

# The State of MFEM

MFEM Community Workshop  
October 26, 2023

Tzanio Kolev  
LLNL



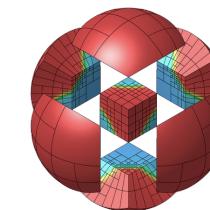
LLNL-PRES-856457

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under contract DE-AC52-07NA27344. Lawrence Livermore National Security, LLC

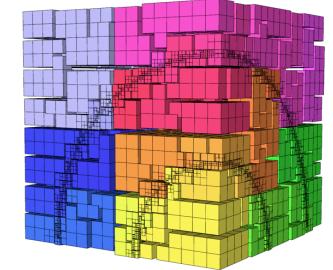
# MFEM

## Cutting-edge algorithms for powerful applications on HPC architectures

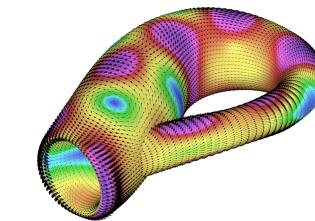
- **Flexible discretizations on unstructured grids**
  - Triangular, quadrilateral, tetrahedral and hexahedral meshes.
  - Local conforming and non-conforming AMR, mesh optimization.
  - Bilinear/linear forms for variety of methods: Galerkin, DG, DPG, ...
- **High-order and scalable**
  - Arbitrary-order H1, H(curl), H(div)- and L2 elements.
  - Arbitrary order curvilinear meshes.
  - MPI scalable to millions of cores and GPU-accelerated.
  - Enables application development from laptops to exascale machines.
- **Built-in solvers and visualization**
  - Integrated with: HYPRE, SUNDIALS, PETSc, SLEPc, SUPERLU, ...
  - AMG preconditioners for full de Rham complex, geometric MG
  - Support for GPU solvers from: HYPRE, PETSc, AmgX
  - Accurate and flexible visualization with VisIt, ParaView and GLVis
- **Open source**
  - Available on GitHub under BSD license. 100+ example codes and miniapps.
  - Part of FASTMath, ECP/CEED, xSDK, OpenHPC, E4S, ...



High-order  
curved elements



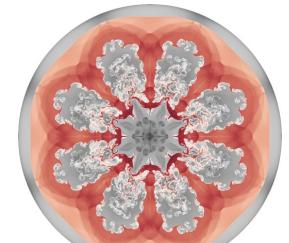
Parallel non-conforming AMR



Surface  
meshes



Heart  
modeling

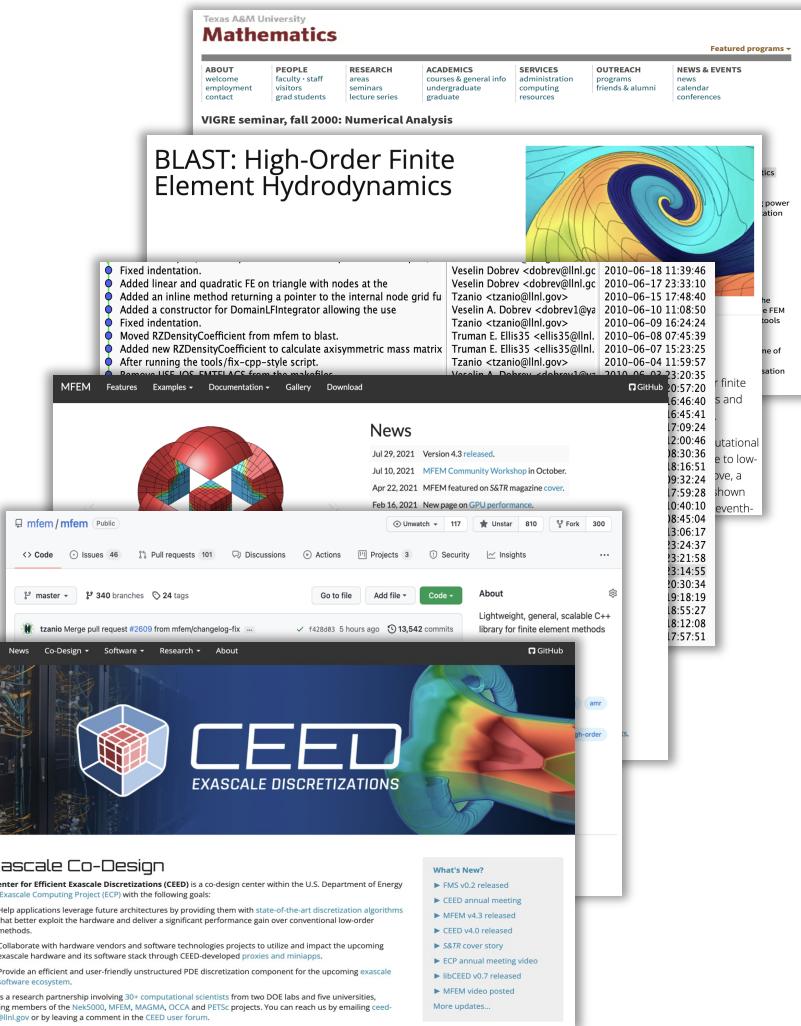


Compressible flow  
ALE simulations

# A Brief History

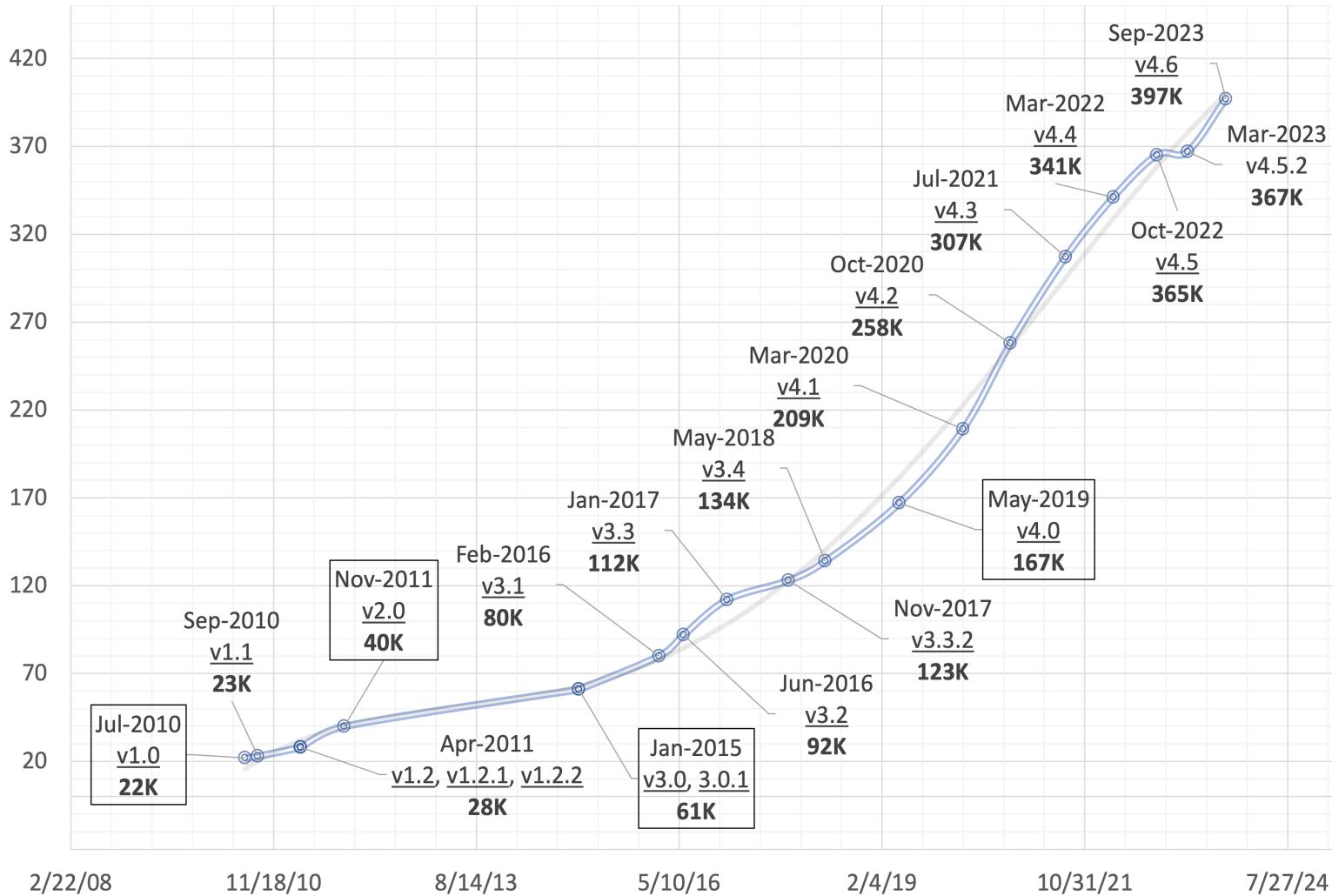
We've been doing this for a long time

- **2000 – “VIGRE seminar: Numerical Analysis,” Texas A&M University**
  - Research code: AggieFEM/aFEM
  - Some of the original contributors: [@v-dobrev](#), [@tzanio](#), [@stomov](#)
  - Used in summer internships at LLNL
- **2010 – BLAST project at LLNL**
  - Motivated high-order, non-conforming AMR and parallel scalability developments
  - MFEM repository created in May 2010
  - Some of the original contributors: [@v-dobrev](#), [@tzanio](#), [@rieben1](#), [@trumanellis](#)
  - Project website [mfem.org](http://mfem.org) goes live in August 2015
- **2017 – Development moved to GitHub**
  - First GitHub commits in February 2017
  - Team expands to include many new developers at LLNL and externally
- **2017 – CEED project in the ECP**
  - Motivated partial assembly, GPU, and exascale computing developments



# The Source Code is Growing

## SLOC in MFEM releases over the last 13 years



mfem-4.6.tgz	v4.6	Sep 2023	3.6M	397K	
mfem-4.5.2.tgz	v4.5.2	Mar 2023	3.3M	367K	
mfem-4.5.tgz	v4.5	Oct 2022	3.3M	365K	
mfem-4.4.tgz	v4.4	Mar 2022	3.0M	341K	
mfem-4.3.tgz	v4.3	Jul 2021	2.8M	307K	
mfem-4.2.tgz	v4.2	Oct 2020	2.4M	258K	
mfem-4.1.tgz	v4.1	Mar 2020	7.9M	209K	
mfem-4.0.tgz	v4.0	May 2019	5.2M	167K	GPU support
mfem-3.4.tgz	v3.4	May 2018	4.4M	134K	
mfem-3.3.2.tgz	v3.3.2	Nov 2017	4.2M	123K	mesh optimization
mfem-3.3.tgz	v3.3	Jan 2017	4.0M	112K	
mfem-3.2.tgz	v3.2	Jun 2016	3.3M	92K	dynamic AMR, HPC miniapps
mfem-3.1.tgz	v3.1	Feb 2016	2.9M	80K	fem ↔ linear system interface
mfem-3.0.1.tgz	v3.0.1	Jan 2015	1.1M	61K	
mfem-3.0.tgz	v3.0	Jan 2015	1.1M	61K	non-conforming AMR
mfem-2.0.tgz	v2.0	Nov 2011	308K	40K	arbitrary order spaces, NURBS
mfem-v1.2.2.tgz	v1.2.2	Apr 2011	240K	28K	
mfem-v1.2.1.tgz	v1.2.1	Apr 2011	240K	28K	
mfem-v1.2.tgz	v1.2	Apr 2011	240K	28K	MPI parallelism based on hypre
mfem-v1.1.tgz	v1.1	Sep 2010	166K	23K	
mfem-v1.0.tgz	v1.0	Jul 2010	160K	22K	initial release

# The Community is Growing

## GitHub, downloads, and workshop stats

### GitHub

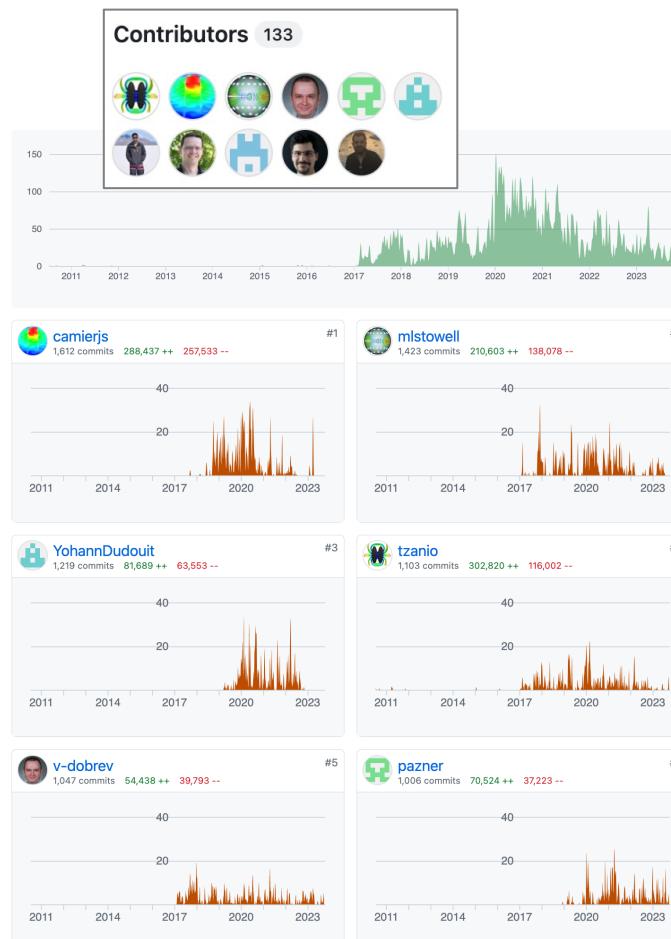
- **133** contributors
- **250** lines of code / day
- **629** people in the mfem organization – *join to contribute + receive announcements*
- **150** unique visitors / day
- **1390** stars – *thank you!*

### Downloads

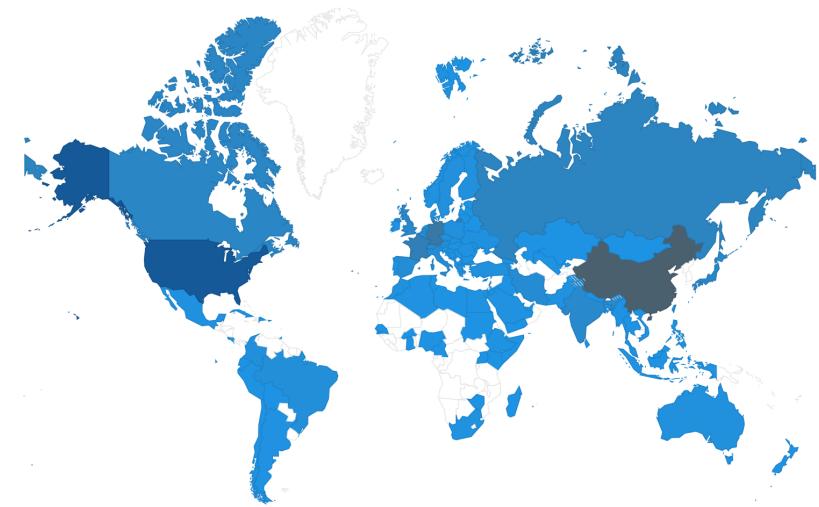
- **250** downloads + clones / day · **91K** / year
- **115** countries total

### 2023 Community Workshop

- **272** researchers
- **134** organizations
- **33** countries



Top contributors as of Oct 2023



MFEM has been downloaded from **115** countries

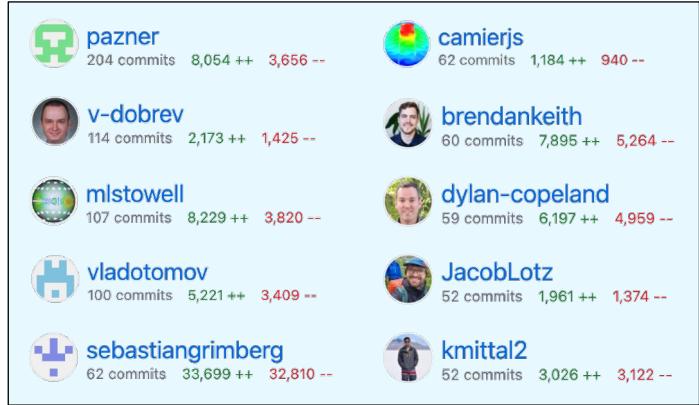
 <a href="https://mfem.org">mfem.org</a>	MFEM Community Workshop	October 2023
001 Aaron Fisher	Lawrence Livermore National Laboratory	fisher47@llnl.gov
002 Abdellatif Semmouri	FST, Sultan Moulay Slimane University	abdellatif.semouri@yahoo.fr
003 Abdelmajid Ezzine	Faculty of Sciences, Mohammed V University in Rabat	abdelmajid.ezzine@usrf.ac.ma
004 Abdesslam Ouaziz	University Sidi Mohammed Ben Abdellah	abdesslam.ouaziz94@gmail.com
005 Achraf El Omari	Hassan II University of Casablanca	achraf.elomari-etu@etu.univ2c.ma
006 Achraf Zinihi	Faculty of Sciences and Technics, Moulay Ismail University of Mekn��	a.zinihi@edu.unim.ac.ma
007 Adel Babah	abdelmalek.essadi.university	a.babah@uae.ac.ma
008 Aditya Parik	Utah State University	aditya.parik@usu.edu
009 Adolfo Rodriguez	Kappa Engineering	adolfr@autodesk.com
010 Adrian Butscher	Autodesk	adrian.butscher@autodesk.com
011 Ahdia Achabbak	Faculty of the science	ahdia.achabbak@etu.uae.ac.ma
012 Alberto Padovan	University of Illinois at Urbana-Champaign	padovan@illinois.edu
014 Alejandro Mu��oz	Universidad de Granada	alumno@ugr.es
015 Alex Lindsay	Idaho National Laboratory	alexander.lindsay@inl.gov
016 Alexander Blair	UK Atomic Energy Authority	alexander.blair@ukea.uk
017 Alexander Grayver	ETH Zurich	grayver@ethz.ch

2022 Community workshop had **258** registrations

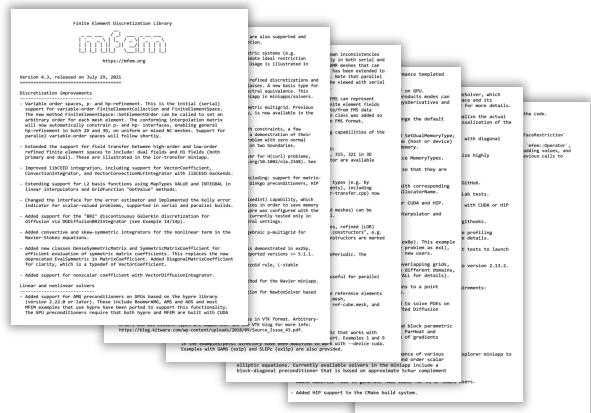
# Latest Releases Were Team Efforts

Versions 4.5.2 + 4.6 stats

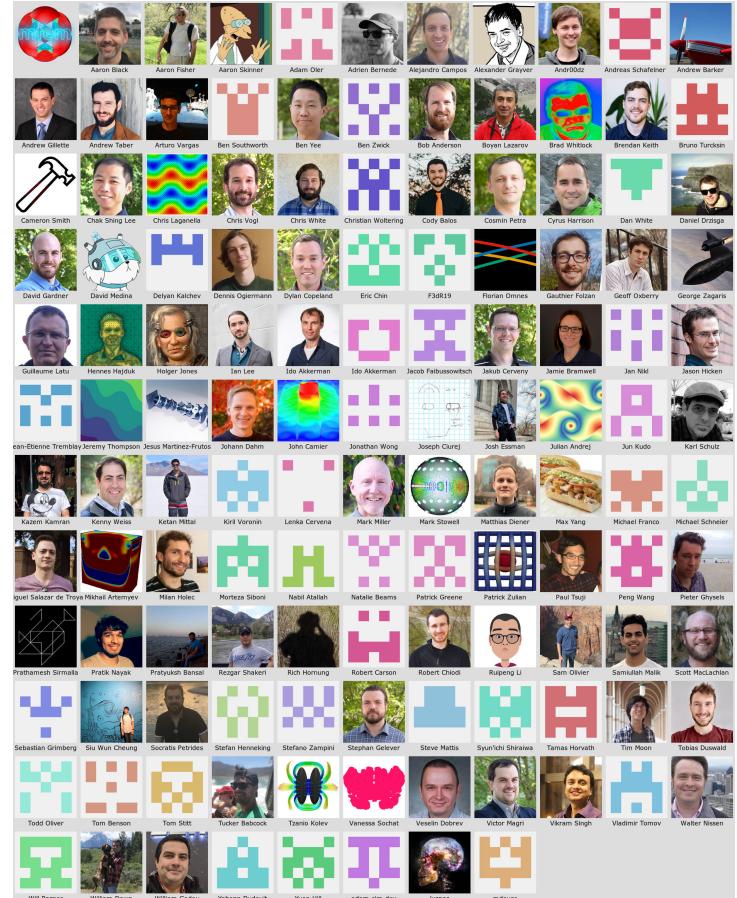
- Released **Mar + Sep, 2023**
- **11 months** in development
- **39 contributors**
- **234 PRs** merged
- **243 issues closed**
- **58K** new lines of code
- **2180** number of commits
- **Many new features:**
  - NURBS + TMOP meshing improvements
  - new H(div) matrix-free solver
  - SubMesh support for H(curl) and H(div)
  - HIP support for PETSc, SUNDIALS
  - stochastic PDEs, k-d tree, ultraweak DPG



Top 10 contributors to the last releases



The mfem-4.5.2+4.6 CHANGELOG has 45 entries



MFEM contributors on GitHub

# Examples

The first stop for new users

The screenshot shows the MFEM website's "Example Codes and Miniapps" page. At the top, there are navigation links: MFEM, Features, Examples, Documentation, GitHub, and Download. Below the navigation, there is a search bar and a "GitHub" button.

**Example Codes and Miniapps**

This page provides a brief overview of MFEM's example codes and miniapps. For detailed documentation of the MFEM sources, including the examples, see the [online Doxygen documentation](#), or the `doc` directory in the distribution.

The goal of the example codes is to provide a step-by-step introduction to MFEM in simple model settings. The miniapps are more complex, and are intended to be more representative of the advanced usage of the library in physics/application codes. We recommend that new users start with the example codes before moving to the miniapps.

Select from the categories below to display examples and miniapps that contain the respective feature. All examples support (arbitrarily) high-order meshes and finite element spaces. The numerical results from the example codes can be visualized using the GLVis visualization tool (based on MFEM). See the [GLVis website](#) for more details.

Users are encouraged to submit any example codes and miniapps that they have created and would like to share. Contact a member of the MFEM team to report bugs or post questions or comments.

**Application (PDE)**: All, Galerkin FEM, Mixed FEM, Discontinuous Galerkin (DG), Discrit. Petrov-Galerkin (DPG), Hybridization, Static condensation, Isogeometric analysis (NURBS), Adaptive mesh refinement (AMR), Partial assembly

**Finite Elements**: All, Linear, Quadratic, Curvilinear, NURBS

**Discretization**: All, Finite Difference, Finite Volume, Finite Element, Spectral Element, Meshfree

**Solver**: All, PETSc, SUNDIALS, STRUMPACK, PUMI, Hypre, Ginkgo, SuperLU, Omega.h, SLEPc, MAGMA, OCCA, libCEED, Laghos, MARBL, Remhos, E3SM, ExaSMR, Urban/Nek, ExaWind, ExaAM

**Example 1: Laplace Problem**

This example code demonstrates the use of MFEM to define a simple isosurface plot of a Laplace problem. The plot shows a 3D volume with a complex internal structure, colored by a scalar field.

$-\Delta u = 1$

with homogeneous Dirichlet boundary conditions. Specifically, we discretize the mesh (linear by default, quadratic for quadratic curvilinear mesh, NURBS for NURBS mesh, etc.)

The example highlights the use of mesh refinement, finite element grid functions, as well as linear and bilinear forms corresponding to the left-hand side and right-hand side of the discrete linear system. We also cover the explicit elimination of essential boundary conditions, static condensation, and the optional connection to the GLVis tool for visualization.

This example has a serial (`ex1.cpp`), a parallel (`ex1.pcpp`), and HPC versions: `performance/ex1.cpp`, `performance/ex1.pcpp`. It also has a PETSc modification in `examples/petsc`, a PUMI modification in `examples/pumi`, and a Ginkgo modification in `examples/ginkgo`. Partial assembly and GPU devices are supported.

**Example 2: Linear Elasticity**

This example code solves a simple linear elasticity problem describing a multi-material cantilever beam. Specifically, we approximate the weak form of

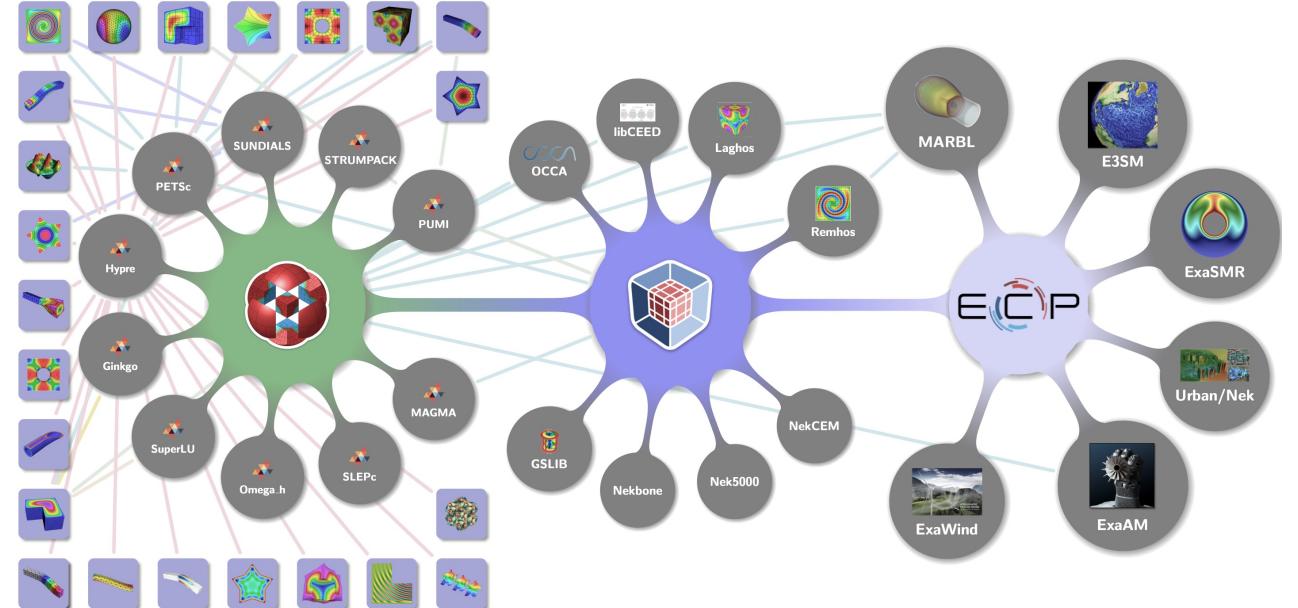
$$-\operatorname{div}(\sigma(u)) = 0$$

where

$$\sigma(u) = \lambda \operatorname{div}(u) I + \mu (\nabla u + \nabla u^T)$$

is the stress tensor corresponding to displacement field  $u$ , and  $\lambda$  and  $\mu$  are the material Lame constants. The boundary conditions are  $u = 0$  on the fixed part of the boundary with attribute 1, and  $\sigma(u) \cdot n = f$  on the remainder with  $f$  being a constant pull down vector on boundary elements with attribute 2, and zero otherwise. The geometry of the domain is assumed to be as follows:

[mfem.org/examples](http://mfem.org/examples)



- 38 example codes, most with both serial + parallel versions
- Tutorials to learn MFEM features
- Starting point for new applications
- Show integration with many external packages, miniapps

# Miniapps

More advanced, ready-to-use physics solvers

## Volta, Tesla, Maxwell and Joule Miniapps

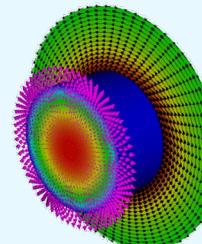
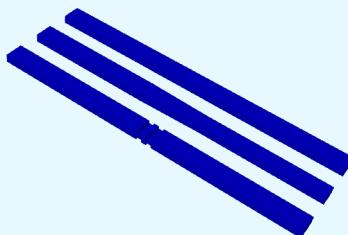
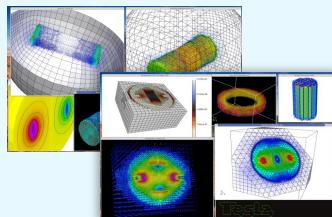
*Static and transient electromagnetics*

- **Volta**  $-\nabla \cdot \epsilon \nabla \varphi = \rho - \nabla \cdot \vec{P}$
- **Tesla**  $\nabla \times \mu^{-1} \nabla \times \vec{A} = \vec{J} + \nabla \times \mu^{-1} \mu_0 \vec{M}$
- **Maxwell** · *transient full-wave EM*

$$\frac{\partial(\epsilon \vec{E})}{\partial t} = \nabla \times (\mu^{-1} \vec{B}) - \sigma \vec{E} - \vec{J}$$

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}$$

- **Joule** · *transient magnetics + Joule heating*
- Arbitrary order elements + meshes
- Adaptive mesh refinement

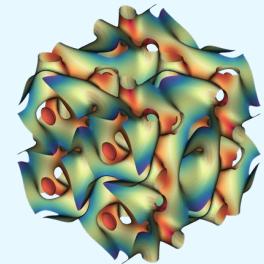


[mfem.org/electromagnetics](http://mfem.org/electromagnetics)

## Navier Miniapp

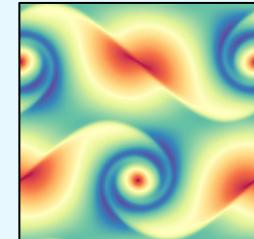
*Transient incompressible Navier-Stokes equations*

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} - \nu \Delta \mathbf{u} + \nabla p = \mathbf{f}$$
$$\nabla \cdot \mathbf{u} = 0$$



3D Taylor-Green vortex, 7<sup>th</sup> order

- Arbitrary order elements
- Arbitrary order curvilinear mesh elements
- Adaptive IMEX (BDF-AB) time-stepping algorithm up to 3<sup>rd</sup> order
- State-of-the-art HPC performance
- GPU acceleration
- Convenient user interface

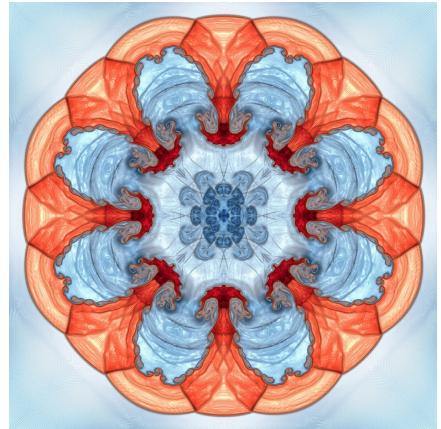


Double shear layer, 5<sup>th</sup> order, Re = 100000

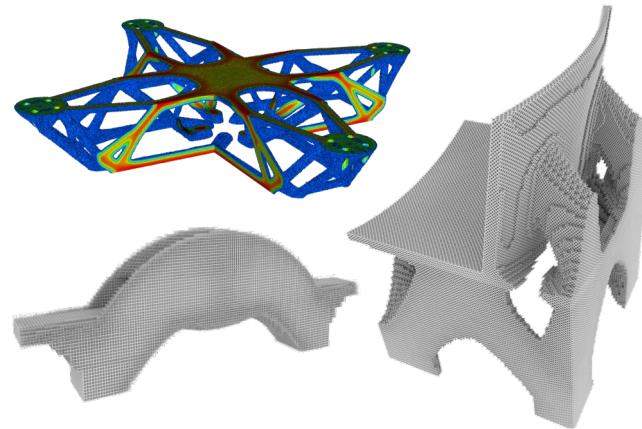
[mfem.org/fluids](http://mfem.org/fluids)

# Applications

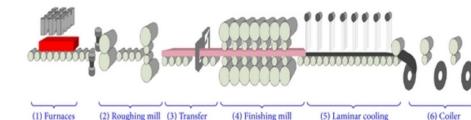
Some of the large-scale simulation codes powered by MFEM



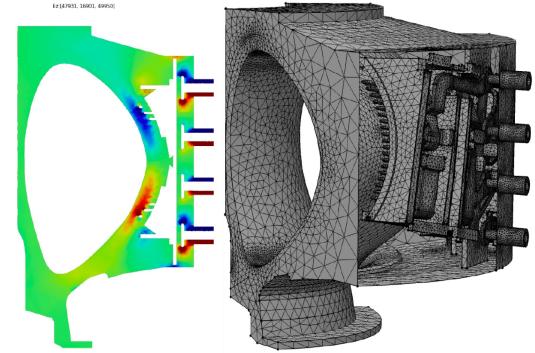
Inertial confinement fusion (BLAST, LLNL)



Topology optimization for additive manufacturing (LiDO, LLNL)



Hot strip mill slab modeling (U.S. Steel)



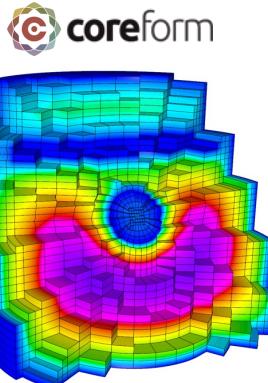
Core-edge tokamak EM wave propagation (SciDAC, RPI)



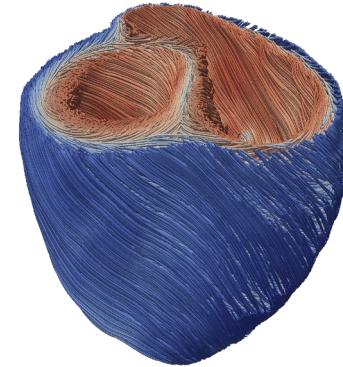
Electric aircraft design (RPI)



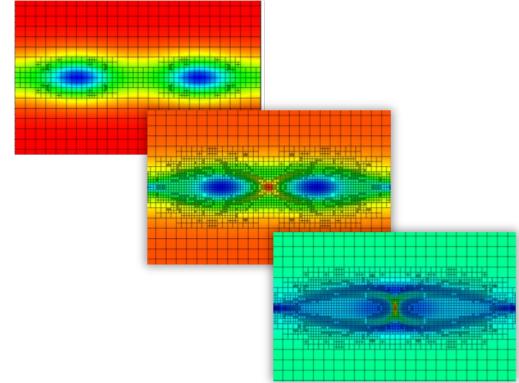
MRI modeling (Harvard Medical)



NURBS meshing and IGA (Coreform LLC, SBIR)



Heart modeling (Cardioid, LLNL/IBM)



Adaptive MHD island coalescence (SciDAC, LANL)

# Adaptive Mesh Refinement

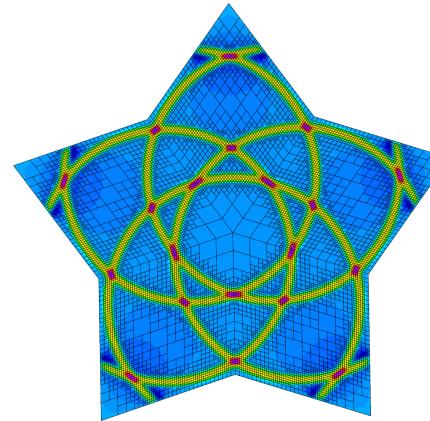
## MFEM's unstructured AMR infrastructure

- **AMR on library level**

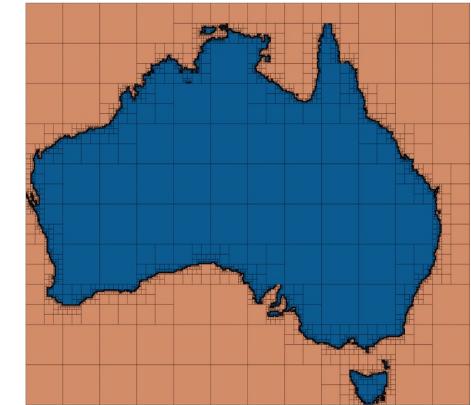
- Conforming local refinement on simplex meshes
- Non-conforming refinement for quad/hex meshes
- Initial hp-refinement

- **General approach**

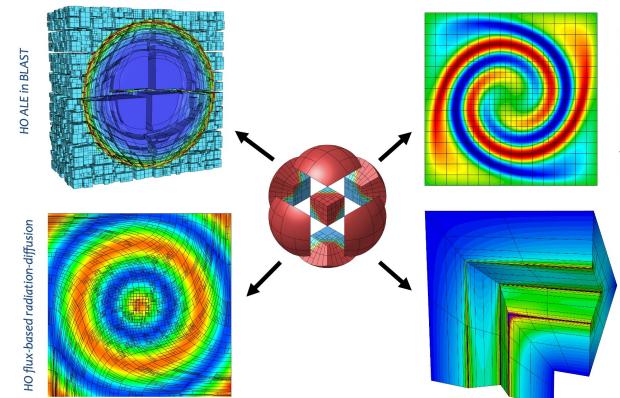
- Any high-order finite element space,  $H^1$ ,  $H(\text{curl})$ ,  $H(\text{div})$ , on any high-order curved mesh
- 2D and 3D · hexes, prisms, tets
- Arbitrary order hanging nodes
- Anisotropic refinement
- Derefinement
- Serial and parallel, including parallel load balancing
- Independent of the physics
- Easy to incorporate in applications



Example 15



Shaper miniapp

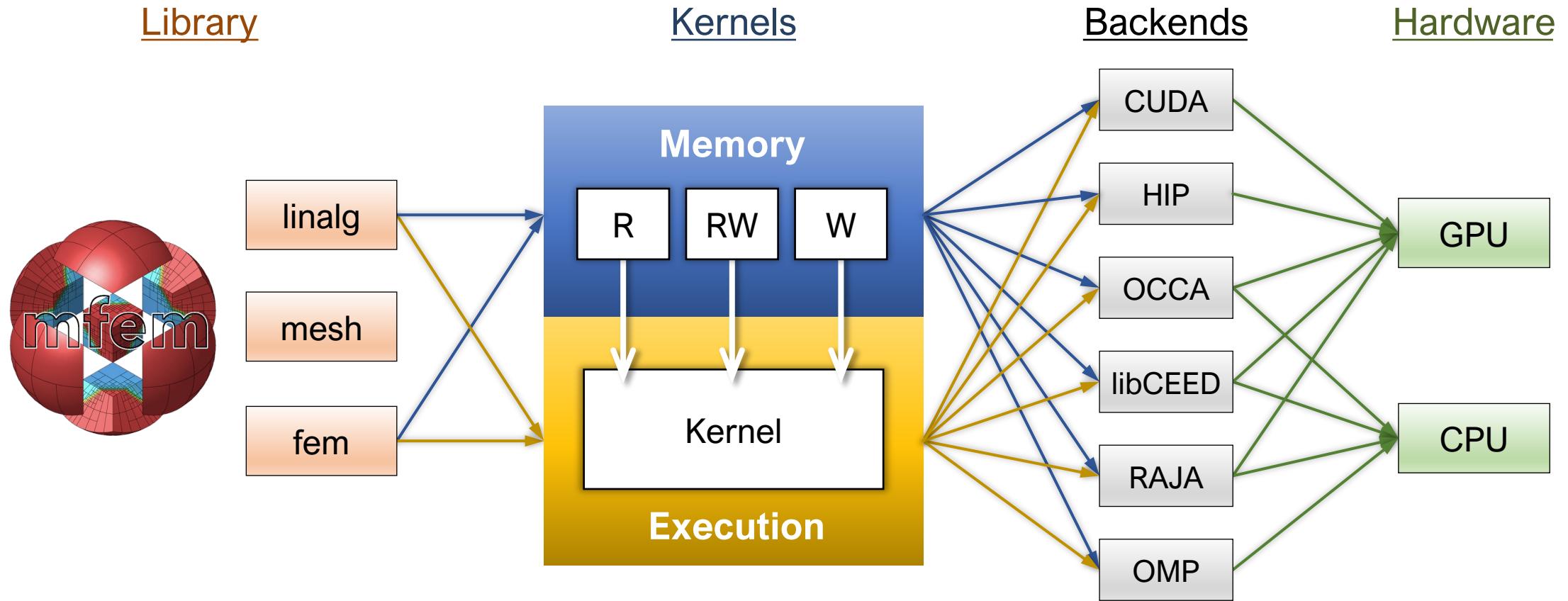


Same AMR algorithms can be applied to a variety of high-order physics



# GPU Support

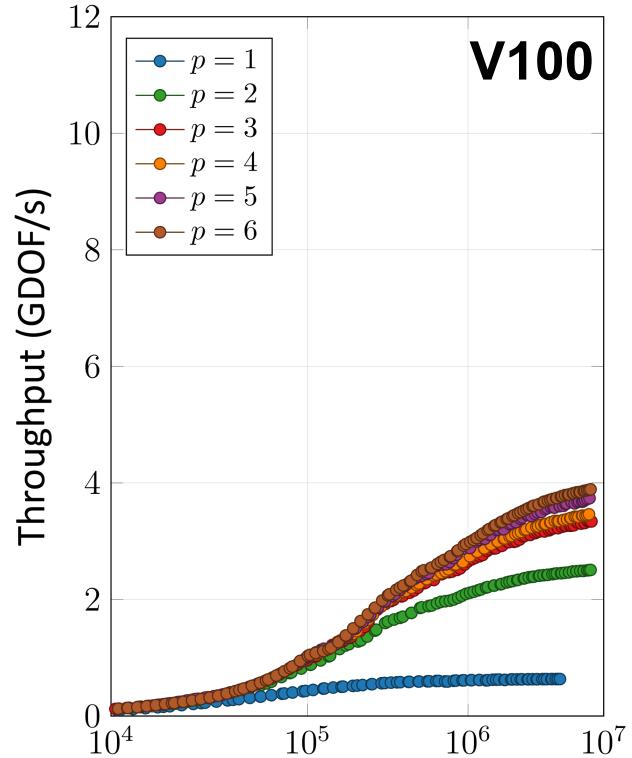
MFEM has provided GPU acceleration for over 5 years (since mfem-4.0)



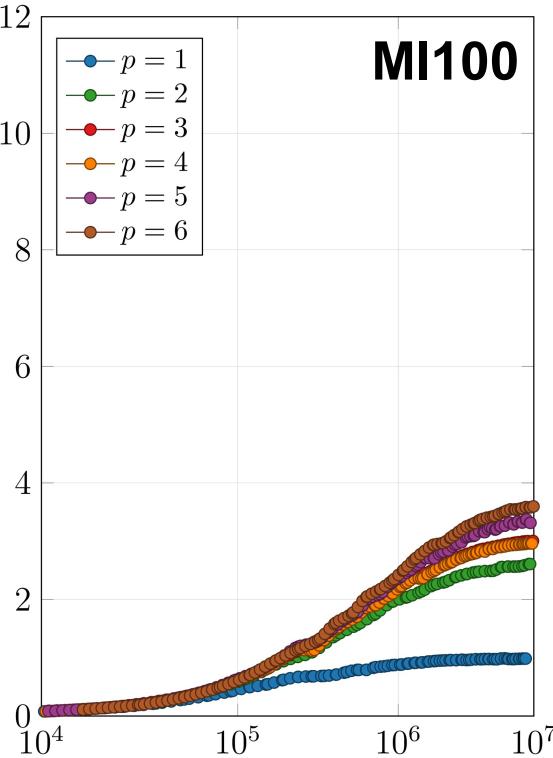
# GPU Support

## Recent GPU kernel improvements in MFEM

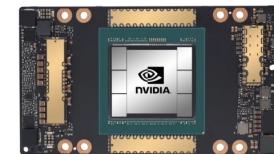
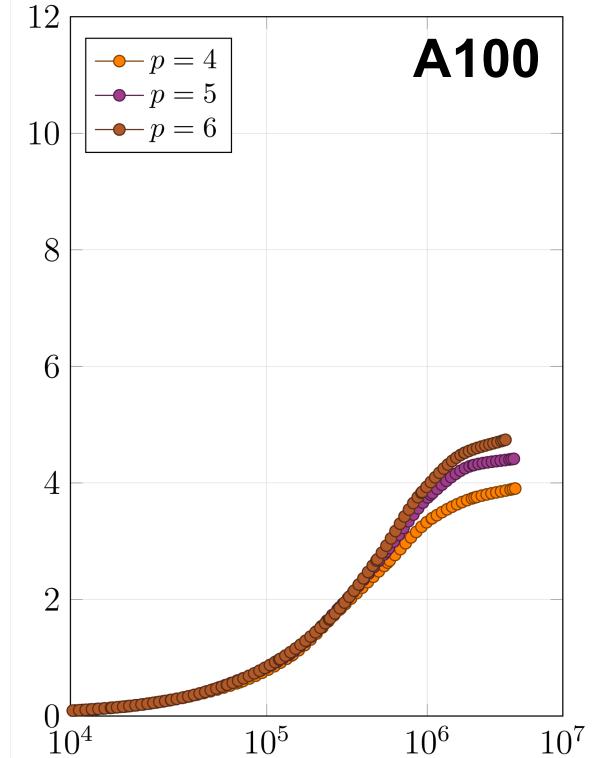
MFEM BP1 (atomics) @ V100



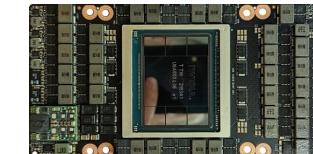
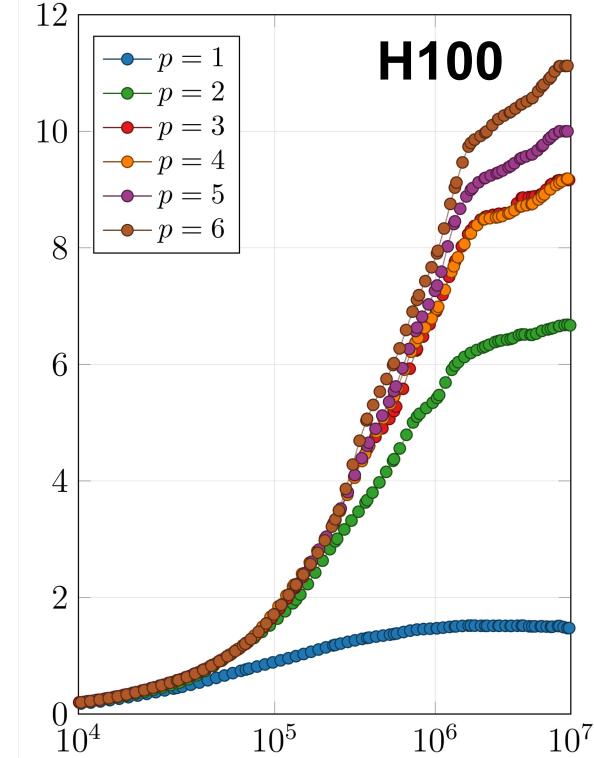
MFEM BP1 (atomics) @ MI100



MFEM BP1 (dmma) @ A100

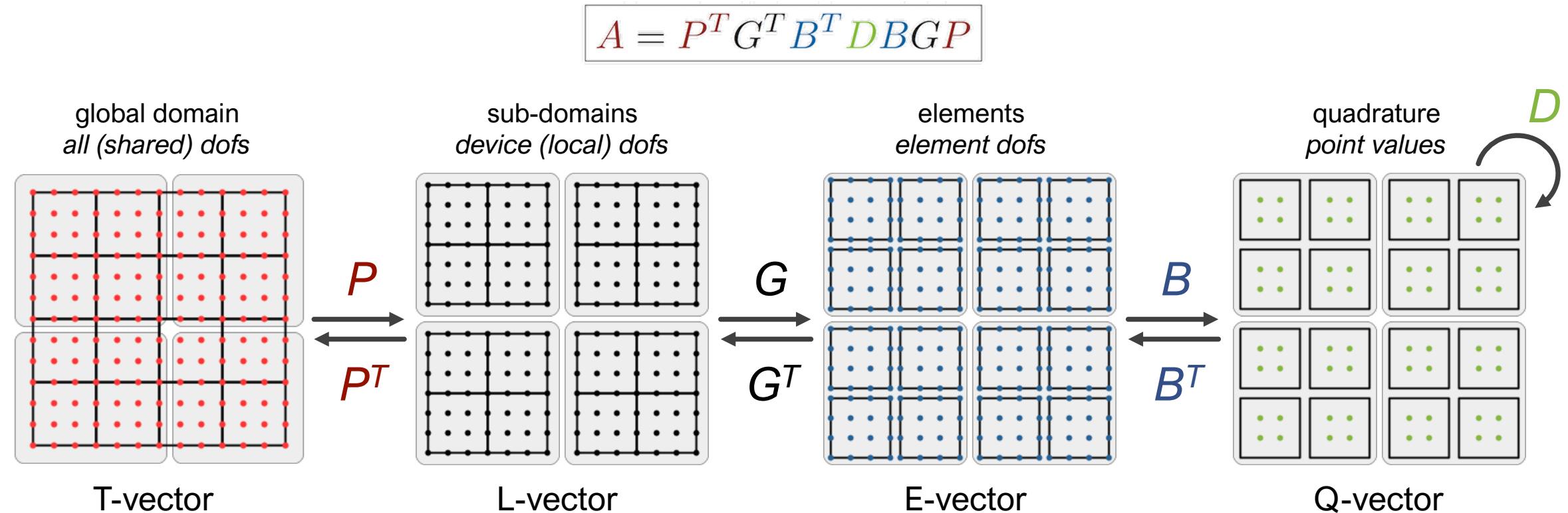


MFEM BP1 (atomics) @ H100

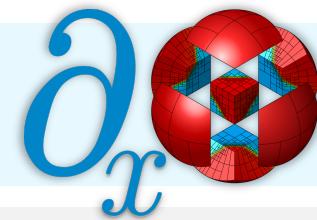


# FEM Operator Decomposition + Partial Assembly

Decompose **A** into parallel, mesh, basis, and geometry/physics parts



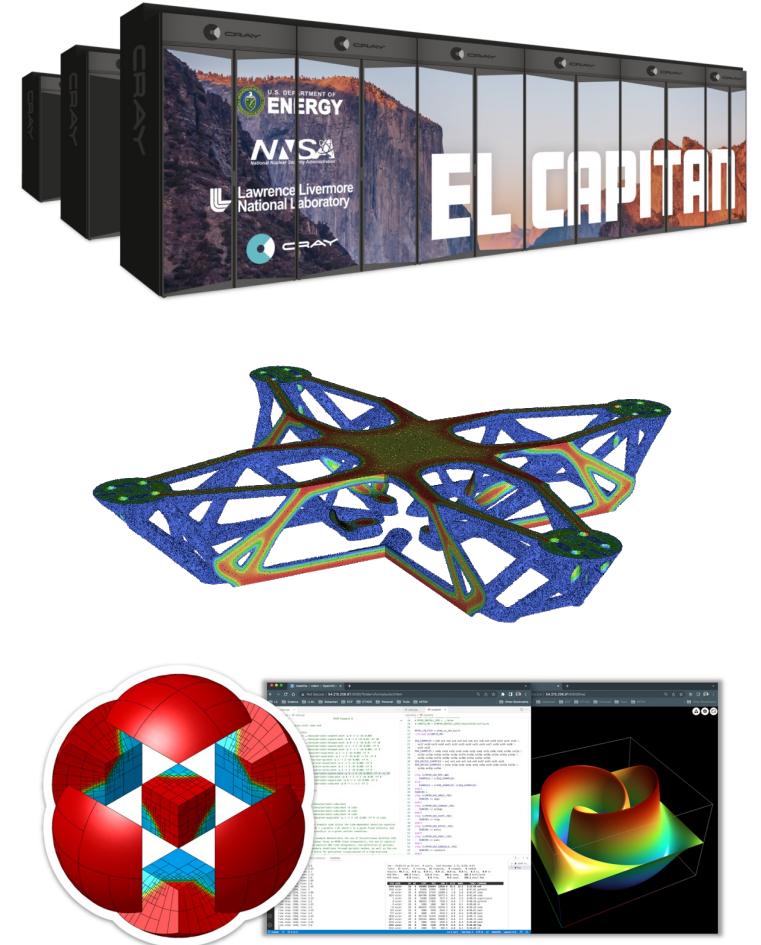
- Partial assembly = store only **D**, evaluate **B**
- Optimal memory, near-optimal FLOPs compared to **A**
- AD-friendly
- MFEM + Enzyme



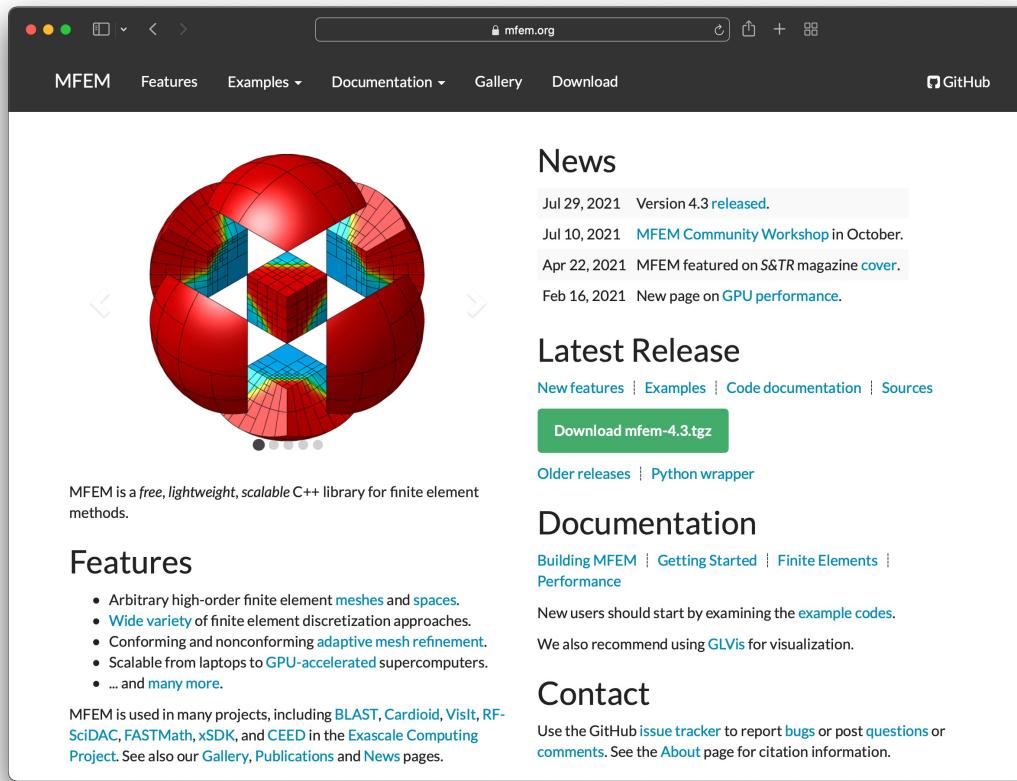
# Roadmap for Next Year

## Plans for FY24

- **GPU support**
  - Performance on AMD GPU: Frontier + El Capitan
  - Continued GPU porting and performance improvements
- **Applications**
  - Automatic differentiation · Design optimization
  - Compressible and incompressible flow
  - Fusion: both magnetic and ICF
  - Contact · 4D · mixed meshes · new collaborations
- **Code quality**
  - Improve documentation, testing
  - Additional examples + miniapps
- **New releases**
  - v4.7 in Mar · start work on v5.0 – *expect breaking changes!*
- **What would you like to see?**
  - Slack: [#meet-the-team](#) · GitHub: [github.com/mfem/mfem/issues](https://github.com/mfem/mfem/issues) · Email: [mfem@llnl.gov](mailto:mfem@llnl.gov)



# MFEM Resources



The screenshot shows the MFEM website homepage. At the top, there's a navigation bar with links for MFEM, Features, Examples, Documentation, Gallery, Download, and GitHub. Below the navigation is a large 3D visualization of a sphere divided into several triangular finite elements, with a color gradient from red to blue. A text overlay below the visualization reads: "MFEM is a free, lightweight, scalable C++ library for finite element methods." The main content area is organized into sections: "News" (with links to Version 4.3 released, MFEM Community Workshop, MFEM featured on S&TR magazine cover, and GPU performance), "Latest Release" (with links to New features, Examples, Code documentation, Sources, and download links for mfem-4.3.tgz and Older releases), "Documentation" (with links to Building MFEM, Getting Started, Finite Elements, and Performance), and "Contact" (with instructions to use the GitHub issue tracker for bugs or questions). A sidebar on the left lists "Features" such as arbitrary high-order finite element meshes and spaces, wide variety of discretization approaches, conforming and nonconforming adaptive mesh refinement, scalability from laptops to GPU-accelerated supercomputers, and many more.

**Website:**  
[mfem.org](http://mfem.org)

**Software:**  
[github.com/mfem](https://github.com/mfem)

**Publications:**  
[mfem.org/publications](http://mfem.org/publications)

**Email:**  
[mfem@llnl.gov](mailto:mfem@llnl.gov)

- Contact us with questions + feedback
- Contribute to the code
- Explore our publications

# Thank you from the MFEM team at LLNL!



**Bob**  
Anderson  
[@rw-anderson](https://twitter.com/rw-anderson)



**Julian**  
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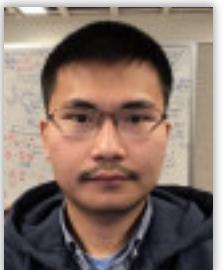
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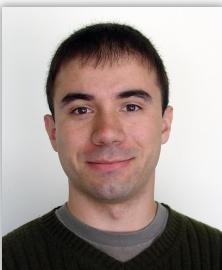
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