2015 D1

27a) $p(x) = \theta_1^{I(|x|=2)}\, \theta_2^{I(x=2)} (1-\theta_2)^{I(x=-2)} \left(\frac{1}{2}-\alpha\right)^{I(|x|=1)} \alpha^{I(|x|=0)} \left(\frac{1-\theta_1}{1-\alpha}\right)^{I(|x|\le 1)}$

Under H_0: $\theta_1 = \alpha$ $\theta_2 = \frac{1}{2}$

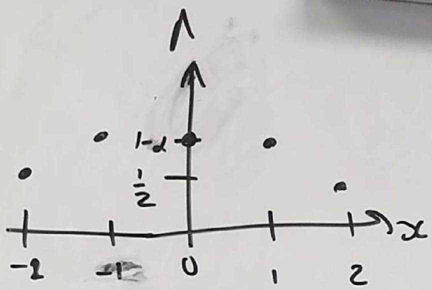
x	-2	-1	0	1	2
$p(x)$	$\frac{\alpha}{2}$	$\frac{1}{2}-\alpha$	α	$\frac{1}{2}-\alpha$	$\frac{\alpha}{2}$

Under H_1

$p(x)$ optains it's supremum under the whole space

at: $\hat{\theta}_1 = \alpha I(|x|=2)$ & $\hat{\theta}_2 = I(x=2)$

x	-2	-1	0	1	2
$p(x)$	α	$\frac{\frac{1}{2}-\alpha}{1-\alpha}$	$\frac{\alpha}{1-\alpha}$	$\frac{\frac{1}{2}-\alpha}{1-\alpha}$	α



SOLRT

x	-2	-1	0	1	2
Λ	$\frac{1}{2}$	$1-\alpha$	$1-\alpha$	$1-\alpha$	$\frac{1}{2}$

$\Rightarrow \Lambda = \left(\frac{1}{2}\right)^{I(|x|=2)} (1-\alpha)^{I(|x|\neq 2)} < k$

$= \left(\frac{1}{2}\right)^{I(|x|=2)} (1-\alpha)^{1-I(|x|=2)} < k$

$= \left(\frac{1}{2(1-\alpha)}\right)^{I(|x|=2)} (1-\alpha) < k$

$\Leftrightarrow \left(\frac{1}{2(1-\alpha)}\right)^{I(|x|=2)} < k_2$

~~$\Leftrightarrow I(|x|=2) < k_3$~~

let $y = I(|x|=2) \sim Bern(p) = Bern(\theta_1)$

y	0		1	
x	$\lvert 1\rvert$	0	-2	2
$p(x)$	$2\left(\frac{1}{2}-\alpha\right)\left(\frac{1-\theta_1}{1-\alpha}\right)$	$\alpha\left(\frac{1-\theta_1}{1-\alpha}\right)$	$\theta_1(1-\theta_2)$	$\theta_1\theta_2$
	$1-\theta_1$		θ_1	

$\Rightarrow -I(|x|=2)\log(2(1-\alpha)) < K_2$

$K_3 = -K_2 \log(2(1-\alpha))$

$\Rightarrow I(|x|=2) > K_3$

2015 D1

assume $c \in \{0,1\}$

2a) cont ...

$\Lambda < k \iff Y > c = k_3 \quad \alpha = \underset{\alpha = \alpha}{E_\alpha}[\phi(x)] = P_\alpha(Y > c) + \gamma P_\alpha(Y = c)$

let $c = 1 \quad = 0 + \gamma\alpha$

$\Rightarrow \gamma = 1$

note $Y > 1$ not possible

$$\phi = \begin{cases} 1 & Y > c \\ \gamma & Y = c \\ 0 & Y < c \end{cases}$$

$$\Rightarrow \phi = \begin{cases} 1 & Y = 1 \\ 0 & Y = 0 \end{cases} \quad (\text{b/c } Y < 1 \iff Y = 0)$$

suppose we choose $c \in (0,1)$

$\alpha = E(\phi(x))\ P_\alpha(Y > c) + \gamma P_\alpha(Y = c)$

$= P_\alpha(Y = 1) + 0 = \alpha$

$$\phi = \begin{cases} 1 & Y > c \\ 0 & Y < c \end{cases} \iff \phi = \begin{cases} 1 & Y = 1 \\ 0 & Y = 0 \end{cases}$$

b/c $Y > c \iff Y = 1$

$Y < c \iff Y = 0$

$$\phi = \begin{cases} 1 & Y > 0 \\ 0 & Y = 0 \\ \cancel{0} & \cancel{Y < 0} \end{cases} = \begin{cases} 1 & Y = 1 \\ 0 & Y = 0 \end{cases}$$

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2015 D1

27a) $p(x) = \theta_1^{I(|X|=2)} \, \theta_2^{I(X=2)} (1-\theta_2)^{I(X=-2)} \left(\frac{1}{2}-\alpha\right)^{I(|X|=1)} \alpha^{I(|X|=0)} \left(\frac{1-\theta_1}{1-\alpha}\right)^{I(|X|\le 1)}$

Under H_0: $\theta_1 = \alpha$ $\theta_2 = \frac{1}{2}$

x	-2	-1	0	1	2
$p(x)$	$\frac{\alpha}{2}$	$\frac{1}{2}-\alpha$	α	$\frac{1}{2}-\alpha$	$\frac{\alpha}{2}$

Under H_1

$p(x)$ obtains it's supremum under the whole space

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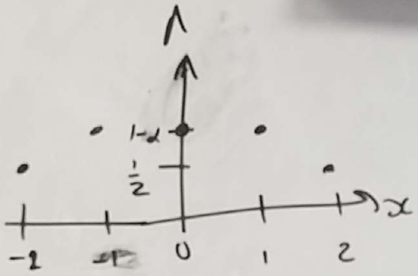
at: $\hat{\theta}_1 = \alpha I(|X|=2)$ & $\hat{\theta}_2 = I(X=2)$

x	-2	-1	0	1	2
$p(x)$	α	$\frac{\frac{1}{2}-\alpha}{1-\alpha}$	$\frac{\alpha}{1-\alpha}$	$\frac{\frac{1}{2}-\alpha}{1-\alpha}$	α



Solrt

x	-2	-1	0	1	2
Λ	$\frac{1}{2}$	$1-\alpha$	$1-\alpha$	$1-\alpha$	$\frac{1}{2}$

$\Rightarrow \Lambda = \left(\frac{1}{2}\right)^{I(|x|=2)} (1-\alpha)^{I(|x|\neq 2)} < K$

$= \left(\frac{1}{2}\right)^{I(|x|=2)} (1-\alpha)^{1-I(|x|=2)} < K$

$= \left(\frac{1}{2(1-\alpha)}\right)^{I(|x|=2)} (1-\alpha) < K$

$\Leftrightarrow \left(\frac{1}{2(1-\alpha)}\right)^{I(|X|=2)} < K_2$

$\cancel{\Leftrightarrow I(|X|=2) < K_3}$

let $Y = I(|X|=2) \sim Bern(p) = Bern(\theta_1)$

Y	0		1	
X	$\lvert 1 \rvert$	0	-2	2
$p(X)$	$2\left(\frac{1}{2}-\alpha\right)\left(\frac{1-\theta_1}{1-\alpha}\right)$	$\alpha\left(\frac{1-\theta_1}{1-\alpha}\right)$	$\theta_1(1-\theta_2)$	$\theta_1\theta_2$
	$1-\theta_1$		θ_1	

$\rightarrow -I(|X|=2) \log(2(1-\alpha)) < K_2$

$K_3 = -K_2 \log(2(1-\alpha))$

$\Rightarrow I(|X|=2) > K_3$

2015 D1

assume $c \in \{0,1\}$

2a) cont ...

$\Lambda < k \iff Y > c = k_3$

$\alpha = E_{\alpha}[\phi(x)] = P_\alpha(Y > c) + \gamma P_\alpha(Y = c)$

$\phi = \begin{cases} 1 & Y > c \\ \gamma & Y = c \\ 0 & Y < c \end{cases}$

let $c = 1$ $\quad = 0 + \gamma\alpha$

$\Rightarrow \gamma = 1$

note $Y > 1$ not possible

$\Rightarrow \phi = \begin{cases} 1 & Y = 1 \\ 0 & Y = 0 \end{cases}$ (b/c $Y < 1 \iff Y = 0$)

Suppose we choose $c \in (0,1)$

$\alpha = E(\phi(x))\ P_\alpha(Y > c) + \gamma P_\alpha(Y = c)$

$= P_\alpha(Y = 1) + 0 = \alpha$

$\phi = \begin{cases} 1 & Y > c \\ 0 & Y < c \end{cases} \iff \phi = \begin{cases} 1 & Y = 1 \\ 0 & Y = 0 \end{cases}$ b/c $Y > c \iff Y = 1$, $Y < c \iff Y = 0$

$\phi = \begin{cases} 1 & Y > 0 \\ 0 & Y = 0 \\ \cancel{0} & \cancel{Y < 0} \end{cases} = \begin{cases} 1 & Y = 1 \\ 0 & Y = 0 \end{cases}$

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2015 D1

2 7 a) $p(x) = \theta_1^{I(|x|=2)} \theta_2^{I(x=2)} (1-\theta_2)^{I(x=-2)} \left(\frac{1}{2}-\alpha\right)^{I(|x|=1)} \alpha^{I(|x|=0)} \left(\frac{1-\theta_1}{1-\alpha}\right)^{I(|x|\le 1)}$

Dist. of X

x	-2	-1	0	1	2
$p(x)$	$\theta_1(1-\theta_2)$	$\left(\frac{1}{2}-\alpha\right)\left(\frac{1-\theta_1}{1-\alpha}\right)$	$\alpha\left(\frac{1-\theta_1}{1-\alpha}\right)$	$\left(\frac{1}{2}-\alpha\right)\left(\frac{1-\theta_1}{1-\alpha}\right)$	$\theta_1\theta_2$

Dist. of $Z = -X$

z	-2	-1	0	1	2
$p(z)$	$\theta_1\theta_2$	$\left(\frac{1}{2}-\alpha\right)\left(\frac{1-\theta_1}{1-\alpha}\right)$	$\alpha\left(\frac{1-\theta_1}{1-\alpha}\right)$	$\left(\frac{1}{2}-\alpha\right)\left(\frac{1-\theta_1}{1-\alpha}\right)$	$\theta_1(1-\theta_2)$

$1-\theta_2 \equiv \theta_2^*$

θ_1^* be parameters for $p_2(z)$

$\bar{g}(\theta)$ is induced fn. on $\Theta = \{(\theta_1, \theta_2)\}$

$\bar{g}(\theta) = \begin{pmatrix}\theta_1^* \\ \theta_2^*\end{pmatrix} = \begin{pmatrix}\theta_1 \\ 1-\theta_2\end{pmatrix}$ for $\theta = \begin{pmatrix}\theta_1 \\ \theta_2\end{pmatrix}$

Test for Θ: $H_0: \theta_1 = \alpha, \ \theta_2 = \frac{1}{2}$ $\quad H_1: \theta_1 < \alpha, \ \theta_2 \ne \frac{1}{2}$

Test for Θ^*: $H_0: \theta_1^* = \alpha, \ \theta_2^* = \frac{1}{2}$ $\quad H_1: \theta_1^* < \alpha, \ \theta_2^* \ne \frac{1}{2}$