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Takami Matsuo
Oita University

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Adaptive Backstepping Control for a Two-Wheeled Autonomous Robot

Tatsuya Nomura, Yuta Kitsuka, Haruo Suemitsu, and Takami Matsuo

Department of Architecture and Mechatronics, Oita University, Oita, Japan
(Tel : +81-97-554-7804; E-mail: matsuo@cc.oita-u.ac.jp)

Abstract: In this paper, we deal with the backstepping control design of the two-wheeled inverted-pendulum-type autonomous robot, the e-nuvo-wheel, made in ZMP Inc. [5]. First, we derive a second-order motion equation of the angle and design an adaptive integral backstepping controller to stabilize the angle in the manner of the modeling in [1, 3]. This controller requires the full-state measurements. In the output feedback case, the K filter or the observer backstepping is needed [7, 8]. However, the structure of the controller becomes complicated. We have presented the non-model-based differentiator based on the adaptive update law citewada. Since the non-model-based differentiator does not need the knowledge of the dynamic structure of the signal, we can use it as a velocity estimator for unknown nonlinear systems. Next, we replace the velocity measurement with the estimates by the non-model-based differentiator. Finally, simulation and experimental results for the proposed controller are presented.

Keywords: Two-wheel inverted pendulum, backstepping control, velocity estimator, adaptive control.

1. INTRODUCTION

The backstepping is one of the most important schemes, that provides a powerful design tool for nonlinear system in the pure feedback and strict feedback forms [2]. However, it cannot be applied to the inverted pendulum system, because the system is not transformable in either of the above two forms [2]. To overcome the fact that the system cannot be written in such a triangular form, Benaskeur et al.[2] proposed an adaptive two-loop cascade controller. The inner loop used a backstepping controller that ensured the stabilization and the convergence towards zero of the angle tracking error. In [4], an LQ controller and an adaptive controller are combined by the backstepping techniques. Ebrahim et al.[3] derives the second-order motion equation of the angle with the force input and derived a state-feedback-based backstepping controller to stabilize the pendulum in the upright position. Moreover, based on this second-order motion equation of the angle, Altinöz [1] designed an adaptive integral backstepping controller to stabilize an inverted pendulum. The adaptive backstepping control allows us to control nonlinear systems with unknown parameters.

In 2000, Segway (Bedford, New Hampshire) developed a human transporter in which the direction and speed were controlled respectively by the shifting weight of the rider and by a manual turning mechanism on the handlebar. Segway implemented a PID controller to stabilize the vehicle, but the main controller was a human on board the machine [6]. The mechanical structure of the vehicle is similar to that of an inverted pendulum, such as is shown in Fig.1. Unlike the situation in an inverted pendulum, the vehicle have to use its two wheels for movement as well as for balance [6]. Kim et al.[6] proposed a LQ controller for a table-size equipment of the two-wheeled inverted-pendulum-type autonomous vehicle.

In this paper, we deal with the backstepping control design of the two-wheeled inverted-pendulum-type autonomous robot, the e-nuvo-wheel, made in ZMP Inc. [5]. First, we derive a second-order motion equation of the angle and design an adaptive integral backstepping controller to stabilize the angle in the manner of the modeling in [1, 3]. This controller requires the full-state measurements. In the output feedback case, the K filter or the observer backstepping is needed [7, 8]. However, the structure of the controller becomes complicated. We have presented the non-model-based differentiator based on the adaptive update law citewada. Since the non-model-based differentiator does not need the knowledge of the dynamic structure of the signal, we can use it as a velocity estimator for unknown nonlinear systems. Next, we replace the velocity measurement with the estimates by the non-model-based differentiator. Finally, simulation and experimental results for the proposed controller are presented.

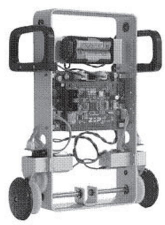

Fig. 1 The structure of the e-nuvo-wheel.

2. DYNAMICS OF TWO-WHEELED AUTONOMOUS ROBOT

The coordinate system for the two-wheeled autonomous robot the e-nuvo-wheel autonomous robot is shown in Fig.2. The dynamics of the two-wheeled autonomous robot are given by

$$M_0(\theta)\ddot{\theta} - M_3(\theta)\dot{\theta}^2 + M_1\ddot{\phi} + c\dot{\phi} = au \quad (1)$$

$$M_2(\theta)\ddot{\theta} - M_3(\theta)\dot{\theta}^2 - M_4(\theta)\theta + M_0(\theta)\ddot{\phi} = 0 \quad (2)$$

where θ is the angle of the body, ϕ is the angle of the wheel, and u is the applied input current. The nonlinear functions $M_0(\theta), M_1, M_2(\theta)$, and $M_3(\theta)$ are defined as

$$M_0(\theta) = (M+m)r_t^2 + mlr_t\cos\theta + J_t + iJ_m$$

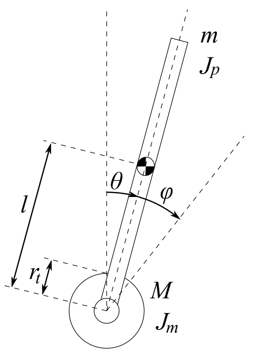

Fig. 2 Cordinate system of the e-nuvo-wheel.

$$
\begin{aligned}
M_1 &= (M+m)r_t^2 + J_t + i^2 J_m \\
M_2(\theta) &= (M+m)r_t^2 + 2mlr_t\cos\theta + ml^2 \\
&\quad + J_p + J_t + J_m \\
M_3(\theta) &= mlr_t\sin\theta \\
M_4(\theta) &= mgl\sin\theta.
\end{aligned}
$$

The parameters are defined as follows:

m:	mass of the body
M:	mass of the cart
J_p:	Moment of inertia of the body [Kgm^2]
J_t:	Moment of inertia of the cart [Kgm^2]
J_m:	Moment of inertia of the motor rotor [Kgm^2]
l:	Length between the wheel axle and the gravity center of the body [m]
r_t:	Radius of the wheel [m]
c:	Friction coefficient of the wheel axle [Kgm^2/s]
a:	Torque gain [Nm/A]
i:	Reduction ratio of the gear.

Substituting (2) into eq:(3), we have

$$
\begin{aligned}
&M_0(\theta)\ddot{\theta} - M_3(\theta)\dot{\theta}^2 + \frac{M_1}{M_0(\theta)} \\
&\Big\{-M_2(\theta)\ddot{\theta} + M_3(\theta)\dot{\theta}^2 + M_4(\theta)\Big\} + c\dot{\phi} = au.
\end{aligned}
$$

Using a new input signal u_0 as $u = u_0 + (c/a)\dot{\phi}$ and substituting (2) to (1), we have the following second-order motion equation of the body angle:

$$g(\theta)\ddot{\theta} = u_0 - h(\theta)$$

where h and g are defined as

$$
\begin{aligned}
g(\theta) &= A(\theta)/a \\
h(\theta) &= \frac{1}{a}\Big(B(\theta)\dot{\theta}^2 + C(\theta)\Big).
\end{aligned}
$$

Also, A,B, and C are given by

$$
\begin{aligned}
A(\theta) &= M_0(\theta) - \frac{M_1 M_2(\theta)}{M_0(\theta)} \\
B(\theta) &= M_3(\theta)\left\{\frac{M_1}{M_0(\theta)} - 1\right\} \\
C(\theta) &= \frac{M_1 M_4(\theta)}{M_0(\theta)}
\end{aligned}
$$

3. DESIGN OF BACKSTEPPING CONTROLLER FOR BODY ANGLE WITH STATE FEEDBACK

By similar method in the inverted pendulum [1], we derive a backstepping controller with the state feedback.

3.1 Case1:g, h are known

Consider the following system:

$$
\begin{aligned}
\dot{\theta} &= \omega && (3) \\
g(\theta)\dot{\omega} &= u_0 - h(\theta). && (4)
\end{aligned}
$$

Define the tracking error as

$$e_1 = \theta_{ref} - \theta, \qquad (5)$$

where θ_{ref} is the reference of θ. In our case, $\theta_{ref} \equiv 0$. Assuming that g, h are known, we derive an integral backstepping controller.

Step1 Regarding ω in (3) as an input, the virtual control α is designed such that $\lim_{t\to\infty} e_1(t) = 0$. The virtual control is selected as

$$\alpha = c_1 e_1 + \dot{\theta}_{ref} + \lambda_1 x_1, \qquad (6)$$

where c_1, λ_1 are positive constants and $x_1 = \int e_1(\tau)d\tau$ is the integral action. The virtual control α attains the asymptotic stability of the tracking error. Assuming that $\omega \equiv \alpha$, (3) is rewritten by

$$
\begin{aligned}
&(\dot{\theta}_{ref} - \dot{\theta}) + c_1(\theta_{ref} - \theta) \\
&+\lambda_1 \int (\theta_{ref} - \theta)d\tau = 0.
\end{aligned}
$$

Since $c_1 > 0, \lambda_1 > 0$, we obtain the following stable differential equation of x_1:

$$\ddot{x}_1 + c_1\dot{x}_1 + \lambda_1 x_1 = 0.$$

Thus, we have

$$
\begin{aligned}
\lim_{t\to\infty} x_1 &= \lim_{t\to\infty} \int (\theta_{ref} - \theta)d\tau = 0, \\
\lim_{t\to\infty} \dot{x}_1 &= \lim_{t\to\infty} (\theta_{ref} - \theta) = 0.
\end{aligned}
$$

Using the Lyapunov function

$$V_1 = \frac{\lambda_1}{2}x_1^2 + \frac{1}{2}e_1^2,$$

its derivative is given by

$$\dot{V}_1 = \lambda_1 x_1 \dot{x_1} + e_1\dot{e}_1 = e_1(\lambda_1 x_1 + \dot{e}_1).$$

Step2 For (4), we design the input u_0 such that $\lim_{t\to\infty}(\alpha - \omega) = 0$. Define the error between the virtual contro and the input, e_2, as

$$e_2 = \alpha - \omega, \qquad (7)$$

Using the virtual input and e_2, the signal $\dot{\theta}$ can be written by

$$
\begin{aligned}
\dot{\theta} &= \omega \\
&= \alpha - e_2 \\
&= c_1 e_1 + \dot{\theta}_{ref} + \lambda_1 x_1 - e_2.
\end{aligned}
$$

Thus, $\dot{e}_1$ is given by

$$
\begin{aligned}
\dot{e}_1 &= \dot{\theta}_{ref} - \dot{\theta} \\
&= \dot{\theta}_{ref} - (c_1 e_1 + \dot{\theta}_{ref} + \lambda_1 x_1 - e_2) \\
&= -c_1 e_1 - \lambda_1 x_1 + e_2.
\end{aligned}
$$

The time derivative of V_1 is

$$
\begin{aligned}
\dot{V}_1 &= e_1(\lambda_1 x_1 + \dot{e}_1) \\
&= e_1(\lambda_1 x_1 - c_1 e_1 - \lambda_1 x_1 + e_2) \\
&= -c_1 e_1^2 + e_1 e_2.
\end{aligned}
$$

With (4) and (6), $g\dot{e}_2$ can be written by

$$\begin{aligned}
g\dot{e}_2 &= g(\dot{\alpha} - \dot{\omega}) \\
&= g(c_1\dot{e}_1 + \ddot{\theta}_{ref} + \lambda_1 e_1 - \dot{\omega}) \\
&= g(c_1\dot{e}_1 + \ddot{\theta}_{ref} + \lambda_1 e_1) - (u_0 - h) \\
&= g(c_1(-c_1e_1 - \lambda_1 x_1 + e_2) + \ddot{\theta}_{ref} \\
&\quad +\lambda_1 e_1) - (u_0 - h) \\
&= g(-c_1^2 e_1 - c_1\lambda_1 x_1 + c_1 e_2 + \ddot{\theta}_{ref} \\
&\quad +\lambda_1 e_1) - (u_0 - h)
\end{aligned}$$

Defining the second Lyapunov function V_2 as

$$V_2 = V_1 + \frac{1}{2}e_2^2,$$

its derivative is obtained by

$$\dot{V}_2 = \dot{V}_1 + e_2\dot{e}_2 = -c_1e_1^2 + e_1e_2 + e_2\dot{e}_2.$$

If $\dot{e}_2$ satisfies

$$\dot{e}_2 = -c_2e_2 - e_1 \tag{8}$$

then

$$\dot{V}_2 = -c_1e_1^2 - c_2e_2^2 < 0.$$

Noting that $g\dot{e}_2 = g(-c_2e_2 - e_1)$, we can see that if the following input is satisfied, (8) is satisfied.

$$u_0 = g\{\xi + \Gamma\} \tag{9}$$

where

$$\begin{aligned}
\xi &= (1 - c_1^2 + \lambda_1)e_1 + (c_1 + c_2)e_2 \\
&\quad -c_1\lambda_1 x_1 + \ddot{\theta}_{ref} \\
\Gamma &= \frac{h(\theta)}{g(\theta)}
\end{aligned}$$

3.2 Case2: g, h are unknown

When g, h are unknown, the function g, h should be replaced by its estimates $\hat{g}, \hat{h}$. Instead of the input u_0, we use the input u_a as

$$u_a = \hat{g}(\xi + \hat{\Gamma})$$

In this case, we have

$$\begin{aligned}
\dot{e}_2 &= -c_1^2e_1 - c_1\lambda_1x_1 + c_1e_2 + \ddot{\theta}_{ref} \\
&\quad +\lambda_1 e_1 - \frac{1}{g}(u_a - h) \\
&= \left\{(1 - c_1^2 + \lambda_1)e_1 + (c_1 + c_2)e_2 \right.\\
&\quad \left. -c_1\lambda_1 x_1 + \ddot{\theta}_{ref}\right\} \\
&\quad -(e_1 + c_2e_2) - \frac{1}{g}(u_a - h) \\
&= \xi - (e_1 + c_2e_2) - \frac{1}{g}(u_a - h).
\end{aligned}$$

The last term of the right-hand side of the above equation can be written by

$$\begin{aligned}
\frac{1}{g}(u_a - h) &= \frac{1}{g}(u_0 - h) + \frac{1}{g}(u_a - u_0) \\
&= \left[\frac{1}{g}\{g(\xi + \Gamma) - h\}\right] \\
&\quad + \left[\frac{1}{g}\left\{\hat{g}(\xi + \hat{\Gamma}) - g(\xi + \Gamma)\right\}\right] \\
&= \left[\xi + \Gamma - \frac{h}{g}\right] \\
&\quad + \left[\frac{1}{g}\left\{\xi(\hat{g} - g) + \hat{g}\hat{\Gamma} - g\Gamma\right\}\right] \\
&= \xi + \left[\frac{1}{g}\left\{-\tilde{g}\xi - (g - \hat{g})\hat{\Gamma} - g\tilde{\Gamma}\right\}\right] \\
&= \xi - \frac{\tilde{g}}{g}\xi - \frac{\tilde{g}}{g}\hat{\Gamma} - \tilde{\Gamma} \\
&= \xi - \frac{\tilde{g}}{g}(\xi + \hat{\Gamma}) - \tilde{\Gamma}
\end{aligned}$$

Hence, we have

$$\begin{aligned}
\dot{e}_2 &= \xi - (e_1 + c_2e_2) - \left\{\xi - \frac{\tilde{g}}{g}(\xi + \hat{\Gamma}) - \tilde{\Gamma}\right\} \\
&= -e_1 - c_2e_2 + \frac{\tilde{g}}{g}(\xi + \hat{\Gamma}) + \tilde{\Gamma},
\end{aligned}$$

where

$$\tilde{g} = g - \hat{g}, \tilde{\Gamma} = \Gamma - \hat{\Gamma}.$$

Defining the third Lyapunov function, V_3, as

$$V_3 = V_2 + \frac{1}{2\gamma_1}\tilde{g}^2 + \frac{1}{2\gamma_2}\tilde{\Gamma}^2,$$

its time derivative is given by

$$\begin{aligned}
\dot{V}_3 &= \dot{V}_2 + \frac{1}{\gamma_1}\tilde{g}\dot{\tilde{g}} + \frac{1}{\gamma_2}\tilde{\Gamma}\dot{\tilde{\Gamma}} \\
&= \left(-c_1e_1^2 + e_1e_2 + e_2\dot{e}_2\right) + \frac{1}{\gamma_1}\tilde{g}\dot{\tilde{g}} + \frac{1}{\gamma_2}\tilde{\Gamma}\dot{\tilde{\Gamma}} \\
&= -c_1e_1^2 + e_1e_2 + e_2\left\{-e_1 - c_2e_2 \right.\\
&\quad \left. +\frac{\tilde{g}}{g}(\xi + \hat{\Gamma}) + \tilde{\Gamma}\right\} + \frac{1}{\gamma_1}\tilde{g}\dot{\tilde{g}} + \frac{1}{\gamma_2}\tilde{\Gamma}\dot{\tilde{\Gamma}} \\
&= -c_1e_1^2 - c_2e_2^2 + \tilde{g}\left\{\frac{1}{g}(\xi + \hat{\Gamma})e_2 + \frac{1}{\gamma_1}\dot{\tilde{g}}\right\} \\
&\quad +\tilde{\Gamma}\left(e_2 + \frac{1}{\gamma_2}\dot{\tilde{\Gamma}}\right).
\end{aligned}$$

If parameter estimation errors satisfy the following equations, $\dot{V}_3$ is negative.

$$\dot{\tilde{g}} = -\gamma_1\frac{1}{g}\left(\xi + \hat{\Gamma}\right)e_2, \dot{\tilde{\Gamma}} = -\gamma_2 e_2$$

where $\gamma_1 > 0, \gamma_2 > 0$. The function g, h are time-varying function. To get the parameter update law, we assume that they are slow time-varying functions. In this case, we have

$$\dot{\tilde{g}} \approx -\dot{\hat{g}}, \dot{\tilde{\Gamma}} \approx -\dot{\hat{\Gamma}}$$

The parameter update laws can be approximated as

$$\dot{\hat{g}} \approx \gamma_3 e_2\left(\xi + \hat{\Gamma}\right), \dot{\hat{\Gamma}} \approx \gamma_2 e_2$$

where γ_2, γ_3 are positive constants.

4. CONTROLLER FOR WHEEL ANGLE

The dynamics of the wheel angle is given by

$$M_0(\theta)\ddot{\phi} = -M_2(\theta)\ddot{\theta} + M_3(\theta)\dot{\theta}^2 + M_4(\theta)$$

Substituting (3) to the above equation, we have

$$M_0(\theta)\ddot{\phi} = -\frac{M_2(\theta)}{g(\theta)}(u_0 - h(\theta)) + M_3(\theta)\dot{\theta}^2 + M_4(\theta)$$

When $\theta \approx 0$, the above equation can be approximated by

$$\ddot{\phi} \approx \delta u_0 \tag{10}$$

where

$$\delta = \frac{aM_2(0)}{M_0(0) - M_1(0)M_2(0)} < 0$$

For simplicity, we use the following PD-type stabilizing input for the wheel angle:

$$u_0 = q_1\phi + q_2\dot{\phi} + q_3\int_0^t \phi(\tau)d\tau$$

where q_1,q_2 and q_3 are positive constants.

Thus, whole adaptive input is given by

$$\begin{aligned} u_0 &= \hat{g}(\xi + \hat{\Gamma}) \\ &\quad +(c/a)\dot{\phi} + q_1\phi + q_2\dot{\phi} + q_3\int_0^t \phi(\tau)d\tau \\ &= \hat{g}(\xi + \hat{\Gamma}) \\ &\quad +q_1\phi + q_2'\dot{\phi} + q_3\int_0^t \phi(\tau)d\tau. \end{aligned}$$

5. OUTPUT FEEDBACK WITH VELOCITY ESTIMATOR

We adopt the non-model based differentiator of a given signal with unknown dynamic structure using the adaptive observer[9]. The velocity measurements $\dot{\theta}$ and $\dot{\phi}$ are replaced by the estimates $\hat{\dot{\theta}}$ and $\hat{\dot{\phi}}$ using the velocity estimators.

Let $x(t)$ be a measurement signal at a time t. We define $\theta_1(t)$ as the derivative with respect to time of the measurement signal, *i.e.*

$$\frac{dx(t)}{dt} = \theta_1(t). \tag{11}$$

The time-varying parameter, $\theta_1(t)$, is written by the following equation:

$$\theta_1(t) = \theta_{10} + \epsilon(t), \tag{12}$$

where θ_{10} is an unknown constant, and $\epsilon(t)$ is assumed that its upper bound is known, *i.e.*

$$|\epsilon(t)| \le \epsilon_0, \tag{13}$$

where ϵ_0 is a known constant.

The design problem of the non-model-based differential filter is defined as follows:

> *Design a differential filter to estimate the derivative, $\theta_1(t)$ with available signal $x(t)$ and without any knowledge of its dynamics.*

We give an adaptive observer to estimate the time-derivative whose upper bound is known. The estimate, $\widehat{\frac{dx(t)}{dt}}$, of the derivative of the signal $x(t)$ is given by the following adaptive observer and the update laws:

$$\dot{\hat{x}} = -k(\hat{x} - x) + \hat{\theta}_1(t) - \hat{\epsilon}(t)\mathrm{sgn}(\hat{x} - x) \tag{14}$$

$$\dot{\hat{\theta}}_1(t) = -\gamma(\hat{x}(t) - x(t)), \dot{\hat{\epsilon}}(t) = |(\hat{x}(t) - x(t))| \tag{15}$$

where $k > 0$. The estimate of $\dot{x}(t)$ is given by

$$\widehat{\frac{dx(t)}{dt}} = \hat{\theta}_1(t) = -\int_0^t \gamma(\hat{x}(\tau) - x(\tau))d\tau. \tag{16}$$

We call this estimator the non-model-based differentiator.

Defining the observer error as $e(t) = \hat{x}(t) - x(t)$, we obtain the error system as

$$\dot{e} = -ke + \tilde{\theta}_1(t) - \hat{\epsilon}(t)\mathrm{sgn}(e) - \epsilon(t) \tag{17}$$

where $\tilde{\theta}_1(t) = \hat{\theta}_1(t) - \theta_{10}$.

6. SIMULATION RESULT

The plant parameters are given by

$$\begin{aligned} &m = 0.5392, M = 0.071, J_p = 2.16 \times 10^{-3}, \\ &J_t = 8.632 \times 10^{-5}, J_m = 1.30 \times 10^{-7}, \\ &l = 0.1073, r_t = 0.02485, c = 1.0 \times 10^{-4} \\ &i = 30, a = 0.0627 \end{aligned}$$

The controller parameters are selected as follows:

- State feedback case:

$$\begin{aligned} &c_1 = 6, c_2 = 6, \lambda_1 = 10^{-4} \\ &\gamma_2 = 6, \gamma_3 = 6, \\ &q_1 = 400, q_2 = 200, q_3 = 200, \\ &k = 100, \gamma = 1; \end{aligned}$$

- Output feedback case with the velocity estimators:

$$\begin{aligned} &c_1 = 6, c_2 = 6, \lambda_1 = 10^{-4} \\ &\gamma_2 = 6, \gamma_3 = 6, \\ &q_1 = 200, q_2 = 50, q_3 = 5, \\ &k = 100, \gamma = 1 \end{aligned}$$

We perform the numerical simulations in the following cases:

- Backstepping controller with the state feedback
- Backstepping controller with the velocity estimators
- LQ controller with the velocity estimators

6.1 Backstepping controller with the state feedback

Figure 3 shows the comparison between small initial value, $\theta_0 = 5$[deg], and large initial value, $\theta(0) = 60$[deg] when other initial conditions are all zeros and all states $\theta(t), \dot{\theta}(t), \phi(t)$, and $\dot{\phi}(t)$ are measurable. The body angle can converge to zero for both initial conditions. Since the controller for the wheel angle is a simple PD-type, the response of $\phi(t)$ has an offset. The estimates $\hat{g}$ and $\hat{\Gamma}$ does not converge to the true values. Their value go to the values near the true values when the initial condition $\theta(0)$ is small.

6.2 Backstepping controller with the velocity estimators

Figure 4 shows the repsonses when the backstepping controller with the velocity estimators, $\hat{\dot{\theta}}$ and $\hat{\dot{\phi}}$. All initial conditions are same as in the state feedback. The performance of the controller with the velocity estimators is not good when the initial value $\theta(0)$ is large.

6.3 LQ controller with the velocity estimators

Figure 5shows the comparison between the backstepping controller with the velocity estimators and LQ controller with the velocity estimators. The feedback gain of LQ controller ,K, is selected as

$$K = \begin{bmatrix} 19.0876 & 0.1732 & 2.25220 & 0.1991 \end{bmatrix}.$$

When the initial value $\theta(0) = 5$[deg], the performance of the LQ controller is better than the backstepping controller. When large initial value $\theta(0) = 35$[deg], the back stepping controller guarantees the boundedness of the states, but the LQ controller causes instability.

7. CONCLUSION

We proposed an adaptive integral backstepping controller to stabilize the two-wheeled autonomous robot with the velocity estimator. The proposed controller is evaluated by computer simulations. The improvement of the controller of the wheel angle should be needed.

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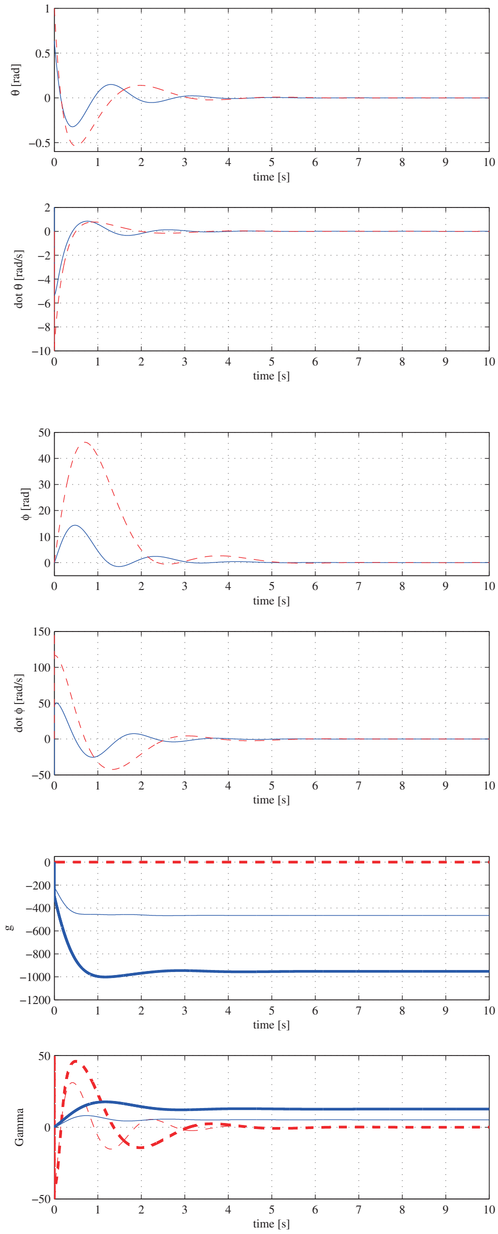

Fig. 3 Comparison of two initial condition, $\theta(0) = 5$[deg] and $\theta(0) = 60$[deg] in the state feedback case. Top:$\theta(t)$ ($\theta(0) = 5$[deg];blue line, $\theta(0) = 60$[deg]; red line),second from the top:$\dot{\theta}(t)$ ($\theta(0) = 5$[deg];blue line, $\theta(0) = 60$[deg]; red line),third from the top:$\phi(t)$ ($\theta(0) = 5$[deg];blue line, $\theta(0) = 60$[deg]; red line),fourth from the top:$\dot{\phi}(t)$ ($\theta(0) = 5$[deg];blue line, $\theta(0) = 60$[deg]; red line), second from the bottom:$g(t)$(red lines) and $\hat{g}(t)$(dbuleu lines) ($\theta(0) = 5$[deg];thin line, $\theta(0) = 60$[deg]; thick line), bottom:$\Gamma(t)$(solid lines) and $\hat{\Gamma)}(t)$(dotted lines) ($\theta(0) = 5$[deg];thin line, $\theta(0) = 60$[deg]; thick line)

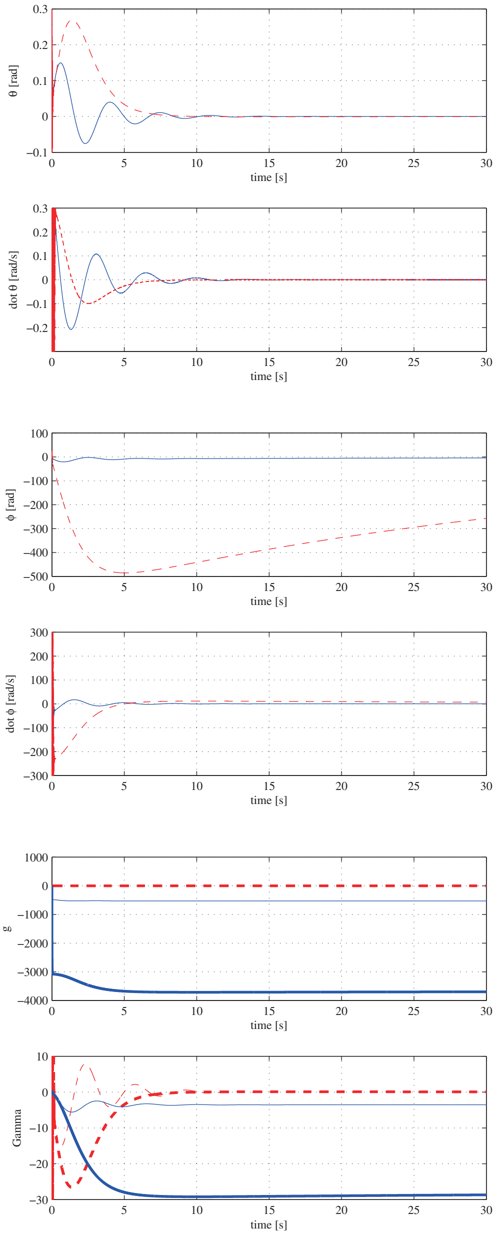

Fig. 4 Comparison of two initial condition, $\theta(0) = 5$[deg] and $\theta(0) = 60$[deg] in the output feedback case with the velocity estimator. Top:$\theta(t)$ ($\theta(0) = 5$[deg];blue line, $\theta(0) = 60$[deg]; red line),second from the top:$\dot{\theta}(t)$ ($\theta(0) = 5$[deg];blue line, $\theta(0) = 60$[deg]; red line),third from the top:$\phi(t)$ ($\theta(0) = 5$[deg];blue line, $\theta(0) = 60$[deg]; red line),fourth from the top:$\dot{\phi}(t)$ ($\theta(0) = 5$[deg];blue line, $\theta(0) = 60$[deg]; red line), second from the bottom:$g(t)$(red lines) and $\hat{g}(t)$(blue lines) ($\theta(0) = 5$[deg];thin line, $\theta(0) = 60$[deg]; thick line), bottom:$\Gamma(t)$(solid lines) and $\hat{\Gamma})(t)$(dotted lines) ($\theta(0) = 5$[deg];thin line, $\theta(0) = 60$[deg]; thick line)

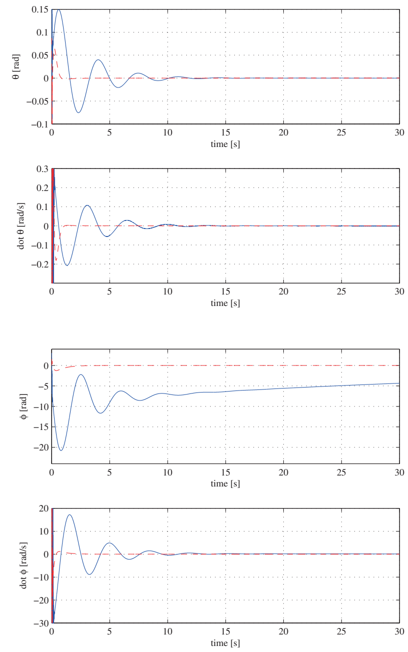

Fig. 5 Comparison between the backstepping controller with velocity estimators and LQ controller with velocity estimators when $\theta(0) = 5$[deg]; backstepping (blue line), LQ (red line). Top:$\theta(t)$, second from the top:$\dot{\theta}(t)$, third from the top:$\phi(t)$, bottom:$\dot{\phi}(t)$.

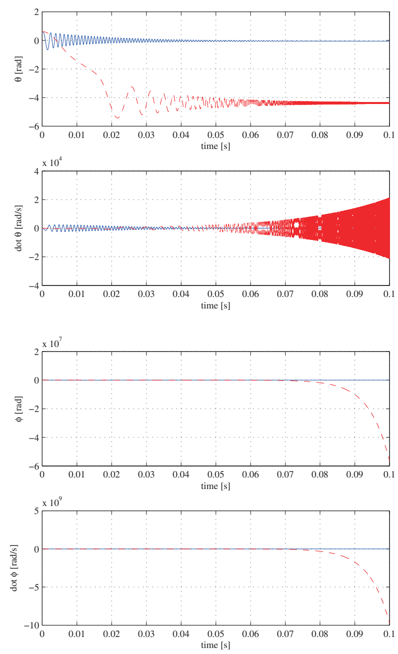

Fig. 6 Comparison between the backstepping controller with velocity estimators and LQ controller with velocity estimators when $\theta(0) = 35$[deg]; backstepping (blue line), LQ (red line). Top:$\theta(t)$, second from the top:$\dot{\theta}(t)$, third from the top:$\phi(t)$, bottom:$\dot{\phi}(t)$.