Machine Learning 1 ICS2207

Assignment Report

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1 Introduction

The Travelling Salesman Problem is a very common problem in the field of operations research. This has been studied extensively by mathematicians, computer scientists, and many great minds yet its complexity is still unknown [1]. The problem statement is given as follows:

"Given a collection of cities and the cost of travel between each pair of them, the travelling salesman problem, or TSP for short, is to find the cheapest way of visiting all of the cities and returning to your starting point."
[1]

Traditional methods for solving this problem come in three types; calculus based methods, exhaustive search methods and random search methods [3]. These methods, however, pose a large number of problems such as the algorithm converging to a local optimum or running in exponential time. Machine Learning Algorithms such as Genetic Algorithms (GAs) or the Ant-Colony Optimisation method (ACO) can be used to find accurate approximations to the solutions to the TSP and other similar optimisation problems.

In order to solve this problem using a Genetic Algorithm (Section 2) and the ACO method (Section 3), a few assumptions were made. Firstly, it was assumed that the data provided will come from a symmetric instance of TSP, i.e. the distance d from city c_i to city c_j $d(c_i, c_j) = d(c_j, c_i) \ \forall i, j \in [1, n]$. Another assumption that's being made is that the "closed" version of TSP will be solved for this task, i.e. the salesman will end in the city where he started.

In Section 2 of this paper, Genetic Algorithms shall be defined and a method to solve the TSP using such an algorithm will be proposed. Section 3 shall define Ant-Colony Optimisation and propose a method for solving the TSP using this method. Section 4 will then compare the outcomes of the two algorithms and these results shall be discussed in detail. All code used will be listed in the Appendix (Section 8).

2 Genetic Algorithms

Genetic algorithms are designed to simulate a biological process [2], thus most of the terminology refers to the algorithm's biological counterpart. The components that make up GAs are as follows:

- an objective (or fitness) function
- a population of chromosomes
- a selection operator on the chromosomes

- a crossover function which produces a new generation of chromosomes
- a random mutation function

The objective function is the function that the algorithm is trying to optimise. In Genetic Algorithms, this is often referred to as a *fitness* function, this term is in fact taken from evolutionary theory [2]. For the TSP, the fitness function is the sum of the distances between the points, the TSP in fact deals with minimising this sum in order to be able to find the shortest path. Given that the data under study is given in coordinate format [4], the fitness function used for the TSP is given in Equation 1 [2].

$$D = \sum_{k=1}^{n} \sqrt{(x_{k+1} - x_k)^2 + (y_{k+1} - y_k)^2}$$
 (1)

A chromosome refers to a value that will be considered as the candidate solution to the optimisation problem. In the case of the TSP, the chromosome could be a permutation of the cities which represent the order that the salesman visits each city. Historically, chromosomes were encoded as a bit string, however it would make more sense not to encode it in this way for the TSP, this however will require a change in the crossover and mutation functions [2].

The selection operator refers to the method used to select which chromosomes are to be chosen for reproduction. In general, a fitter chromosome should be more likely to be selected. In the case of TSP, the probability function must be changed slightly from the original definition since the objective of the TSP is to minimize the fitness function (cost) and not maximize it, thus care must be taken to reverse the probability function.

Once the "fittest" chromosomes are selected, the crossover operator is then used to "combine" them. This operator "resembles the biological crossing over and recombination of chromosomes to create two offspring" [2]. In the case of the TSP, special care needs to be taken when designing this function and the classical way of defining this operator will not work since each city needs to be visited only once [5]. The technique which shall be used in the implementation for the TSP is known as the "cycle" crossover (as taken from [2]) and works as follows:

1. Initially, a random location is chosen in the length of the chromosome.

- 2. The two parent chromosomes exchange integers at this point to create the offspring.
- 3. If the integers are the same value then the offspring is the same as the parent and the algorithm terminates.
- 4. Otherwise, each offspring now has a duplicate integer, so switch the duplicate integer in the first offspring with the integer in the same location in the second offspring.
- 5. Repeat the above step until there are no duplicates in the first offspring (and thus the second offspring).
- 6. Both are now valid permutations.

The importance of the mutation step is to reduce the probability of the algorithm converging to a local optimum [5]. This is achieved by causing the algorithm to maintain diversity in the population, however it can cause it to converge more slowly [2]. Again, since the chromosome for the TSP is not encoded as a bit string, the mutation operator changes slightly from that described by Haupt in the original definition of a Genetic Algorithm. The implemented mutation operator randomly chooses two integers in a chromosome from the new generation and swaps them [2]. This will happen with a rather low probability in order to increase the rate of convergence.

3 Ant-Colony Optimisation

The Ant-colony Optimisation algorithm forms part of a class of population-based algorithms known as swarm intelligence (SI) that considers the collective behaviour of the population and individual solutions [6]. A general ACO algorithm has the following form [7]:

Set parameters, initialize pheromone trails SCHEDULE ACTIVITIES

 $\begin{array}{c} ConstructAntSolutions \\ DaemonActions & \{optional\} \\ UpdatePheromones \\ END_SCHEDULE_ACTIVITIES \end{array}$

The parameters initialised for my implementation of the Ant Colony Optimisation Algorithm are as follows:

- 1. no_of_ants: the number of ants in each iteration.
- 2. max_iter: the maximum number of iterations.
- 3. evaporation_rate: the rate at which the pheromones evaporate.
- 4. <u>alpha</u>: a parameter for calculating the probability of an ant selecting a certain route. Alpha gives more weighting to the pheromone values.
- 5. <u>beta</u>: a parameter for calculating the probability of an ant selecting a certain route. Beta gives more weighting to the heuristic value.
- 6. <u>q0</u>: this parameter gives the probability of using the previous ants' experience over selecting a random path.

Since the task at hand is to write an algorithm to solve the symmetric TSP and R is optimised for working with matrix algebra, the distances between each city were calculated as an initialisation step in order to avoid having to compute these values each time on every ant run. The distances between each city were stored in a symmetric matrix thus only the lower triangular matrix needed to be worked out using for loops.

In order to construct the ant solutions, initially, each ant is placed at random on a city. At each step, the ant then chooses the next city based on the probability matrix given in Equation 2.

$$p_{ij}^{k}(t) = \frac{[\tau_{ij}(t)]^{\alpha} \cdot [\eta_{ij}]^{\beta}}{\sum_{l \in \mathcal{N}_{i}^{k}} [\tau_{il}(t)]^{\alpha} \cdot [\eta_{il}]^{\beta}} \ if \ j \in \mathcal{N}_{i}^{k}$$
 (2)

Where $p_{ij}^k(t)$ is the probability that ant k goes to city j from city i at iteration t, $\tau_{ij}(t)$ is the pheromone value on edge ij at iteration t, η_{ij} is the heuristic value on edge ij and \mathcal{N}_i^k is the set of all possible nodes reachable by ant k from node i which in our case will be the set of all possible nodes not yet visited by ant k [8].

The ant with the best solution after each iteration will then update the

pheromone values of each edge and is defined as seen in Equation 3 [6].

$$\tau_{ij} = (1 - \rho)\tau_{ij} + \rho \sum_{k=1}^{m} \triangle \tau_{ij}^{k},$$

$$\triangle \tau_{ij}^{k} = \frac{1}{L^{k}}$$
(3)

Where m is the number of ants, and L^k is the length of the best tour of that iteration. All pheromone values also evaporate at a rate defined by the parameter after each iteration.

4 Results and Comparisons

In general, when comparing the performance of the ACO Algorithm to the GA algorithm, one can note that the ACO algorithm seems to find a better approximation to the solution than the GA, and it does it in much less time using less iterations. This section compares the results of both algorithms on specific instances. By simply looking at the plots of the standard error of our estimated value compared with the iteration number, one can note the significant difference in performance. Note that with very large numbers, the Genetic Algorithm would take a very long time to compute and thus all the points were normalised between 0 and 1 in order to improve the speed of the algorithm. This step was then also repeated in the ACO Algorithm in order to be able to accurately compare the 2 methods.

Instance name: bays29.tsp

GA method: see listing 1 ACO method: see listing 2

GA Performance: ACO Performance:

<u>Time Taken:</u> 177127.73ms <u>Time Taken:</u> 7170.33ms

<u>Best Distance Found:</u> 6.129391 <u>Best Distance Found:</u> 4.601584

Actual Optimal Distance: 4.581227 Actual Optimal Distance: 4.581227

Standard Error: 2.965736 Standard Error: 0.000414

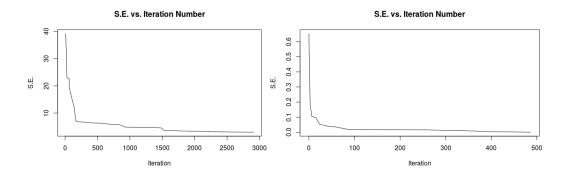


Figure 1: Genetic Algorithm

Figure 2: ACO Algorithm

Instance name: att48.tsp

GA method: see listing 1 ACO method: see listing 2

GA Performance:

 $\underline{\text{Time Taken:}}\ 475314.39\text{ms}$

Best Distance Found: 8.432387

Actual Optimal Distance: 5.377854

Standard Error: 9.330172

ACO Performance:

<u>Time Taken:</u> 49082.17ms

Best Distance Found: 5.431998

Actual Optimal Distance: 5.377854

Standard Error: 0.00101

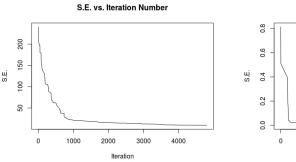


Figure 3: Genetic Algorithm

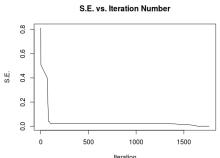


Figure 4: ACO Algorithm

Instance name: ch130.tsp

GA method: see listing 1 ACO method: see listing 2

GA Performance:

Time Taken: 1292756.23ms

Best Distance Found: 26.76519

Standard Error: 326.2241

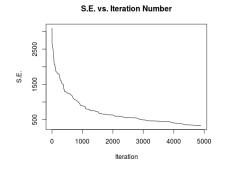
ACO Performance:

Time Taken: 57314.04ms

Best Distance Found: 9.542707

Actual Optimal Distance: 8.703517 Actual Optimal Distance: 8.703517

Standard Error: 0.5555441



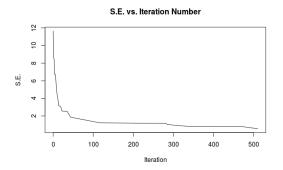


Figure 5: Genetic Algorithm

Figure 6: ACO Algorithm

Conclusion 5

Statement of Completion 6

7 Plagiarism Declaration Form

Appendix 8

```
1
    rm(list=ls()) #Clear Environment
2
3
    require (readxl) #needed to import data
4
    Data = read_xlsx(file.choose(),col_names=FALSE) #
     Choose the data file you want to open
    X = Data[2] #X coordinates indexed by city
5
    Y = Data[3] #Y coordinats indexed by city
6
    rm(Data) #remove Data (not needed)
9
    #Normalize points to be between 0 and 1
    #This is an optimisation step to improve the speed
10
     of the function
```

```
11
    X = X/max(X)
12
    Y = Y/max(Y)
13
14
    #fitnessFunction:
15
    #takes a population of chromosomes in Matrix form as
       an argument
16
    #returns a vector of distances each element
      corresponding to a chromosome
    #Example of use:
17
18
    \#a = c(1,2,3,4,5,6,7)
    \#b = c(2,4,3,1,5,15,12)
19
20
    \#c = c(10,2,3,1,5,6,11)
21
    \#pop = t(cbind(a,b,c))
    #distances=fitnessFuntion(pop)
22
23
    fitnessFunction <- function(pop) {
      Npop = nrow(pop)#size of population
24
25
       Ncity = ncol(pop)#no. of cities
26
      #append first city to the end in order to be able
      to calculate the distance
27
       tour = cbind(pop, pop[,1])
28
      #Empty vector to store distances of each
      chromosome
29
       distances = mat.or.vec(Npop, 1)
30
      #loop over every tour in the population
31
32
       for (i in 1:Npop) {
33
         chromosome = tour[i,]
34
         #for every city in the chromosome
35
         distance = 0
36
         for (j in 1: Ncity) {
37
           #fitness function as defined in Equation 1 of
     my Documentation
           temp =
38
                      \mathbf{sqrt}(((X[\operatorname{chromosome}[j+1],1]-X[
     chromosome[j],1])^2+(Y[chromosome[j+1],1]-Y[
      chromosome [ j ] ,1]) ^2))
           distance = distance + temp
39
```

```
40
         }
41
         distances [i]=distance
42
43
       distances
44
    }
45
46
    #mate function, mates 2 chromosomes given a pointer
    #the starting pointer and returns a list of the 2
47
     new
    #child chromosomes
48
49
    mate <- function(mate1, mate2, pointer) {</pre>
       temp = mate1
50
       change = TRUE
51
52
      #loop until all values are unique
53
       while (change) {
54
         #swap values
55
         mate1 [pointer] = mate2 [pointer]
         mate2 [pointer]=temp[pointer]
56
57
         #returns an array of indices which match the
      value at mate1 [pointer]:
58
         pointers = which (mate1 = mate1 [pointer], arr.ind =
      TRUE)
59
         #check if there exists an duplicate in the
      chromosome
         change = FALSE
60
61
         tempPointer = pointer
         for (j in 1:length(pointers)) {
62
63
           if (pointers[j]!=pointer) {
64
             #if there's a duplicate, point to it
65
             tempPointer = pointers[j]
             change = TRUE
66
67
           }
68
         }
69
         pointer = tempPointer
70
       }
```

```
71
      return (unname (rbind (mate1, mate2)))
72
    }
73
74
    #function that mutates the chromosome
75
    mutate <- function(chromosome) {
76
      indices = sample(1:length(chromosome),2,replace=
     FALSE) #choose 2 random indices in the chromosome
77
      #swap the elements
78
      swapTemp = chromosome[indices[1]]
      chromosome [indices [1]] = chromosome [indices [2]]
79
      chromosome [indices [2]] = swapTemp
80
81
      return(chromosome)
82
    }
83
84
    #Main Function:
85
    #Initialize Genetic Algorithm Parametes:
86
    NoOfCities = nrow(X) #Number of Cities
87
88
    popSize = 10 #no of chromosomes in each population
    pop = mat.or.vec(popSize, NoOfCities)
89
90
    pop2 = mat.or.vec(popSize, NoOfCities)
91
    best = mat.or.vec(1, NoOfCities) #vector to keep the
     best chromosome so far
92
    keep = 6 #no of chromosomes to be chosen as parents
    mutationRate = 0.4 #probability of mutation
93
94
    #noMutations = ceiling((popSize-1)*mutationRate) #
      total number of mutations
95
    Matings = ceiling((popSize-keep)/2) #number of
     matings
96
    maxit = 5000 #maximum number of iterations
97
98
    \#\text{optTour} = c
      (1,28,6,12,9,5,26,29,3,2,20,10,4,15,18,17,14,22,11,19,25,7,23,27,8,24,
     #bays29
99
    \#optTour = c
      (1,8,38,31,44,18,7,28,6,37,19,27,17,43,30,36,46,33,20,47,21,32,39,48,5)
```

```
#att48
100
     optTour = c
      (1,41,39,117,112,115,28,62,105,128,16,45,5,11,76,109,61,129,124,64,69,
101
      79,87,12,81,103,77,94,89,110,98,68,63,48,25,113,32,36,84,119,111,123,1
102
      #ch130
     optDistance = 0
103
104
     for (i in 2:NoOfCities) {
105
       optDistance = optDistance + sqrt((X[optTour[i
      -1],1]-X[optTour[i],1])^2+(Y[optTour[i-1],1]-Y[
      optTour[i],1])^2)
106
     }
107
108
     bestVal = 1e+22 #Arbitrary large number
     se = (bestVal-optDistance)^2
109
110
     iteration = 0
111
112
     #chromosomes that will survive and mate:
113
     kept = mat.or.vec(keep, NoOfCities)
114
     #stores the probability of each chromosome to
      survive
115
     prob = mat.or.vec(popSize, 1)
116
117
     #populate initial population with random chromosomes
     for (i in 1:popSize) {
118
119
       pop[i,] = sample(1: NoOfCities, NoOfCities) #creates
       a chromosome
120
     }
121
122
     #set starting time:
     start_time = as.numeric(Sys.time())*1000;
123
```

```
124
      #MAIN LOOP:
      for (gen in 1:maxit) {
125
        #compute the fitness function on the population
126
127
        Lengths = fitnessFunction(pop)
128
        #Save the best solution
129
        if (min(Lengths) < bestVal) {
130
           best[1,] = pop[which.min(Lengths),]
           bestVal = min(Lengths)
131
          \mathbf{se} = \mathbf{c}(\mathbf{se}, (\mathbf{bestVal-optDistance})^2) \#\mathbf{standard}
132
       error for graph
           iteration = c(iteration, gen) #iter number for
133
       graph
          end_time=as.numeric(Sys.time())*1000
134
135
          print(bestVal)
        }
136
137
138
        #selection
        total = sum(Lengths)
139
        #Probability of each chromosome to be selected
140
        for (i in 1:popSize) {
141
142
           \operatorname{prob}[i] = 1 - (\operatorname{Lengths}[i]/\operatorname{total})
143
          #This gives a higher probability value to the
       shortest lengths
144
          #Thus we need to select the chromosomes with the
        highest probability
145
146
        #selects the elements of the population to be kept
        based on the probability distribution
147
        #as defined above
148
        odds = sample(1:popSize, keep, replace=TRUE, prob =
       prob)
149
        #choose the chromosomes to be kept and store them
       in the new population
150
        #keep the best and second best solutions
151
        pop2[1,] = pop[\mathbf{which}.\mathbf{min}(Lengths),]
        pop2[2,]=pop[which(Lengths=sort(Lengths,
152
```

```
decreasing=TRUE) [popSize-1]) [1],
       #choose parents, we'll choose 3 mums and 3 dads,
153
       to have 6 kids:
154
        for (i in 1:keep) {
155
          #keep a record of the parents kept:
          kept[i,] = pop[odds[i],]
156
157
       }
158
159
       index = 3
        while (index < 9) {
160
161
          #mate1, mate2 are random integers between 1 and
       keep (index)
          mate1 = ceiling(runif(1, min=0, max=keep-1))
162
          mate2=ceiling(runif(1, min=0, max=keep-1))
163
          pointer = ceiling (runif(1,0, NoOfCities)) #random
164
        int between 1 and NoOfCities
165
          children = mate(kept[mate1,],kept[mate2,],
       pointer) #call the mate function
166
          #mutate with probability and save to population
167
          if (runif(1)<=mutationRate) {</pre>
168
            pop2[index,] = mutate(children[1,])
169
          } else {
170
            pop2[index,] = children[1,]
171
          if (runif(1)<=mutationRate) {</pre>
172
            pop2[index+1,] = mutate(children[2,])
173
174
          } else {
            pop2[index+1,] = children[2,]
175
176
177
          index=index+2
178
179
       #Randomly fill remaining part of population with
       chromosomes
180
        for (i in 9:10) {
181
          pop2[i,] = sample(1: NoOfCities, NoOfCities) #
       creates a chromosome
```

```
182
        }
183
        #Print iteration number
184
185
        if (gen%/100==0) {
186
          print(gen)
187
188
        pop = pop2
189
      #compute the fitness function on the population
190
      Lengths = fitnessFunction(pop)
191
      #Save the best solution
192
193
      if (min(Lengths)<bestVal) {</pre>
        best[1,] = pop[which.min(Lengths),]
194
195
        bestVal = min(Lengths)
        \mathbf{se} = \mathbf{c}(\mathbf{se}, (\mathbf{bestVal-optDistance})^2) \ \# \mathrm{standard} \ \mathrm{error}
196
        for graph
197
        iteration = c(iteration, gen) #iter number for
       graph
        end_time=as.numeric(Sys.time())*1000
198
        print(bestVal)
199
200
      paste("Time Taken: ",end_time-start_time)
201
202
      print ( best [1,])
      plot (iteration [-1], se [-1], type="l", main="S.E. vs.
203
       Iteration Number", xlab="Iteration", ylab="S.E.")
204
                    Listing 1: Genetic Algorithm Code
 1
     rm(list=ls()) #Clear Environment
  2
  3
      require(readxl) #needed to import data
      Data = read_xlsx(file.choose(),col_names=FALSE) #
       Choose the data file you want to open
     X = Data[2] #X coordinates indexed by city
  6
     Y = Data[3] #Y coordinats indexed by city
     rm(Data) #remove Data (not needed)
```

```
8
9
    #Normalize points to be between 0 and 1
    #This is an optimisation step to improve the speed
10
     of the function
    X = X/max(X)
11
12
    Y = Y/max(Y)
13
14
    15
    #Parameters# <- adjusting these would change the
     performance of the algorithm
16
    17
    no\_of\_ants = 10 \# no of ants in each iteration
    max_iter = 3000 #maximum number of iterations
18
    evaporation_rate = 0.15 #evaporation rate of
19
     pheromones
20
    alpha = 1 #alpha and beta are parameters for
     calculating the prob matrix
21
    beta = 4
22
    q0 = 0.6 #probability of using other ant's
     experience
23
24
    25
    #Functions#
26
    #this function returns the reciprocal of a number,
27
     unless that number is 0
28
    probValues <- function(number) {</pre>
29
      if (number == 0) {
30
        number
31
      } else {
32
        1/number
33
      }
34
    }
35
36
    #function that accepts a matrix with the first
     column being the starting point of each ant and the
```

```
other columns all being 0
37
    #a Heuristic Matrix and the pheromone value Matrix
     Tau and returns tours of each ant.
38
    setRoutes <- function (Routes, Heuristic, Tau) {
39
      #for every ant:
      for (i in (1:nrow(Routes))) {
40
41
         Memory = mat.or.vec(ncol(Routes),1) #Set Ant's
     memory to 0s
         Score = (Tau^alpha)*(Heuristic)^beta #Calculate
42
      score matrix
43
         Memory[1] = Routes[i,1] #Add starting node to
     memory
        #Loop until all cities have been visited:
44
45
         for (j in 2:ncol(Routes)) {
           currentCity = Memory[j-1]
46
           #set score to 0 for current city:
47
           Score[, currentCity] = 0
48
           Particular Score = Score [current City,]
49
50
           if (runif(1)<=q0) {
             #find the city with the largest score:
51
52
             Memory[j] = which.max(ParticularScore)
53
           } else {
54
             #Choose the next node based on probability
             Probability = Particular Score/sum(
55
      Particular Score)
56
             Memory[j] = sample(1:length(Probability),1,
      prob=Probability)
           }
57
58
         }
59
         Routes [i,] = Memory
      }
60
      Routes
61
62
    }
63
64
    #function that calculates the length of each tour
    fitnessFunction <- function(Routes, Distances) {</pre>
65
```

```
66
       sum = mat.or.vec(nrow(Routes), 1)
67
       #for each ant j
       for (j in 1:nrow(Routes)) {
68
69
         cities = Routes[j,]
         #for each city visited
70
71
         for (i in 1:(\mathbf{ncol}(Routes)-1)) {
72
           #add up the distances
73
           sum[j] = sum[j] + Distances[cities[i], cities[i
      +1]]
74
         }
         sum[j] = sum[j] + Distances[cities[i+1], cities]
75
      [1]]
76
77
    sum
78
     }
79
80
    #function that updates pheromones after an iteration
     updatePheromones <- function(path, length,
81
      evaporation_rate, Tau) {
82
       for (i in 1: (length(path)-1)) {
83
         Tau[\mathbf{path}[i], \mathbf{path}[i+1]] = (1-evaporation_rate) *
      Tau[path[i], path[i+1]]+evaporation_rate*(length)
      \hat{}(-1);
         Tau[\mathbf{path}[i+1], \mathbf{path}[i]] = Tau[\mathbf{path}[i], \mathbf{path}[i+1]]
84
85
       Tau
86
87
     }
88
89
    90
    #Other Variables#
91
    92
     start_time=as.numeric(Sys.time())*1000;
93
     no\_of\_cities = nrow(X)
     Tau = matrix(0.00001, nrow = no_of_cities, ncol = no_of_cities)
94
      of_cities) #initial pheromone matrix
     starting\_nodes = mat.or.vec(no\_of\_ants, 1)
95
```

```
96
     dontStop = TRUE
97
     #generate distance between cities matrix
     Distances = matrix(0, nrow = no_of_cities, ncol = no_
98
      of_cities)
     Heuristic = Distances
99
     iter = 0
100
101
     #Note that Distances is a symmetric matrix since we'
      re working out the Symetric TSP
     #, thus in order to improve complexity first the
102
      lower triangular part is worked out,
     #then its transpose is added to itself.
103
104
     for (i in 2:no_of_cities) {
       for (j in 1:i) {
105
106
          Distances [i, j] = \mathbf{sqrt} ((X[i,1] - X[j,1])^2 + (Y[i,1] -
      Y[j,1])^2
       }
107
108
     Heuristic = apply (Distances, c(1,2), probValues) #
109
      Genarate the Heuristic Matrix (see function
      probValues)
110
     Distances = Distances + t(Distances)
111
     Heuristic = Heuristic + t(Heuristic)
112
     bestRoute = mat.or.vec(no_of_cities,1)
113
     bestLength = 1e27 #Arbitrarily large number
114
115
116
     \#optTour = c
      (1,28,6,12,9,5,26,29,3,2,20,10,4,15,18,17,14,22,11,19,25,7,23,27,8,24,
      #bays29
117
     \#optTour = c
       (1,8,38,31,44,18,7,28,6,37,19,27,17,43,30,36,46,33,20,47,21,32,39,48,5)
      #att48
118
     optTour = c
       (1,41,39,117,112,115,28,62,105,128,16,45,5,11,76,109,61,129,124,64,69,
```

```
120
      10, 14, 67, 13, 96, 122, 55, 60, 51, 42, 44, 93, 37, 22, 47, 40, 23, 33, 21, 126, 121, 78, 6
      #ch130
121
     optDistance = 0
122
     for (i in 2:no_of_cities) {
       optDistance = optDistance + Distances[optTour[i
123
      -1], optTour[i]]
124
     optDistance = optDistance + Distances[optTour[i],
125
      optTour[1]]
     se = (bestLength-optDistance)^2
126
     iteration = 0
127
128
129
     #Main Algorithm Loop#
130
     131
132
     while (dontStop) {
       #initialize each ant in a starting node
133
134
       starting_nodes = sample(1:no_of_cities, no_of_ants,
      replace = TRUE)
135
       routes = matrix(0, nrow = no_of_ants, ncol = no_of_
      cities) #routes will store the routes of the ants
       routes [,1] = starting_nodes
136
137
138
       #generate the routes for each ant
139
       routes = setRoutes (routes, Heuristic, Tau)
140
141
       #build a solution
142
       lengths = mat.or.vec(no\_of\_ants,1)
143
       lengths = fitnessFunction(routes, Distances)
144
145
       #save best route
146
       bestAntIndex = which.min(lengths)
       if (lengths[bestAntIndex] < bestLength) {</pre>
147
```

79,87,12,81,103,77,94,89,110,98,68,63,48,25,113,32,36,84,119,111,123,1

```
148
            bestLength = lengths [bestAntIndex]
            bestRoute = routes [bestAntIndex,]
149
            \mathbf{se} = \mathbf{c}(\mathbf{se}, (\mathbf{bestLength-optDistance})^2) \ \# \mathbf{standard}
150
        error for graph
            iteration = c(iteration, iter) #iter number for
151
        graph
            end_time = as.numeric(Sys.time())*1000
152
153
            print(bestLength)
154
         }
155
         #update pheromone values
156
157
         Tau = updatePheromones(routes[bestAntIndex,],
        lengths[bestAntIndex], evaporation_rate, Tau)
158
         #Stopping Criteria:
159
         iter = iter+1
160
161
          if (iter%%10==0) {print(iter)}
162
         dontStop = iter <= max_iter
163
164
       paste("Time Taken: ",end_time-start_time)
165
       print(bestRoute)
       \mathbf{plot} \, (\, \mathrm{iteration} \, [\, -1] \, , \mathbf{se} \, [\, -1] \, , \mathrm{type} = \mathrm{"l"} \, , \mathrm{main} = \mathrm{"S.E.} \quad \mathrm{vs.}
166
        Iteration Number", xlab="Iteration", ylab="S.E.")
167
```

Listing 2: ACO Code

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