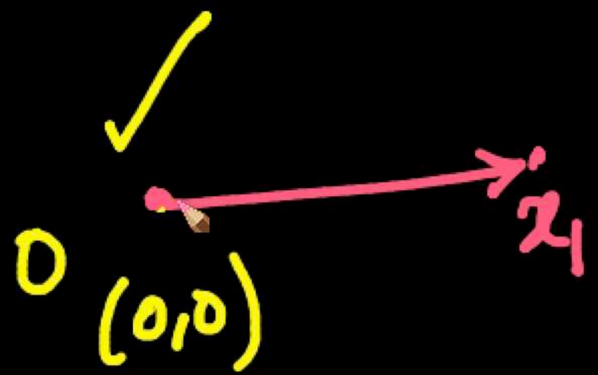
ML

Tld: $w^T x + w_0 = 0$

$w \cdot x$ (dot-product)

(Q)

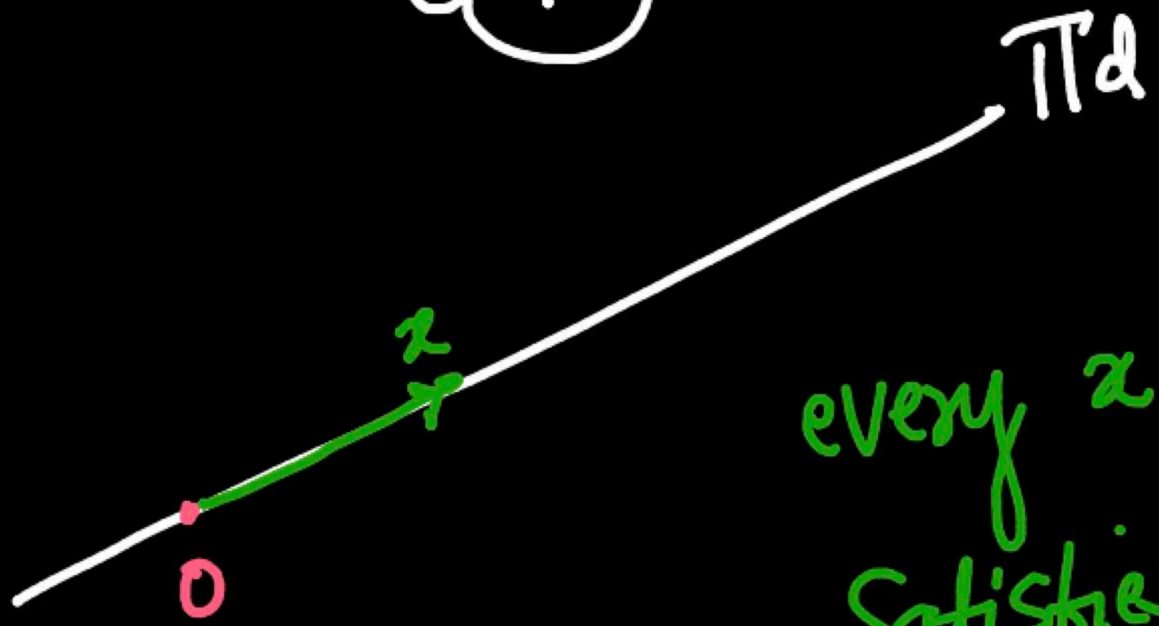
which vec has len = 0



$x_1 \cdot 0 = 0$

π_d: $w^T x + w_0 = 0$

Plane through origin



every x on π_d satisfies $w^T x + w_0 = 0$

$$w^T x + w_0 = 0$$

$$[w_1 \; w_2 \cdots \; w_d] \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + w_0 = 0$$

$$= 0$$

$y = mx + c \to 0$ if line through origin

π_d

$\pi_d: w^T x + w_0 = 0$

0

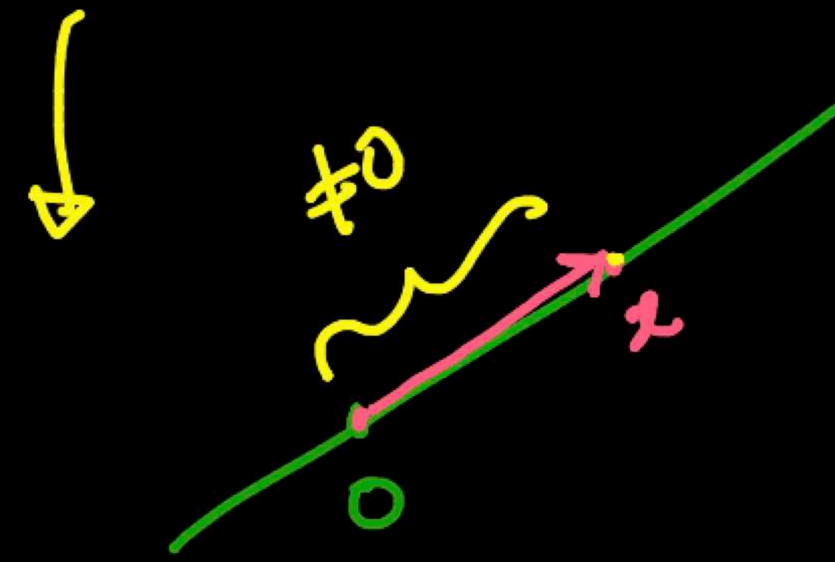
1

[$w_0 = 0$ for a plane through origin

every x on Πd satisfies

$\Pi d : w^T x = 0$

Plane through origin



$\Pi d : w^T x = 0$

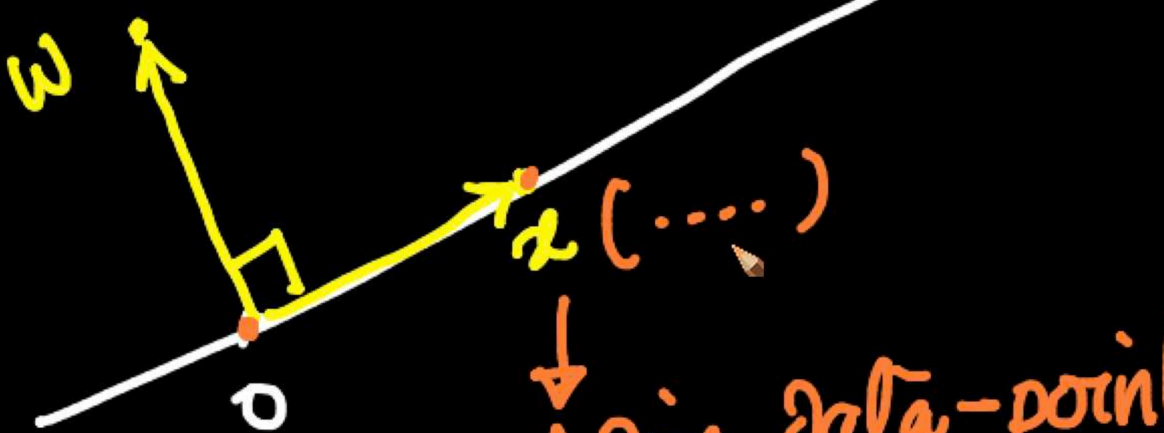
(?)

$w \cdot x = 0$

$\|w\| \|x\| \cos\theta = 0$

$\|w\| \neq 0$, $\|x\| \neq 0$, $\cos\theta = 0$

$w \perp x$



d-dim data-point (ML)

✓ $w^T x = w \cdot x = \|w\| \, \|x\| \cos\theta$

$x_1^T x_2 = x_1 \cdot x_2 = \|x_1\| \, \|x_2\| \cos\theta$

$w^T x = w \cdot x$ → d-dim point (_, _, _)

$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_d \end{bmatrix}_{d \times 1} \quad \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix}_{d \times 1}$$

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$[w_1, w_2 \cdots w_d]$

w

π_d

x

w

d-dim point

{ Plane through origin

$\pi_d : w^T x = 0$

$\|w\| = 0$

$\Rightarrow w = \vec{0}$

$w \cdot x$

$\Pi_d : w^T x + w_0 = 0$

if Π_d passes through origin $w_0 = 0$

$w \perp \Pi_d$

$\pi_d : w \cdot x + w_0 = 0$

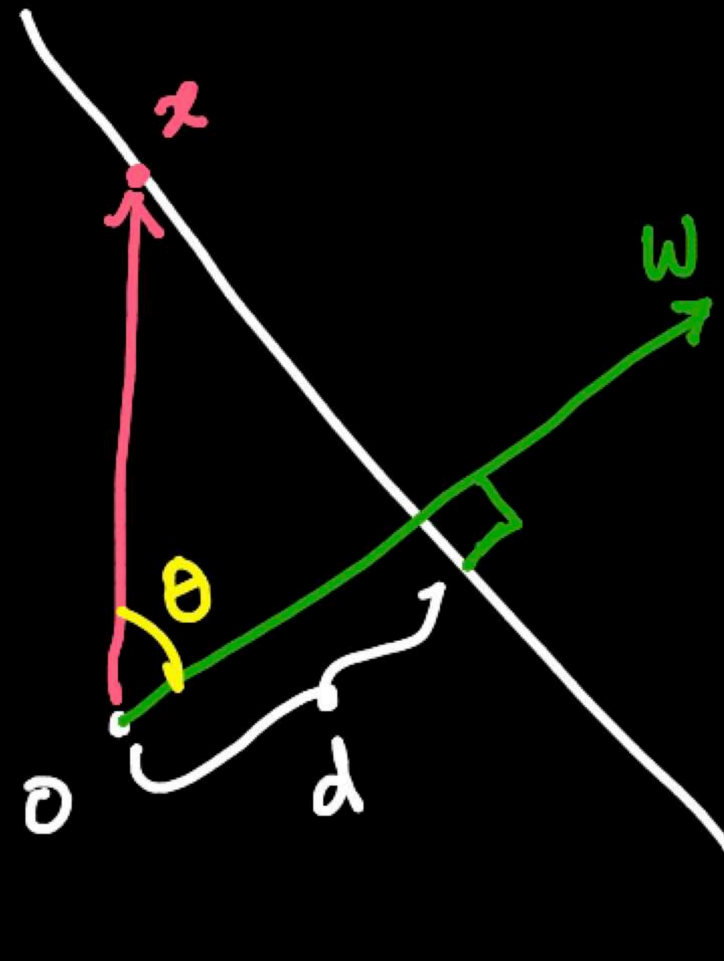
need not be

$w \not\perp x$

1. $w \perp \pi_d$
2. w passes through origin

$w \perp \Pi_d$: w through origin

[Plane not through origin

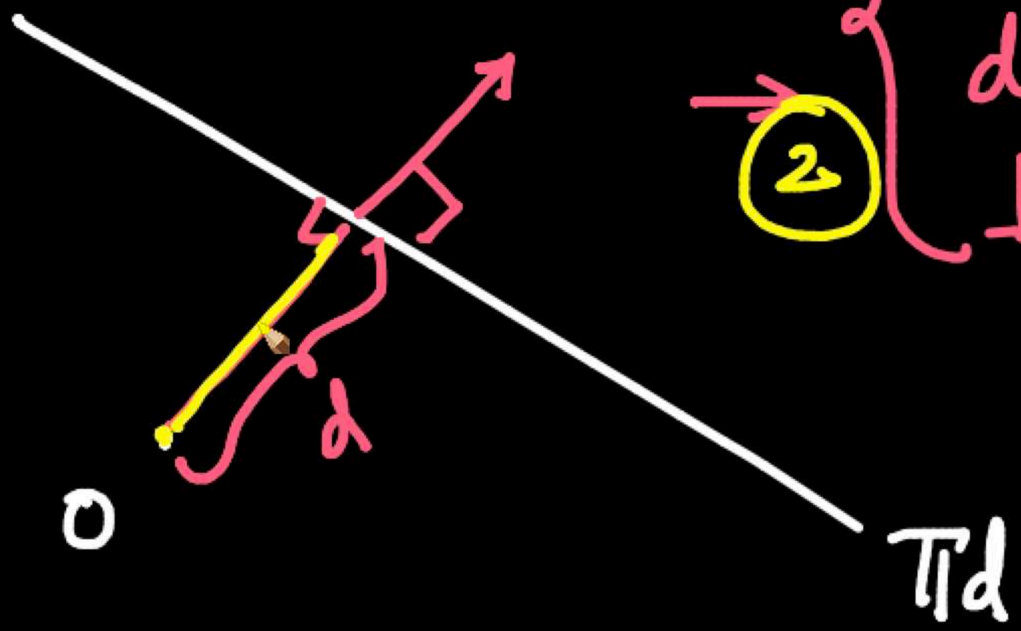


$w \not\perp x$

$\Pi_d : w^T x + w_0 = 0$

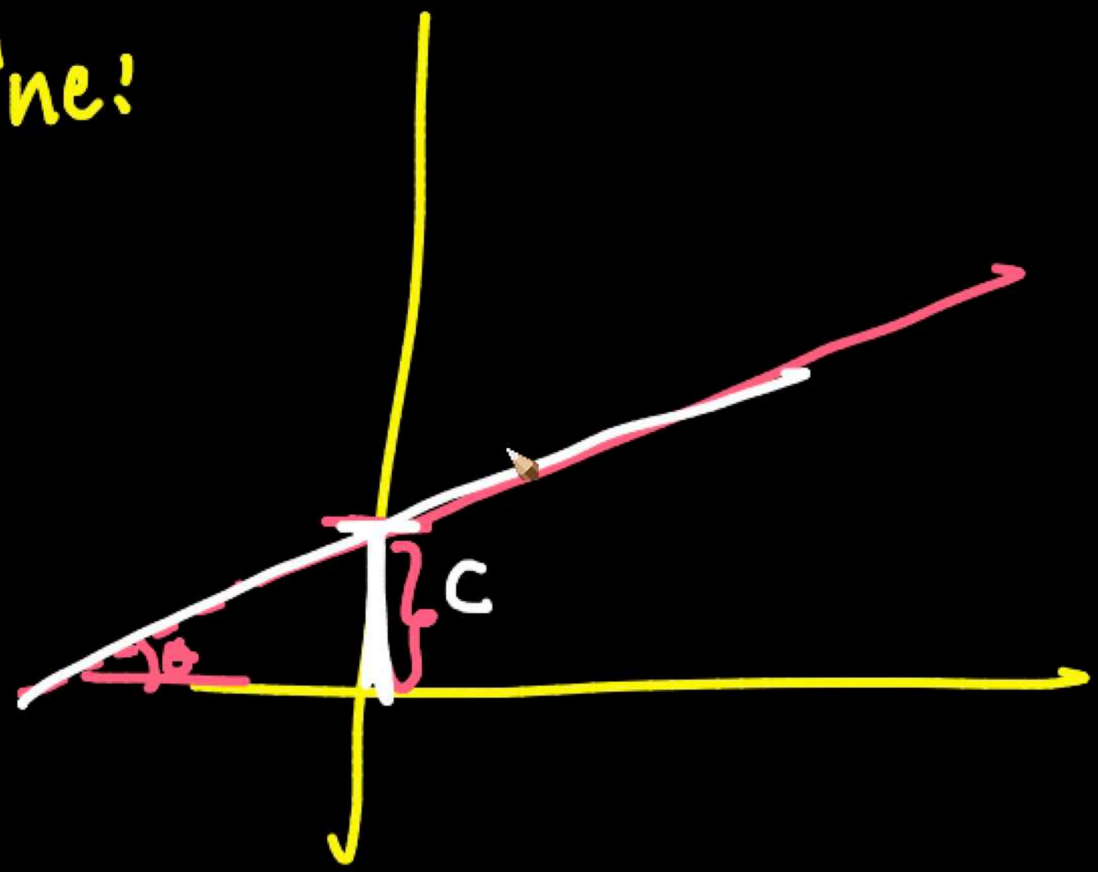
$\neq 0$

define a plane:

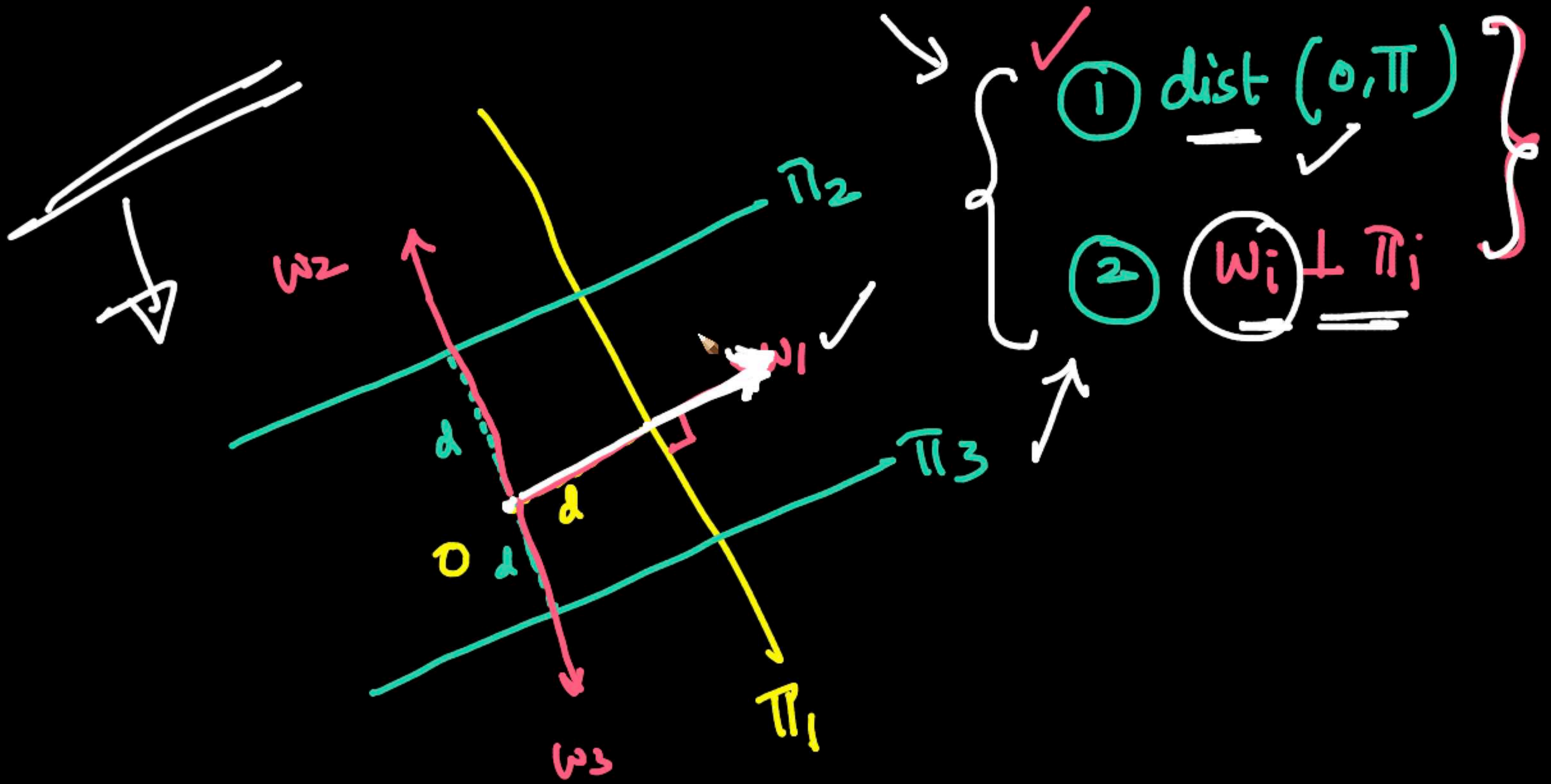


1. dist (O, Π_d) ← wo
2. direction that is ⊥ to Π_d

Line:



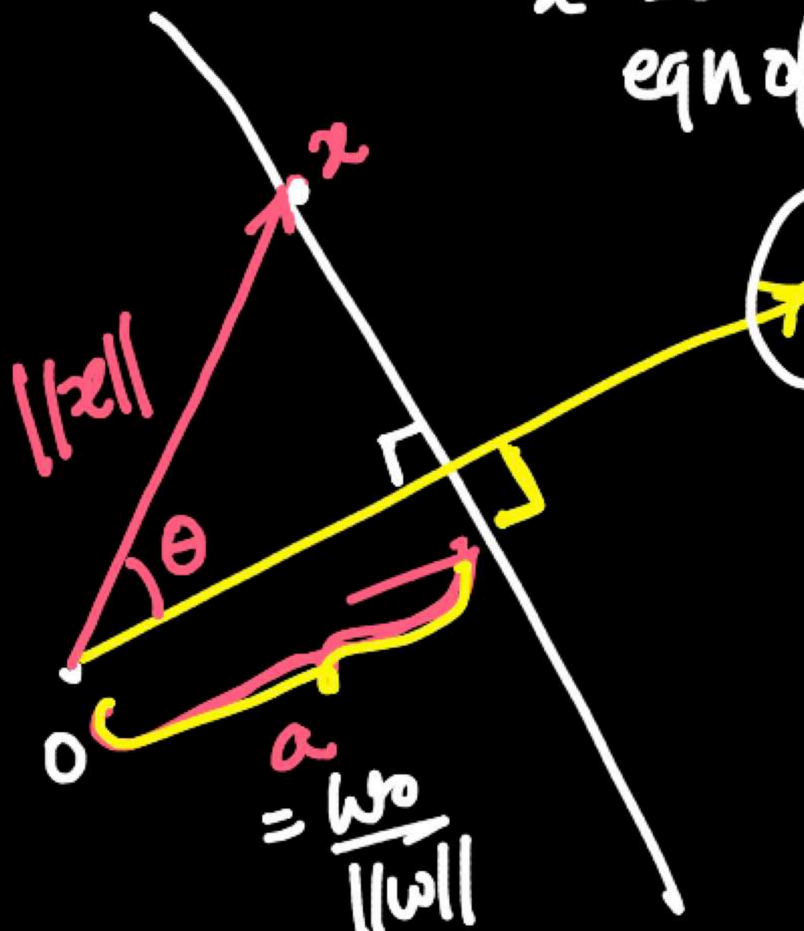
✓ m, c ✓



x satisfies eqn of π_d

$w \perp \pi_d$

$w \not\perp x$



$w^T x + w_0 = 0$

$||w||\,||x||\cos\theta + w_0 = 0$

$\frac{a}{||x||} = \cos\theta$

$a = -\frac{w_0}{||w||}$

$\pi_d : w^T x + w_0 = 0$

d-dim

scalar

① $w \perp \pi_d$

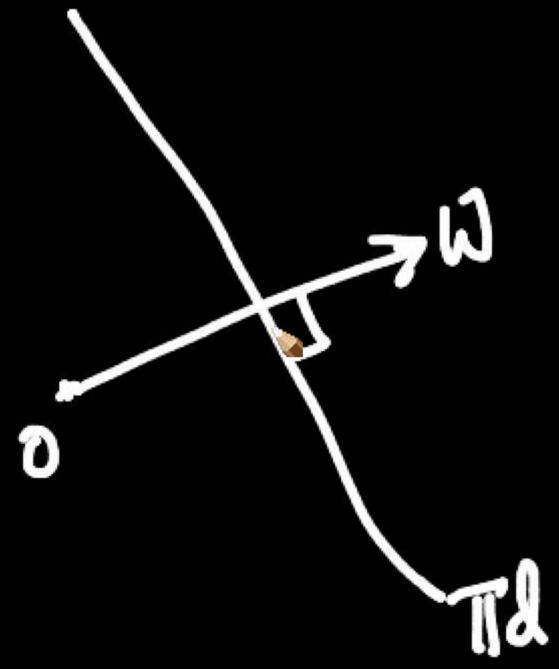
② $\text{dist}(0, \pi_d) = \frac{|w_0|}{\|w\|}$

if $w_0 = 0$ ↓ $\text{dist}(0, \pi_d) = 0$

$[\ldots\ldots]$

$[1, w, \ldots]$

Till now → $\pi_d: w^T x + w_0 = 0$



dot product

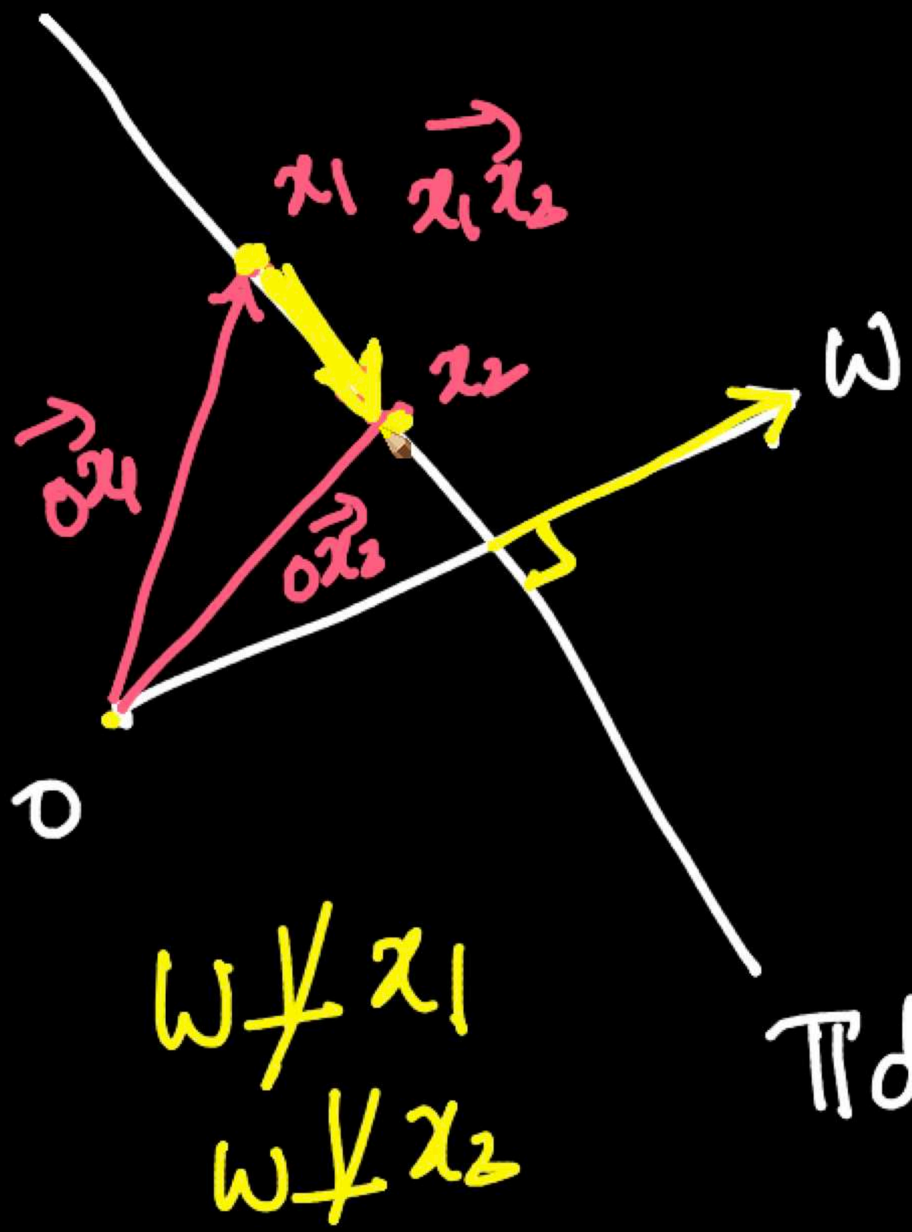
$x_1 \cdot x_2$

{ abc x5 x

5x6 ✓

5x 6.2 ✓

$\vec{x_1x_2} \perp w$



$\vec{0x_1} + \vec{x_1x_2} = \vec{0x_2}$

$x_1 + \vec{x_1x_2} = x_2$

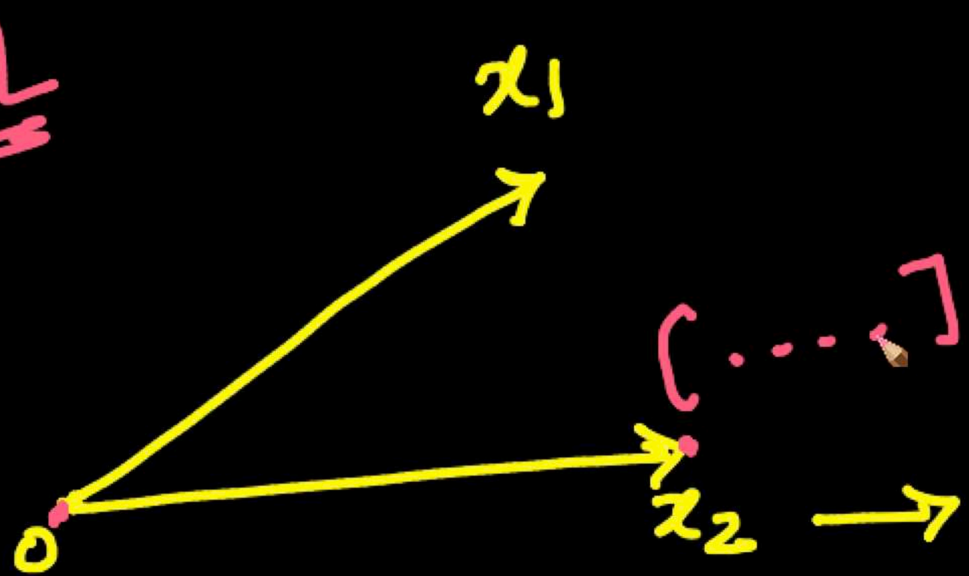
$\Rightarrow \vec{x_1x_2} = x_2 - x_1$

$x_2 - x_1 \perp w$ ✓

$w \nparallel x_1$

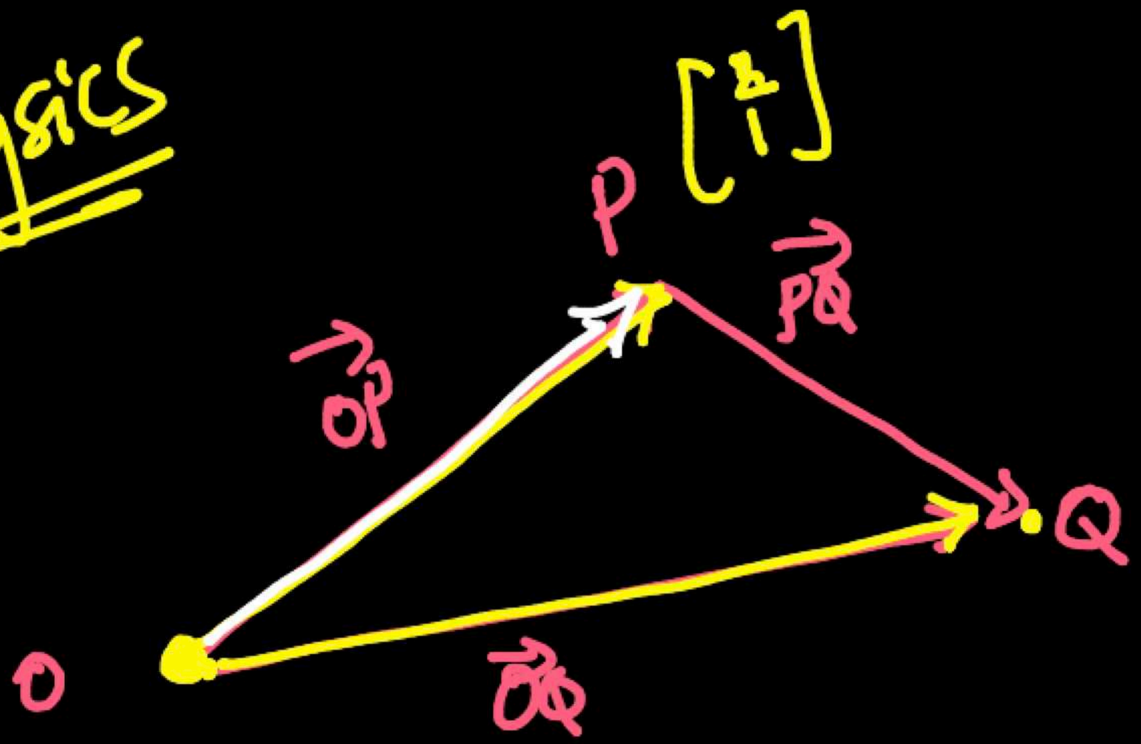
$w \nparallel x_2$

ML



→ not a ∂-dim data point

Physics



$\vec{OP} + \vec{PQ} = \vec{OQ}$

$\vec{PQ} = \vec{OQ} - \vec{OP}$

$\begin{bmatrix} 3 \\ 4 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

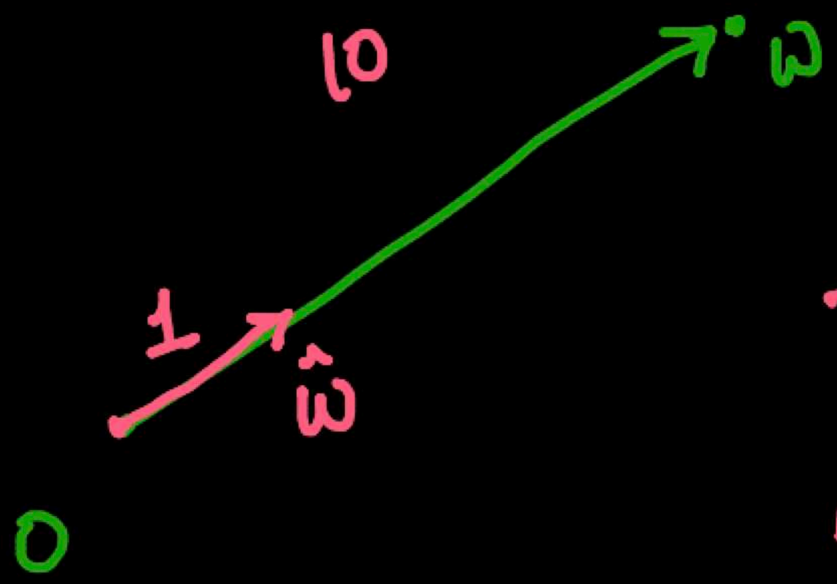
$\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

(0,0)

✓ $\vec{PQ} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$\vec{OP} + \vec{PQ} = \vec{OQ}$

$\begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$



ŵ : unit vector ← one

$$\{\pi d : \hat{w} \cdot x + w_0 = 0$$

w = unit vector

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2

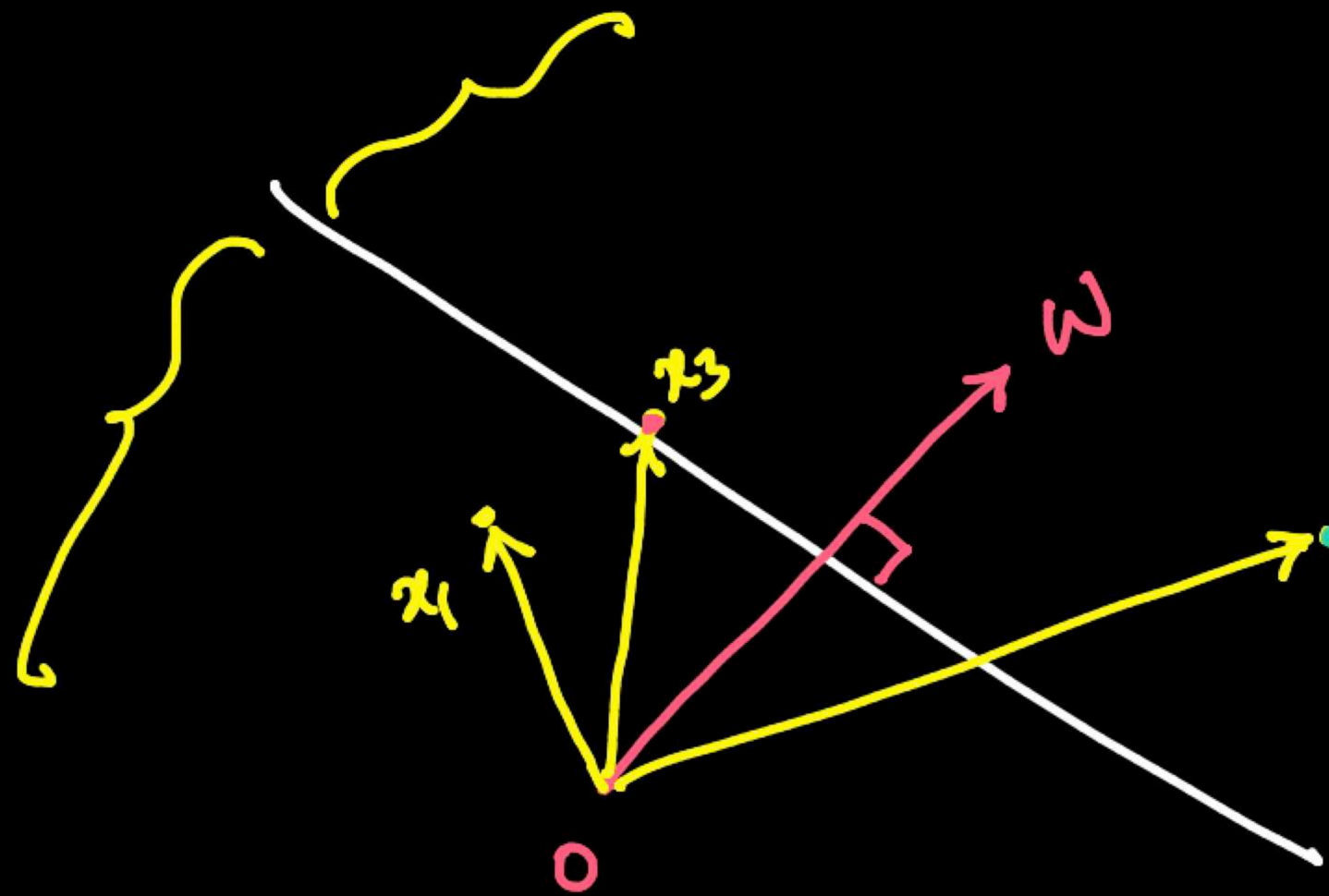
$x_{qj} : [10, 2, \dots]$

$\cdot x_{qj}$

half-spaces

$\pi_d : \underline{w}^T \underline{x} + w_0 = 0$

$[1, w, \dots]$



$w^T x_3 + w_0 = 0$ ✓

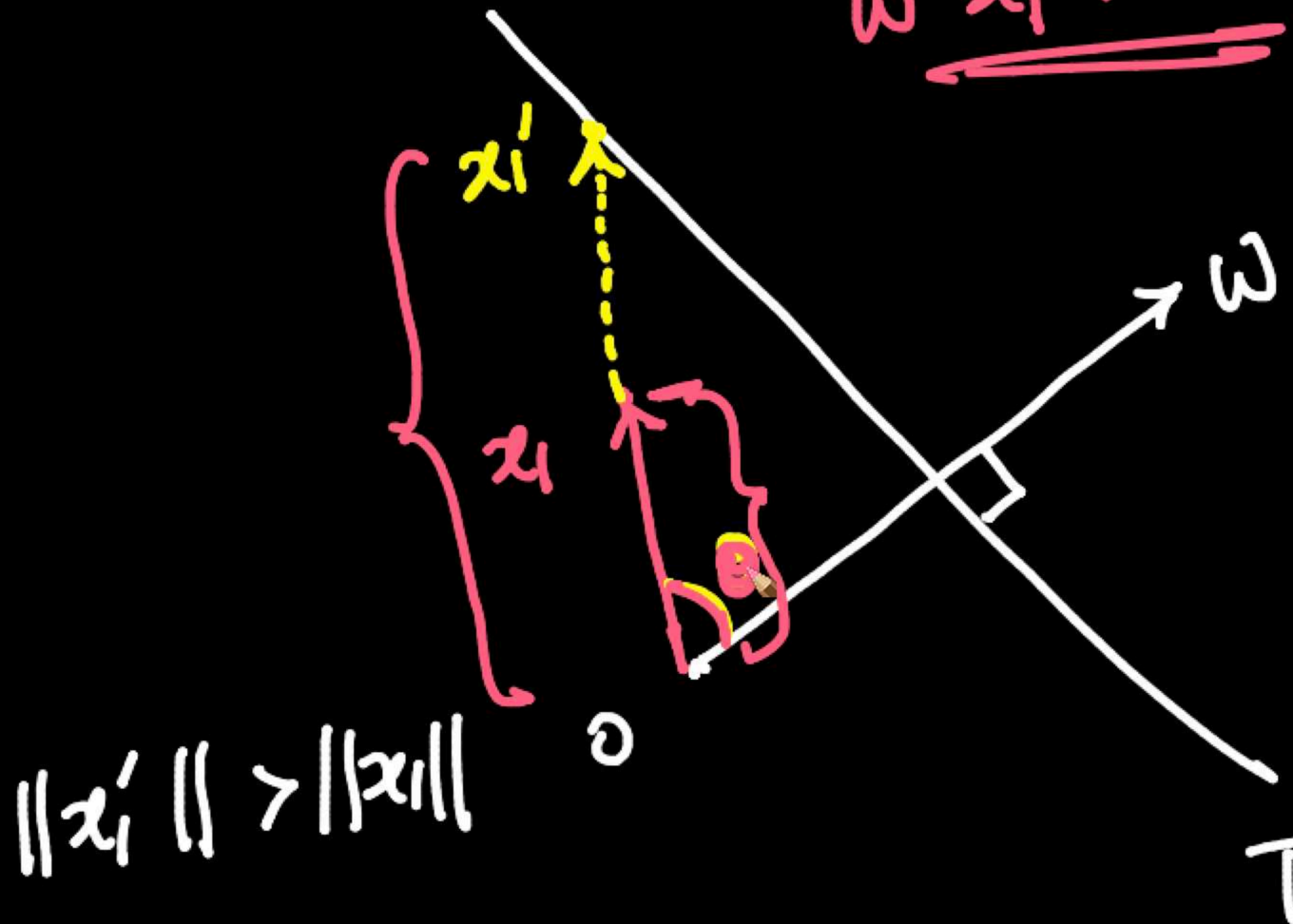
$\{ w^T x_1 + w_0 \leq 0$

$w^T x_2 + w_0 > 0$

$\pi d: w^T x + w_0 = 0$

$w^T x_i + w_0 < 0$

$w^T x_i + w_0 < 0$



$$= w^T x_i' + w_0 = 0$$

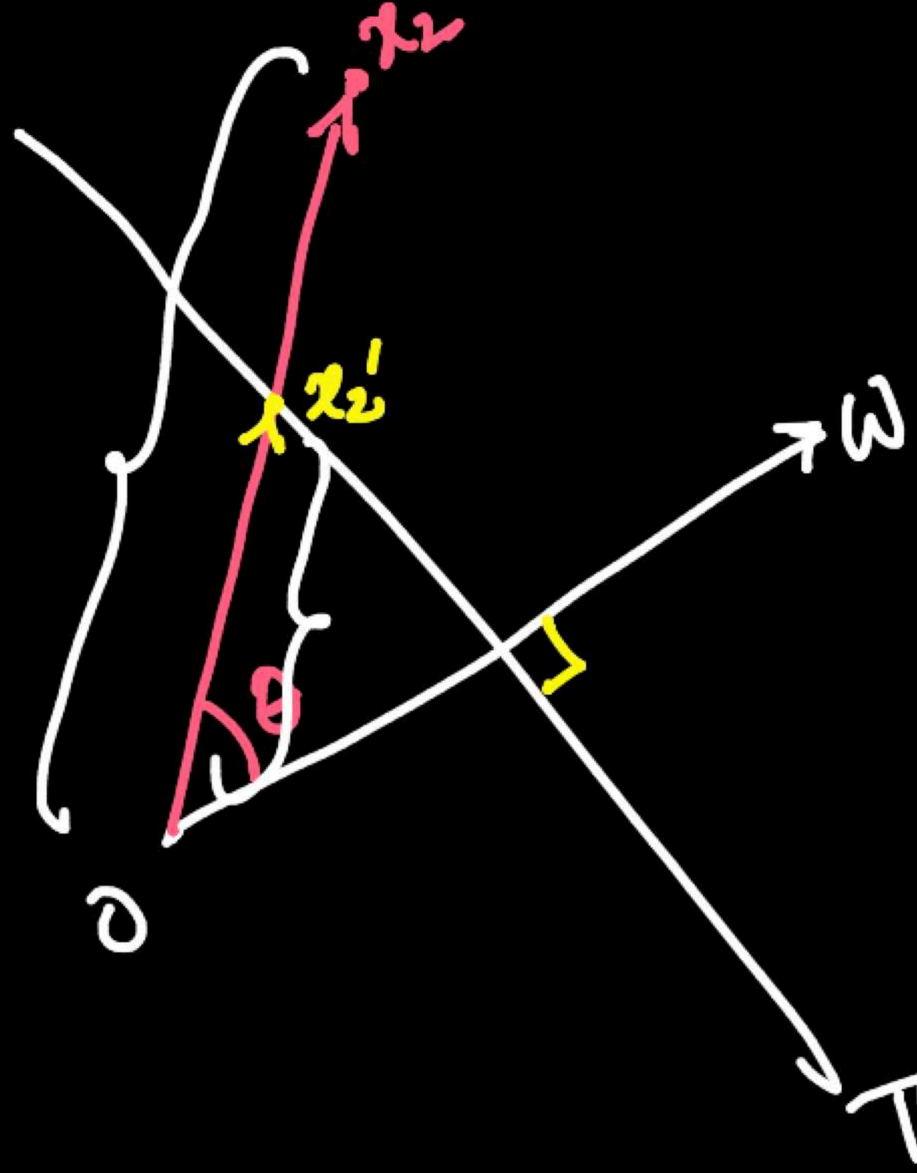
$$= \|w\| \|x_i'\| \cos\theta + w_0 = 0$$

-ve

$\|x_i\|$

$2 \cdot 5 \cdot \_ - 10 = 0$

$2 \cdot 3 \_ - 10 < 0$

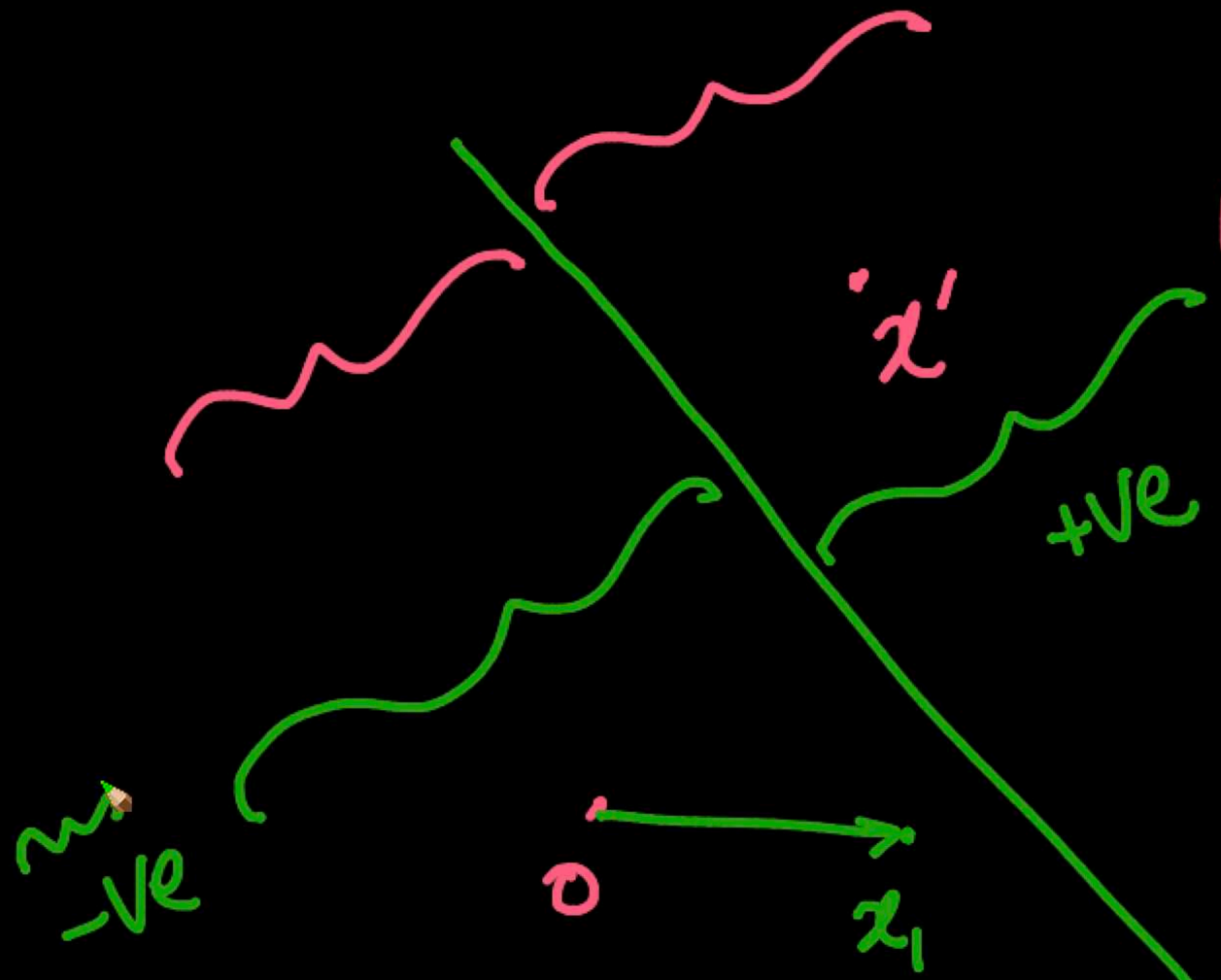


$$w^T x_2' + w_0 = 0$$

$$\|w\| \|x_2'\| + w_0 = 0$$

$$\cos\theta$$

$$w^T x_2 + w_0 > 0$$

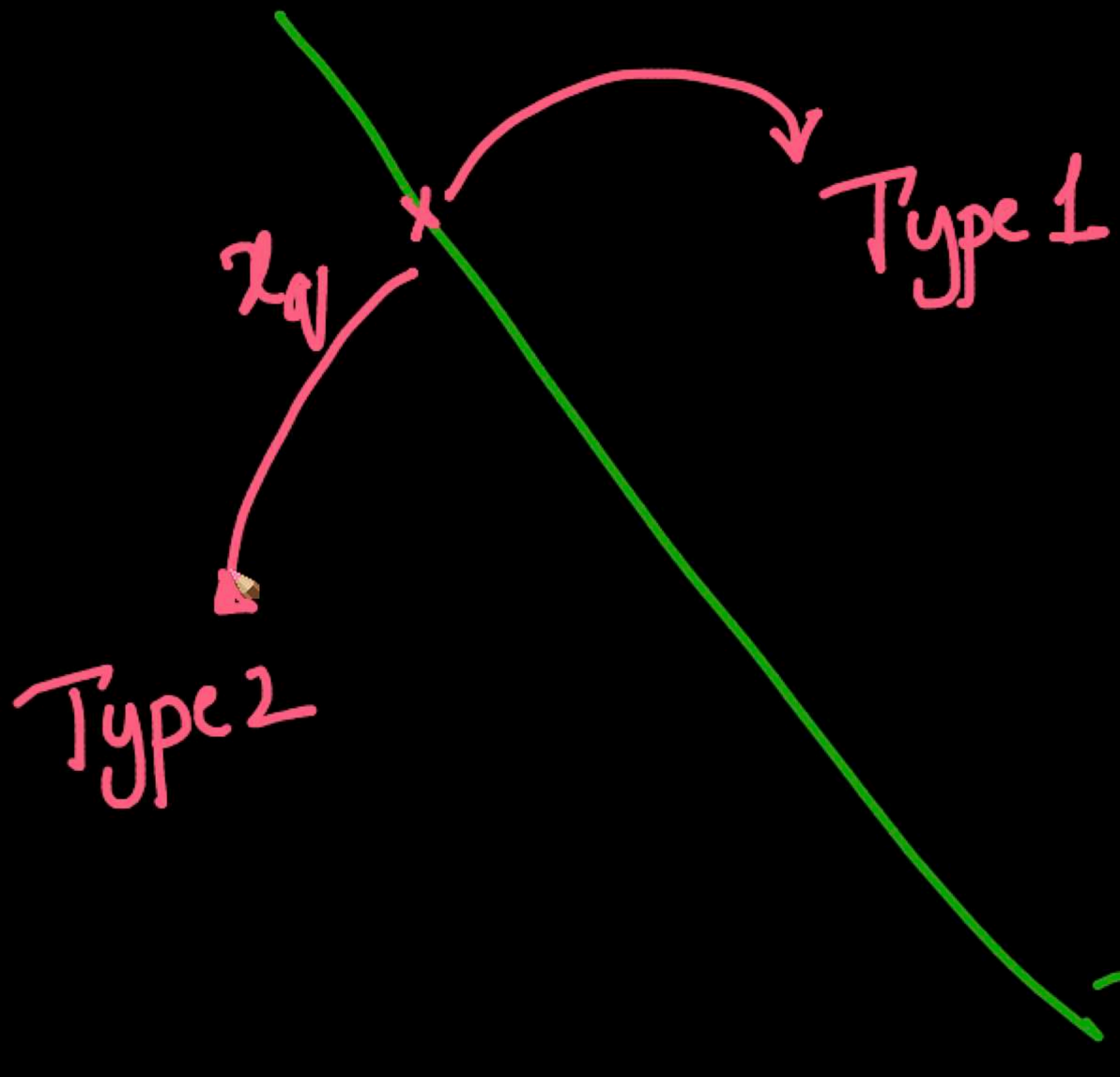


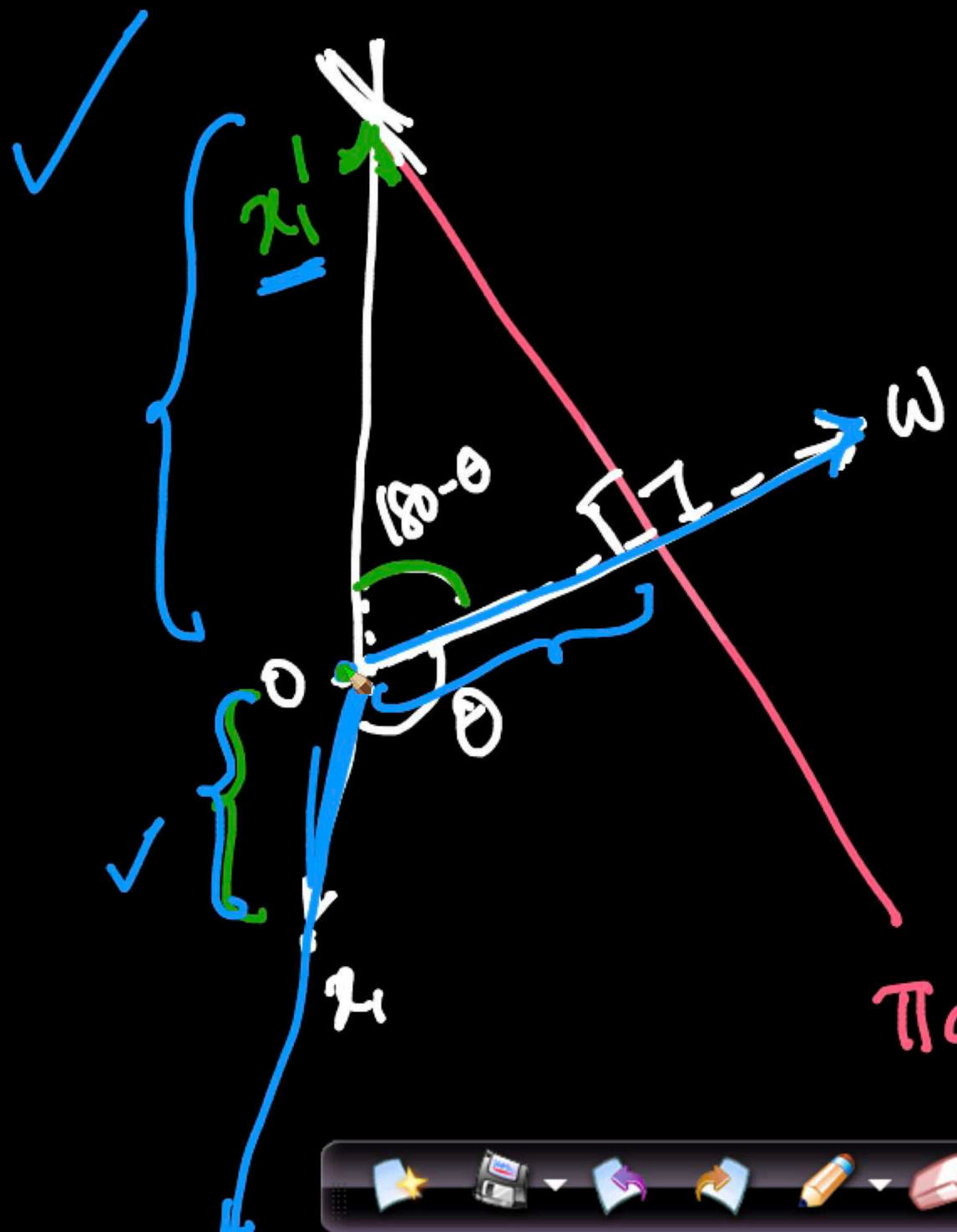
$w^T x' + w_0 = 0$ → on

→ < 0 : -ve HS

→ > 0 : +ve HS

Plane: $w^T x + w_0 = 0$





$\cos(180-\theta) = -\cos\theta$

$\|w\| \|x_1'\| (\cos\theta) + w_0 = 0$

$w \cdot x_1 + w_0$

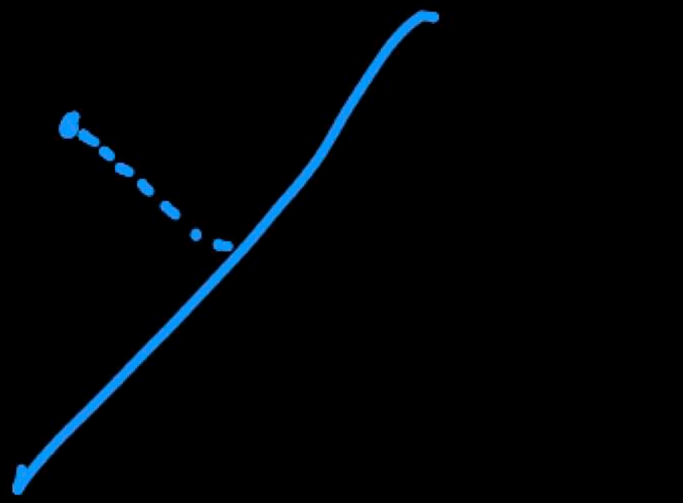
$\|w\| \|x_1\| (-ve) + w_0 = 0$

recap:

1. Π_d: $w^T x + w_0 = 0$ ✓

2. half-spaces: which side ✓ Type 1 or 2

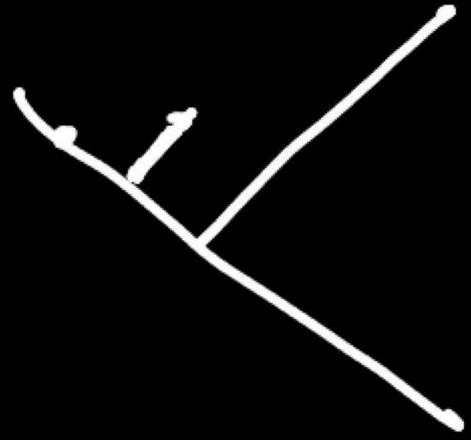
3. distance

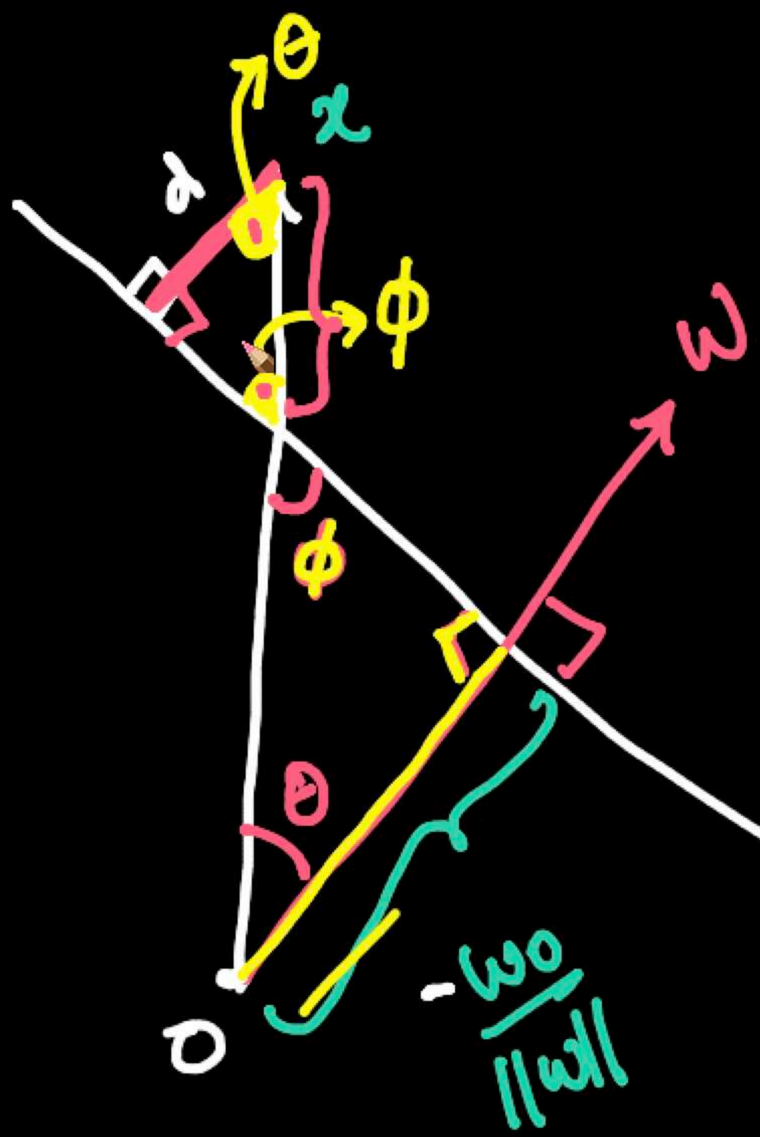


distances from

x to πd

Confident





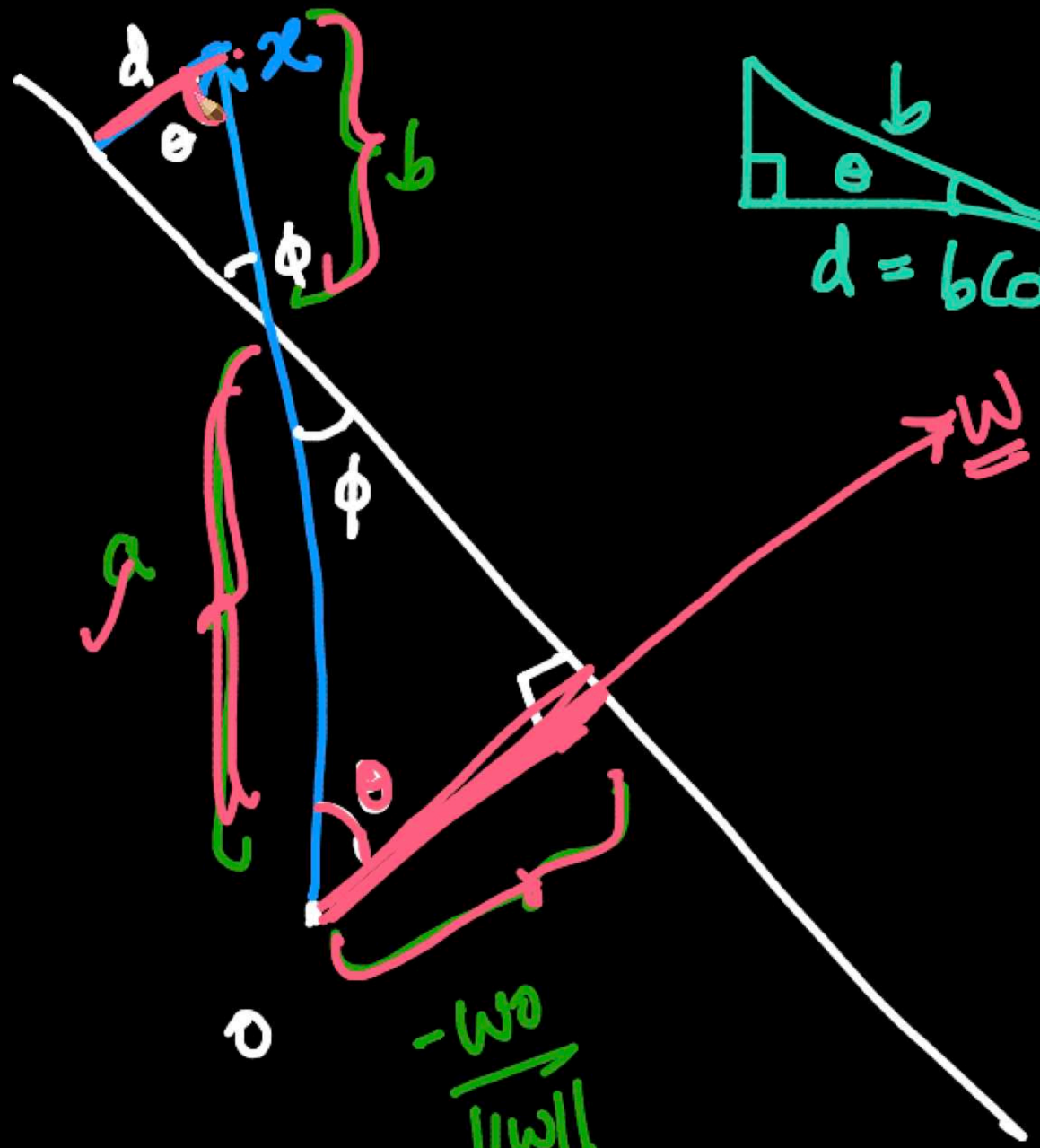
w, w_0, x ← given

d ← find

θ

$\phi = 90 - \theta$

$\pi_d : w^T x + w_0 = 0$



$w, w_0, \underline{z}$ ← given

$$a\cos\theta = \frac{-w_0}{\|w\|}$$

$$a = \frac{-w_0}{\|w\| \cos\theta}$$

$$b = \|x\| - w$$

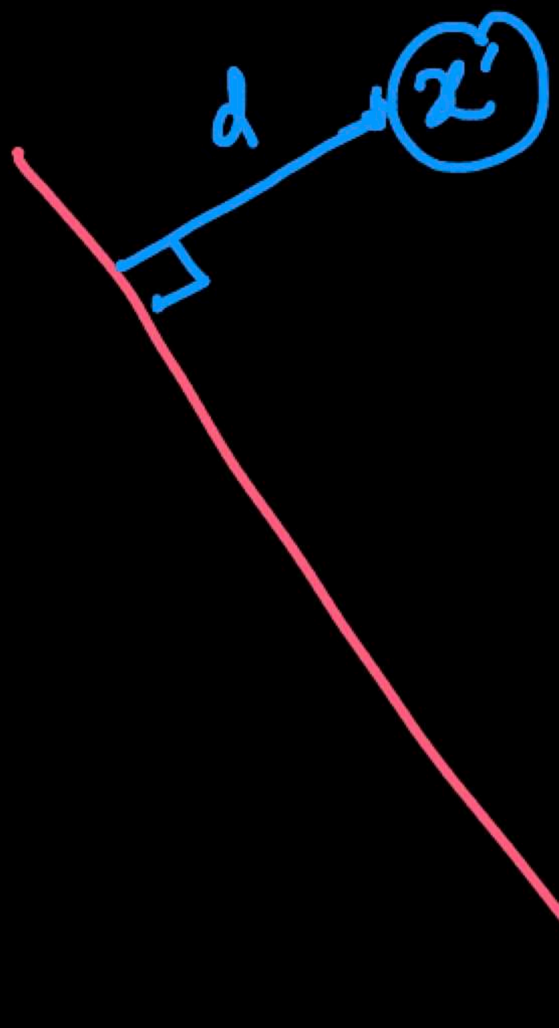
$$b = \|x\| + \frac{w_0}{\|w\| \cos\theta}$$

$$d = b\cos\theta = \|x\|\cos\theta + \frac{w_0}{\|w\|}$$

$$d = \|x\| \cos\theta + \frac{w_0}{\|w\|}$$

$$= \frac{\|w\| \, \|x\| \cos\theta + w_0}{\|w\|}$$

$$d = \frac{w^T x + w_0}{\|w\|}$$



$$d = \frac{w^T x' + w_0}{\|w\|}$$

$$\pi: w^T x + w_0 = 0$$

Try it out on your own!

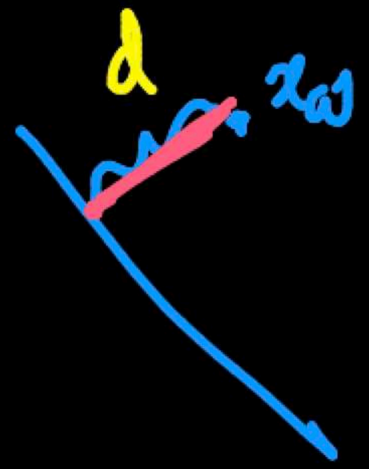
1 Pd: $w^T x + w_0 = 0$

$\frac{w_0}{\|w\|}$

2 half-spaces! x_q

$w^T x_q + w_0$ → >0: +ve, $=0$ → plane, <0: -ve

3



$\frac{w^T x_q + w_0}{\|w\|} = d$

$\|w\| = 3$

$\frac{2}{3}$

scalar

w

w

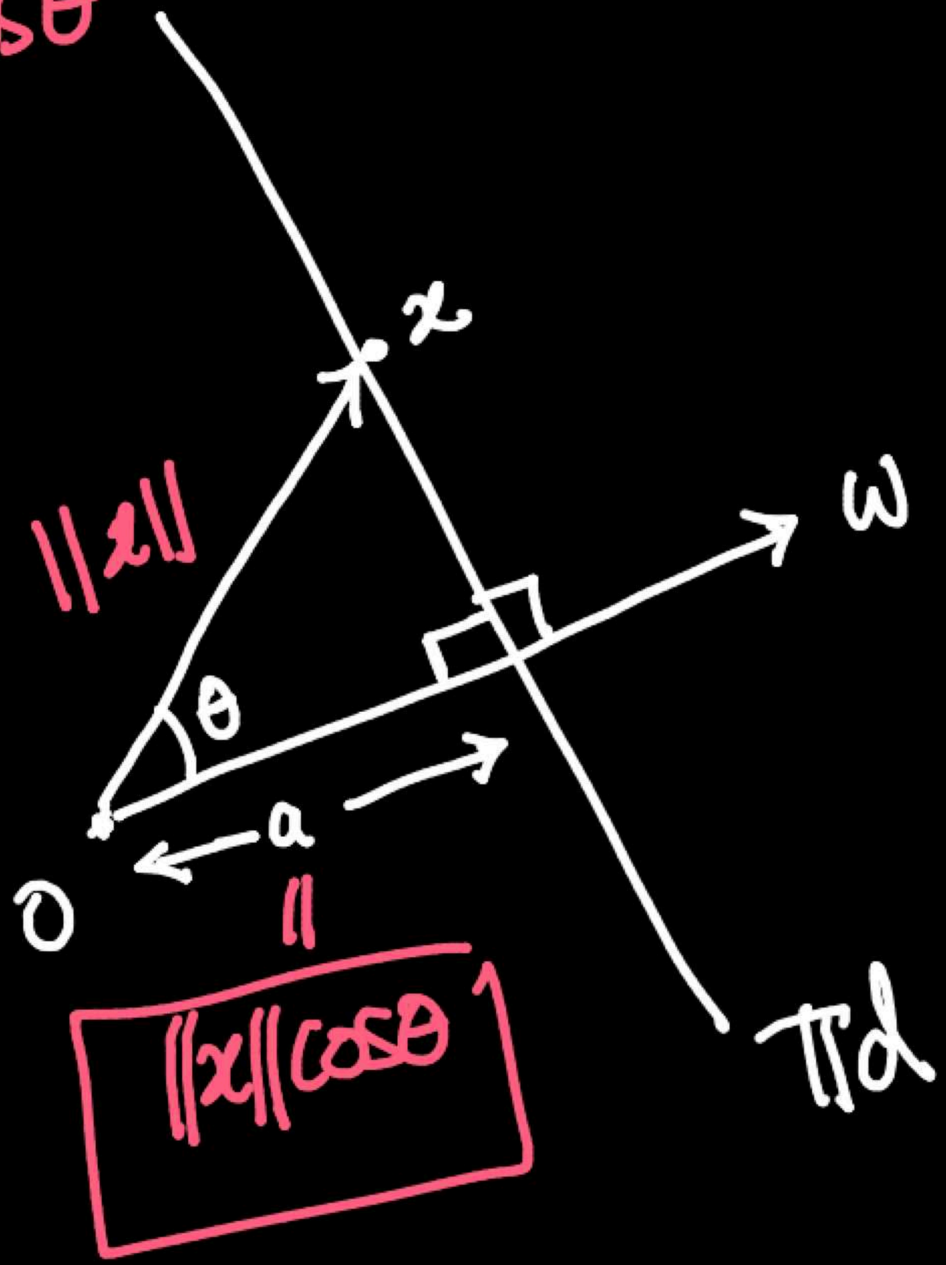
0

Td: $\underline{w}^T x + w_0 = 0$

w_0^2

(w : unit-vec)

$$\frac{a}{\|x\|} = \cos\theta$$

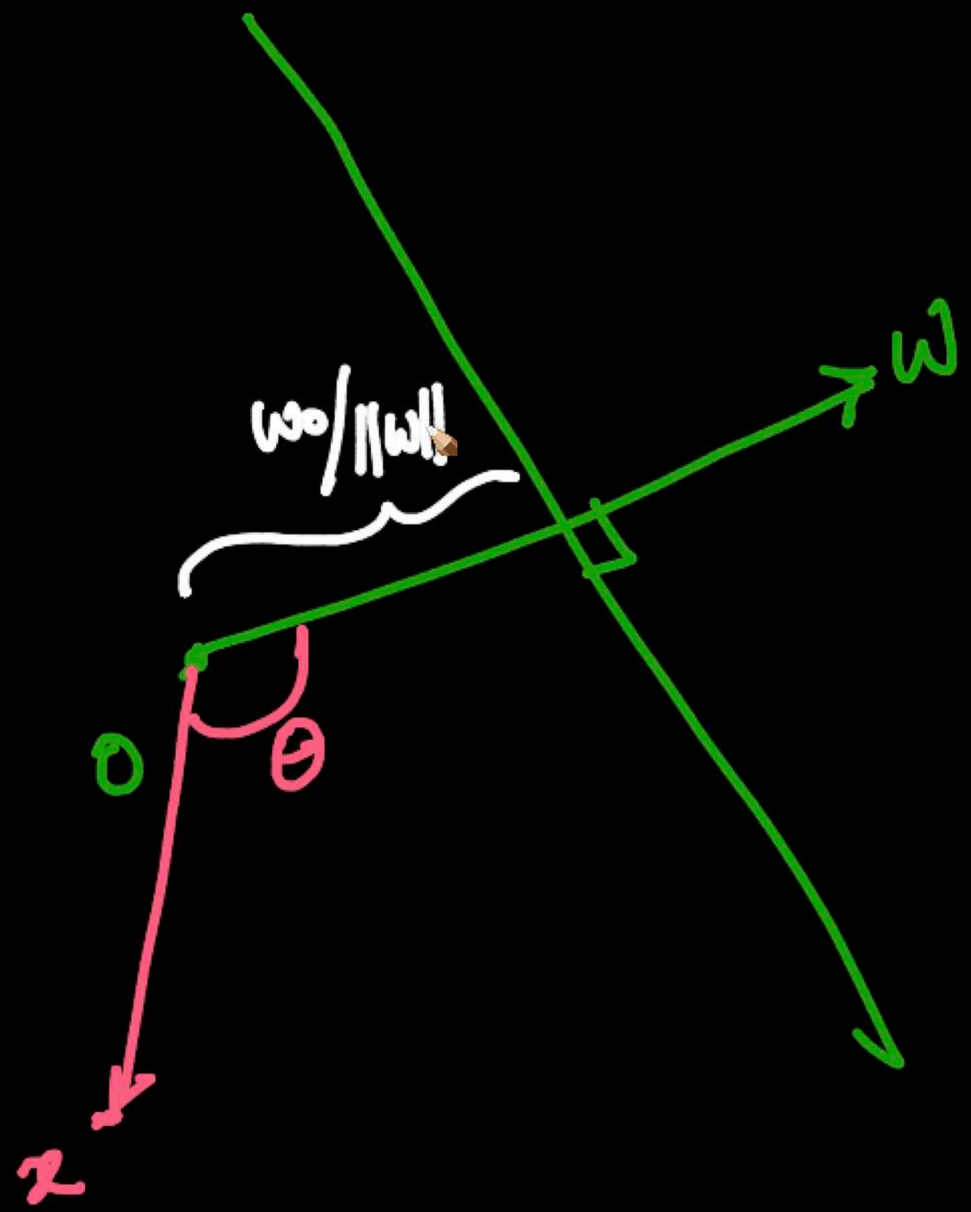


$$w^T x + w_0 = 0$$

$$\|w\| \|x\| \cos\theta + w_0 = 0$$

$$\|w\| \cdot a + w_0 = 0$$

$$a = \frac{-w_0}{\|w\|}$$



$$\|w\|\|x\|\underbrace{\cos\theta}_{<0} + w_0 = 0$$

π_d : w_1 w_2 ← scalar

↑ d-dim

d+1 numbers →

Optimization

Calculus

differentiation

(Maxima & minima)

Brute-force!

1 Million

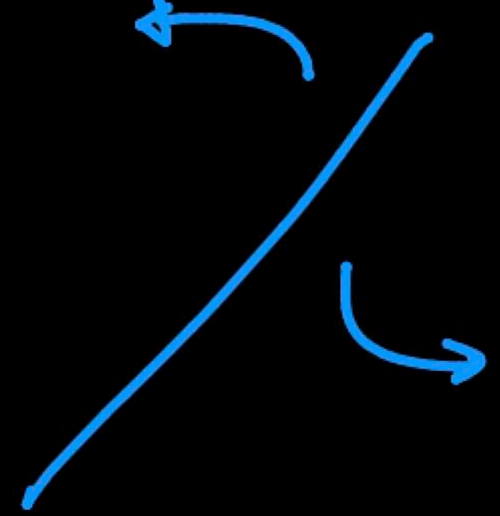
try out various combinations for d+1 numbers

Terrible

{ # misclassification for every combination of d+1 value

1

Data → good hyperplane



Logistic Regssn →
SVM →
DT; RF; GBDT →

DL: not X
Naive Bayes: X

✓ { K-NN : X

Fish sorting

5-dim

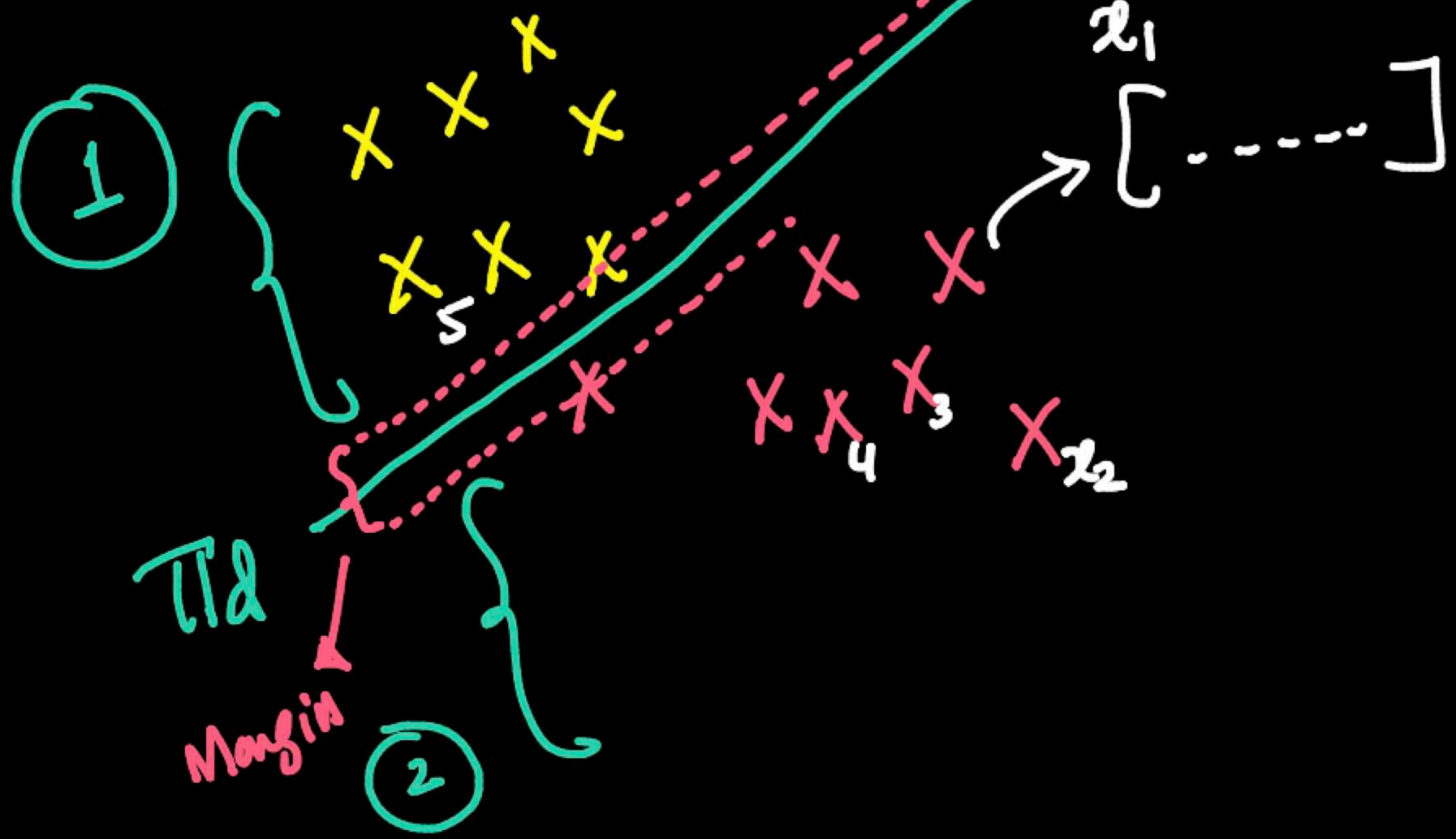
data-point

x: [l, w, wg, ...]

a fish

Margin $= \frac{2}{\|w\|}$ SVM

$\pi_d : \langle w, w_0 \rangle$



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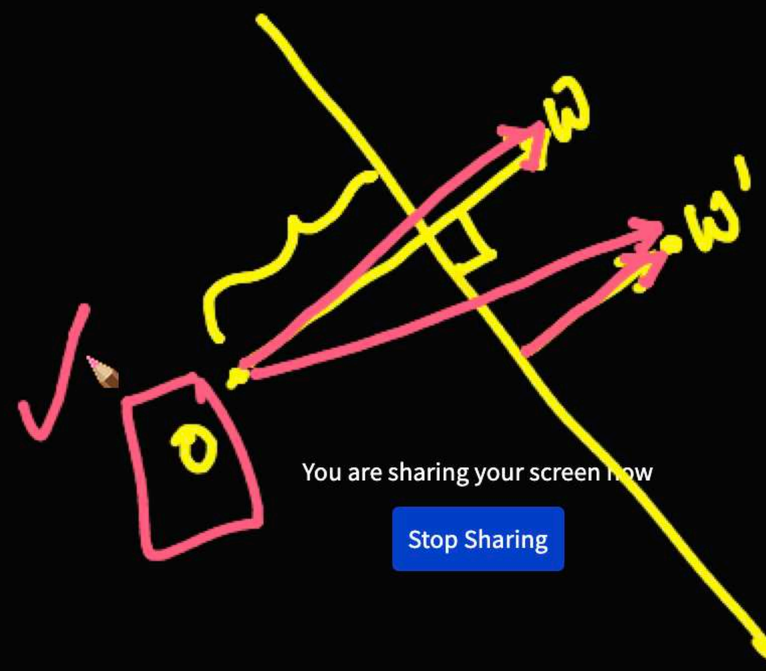

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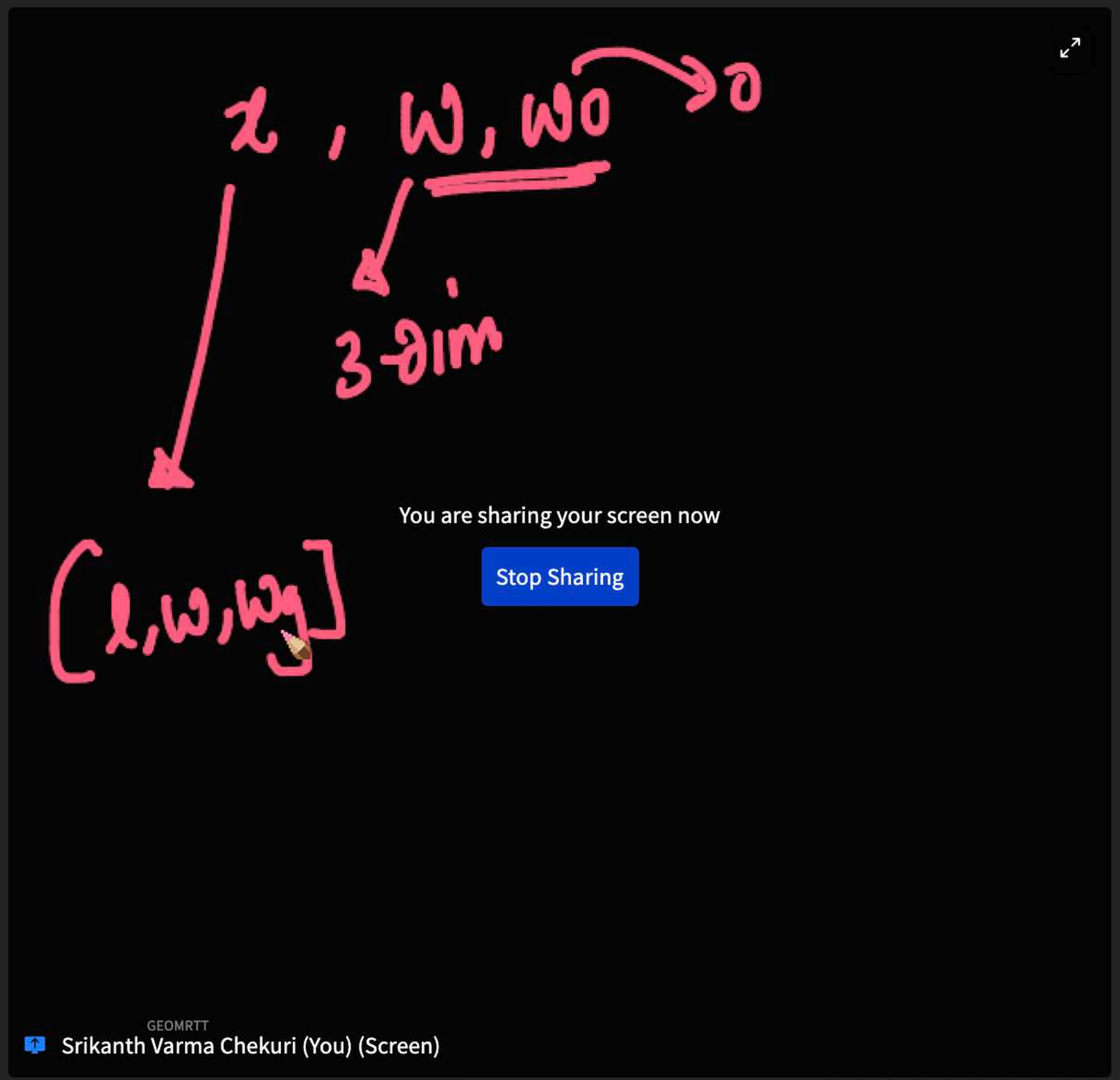

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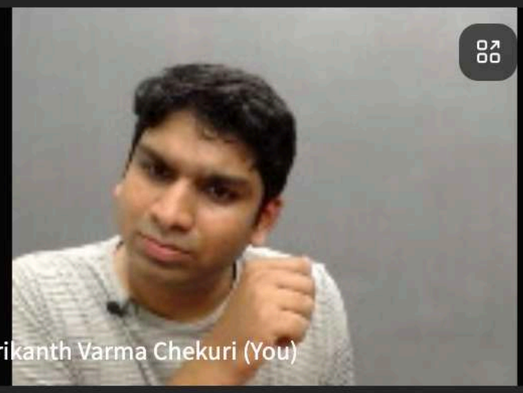

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Gowtham

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$x_1[\ ,\ ,\ ]$

w

$x_3 \cdot x_1$

0

x_4

x_2

π_d

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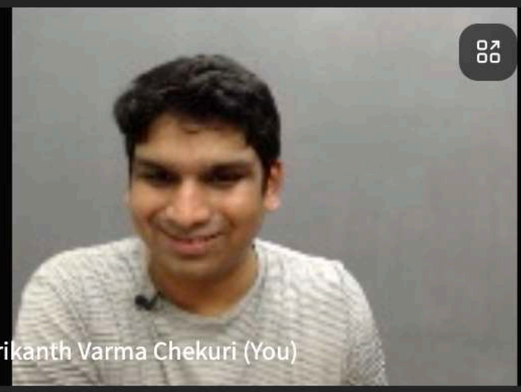

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