d) $\underset{\sim}{Y} = f_x(\underset{\sim}{x} | \underset{\sim}{\xi})$

$H_0 : h(\underset{\sim}{\xi}) = \underset{\sim}{b}_0 \iff H_0 : \mu = \mu_0$

$H_1 : h(\underset{\sim}{\xi}) \neq \underset{\sim}{b}_0 \quad\quad H_1 : \mu \neq \mu_0$

Wald test is

$$W_n = (h(\hat{\underset{\sim}{\xi}}) - \underset{\sim}{b}_0)' \left[H(\underset{\sim}{\xi}) \, I_n(\hat{\underset{\sim}{\xi}})^{-1} H'(\underset{\sim}{\xi}) \right]^{-1} (h(\hat{\underset{\sim}{\xi}}) - \underset{\sim}{b}_0)$$

$$H(\underset{\sim}{\xi}) = \frac{\partial h(\underset{\sim}{\xi})}{\partial \underset{\sim}{\xi}}$$

$\underset{\sim}{\xi} = \begin{pmatrix} \mu \\ \phi \end{pmatrix}$, $h(\underset{\sim}{\xi}) = (\mu)$, $\underset{\sim}{b}_0 = (\mu_0)$, $H(\underset{\sim}{\xi}) = (1 \quad 0)$

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2c) suppose μ_1, μ_0 known

Then by NPL, $\exists$ a UMP test ϕ of

$H_0: \gamma = \gamma_0$ v $H_1: \gamma = \gamma_1$ st $\underline{\gamma_1 > \gamma_0}$ of the form

$$\phi(x) = \begin{cases} 1 & p(X|\gamma_1) > k\, p(X|\gamma_0) \\ 0 & p(X|\gamma_1) \leq k\, p(X|\gamma_0) \end{cases} = \begin{cases} 1 & p(X|\gamma_1)/p(X|\gamma_0) > k \\ 0 & p(X|\gamma_1)/p(X|\gamma_0) < k \end{cases}$$ when $E_{\gamma_0}[\phi] = \alpha$

our rejection region is: $$\frac{p(X|\gamma_1)}{p(X|\gamma_0)} = \left(\frac{\gamma_0 \mu_0^2}{\gamma_1 \mu_1^2}\right)^{\frac{n}{2}} \exp\left\{ -\frac{1}{2\gamma_1\mu_1^2}\sum(X_i-\mu_1)^2 + \frac{1}{2\gamma_0\mu_0^2}\sum(X_i-\mu_0)^2 \right\} > K$$

$$\Longleftrightarrow \underset{\uparrow}{\frac{1}{\gamma_1\mu_1^2}}\sum(X_i-\mu_1)^2 - \frac{1}{\gamma_0\mu_0^2}\sum(X_i-\mu_0)^2 > k^*$$

Thus our rejection region depends on γ_1 & Thus a UMP doesn't exist for $H_0: \gamma = \gamma_0$ v $H_1: \gamma > \gamma_0$ when μ_0, μ_1 known

suppose μ_0, μ_1 aren't known $\Rightarrow$ use their estimates

in part (b) it is seen that $\hat{\mu}_1$ depends on $\hat{\gamma}_1$ (aka $\hat{\mu}_1$ depends on H_1)

so it still doesn't exist