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# *Optimal Control of Segway Personal Transporter*

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***Abstract*— In this paper an optimal controller is proposed for a self-balancing electrical vehicle called Segway PT. This vehicle has one platform and two wheels on the sides and the rider stands on the platform. A handlebar, as a navigator, is attached to the body of Segway, with which the rider controls the vehicle. Since Segway uses electrical energy produced by batteries, resource consumption management is of utmost importance. On the other hand, complex nonlinear dynamics cause difficulties in controlling the vehicle. Our proposed controller reduces energy consumption and enhance response speed of system instead of classic PID controller which proposed before. Simulation results show the desired performance of the proposed controller.**

***Keywords—Optimal Control; Segway Personal Transporter; State Feedback controller; styling; Riccati Equation***

## I. INTRODUCTION

There is a growing interest in automatic control of mechanical systems. Control methods become more complex and at the same time, more efficient, and all the systems are automated. Segway PT or electrical scooter, which is a one-person horizontal transporter, was invented and manufactured by Dean Kamen in 2001. Because of unique structure and capabilities, it was popular in no time. Also the interesting structure of Segway was considered by academic community, especially control engineers, to propose more accurate models and control methods for the vehicle [1].

Segway has one horizontal platform and two wheels on its sides, which are derived by electrical motor force, and the rider stands on the platform. There is a vertical handlebar for its control. The first and most important control objective is maintaining vertical balance of the system and preventing the vehicle and rider’s crash. This objective is met by gyroscopic velocity and angle sensors. On the other hand, it is expected that the system responds well to rider’s commands and move to the desired point. For this purpose, the rider can move forward or backward by shifting their weight forward or backward on the platform.

Segway has two wheels that are derived by direct current electrical motors. The energy sources of Segway are rechargeable batteries, which can be charged by normal AC outlets with special chargers. DC motors are used in Segways because of their capabilities and their easy application with batteries. Both wheels and motors are located at each side of the platform. Normally when the rider is on the platform and handles the handlebar, if the control system and sensors operate, Segway maintains its balance. The rider can move to a desired direction backward or forward, by changing the angle of handlebar. Fig. 1 shows the control and function of a Segway [1], [3].

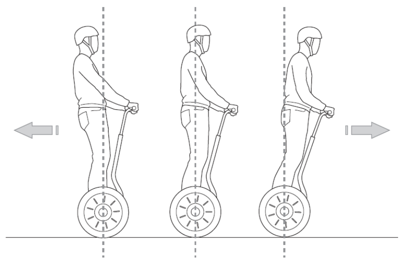

Fig. 1. Control and navigation of a Segway

When the system is on, it can maintain the balance and prevent crash. This task is done by applying electro motive force to the wheels with DC motors. In [2] a robust controller is proposed for the system. In [3] a stabilizing controller and in [6] a tracking method is proposed. Balancing stability of Segway is considered in [9].

The rest of the paper is as follows. In section II the combination of state space model of DC motor and dynamic model of cart, results in nonlinear state space model of Segway. In section III optimal control method is proposed. In section IV simulation results are shown and finally section V concludes the paper.

## II. SEGWAT MODEL

Researchers have long considered Modelling and system behavior description, with mathematical equations and it has been a great effort to find an appropriate model to describe systems. It seems that all scientific activities need modelling and understanding of the system, whether it is from natural phenomena such as fuzzy and neural network intelligent models or, mathematical models based on differential equations. In fact, all control methods are implemented on system models and without correct and reliable models, results cannot be trusted. Therefor in order to have an accurate control system, we need a complete and accurate model.

Segway PT model needs lots of attention because of its dynamics and physical structure. Since DC motors are used in

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Segways and because of their widespread applications, first the dynamic model of DC motor is presented and then physical dynamic model of Segway PT is obtained. By combining these two models a comprehensive model is obtained in which, the input is DC motor voltage and the output is the physical pose of Segway PT and this model is used for controller design in section III. [3],[5],[6].

### *A. Linear Model of a DC Motor*

Segway is supplied with two DC motors. The state space model of a DC motor is as follows.

$$\begin{bmatrix} \dot{i} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \frac{R}{L} & \frac{k_e}{L} \\ \frac{k_m}{I_R} & \frac{-k_e}{I_R} \end{bmatrix} \begin{bmatrix} i \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & -\frac{1}{I_R} \end{bmatrix} \begin{bmatrix} V_a \\ \tau_a \end{bmatrix} \tag{1}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i \\ \omega \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} V_a \\ \tau_a \end{bmatrix} \tag{2}$$

In which V and I are voltage, current respectively, and ω in angular velocity.

### *B. Dynamic Model of Segway*

Although personal transporter scooter is more complicated than dynamic system of cart and inverted pendulum, they are similar in many ways. We begin the modelling by analyzing the dynamic behavior of the system, which leads to two motion equations describing Segway performance. This system's behavior can be described by applying motor torque input to the system [2] , [4].

Fig. 2 shows forces applied to left and right wheels. Since both equations for left and right wheels are the same, here only right wheel is considered for deriving the equations.

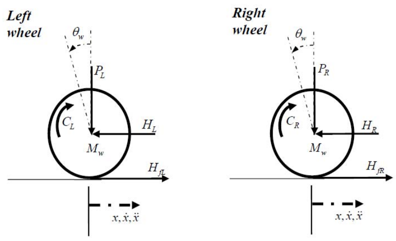


Fig. 2. Applied forces to wheels

Using Newton's second law we have:

$$\sum F_x = Ma \Rightarrow M_w \ddot{x} = H_{fR} - H_R \tag{3}$$

Sum of torques around the wheel axis is:

$$\sum M_o = I\alpha \Rightarrow I_w \ddot{\theta}_w = C_R - H_{fR} r \tag{4}$$

The motor torque can be stated as:

$$\tau_m = I_R \frac{d\omega}{dt} + \tau_a \tag{5}$$

Using DC motor parameters, applied torque to wheels equals:

$$C = I_R \frac{d\omega}{dt} = -\frac{k_m k_e}{R} \dot{\theta}_\omega + \frac{k_m}{R} V_a \tag{6}$$

Therefore (4) can be rewritten as:

$$I_w \ddot{\theta}_w = -\frac{k_m k_e}{R} \dot{\theta}_\omega + \frac{k_m}{R} V_a - H_{fR} r \tag{7}$$

Hence:

$$H_{fR} = -\frac{k_m k_e}{Rr} \dot{\theta}_\omega + \frac{k_m}{Rr} V_a - \frac{I_w}{r} \ddot{\theta}_w \tag{8}$$

Substituting (8) in (3) the right and left wheels' equations are obtained.

Right wheel:

$$M_w \ddot{x} = -\frac{k_m k_e}{Rr} \dot{\theta}_\omega + \frac{k_m}{Rr} V_a - \frac{I_w}{r} \ddot{\theta}_w - H_R \tag{9}$$

Left Wheel:

$$M_w \ddot{x} = -\frac{k_m k_e}{Rr} \dot{\theta}_\omega + \frac{k_m}{Rr} V_a - \frac{I_w}{r} \ddot{\theta}_w - H_L \tag{10}$$

We can change angular rotation to a linear motion by a simple conversion:

$$\ddot{\theta}_w r = \ddot{x} \Rightarrow \ddot{\theta}_w = \frac{\ddot{x}}{r} \tag{11}$$

$$\dot{\theta}_w r = \dot{x} \Rightarrow \dot{\theta}_w = \frac{\dot{x}}{r} \tag{12}$$

Substituting into (9) and (10) we get:

Right wheel:

$$M_w \ddot{x} = -\frac{k_m k_e}{Rr^2} \dot{x} + \frac{k_m}{Rr} V_a - \frac{I_w}{r^2} \ddot{x} - H_R \tag{13}$$

Left Wheel:

$$M_w \ddot{x} = -\frac{k_m k_e}{Rr^2} \dot{x} + \frac{k_m}{Rr} V_a - \frac{I_w}{r^2} \ddot{x} - H_L \tag{14}$$

By combining (13) and (14) we get:

$$2\left(M_w + \frac{I_w}{r^2}\right) \ddot{x} = -\frac{2k_m k_e}{Rr^2} \dot{x} + \frac{2k_m}{Rr} V_a - (H_L + H_R) \tag{15}$$

In addition, the rider and handlebar can be considered accurately as a rigid body with following forces applied to it:

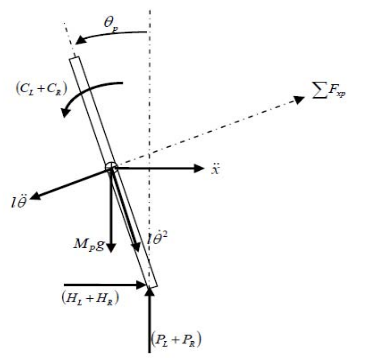


Fig. 3. Forces applied to the rider and handlebar

Again using Newton's second law on x axis we have:

$$\begin{aligned}&\sum F_x = M_p\ddot{x}\\&\Rightarrow (H_L+H_R)-M_pl\ddot{\theta}_p\cos\theta_p+M_pl\dot{\theta}_p^2\sin\theta_p=M_p\ddot{x}\end{aligned} \tag{16}$$

This results in:

$$(H_L+H_R)=M_pl\ddot{\theta}_p\cos\theta_p+M_pl\dot{\theta}_p^2\sin\theta_p+M_p\ddot{x} \tag{17}$$

The sum of aplied forces to rider and handlebar is:

$$\begin{aligned}&\sum F_{xp}=M_p\ddot{x}\cos\theta_p\\&\Rightarrow (H_L+H_R)\cos\theta_p+(P_L+P_R)\sin\theta_p-M_pl\ddot{\theta}_p-M_pg\sin\theta_p=M_p\ddot{x}\cos\theta_p\end{aligned} \tag{18}$$

The sum of torques around center of the mass of rider and handlebar is:

$$\begin{aligned}&\sum M_o = I\alpha\\&\Rightarrow -(H_L+H_R)l\cos\theta_p+(P_L+P_R)l\sin\theta_p+(C_L+C_R)=I_p\ddot{\theta}_p\end{aligned} \tag{19}$$

Therefore, using (15) and linear transformation in (11) and (12) the applied torque to the rider and handlebar with motor parameters equals:

$$(C_L+C_R)=-\frac{2k_mk_e}{Rr}\dot{x}+\frac{2k_m}{R}V_a \tag{20}$$

Substituting (20) into (19), we get:

$$-(H_L+H_R)lcos\theta_p+(P_L+P_R)lsin\theta_p+\left(-\frac{2k_mk_e}{Rr}\dot{x}+\frac{2k_m}{R}V_a\right)=I_p\ddot{\theta}_p \tag{21}$$

Multiplying (18) by -1 and substituting it into (21) we get:

$$I_p\ddot{\theta}_p-\frac{2k_mk_e}{Rr}\dot{x}+\frac{2k_m}{R}V_a+M_pl^2\ddot{\theta}_p+M_pglsin\theta_p=-M_pl\ddot{x}cos\theta_p \tag{22}$$

To eliminate $(H_L+H_R)$ from the dynamic model of motor, (17) is substituted in (15) and we have:

$$2\left(M_w+\frac{I_w}{r^2}\right)\ddot{x}=-\frac{2k_mk_e}{Rr^2}\dot{x}+\frac{2k_m}{Rr}V_a-M_pl\ddot{\theta}_pcos\theta_p+M_pl\dot{\theta}_p^2sin\theta_p-M_p\ddot{x} \tag{23}$$

Equations (22) and (23) give the nonlinear motion equation of the system:

$$\left(M_pl^2+I_p\right)\ddot{\theta}_p-\frac{2k_mk_e}{Rr}\dot{x}+\frac{2k_m}{R}V_a+M_pglsin\theta_p=-M_pl\ddot{x}cos\theta_p \tag{24}$$

$$\frac{2k_m}{Rr}V_a=\left(2M_w+\frac{2I_w}{r^2}+M_p\right)\ddot{x}+\frac{2k_mk_e}{Rr^2}\dot{x}+M_pl\ddot{\theta}_pcos\theta_p-M_pl\dot{\theta}_p^2sin\theta_p \tag{25}$$

In order to obtain state space model of the system, we should linearize the above equations. Suppose $\theta_p=\pi+\phi$ in which, $\phi$ is a small angular deviation from the vertical axis, so we have:

$$cos\theta_p=-1,\quad sin\theta_p=-\phi,\quad \left(\frac{d\theta_p}{dt}\right)^2=0 \tag{26}$$

The linearized state space model of the system becomes:

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{\phi}\\ \ddot{\phi}\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & \dfrac{2k_mk_e\left(M_plr-I_p-M_pl^2\right)}{Rr^2\alpha} & \dfrac{M_p^2gl^2}{\alpha} & 0\\ 0 & 0 & 0 & 1\\ 0 & \dfrac{2k_mk_e\left(r\beta-M_pl\right)}{Rr^2\alpha} & \dfrac{M_pgl\beta}{\alpha} & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \phi\\ \dot{\phi}\end{bmatrix}+\begin{bmatrix}0\\ \dfrac{2k_m\left(-M_plr+I_p+M_pl^2\right)}{Rr\alpha}\\ 0\\ \dfrac{2k_m\left(-r\beta+M_pl\right)}{Rr\alpha}\end{bmatrix}V_a \tag{27}$$

In which:

$$\begin{aligned}\beta&=\left(2M_w+\frac{2I_w}{r^2}+M_p\right)\\ \alpha&=\left[I_p\beta+2M_pl^2\left(M_w+\frac{I_w}{r^2}\right)\right]\end{aligned} \tag{28}$$

## III. CONTROL OF SEGWAY

In all control systems, the purpose is to reach a desired response (output or state). Measuring the output or states of the system and giving the control input to the system is the base of control theory. According to various control methods, the control input is calculated in many different ways.

Various methods have been developed in control theory and each are efficient in a particular area. We have used optimal control theory in order to control the Segway PT. In this system there are four states including position of Segway, vertical deviation angle and both their derivatives. Measuring these states by gyroscopic and optical sensors and calculating the control input, we could control the system [3], [6], [10].

### *A. Optimal Control*

Optimal control is one of the practical and widespread methods in control engineering. The objective is to minimize a weighted function of states and inputs, in order to reach the required performance of the control system [10].

The state space model of the system is considered as:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \\ z &= Gx + Hu \end{aligned} \tag{29}$$

This consists of two separate outputs:

1. Measured outputs $y(t) \in \Re^k$: measures and available signals for control.

2. Control outputs $z(t) \in \Re^l$: signals to be controlled.

### *B. Optimal Regulator*

The optimal control problem is actually finding a controller, which minimizes the following functional:

$$J_{LQR} = \int_0^\infty z'(t)Qz(t) + \rho u'(t)Ru(t)dt \tag{30}$$

R and Q are positive definite symmetric matrices.

If all the states are available, control problem becomes an optimal tracking problem with state feedback. This way all states are measured and used in control law. Control input and controller gain matrix equal:

$$\begin{aligned} u &= -Kx \\ K &= (H'QH + \rho R)^{-1}(B'P + H'QG) \end{aligned} \tag{31}$$

Matrix P is a unique positive definite solution of algebraic Riccati equation:

$$A'P + PA + G'QG - (PB + G'QH)(H'QH + \rho R)^{-1}(B'P + H'QG) = 0 \tag{32}$$

### *C. Tracking Optimal Control*

In many applications, the objective is to make the output reach a desired point, in this we need to focus on tracking. In this case, if r is the reference, the states and inputs are converged to $x^*$ and $u^*$ in a way that:

$$\begin{aligned} Ax^* + Bu^* &= 0 \\ r &= Gx^* + Hu^* \end{aligned} \tag{33}$$

This equation guarantees the convergence of the output to the reference input when states and control input converge to $x^*$ and $u^*$. In addition it is understood from the above equation that $x^*$ and $u^*$ is the equilibrium point of the system. Solving two equations, we get:

$$\begin{aligned} x^* &= Fr \\ u^* &= Nr \end{aligned} \tag{34}$$

In which:

$$\begin{aligned} &\begin{bmatrix} A & B \\ G & H \end{bmatrix} \begin{bmatrix} x^* \\ u^* \end{bmatrix} = \begin{bmatrix} 0 \\ r \end{bmatrix} \\ &\Rightarrow \begin{bmatrix} x^* \\ u^* \end{bmatrix} = \begin{bmatrix} A & B \\ G & H \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ r \end{bmatrix} \end{aligned} \tag{35}$$

It should be noted that if number of inputs were less than number of outputs, above equations would have no solution and if number of inputs were more than number of outputs, we would have multiple solutions. When output convergence to a reference is desired, control law becomes:

$$u = -K(x - x^*) + u^* = -Kx + (KF + N)r \tag{36}$$

K is the optimal regulator gain of the system. Fig. 4 shows the block diagram of the optimal tracking state feedback controller.

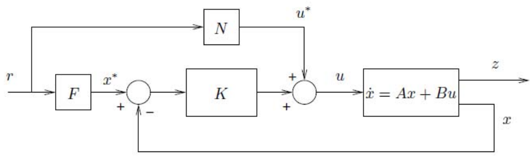


Fig. 4. Block Diagram of the Optimal Tracking State Feedback controller

The final closed-loop equations of systems are:

$$\begin{aligned} \dot{x} &= Ax + Bu = (A - BK)x + B(KF + N)r \\ z &= Gx + Hu = (G - HK)x + H(KF + N)r \end{aligned} \tag{37}$$

## IV. Simulation Results

The nonlinear system of Segway PT is simulated in MATLAB/Simulink. Fig. 5 shows the Simulink model of the system.

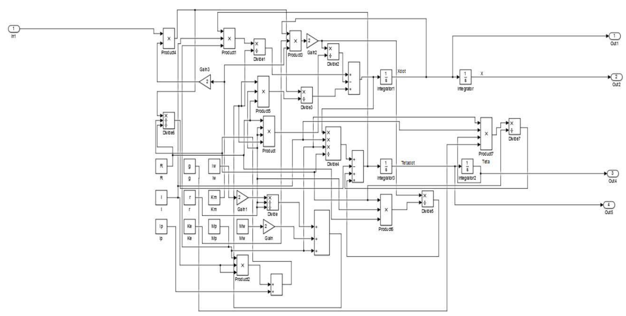

Fig. 5. Simulink Model of The System

The optimal controller is applied to the system and results are depicted in fig. 6. The control objectives are to maintain the balance of scooter while it tracks reference input. The results show great performance of optimal controller. The advantages of the proposed controller to other methods are desired speed in response and great performance.

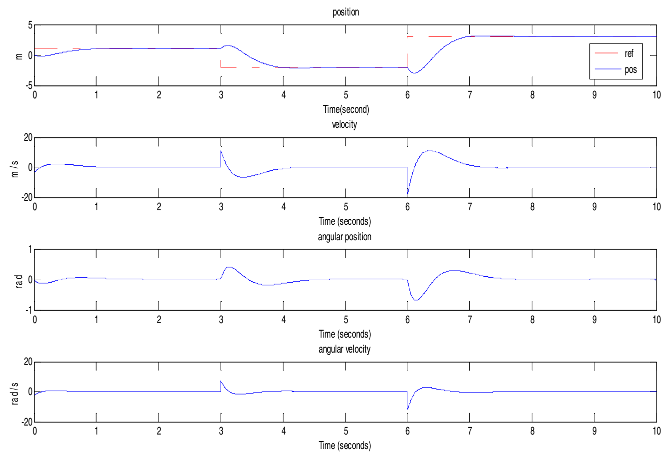


Fig. 6. System performance with tracking controller

## V. CONCLUSION

The proposed optimal controller for the nonlinear system of Segway PT resulted in a great performance. The controller showed the balance between speed response and control cost. To achieve faster response results in higher cost and vice versa. Decreasing the control cost leads to a slower response. The better speed of response is obvious compared to other methods [2-3-6]. In order to analyze the effect of disturbance, the result in figure (7) indicates the rejection of disturbance applied at $t = 4$ with optimal controller.

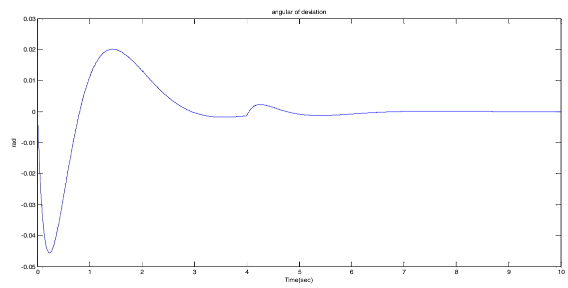


Fig. 1: Robustness of proposed controller to applied step disturbance at t=4

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