Linear Quadratic Gaussian (LQG) Control of Wind Turbines

Abdulrahman Kalbat
Electrical Engineering Department
Columbia University in the City of New York
New York, NY 10027
Email: ak3369@columbia.edu

Abstract—With the increasing deployment of wind energy technologies, innovative modern control theories, which utilize linear time invariant models, are being developed. The potential for the success of utilizing these theories is better with the availability of concepts such as the variable speed wind trubines. Such control theories are not only capable of increasing the energy capture efficiency of the wind turbines, but also reducing the number of measurements needed which in turn reduces the operation and maintenance costs associated with the system. In this paper, the general formulation of the Linear Quadratic Gaussian Control (LQG) along with the design procedures and the assumptions needed will be stated. Then, a numerical model for the Controls Advacned Research Turbine (CART) that was created and field tested by the National Renewable Energy Laboratory (NREL) will be used to simulate LQG regulator using MATLab script. At the end, the results obtained from the simulation will be presented and the limitations of the LQG controller will be discussed.

Index Terms—Linear Quadratic Gaussian, LQG, Kalman Filter, Estimation, Wind Turbine.

I. INTRODCUTION

The future plans for smart grid deployment , which mainly depend on the concept of distributed generation, make wind turbines more attractive solution, especially in regions with the high average wind speed. The control methods of wind turbines started by PI controllers which imposed many limitations and operation and maintenance costs to the system. With the advancement of modern control theory, it is now possible to use linear state space models of the wind turbines to implement control systems which are more powerful than PI controllers. These modern control methods also reduced the maintenance and operation cost by the utilization of Kalman Filters for state estimation that reduced the number of the sensors necessary to implement full state feedback controllers.

In section II of this paper, the general formulation of the Linear Quadratic Gaussian Control (LQG) along with the design procedures and the assumptions needed will be stated. In section III, National Renewable Energy Laboratory's (NREL) Controls Advacned Research Turbine (CART) features and capabilities will be presented. In section IV, a numerical model for CART that was created and field tested by NREL will be used to simulate LQG regulator using MATLab script. In section V, the results obtained from the simulation will be

presented and the limitations of the LQG controller will be discussed.

II. LQG GENERAL FORM

Linear Quadratic Gaussian (LQG) design problem is rooted in optimal stochastic control theory and has many applications in the modern world which ranges from flight and missile navigation control systems, medical porocesses controllers and even nuclear power plants. It combines both concepts of Linear Quadratic Regulators (LQR) for full state feedback and Kalman Filters for state estimation. In this section, the state space equations defining the LQG regulator along with the statistical assumptions will be presented first. Then, the design steps for finding the optimal state feedback gain (K_f) and optimal state estimation gain (K_k) will be listed. Finally, the state space equations of the augmented system which closes the loop will be presented.

The state space equations of the open loop plant for a standard LQG problem is shown in 1.

$$\dot{x}(t) = Ax(t) + Bu(t) + Gw(t)$$
$$y(t) = Cx(t) + v(t) \tag{1}$$

where $x(t)$ is the state vector, $u(t)$ is the control input vector, $y(t)$ is the measured output vector, $w(t)$ and $v(t)$ are stochastic white noise processes assoctiated with the process and the measurement, respectively. The matrices A(state matrix), B(control input gain matrix), G(plant noise gain matrix), C(measured state matrix) are all Linear Time Invariant (LTI). As shown in 2, both $w(t)$ and $v(t)$ are assumed to be white gaussian noises with zero mean and the expected values of the initial values of the states $x(0)$ are assumed to be equal to $\hat{x}_o$. From 3, $w(t)$ and $v(t)$ have covariance matrices of W and V, respectively, and the cross covariance R_{12} is assumed to be zero ($w(t)$ and $v(t)$ are uncorrelated) in order to simplify many expressions and derivations. Finally, the initial values of the states $x(0)$ are assumed to be uncorrelated with both $w(t)$ and $v(t)$, as shown in 4.

$$E[x(0)] = \hat{x}_o$$
$$E[w(t)] = 0 \tag{2}$$
$$E[v(t)] = 0$$

$$E[w(t)w(\tau)^T] = \begin{cases} W, & \text{if } t = \tau \\ 0, & \text{if } t \neq \tau \end{cases}$$

$$E[v(t)v(\tau)^T] = \begin{cases} V, & \text{if } t = \tau \\ 0, & \text{if } t \neq \tau \end{cases} \tag{3}$$

$$E[w(t)v(\tau)^T] = \begin{cases} R_{12}, & \text{if } t = \tau \\ 0, & \text{if } t \neq \tau \end{cases}$$

$$\begin{aligned} E[x(0)w(t)^T] &= 0 \\ E[x(0)v(t)^T] &= 0 \end{aligned} \tag{4}$$

In real world control design problems, it is rarely possible to have access to all states of the system which are needed for full state feedback. Instead, access is only possible to specific measured outputs of the system. If these measurements carry enough information about the states of the system, then a state observer using Kalman Filter could be implemented to estimate all states of the system. This observer is capable of rejecting disturbances of the system by acting as a low pass filter. The main inputs to the observer are the control input $(u(t))$ and the system output $(y(t))$. The state space equations of the Kalman Filter are shown in 5. It should be noticed that it uses the same state space matrices (A, B and C) as the main system and the estimated states $(\hat{x}(t))$ are used as the system states [1].

$$\begin{aligned} \dot{\hat{x}}(t) &= (A - K_f C)\hat{x}(t) + Bu(t) + K_f y(t) \\ \hat{y}(t) &= C\hat{x}(t) \end{aligned} \tag{5}$$

From Fig.1, it should be noticed that LQG is formed by connecting the system and the Kalman Filter through the optimal state estimation gain (K_k) and then creating full state feedback by using the estimated states $(\hat{x}(t))$ which passed through the optimal feedback gain (K_f). Because of the stochastic seperation principle, the previoulsy mentioned gain could be designed individually.

The design process starts with checking controllability and observability of the pairs (A, B) and (A, C), respectively, as shown in 6. These criterias are necessary for the existence of the solutions for the equations used to find the optimal gains. Then, the optimal state estimation gain (K_k) is calculated as shown in 7c, where P_k is a positive semi-definite mtarix and the solution of the Filter Algebraic Riccati Equation (FARE) shown in 7b. This solution ensures a minimum value of the cost function shown in 7a. After that, the optimal state feedback gain (K_f) is calculated as shown in 8c, where P_f is a positive semi-definite matrix and the solution of the Control Algebraic Riccati Equation (CARE) shown in 8b. This solution ensures a minimum value of the cost function shown in 8a.

Unlike FARE, which requires only noise covariances W and V, CARE requires two weighing matrices: Q_f (symmetric positive semi-definite) and R_f (symmetric positive definite). These weighing matrices provide a means to trade-off opposing objectives: state regulation and control usage. They could be selected based on trial and error approach with some

engineering sense, or by using the method shown in 9 or by using Bryson's Rule shown in 10. Bryson's Rule related the reciprocal of the maximum squared values of the states with Q_f and the reciprocal of the maximum squared values of the control inputs with R_f.

After finding the optimal gains, the closed loop system could be created by augmenting the system with Kalman Filter resulting in the new model shown in 11. The augmented system model which include the dynamics of the system and the state estimation error $e(t)$ [2].

Step 1: Optimal gain existance criteria:

$$\begin{aligned} &(A, B) \; is \; Controllable \\ &(A, C) \; is \; Observable \end{aligned} \tag{6}$$

Step 2: Optimal State Estimation Gain Calculation:

$$J_k = E\left\{(x - \hat{x})^T(x - \hat{x})\right\} \tag{7a}$$

$$AP_k + P_k A^T + GWG^T - P_k C^T V^{-1} C P_k = 0 \tag{7b}$$

$$K_k = P_k C^T V^{-1} \tag{7c}$$

Step 3: Optimal State Feedback Gain Calculation and Weighing Matrices Selection:

$$J_f = \int_0^T (z^T Q_f z + u^T R_f u)dt \tag{8a}$$

$$A^T P_f + P_f A - P_f B R_f^{-1} B^T P_f + Q_f = 0 \tag{8b}$$

$$K = R_f^{-1} B^T P_f \tag{8c}$$

$$\begin{aligned} Q &= C^T C \\ R &= \rho I \end{aligned} \tag{9}$$

$$\begin{aligned} Q_{ii} &= \frac{1}{Max \; (x_{ii}^2)} \\ R_{ii} &= \frac{1}{Max \; (u_{ii}^2)} \end{aligned} \tag{10}$$

Step 4: Linear Quadratic Gaussian Regulator by combining Optimal State Estimation and Optimal State Feedback:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{e}(t) \end{bmatrix} = \begin{bmatrix} A - BK_f & BK_f \\ 0 & A - K_k C \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} G & 0 \\ G & -K_k \end{bmatrix} \begin{bmatrix} w(t) \\ v(t) \end{bmatrix} \tag{11}$$

$$y(t) = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} w(t) \\ v(t) \end{bmatrix}$$
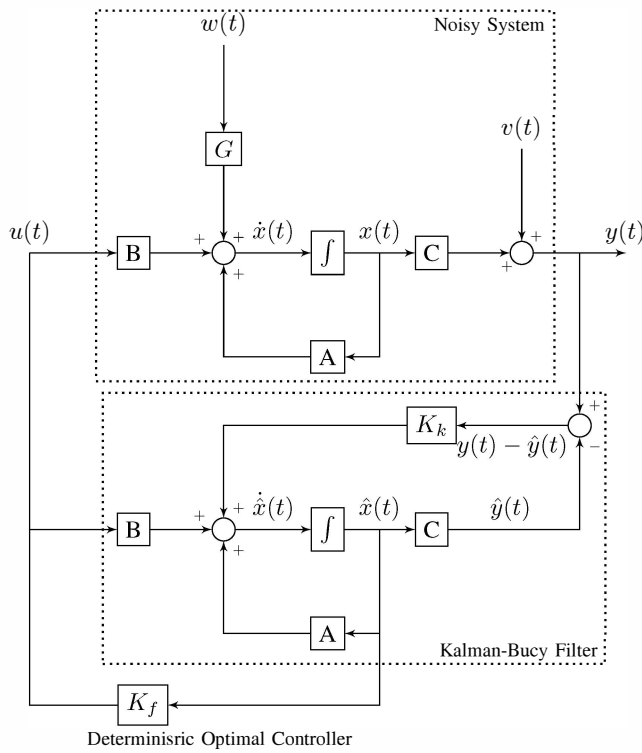
Fig. 1. LQG regulator block diagram

III. Controls Advacned Research Turbine (CART)

The numerical example that will be discussed in the next section is based on a linear model of the Controls Advacned Research Turbine (CART) which is used by the National Wind Technology Center (NWTC) that is operated by the National Renewable Energy Laboratory (NREL) and located at Boulder, Colorado. Researchers at NWTC tackles wind industry engineering challenges ranging from atmospheric fluid mechanics and aerodynamics, dynamics, structures, fatigue, power systems and electronics and wind turbine engineering applications [3].

CART is a modified Westinghouse WGT-600, 600-KW, 2-bladed, horizontal-axis research wind turbine capable of operating in constant speed or variable speed mode [4]. It is used by the researchers at NREL's NWTC as a test bed for exploring potential control innovations and field test advanced control systems. These control designs could be implemented by writing an ANSI-C based program on a dedicated personal computer that is connected to the turbine sensors and actuators. The turbine is instrumneted with more than 80 sensors with measurements being recorded at a rate of 100 Hz. These sensors are used to record the dynamic state of the turbine and they fall in one of three categories: Performance (torque and power sensors), loads (strain gauges, accelerometers and position sensors) or meteorological (wind speed, direction, temperature and pressure) [5].

One of the unique capabilities of CART is that beside the conventional collective pitching technique, it is possible to implement the individual pitching technique. In the collective pitching technique each of the blades are picthed by the same amount and this controlls the symmetric loading on the rotor. But in the individual pitching technique, each blade is individually pitched and this offers a control over both the symmetric and asymmetric loading on the rotor [6]. Because of the access to all sensors data and actuators and the flexible capabilities of CART, researchers at NWTC could implement infinite combinations of control systems.

IV. Numerical Example Using MATLAB

In [7], several Linear Time Invariant (LTI) state space models of CART were created which varied in their complexities between 1-state model to 9-state model. In this paper, the 3-state model was considered because of its simplicity and because this model is both controllable and observable (minimal) which are both conditions needed for designing LQG controllers. The numerical model shown below was created for CART by NWTC after linearizing the motion equations of the turbine's considered states at a control design point of 18 m/s for wind speed, 12 degress for rotor collective pitch and 42 RPM for rotor speed. The main objective of the controller is to operate the machine as a variable speed wind turbine in region 3 by applying constant torque to the generator by maintaining a constant rotor speed through the collective rotor blade pitching.

The control input u(t) of the considered model is the collective blade pitch angle and the states are the rotor speed $x_1(t)$, drive train (shaft) torsional spring force $x_2(t)$ and the generator speed $x_3(t)$. The disturbances are the turbine system noise $w(t)$ and the measurement noise $v(t)$. For the sake of simplicity, both turbine and measurement noises were assumed to have the same magnitude as the wind but in the case of turbine system noise, it was scaled by the matrix G. The non-zero first row of the control input gain B indicates that the control input (rotor collective pitch) affects the system by changing the rotor speed which affetcs the other states through the coupling caused by the state gain matrix A. Also, from matrix C, it should be noticed that the only measurement taken is the generator speed.

The simualtion of LQG controller for CART was done using MATLab by following the design steps shown in section II and by using the numerical model's matrices shown below. The design started by checking the controllability and the observability of the pairs (A, B) and (A, C), respectively. Then, the optimal state estimation gain (K_k) was calculated using the noise covariances shown below. Sizing the optimal state feedback gain was found using a trial and error approach by changing the weighing matrices (Q_f and R_f). The best response of the generator speed at which it stabilized at steady state when perturbed by step changes in wind speed was found at the values of the weighing matrices shown below. If the maximum limits of all states and control inputs were available, which was not the case for CART, then Bryson's rule could be used to initialize the weighting matrices as shown below [8]. After that, the closed loop system of the LQG regulator

was created by augmenting the turbine system with the state estimator. The inputs to the closed loop regulating system are turbine system noise $w(t)$ and the measurement noise $v(t)$ which were generated as a normally distributed random number by MATLab to represent the white gaussian noise. The peroformance of the closed loop LQG system will be discussed next.

MATLab Simulation inputs for CART

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \quad \text{where} \quad \begin{cases} x_1(t) : \text{is rotor speed} \\ x_2(t) : \text{is drive train torsion} \\ x_3(t) : \text{is generator speed} \end{cases}$$

$$u(t) = \begin{bmatrix} w(t) \\ v(t) \end{bmatrix} \quad \text{where} \quad \begin{cases} w(t) : \text{is turbine system noise} \\ v(t) : \text{is measurement noise} \end{cases}$$

$$A = \begin{bmatrix} -1.4454\text{x}10^{-1} & -3.1078\text{x}10^{-6} & 0.0 \\ 2.6910\text{x}10^{7} & 0.0 & -2.6910\text{x}10^{7} \\ 0.0 & 1.5601\text{x}10^{-5} & 0.0 \end{bmatrix} \quad (12)$$

$$B = \begin{bmatrix} -3.4559 & 0.0 & 0.0 \end{bmatrix}^T$$

$$C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$G = \begin{bmatrix} 7.8938\text{x}10^{-2} & 0.0 & 0.0 \end{bmatrix}^T$$

$$W = E[w(t)w(\tau)^T] = 0.1 \text{ (Turbine system noise covariance)}$$

$$V = E[v(t)v(\tau)^T] = 0.1 \text{ (Measurement noise covariance)}$$

Results

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1\text{x}10^{-13} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R = 1$$

$$\quad (13)$$

$$K_k = \begin{bmatrix} 7.6282\text{x}10^{-3} & 1.2663\text{x}10^{2} & 6.2859\text{x}10^{-2} \end{bmatrix}^T$$

$$K_f = \begin{bmatrix} -2.0336 & -2.1225\text{x}10^{-7} & 6.6055\text{x}10^{-1} \end{bmatrix}$$

V. RESULTS AND DISCUSSION

Fig. 2 shows the generator speed and the pitch angle when the wind speed is varying between 14 m/s to 20 m/s. As the wind speed is increasing, the pitch angle is also increasing to keep the generator speed at a constnt value of 42 RPM. From the figure, it is clear that the generator is operating at 42 RPM only at 18 m/s and when the pitch angle is 12 degrees, the control design point. So, LQG regulator is not able to keep the generator speed at 42 RPM even when the wind speed is varying. This example clearly shows the robustness problem of LQG controllers. According to [9] , control system robustness is defined as the ability to maintain satisfactory stability or performance characteristics in the presence of all conceivable system parameter variations. In other words,a robust control system works not only for the linear system which serves as the plant model but it also works for the real physical system with minor performance degradation [10]. Another
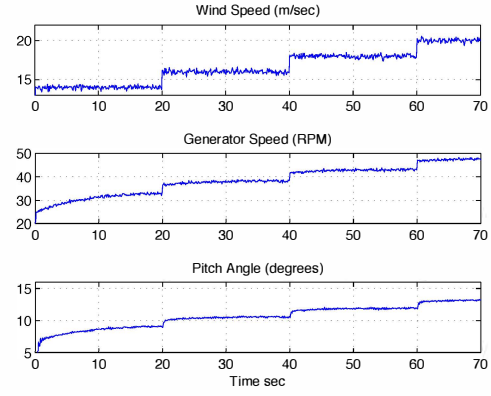

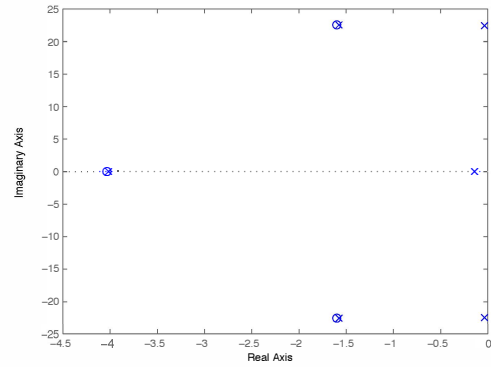Fig. 2. Wind turbine's response to different wind speeds

Fig. 3. Poles/Zeros plot of the closed loop system

reason might be models' simplicity and uncertainty because of the neglected high frequecny dynamics of the turbine. In order to improve the robustness of the LQG regulator, one could go further and design a Loop Transfer Recovery (LTR) which recovers the robustness of Linear Quadratic Regulators (LQR) and Kalman Filters.

Fig. 3 shows the pole/zero map of the closed loop system. Since all poles lies in the left half plane, then the closed loop system is stable. From Fig. 4, one should notice how the LQG regulator successfully reduced the effect of the system white gaussian noise $w(t)$ (input 1) on the closed loop system. Another observation is that the LQG at high frequencies the effect of the measurement white gaussian noise $v(t)$ is higher in the closed loop system than in open loop system. One explanation might be because the $v(t)$ is being multiplied by the optimal state estimation gain K_k in the final augmented model shown in 11.

VI. CONCLUSION

In this paper, the general formulation of the Linear Quadratic Gaussian Control (LQG) along with the design procedures and the assumptions needed were stated. Then, a numerical model for the Controls Advacned Research Turbine (CART) was used to simulate LQG controller using MATLab script. Then, the peroformance of the LQG regulator was evaluated by plotting the time response of the system and the
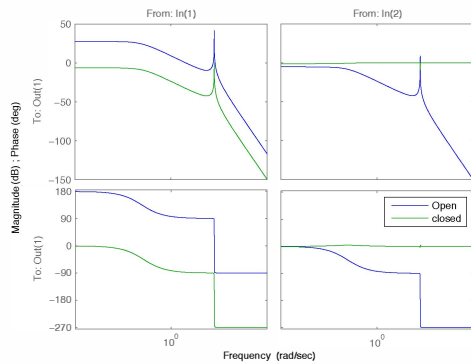
Fig. 4. bode plot of the open and closed loop systems

pole/zero map of the closed loop system. Finally, the bode plot of both the open loop and the closed loop systems were compared.

REFERENCES

[1] C.-H. Won and K. Gunaratne, "Performance study of lqg, mcv, and risk-sensitive control methods for satellite structure control," in *American Control Conference, 2002. Proceedings of the 2002*, vol. 3, pp. 2481–2486 vol.3.

[2] E. E. Ostertag, "Optimal stochastic control," in *Mono- and Multivariable Control and Estimation*, ser. Mathematical Engineering. Springer Berlin Heidelberg, 2011, vol. 2, pp. 259–266. [Online]. Available: http://dx.doi.org/10.1007/978-3-642-13734-1-5

[3] National wind technology center (nwtc). [Online]. Available: http://www.nrel.gov/wind/pdfs/53405.pdf

[4] K. Johnson, L. Fingersh, and A. Wright, "Controls advanced research turbine: Lessons learned during advanced controls testing," National Renewable Energy Laboratory, NREL/TP 500-38130, June 2005.

[5] L. Fingersh and K. Johnson, "Controls advanced research turbine (cart) commissioning and baseline data collection," National Renewable Energy Laboratory, NREL/TP 500-32879, October 2002.

[6] K. A. Stol, W. Zhao, and A. D. Wright, "Individual blade pitch control for the controls advanced research turbine (cart)," *Journal of solar energy engineering*, vol. 128, no. 4, pp. 498 – 505, 2006.

[7] A. D. Wright, "Modern control design for flexible wind turbines," National Renewable Energy Laboratory, NREL/TP 500-35816, July 2004.

[8] . Oral, L. etin, and E. Uyar, "A novel method on selection of q and r matrices in the theory of optimal control," *International Journal of Systems Control*, vol. 1, no. 2, pp. 84–92, 2010.

[9] R. Stengel and L. Ryan, "Stochastic robustness of linear time-invariant control systems," *Automatic Control, IEEE Transactions on*, vol. 36, no. 1, pp. 82–87, 1991.

[10] U. Mackenroth, *Control Systems: Basic Definitions and Concepts*. Springer Berlin Heidelberg, 2004, pp. 4–5.