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Research on Control Method of Two-wheeled Self-balancing Robot

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Abstract—**Based on Newton dynamics mechanics theory make a study of two-wheeled self-balancing robot, a detailed mathematical model of the modeling process is provided, and then, using the reasonable method, a linear state-space equations is built up. After that, the LQR controller and state-feedback controller based on pole placement theory are both designed. After a number of simulation experiments, we get the best closed-loop poles and Q, R matrix, both of which have good simulation curves at the same disturbance force. The results of experiments prove that both of them have good dynamic performance and robustness, which also prove the system modeling and controller design are reasonable and effective via these methods. The curves from LQR controller have a better dynamic performance compare with pole placement state-feedback controller.**

Keywords-LQR;Pole Placement;Two-wheeled Self-balancing Robot; Control Theory

I. INTRODUCTION

The two-wheeled self-balancing robot [1] system possesses the characteristics of high order, non-linear, coupled, multiple-variables, and instability, as it is considered as a typical research target by many modern control theory researchers [2], and a lot of the abstract control concepts such as system stabilities, robustness, controllability, system anti-interference property etc can be displayed via the two-wheeled self-balancing robot system experiments. Therefore, the robot has become an essential and classical experiment facility to prove various kinds of control theory and control method. The two-wheeled self-balancing robot has the characteristics of sports flexible and simple structure, working well in a small space, especially in the poor working conditions and complexity task places, such as space exploration, topographic investigation, the field of transportation of dangerous goods etc. At present, research on the two self-balancing robot is in its infancy, so it has an important prospects for theoretical and practical significance to carry out the study of two-wheeled self-balance robot ,which can also improve our level of scientific research and extend the application of the robot in this field.

This paper makes a study of the GBOT1001 two-wheeled self-balancing robot produced by Googol Technology (Shenzhen) Limited, establishing the mathematical model of this system, suing LQR and pole placement control method to control the position and speed of the robot, at the same time we get a good control effect in MATLAB.

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II. DESCRIPTION OF THE SYSTEM

A. Structural Analysis of The Robot System

We can get a linear model with these assumptions [3]:

- The robot is a rigid-body and does not distort during moving.
- The left hand wheel and the ones for the right-hand wheel are completely analogous.
- Cornering forces are considered negligible.
- The friction is neglected during analyzed.
- Since the time constant of the electric motors is small compared to the system’s time constants, the motors dynamics have been neglected in the model.

The design of two-wheeled self-balancing robot is based on mobile single inverted pendulum, with two wheels. The robot is composed of a chassis carrying a DC motor coupled to a planetary gearbox for each wheel, the DSP board used to implement the controller, the power amplifiers for the motors, the necessary sensors to measure the robot’s states, the receiver for the radio control unit as well as a vertical bar [1]. Figure 1 is the schematic diagram of the robot system.

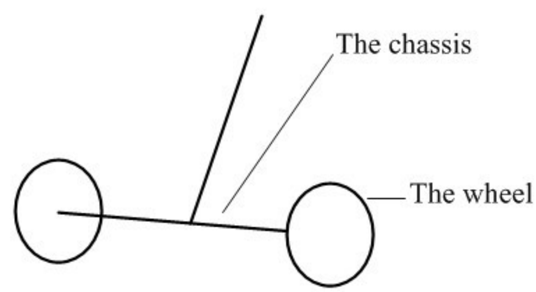

Figure 1. Schematic diagram of the robot system .

B. Dynamics Model of The System

Reference [1], Figure 2 and Figure 3 show the free body diagram of the robot. The linear movement of the chassis is characterized by the position x_r and the speed, It $\dot{x}_r$ is able to rotate around the axis (pitch), a movement described by the angle and θ_p the corresponding angular velocity $\dot{\theta}_p$.

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Additionally, the vehicle can rotate around its vertical axis (yaw) with the associated angle δ and angular velocity $\dot{\delta}$. J_{RL} and J_{RR} are the moment of inertia of the rotating masses with respect to the *z* axis. M_r is the mass of rotating masses connected to the left and right wheel, M_p is the mass of the chassis, J_p is the moment of inertia of the chassis with respect to *z* axis, J_δ is the moment of inertia of the chassis with respect to the *y* axis, *R* is the radius of the wheel, *L* is the distance between the *z* axis and the center of

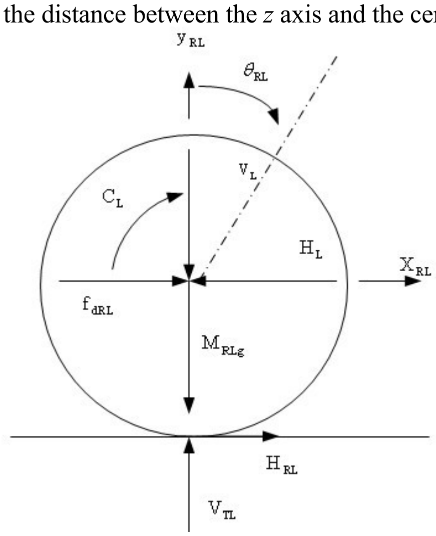

Figure 2. The left hand wheel diagram of the robot .

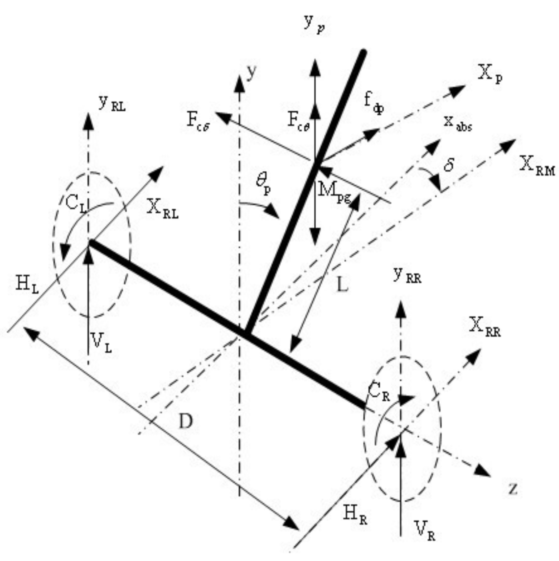

Figure 3. The chassis diagram of the robot .

gravity of the chassis. *D* is the lateral distance between the contact patches of the wheels. y_r is the shift position of the wheel with respect to the y axis. x_p is the shift position of the chassis with respect to the x axis.

For the left hand wheel:

$$\ddot{x}_{RL} M_r = H_{TL} - H_L + (f_{dRL} + f_{dRR}) \tag{1}$$

$$\ddot{y}_{RL} M_r = V_{TL} - V_L - M_r g \tag{2}$$

$$\ddot{\theta}_{RL} J_{RL} = C_L - H_{TL} R \tag{3}$$

$$\dot{x}_{RL} = R \dot{\theta}_{RL} \tag{4}$$

$$\dot{y}_p = -\dot{\theta}_p L \sin\theta_p \tag{5}$$

$$\dot{x}_p = \dot{\theta}_p L \cos\theta_p + \frac{\dot{x}_{RL} + \dot{x}_{RR}}{2} \tag{6}$$

$$\dot{\delta} = \frac{\dot{x}_{RL} - \dot{x}_{RR}}{2f} \tag{7}$$

For the chassis：

$$\dot{x}_p M_p = (H_R + H_L) + f_{dp} \tag{8}$$

$$\ddot{y}_p M_P = V_R + V_L - M_p g + F_{C\theta} \tag{9}$$

$$\ddot{\theta}_p J_p = (V_R + V_L) L \sin\theta_p - (H_R + H_L) L \cos\theta_p - (C_L + C_R) \tag{10}$$

$$\ddot{\delta} J_\delta = (H_L - H_R)\frac{D}{2} \tag{11}$$

Where $H_{TL}, H_{TR}, H_L, H_R, V_{TL}, V_{TR}, V_L, V_R$ and represent reaction forces between the different free bodies. Reference [4], modifying the equations above and then linearizing the result around the operating point ($\theta_p = 0, x_r = 0, \delta = 0$ then $\sin\theta_p = \theta_p, \cos\theta_p = 1$) the system's state-space equations can be written in matrix form as [12]:

In order to get a simpler mathematical model, the state-space equations for the vehicle can be written as two different systems: one system is the model (13) of pendulum which describes the rotation about the *z* axis; the other one is the model (14) of rotation which describes the rotation about the *y* axis.

$$\begin{bmatrix}\dot{x}_r\\ \ddot{x}_r\\ \dot{\theta}_p\\ \ddot{\theta}_p\\ \dot{\delta}\\ \ddot{\delta}\end{bmatrix} = \begin{bmatrix}0&1&0&0&0&0\\0&0&A_{23}&0&0&0\\0&0&0&1&0&0\\0&0&A_{43}&0&0&0\\0&0&0&0&0&1\\0&0&0&0&0&0\end{bmatrix}\begin{bmatrix}x_r\\ \dot{x}_r\\ \theta_p\\ \dot{\theta}_p\\ \delta\\ \dot{\delta}\end{bmatrix} + \begin{bmatrix}0&0\\B_{21}&B_{22}\\0&0\\B_{41}&B_{42}\\0&0\\B_{61}&B_{62}\end{bmatrix}\begin{bmatrix}C_L\\C_R\end{bmatrix} \tag{12}$$

$$\begin{bmatrix}\dot{x}_r\\ \ddot{x}_r\\ \dot{\theta}_p\\ \ddot{\theta}_p\end{bmatrix} = \begin{bmatrix}0&1&0&0\\0&0&A_{23}&0\\0&0&0&1\\0&0&A_{43}&0\end{bmatrix}\begin{bmatrix}x_r\\ \dot{x}_r\\ \theta_p\\ \dot{\theta}_p\end{bmatrix} + \begin{bmatrix}0\\B_2\\0\\B_4\end{bmatrix}\left[C_\theta\right] \tag{13}$$

$$\begin{bmatrix}\dot{\delta}\\ \ddot{\delta}\end{bmatrix} = \begin{bmatrix}0&1\\0&0\end{bmatrix}\begin{bmatrix}\delta\\ \dot{\delta}\end{bmatrix} + \begin{bmatrix}0\\B_6\end{bmatrix}\left[C_\delta\right] \tag{14}$$

For a premise:

$$X = \frac{1}{3}\frac{M_p(M_p+6M_r)L}{M_p+\frac{3}{2}M_r}, Y = \frac{M_p}{(M_p+\frac{3}{2}M_r)R} + \frac{1}{L},$$

where $A_{23} = g(1-\frac{4}{3}L\frac{M_p}{X})$, $A_{43} = \frac{gM_p}{X}$,

$$B_2 = (\frac{4LY}{3X} - \frac{1}{M_pL}), B_4 = -\frac{Y}{X},$$

$$B_6 = \frac{6}{(9M_r+M_p)RD}$$

III. DESIGN OF THE CONTROLLER

A. Pole Placement

Since both state feedback and output feedback change the system matrices of the closed-loop system , they can be used to change the pole positions of a system .Especially, the state feedback can be used to achieve the desired pole positions. Self-balancing robot is similar to a mobile inverted pendulum, in fact we can control its speed and rotation angle to control the robot. Using the motor to control two independent wheels so that it can move according to the given movement speed and rotation angle and keep balance. Actually, it is transformed into a signal tracking problem. Using of the system state and output feedback , we can get the state feedback matrix through the pole placement method, therefore, the left and right wheel driving torque we obtain.

Some parameters of GBOT1001 robot produced by Googol Technology Limited are as follows: M_p=21kg, M_r=0.420kg, R=0.106m, L=0.3m, g=9.8 m/s^2 . so we get:

$$\begin{bmatrix}\dot{x}_r\\ \ddot{x}_r\\ \dot{\theta}_p\\ \ddot{\theta}_p\end{bmatrix} = \begin{bmatrix}0&1&0&0\\0&0&-26.25&0\\0&0&0&1\\0&0&90.125&0\end{bmatrix}\begin{bmatrix}x_r\\ \dot{x}_r\\ \theta_p\\ \dot{\theta}_p\end{bmatrix} + \begin{bmatrix}0\\2.0296\\0\\-5.4708\end{bmatrix}\left[C_\theta\right] \tag{15}$$

$$\begin{bmatrix}\dot{\delta}\\ \ddot{\delta}\end{bmatrix} = \begin{bmatrix}0&1\\0&0\end{bmatrix}\begin{bmatrix}\delta\\ \dot{\delta}\end{bmatrix} + \begin{bmatrix}0\\5.1917\end{bmatrix}\left[C_\delta\right] \tag{16}$$

At first, examine the controllablility of the open-loop system. After calculating the system is controllable and the poles of the state feedback system can be assigned arbitrarily. After a number of simulation experiments, the appropriate closed-loop pole is:

$$P = [-1.5-j;-1.5+j;-3.5-4*j;-3.5+4*j],$$

Then the state feedback matrix is:

$$K = [-2.3357 \quad -2.7348 \quad -26.9368 \quad -2.8425].$$

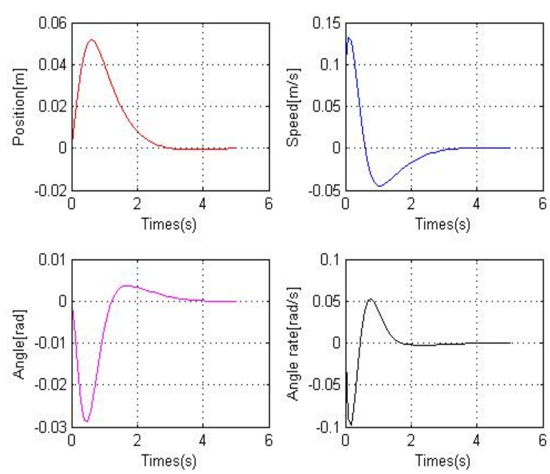

Figure 4. Pole placement of the system and associated response to a disturbance .

When a disturbance $x_0 = \begin{bmatrix} 0 & 0.1 & 0 & 0 \end{bmatrix}$ is given to the system, the response curves are followed as Figure 4.

Figure 4 shows that: when a disturbance is given to the system, using the method of pole placement, the robot's position, speed, angular, and angular velocity return to the origin point after 3 seconds more or less, and the system is stable all the time with a good dynamic performance and robustness, which also proves the method of pole placement is effective.

B. LQR

LQR [5] is a method in modern control theory that uses state-space approach to analyze such a system. In this section, the basic design process for LQR will be illuminated. From the state equation of the controlled system (15)-(16) and the initial condition, the performance index is given by:

$$J = \frac{1}{2}\int_0^{\infty}(x^T Q x + u^T R u)dt \,,$$

Where Q and R are the weight matrices, Q is required to be positive definite or positive semi-definite symmetry matrix, R is required to be positive definite symmetry matrix. The value of the elements in Q and R is related to its contribution to the cost function J. According to the optimal control law, its optimality is totally depended on the selection of Q and R. However, there is no resolving method to choose these two matrices. The widespread method used to choose Q and R is simulation and trial. Generally, Q and R should be both diagonal matrix. If we hope smaller input, larger R is needed [6]; if we hope the input of a state is smaller, the element in the corresponding column of Q needs to be larger. We choose the weighting matrices $Q = [1000,0,0,0;0,0,0,0;0,0,1000,0;0,0,0,0]$ and R=1 after a lot of simulations. Then, we use MATLAB function K=lqr (A, B, Q, R) to solve the optimal problem. Run K=lqr (A, B, Q, R) in MATLAB, we can get:

$$K = \begin{bmatrix} -31.6228 & -24.4088 & -84.1595 & -14.2010 \end{bmatrix}$$

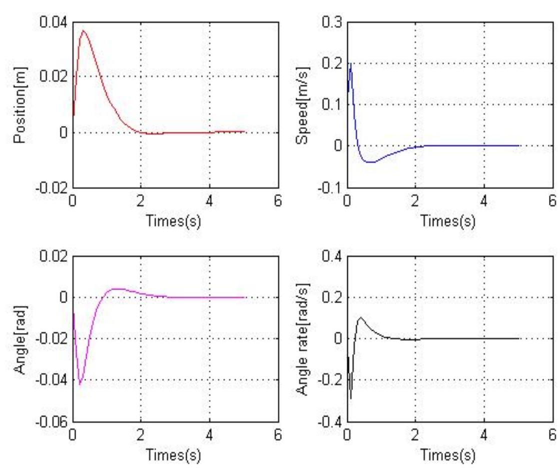

Figure 5. LQR of the system and associated response to a disturbance .

Just like the experiment we do before, a disturbance $x_0 = \begin{bmatrix} 0 & 0.1 & 0 & 0 \end{bmatrix}$ is given to the system, the response curves are followed as Figure 5.

As shown in Figure 5, the system are successfully stabilized as required with a good dynamic performance. The robot's position, speed, angular, and angular velocity return to the origin point almost no more than 2s with the overshoot a little high compare with using the method of pole placement.

IV. CONCLUSION

In this paper, based on the construction of the structure model for the two-wheeled self-balancing robot, the systematic mathematical model is worked out according to the dynamic mechanics theory. And then, a controller based on pole placement and optimal controller design LQR based on state-space method are adopted, both of which coordinate quite well the robust stability and the speediness of system. Validity and rationality of the system modeling and the controller designing are verified through the performance experiments of the prototype. The simulation results prove that the two control method both have good performance in maintaining stability, yielding short settling time and also low overshoot. The lowest overshoot was achieved by the pole placement controller, and the settling time was nearly comparable to that achieved by the LQR controller.

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