2009 Qualifying Exam Section 2

February 21, 2019

Question 1

1.a

Question 3

3.a.

Let $\boldsymbol{x} = (x_{11}, x_{12}, x_{21}, x_{22})$

$$f(\boldsymbol{x}) = \frac{n!}{\prod_{i,j} x_{ij}!} \prod_{i=1}^{2} \prod_{i=1}^{2} p_{ij}^{x_{ij}}$$

$$= \exp\left\{ x_{11} \log p_{11} + x_{12} \log p_{12} + x_{21} \log p_{21} + x_{22} \log p_{22} \right\}$$

$$= \exp\{ x_{11} \log \frac{p_{11}}{1 - p_{11} - p_{12} - p_{21}} + x_{12} \log \frac{p_{12}}{1 - p_{11} - p_{12} - p_{21}} + x_{21} \log \frac{p_{21}}{1 - p_{11} - p_{12} - p_{21}}$$

$$+ n \log p_{22} + c(x) \}$$

$$= \exp\left\{ \boldsymbol{Q}(\boldsymbol{x})^T \boldsymbol{T}(\boldsymbol{\theta}) + d(\boldsymbol{\theta}) + d(\boldsymbol{x}) \right\}$$

where $\boldsymbol{Q}(\boldsymbol{x}) = (x_{11}, x_{12}, x_{21})^T$, $\boldsymbol{T}(\boldsymbol{\theta}) = (\theta_{11}, \theta_{12}, \theta_{21})^T$, and $\theta_{ij} = \log \frac{p_{ij}}{1 - p_{11} - p_{12} - p_{21}}$ for $(i, j) \in \{(1,1), (1,2), (2,1)\}$