Math 362 Assignment 6

Due: Tuesday, December 3

- Answer all questions. Each question is worth 5 marks. Full marks will be awarded only for answers that are both mathematically correct and coherently written.
- Please consider the markers and write neatly and legibly! I have instructed the markers to ignore work they cannot read. (And I won't read it, either.)

1. Find **seven(!)** Pythagorean triangles with a side of length 24. Does there exist one with a hypotenuse of length 24?
2. Find two pairs of relatively prime positive integers (x, y) such that $x^2 + 5929 = y^2$.
3. A (finite or infinite) sequence of numbers $a, a+d, a+2d, \ldots$, where d is a constant, is said to be "in arithmetic progression". Show that $3n, 4n, 5n$, where $n = 1, 2, \ldots$, are the only Pythagorean triples whose terms are in arithmetic progression.
4. Show that the equation $x^2 + y^2 + z^2 = 2xyz$ has no nontrivial integer solutions.
5. Find all possible ways of writing $9945 = 3^2 \times 5 \times 13 \times 17$ as the sum of two squares. (No marks for using trial and error!)
6. Prove that
   (a) a positive integer n is representable as the difference of two squares if and only if n is the product of two numbers that are either both even or both odd;
   (b) a positive even integer n can be written as the difference of two squares if and only if $n \equiv 0 \pmod{4}$.
7. Find all primes p for which $29p + 1$ is a perfect square.