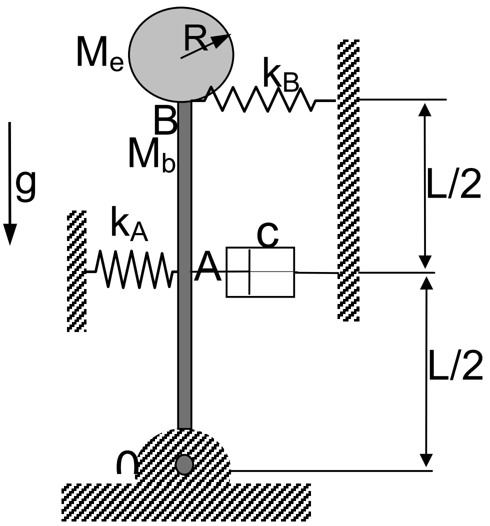
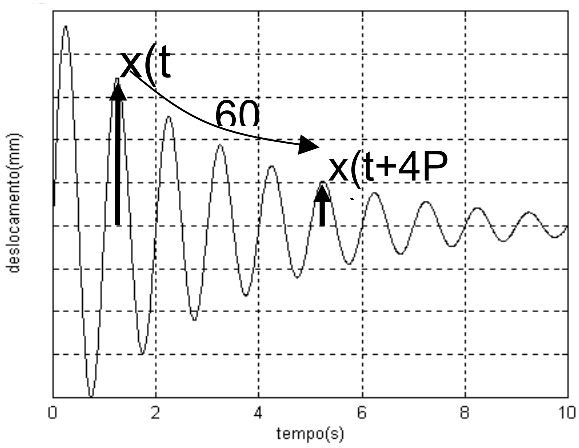
Determinar a constante de amortecimento c para que, uma vez deslocada da posição de equilíbrio, o corpo composto (barra uhd, esfera) tenha um decaimento de 60% em 4 ciclos.

**I – Pela equação diferencial;**
**II – Por massa e rigidez efetivas.**



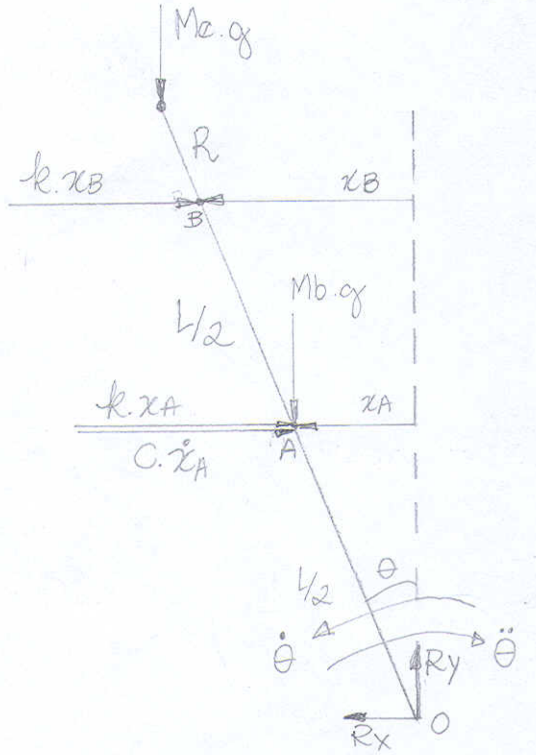




$k_A = k_B = k = 2000$N/m
Esfera: massa $M_e = 1$ kg; raio R = 5 cm;
$J_{CM} = 2/5$.massa.Raio²
g = 9,80 m/s$^2$

Barra: uniforme, homogênea e delgada
massa $M_b = 9$ kg ; comprimento = L =1 m;
$J_{CM} = 1/12$.massa.compr²

I-Pela equação diferencial

DCL




$\Sigma M_O = J_O.\ddot{\theta}$

$J_O = J_{O_B} + J_{O_e}$ $\qquad J_{O_B} = J_{B_{CM}} + m.d_{zz}^2 = \frac{1}{12}.Mb.L^2 + Mb.\frac{L^2}{4}$

$J_O = 3 + 1{,}1035$

$J_O = 4{,}1035$ $\qquad J_{O_B} = \frac{1}{12}.9.1^2 + 9.\frac{1^2}{4} = \frac{9}{12} + \frac{9}{4} = \frac{9+27}{12} = \frac{36}{12} = 3$

$\theta << 1\,rad$

$\begin{cases} sen\,\theta \simeq \theta \\ \cos\theta \simeq 1 \end{cases}$ $\qquad J_{O_e} = J_{e_{CM}} + md_{zz}^2 = \frac{2}{5}.1.0{,}05^2 + 1.(0{,}05+1)^2 = 1{,}1035$

$\Sigma M_O = J_O.\ddot{\theta}$

$Me.g.(R+L).sen\,\theta + Mb.g.\frac{L}{2}.sen\,\theta - k.x_B.L.\cos\theta - k.x_A.\frac{L}{2}.\cos\theta - C.\dot{x}_A.\frac{L}{2}.\cos\theta = J_O.\ddot{\theta}$

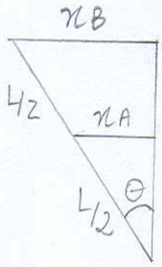
$10{,}3005.sen\,\theta + 44{,}145.sen\,\theta - 2000.x_B\cos\theta - 1000.x_A.\cos\theta - 0{,}5.C.\dot{x}_A.\theta = J_O.\ddot{\theta}$

$10{,}3005.\theta + 44{,}145.\theta - 2000.\theta - 500.\theta - 0{,}25.C.\dot{\theta} = 4{,}1035.\ddot{\theta}$

$-2445{,}55.\theta - 0{,}25.C.\dot{\theta} = 4{,}1035.\ddot{\theta}$

$4{,}1035.\ddot{\theta} + 0{,}25.C.\dot{\theta} + 2445{,}55.\theta = 0 \quad (\div 4{,}1035)$

$\ddot{\theta} + 0{,}0609.\dot{\theta} + 595{,}9668.\theta = 0$




$x_B = L.sen\,\theta = L\theta$

$x_A = \frac{L}{2}.sen\,\theta = \frac{L}{2}.\theta$

$x_B = 1.\theta \Rightarrow x_A = 0{,}5\theta$

$\dot{x}_A = 0{,}5.\dot{\theta}$

$$\ddot{\theta} + 0{,}0609.\dot{\theta} + 595{,}9668.\theta = 0$$

comparando c/

$$\ddot{\theta} + 2\xi\omega_N.\dot{\theta} + \omega_N^2.\theta = 0$$

Temos,

$$\omega_N^2 = 595{,}9668$$

$$\omega_N = \sqrt{595{,}9668} = 24{,}41 \text{ rad/s}$$

$$D = \frac{1}{n}.\ln\left(\frac{x(t)}{x(t+nP_D)}\right) = \frac{1}{4}.\ln\left(\frac{100}{40}\right) = 0{,}229$$

$$D = \frac{2\pi.\xi}{\sqrt{1-\xi^2}} \Rightarrow \sqrt{1-\xi^2} = \frac{2\pi\xi}{D} \Rightarrow 1-\xi^2 = \left(\frac{2\pi}{D}\right)^2.\xi^2$$

$$\frac{1-\xi^2}{\xi^2} = 752{,}81 \Rightarrow 1-\xi^2 = 752{,}81\xi^2$$

$$752{,}81\xi^2 + \xi^2 = 1 \Rightarrow \xi^2 = \frac{1}{753{,}81} \Rightarrow \xi = 0{,}036$$

$$0{,}0609.c = 2\xi.\omega_N$$

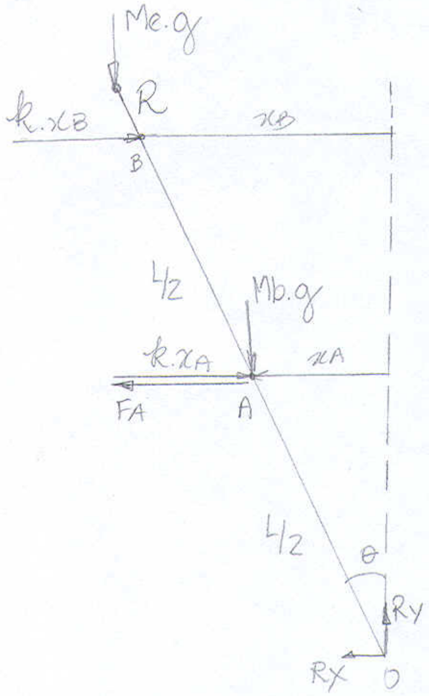
$$c = \frac{2.0{,}036.24{,}41}{0{,}0609} = 28{,}86 \text{ kg/s}$$

$$\xi = \frac{c}{C_c} \Rightarrow C_c = \frac{c}{\xi} = \frac{28{,}86}{0{,}036} = 801{,}67$$

$$C_c = 801{,}67 \text{ kg/s}$$

II – Por massa e rigidez efetivas

DCL



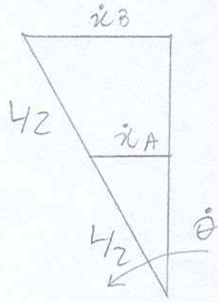

$T = T_b + T_e$

$\frac{1}{2}(m_A).\dot{x}_A^2 = \frac{1}{2}.J_{O_B}.\dot{\theta}^2 + \frac{1}{2}.J_{O_e}.\dot{\theta}^2$

$\frac{1}{2}(m_A).\dot{x}_A^2 = \frac{1}{2}(J_{OB} + J_{Oe}).\dot{\theta}^2$

$\frac{1}{2}(m_A).\dot{x}_A^2 = \frac{1}{2}(3 + 1{,}1035).2^2.\dot{x}_A^2$

$m_A = 4.4{,}1035 = 16{,}414 \text{ kg}$




$\dot{x}_A = \frac{L}{2}.\dot{\theta}$

$\dot{\theta} = 2.\dot{x}_A$

$\theta << 1 \text{ rad} \begin{cases} \operatorname{sen}\theta \simeq \theta \\ \cos\theta \simeq 1 \end{cases}$

$\Sigma M_O = 0$

$F_A.\frac{L}{2}\cos\theta - k.x_A.\frac{L}{2}.\cos\theta - k.x_B.L.\cos\theta + Me.g(R+L).\operatorname{sen}\theta + M_B.g.\frac{L}{2}.\operatorname{sen}\theta = 0$

$F_A.\frac{L}{2} - k.x_A.\frac{L}{2} - k.x_B.L + Me.g(R+L).\theta + M_B.g.\frac{L}{2}.\theta = 0$

$F_A.\frac{1}{2} - 1000.x_A - 2000.x_B + 10{,}85.\theta + 44{,}1.\theta = 0$

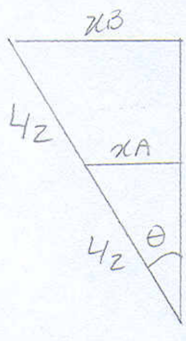
$\frac{x_A}{L/2} = \frac{x_B}{L} \Rightarrow x_B = 2x_A$

$\frac{F_A}{2} = 1000.x_A + 2000.x_B - 54{,}95\theta \quad (\times 2)$

$F_A = 2000.x_A + 4000.x_B - 109{,}9.\theta$

$F_A = (2000 + 8000 - 219{,}8).x_A$

$\frac{F_A}{x_A} = 9780{,}2 = K_A$




$x_A = \frac{L}{2}.\operatorname{sen}\theta = \frac{L}{2}.\theta$

$\theta = \frac{2.x_A}{L} = 2x_A$

$$\omega_N = \sqrt{\frac{K_A}{m_A}} = \sqrt{\frac{9780{,}2}{16{,}41}} = 24{,}41 \ rad/s$$

$$C_c = 2 \cdot m_A \cdot \omega_N = 2 \cdot 16{,}41 \cdot 24{,}41 = 801{,}14 \ \frac{kg}{s}$$

$$\zeta = \frac{C}{C_c}$$

$$C = \zeta \cdot C_c = 0{,}036 \cdot 801{,}14 = 28{,}84 \ \frac{kg}{s}$$