1) a.

$$f_{Y_{i1}Y_{i2}} = \frac{\mu_{i1}^{y_{i1}} e^{-\mu_{i1}}}{y_{i1}!} \cdot \frac{\mu_{i2}^{y_{i2}} e^{-\mu_{i2}}}{y_{i2}!} = \frac{(\psi\mu_{i2})^{y_{i1}} e^{-\psi\mu_{i2}}}{y_{i1}!} \cdot \frac{\mu_{i2}^{y_{i2}} e^{-\mu_{i2}}}{y_{i2}!}$$

$$L(\psi, \mu_{i2}) = \prod_{i=1}^{n} \frac{\psi^{y_{i1}} \mu_{i2}^{y_{i1}+y_{i2}} e^{-\mu_{i2}(1+\psi)}}{y_{i1}!\, y_{i2}!}$$

$\vdots$

$$\frac{\delta}{\delta\psi} l = \frac{\sum y_{i1}}{\psi} - \sum \mu_{i2}$$

$$\frac{\delta}{\delta\psi^2} l = \frac{-\sum y_{i1}}{\psi^2} \qquad O.I. = \frac{\sum y_{i1}}{\hat{\psi}^2} = \frac{\left(\sum y_{i2}\right)^2}{\sum y_{i1}}$$

$y_{i1} | y_{i1}+y_{i2}=m_i \sim Bin(m_i, \frac{\psi}{\psi+1})$

b. $$p(y_{i1} | y_{i1}+y_{i2}=m_i) = \binom{m_i}{y_{i1}} \left(\frac{\mu_{i1}}{\mu_{i1}+\mu_{i2}}\right)^{y_{i1}} \left(1 - \frac{\mu_i}{\mu_{i1}+\mu_{i2}}\right)^{m_i - y_{i1}}$$

$$= \binom{m_i}{y_{i1}} \left(1 + \frac{1}{\psi}\right)^{-y_{i1}} (1+\psi)^{y_{i1}-m_i}$$

$\vdots$

$$l(\psi) = \sum -y_{i1} \log\left(1+\frac{1}{\psi}\right) + \sum (y_{i1}-m_i) \log(1+\psi) + c(\psi)$$

$\vdots$

$$\hat{\psi}_c = \frac{\sum y_{i1}}{\sum y_{i2}} \qquad \left(= \frac{\sum y_{i1}}{\sum m_i - y_{i1}}\right)$$

Scanned by CamScanner

$$\frac{d}{d\psi} l = \frac{\sum y_{i1}}{\psi} - \frac{\sum m_i}{\psi+1}$$

$$\frac{d^2}{d\psi^2} l = \frac{-\sum y_{i1}}{\psi^2} + \frac{\sum m_i}{(\psi+1)^2}$$

$$I_n(\psi) = \frac{1}{\psi^2} \sum E(y_{i1} \mid y_{i1}+y_{i2}=m_i) - \frac{\sum m_i}{(\psi+1)^2}$$

$$= \frac{1}{\psi^2} \sum m_i \frac{\psi}{\psi+1} - \frac{\sum m_i}{(\psi+1)^2}$$

$$= \frac{\sum m_i}{\psi(\psi+1)} - \frac{\sum m_i}{(\psi+1)^2} = \frac{\sum m_i}{(\psi+1)}\left(\frac{1}{\psi} - \frac{1}{\psi+1}\right)$$

$$= \frac{\sum m_i}{(\psi+1)}\left(\frac{\cancel{\psi}+1-\cancel{\psi}}{\psi(\psi+1)}\right) = \boxed{\frac{\sum m_i}{\psi(\psi+1)^2}}$$

d) From a)

$$f_x(x \mid \mu_{i2}, \psi) = \exp\left\{-(\psi+1)\sum_{i=1}^{n} \mu_{i2} + \ln\left(\prod_{i=1}^{n} y_{i1}! \, y_{i2}!\right) + \psi \sum_{i=1}^{n} Y_{i1} + \sum_{i=1}^{n} (Y_{i1}+Y_{i2}) \ln(\mu_{i2})\right\}$$

$$T(x) = \sum_{i=1}^{n} Y_{i1}, \quad \theta = \psi$$

$$U_i(x) = Y_{i1} + Y_{i2} \quad \rho_i = \ln(\mu_{i2}) \quad i = 1, \dots, n$$

for $H_0: \psi = \psi_0$

$H_1: \psi \neq \psi_0$

UMPU test of level α

$$\phi(x) = \begin{cases} 1 & \sum Y_{i1} < k_1 \text{ or } \sum Y_i > k_2 \\ \gamma_2 & \sum_{i=1}^{n} Y_{i1} = k_1 \\ \gamma_1 & \sum_{i=1}^{n} Y_{i1} = k_2 \\ 0 & k_1 < \sum_{i=1}^{n} Y_{i1} < k_2 \end{cases}$$

$$\alpha = E\left[\phi(x) \mid U, \psi_0\right]$$

$$\alpha E\left[T(x) \mid U, \psi_0\right] = E\left[\phi(x) T(x) \mid U, \psi_0\right]$$

then to construct the interval,

use acceptance region

$$\text{Set} = \left\{\psi_0 : k_{10} \leq \sum_{i=1}^{n} Y_{i1} \leq k_2\right\}$$

Scanned by CamScanner

1) e) $\mu_{i1} = e^{x_i^T\beta}\mu_{i2}$

$$f(y_{i1}, y_{i2}) = \left(\frac{1}{y_{i1}!\,y_{i2}!}\right) e^{y_{i1}x_i^T\beta} e^{-\mu_{i2}(e^{x_i^T\beta}+1)} \mu_{i2}^{\,y_{i1}+y_{i2}}$$

$$\ell(\beta, \mu_{12}, \ldots, \mu_{n2} \mid \vec{y}) \propto \sum y_{i1}x_i^T\beta - \sum \mu_{i2}(e^{x_i^T\beta}+1) + \sum (y_{i1}+y_{i2}) \log \mu_{i2}$$

$$\frac{d}{d\beta}\ell = \sum y_{i1}x_i - \sum \mu_{i2} e^{x_i^T\beta} x_i$$

$$\frac{d^2}{d\beta d\beta^T} = -\sum \mu_{i2} e^{x_i^T\beta} x_i x_i^T$$

$$\frac{d^2}{d\beta d\mu_{i2}} = -e^{x_i^T\beta} x_i$$

$$\frac{d}{d\mu_{i2}}\ell = -(e^{x_i^T\beta}+1) + \frac{y_{i1}+y_{i2}}{\mu_{i2}}$$

$$\frac{d^2}{d\mu_{i2}^2}\ell = \frac{-(y_{i1}+y_{i2})}{\mu_{i2}^2}$$

$$\frac{d^2}{d\mu_{i2}d\mu_{j2}}\ell = 0$$

let $\xi = (\beta, \mu_{12}, \ldots, \mu_{n2})$

$$\frac{d}{d\xi}\ell = \begin{pmatrix} \sum y_{i1}x_i - \sum \mu_{i2} e^{x_i^T\beta} x_i \\ -(e^{x_1^T\beta}+1) + \frac{y_{11}+y_{12}}{\mu_{12}} \\ \vdots \\ -(e^{x_n^T\beta}+1) + \frac{y_{n1}+y_{n2}}{\mu_{n2}} \end{pmatrix}$$

$$-\frac{d^2}{d\xi d\xi^T} = \begin{pmatrix} \sum \mu_{i2} e^{x_i^T\beta} x_i x_i^T & (e^{x_1^T\beta}x_1)^T & \cdots & (e^{x_n^T\beta}x_n)^T \\ (e^{x_1^T\beta}x_1) & \frac{y_{11}+y_{12}}{\mu_{12}} & & 0 \\ \vdots & & \ddots & \\ e^{x_n^T\beta}x_n & 0 & & \frac{y_{n1}+y_{n2}}{\mu_{n2}} \end{pmatrix}$$

$$\Rightarrow \xi^{(c+1)} = \xi^{(c)} - \left[\frac{d^2}{d\xi d\xi^T}\ell\right]^{-1} \frac{d}{d\xi}\ell \Big|_{\xi^{(c)}}$$

and then 1st 2 components of ξ are $(\hat{\beta}, \hat{\beta}_2)$