2015, Part 1

7/10/17

#3 a) We know $E\left[\begin{pmatrix} X_i \\ Y_i \end{pmatrix}\right] = \begin{pmatrix} \mu_X \\ \mu_Y \end{pmatrix}$ are identifiable (b/c of Golden Rule of estimability/identifiability).

All params of $\underbrace{\left\{\begin{matrix} R_i \\ R_i Y_i \\ R_i X_i \end{matrix}\right\}}_{\text{observed data.}}$ are estimable (b/c we have the joint likelihood & all data is observed)

$$E[R_i Y_i] = E[R_i]E[Y_i] = \overset{\text{known}}{\pi}\mu_Y \quad ✓$$

$$E[R_i X_i] = E[R_i]E[X_i] = \overset{\text{known}}{\pi}\mu_X \quad ✓$$

$$\begin{aligned} Var(R_i Y_i) &= E[R_i^2 Y_i^2] - (E[R_i]E[Y_i])^2 \\ &= E[R_i]E[Y_i^2] - \pi^2\mu_Y^2 \\ &= \pi\left(Var(Y_i) + (E[Y_i])^2\right) - \pi^2\mu_Y^2 \\ &= \pi(\sigma_{22} + \mu_Y^2) - \pi^2\mu_Y^2 = \underbrace{\pi\mu_Y^2(1-\pi)}_{\text{known/identifiable}} + \pi\sigma_{22} \end{aligned}$$

$\Rightarrow$ identifiable ✓

We know (since this is the obs. data vector) that we can get all expectations, variances, & covariances of the RVs (R_i, R_iY_i, & R_iX_i) in this vector.

$$Var(R_i X_i) = \underset{\text{(same process)}}{\ldots} = \pi\mu_X^2(1-\pi) + \pi\sigma_{11}$$

$\Rightarrow$ identifiable.

$$Cov(R_i Y_i, R_i X_i) = E(R_i^2 Y_i X_i) - \underbrace{E[R_i Y_i]}_{\pi\mu_Y}\underbrace{E[R_i X_i]}_{\pi\mu_X}$$

not indep

$$\begin{aligned} &= E[R_i]E[Y_i X_i] - \pi^2\mu_Y\mu_X \\ &= \pi\left(Cov(X_i, Y_i) + E[Y_i]E[X_i]\right) - \pi^2\mu_Y\mu_X \\ &= \pi(\sigma_{12} + \mu_Y\mu_X) - \pi^2\mu_Y\mu_X \end{aligned}$$

only unknown here $\Rightarrow$ identifiable.