2013 Qualifying Exam Section 1

February 21, 2019

1 Question 1

2 Question 2

2.a

For each $\theta_0 \in \Theta$, let T_{θ_0} be a test of $H_0 : \theta = \theta_0$ (versus some H_1) with significance level α and acceptance region $A(\theta_0)$. For each y in the range of the random variable Y , define $C(y) = \{\theta : y \in A(\theta)\}$. Show that $C(Y)$ is a level $1 - \alpha$ confidence set for θ.

Solution:
Note that $\theta \in C(Y)$ if and only if $Y \in A(\theta)$.

$$P_\theta(\theta \in C(Y)) = P_\theta(Y \in A(\theta)) \geq 1 - \alpha$$

where the inequality follows because $A(\theta)$ is a level-α test, so the probability that we reject H_0 is at most α, for any $\theta \in \Theta$ by definition of power.

2.b

Suppose $X_1, \dots, X_n$ is a random sample from $N(\mu, \sigma^2)$, where $\sigma^2 = \gamma\mu^2$ and $-\infty < \mu < \infty$ and $\gamma > 0$ are both unknown scalar parameters and $\mu \neq 0$. Using part (a), derive a confidence set for γ with confidence coefficient $1-\alpha$ by inverting the acceptance region of the likelihood ratio test for testing $H_0 : \gamma = \gamma_0$ versus $H_1 : \gamma \neq \gamma_0$.

Solution: