System F Type Systems: From Theory to Practice

Theory
- Background on STLC
- Why isn't STLC enough?
- Why isn't System F enough?

Practice
- How to leverage these ideas in mainstream PLs?
- How do pragmatic concerns affect design decisions?

Simply Typed Lambda Calculus

terms

$$
\begin{aligned}
e ::=\ & x \\
\mid\ & \lambda\{x:t\}.\ e \\
\mid\ & e\ \ e \\
\mid\ & n. \\
\mid\ & \text{add1}\ \ e
\end{aligned}
$$

types

$$
\begin{aligned}
t ::=\ & \text{int} \\
\mid\ & t \to t.
\end{aligned}
$$

Ex

$(\lambda\{x:\text{int}\}.\ \text{add1}\ x)\ 42$ ✓.

$(\lambda\{x:\text{int}\}.\ \text{add1}\ x)\ (\lambda\{x:\text{int}\}.\ x)$ ✗

Limitation of STLC

- An Identity function.

$(\lambda\{x:int\}.\ x)\ 74$ ✓.

$(\lambda\{x:int\to int\}.\ x)\ (\lambda\{x:int\}.x)$ ✓.

$(\lambda\{x:int\to int\}.\ x)\ 74$ ✗

- But without types, they all reduce to values.

$(\lambda x.\ x)\ 74 \Rightarrow 74$

$(\lambda x.\ x)\ (\lambda x.\ x) \Rightarrow (\lambda x.x)$

$(\lambda x.\ x)\ 74. \Rightarrow 74$

- Too Conservative.

$\Rightarrow$ Code Duplication.

- sort
- map
- ......

System F (Reynolds 74)
(Girard 72).

Intuition

$(\lambda\{x : int\}.\ x)$

$(\lambda\{x : int \to int\}.\ x)$.

What's varying? Types.

- λ abstracts over varying <u>terms</u>
  - term $\to \boxed{\lambda} \to$ term.
- Λ abstracts over varying <u>types</u>
  - type $\to \boxed{\Lambda} \to$ term.

Big Lambda Λ Grammar

$$e ::= x \mid \lambda\{x:t\}.\, e \mid e\ e \mid n \mid \text{add1}\ e$$
$$\mid \Lambda T.\, e \qquad \text{(Big Lambda)}$$
$$\mid e[t] \qquad \text{(Type application)}$$

$$t ::= \text{int} \mid t \to t$$
$$\mid \forall T.\, t \qquad \text{(Universal/forall type)}$$
$$\mid T \qquad \text{(Type variable)}$$

$\boxed{\text{Big Lambda Semantics}}$

- Type application is similar to term application
  $\boxed{\text{Substitution}}$

$$\text{let } id = \Lambda T.\ \lambda\{x:T\}.\ x.$$
$$\text{in } (id\ [int \to int]\ (\lambda\{x: int\}.\ x))$$
$$(id\ [int]\ 74)$$

$$\Rightarrow ((\Lambda T.\ \lambda\{x:T\}.\ x)\ [int \to int]\ (\lambda \ldots))$$
$$((\Lambda T.\ \lambda\{x:T\}.\ x)\ [int]\ 74)$$

$$\Rightarrow ((\lambda\{x: int \to int\}.\ x)\ (\lambda\{x: int\}.\ x))$$
$$((\lambda\{x:int\}.\ x)\ 74).$$

$$\Rightarrow (\lambda\{x:int\}.\ x)\ 74$$

$$\Rightarrow 74$$

User Defined Types

Warmup example

```
class Counter {
    Counter() {...}
    Counter inc() {...}
    int   get() {...}}
----------------------------------------
new Counter().inc().get()
```

System F Example

$$(\Lambda\,Counter.\ \lambda\{new: unit \to Counter\}.$$
$$\lambda\{inc: Counter \to Counter\}.$$
$$\lambda\{get: Counter \to int\}.$$
$$get\ (inc\ (new\ ()))\ )$$

int

$(\lambda\{x: unit\}.\ \cdots)$

$(\lambda\{x: int\}.\ \cdots)$

$(\lambda\{x: int\}.\ \cdots)$

$\boxed{\text{Representation Theorem}}$

Main idea:

$\boxed{\text{Representations of primitive types shouldn't affect program behavior.}}$

For compiler writers:

- runtime representation of integer, etc.
- type Direction = North
  | South
  | East
  | West.

Background: Record types & Subtyping

Record types

$$e ::= \ldots \qquad\qquad t ::= \ldots$$
$$\mid \{l_1 : e_1, \ldots, l_n : e_n\} \qquad \mid \{l_1 : t_1, \ldots, l_n : t_n\}$$
$$\mid e.l_i$$

$$\{count : 74\}.count \Rightarrow 74$$

Why Subtyping?

$$(\lambda\{x : \{count : int\}\}.\, x.count)\ \underbrace{\{count : 5, extra : 85\}}_{\{count : int,\ extra : int\}}$$

Won't typecheck!

$$\frac{\forall i \in [1, n],\ \Gamma \vdash s_i \leq t_i}{\Gamma \vdash \{l_1 : s_1, \ldots, l_n : s_1, \ldots\} \leq \{l_1 : t_1, \ldots, l_n : t_n\}}$$

Adding Subtyping to System F

term: $\lambda\{x:\{count:int\}\}.(x.count, x)$

type: $\{count:int\} \to int \times \{count:int\}$

Implicit Subtyping "forgets" actual type.

term: $\Lambda T. \lambda\{x:T\}. (x.count, x)$ ✗

type: $\forall T. T \to int \times T$

$(\Lambda T. \ldots)$ can't "see through" T.

So close...

Bounded Quantification — Cardelli & Wegner 85

idea: Explicit Subtyping in $(\Lambda T. \cdots)$

$e ::= \ldots$
$\quad \mid \Lambda T \leq t. e$

$t ::= \ldots$
$\quad \mid \forall T \leq t. t$

Ex

$$(\Lambda\ T \leq \{count: int\}. \lambda \{x : T\}. (x.count, x))$$
$$[\{count: int, extra: int\}]$$
$$\{count: 74, extra: 85\}$$

$$\Rightarrow (\lambda \{x : \{count: int, extra: int\}\}. (x.count, x))$$
$$\{count: 74, extra: 85\}$$

$\Rightarrow \ldots$

Background: Objects & Recursive Types

- Goal: Model Objects.

```
class Point {
    int x, y;
    Point move(int dx, int dy) {...}
    boolean lesseq(Point other) {...}
}
```

---

not in scope

$$\text{Point} = \text{Rec pnt. } \{ \\ x : \text{int}, \; y : \text{int}, \\ \text{move} : \text{int} \times \text{int} \to \text{pnt}, \\ \text{lesseq} : \text{pnt} \to \text{bool} \}$$

$\boxed{\text{Background: Function Subtyping}}$

- Conditions for $A_1 \to B_1 \leq A_2 \to B_2$

$$A_1 \leq A_2 \wedge B_1 \leq B_2 \; ?$$

$$(\lambda \{f : Fruit \to Fruit\}. \; (f \; (f \; orange)))$$

$$: (Fruit \to Fruit) \to Fruit$$

$\boxed{\text{What types of 'f' is acceptable?}}$

$$f_1 : Apple \to Fruit \not\leq Fruit \to Fruit$$

$$f_2 : Top \to Orange \leq Fruit \to Fruit$$

$$f_3 : Fruit \to Top \not\leq Fruit \to Fruit$$

Limitations of Bounded Quantification, Canning et al. 89

Function subtyping:

$$\frac{\Gamma \vdash A_2 \le A_1 \qquad \Gamma \vdash B_1 \le B_2}{\Gamma \vdash A_1 \to B_1 \le A_2 \to B_2}$$

Recursive type subtyping

$$\frac{\Gamma, s \le t \vdash T_1 \le T_2}{\Gamma \vdash \text{Rec } s.T_1 \le \text{Rec } t.T_2}$$

Model "Interface"

- Moveable = Rec mv. {move: int × int → mv}

  Point ≤ Moveable ✓.

- $\Lambda T \le \text{Moveable}.\ \lambda\{x:T\}.\ x.\text{move}\ (1,1)$

  $\forall T \le \text{Moveable}.\ T \to$ Moveable (want T)

- Comparable = Rec cp. { lesseq : cp → bool }

  Point ≤ Comparable requires cp ≤ pnt ✗

Limitations of Bounded Quantification Cont

- Where's the problem?
  - Moveable = Rec mv. { move: int × int → mv }
  - Comparable = Rec cp. { lesseq: cp → bool }

$\boxed{\text{Type system is right!}}$

mv/cp could be any Moveable/Comparable

$\Rightarrow$ $\boxed{\text{need more specific types.}}$

Limitation of Bounded Quantification Cont.

Attempt 2: Leave it to the users

- $Moveable[mv] = \{move : int \times int \to mv\}$
- $Comparable[cp] = \{lesseq : cp \to bool\}$

$Point \leq Moveable[Point]$ ✓.

$Point \leq Comparable[Point]$ ✓.

But we can't use it as we'd like to

$\Lambda\, T \leq Moveable[T].\ \lambda\{x : T\}\ x.move(1, 1)$

not in scope

F-bounded Quantification Canning et al. 89

Def

$$\forall t \leq \underbrace{F[t]}_{\text{Allows } t \text{ in the bound.}} . \sigma$$

Ex

$$\Lambda\, T \leq \text{Comparable}[T].\ \lambda\{x : T\}.\ \lambda\{y : T\}.$$
$$\text{if } x.\text{lesseq}\ y \text{ then } x \text{ else } y$$

$$: \forall\, T \leq \text{Comparable}[T].\ T \to T \to T$$

Into the practical world

$$\text{Comparable} = \Lambda T.\ \{ \text{lesseq} : T \to \text{bool} \}$$

```
interface Comparable<T> {
    int compare(T other);
}
```

Java Generics

But it wasn't always like this

One path of attempts:

- Pizza (Odersky & Wadler 97)
- GJ (Odersky et al. 98).

Java pre-generics history

```
interface Comparable {
    int compare(Object other);
}

class Num implements Comparable {
    int val;

    int compare(Object other) {
        Num otherNum = (Num) other;
        ... this.val ... otherNum.val ...
    }
}
```

- Programmers must be careful...

Extending Java with generics Odersky & Wadler 97

- new syntax
- translates to old Java.

Ex

```
interface Comparable<T> {
    int compare(T other);
}
class Num implements Comparable<Num> {
    int val;
    int compare (Num otherNum) {
        ... this.val ... otherNum.val ...
    }
}
```

Homogeneous Translation

```
interface Comparable {
    int compare$(Object other); }

class Num implements Comparable {
    int val;
    int compare(Num o) {...}
    int compare$(Object o) {
        return this.compare((Num) o); }
```

Heterogeneous Translation

```
interface Comparable_num {
    int compare(Num o); }

class Num implements Comparable_num {
    int val;
    int compare(Num o) {...} }
```

Generic Methods

```
<T implements Comparable<T>> T min(T a, T b){
    if (a.compare(b) ≤ 0) return a;
    else return b;}.
```

```
Num x = min(new Num(89), new Num(97))
```

Homogeneous Translation

```
Comparable min(Comparable a, Comparable b){
    ... compare$ ... }.
```

```
Num x = (Num) min(new Num(89), new Num(97))
```

Heterogeneous Translation

```
Num min_Num(Num a, Num b){...}
```

```
Num x = min(new Num(89), new Num(97));
```

Homogeneous Translation and Java Array

```
<T> T[] copy(T[] arr){
    T[] cpy = new T[arr.length];
    for(...){...}
    return cpy; }
```

$\Longrightarrow$

```
Object[] copy(Object[] arr) {
    Object[] cpy = new Object[arr.length];
    ...}
```

- ISSUE: int[] $\not\leq$ Object[]

Not generic enough.

Homogeneous Translation & Java Array Cont.

- Wrapper Array class

```
abstract class Array {
  int length();
  Object get(int i);
  void set(int i, Object o);
}
```

Array_obj   Array_int   Array_double   ...

Now Array_int ≤ Array

```
Array copy(Array arr) {
  Array cpy = new Array_obj(arr.length());
  ... }.
```

Generics & Subtyping

- $(X \leq Y) \overset{?}{\Rightarrow} (List\langle X\rangle \leq List\langle Y\rangle)$

Assume yes

```
class Cell<T>{
    T x;
    Cell(T x){ this.x = x;}
    T get() { return this.x;}
    void set(T x){ this.x = x;}
}

Cell<String> str = new Cell("97");
Cell<Object> obj = str;
obj.set(new Integer(89));
String s = str.get(); //ERROR.
```

- Generics must be invariant.

  type variables must match exactly

Homogeneous translation & Casting

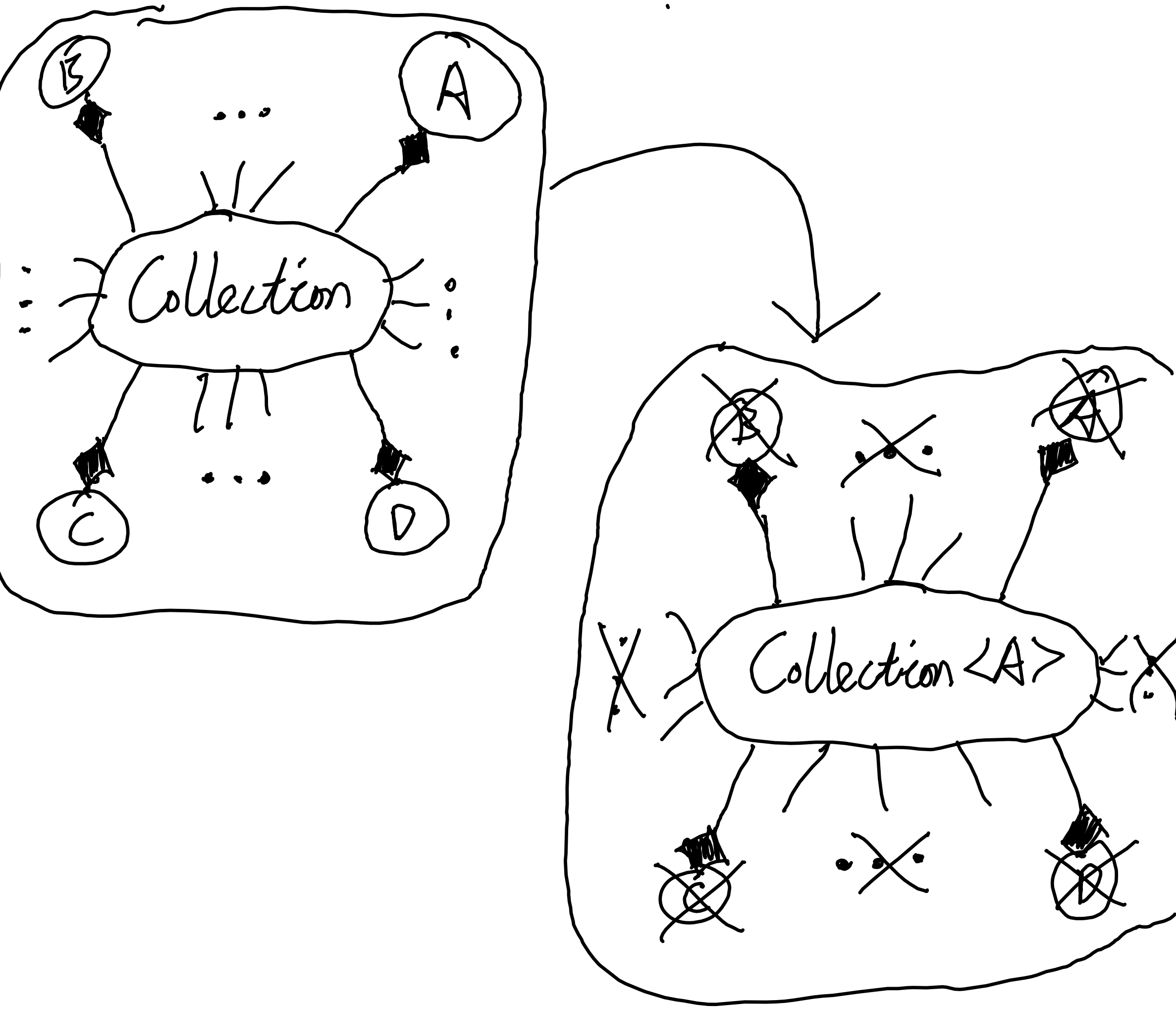
- Generics have no RTTI about parameters after homogeneous translation

```
class Cell<T> {
    ...
    boolean equals(Object o) {
        if (o instanceof Cell) {
            Cell<X> other = (Cell) o;
            return this.x.equals(other.x);
        }
    ......}
```

- $\Gamma \vdash (\text{Cell})\, o : \exists x.\ \text{Cell}\langle x \rangle$

Pizza seems to be self-consistent in terms of language features... BUT

The generic legacy Problem Odersky et al. 98



Odersky et al. 98 proposes $\boxed{GJ}$ to

- Add generics to Java
- Address legacy code compatibility issues

Raw Types in GJ

Goal:

Old code + new library.

Observation:

Legacy code uses "Object & Casting" idiom

Idea:

Allow this idiom with generics

```
class ArrayList<T> {...}
ArrayList<Integer> ns = new ArrayList<Integer>();
ArrayList          raw = new ArrayList();
```

Warning Generation

Ex

```
Cell<String> str = new Cell<String>("98");
Cell raw = str;
Cell<Integer> i = raw; // warning!
raw.set(new Integer(98)); // warning!
```

Generates unchecked warning on:

- assignment to raw type
  if lhs type contains type variable.

- method call to raw type
  if parameter contains type variable.

Raw Type and casting

```
class Cell<T> {
    ......
    boolean equals(Object o) {
      if (o instanceof Cell) {
        Cell other = (Cell) o;
        return this.x.equals(other.x);
    ......}
}.
```

- Naturally replaces existential type in Pizza.
- Cell behaves like a restricted version of Cell<Object>.

Retrofitting in GJ

- Goal:
  New code + old libraries
- Observation:
  Generics translates to old idiom anyway
- Idea:
  Trust old libraries. leave them alone

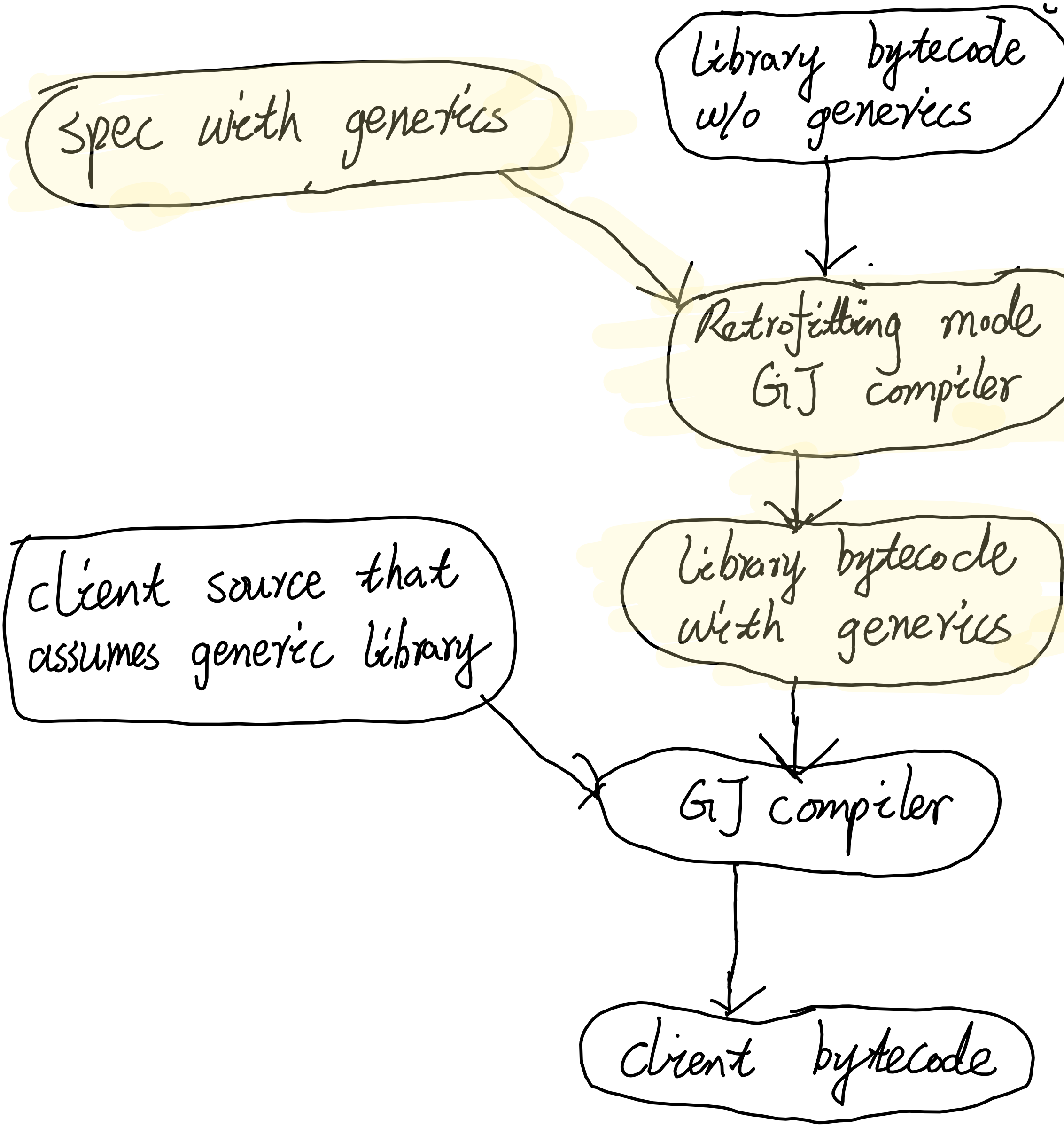
How does compiler know?

- "retrofitting mode"

$\boxed{\text{retrofitting spec file.}}$

```
class ArrayList<T> {
    void add(T x);
    T    get(int i);
    ......}
```

Retrofitting Pipeline



Summary on code Compatibility

Library State	Client Assumption	How
Collection<T>	Collection	Raw Types
Collection	Collection<T>	Retrofitting
Collection	Collection	Programmers' caution
Collection<T>	Collection<T>	compiler checks

Recap

STLC — 1940

↓ No polymorphism, code duplication

System F — 1974

↓ Not orthogonal with subtyping

Bounded Quantification — 1985

↓ Can't model objects with recursive type

F-Bounded Quantification — 1989

↓ Java wants generics.

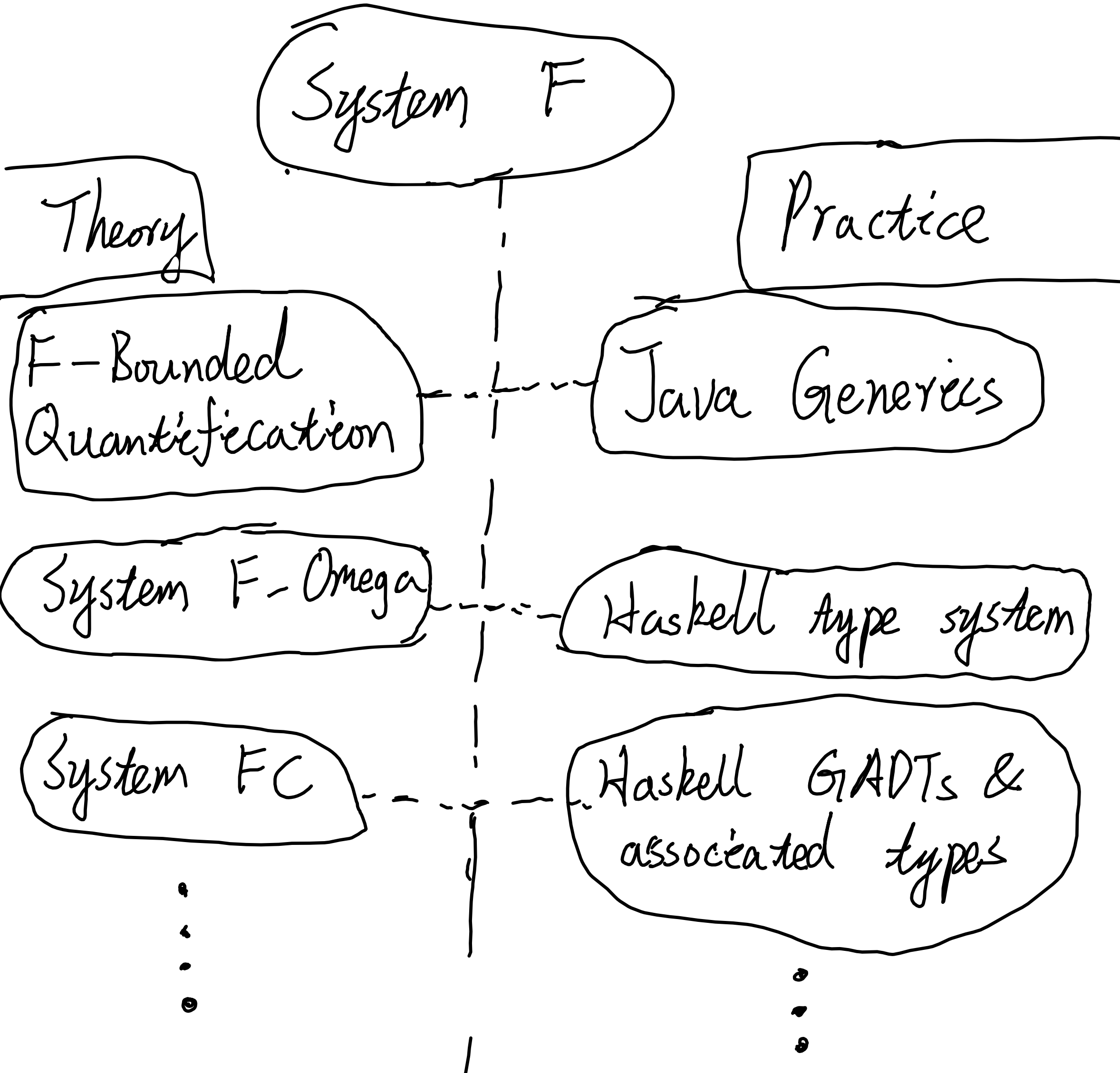
Pizza — 1997

↓ Compatible in terms of language features but not well with legacy code.

GJ — 1998

↓ ......

Zoom Out


System F
Theory
Practice
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Java Generics
System F-Omega
Haskell type system
System FC
Haskell GADTs & associated types