1) $N \sim Pois(\mu)$ $X_1, X_2, \ldots \overset{iid}{\sim} Pois(\lambda)$ $0 < \mu, \lambda < \infty$

let $U = I(N>0) \sum_{i=1}^{N} X_i$

a) $E(U) = E\Big[E[U|N]\Big] = E\Big[E\big[I(N>0)\sum_{i=1}^{N} X_i \,|\, N\big]\Big]$

$= E\Big[I(N>0)\sum_{i=1}^{N} E(X_i)\Big] = E\big[I(N>0)N\lambda\big]$

$= \lambda E_N\big[I(N>0)N\big] = \lambda E[N] = \mu\lambda$

$Var(U) = Var(E(U|N)) + E(Var(U|N))$

$= Var(\lambda N I(N>0)) + E(I(N>0)N\,\underbrace{Var(X_i)}_{\lambda})$

$= \lambda^2 Var(N I(N>0)) + \lambda E\big[I(N>0)N\big]$

$= \lambda^2 Var(N) + \lambda E(N)$

$= \lambda^2\mu + \lambda\mu = \mu\lambda(\lambda+1)$

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