BIOS 779 Notes

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1 Prior and Posterior Expectation and Variance

$$E(\theta) = E\Big[E[\theta|Y]\Big]$$

Prior mean of θ = Average posterior mean of θ over data distribution.

$$Var(\theta) = E\Big[Var(\theta|Y)\Big] + Var\Big[E[\theta|Y]\Big]$$

Posterior variance of θ is, on average, less than prior variance of θ.

2 Prior Distribution

When we do not know the precision, we can use the normal-gamma conjugate prior for θ, P

3 Bayesian in linear regression

(i) If we start with the non-estimable $X'X$ is not full rank, $X'X$ inverse does not exist. But if we have a proper prior, we could still estimate.

(ii) sensitivity analysis based on different variances, assume the a_0 to incorporate the variance, there is no just one the prior

(iii) Use discount likelihood as the prior specification, the power prior. Need to understand how we go from discount likelihood to discount likelihood times initial prior.

Bayesian Update It is a hierarchy, prior today is the posterior of yesterday.

Stage 0: haven't observed historical data. $\pi_0(\beta, \gamma)$ - non-informative Stage 1: $D_0 = (n_0, Y_0, X_0), a_0 = 1$, observed historical data, natural bayesian update. Then it is posterior of β based on on D_0 with $\pi_0(\beta, \gamma)$ as initial prior. Stage 2: $D = (n, Y, X), a_0 = 1$, posterior of β uses the posterior in stage 1 as the prior for β

Question: When $\alpha = 0$, the HPD included H_0, and as α increase, the HPD does not include H_0. Then how do we interpret it?

4 L measure

1. Y needs to be in the same scale, not only the X is the same.