

Optimization - 2

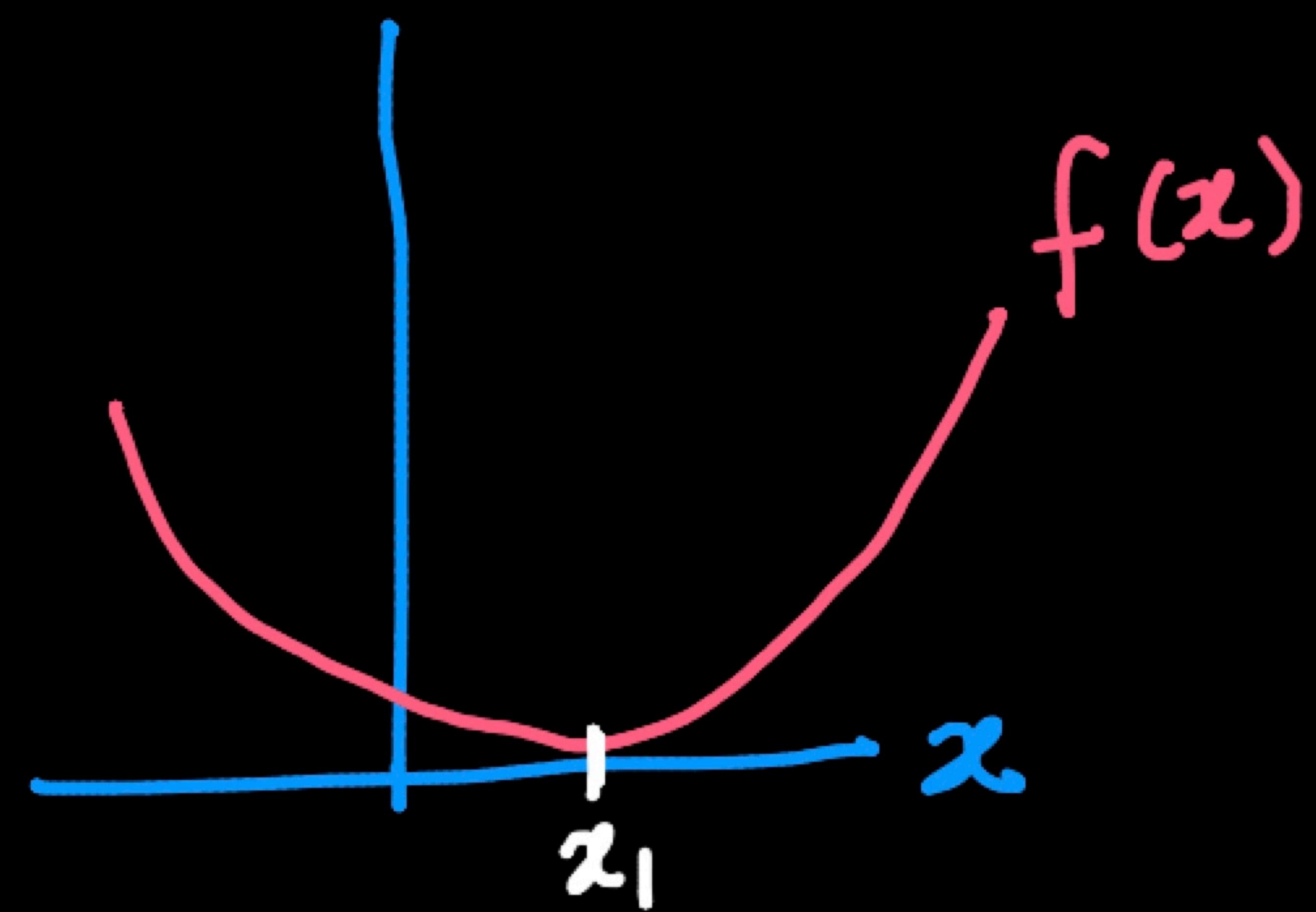
(Towards Gradient Descent)

class
@ 9:03 PM

- Maxima & minima
 - ↳ intuition ; geometry ; Math
- Partial derivatives & optima
- Saddle point
- Gradient descent - intuition

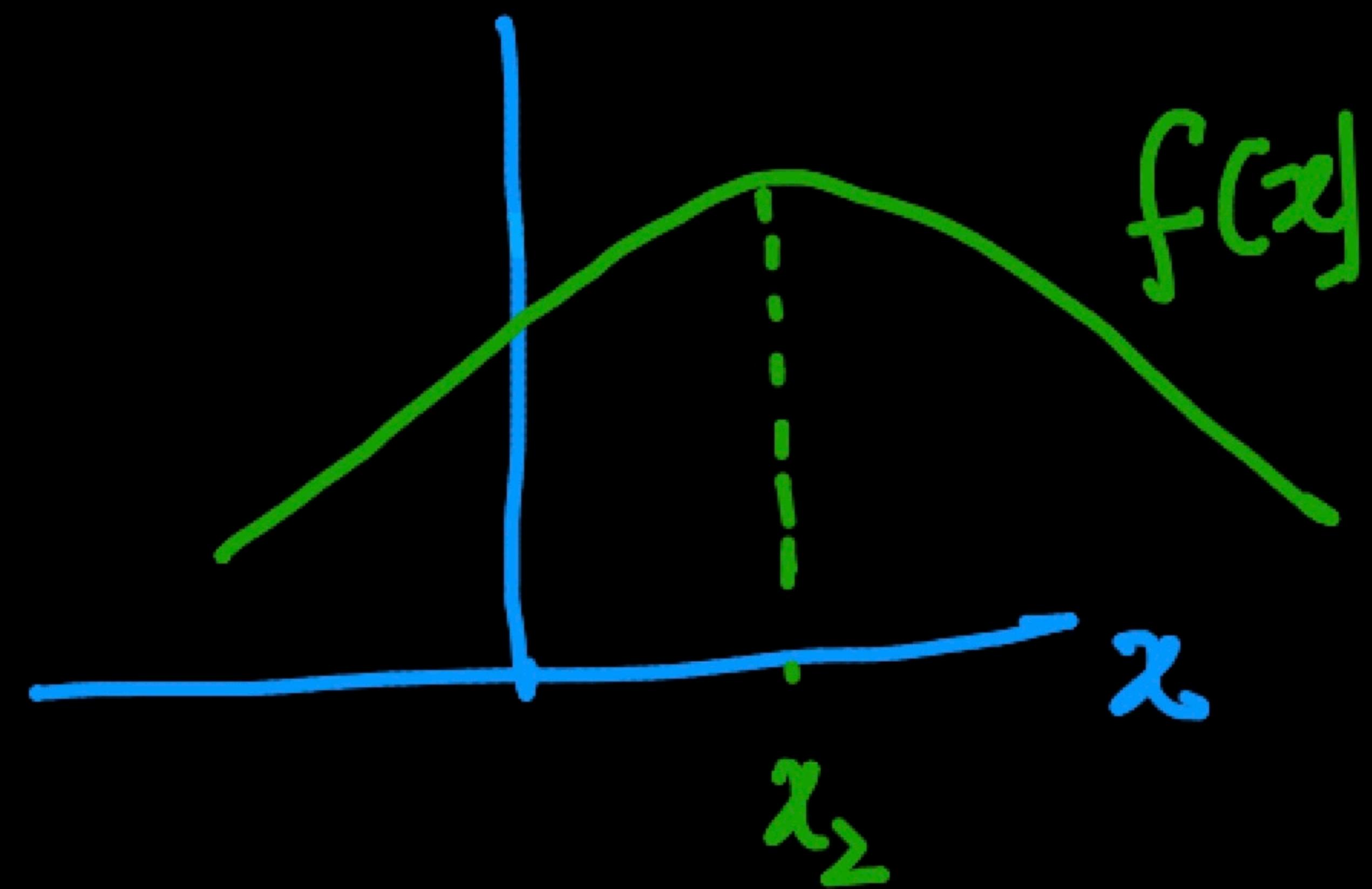
Maxima &
minima

Optima



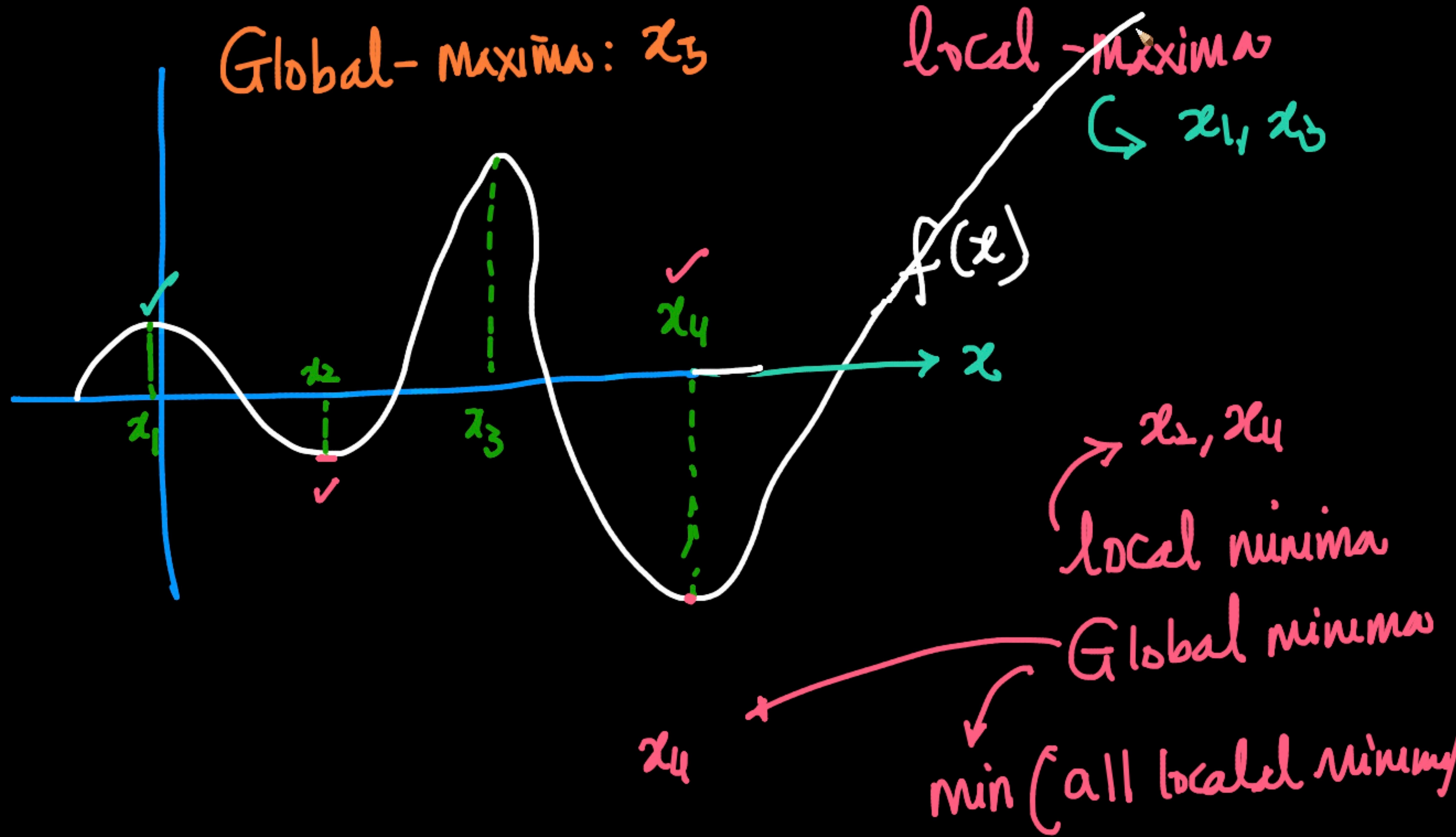
minimum @ x_1

$$\min f(x) = f(x_1)$$

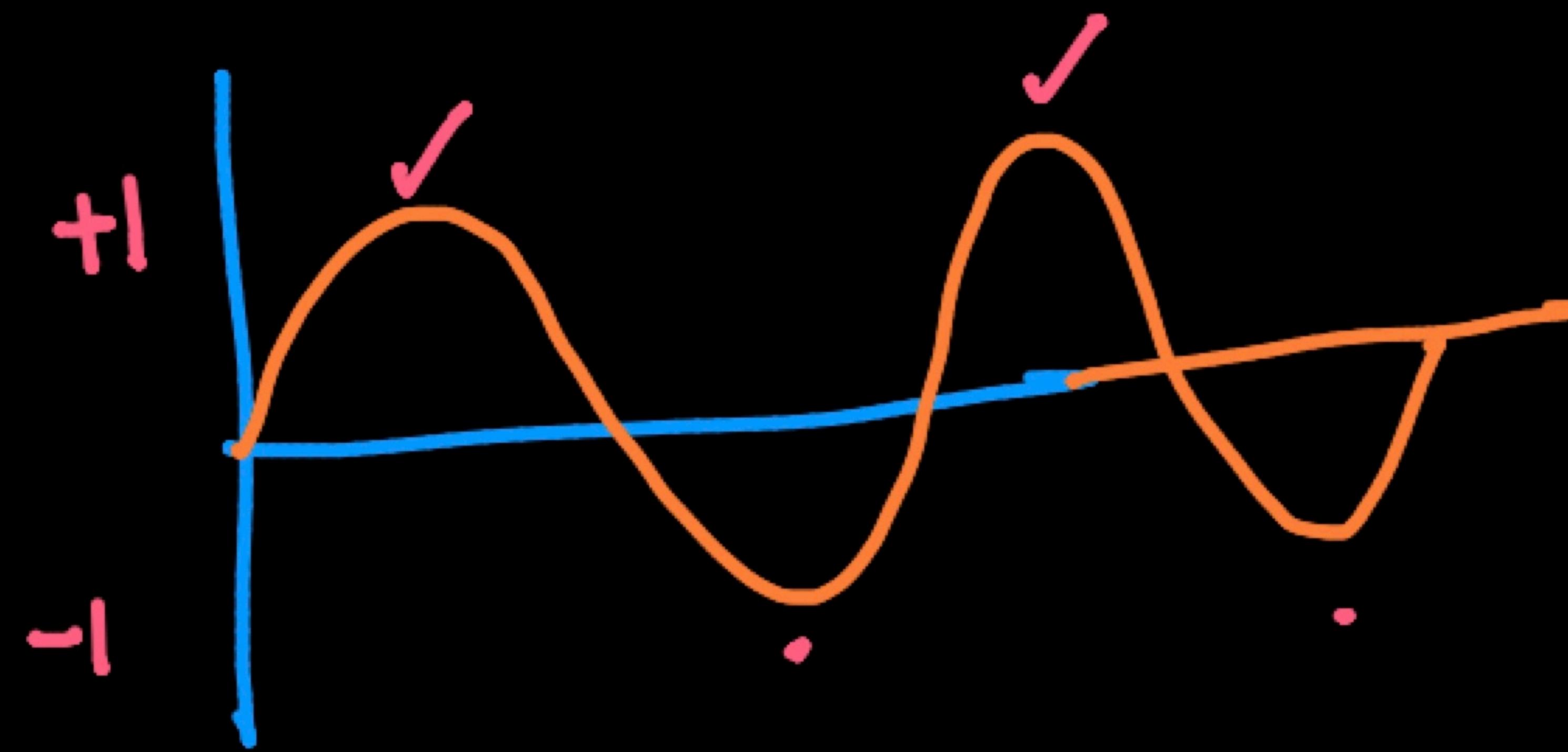


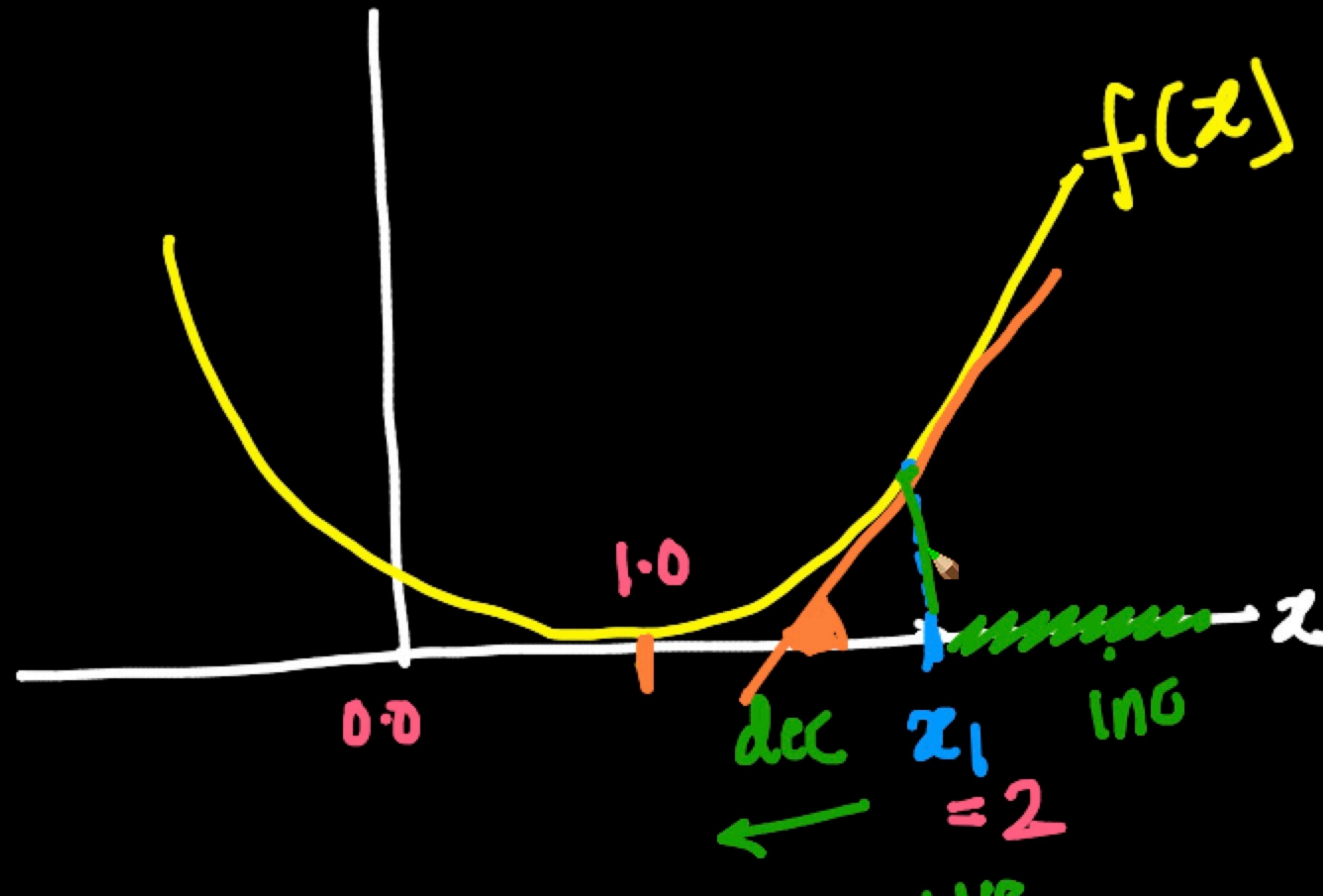
maxima @ x_2

$$\max f(x) = f(x_2)$$



(Q) $f(x)$: multiple global minima / maxima





$$\frac{df}{dx} \Big|_{x_1} = +ve$$

How to find
minima?

$$\frac{df}{dx} = 0$$

$$f = \frac{x \sin x + x^2 \tan x}{x^3 \sqrt{\sin x}}$$

(Let)

= NOT always easy to solve the

eqn $\frac{df}{dx} = 0 =$

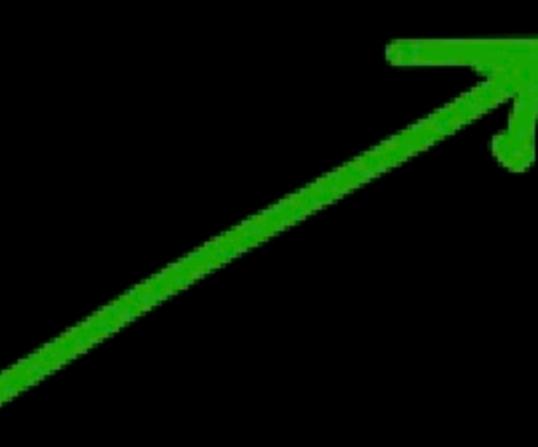
Code

① randomly pick x_1 on z-axis

②

$$\frac{df}{dx} \Big|_{x_1}$$

+ve



+ve

→ move in +ve dir



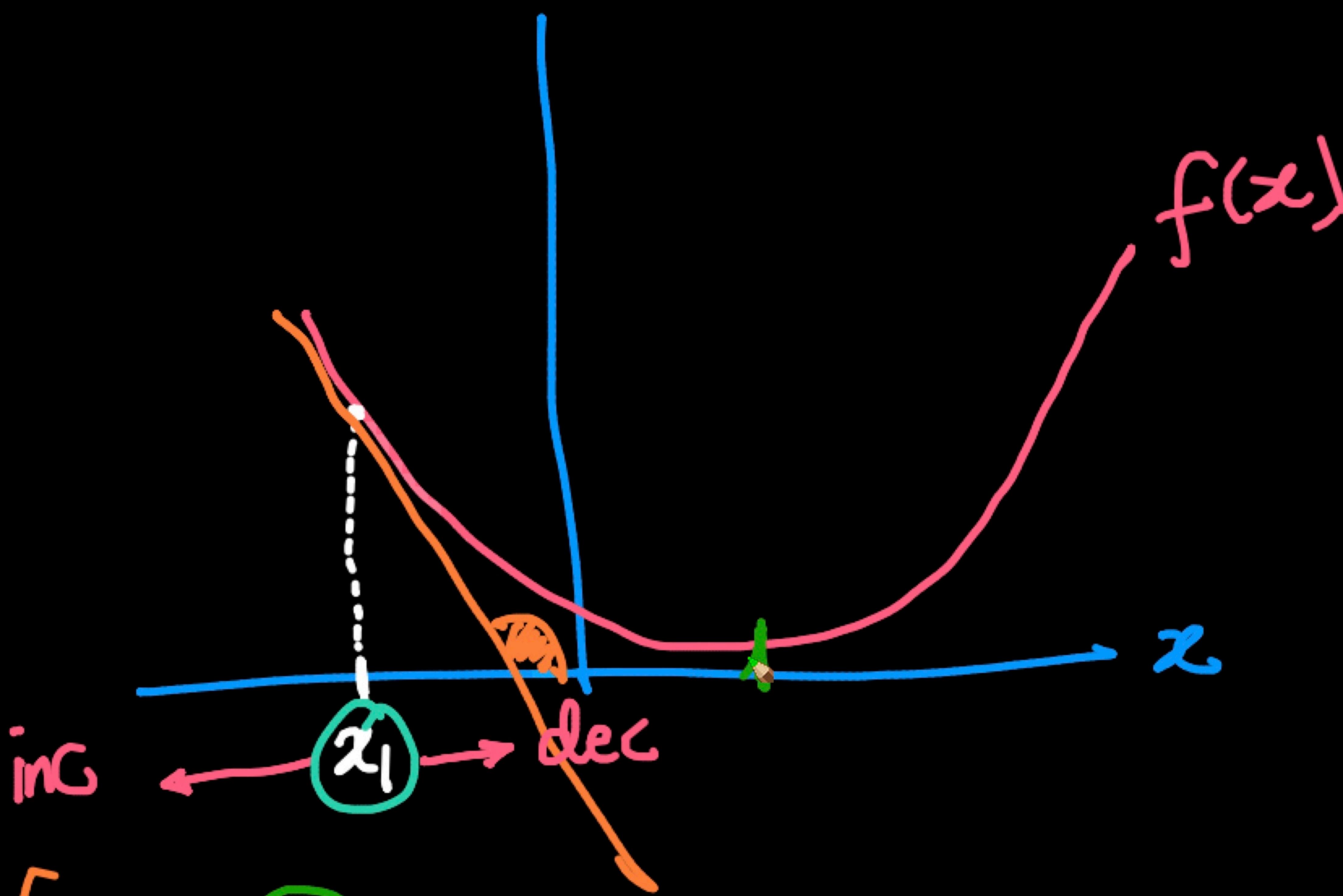
-ve

→ move in -ve dir



0

→ minima



$$\frac{df}{dx} \Big|_{x_1}$$

$$\frac{dF}{dx} \Big|_{x_1} = -\text{rc}$$

MINIMA

$$\frac{df}{dx} \Big|_{x_1}$$

+ve



negative dir
-elt

$$\begin{matrix} - \\ \diagdown \end{matrix}$$

-ve

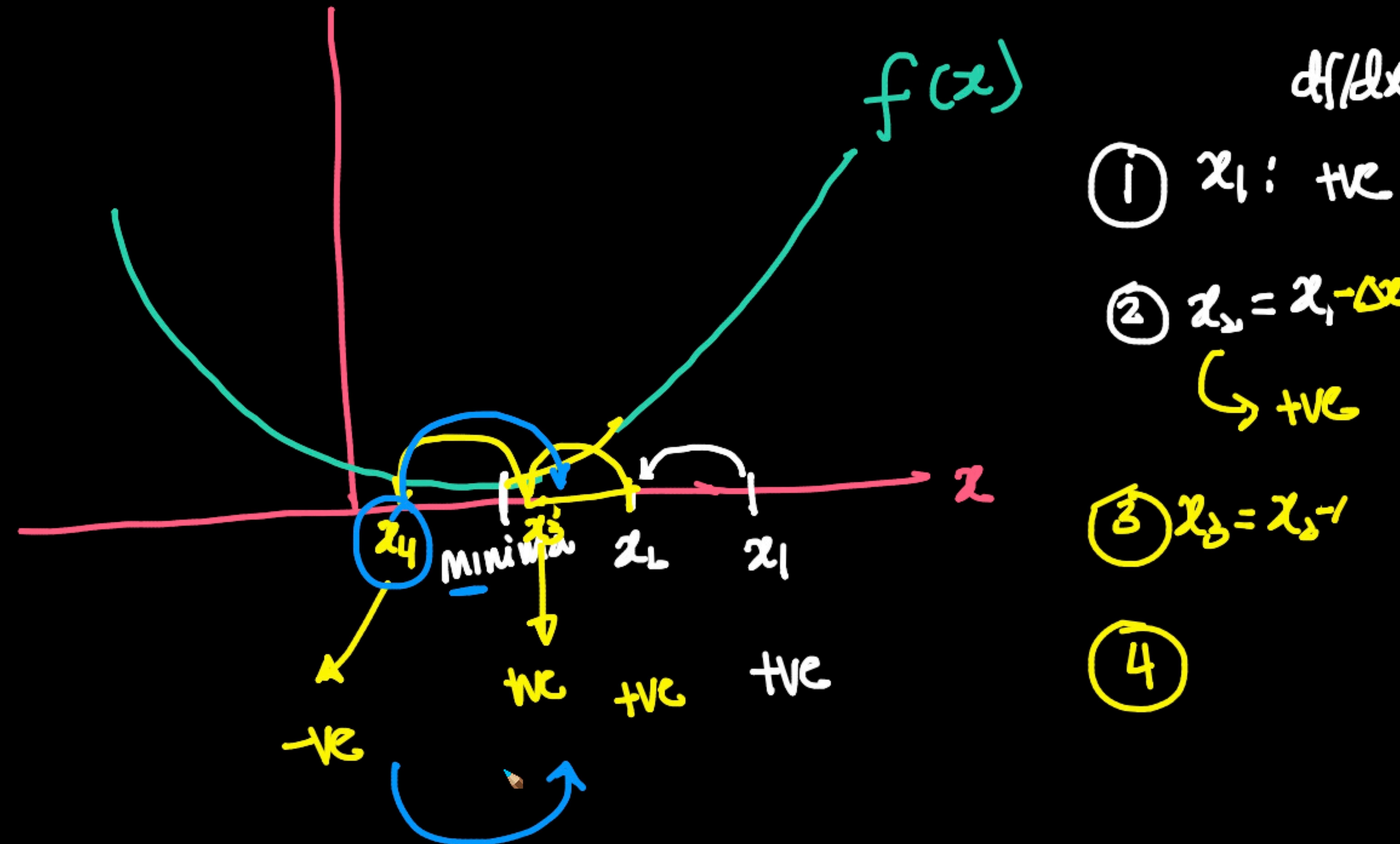


positive dir redn

0



minima ✓



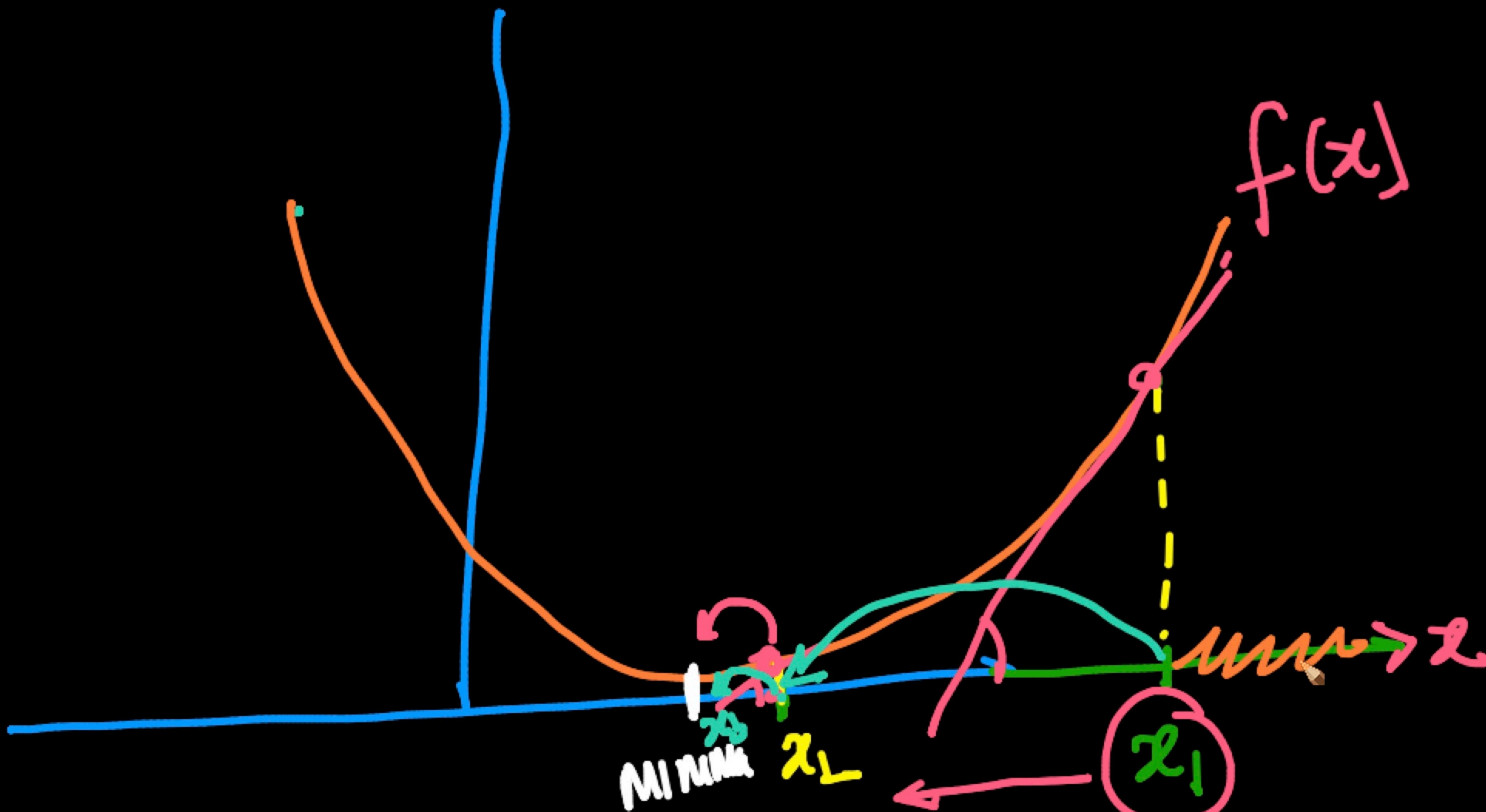
$$x_1 - \Delta x = x_2$$

$$x_3 = x_2 - \Delta x$$

$$x_4 = x_3 - \Delta x$$

$$x_5 = x_4 + \Delta x$$

Cant have
same Δx
everytime



$$\frac{df}{dx} \Big|_{x_1} > \frac{df}{dx} \Big|_{x_2}$$

x_1

Randomly

$$\underline{x}_2 = \underline{x}_1 - \eta \cdot \frac{df}{dx} \Big|_{x_1}$$

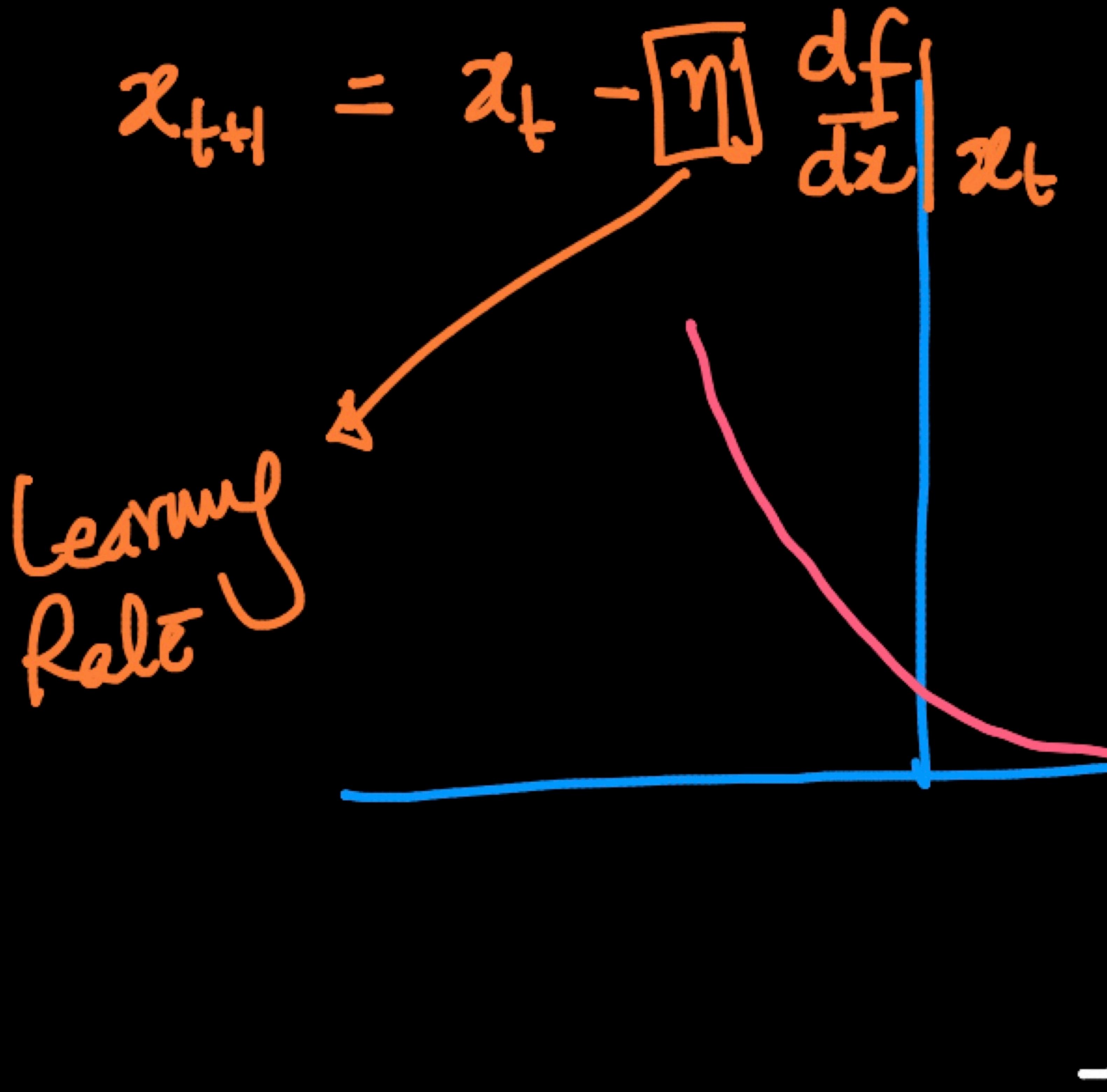
1
 η : Constant

 x_d

$$x_d = x_2 - \eta \cdot \frac{df}{dx} \Big|_{x_2}$$

 $x_u =$

⋮



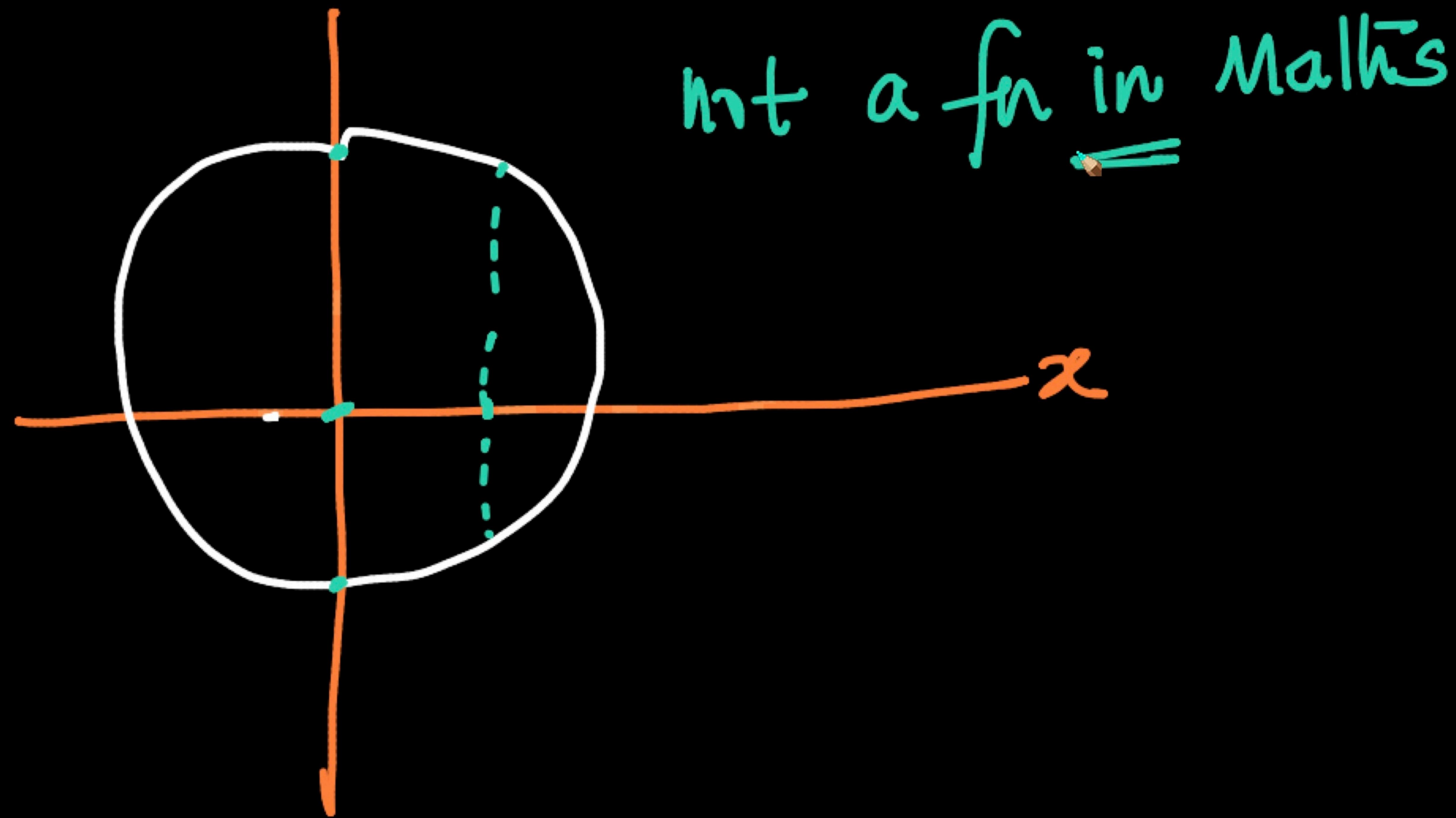
$f(x)$

oscillation problem

Soln: reduce η

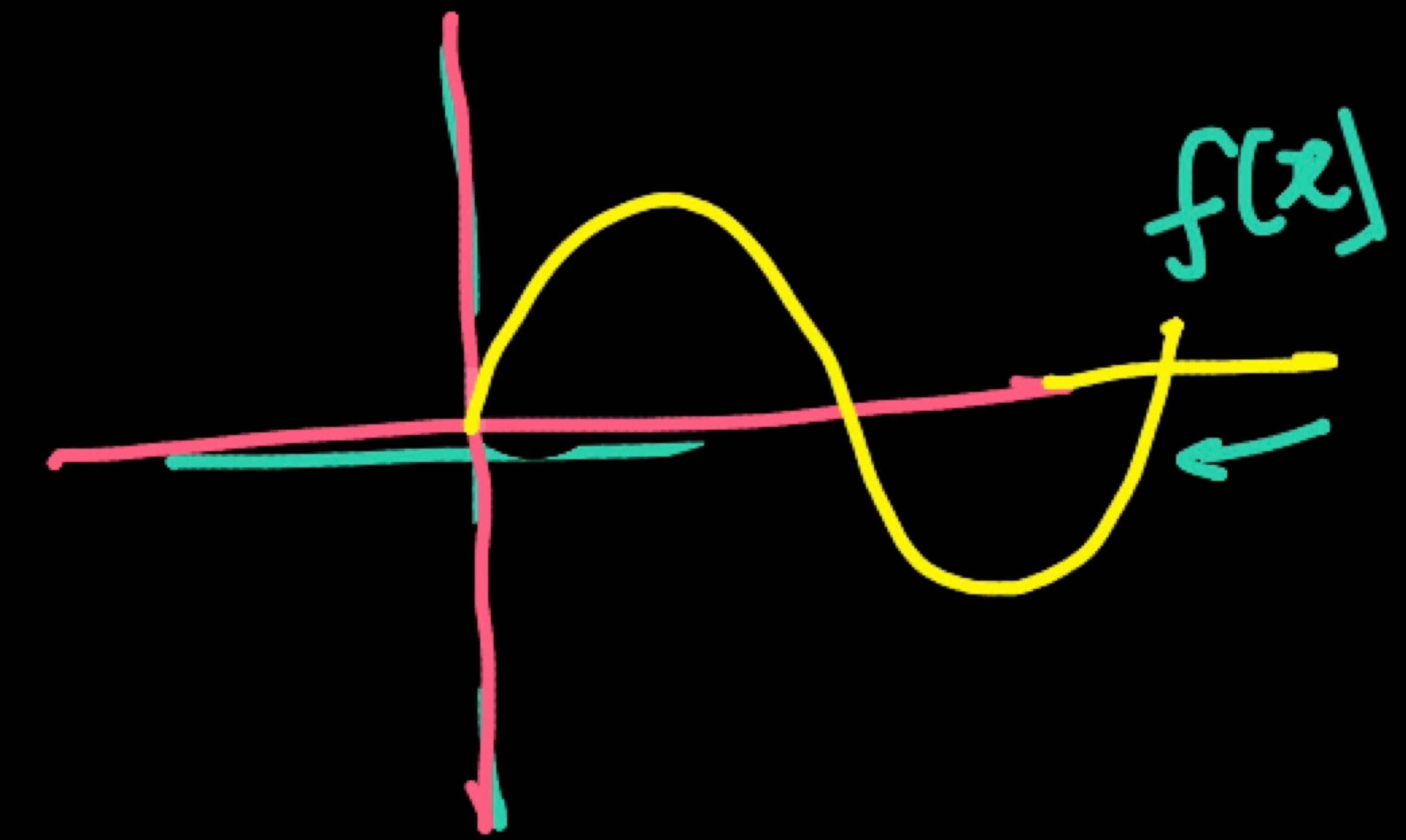
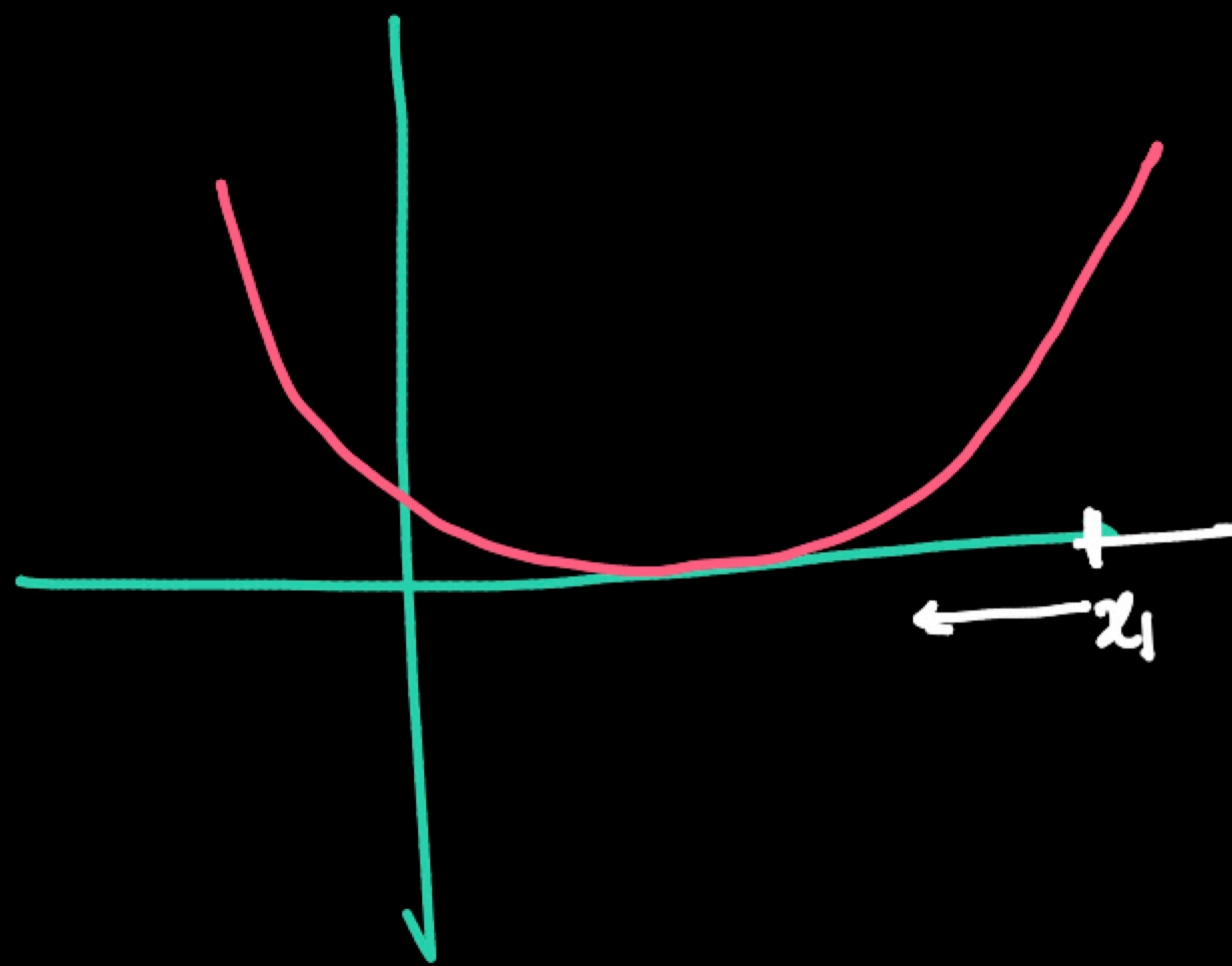
as t increases





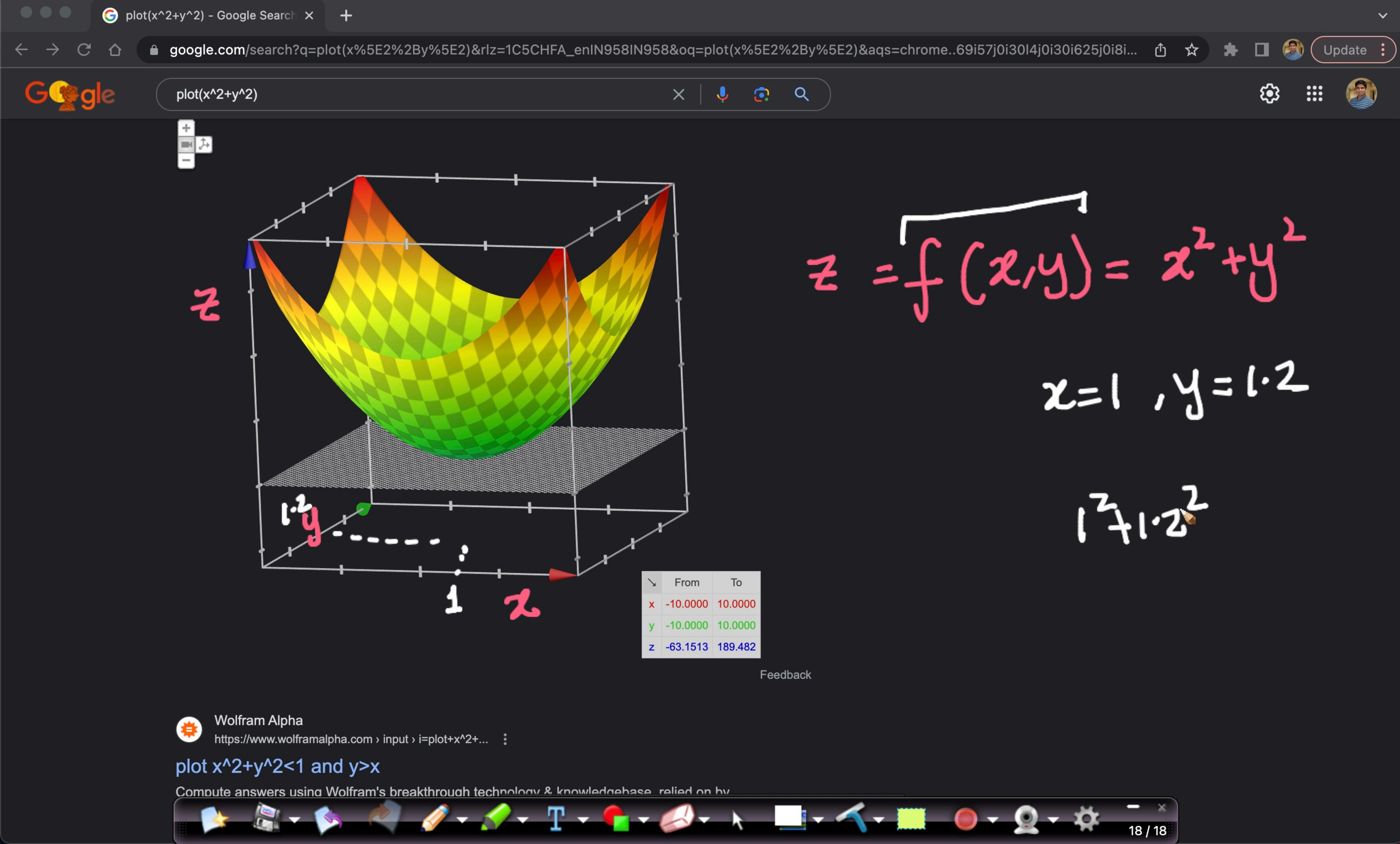
Int a fn in Maths

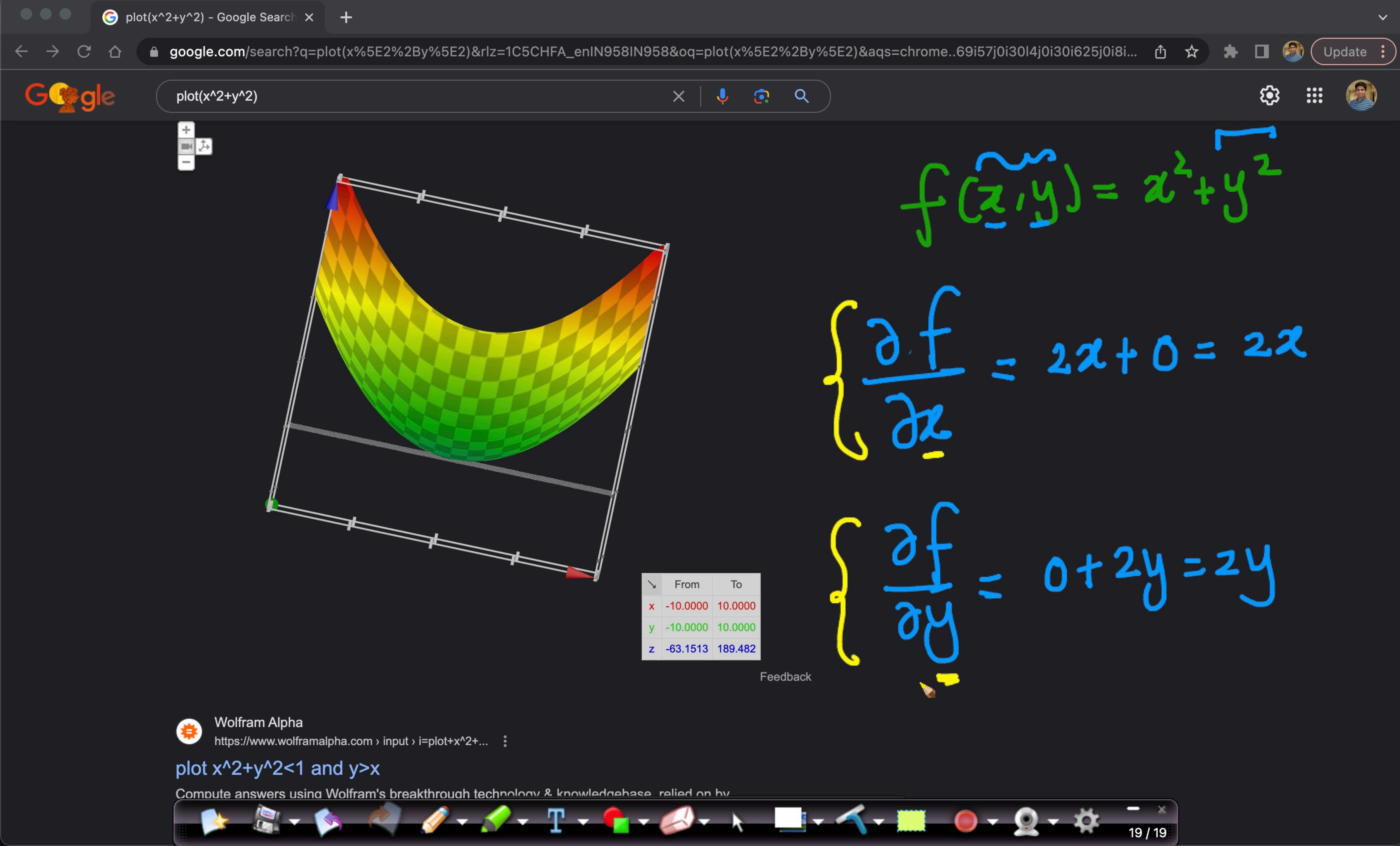
$$\frac{d \sin(x)}{dx} = \cos x$$



$\min f(x)$
↳ one variable

$\min f(w, w_0)$
↳ d+1 variables





$$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$$

$$f(x,y) = x^2 + y^2$$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

partial derivatives

2-dim vector

$f(\overset{w}{\overbrace{w_0, w_1, w_2, \dots, w_d}})$

min $f(\underline{w_0}, \underline{w_1})$

① Random-guess

$$\underline{w_0} = 1.0, \underline{w_1} = 1.8$$

②

$$\underline{w_0}^2 = \underline{w_0} - \eta \cdot \frac{\partial f}{\partial w_0} \Big|_{\underline{w_0}}$$
$$\underline{w_1}^2 = \underline{w_1} - \eta \cdot \frac{\partial f}{\partial w_1} \Big|_{\underline{w_1}}$$

$\underline{w_1}$ as ∞ const
 $\underline{w_0}$ as ∞ const

repeat

plot(x^2+y^2) - Google Search +

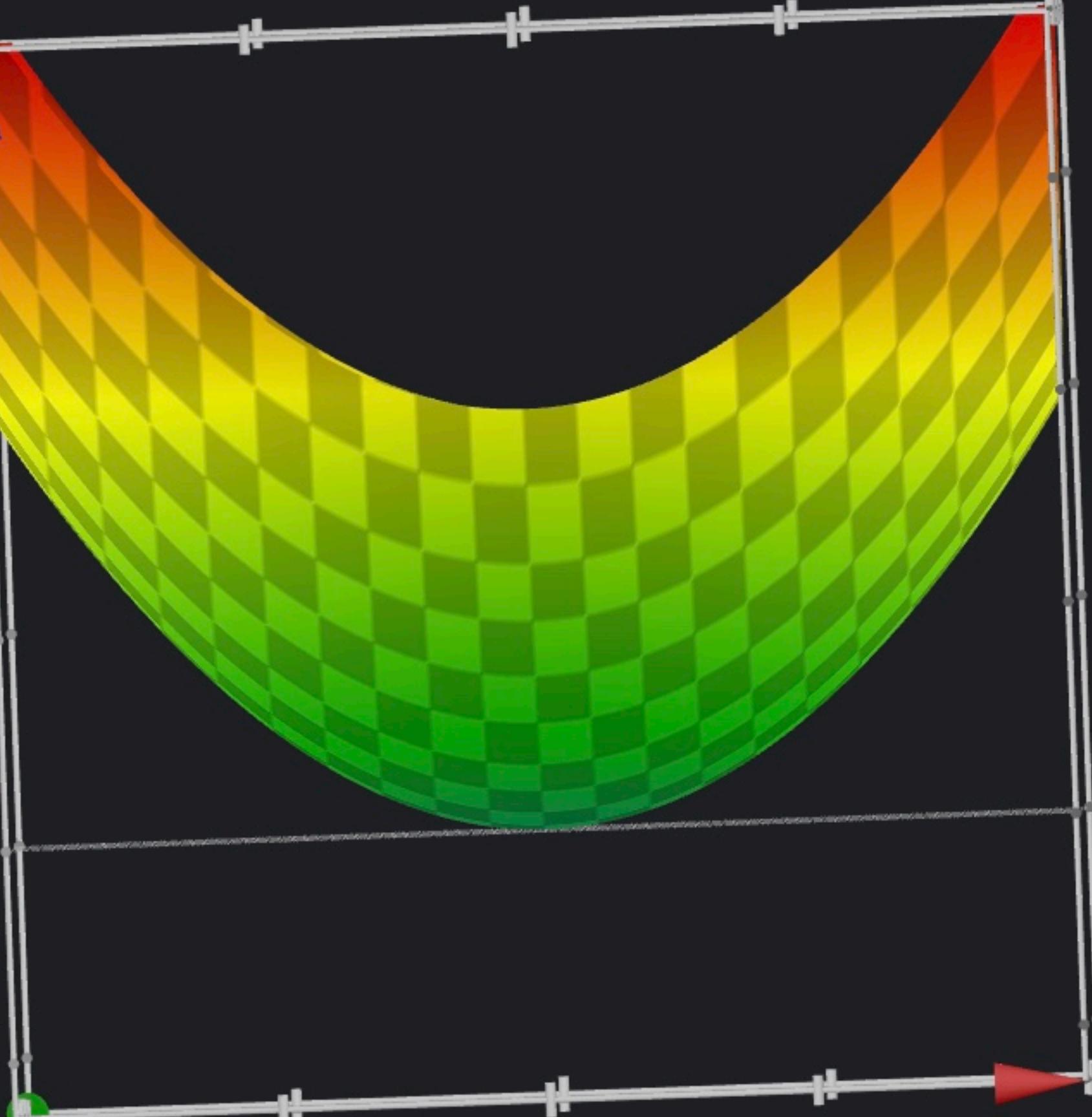
google.com/search?q=plot%28x%5E2%2By%5E2%29&rlz=1C5CHFA_enIN958IN958&ei=CGy1ZP3BEfiXjuMPsrSniAM&ved=0ahUKEwj95cW9IJaAAx... Update :

Google plot(x^2+y^2)

Images Formula Equation Calculator Answer 2 1 Z^2 4 All filters Tools SafeSearch

About 68,40,00,000 results (0.43 seconds)

Graph for $x^2 + y^2$



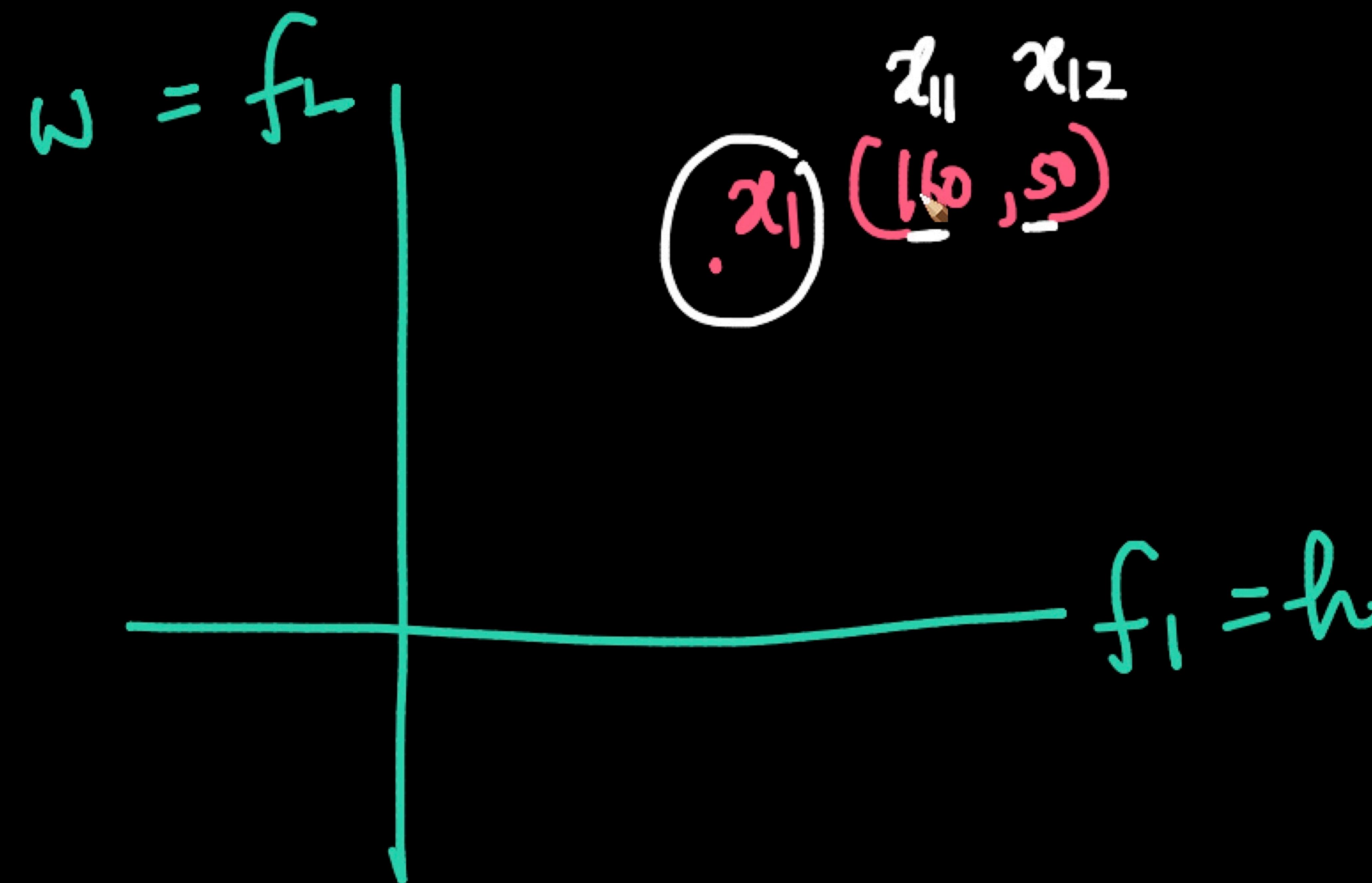
∂f
 ∂w_1

w_1^3
 w_1^2
 w_1^1

$f(w_0, w_1)$

From To

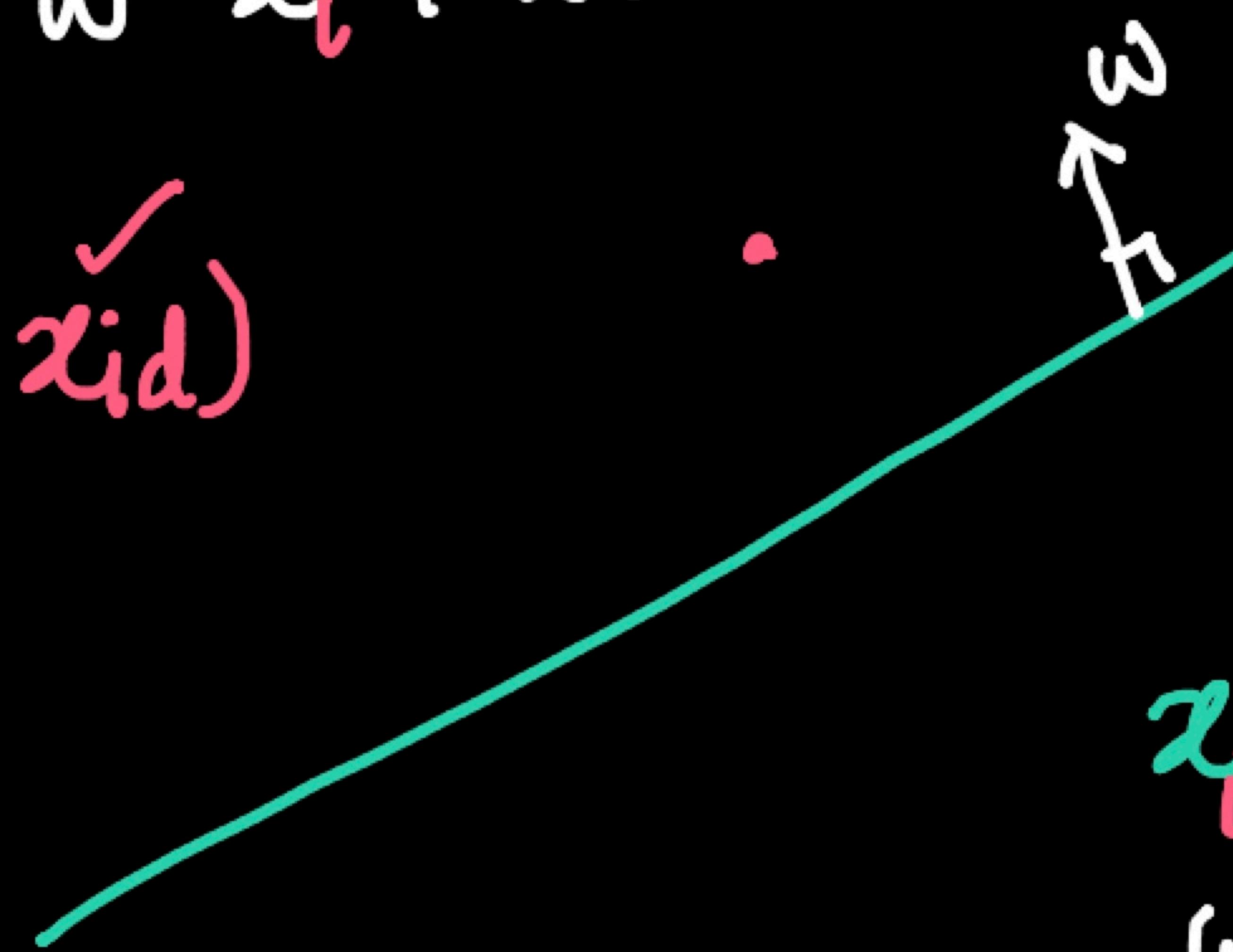
24 / 24



$$\omega^T x_i + w_0$$

$$x_i = (x_{i1}, x_{i2}, \dots, x_{id})$$

$$\pi: \underline{\omega^T x + w_0} = 0$$



$$x_i \in \mathbb{R}^d$$

$$\underline{w} \in \mathbb{R}^d$$

(8)

$$\frac{\partial f}{\partial w_0} = 1 \cdot w_2 + 0$$

$$f = w_0 w_2 + l_0$$

$$f(w_0, \underline{w_1}, \underline{w_2}, \dots, w_d)$$

- ① w_0^1, w_1^1, w_2^1
- ② $w_0^2, \underline{w_1^2}, w_2^2$



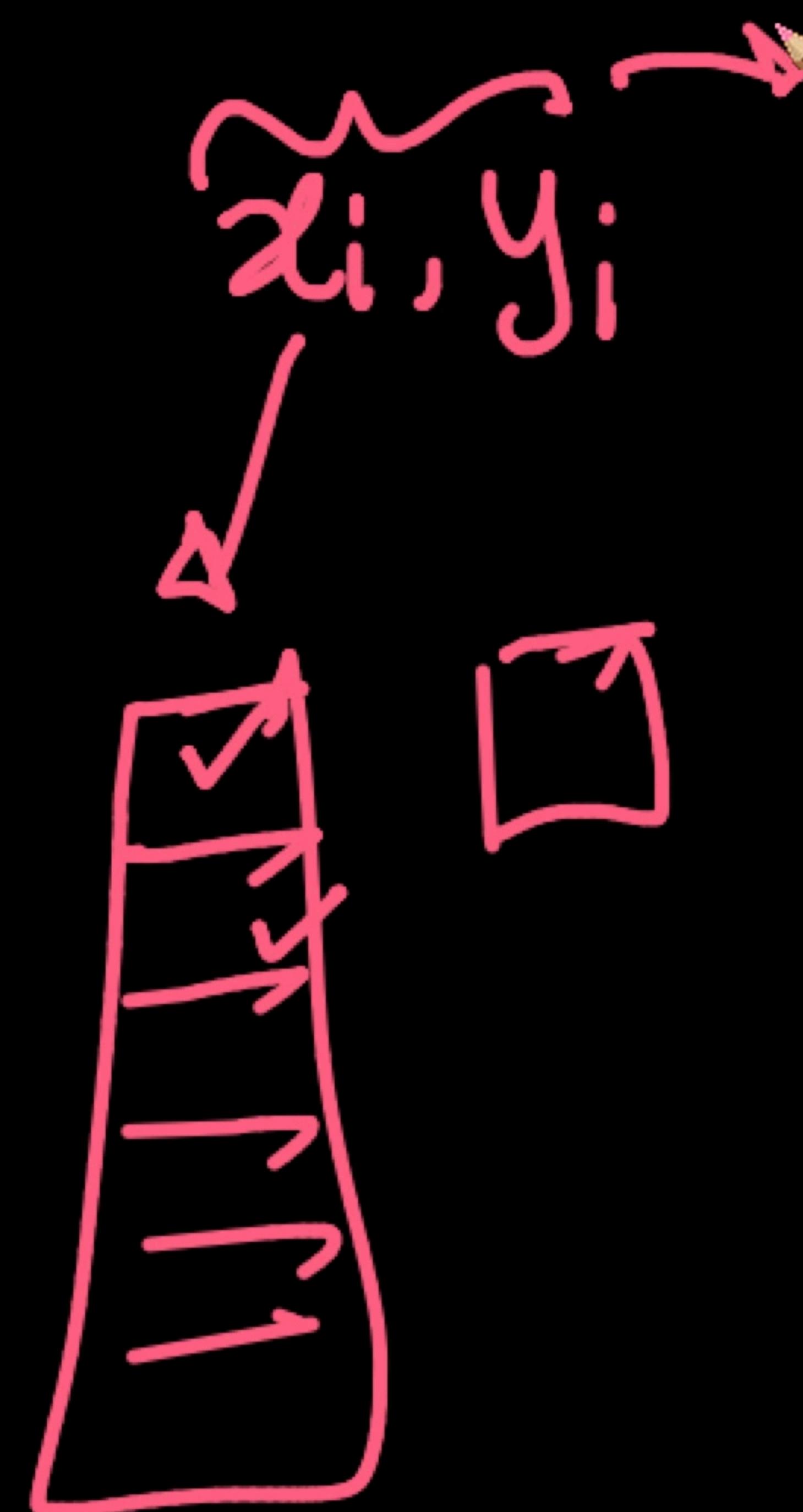
$$w_0^2 = w_0^1 - \eta \left[\frac{\partial f}{\partial w_0} \right]_{w_0^1}$$

$$w_2^2 = w_2^1 - \eta \left[\frac{\partial f}{\partial w_2} \right]_{w_0^1, w_2^1}$$

Math $\min f(x)$

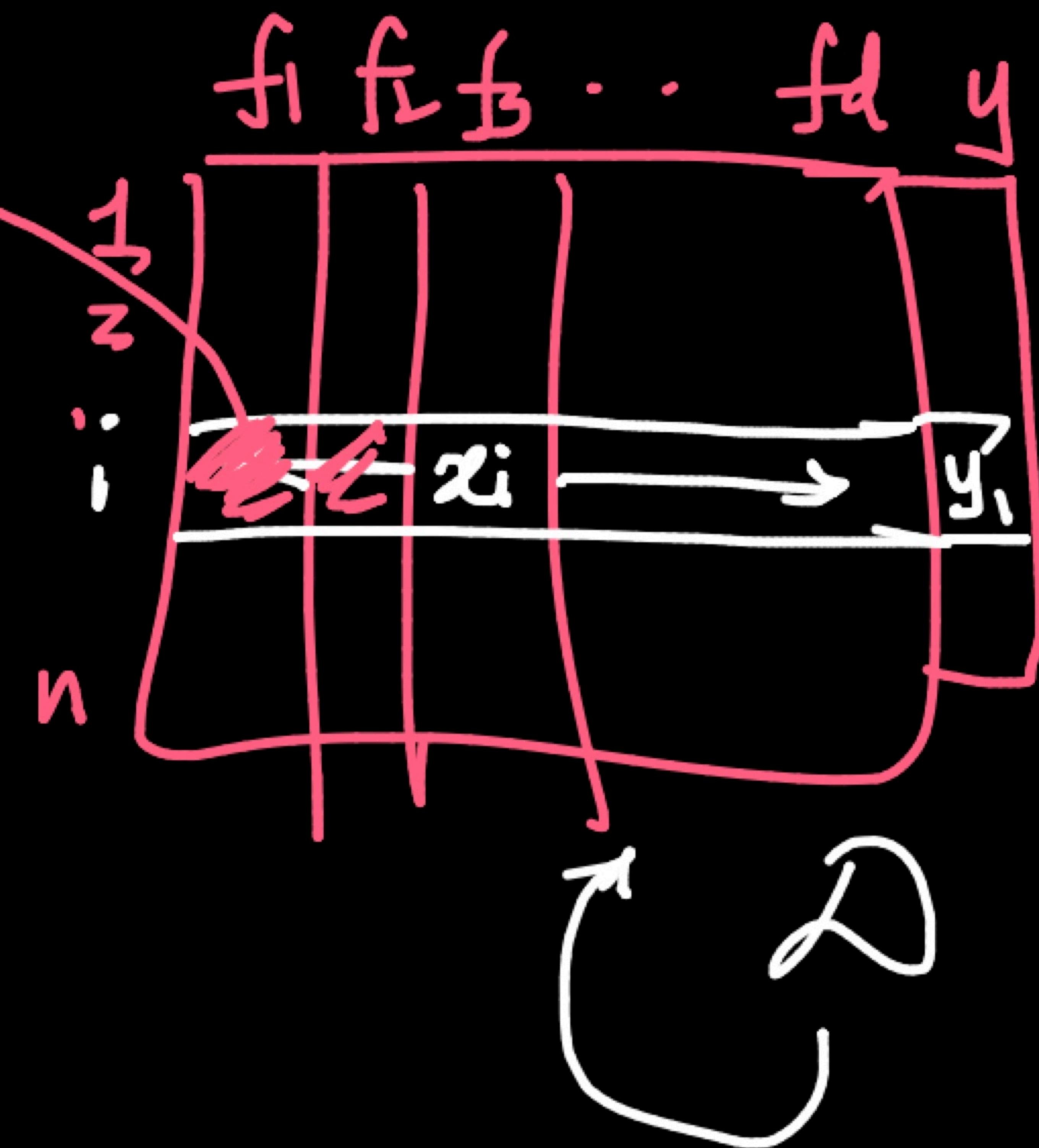
ML

$\min f(w_0, w_1, w_2, \dots, w_d)$



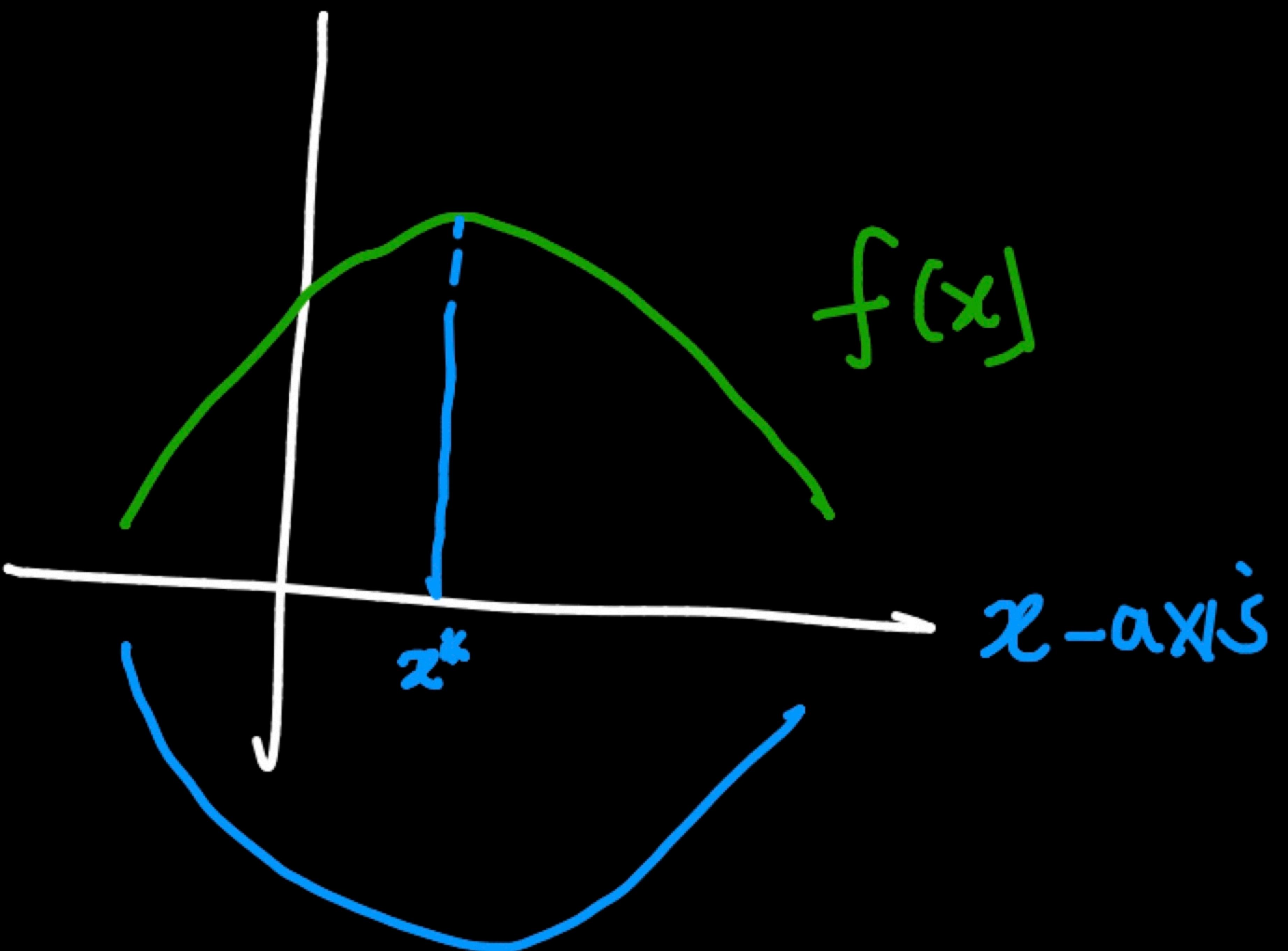
$$= \min_{\mathbf{w}} \sum_{i=1}^n \frac{w_1 x_1 + w_2 x_2 + \dots + w_d x_d + w_0}{\|\mathbf{w}\|} y_i$$

$$\max \sum_{i=1}^n \frac{\mathbf{w}^\top \mathbf{x}_i + w_0}{\|\mathbf{w}\|} \cdot y_i$$

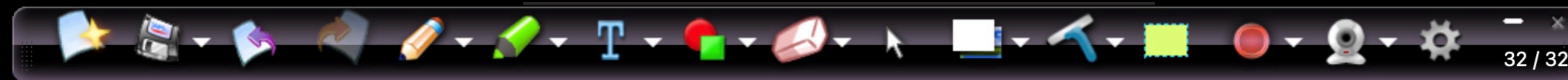


Math:
Max $f(x)$

$\min (-f(x))$



$$f(x) = x^3$$



plot(x^3) - Google Search

google.com/search?q=plot%28x%5E3%29&rlz=1C5CHFA_enIN958IN958&ei=rW61ZJenKPzi4-EPnoCFiAo&ved=0ahUKEwjXscWAj5aAAxV88TgGHR5... Update

Google

plot(x^3)

https://byjus.com/question-answer/the-graph-of-x-3-i...

The graph of $x = 3$ is a line: - Byju's

About featured snippets • Feedback

Graph for x^3

$\frac{d}{dx} x^3 \Big|_{x=0}$

Neither Max nor Minimum

Saddle points

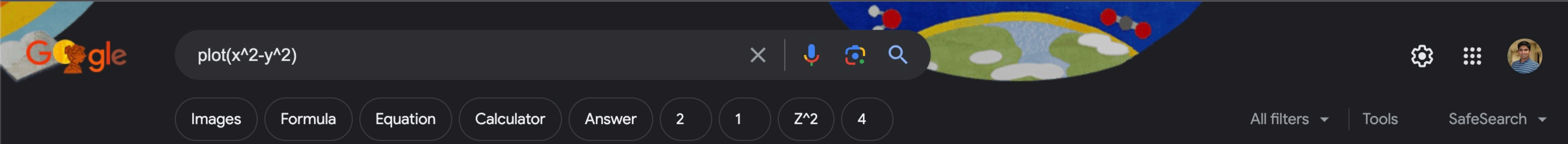
Mathway

Graph $x=3$

Toppr

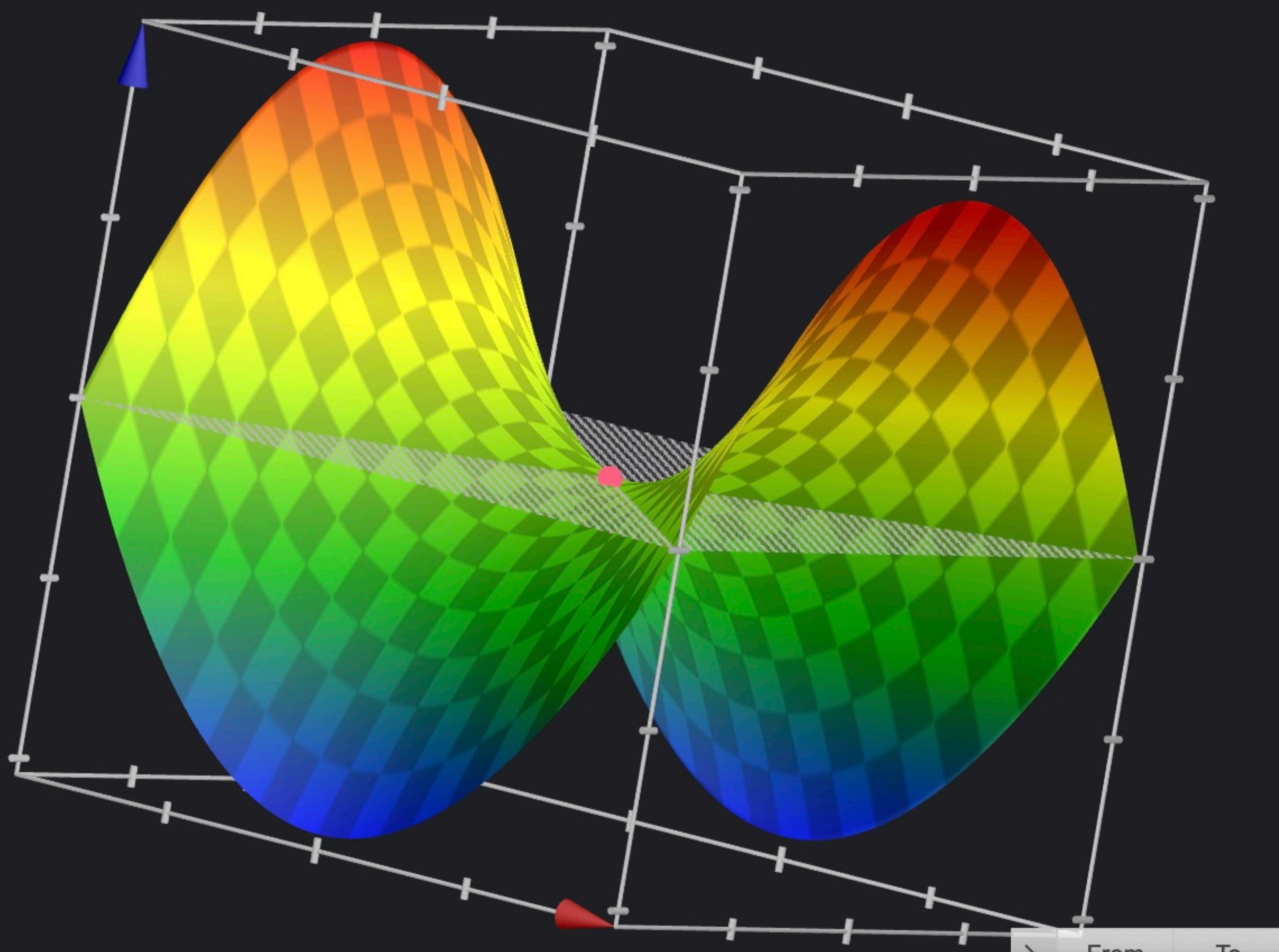
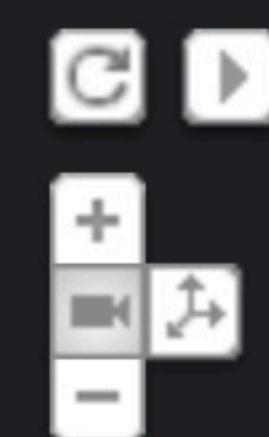
33 / 33

Since $x=3$ $x = 3$ is a vertical line, there is no y-intercept and the slope is undefined. ... Find two points on the line. ... Graph the line using the slope, y- ...



About 67,20,00,000 results (0.38 seconds)

Graph for $x^2 - y^2$



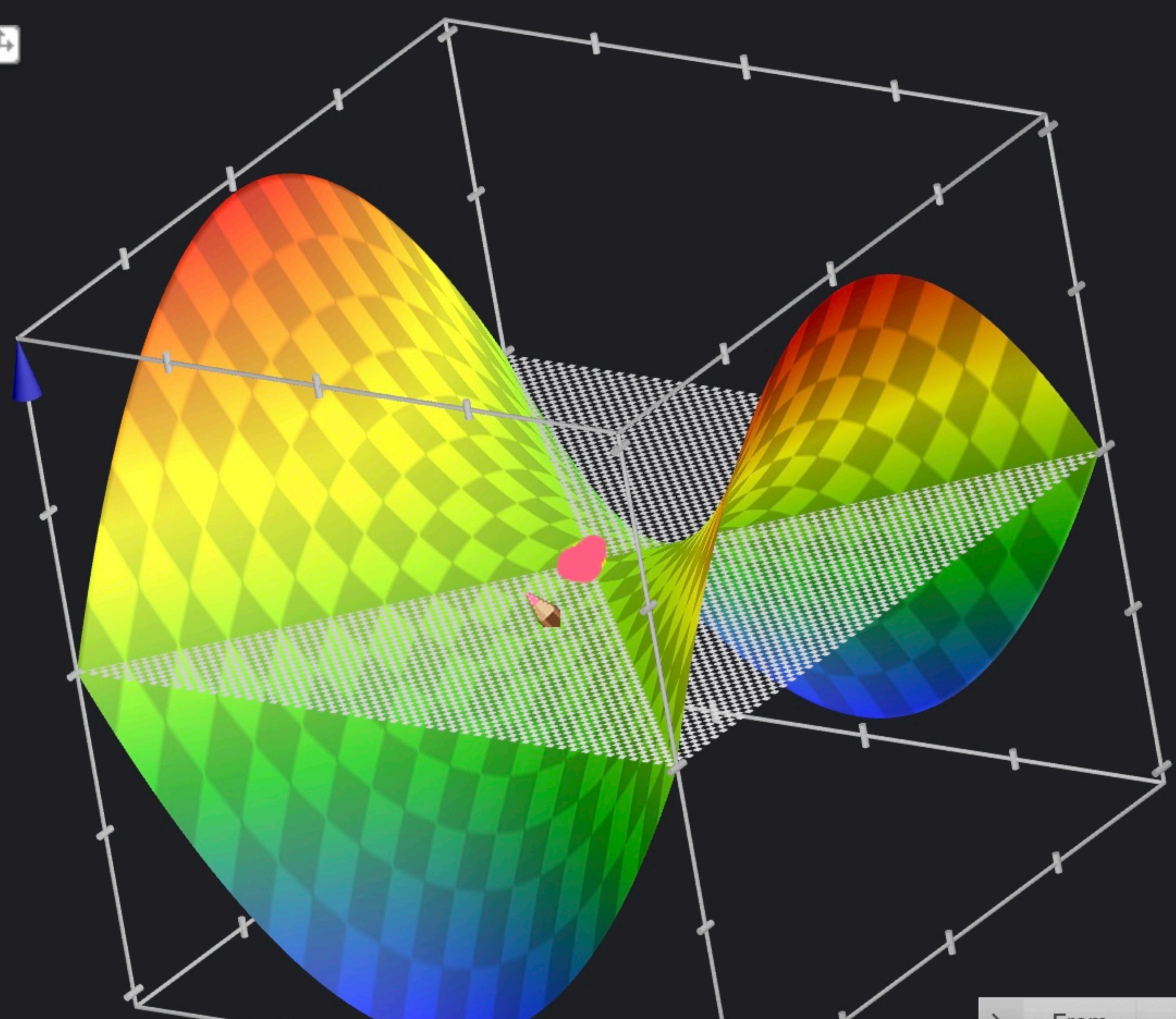
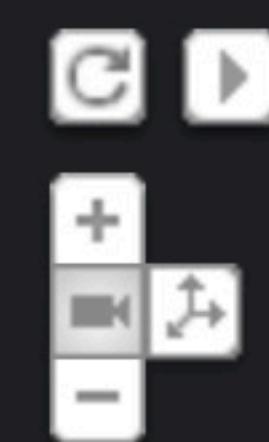
$$x^2 - y^2 = f(x,y)$$

$$\frac{\partial f}{\partial x} \Big|_{x=0} = 0$$

$$\frac{\partial f}{\partial y} \Big|_{y=0} = 0$$

About 67,20,00,000 results (0.38 seconds)

Graph for $x^2 - y^2$



Mark
(a)

$$f(x,y)$$

x_i, y_i

21

1

A diagram illustrating a mapping from a point (x_t, y_t) to a saddle point $(-, -)$. On the left, a pink circle contains the text x_t and y_t , with a bracket below it labeled x_{t+1}, y_{t+1} . A yellow arrow points to a pink circle containing the text $(-, -)$, which is labeled "Saddle pt".

plot(x^3) - Google Search plot(x^2-y^2) - Google Search horse saddle - Google Search

google.com/search?q=plot%28x%5E2-y%5E2%29&rlz=1C5CHFA_enIN958IN958&ei=-nS1ZN2PL_asseMPttSx-Ac&ved=0ahUKEwjd9N2BnZAAxV2... Update

Google plot(x^2-y^2)

Images Formula Equation Calculator Answer 2 1 Z^2 4 All filters Tools SafeSearch

About 67,20,00,000 results (0.38 seconds)

Graph for $x^2 - y^2$

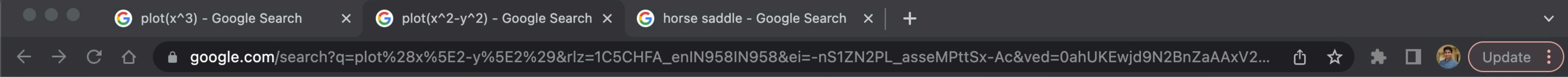
$f(x_t, y_t) = L$

x_t, y_t

$x_t + 0.01, y_t + 0.01$

$f(x_t + 0.1, y_t + 0.1) = 1.2$

3D surface plot of the function $f(x, y) = x^2 - y^2$. The plot shows two saddle-shaped surfaces along the x-axis. A point (x_t, y_t) is marked on one surface, and a nearby point $(x_t + 0.01, y_t + 0.01)$ is marked on the other surface. Handwritten annotations show the function values at these points: $f(x_t, y_t) = L$ and $f(x_t + 0.01, y_t + 0.01) = 1.2$.



Google

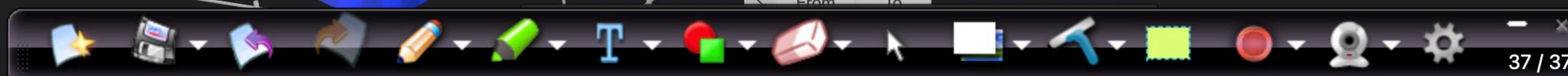
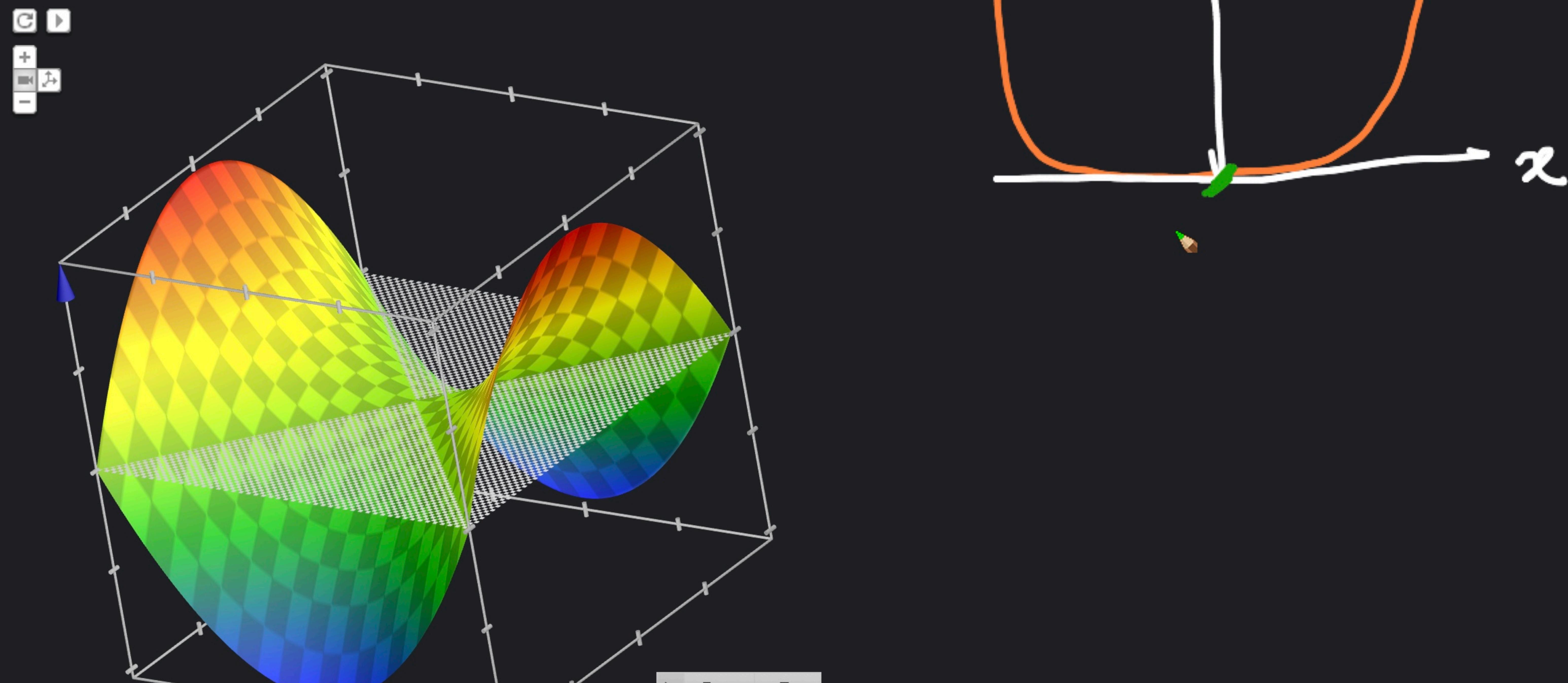
plot(x²-y²)

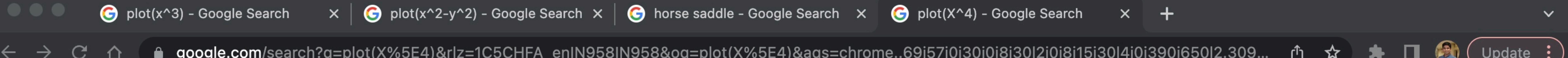
Images Formula Equation Calculator Answer 2 1 Z² 4

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Graph for $x^2 - y^2$





plot(X^4)

X | ⚡ 🔍

4, regardless of the y -value. Possible points could be (4;-5), (4;-2), (4;0), (4;7) etc. Plotting the points will give you a vertical line passing through the x -axis at 4.

03-May-2016



Socratic

<https://socratic.org> › Algebra › Graphs of Linear Equations

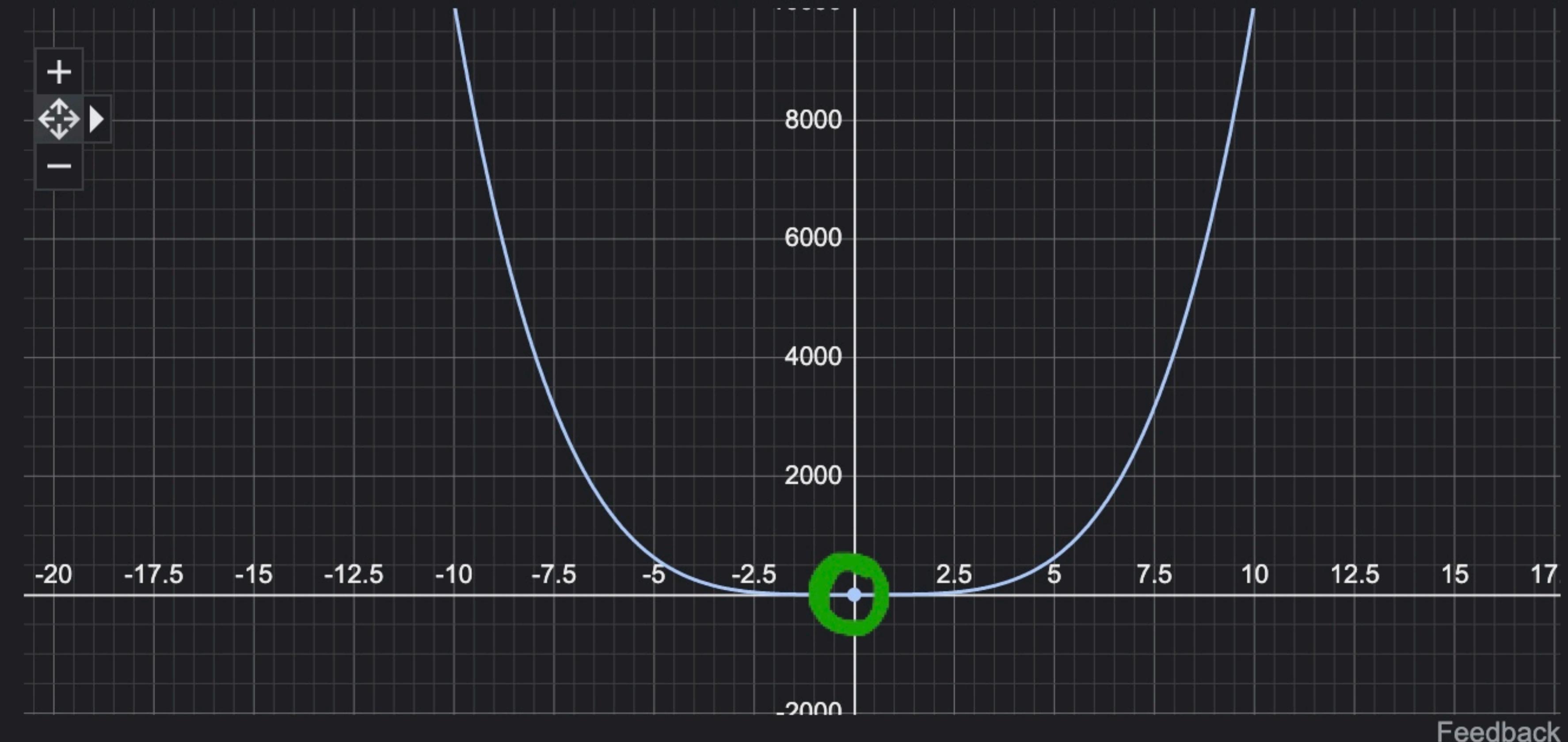
⋮

How do you graph the line $x = 4$? - Socratic

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Graph for x^4

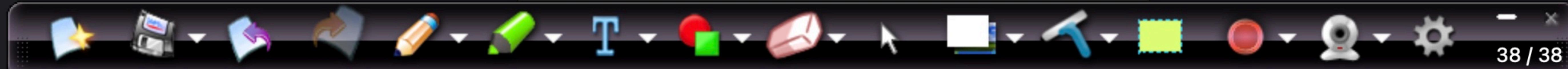


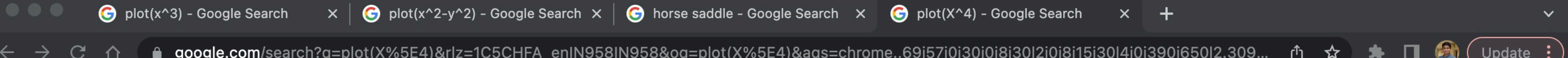
Mathway

<https://www.mathway.com> › Algebra

⋮

Graph $x=4$



plot(X⁴)

X | ⚡ 🔍

4, regardless of the y -value. Possible points could be (4;-5), (4;-2), (4;0), (4;7) etc. Plotting the points will give you a vertical line passing through the x -axis at 4. 03-May-2016



Update



Socratic

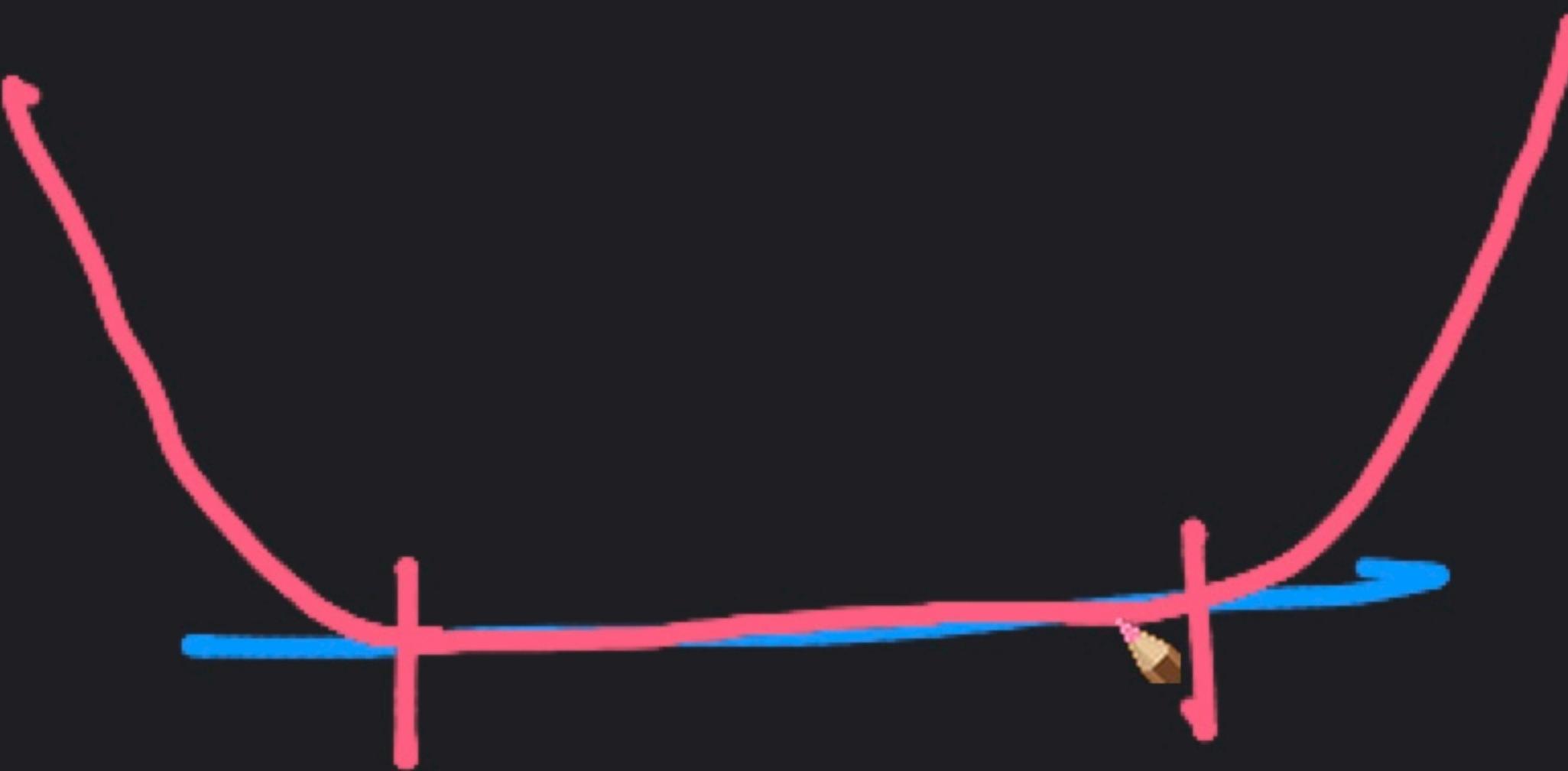
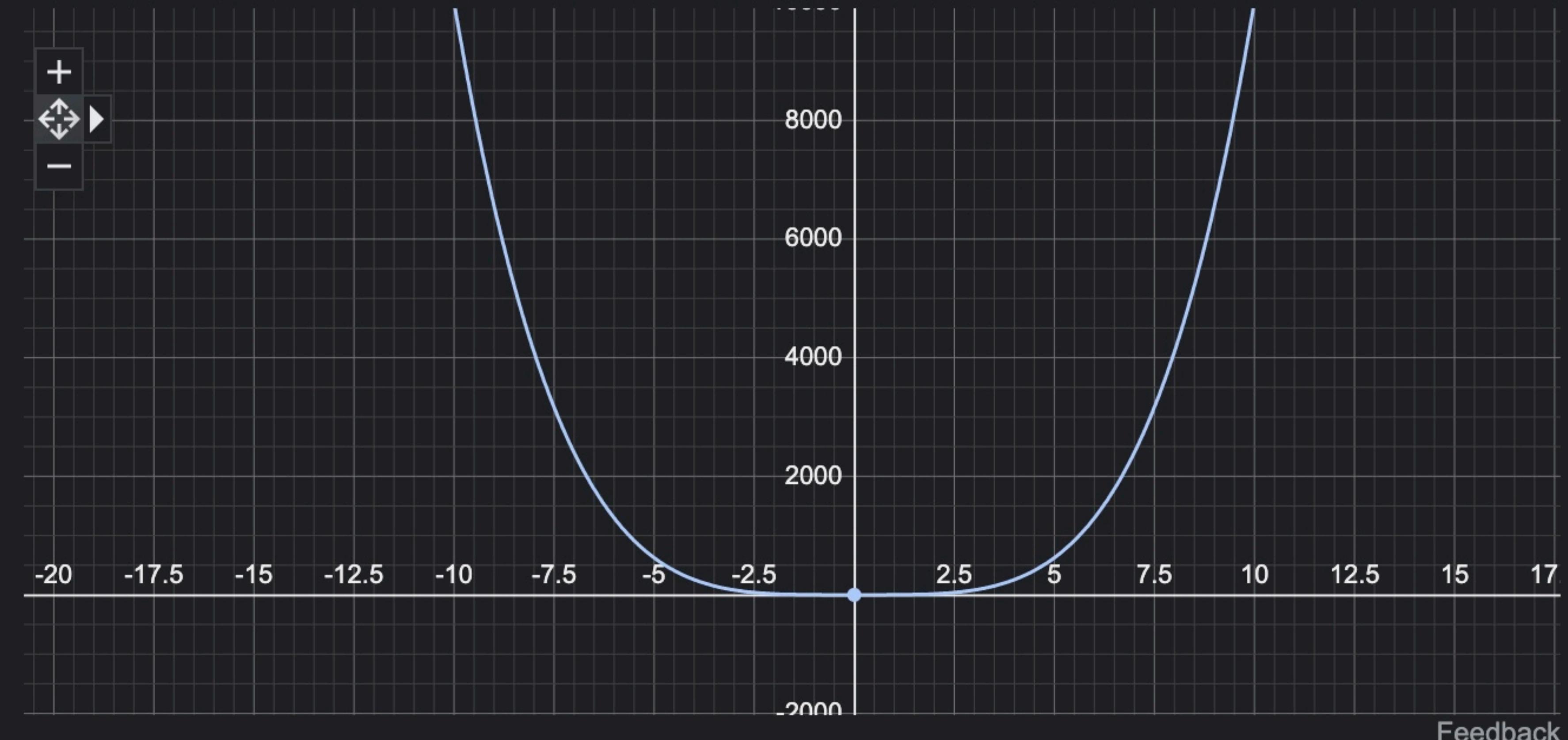
<https://socratic.org> › Algebra › Graphs of Linear Equations

⋮

How do you graph the line $x = 4$? - Socratic

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Graph for x^4

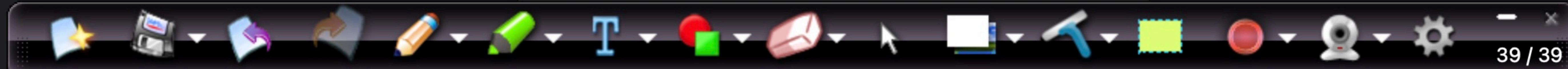


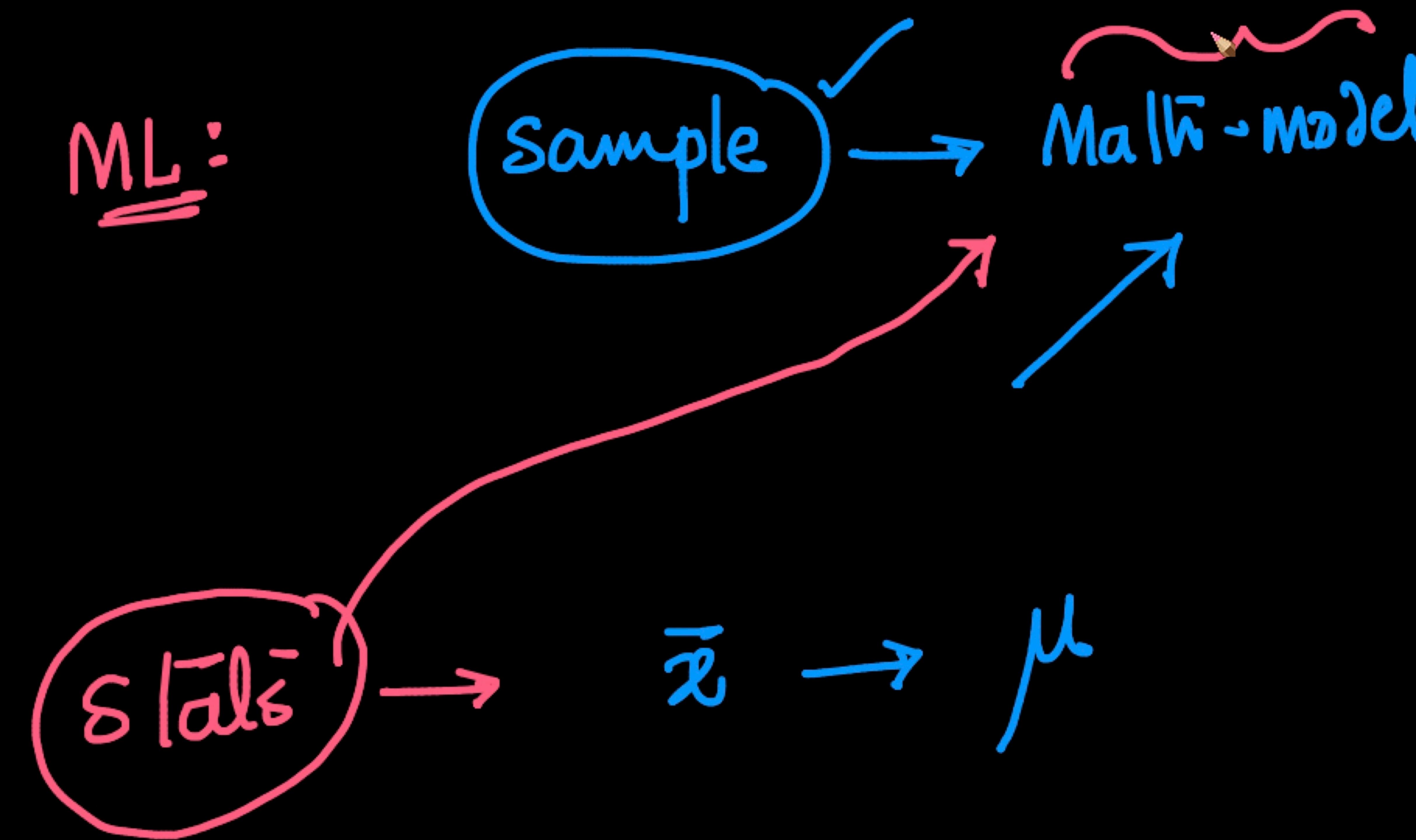
Mathway

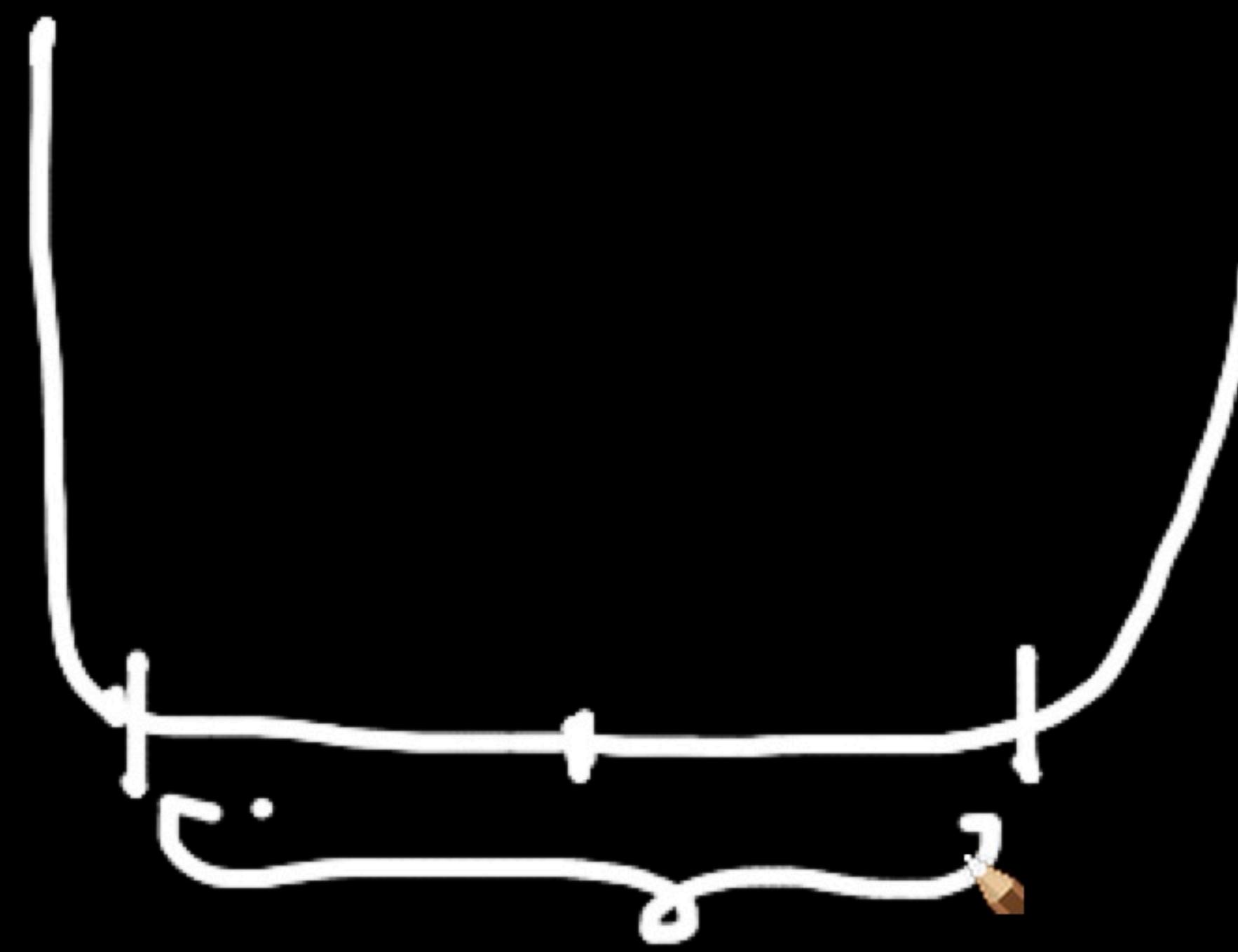
<https://www.mathway.com> › Algebra

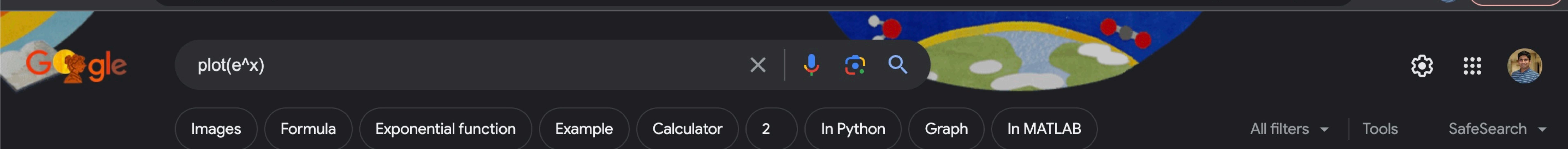
⋮

Graph $x=4$



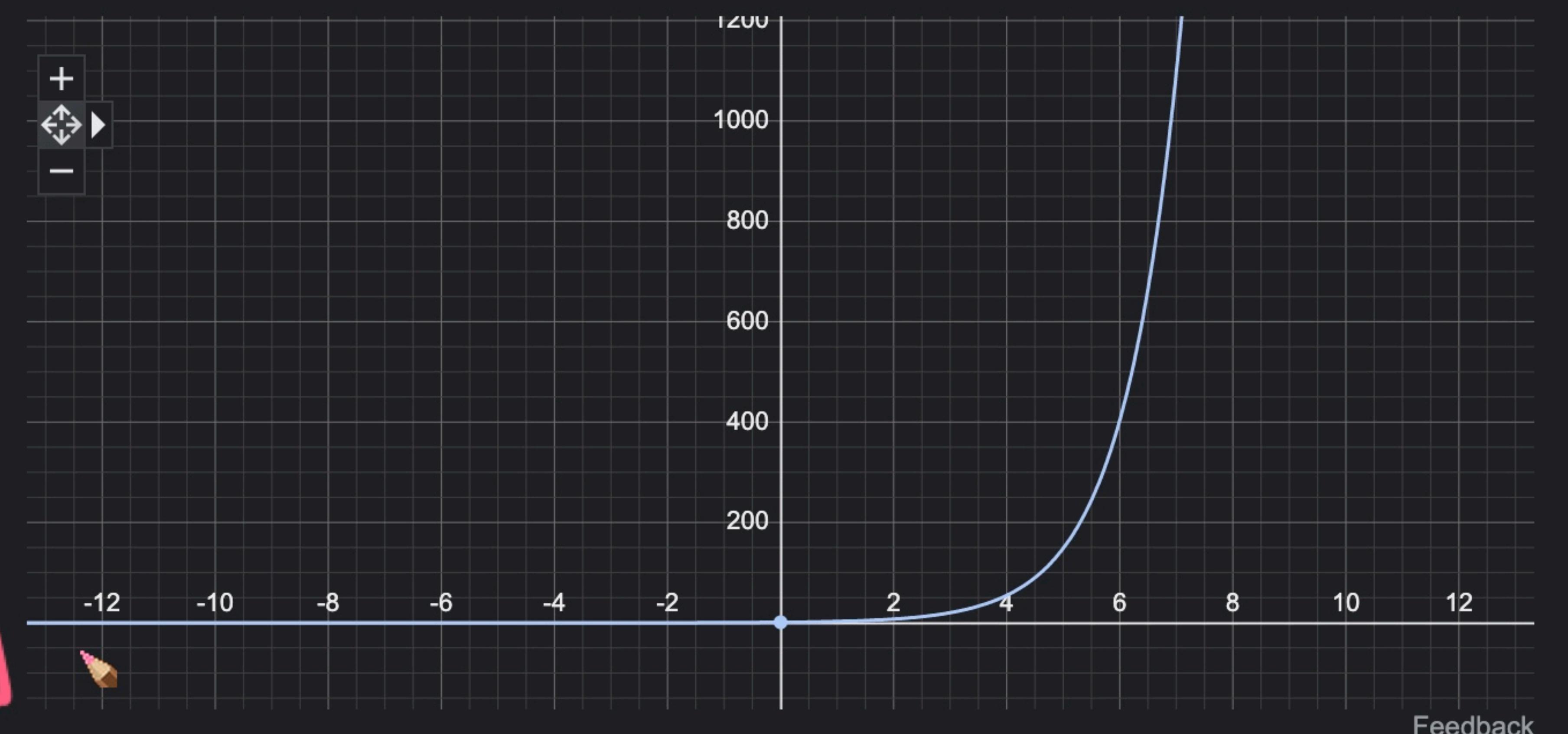






About 1,09,00,00,000 results (0.46 seconds)

Graph for e^x



$$\frac{d e^x}{dx} = e^x = 0$$



Wolfram Alpha

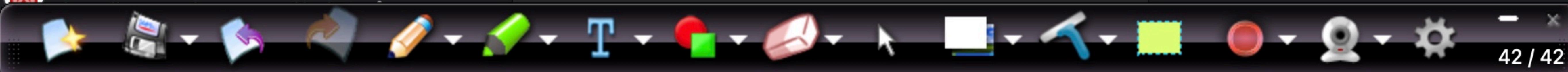
<https://www.wolframalpha.com> › input › i=plot+e^x

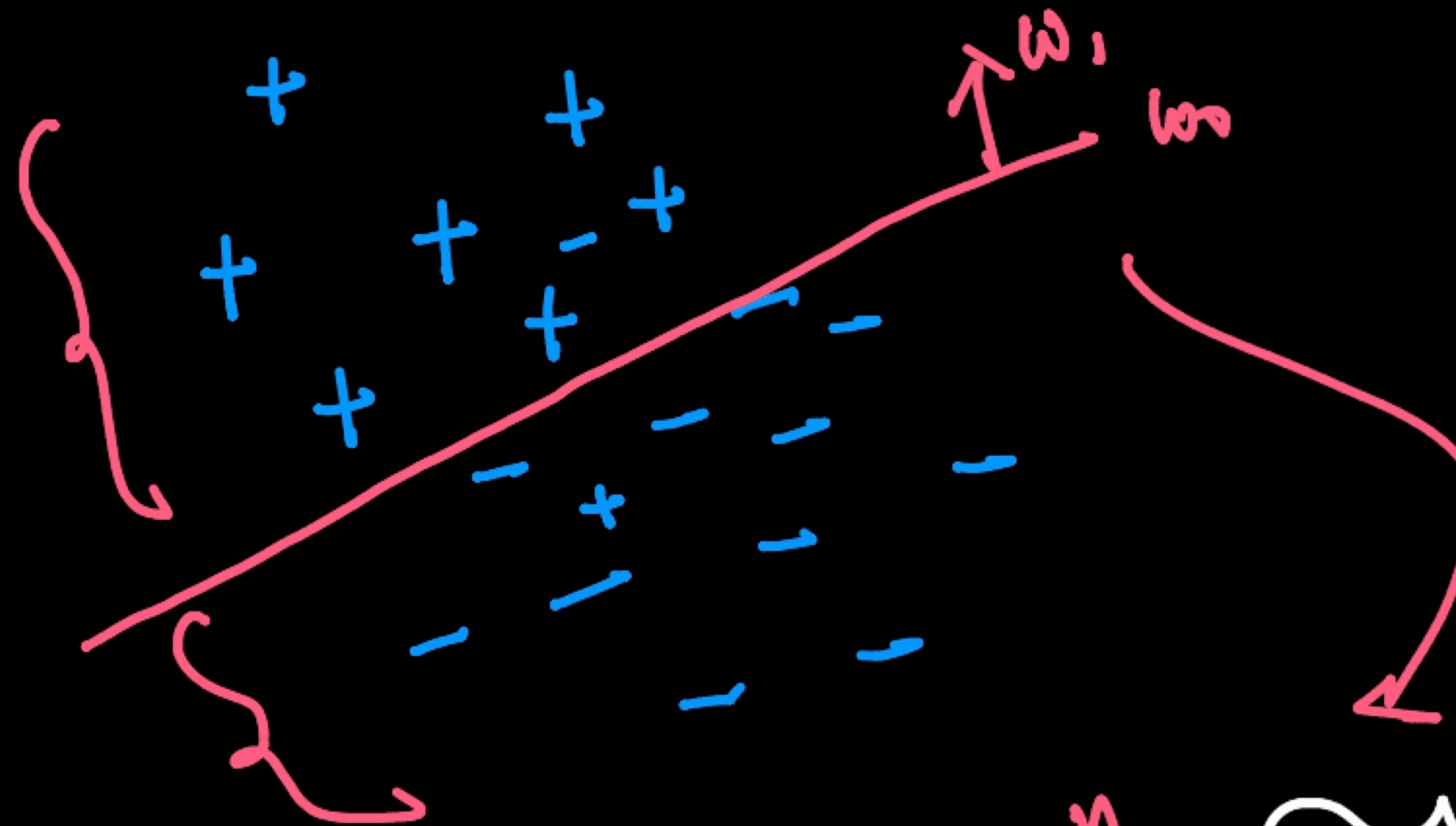
plot e^x

Compute answers using Wolfram's breakthrough technology & knowledgebase, relied on by millions of students & professionals. For math, science, nutrition, ...



Mathway





min ~~Max~~ $\sum_{i=1}^n \frac{w^\top u_i + w_0}{\|w\|} \cdot y_i$

ML

Max $\sum_{i=1}^n \frac{\tilde{w}^T \cancel{x_i + w_0}}{\|w\|} y_i$

$f(w_0, w_1, w_2, \dots, w_d)$

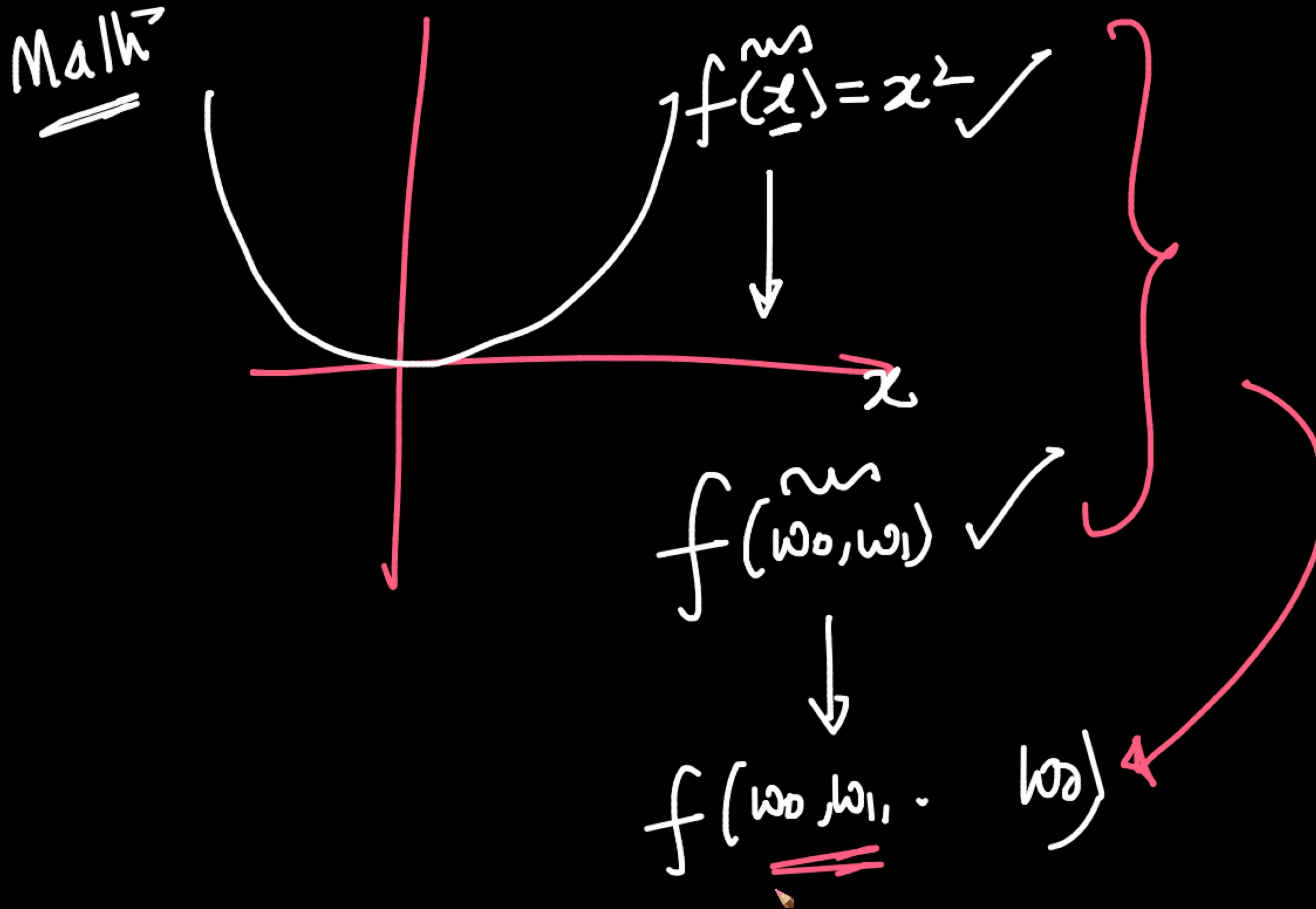
d -dim $\{+1, -1\}$

x_1, y_1

x_2, y_2

x_i, y_i

x_n, y_n



$f(x)$

