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2) a) see 761 Ch 1 Th 1.9 proof (p 74)

b) Suppose there exists a rule d s.t. $R(\theta_i, d) \le R(\theta_i, d_B)$ for all i, with strict inequality for some i. For those i with strict inequality, let $\lambda_i = 0$. Let all other $\lambda_i > 0$, with corresponding $R(\theta_i, d) = R(\theta_i, d_B)$.

Then $\sum \lambda_i R(\theta_i, d) = \sum \lambda_i R(\theta_i, d_B) \Rightarrow \mathcal{R}(\Lambda, d) = \mathcal{R}(\Lambda, d_B)$

So d and d_B are both Bayes rules, but by construction, d_B is not admissible.

Another idea:

If d_B is inadmissible then $\exists$ rule d st $R(\theta_i, d) \le R(\theta_i, d_B)$ $\forall i$ & $R(\theta_i, d) < R(\theta_i, d_B)$ some i

$\Rightarrow R(\theta_i, d)\lambda_i \le R(\theta_i, d_B)\lambda_i \quad \forall i$

$\Rightarrow \sum_{i=1}^{l} R(\theta_i, d)\lambda_i \le \sum_{i=1}^{l} R(\theta_i, d_B)\lambda_i$

$\Rightarrow \mathcal{R}(\Lambda, d) \le \mathcal{R}(\Lambda, d_B)$ But since d_B is Bayes

$\Rightarrow \mathcal{R}(\Lambda, d_B) \le \mathcal{R}(\Lambda, d) \quad \forall d \in \mathcal{D}$

$\Rightarrow \mathcal{R}(\Lambda, d) = \mathcal{R}(\Lambda, d_B)$

$\Rightarrow$ Multiple Bayes rules $\Rightarrow$ d_B may be inadmissible

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WLOG $R(\theta_i, d_B) = c$

2) c) Sps $R(\theta, d_B) < \infty$ & constant on θ_i's w/ $\lambda_i > 0$.

Then $\mathcal{R}(\theta, d_B) = \underbrace{\sum_{\substack{\theta_i\text{'s w/}\\ \lambda_i > 0}} \underbrace{R(\theta_i, d_B)}_{c} \lambda_i}_{= \text{constant}} + \underbrace{\sum_{\substack{\theta_i\text{'s w/}\\ \lambda_i = 0}} R(\theta_i, d_B) \lambda_i}_{0}$

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$\therefore$ $c\sum \lambda_i = c \cdot 1 + 0 = \sup_{\substack{\theta_i \text{ st}\\ \lambda_i > 0}} R(\theta, d_B)$

By Th 1.12 d_B is minimax over all θ_i w/ $\lambda_i > 0$

d) We cannot b/c its possible that $R(\theta_i, d_B) > c$ when $\lambda_i = 0$

$\Rightarrow \mathcal{R}(\theta, d_B) \neq \sup_{\substack{\theta_i\\ \lambda_i \geq 0}} R(\theta_i, d_B)$

But if $R(\theta_i, d_B) \leq c$ then d_B is minimax over all θ_i

ie $\mathcal{R}(\theta, d_B) = \sup_{\substack{\theta_i\\ \lambda_i \geq 0}} R(\theta_i, d_B)$

Note:

d_B is Bayes if $\mathcal{R}(\Lambda, d_B) = \inf_{d \in \mathcal{D}} \{ \mathcal{R}(\Lambda, d) \}$

d_M is minimax if $\sup_{\theta} R(\theta, d_M) = \inf_{d \in \mathcal{D}} \{ \sup_{\theta \in \Theta} R(\theta, d) \}$

Connection in Thm 1.12

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