$$\vec{F} = -k\nabla T$$ -- (1)

$$\rho C_v \frac{dT}{dt} = -\nabla \cdot \vec{F}$$

$$C\frac{dT}{dt} = -\nabla \cdot \vec{F}$$ -- (2)

Neglected

Derivating the eq. 1 $$\frac{d\vec{F}}{dt} = -k\nabla\frac{dT}{dt} - \nabla T\frac{dk}{dt}$$ -- (3)

From 2 & 3 $$\frac{d\vec{F}}{dt} = -k\nabla\left[\frac{-1}{C}\nabla \cdot \vec{F}\right]$$

$$\frac{d\vec{F}}{dt} = k\nabla\left[\frac{1}{C}\nabla \cdot \vec{F}\right]$$

Multiplying both sides with test function ‘v’ and integrating over domain ‘Ω’

$$\int_\Omega \frac{d\vec{F}}{dt}\vec{v} = \int_\Omega k\nabla\left[\frac{1}{C}\nabla \cdot \vec{F}\right]\vec{v}$$

$$\int_\Omega \frac{1}{k}\frac{d\vec{F}}{dt}\vec{v} = \int_\Omega \nabla\left[\frac{1}{C}\nabla \cdot \vec{F}\right]\vec{v}$$

$$\left(\frac{1}{k}\vec{u},\vec{v}\right)\left(\frac{\vec{F}^{n+1}-\vec{F}^{n}}{dt}\right) = -\left(\frac{1}{C}\nabla\cdot\vec{u},\nabla\cdot\vec{v}\right)\vec{F}^{n+1}$$ Assuming zero boundary condition

$$\left[\left(\frac{1}{k}\vec{u},\vec{v}\right)+\left(\frac{dt}{C}\nabla\cdot\vec{u},\nabla\cdot\vec{v}\right)\right]\vec{F}^{n+1}=\left(\frac{1}{k}\vec{u},\vec{v}\right)\vec{F}^{n} \quad \text{-- (4)}$$

$$\vec{F}^{n}=-k\nabla T^{n}$$

Multiplying both sides with test function ‘v’ and integrating over domain ‘Ω’

$$\int_{\Omega}\frac{1}{k}\vec{F}^{n}\vec{v}=-\int_{\Omega}\nabla T^{n}\vec{v}$$

$$\left(\frac{1}{k}\vec{u},\vec{v}\right)\vec{F}^{n}=-(\nabla u,\vec{v})T^{n}$$

$$\vec{F}^{n}=-\left(\frac{1}{k}\vec{u},\vec{v}\right)^{-1}(\nabla u,\vec{v})T^{n} \quad \text{-- (5)}$$

From eqns 4 & 5

$$\left[\left(\frac{1}{k}\vec{u},\vec{v}\right)+\left(\frac{dt}{C}\nabla\cdot\vec{u},\nabla\cdot\vec{v}\right)\right]\vec{F}^{n+1}=\left(\frac{1}{k}\vec{u},\vec{v}\right)\left(-\left(\frac{1}{k}\vec{u},\vec{v}\right)^{-1}(\nabla u,\vec{v})T^{n}\right)$$

$$\left[\left(\frac{1}{k}\vec{u},\vec{v}\right)+\left(\frac{dt}{C}\nabla\cdot\vec{u},\nabla\cdot\vec{v}\right)\right]\vec{F}^{n+1}=-(\nabla u,\vec{v})T^{n}$$

Final equation

$$\vec{F}^{n+1}=-\left[\left(\frac{1}{k}\vec{u},\vec{v}\right)+\left(\frac{dt}{C}\nabla\cdot\vec{u},\nabla\cdot\vec{v}\right)\right]^{-1}(\nabla u,\vec{v})T^{n}$$

$\left(\frac{1}{k}\vec{u},\vec{v}\right)$ = VectorFEMassIntegrator (InvTCond)

$(\nabla u,\vec{v})$ = WeakDiv->MultTranspose

$\left(\frac{dt}{C}\nabla\cdot\vec{u},\nabla\cdot\vec{v}\right)$ = DivDivIntegrator(dt* InvTCap)