Thermal flux $\bar{F} = -k \nabla T$ —— ①

$$\rho C_v \frac{dT}{dt} = \nabla \cdot \bar{F} + \cancel{w}^{\;0 \text{ neglected}} \quad \text{(Joule heating)}$$

$$\rho C_v \frac{dT}{dt} = \nabla \cdot \bar{F}$$

$$\boxed{\frac{dT}{dt} = \frac{1}{C} \nabla \cdot \bar{F}} \quad \text{—— ②}$$

differentiate eqn ①

$$\frac{d\bar{F}}{dt} = \frac{d}{dt}(-k \nabla T)$$

$$= -k \cdot \nabla\left(\frac{dT}{dt}\right) - \cancel{\nabla T \frac{dk}{dt}}^{\text{neglected}}$$

From eqn ①

$$\frac{d\bar{F}}{dt} = -k \nabla\left(\frac{1}{C} \nabla \cdot \bar{F}\right)$$

$$\boxed{\frac{1}{k} \frac{d\bar{F}}{dt} = \nabla\left(\frac{1}{C} \nabla \cdot \bar{F}\right)} \quad \text{— ③}$$

Multiply eqn ③ with test function & integrate over domain 'Ω'.

$$\int_\Omega \frac{1}{k} \frac{d\bar{F}}{dt} \bar{v}_j \, d\Omega = \int_\Omega \nabla\left(\frac{1}{C} \nabla \cdot \bar{F}\right) \bar{v}_j \, d\Omega.$$

$$\int_\Omega \frac{1}{k} \cdot \frac{1}{dt} (\bar{F}^{n+1} - \bar{F}^n) \bar{v}_j \, d\Omega = \int_\Omega \nabla\left(\frac{1}{C} \nabla \cdot \bar{F}^{n+1}\right) \bar{v}_j \, d\Omega$$

Assume $\bar{F} = \sum_i F \bar{u}_i$

$$\left[\left(\frac{1}{k}\bar{u}, \bar{v}\right) - dt_{+}\left(\frac{1}{c}\nabla\cdot\bar{u}, \nabla\cdot\bar{v}\right)\right] F^{n+1}$$

$$= \left(\frac{1}{k}\bar{u}, \bar{v}\right) F^{n}$$

(Assume Zero boundary condition on flux)

$\left(\frac{1}{k}\bar{u}, \bar{v}\right) \longrightarrow$ VectorFEmass Integrator$\left(\frac{1}{k}\right)$

$dt_{+}\left(\frac{1}{c}\nabla\cdot\bar{u}, \nabla\cdot\bar{v}\right) \longrightarrow$ DivDiv Integrator$\left(\frac{1}{c}\right)$

$$F^{n} = -k\nabla T^{n}$$