Introduction to Approximation Algorithms - 2014/2 Practical Project Proposal

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Abstract

The main objective of this Practical Project is to extend and validate the propositions of the Seminar chosen paper. The extension proposed is to solve other problem with the approximation algorithm studied and the validation will be obtained through simulations on both best and worst case scenarios.

I. Introduction

The chosen paper [2] solves the Maximum-Weighted Independent Set of Links Problem (**MWIS**) using algorithms designed under the *protocol* interference model. In general, a protocol interference model specifies a pairwise conflict relations among all links. A subset is independent if all links are pairwise conflict-free. This paper describes several algorithms, we are interested in the orientation one named **OrientWIS**.

There is a related work on Wireless Communications [1] in which the authors made a proof (Theorem 4.5) that any algorithm to solve **MWIS** is a constant approximation for the Multi-Rate Scheduling Problem (**MRS**).

So, our proposed contributions are:

1) obtains a constant approximation algorithm to **MRS**
2) implements **OrientWIS**
3) simulates both best and worst case scenarios

II. Practical Project Tasks

This project is divided in two parts: analytical and experimental.

A. Analytical

Into this part, the main task will be map the **MRS** [1] into a kind of input to the **OrientWIS** described at [2]. This map must preserve the approximation guarantees.

B. Experimental

Simulations will be used to validate **OrientWIS** in best and worst case scenarios. The instances used by simulations for best and worst case scenarios will be manually crafted based on the nature of the **OrientWIS** algorithm. The implementation code will be available at github [3].

III. Equations

$$\rho = 2(1+\epsilon)\alpha_D^{in} = C\left(1+\frac{2a}{b}\right)^2 \implies \epsilon = \frac{C\left(1+\frac{2a}{b}\right)^2}{2\alpha_D^{in}} - 1 \tag{1}$$

$$\left.\begin{array}{r}\alpha_D^{in} = \left\lceil \frac{\pi}{\arcsin\left(\frac{c-1}{2c}\right)}\right\rceil \\ c = z_{min}\end{array}\right\}\alpha_D^{in} = \left\lceil \frac{\pi}{\arcsin\left(\frac{z_{min}-1}{2z_{min}}\right)}\right\rceil \tag{2}$$

$\epsilon = \frac{C}{2\pi}\left(1+\frac{2a}{b}\right)^2 \arcsin\left(\frac{z_{min}-1}{2z_{min}}\right) - 1$

$$2(1+\epsilon)\alpha_D^{in} = C\left(1+\frac{2a}{b}\right)^2 \tag{3}$$

$$\left.\begin{array}{r}\rho = 2(1+\epsilon)\alpha_D^{in} = C\left(1+\frac{2a}{b}\right)^2 \implies \epsilon = \frac{C\left(1+\frac{2a}{b}\right)^2}{2\alpha_D^{in}} - 1 \\ \left.\begin{array}{r}\alpha_D^{in} = \left\lceil \frac{\pi}{\arcsin\left(\frac{c-1}{2c}\right)}\right\rceil \\ c = z_{min}\end{array}\right\}\alpha_D^{in} = \left\lceil \frac{\pi}{\arcsin\left(\frac{z_{min}-1}{2z_{min}}\right)}\right\rceil\end{array}\right\}\epsilon = \frac{C}{2\pi}\left(1+\frac{2a}{b}\right)^2 \arcsin\left(\frac{z_{min}-1}{2z_{min}}\right) - 1 \tag{4}$$

$$\epsilon(C, a, b, z_{min}) \tag{5}$$

$a \le 2\Delta_l d_{min} z_{min}\left\{\frac{2\sqrt{3}\pi\alpha(\alpha-1)}{3(\alpha-2)}\frac{\Delta_\beta \beta_{min}(\Delta_l d_{min})^\alpha}{(d_{min}z_{min})^\alpha}\right\}^{\frac{1}{\alpha-2}}$

$$\le 2\Delta_l\left\{\frac{2\sqrt{3}\pi\alpha(\alpha-1)}{3(\alpha-2)}\Delta_l^\alpha \Delta_\beta \frac{\beta_{min}(d_{min})^\alpha (d_{min}z_{min})^{\alpha-2}}{(d_{min}z_{min})^\alpha}\right\}^{\frac{1}{\alpha-2}}$$

$\frac{(d_{min})^\alpha (d_{min}z_{min})^{\alpha-2}}{(d_{min}z_{min})^\alpha} = \frac{d_{min}^{\alpha-2}}{z_{min}^2} = d_{min}^{\alpha-2}\left(\frac{(\alpha-2)6}{\beta_{min}\alpha 4\pi\sqrt{3}}\right)^{\frac{2}{\alpha}}$

$$\le 2\Delta_l\left\{(\alpha-1)\frac{2\sqrt{3}\pi\alpha}{3(\alpha-2)}\Delta_l^\alpha \Delta_\beta d_{min}^{\alpha-2}\beta_{min}^{\frac{\alpha-2}{\alpha}}\left(\frac{(\alpha-2)3}{\alpha 2\pi\sqrt{3}}\right)^{\frac{2}{\alpha}}\right\}^{\frac{1}{\alpha-2}}$$

$$\le 2\Delta_l\left\{\left(\frac{2\sqrt{3}\pi\alpha}{3(\alpha-2)}\right)^{\frac{\alpha-2}{\alpha}}\Delta_l^\alpha \Delta_\beta d_{min}^{\alpha-2}\beta_{min}^{\frac{\alpha-2}{\alpha}}\right\}^{\frac{1}{\alpha-2}}$$

$$\epsilon = \frac{C}{2\pi}\left(1+\frac{2a}{b}\right)^2 \arcsin\left(\frac{z_{min}-1}{2z_{min}}\right) - 1 \tag{6}$$

$$C = \frac{\sqrt{3}\pi}{6} \qquad a \le 2\Delta_l^{\frac{2(\alpha-1)}{\alpha-2}}\Delta_\beta^{\frac{1}{\alpha-2}}\left(\frac{2\sqrt{3}\pi\alpha}{3(\alpha-2)}\right)^{\frac{1}{\alpha}} d_{min}\beta_{min}^{\frac{1}{\alpha}} \qquad b = \beta_{min}^{\frac{1}{\alpha}} - 1 \tag{7}$$

$$\frac{2a}{b} \le 4\Delta_l^{\frac{2(\alpha-1)}{\alpha-2}}\Delta_\beta^{\frac{1}{\alpha-2}}\left(\frac{2\sqrt{3}\pi\alpha}{3(\alpha-2)}\right)^{\frac{1}{\alpha}} d_{min}\left(1-\frac{1}{\beta_{min}^{\frac{1}{\alpha}}}\right)^{-1} \tag{8}$$

$$\epsilon(\Delta_l, \Delta_\beta, d_{min}, \beta_{min}, \alpha) \tag{9}$$

$$\epsilon \le \frac{\sqrt{3}}{12}\left[1 + 4\Delta_l^{\frac{2(\alpha-1)}{\alpha-2}}\Delta_\beta^{\frac{1}{\alpha-2}}\left(\frac{2\sqrt{3}\pi\alpha}{3(\alpha-2)}\right)^{\frac{1}{\alpha}} d_{min}\left(1-\frac{1}{\beta_{min}^{\frac{1}{\alpha}}}\right)^{-1}\right]^2 \arcsin\left(\frac{1}{2} - \left(\frac{3(\alpha-2)}{\alpha 2\pi\sqrt{3}}\right)^{\frac{1}{\alpha}}\right) \tag{10}$$

REFERENCES

[1] Olga Goussevskaia et al, *Wireless Multi-Rate Scheduling: From Physical Interference to Disk Graphs*, 3rd ed. Harlow, England: Addison-Wesley, 1999.

[2] Peng-Jun Wan et al, *Fast and Simple Approximation Algorithms for Maximum Weighted Independet Set of Links*, IEEE INFOCOM 2014 - IEEE Conference on Computer Communications.

[3] Flavio Haueisen *and* Manasses Ferreira, *Multi-Rate Scheduling source-code*, GitHub Repository, https://github.com/mfer/mrs.git