

Part c

$$\hat{\mu}_g = \frac{1}{n} \left[\sum R_i Y_i / \pi(x_i) + \sum \left(1 - \frac{R_i}{\pi(x_i)}\right) g(x_i) \right] = \hat{\mu}_g + \frac{1}{n} \sum \left(1 - \frac{R_i}{\pi(x_i)}\right) g(x_i)$$

$$E\left[\left(1 - \frac{R_i}{\pi(x_i)}\right) g(x_i)\right] = E\left[E\left[\left(1 - \frac{R_i}{\pi(x_i)}\right) g(x_i) \mid X_i\right]\right] = E\left[g(x_i) \left(1 - \frac{1}{\pi(x_i)} E(R_i \mid X_i)\right)\right]$$

$$= E[g(x_i)(1-1)] = 0$$

$$\text{Var}\left[\frac{R_i Y_i}{\pi(x_i)} + \left(1 - \frac{R_i}{\pi(x_i)}\right) g(x_i)\right] = \text{Var}\left(\frac{R_i Y_i}{\pi(x_i)}\right) + \text{Var}\left(\left(1 - \frac{R_i}{\pi(x_i)}\right) g(x_i)\right) + 2 \text{Cov}\left(\frac{R_i Y_i}{\pi(x_i)}, \left(1 - \frac{R_i}{\pi(x_i)}\right) g(x_i)\right)$$

$$\text{① } \text{Var}\left(\left(1 - \frac{R_i}{\pi(x_i)}\right) g(x_i)\right) = E\left[\text{Var}\left(\left(1 - \frac{R_i}{\pi(x_i)}\right) g(x_i) \mid X_i\right)\right] + 0 = E\left[\frac{g(x_i)^2}{\pi(x_i)^2} \text{Var}(R_i \mid X_i)\right]$$

$$= E\left[\frac{g(x_i)^2}{\pi(x_i)^2} \pi(x_i)(1-\pi(x_i))\right] = E\left[\frac{g(x_i)^2 (1-\pi(x_i))}{\pi(x_i)}\right]$$

$$\text{② } \text{Cov}\left(\frac{R_i Y_i}{\pi(x_i)}, \left(1 - \frac{R_i}{\pi(x_i)}\right) g(x_i)\right) = E\left[\frac{R_i Y_i}{\pi(x_i)} \left(1 - \frac{R_i}{\pi(x_i)}\right) g(x_i)\right] + 0 = E\left[\frac{g(x_i)}{\pi(x_i)} E(R_i Y_i (1 - \frac{R_i}{\pi(x_i)}) \mid X_i)\right]$$

$$= E\left[\frac{g(x_i)}{\pi(x_i)} E(Y_i \mid X_i) E\left(R_i - \frac{R_i^2}{\pi(x_i)} \mid X_i\right)\right] = E\left[\frac{g(x_i)}{\pi(x_i)} E(Y_i \mid X_i) \left[\pi(x_i) - \frac{\pi(x_i)}{\pi(x_i)}\right]\right]$$

$$= E\left[\frac{g(x_i)}{\pi(x_i)} E(Y_i \mid X_i) \left[\pi(x_i) - \pi(x_i)\right]\right] = E\left[\frac{g(x_i)}{\pi(x_i)} E(Y_i \mid X_i) \cdot 0\right] = 0$$

$$EY = 0$$

$$C(X, Y)$$

$$= E(XY) -$$

Part d

$$\begin{aligned} & E \left[\frac{g^2(x)(1-\pi(x))}{\pi(x)} - 2 \frac{g(x)(1-\pi(x))}{\pi(x)} E(Y|x) \right] \\ &= E \left[\frac{(1-\pi(x))}{\pi(x)} \left(g^2(x) - 2g(x)E(Y|x) + E^2(Y|x) \right) \right] - E \left[\frac{(1-\pi(x))}{\pi(x)} E^2(Y|x) \right] \\ &= E \left[\frac{(1-\pi(x))}{\pi(x)} \left(E(Y|x) - g(x) \right)^2 \right] \\ & \quad g(x) = E(Y|x) \end{aligned}$$