Functional Reactive Programming (FRP)

- New Domain
- New Abstractions

} Prehistory

- Denotational Semantic Design
- What is FRP?

} Early Design Choices

- Design space exploration
- Integrate with widespread languages
- Web GUI domain

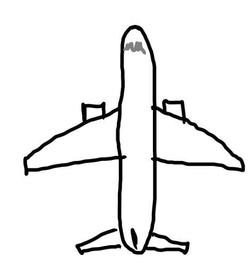
} Modern FRP Solutions

Prehistory : Reactive Systems

1968 — 1972 — ... — 1988

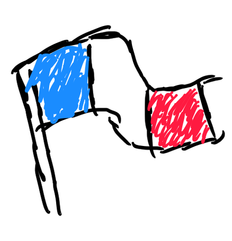
Analog Flight: Apollo 8 fly-by-wire → Military Planes → ... → Airbus A320

Joystick → [Levers / Software] → Rudder Control



Temp → [Circuits / Software] → Control Rods

1980s : Domain Specific Languages For Reactive Systems

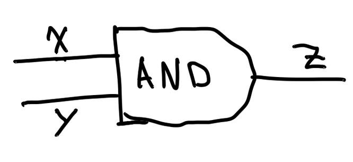


Esterel (Berry & Gonthier), Lustre (Halbwachs et. al.), Signal (Benveniste et. al.) → industry

Lucid (Wadge & Ashcroft) — "Stream Programming"

Lustre Language: (Caspi, Pilaud, Halbwachs, Plaice) 1987 POPL

- Equations to relate variables for all of time



$z = x$ and y $\implies \forall t.\ z_t = x_t$ and y_t

- Discrete & implicit time

$$eqns ::= (id = e)^+$$ → Recursive Equations!

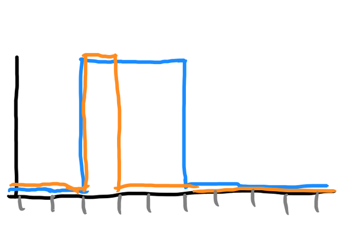
$$e ::= \text{if } e_1 \text{ then } e_2 \text{ else } e_3 \mid e_1 + e_2 \mid \ldots \mid c \mid id$$

- Time Manipulation Primitives

$$e ::= \ldots \mid e_1 \rightarrow e_2 \mid \text{pre}(e_1) \mid e_1 \text{ when } e_2 \mid \text{current}(e_1)$$

Ex 1: Rising Edge of a signal (→ and pre)

x → [EDGE] → y



```
node EDGE (x: bool) returns (y: bool);
let
  y = false -> x and not pre(x);
tel
```

x	x_1	x_2	x_3	x_4 ...
pre(x)		x_1	x_2	x_3
y	F	$x_2 \wedge (\neg x_1)$	$x_3 \wedge (\neg x_2)$	$x_4 \wedge (\neg x_3)$

Ex 2: Sum of a signal (recursion)

```
node SUM(x: int) returns (y: int);
let
  y = x -> pre(y) + x;
tel
```

x	2	3	1	4	...
y	2	5	6	10	...
pre(y)		2	5	6	...

Ex 3: Sampling

when

x	x_0	x_1	x_2	x_3	x_4 ...
b	T	F	T	F	T ...
e = x when b	$e_0 = x_0$		$e_1 = x_2$		$e_2 = x_4$ ...

current

x	x_0		x_1		x_2
current(x)	x_0	x_0	x_1	x_1	x_2

What can go wrong?

1. "Nonsense": $x = 3x + 1$
   - ↳ Does not denote [Kahn 1974] & later
   - ↳ Cycles must have pre
2. Sensical, but "bad". Ex:
   - ↳ Unbounded Memory!

x	x_0	x_1	x_2	x_3	x_4
e	$e_0 = x_0$		$e_1 = x_2$		$e_2 = x_4$
z = x + e	$z_0 = x_0 + x_0$		$z_1 = x_1 + x_2$		$z_2 = x_2 + x_4$

- ↳ Domain-Specific!
- ↳ Semantics-Specific!
- ↳ "clock calculus" ensures compatible clocks

Semantics: Labelled Transition Systems + external stimuli

(Recall: π-calculus)

$$P_0 \xrightarrow{} P_1 \xrightarrow{} P_2 \xrightarrow{} \ldots$$

(inputs I_0, I_1, I_2; outputs R_0, R_1, R_2)

Equations:

$$eqn \xrightarrow[\sigma]{} eqn'$$

⋮

Expressions:

$$\sigma \vdash e \xrightarrow{v} e'$$

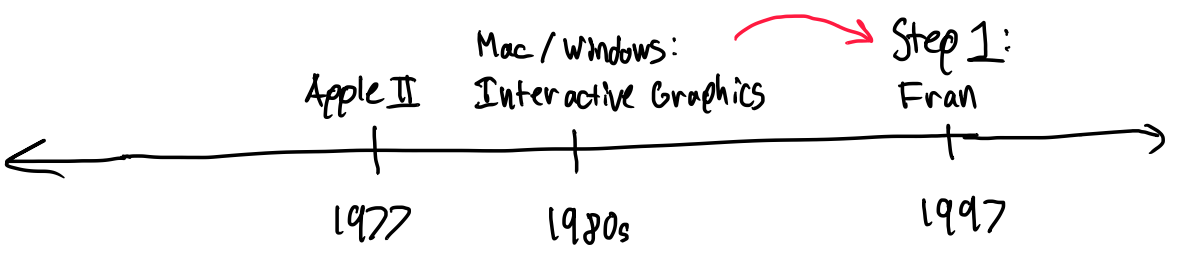
$$\frac{e_1 \xrightarrow{v} e_1'}{\sigma \vdash e_1 \rightarrow e_2 \xrightarrow{v} e_2}$$

⋮

Implementation: Compile Lustre, Esterel, Signal → Finite Automaton in C, then call from driver C code

Prehistory Recap:

1. Declaratively state reactions to the world
2. Time is implicit and discrete
3. Time manipulation primitives
4. Rule-out terms: nonsense vs. undesirable — Subjectively Depends on Domain!
5. Semantics by LTS
6. Compiles to finite automata



Step 1: Functional Reactive Animation (Fran)

Elliott and Hudak, 1997

- Question: How to make interactive graphics functional language?
- Elliott: Seems a lot like reactive systems!
  - Continuous time rather than discrete
    → Composability for modelling animations
    → Hopefully avoid clock calculus mess
  - Embedded DSL in Haskell
  - Denotations first, implementation second

Fran Types:

```
data Behavior a = (--hidden--)
data Event    a = (--hidden--)

T = ℝ   (double)
```

Ex: Behavior Shape is an animation

Fran Denotation & Primitives ("Combinators")

```
at  : Behavior a → (T → a)
occ : Event a → List (T, a)
```

```
time :: Behavior T
at[[time]] = λt. t
```

```
lift0 :: a → Behavior a
at[[lift0 x b]] = λt. x
```

```
lift1 :: (a1 → a2) → Behavior a1 → Behavior a2
at[[lift1 f b]] = λt. f (at[[b]] t)
```

⋮

```
(⟹) :: Event a → (T → a → b) → Event b
predicate :: Behavior Bool → Event ()
snapshot :: Event a → Behavior b → Event (a, b)
until :: Behavior a → Event (Behavior a) → Behavior a
```

⋮

Fran Example

```
wiggle :: Behavior Double
wiggle = (lift1 sin) ((lift1 (*)) (lift0 pi) time)
wiggle = sin (pi * time)
```

Haskell Typeclass Magic

```
ball = move (integral wiggle, 0) circle
```

```
cycle :: Color → Color → T → Behavior Color
cycle x y t0 = x `until` (lbp t0 ⟹
                λt. λ_. cycle y x t)
```

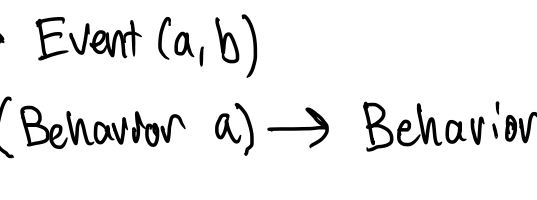
```
ballAnim t0 = withColor (cycle red blue t0) ball
```

Implementation

Strategy: 1. Choose a concrete (efficient) representation for Behavior a and Event a

2. Pull-based (polling)

b `until` e:



Reflection:

- Potentially good abstraction?
- Pull-based implementation ☹
- Denotational semantics is nice, but continuous functions $\mathbb{R} \to a$ hide a lot of complexity:
  → Contains many crazy terms
    - Implementation ≠ Semantics
    - Space leaks, etc.

Step 1.1: Are Continuous Time Semantics Really a good idea?

Functional Reactive Programming from First Principles
Wan and Hudak, 2000

- Compare continuous vs. discrete sampling semantics
- Take limit as sampling interval $\to 0$

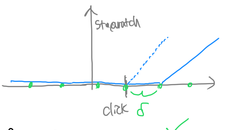
$\widetilde{at}$: Behavior $a \to \mathbb{R} \to \mathbb{R} \to a$ (start time, sample time) — Continuous Semantics ↪ Elliott

vs.

$\widetilde{at}^{\delta}$: Behavior $a \to \mathbb{R} \to \mathbb{R} \to a$ — Limit of Sampling Implementation

$\widetilde{at}^{\delta}[\![b]\!]\ t_0\ t = \lim_{\delta \to 0} \langle \text{sampling implementation} \rangle$

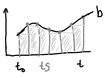
Ex 1: stopwatch = 0 `until` click $\Rightarrow \lambda t_c.$ time $- t_c$ — Time Capture Primitive



$\lim_{\delta \to 0} \widetilde{at}^{\delta}[\![\text{stopwatch}]\!] = \underline{at}[\![\text{stopwatch}]\!]$ ✓

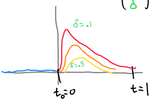
Ex 2: Integral Primitive

integral b = $\lambda t_s.$ cumulative sum of rectangles at sample points



$\underline{at}[\![\text{integral } b]\!]\ t_0\ t = \int_{t_0}^{t} (\underline{at}[\![b]\!]\ t_0\ t)\, dt$

bizarre = 0 `until` (time > 0) $\Rightarrow \lambda \delta.$ $\left(\frac{1}{\delta}\right)^2 \text{time}(1-\text{time})^{1/\delta}$



bad = integral bizarre

$\widetilde{at}^{\delta}[\![\text{integral bizarre}]\!]\ 0\ 1 = \lim_{\delta \to 0} \int_0^1 \left(\frac{1}{\delta}\right)^2 t(1-t)^{1/\delta} dt = \lim_{\delta \to 0} \frac{(1/\delta)^2}{(1/\delta+1)(1/\delta+2)} = 1$

$\stackrel{?}{=}$

$\int_0^1 (\widetilde{at}^{\delta}[\![\text{bizarre}]\!]\ 0\ t)\, dt = \int_0^1 \lim_{\delta \to 0} \left(\frac{1}{\delta}\right)^2 t(1-t)^{1/\delta} dt = \int_0^1 0\, dt = 0$

$\Rightarrow$ Sampling $\not\to$ Semantics!

Lesson:

- Simple on paper $\neq$ Simple in practice

Prehistory: 1980s Reactive Systems / Esterel, Lustre
- discrete
- push-based
- Efficient/bounded mem
- restricted

1: 1997 (Elliott) Interactive Graphics / Fran
- continuous
- pull-based
- Slow(?)

1.1: 2000 (Wan & Hudak) Continuous Semantics are weird...

1.2: 2001 (Wan, Taha, Hudak) RT-FRP

Step 1.2: RT-FRP

Wan, Taha, Hudak, 2001

- Discrete time
- Fewer primitives:

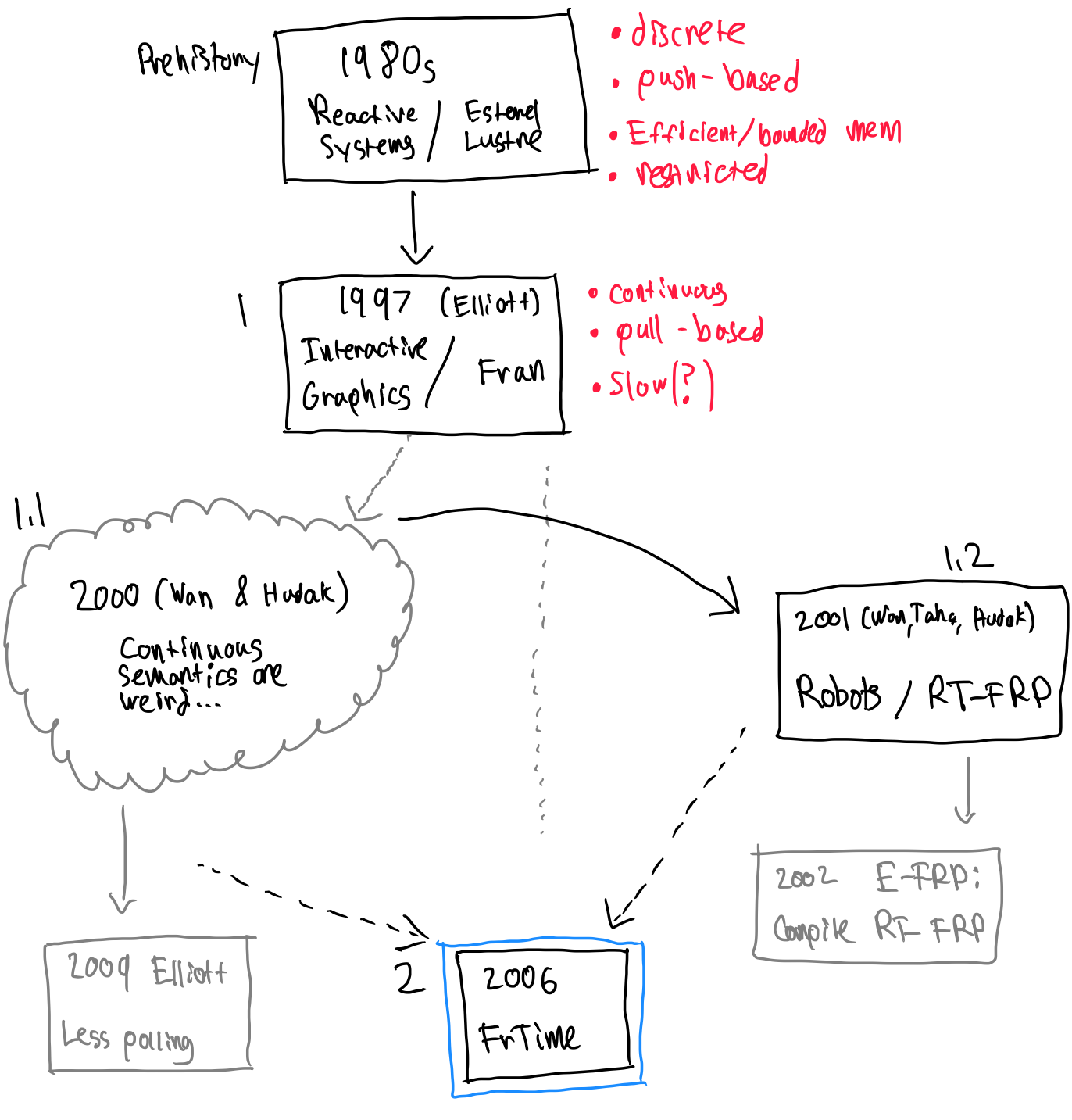
	Fran	Lustre
• time	time	X
• delay c s	X	c → pre(s)
• let snapshot x ← s_1 in s_2	lift	(implicit)
• s_1 switch on x ← e in s_2	until	if
• Recursion		

- Recursion:
  - Syntactically require tail calls during mutual recursion
    +
  - Wild, ad-hoc type system
    ⇓
  - Bounded Memory Usage → same guarantee as Esterel / Lustre
- Operational Semantics: Labelled Transition Systems!

$$\frac{\mathcal{E};k \xrightarrow{t,i} s \quad \xrightarrow{c'} \quad s'}{\mathcal{E};k \vdash \text{delay } c\ s \xrightarrow[c]{} \text{delay } c'\ s'}$$

→ Same as Lustre!

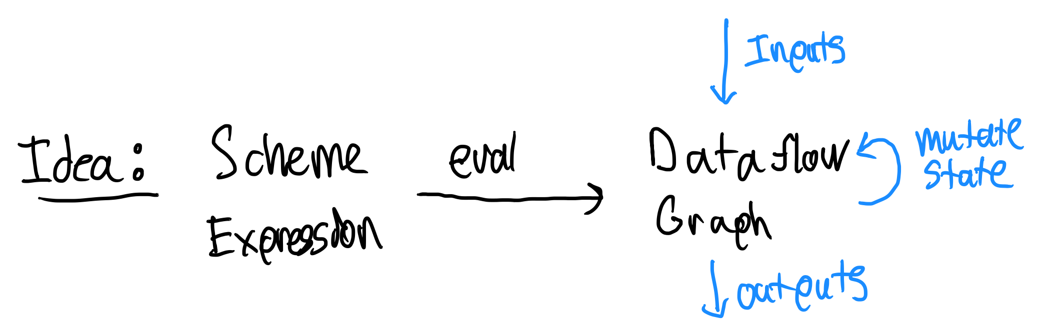
- Never gave examples demonstrating that it is more general than Esterel / Lustre!



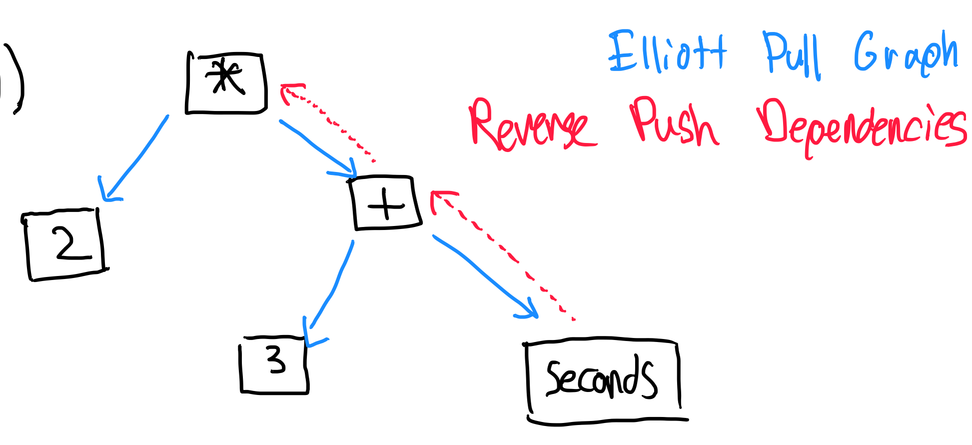
Step 2: Embedding Dynamic Dataflow in a Call-by-Value Language (FrTime)

Cooper & Krishnamurthi, 2006

- ~~Continuous semantics~~ discrete updates
- ~~Polling~~ push-based updates
- Higher-order ✓
- ~~Call-by-need~~ Scheme/Racket (use mutable state)
- Efficient ✓ (but weaken semantics)



(* 2 (+ 3 seconds))



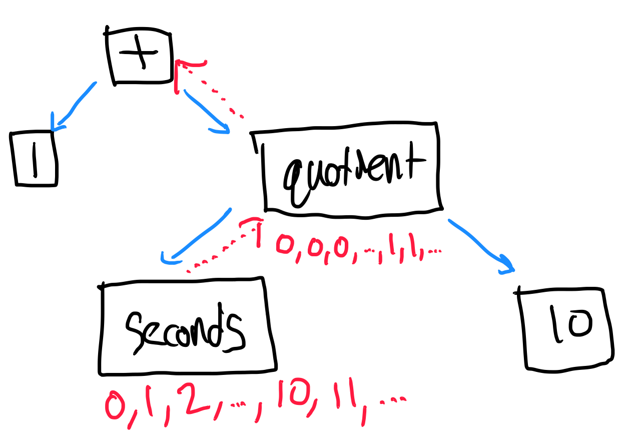
1. How to evaluate to a graph?

Redefine primitives. Ex:

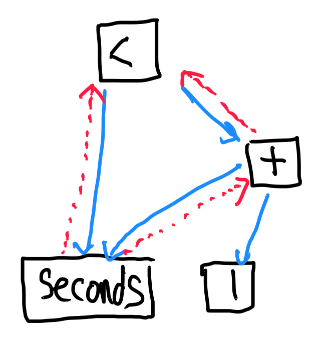
```
(+ x y) ==> (if (or (signal? x) (signal? y))
                (make-signal (λ () (+ (current-value x)
                                      (current-value y))) x y)
                (+ x y))
```

2. Once graph is built, propagate changes as needed.

Ex: (+ 1 (quotient seconds))



Ex: (< seconds (+ 1 seconds))



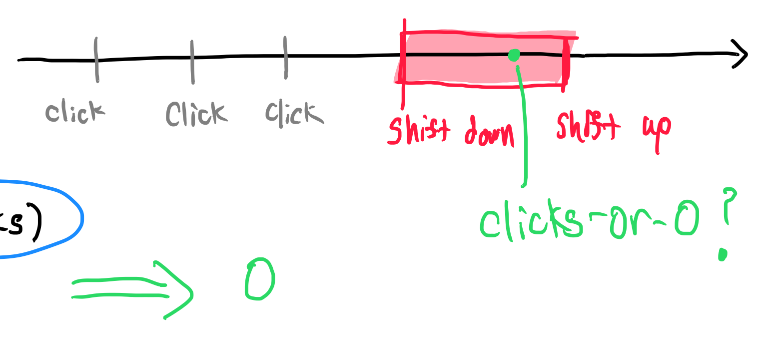
⟹ Update in Topological Order

3. Complications:
   - Cycles?
     ↳ Add a special delay primitive
   - Dynamically Reconfigure the Graph
     ↳ Handle very carefully, avoid space leaks
     with some semantic "sacrifices"

Ex:

```
(define (count e) ...)      : Event → Behavior Int
(define shift-down ...)     : Behavior Bool
(define left-clicks ...)    : Event
```

```
(define clicks-or-0
  (if shift-down
      (count left-clicks)
      0 ))
```

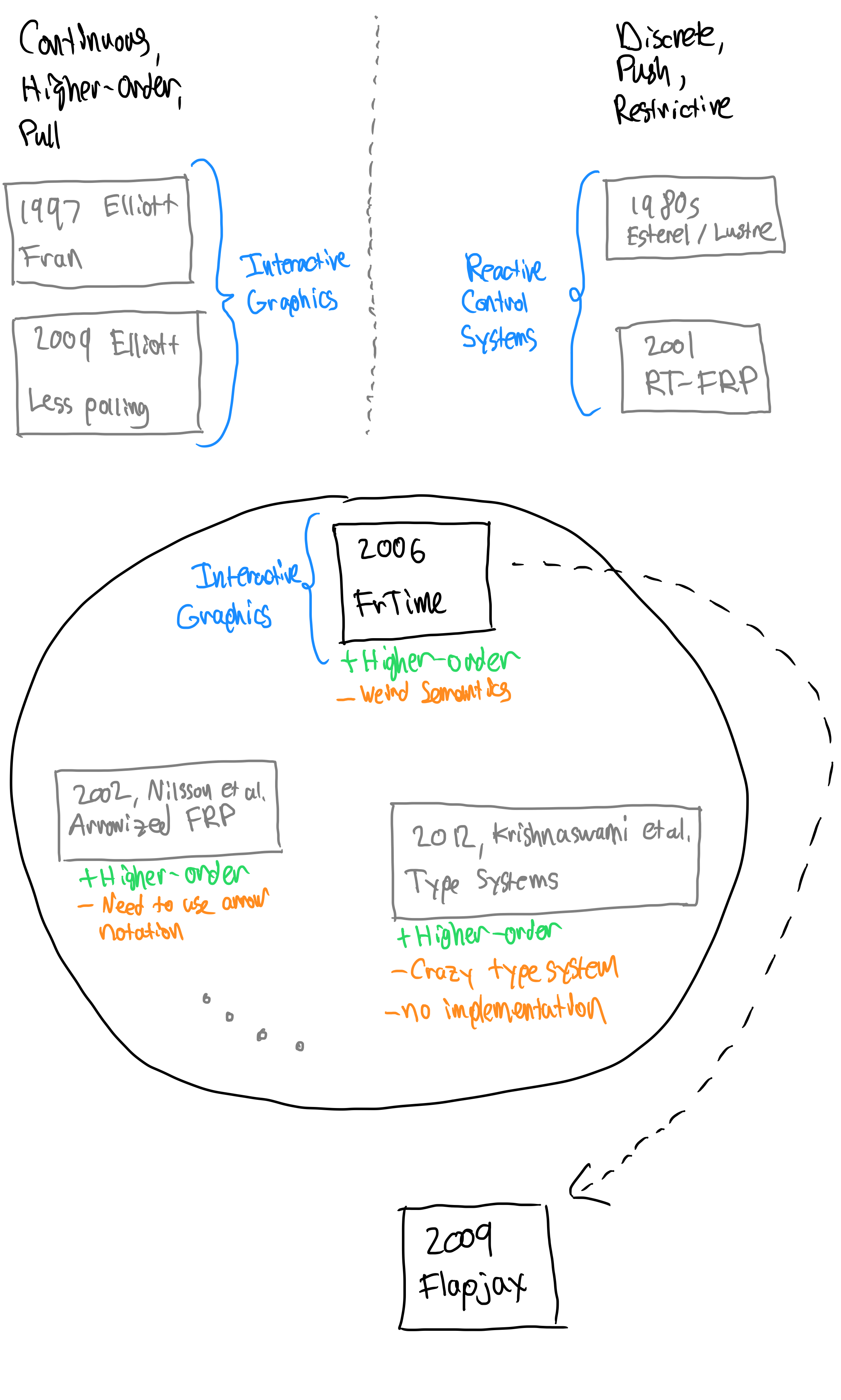


⟹ 0

```
(define click-count (count left-clicks))
(define clicks-or-0-2
  (if shift-down
      click-count
      0))
```

⟹ 3

→ Fran should give 3 for both



Step 3: Flapjax: A Programming Language for Ajax Applications

Meyerovich et al. 2009

— FrTime got the algorithm for push-update

— But for what domain?

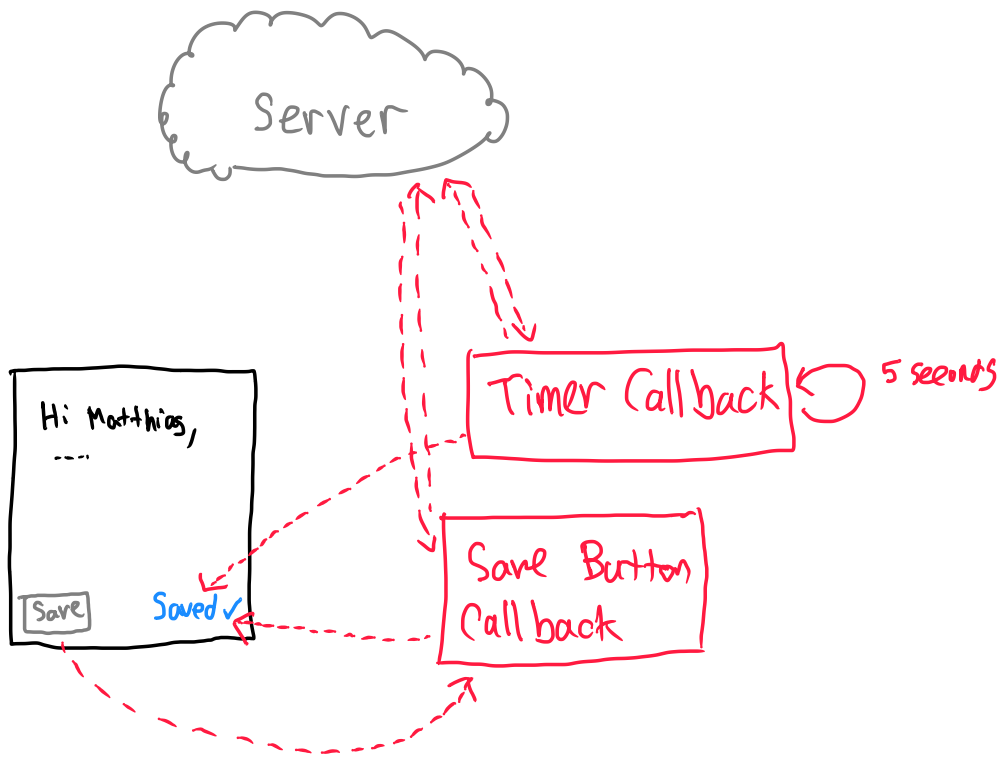
→ Interactive graphics? → cool for teaching!

→ GUIs?

→ Reactive control systems?

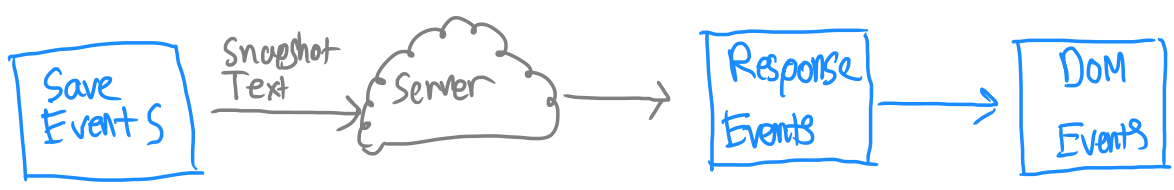
— Web GUIs! Callbacks = bad abstraction!

Ex:



- Complex policies for when to trigger save
- Policy defined by state mutation spread across multiple callbacks

— Use Events / Behaviors as abstractions



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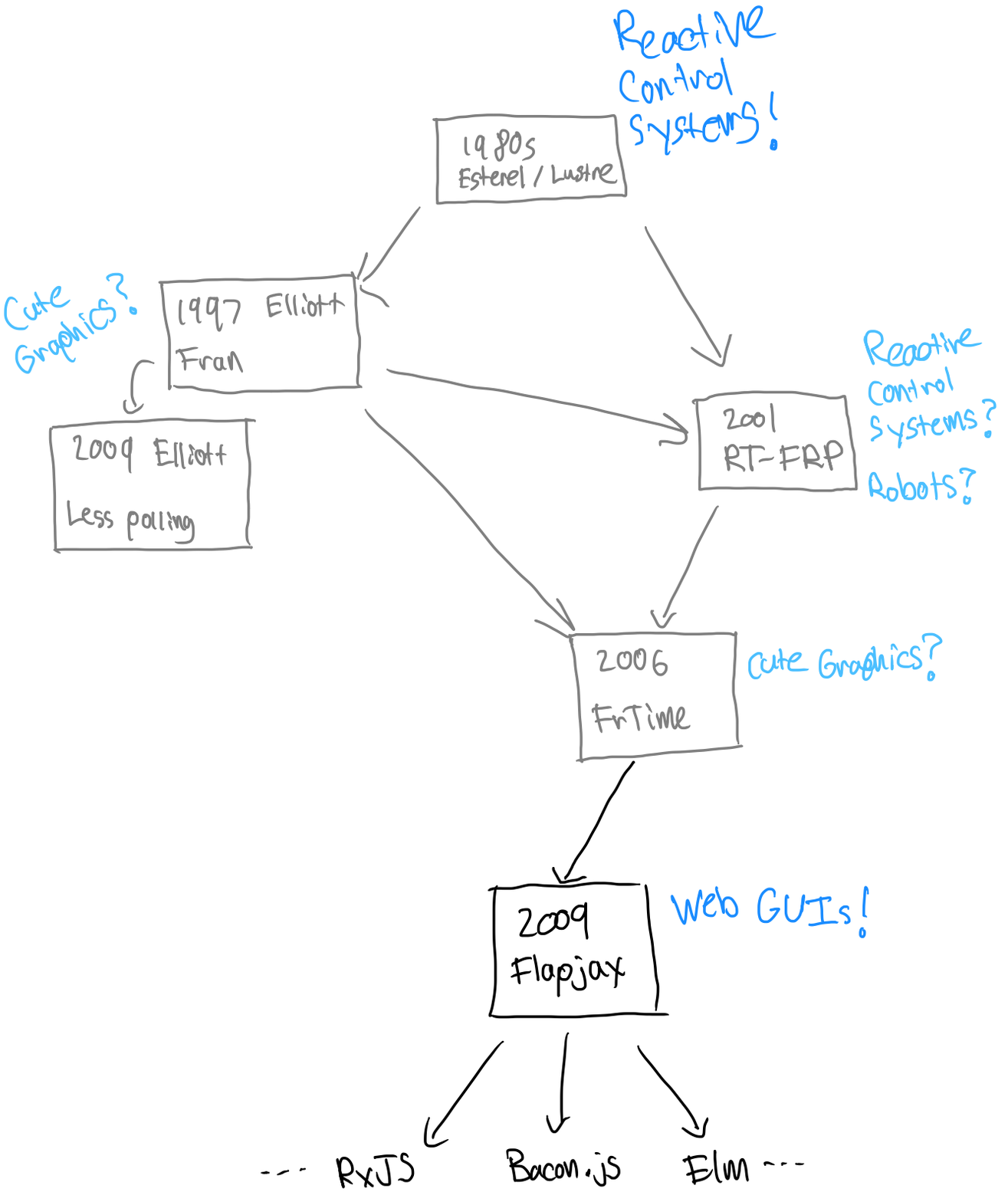
e.g. merge(timer(5000), clicks(button))

— FrTime algorithm for Javascript

— Lots of Web-specific combinators!!



Reflections:

1. Don't be fooled by "simple" semantics

   → Implementation is what gets adopted into industry!

2. Abstractions follow Domains

   ↳ Take the domain seriously