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Cognitive interference channel: achievable rate region and power allocation

Zouhair Al-qudah[1], *Mohammad Al Bataineh*[2]

[1]*Department of Electrical and Communication Engineering, Al-Hussein bin Talal University, Ma'an, Jordan*
[2]*Telecommunication Engineering Department, Yarmouk University, Irbid, Jordan*
E-mail: qudahz@ahu.edu.jo

Abstract: In this study, the authors consider a state-dependent two user interference channel. The two users sharing the spectrum are assumed to be cognitive and each user has a non-causal access to the signal from the other user. For this channel model, an achievable rate region is established for both discrete memoryless model and Gaussian channel. In particular, the achievable rate region is obtained by combining Han–Kobayashi rate splitting coding scheme, superposition coding, Gelfand–Pinsker coding scheme and zero-forcing dirty paper coding. Furthermore, the sum rate maximisation and the associated power allocation problem are studied, numerically and theoretically. The corresponding numerical examples show that the proposed combined coding scheme outperforms the existing schemes in the sense of achievable rate region. Moreover, the effectiveness of the optimal power allocation between the two cognitive nodes is also shown.

1 Introduction

Interference channel (IC) with cognition capabilities and interference avoidance have been extensively investigated. A simple model of the IC is composed of two senders with their corresponding receivers. For a given user, the achievable rate is mainly limited by the other user's signal sharing the spectrum. The capacity region for this IC channel model is available only in the case of very strong Gaussian IC [1] and strong Gaussian IC [2]. In these cases, each receiver can perfectly decode the interference signal from the other user and then remove it. In the non-strong interference, in which the gain of the crossover channels is <1, the interference signal can't be decoded by the non-intended receiver. In this case, Han and Kobayashi (HK) [3] derived the best ever known achievable rate region in which the signal for each user is split into private and public parts. Apparently, the public signal can be decoded by both destinations while the private part can be decoded by only the intended receiver.

There are many design techniques that were proposed to increase the achievable rate region. These techniques include conferencing transmitters [4–6], conferencing receivers [6, 7] and cognitive radio. For instance, in conferencing transmitters, the two sources exchange their signals completely or partially before these senders start transmission to their destination. Thus, either a common message is sent by the two transmitters or each sender may work as a relay for the other one. On the other technique, in the conferencing receivers, the receiver's exchange the received signals before the destinations start decoding the messages. In addition, the cognitive radio enables the cognitive sender to adapt its signal based on the interference signal from the other sources sharing the spectrum.

The cognitive IC, in which one of two users or the two users can work in cognitive mode, has been extensively considered. For instance, the authors in [8–10] studied the state-dependent IC in which a uni-directional cognitive link is employed to reduce the effect of the other user's signal. In addition, the authors in [11] considered the case in which both users are cognitive. In another scenario, the IC which is augmented by a cognitive relay was studied in [12–15]. In these scenarios, a combination of rate splitting [3], Gelfand–Pinsker (GP) encoding [16] and superposition encoding [17] are used at the two senders to transmit independent data. Furthermore, the authors in [5, 11, 18] studied the case wherein the two users are cognitive.

In this work, we focus on the cognitive Gaussian IC with bidirectional cognitive links, as depicted in Fig. 1. In particular, we consider the IC model in which the two users can operate in cognitive mode. For instance, interference cancellation for cognitive radio network [8, 9] enables the cognitive user to improve its rate by employing interference mitigation technique such as dirty paper coding (DPC) [8, 9, 19] and zero-forcing-DPC (ZF-DPC) [5, 18, 20–22] By employing DPC at a given transmitter, an interference-free Gaussian channel rate is obtained as long as this interference is known non-causally at the transmitter [19]. In contrast to the optimal DPC, ZF-DPC is a suboptimal interference mitigation by either (i) triangularising the channels between a transmitter and receivers in the multiple input broadcast channel [20–22] or (ii) a transmitter uses a portion of its power to mitigate the effect of interference from the other user in the IC [5, 18]. Furthermore, the state-dependent two user IC, in which there is a common

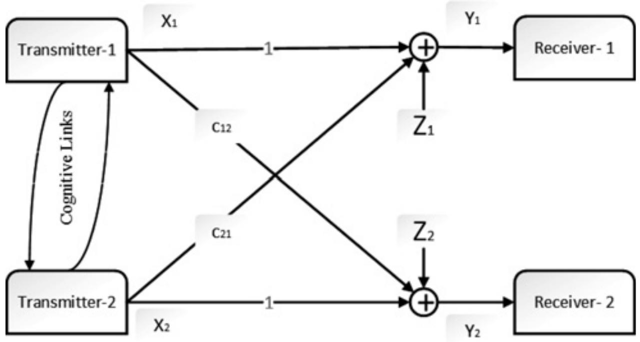

Fig. 1 *Realisation of the interference radio channel with bidirectional cognitive links*

additive interference, known to both transmitters but not to the receivers was studied in [23–25].

In their work, GP showed that the capacity, C, of a discrete memoryless channel $p(y|x, s)$ with a non-causal additive interference, S, available to the transmitter but not to the receiver is given by [16, 26]

$$C = \max_{p(u,x|s)} \{I(U; Y) - I(U; S)\} \tag{1}$$

where U is an auxiliary random variable, that has to be designed and Y is the channel output signal. Subsequently, Costa [19] extended the previous capacity formula to the additive.

Gaussian channel to show that an interference free channel capacity can be attained. Specifically, Costa considered the problem of communicating over a channel modelled as

$$Y = X + S + Z \tag{2}$$

where Y represents the channel output, X represents the channel input and finally S and Z are the additive interference signal and the additive white Gaussian noise (AWGN) signal, respectively. In his work, Costa made two main assumptions to achieve the channel capacity: (i) a non-causal version of the interference is only available at the transmitter and (ii) the random variable U is designed to be $U = X + \beta S$ where β is the power inflation factor. Recall that, for such a model, $\beta = (P_X/(P_X + P_Z))$ where P_X and P_Z are the power variance of both the desired signal X and the additive noise signal Z, respectively. In the case that both the transmitter and the channel have the same noisy version of the side information, the rate given in (1) can be reduced to [27]

$$C = \max_{p(x|s)} I(X; Y|S) \tag{3}$$

Rate splitting encoding, which was originally proposed by HK in [3] for a non-strong IC, is the best ever known inner bound of the IC achievable rate region. In this encoding scheme, each user's message is divided into two parts, the private and the public. Hence, the private part can only be decoded by the intended receiver whereas the public one is decoded by both users in order to reduce the interference from the other user at each destination. In a more recent study, Chong–Motani–Garg [28, 29] established a new simplified description of the Han–Kobayashi rate region. In this new approach, a new equivalent representation of the Han–Kobayashi was introduced. Furthermore, Kramer [30] showed that the sum rate bound of the Chong–Motani–Garg is the same as the Han–Kobayashi rate region. In a subsequent work, the capacity region of the two users Gaussian IC was studied for some certain scenarios in [10] based on the Chong–Motani–Garg rate region.

In this paper, we first derive the achievable rate region of the general state-dependent IC with cognition at both the transmitters. In particular, each sender has a non-causal access to the other user's signal. Indeed, Han–Kobabyshi rate splitting, GP binning encoding and ZF-DPC are employed to derive the achievable rate region. In addition, the derived achievable rate region is also extended to the additive Gaussian setting. Specifically, based on the crossover channel gains, one of the two users has to employ DPC whereas the other user has to employ ZF-DPC. Numerical results show that the proposed encoding scheme outperforms the best existing encoding scheme for the cognitive Gaussian IC. In addition, by fixing the sum power of both cognitive transmitters, the effectiveness of varying the power between both of them are illustrated, numerically and mathematically.

The remainder of this paper is outlined as follows. Section 2 describes the communication model that we study in this work. The achievable rate region, in which a non-causal knowledge of the other user's signal sharing the spectrum is available, is studied in Section 3. Next, these results are extended to the Gaussian cognitive IC in Section 4. Then, sum rate maximisation and the associated power allocation problem is studied in Section 5. Afterwards, numerical examples that validate our theoretical results are presented in Section 6. Finally, we conclude the paper in Section 7.

2 System model

We consider the state-dependent IC model consists of two finite channel input alphabets $\mathcal{X}_1$ and $\mathcal{X}_2$, two finite channel output alphabets $\mathcal{Y}_1$ and $\mathcal{Y}_2$ and a transition probability distribution $P_{Y_1,Y_2|X_1,X_2}$ as illustrated in Fig. 2. In this scenario, $X_1 \in \mathcal{X}_1$ and $X_2 \in \mathcal{X}_2$ are the input signals associated with the first and second transmitters, respectively. Further, $Y_1 \in \mathcal{Y}_1$ and $Y_2 \in \mathcal{Y}_2$ represent the output alphabets at the first and second destinations, respectively. In this channel model, the first transmitter has a message $M_1 \in \{1, \ldots, 2^{nR_1}\}$ to be communicated to the first receiver over n channel uses, and the second sender has a message $M_2 \in \{1, \ldots, 2^{nR_2}\}$ intended for the second receiver, where R_1 and R_2 are the rates at which the first and second senders can communication with their destinations, respectively.

In this channel model, for positive integers n, M_1 and M_2, an (M_1, M_2, n, P_e) code for the cognitive IC consists of the following mapping:

- The encoding at both tranmitters consists of

$$f_1{:}M_1 \mapsto \mathcal{X}_1^n \quad f_2{:}M_1 \times M_2 \mapsto \mathcal{X}_2^n \tag{4}$$

- The two destinations have the following decoding functions

$$g_1{:}\mathcal{Y}_1 \mapsto M_1 \quad g_2{:}\mathcal{Y}_2 \mapsto M_2 \tag{5}$$

In this formulation, a rate pair (R_1, R_2) is said to be achievable if, for any $\epsilon > 0$, there exist a sequence of (M_1, M_2, n, P_e)

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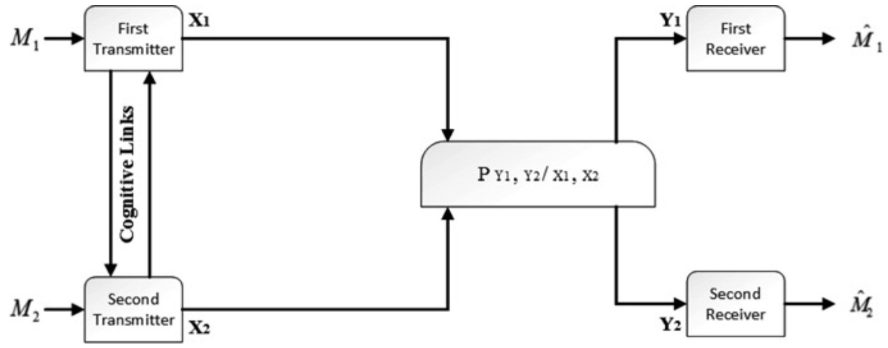

Fig. 2 *General memoryless cognitive IC model considered in this work*

code with $P^n_{e,i}$ vanishes for sufficiently large n. In this definition, the average error probability for user $i = 1, 2$ is defined as

$$P_{e,i} = \sum_{m_1, m_2} \frac{1}{2^{n(R_1+R_2)}} P[g_i(Y_i) \neq m_i | m_1, m_2 \text{ was sent}] \quad (6)$$

where $M_1 = m_1$ and $M_2 = m_2$ are the two transmitted messages from the first and second transmitters, respectively.

3 Non-causal cognitive IC

In this section, an achievable rate region of the state-dependent IC is derived. In this derivation, for a given transmitter, the signal from the other user is known non-causally. In this case, the achievable rate region is obtained using the well-known random coding techniques: rate splitting, superposition encoding scheme, zero-forcing DPC and GP encoding. Furthermore, each user divides its signal m_i, $i \in \{1, 2\}$ into two parts, the private m_{ii} and the public m_{ij}, $i \neq j$ such that the total rate for user i is $R_i = R_{ii} + R_{ij}$. Normally, the private part is superimposed on the public one. Further, the first user employs ZF-DPC in order to minimise the effect of interference from the second transmitter at the first destination. In addition, the second transmitter employs GP binning coding scheme such that the effect of the interference from the first sender is completely cancelled at the second destination. The selection of encoding technique at each transmitter will be further explained in the next section. After this introduction, we are ready now to state the following theorem.

Theorem 1: The achievable rate region for the memoryless state-dependent IC is the convex-hull of the positive rate pairs (R_1, R_2)

$$R_1 \leq \psi_3 \quad (7a)$$

$$R_2 \leq \psi'_7 \quad (7b)$$

$$R_1 + R_2 \leq \min\{\psi_1 + \psi'_8\ \psi_2 + \psi'_6\ \psi_4 + \psi'_5\} \quad (7c)$$

$$2R_1 + R_2 \leq \psi_1 + \psi_4 + \psi'_6 \quad (7d)$$

$$R_1 + 2R_2 \leq \psi_2 + \psi'_5 + \psi'_8 \quad (7e)$$

for some joint probability distribution that factors as

$$P^* = p(u_{12})p(u_{11}|u_{12})p(x_1|u_{11}, u_{12}) \times p(u_{21})p(u_{22}|u_{21}, x_1)p(x_2|u_{21}, u_{22})$$

and for some positive rate vectors $(R_{11}, R_{12}, R_{21}, R_{22})$ satisfying

$$R_{11} \leq \psi_1 = I(X_1; Y_1|U_{12}, U_{21}) \quad (8a)$$

$$R_{21} + R_{11} \leq \psi_2 = I(U_{21}, X_1; Y_1|U_{12}) \quad (8b)$$

$$R_{12} + R_{11} \leq \psi_3 = I(X_1; Y_1|U_{21}) \quad (8c)$$

$$R_{21} + R_{12} + R_{11} \leq \psi_4 = I(U_{21}, X_1; Y_1) \quad (8d)$$

$$R_{22} \leq \psi'_5 = I(U_{22}; Y_2|U_{12}, U_{21}) - I(U_{22}; X_1|U_{12}, U_{21}) \quad (8e)$$

$$R_{12} + R_{22} \leq \psi'_6 = I(U_{12}, U_{22}; Y_2|U_{21}) - I(U_{22}; X_1|U_{12}, U_{21}) \quad (8f)$$

$$R_{21} + R_{22} \leq \psi'_7 = I(U_{22}; Y_2|U_{12}) - I(U_{22}; X_1|U_{12}, U_{21}) \quad (8g)$$

$$R_{12} + R_{21} + R_{22} \leq \psi'_8 = I(U_{12}, U_{22}; Y_2) - I(U_{22}; X_1|U_{12}, U_{21}) \quad (8h)$$

Proof: The proof is given in Appendix 1.
In brief, by using GP encoding, the second sender can bin its signal U_{22} such that the effect of the primary signal X_1 is mitigated at the second destination. Moreover, the first transmitter can allocate part of its power to diminish the interference (private part) effect from the second transmitter. □

Remarks 1: The achievable rate region in Theorem 1 is equivalent to the one in the following scenario: suppose the two senders in the IC are both cognitive and affected by a common interference source. Thus, by employing GP encoding, the effect of this common interference source can be completely mitigated if it is available non-causally at both the transmitters. Furthermore, one of the two cognitive users can employ DPC whereas the second may employ ZF-DPC.

In this section, the achievable rate region for the memoryless IC is obtained. The derivation assumes that, for a given user, a non-causal knowledge of the other user's signal is available. Next, these results are extended to the Gaussian cognitive IC. Furthermore, numerical examples are provided to illustrate the value of our theoretical results.

4 Gaussian cognitive IC

In this section, we develop the achievable rate region for the cognitive Gaussian IC. In particular, in this channel model, the received signal is a combination of the desired signal, additive interference from the other user and AWGN, as depicted in Fig. 1. In deriving the achievable rate region, a combination of DPC, ZF-DPC, HK-rate splitting and superposition encoding are used. In addition, we assume that a non-causal version of each user's signal is available to the other user. Furthermore, a non-equal crossover channel gains $|c_{21}| < |c_{12}|$ are assumed given that equal power is allocated to each sender. In this case, the first user can employ ZF-DPC and HK-rate splitting whereas the second user can combine DPC with HK-rate splitting. In other words, the user that has more interference from the other has to employ DPC at its sender.

Now, the Gaussian IC can be described by the following set of equations

$$Y_1 = X_1 + c_{21}X_2 + Z_1 \tag{9a}$$

$$Y_2 = X_2 + c_{12}X_1 + Z_2 \tag{9b}$$

where Y_i, $i \in \{1, 2\}$ is the received signal at the two cognitive receivers. In addition, Z_i, $i \in \{1, 2,\}$ is assumed to be independent and identically distributed with zero mean and variance that is, $N(0, N_i)$ Gaussian noise. Furthermore, the constants c_{21}, c_{12} represent the crossover channel gains as depicted in Fig. 1. Moreover, the channel state information is assumed to be globally available at all nodes. In this system, all transmit and receive nodes are equipped with a single antenna for transmission and reception. Finally, the average power of the transmitted signals is constrained by

$$E[X_j^2] \le P_j, \quad j = 1, 2 \tag{10}$$

We now characterise the achievable rate region of the Gaussian cognitive IC. In this construction, the first cognitive transmitter uses part of its power to reduce the effect of interference, $c_{21}X_2$, from the second cognitive interference by employing ZF-DPC. On the contrary, the second cognitive transmitter employs DPC such that the interference signal, $c_{12}U_{11}$, is completely cancelled. Thus, the interference at the second destination is completely removed. Thus, the two transmitters generate the following codewords

$$U_{12} = \sqrt{\alpha_{12}P_1}V_{12}(m_{12}) \tag{11a}$$

$$U_{11} = \sqrt{\alpha_{11}P_1}V_{11}(m_{11}) + \sqrt{\alpha_{12}P_1}V_{12}(m_{12}) \tag{11b}$$

$$X_1 = \sqrt{\alpha_{11}P_1}V_{11}(m_{11}) + \sqrt{\alpha_{12}P_1}V_{12}(m_{12}) - \sqrt{\frac{\gamma P_1}{P_2}}X_2 \tag{11c}$$

$$U_{21} = \sqrt{\alpha_{21}P_2}V_{21}(m_{21}) \tag{11d}$$

$$U_{22} = \sqrt{\alpha_{22}P_2}V_{22}(m_{22}) + \sqrt{\alpha_{21}P_2}V_{21}(m_{21}) \tag{11e}$$

$$U_{22} = X_2 + \beta c_{12}U_{11} \tag{11f}$$

where V_{12}, V_{11}, V_{21} and V_{22} are independent random variables with zero mean and unity variance, that is, $N(0, 1)$ used to encode m_{12}, m_{11}, m_{21} and m_{22}, respectively. Furthermore, the following parameters α_{12}, α_{11}, α_{21}, α_{22} and γ are the power allocation factors satisfying the following constraints

$$\alpha_{12} + \alpha_{11} + \gamma \le 1 \tag{12a}$$

$$\alpha_{21} + \alpha_{22} \le 1 \tag{12b}$$

Moreover, we remind that the received signals at the IC output destinations are as given in (9). Now, using the constructed codewords in (11) and (8), the achievable rate region in Theorem 1 can easily be derived as follows

$$\psi_1 = C\left(\frac{\alpha_{11}P_1}{N_1 + \left(|c_{21}|\sqrt{\alpha_{22}P_2} - \sqrt{\gamma P_1}\right)^2}\right)$$

$$\psi_2 = C\left(\frac{\alpha_{11}P_1 + |c_{21}|^2\alpha_{21}P_2}{N_1 + \left(|c_{21}|\sqrt{\alpha_{22}P_2} - \sqrt{\gamma P_1}\right)^2}\right)$$

$$\psi_3 = C\left(\frac{(\alpha_{11} + \alpha_{12})P_1}{N_1 + \left(|c_{21}|\sqrt{\alpha_{22}P_2} - \sqrt{\gamma P_1}\right)^2}\right)$$

$$\psi_4 = C\left(\frac{(\alpha_{11} + \alpha_{12})P_1 + |c_{21}|^2\alpha_{21}P_2}{N_1 + \left(|c_{21}|\sqrt{\alpha_{22}P_2} - \sqrt{\gamma P_1}\right)^2}\right)$$

$$\psi'_5 = C\left(\frac{\left(\sqrt{\alpha_{22}P_2} - |c_{12}|\sqrt{\gamma P_1}\right)^2}{N_2}\right)$$

$$\psi'_6 = C\left(\frac{\left(\sqrt{\alpha_{22}P_2} - |c_{12}|\sqrt{\gamma P_1}\right)^2 + |c_{12}|^2\alpha_{12}P_1}{N_2}\right)$$

$$\psi'_7 = C\left(\frac{\left(\sqrt{\alpha_{22}P_2} - |c_{12}|\sqrt{\gamma P_1}\right)^2 + \alpha_{21}P_2}{N_2}\right)$$

$$\psi'_8 = C\left(\frac{\left(\sqrt{\alpha_{22}P_2} - |c_{12}|\sqrt{\gamma P_1}\right)^2 + \alpha_{21}P_2 + |c_{12}|^2\alpha_{12}P_1}{N_2}\right)$$

where $C(x) = 0.5\log_2(1 + x)$.

Remarks 2: The value of γ is bounded as $0 \le \gamma \le |c_{21}|^2\alpha_{22}(P_2/P_1)$.

- The case of $\gamma = 0$ returns to the case of IC with unidirectional transmitters cooperation.
- The case of $\gamma = |c_{21}|^2\alpha_{22}(P_2/P_1)$ means that the interference signal is completely cancelled at the first destination by using ZF-DPC.

5 Sum rate maximisation and power allocation

In this section, we introduce the power allocation problem between the two cognitive users to maximise the achievable sum rate. In this problem, the total transmit power allocated to both cognitive transmitters is constrained to P_T, but varying the power allocated to each sender such that the sum rate is maximised. In the next section, we introduce the problem and the potential solutions. Then, we theoretically analyse this optimisation problem in Section 5.2.

5.1 Power allocation with numerical solution

In this section, we introduce the problem of allocating the transmit power to the two users. Mathematically, this optimisation problem can be expressed as

$$\begin{array}{ll} \text{maximise} & R_1 + R_2 \\ & 2R_1 + R_2 \\ & R_1 + 2R_2 \\ \text{subject to} & P_1 + P_2 \le P \end{array} \tag{13}$$

Clearly, this is a multi-objective optimisation problem [31]. In such a problem, where more than one objective function is available, mathematical theory reveals that there exists a set of solution points rather than a single optimal point [31, 32]. This solution can help the decision maker find the most preferred solution as the final one. Moreover, there is no mathematical solution to the optimisation problem in (14) since the first objective function can be only numerically solved. Therefore this problem will be numerically solved in Section 6, the numerical results. In addition, in the next section, this problem will be mathematically analysed after omitting the first objective function.

There are many techniques used to solve a multi-objective optimisation problem. These methods include, but are not limited to: (i) weighted sum method, (ii) lexicographic method and (iii) weighted product method. In brief, in weighted sum method, each objective function is given a weight then all of these objective functions are summed together to form an objective function. In lexicographic method, the objective functions are arranged based on their importance. Then, the optimisation problem under consideration is split into m optimisation problems where m is the number of the objective functions. Further, in weighted product method, each objective function is given an exponent based on the relative significance of the objective functions. Then, the product of all the objective functions may represent the problem's objective function. Finally, for a detailed treatment of the topic of optimisation with multi-objective function, we refer the reader to [31, 33].

5.2 Power allocation with a mathematical solution

In this section, we reformulate the optimisation problem in (14) to find a mathematical solution. Specifically, the first objective function is omitted then a mathematical solution is obtained

$$\begin{array}{ll} \text{maximise} & 2R_1 + R_2 \\ & R_1 + 2R_2 \\ \text{subject to} & P_1 + P_2 \le P \end{array} \tag{14}$$

In general, solving this optimisation problem contains two steps. Specifically, by using the lexicographic method [31, 32], the objective functions are arranged in order of importance. Initially, in the first step, the first objective function is optimised with regard to the set of constraints. In this step, the set of points and the segments between them, that maximise the first objective function, are determined [32]. Then, in the second step, we search for points belonging to these segments to find a solution that maximises the second objective function in the lexicographic order. Thus, the optimisation problem in (15) is divided into two separate optimisation problems. The first optimisation problem is

$$\begin{array}{ll} \text{maximise} & 2R_1 + R_2 \\ \text{subject to} & 0 \le P_1 \le P \end{array} \tag{15}$$

In this optimisation problem, the objective function $2R_1 + R_2$ is given as

$$\begin{aligned} 2R_1 + R_2 &\le \psi_1 + \psi_4 + \psi'_6 \\ &= C\left(\frac{\alpha_{11}P_1}{1 + (|c_{21}|\sqrt{\alpha_{22}P_2} - \sqrt{\gamma P_1})^2}\right) \\ &\quad + C\left(\frac{(\alpha_{11} + \alpha_{12})P_1 + c_{12}^2\alpha_{21}P_2}{1 + (|c_{21}|\sqrt{\alpha_{22}P_2} - \sqrt{\gamma P_1})^2}\right) \\ &\quad + C\left(\left(\sqrt{\alpha_{22}P_2} - c_{12}\sqrt{\gamma P_1}\right)^2 + c_{12}^2\alpha_{12}P_1\right) \\ &= 0.5\log_2\left(1 + \frac{\alpha_{11}P_1}{1 + (|c_{21}|\sqrt{\alpha_{22}P_2} - \sqrt{\gamma P_1})^2}\right) \\ &\quad + 0.5\log_2\left(1 + \frac{(\alpha_{11} + \alpha_{12})P_1 + c_{12}^2\alpha_{21}P_2}{1 + (|c_{21}|\sqrt{\alpha_{22}P_2} - \sqrt{\gamma P_1})^2}\right) \\ &\quad + 0.5\log_2\left(1 + \left(\sqrt{\alpha_{22}P_2} - c_{12}\sqrt{\gamma P_1}\right)^2 + c_{12}^2\alpha_{12}P_1\right) \end{aligned} \tag{16}$$

where, for simplicity, the noise variances N_1 and N_2 are normalised to 1. Because the logarithm is monotonically increasing function, it is sufficient to incorporate the first objective function as

$$\begin{aligned} J(P_1) = &\left(1 + \frac{\alpha_{11}P_1}{D}\right)\left(1 + \frac{(\alpha_{11} + \alpha_{12})P_1 + c_{12}^2\alpha_{21}(P - P_1)}{D}\right) \\ &\times \left(1 + \left(\sqrt{\alpha_{22}(P - P_1)} - c_{12}\sqrt{\gamma P_1}\right)^2 + c_{12}^2\alpha_{12}P_1\right) \end{aligned} \tag{17}$$

where $D = 1 + \left(|c_{21}|\sqrt{\alpha_{22}(P - P_1)} - \sqrt{\gamma P_1}\right)^2$. We need to note that $J(P_1)$ is not guaranteed to be a concave function of P_1. Thus, all possible boundary points and the extreme points should be searched to find the optimal power point (P_1^*, P_2^*). Therefore the values of $J(P_1)$ at the boundary points are given as

$$J(P_1 = 0) = \left(1 + \frac{c_{12}^2\alpha_{21}P}{D}\right)(1 + \alpha_{22}P) \tag{18a}$$

$$\begin{aligned} J(P_1 = P) = &\left(1 + \frac{\alpha_{11}P}{D}\right)\left(1 + \frac{(\alpha_{11} + \alpha_{12})P}{D}\right) \\ &\times \left(1 + c_{12}^2(\gamma + \alpha_{12})P\right) \end{aligned} \tag{18b}$$

Now, to find the extreme points, let us define $J(P_1) = C/D^2$, then $J' = (\partial J/\partial P_1) = ((D^2C' - CD^{2'})/D^4)$. Hence, since D^4 is always positive, then $\partial J/\partial P_1 = 0$ is equivalent to solving $D^2C' - CD^{2'} = 0$. In particular, solving $D^2C' - CD^{2'} = 0$ is equivalent to find the roots of (20), where

$$c_1 = (g_1g_3 - 4\gamma\alpha_{22}c_{21}^2)g_5 + 2\gamma\alpha_{22}c_{21}(g1 + g3) \quad (19a)$$

$$c_2 = (g_1g_3 - 4\gamma\alpha_{22}c_{21}^2)g_6 + g_5(g_1g_4 + g_2g_3 - 4\gamma\alpha_{22}c_{21}^2P) + 2c_{21}(g_4 + g_2) - 2c_{21}(g_1 + g_3)P \quad (19b)$$

$$c_3 = g_6(g_1g_4 + g_2g_3 - 4\gamma\alpha_{22}c_{21}^2P) + g_2g_4g_5 - 2c_{21}(g_4 + g_2)P \quad (19c)$$

$$c_4 = -2c_{12}(g_1g_3 - 4\gamma\alpha_{22}c_{21}^2) - 2c_{21}(g_1 + g_3)g_5 \quad (19d)$$

$$c_5 = -2c_{12}(g_1g_4 + g_2g_3 - 4\gamma\alpha_{22}c_{21}^2P) - 2c_{21}(g_4 + g_2)g_5 - 2c_{21}g_6(g_1 + g_3) \quad (19e)$$

$$c_6 = -2g_6c_{21}(g_4 + g_2) - 2c_{12}g_2g_4 \quad (19f)$$

and

$$\begin{aligned} g_1 &= \gamma - c_{21}^2\alpha_{22} \\ g_2 &= 1 + c_{21}^2\alpha_{22}P \\ g_3 &= 1 - c_{12}^2\alpha_{21} - c_{21}^2\alpha_{22} \\ g_4 &= 1 + (c_{21}^2\alpha_{22} + c_{12}^2\alpha_{21})P \\ g_5 &= (c_{12}^2(\gamma + \alpha_{12}) - \alpha_{22}) \\ g_6 &= 1 + \alpha_{22}P \end{aligned}$$

Now, from (20), the extreme points P_1 which are belonging to $[0, P]$ can be identified and then $J(P_1)$ at those points can be computed using (17). In the case that there is more than one extreme point, then these extreme points and the segments between them forms the solution. Then, the points which are belonging to the given line segments that maximise the second objective function are selected. We note that if the solution of the first optimisation problem is a single point, then this point is the solution of the whole optimisation problem. Let $P_1^{(1)}$ and $P_1^{(2)}$ be two extreme points that maximise the first objective function, $2R_1 + R_2$, then the line segment represented by $\lambda P_1^{(1)} + (1 - \lambda)P_1^{(2)}$, where $0 \le \lambda \le 1$, is the constraint for the second optimisation problem. In particular, the second optimisation problem is given as

$$\begin{aligned} &\text{maximise} \quad R_1 + 2R_2 \\ &\text{subject to} \quad 0 \le P_2 \le (P - P_1) \\ &P_2 \in \Omega_{P_1} \end{aligned} \quad (21)$$

where Ω_{P_1} is the set of all real roots and the segments between them. Now, the second the objective function of this optimisation problem is given in (22)

where $M = 1 + (|c_{21}|\sqrt{\alpha_{22}P_2} - \sqrt{\gamma P_1})^2$ and $T = (\sqrt{\alpha_{22}P_2} - |c_{12}|\sqrt{\gamma P_1})^2$. Again, since the logarithm is monotonically increasing function, it is sufficient to incorporate the second objective function given in (23)

Here, $J(P_2)$ values at the boundary at the boundary are given as follows

$$J(P_2 = 0) = \left(1 + \frac{\alpha_{11}P}{M}\right)(1 + c_{12}^2\gamma P)\left(1 + c_{12}^2(\gamma + \alpha_{12})P\right) \quad (24a)$$

$$J(P_1 = P) = \left(1 + \frac{c_{21}^2\alpha_{21}P}{M}\right)(1 + \alpha_{22}P)(1 + P) \quad (24b)$$

To find the extreme points, let us define $J(P_2) = N/M$, then $J' = (\partial J/\partial P_1) = (MN' - NM')/M^2$. In this formulation, M^2 is always positive. Thus, it is sufficient to solve $MN' - NM' = 0$. Specifically, solving $D^2C' - CD^{2'} = 0$ is equivalent to

$$\left(d_1P_1 + d_1^2 - 2c_{21}\sqrt{\gamma\alpha_{22}P_1(P - P_1)}\right)^2\left(c_1P_1^3 + c_2P_1^2 + c_3P_1 + \sqrt{\gamma\alpha_{22}P_1(P - P_1)}[c_4P_1^2 + c_5P_1 + c_6] + g_2k_2l_2\right)' - \left(d_1P_1 + d_1^2 - 2c_{21}\sqrt{\gamma\alpha_{22}P_1(P - P_1)}\right)^{2'}\left(c_1P_1^3 + c_2P_1^2 + c_3P_1 + \sqrt{\gamma\alpha_{22}P_1(P - P_1)}[c_4P_1^2 + c_5P_1 + c_6] + g_2k_2l_2\right) = 0 \quad (20)$$

$$\begin{aligned} R_1 + 2R_2 &\le \psi_2 + \psi'_5 + \psi'_8 \\ &= C\left(\frac{\alpha_{11}P_1 + |c_{21}|^2\alpha_{21}P_2}{M}\right) + C\left(\frac{\left(\sqrt{\alpha_{22}P_2} - |c_{12}|\sqrt{\gamma P_1}\right)^2}{1}\right) + C\left(\frac{\left(\sqrt{\alpha_{22}P_2} - |c_{12}|\sqrt{\gamma P_1}\right)^2 + \alpha_{21}P_2 + |c_{12}|^2\alpha_{12}P_1}{1}\right) \\ &= 0.5\log_2\left(1 + \frac{\alpha_{11}P_1 + |c_{21}|^2\alpha_{21}P_2}{M}\right) + 0.5\log_2(1 + T) + 0.5\log_2\left(1 + T + \alpha_{21}P_2 + |c_{12}|^2\alpha_{12}P_1\right) \end{aligned} \quad (22)$$

$$J(P_2) = \left(1 + \frac{\alpha_{11}P_1 + |c_{21}|^2\alpha_{21}P_2}{M}\right)\left(1 + \frac{\left(\sqrt{\alpha_{22}P_2} - |c_{12}|\sqrt{\gamma P_1}\right)^2}{1}\right)\left(1 + \frac{\left(\sqrt{\alpha_{22}P_2} - |c_{12}|\sqrt{\gamma P_1}\right)^2 + \alpha_{21}P_2 + |c_{12}|^2\alpha_{12}P_1}{1}\right) \quad (23)$$

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find the roots of (27), where

$$m_1 = -2\sqrt{\gamma\alpha_{22}P_1}(c_{12}k_1 + c_{21}k_3) - 2c_{12}k_1k_3 \tag{25a}$$

$$\begin{aligned} m_2 = {} & k_1k_3k_5 + (k_1k_4 + k_2k_3 + 4c_{12}c_{21}\gamma\alpha_{22}P_1) \\ & + 4\gamma\alpha_{22}P_1(c_{12}k_2 + c_{21}k_4)c_{12} \end{aligned} \tag{25b}$$

$$\begin{aligned} m_3 = {} & -2\sqrt{\gamma\alpha_{22}P_1}\big((c_{12}k_1 + c_{21}k_3)k_5 + (c_{12}k_2 + c_{21}k_4)\big) \\ & - 2c_{12}\sqrt{\gamma\alpha_{22}P_1}(k_1k_4 + k_2k_3 + 4c_{21}c_{12}\gamma\alpha_{22}P_1) \end{aligned} \tag{25c}$$

$$\begin{aligned} m_4 = {} & (k_1k_4 + k_2k_3 + 4c_{21}c_{12}\gamma\alpha_{22}P_1)k_5 \\ & + 4c_1(\gamma\alpha_{22}P_1)(c_{12}k_2 + c_{21}k_4) + k_2k_4 \end{aligned} \tag{25d}$$

$$m_5 = -2\sqrt{\gamma\alpha_{22}P_1}(c_{12}k_2 + c_{21}k_4)k_5 - 2c_{12}k_2k_4\sqrt{\gamma\alpha_{22}P_1} \tag{25e}$$

and

$$k_1 = c_{21}^2 \tag{26a}$$

$$k_2 = (\gamma + \alpha_{11})P_1 + 1 \tag{26b}$$

$$k_3 = \alpha_{22} \tag{26c}$$

$$k_4 = c_{12}^2\gamma P_1 + 1 \tag{26d}$$

$$k_5 = c_{12}^2(\gamma + \alpha_{12})P_1 + 1 \tag{26e}$$

Finally, after solving (20) and then (27), the point (P_1^*, P_2^*) that maximises the objective functions in (15) is obtained. We have to recall that this point should satisfy the condition $|c_{21}|^2P_2 < |c_{12}|^2P_1$. Otherwise, the first user should employ DPC and the second user may use ZF-DPC.

6 Numerical examples

In this section, we numerically evaluate the achievable rate region over the two users cognitive Gaussian IC. Unless otherwise specified, we initially set the channel gains as follows: $c_{11} = c_{22} = 1$, $c_{12} = 0.8$ and $c_{21} = 0.6$. Further, we set the power constraints P_1 and P_2 to 100. Finally, we note that the achievable rates here are given in bits per second/channel use (bps).

Fig. 3 shows the achievable rate region for four different scenarios. First, we compare our results with the best ever known achievable rate region for the Gaussian IC, which was derived by Han and Kobabyshi in [3]. Second, the achievable rate region for the cognitive Gaussian IC with unidirectional cooperation, which was derived by Chung *et al.* in [34], is shown. Third, the Gaussian IC with conferencing transmitters [5] is also presented. Fourth, the achievable rate region, which is derived in this paper is also shown. From this figure, it is clear that the achievable rate region can be improved by using two cognitive nodes instead of one.

Now, we turn to the sum rate maximisation, numerically and theoretically. In this example, the total power of both cognitive senders is fixed to $P_T = 20$ but varying the power allocated to each transmitter. Figs. 4 and 5 show how the sum rate and the associated power allocation are varying with the cross-over channel gain, c_{21}. The sum rate reaches its maximum for the case that $c_{21} = 0$. In this case, the interference from the first user is completely removed by employing DPC at the second sender. In addition, the first user does not suffer any interference from the second user. Therefore the total power is equally divided between the two users. On the contrary, for $c_{21} = 1$, the first user has to spend more power in order to cancel the interference that is introduced by the second user. Thus, the total power is only allocated to the first user. In this case, the sum rate reaches its minimum, which is $0.5\log_2(1 + 20) = 2.1962$.

Further, Fig. 5 indicates that the numerical solution is matching the theoretical result-1. In the theoretical work,

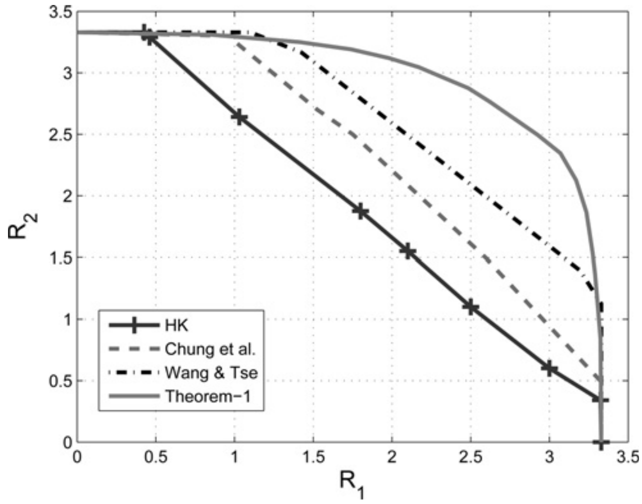

Fig. 3 *Achievable rate region of the cognitive Gaussian dependent IC*

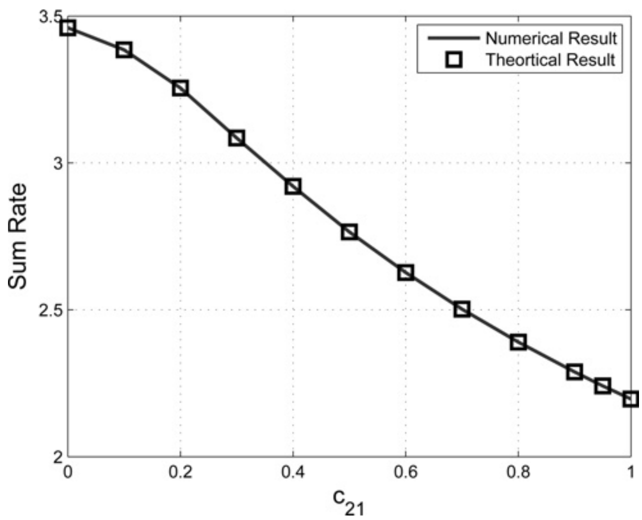

Fig. 4 *Achievable sum rate of the cognitive Gaussian IC as a function of the crossover channel gain c_{21}*

$$\begin{aligned} & \left(c_{21}\alpha_{22}u^2 - 2c_{21}\sqrt{\gamma\alpha_{22}P_1}u + (\gamma P_1 + 1)\right)^2 (g_1k_1l_1u^6 + m_1u^5 + m_2u^4 + m_3u^3 + m_4u^2 + m_5u + g_2k_2l_2) \\ & - \left(c_{21}\alpha_{22}u^2 - 2c_{21}\sqrt{\gamma\alpha_{22}P_1}u + (\gamma P_1 + 1)\right)^{2'} (g_1k_1l_1u^6 + m_1u^5 + m_2u^4 + m_3u^3 + m_4u^2 + m_5u + g_2k_2l_2) = 0 \end{aligned} \tag{27}$$

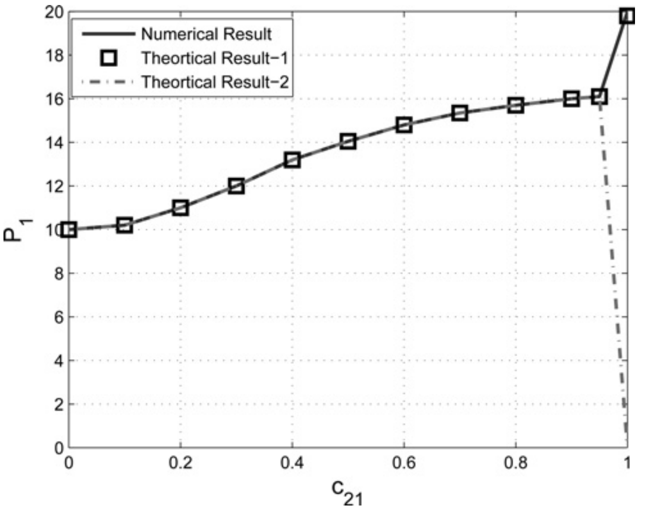

Fig. 5 *Allocation of the total power between the two users such that the sum rate is maximised*

the obtained point $\left(P_1^*, P_2^*\right)$ is substituted in all the objective functions in (14). Theoretical result-1 refers to the case of maximising $2R_1 + R_2$ as a function of P_1 at first whereas theoretical result-2 refers to the case of maximising $R_1 + 2R_2$ as function of P_2 at first. Thus, optimising $2R_1 + R_2$ at first allocates the total power to the first user in the case of $c_{21} = 1$. However, optimising $R_1 + 2R_2$ at first, the total power is allocated to the second user in the case of $c_{21} = 1$. At the end, for $c_{21} = 1$, it does not matter which user should use the total power since c_{11} and c_{22} are normalised to 1. Thus, the total power is allocated to only one user.

7 Concluding remarks

In this paper, the achievable rate region of the cognitive IC, in the case that a non-causal version of a given user's signal is available at the other user, has been characterised. In particular, a combination of random coding techniques such as rate splitting, GP encoding scheme superposition encoding and ZF-DPC have been used to develop the achievable rate region. Then, the achievable rate region has been extended to the additive Gaussian case. In addition, the sum rate maximisation and the associated power allocation between the two users have numerically and theoretically investigated. Finally, numerical results that demonstrate the validity of our construction have been shown.

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9 References

1 Carleial, A.: 'A case where interference does not reduce capacity (Corresp.)', *IEEE Trans. Inform. Theory*, 1975, **21**, pp. 569–570
2 Sato, H.: 'The capacity of the Gaussian interference channel under strong interference (Corresp.)', *IEEE Trans. Inform. Theory*, 1981, **27**, pp. 786–788
3 Han, T., Kobayashi, K.: 'A new achievable rate region for the interference channel', *IEEE Trans. Inform. Theory*, 1981, **27**, pp. 49–60
4 Maric, I., Yates, R., Kramer, G.: 'Capacity of interference channels with partial transmitter cooperation', *IEEE Trans. Inform. Theory*, 2007, **53**, pp. 3536–3548
5 Wang, I.-H., Tse, D.: 'Interference mitigation through limited transmitter cooperation', *IEEE Trans. Inform. Theory*, 2011, **57**, (5), pp. 2941–2965
6 Prabhakaran, V., Viswanath, P.: 'Interference channels with source cooperation', *IEEE Trans. Inform. Theory*, 2011, **57**, (1), pp. 156–186
7 Wang, I.-H., Tse, D.: 'Interference mitigation through limited receiver cooperation', *IEEE Trans. Inform. Theory*, 2011, **57**, pp. 2913–2940
8 Devroye, N., Mitran, P., Tarokh, V.: 'Achievable rates in cognitive radio channels', *IEEE Trans. Inform. Theory*, 2006, **52**, pp. 1813–1827
9 Jovicic, A., Viswanath, P.: 'Cognitive radio: an information-theoretic perspective', *IEEE Trans. Inform. Theory*, 2009, **55**, pp. 3945–3958
10 Motahari, A., Khandani, A.: 'Capacity bounds for the Gaussian interference channel', *IEEE Trans. Inform. Theory*, 2009, **55**, pp. 620–643
11 Li, Q., Li, K.H., The, K.C.: 'An achievable rate region for the cognitive interference channel with causal bidirectional cooperation', *IEEE Trans. Veh. Technol.*, 2010, **59**, pp. 1721–1728
12 Sridharan, S., Vishwanath, S., Jafar, S., Shamai, S.: 'On the capacity of cognitive relay assisted Gaussian interference channel'. IEEE Int. Symp. Inform. Theory, Jul. 2008, pp. 549–553
13 Rini, S., Tuninetti, D., Devroye, N., Goldsmith, A.: 'On the capacity of the interference channel with a cognitive relay'. CoRR, vol. abs/1107.4600, 2011
14 Sahin, O., Erkip, E.: 'Achievable rates for the gaussian interference relay channel'. IEEE Global Telecommunication Conf., November 2007, pp. 1627–1631
15 Chaaban, A., Sezgin, A.: 'Achievable rates and upper bounds for the interference relay channel'. Asilomar Conf. on Signals, Systems and Computers (ASILOMAR), November 2010, pp. 267–271
16 Gelfand, S., Pinsker, M.: 'Coding for channel with random parameters'. Problems of Control and Inform. Theory, 1980
17 Cover, T.: 'Broadcast channels', *IEEE Trans. Inform. Theory*, 1972, **18**, pp. 2–14
18 Bagheri, H., Motahari, A., Khandani, A.: 'Zero-forcing for the symmetric interference channel with conferencing encoders'. IEEE Int. Symp. on Information Theory, 2010, pp. 370–374
19 Costa, M.: 'Writing on dirty paper (Corresp.)', *IEEE Trans. Inform. Theory*, 1983, **29**, pp. 439–441
20 Dabbagh, A., Love, D.: 'Precoding for multiple antenna gaussian broadcast channels with successive zero-forcing', *IEEE Trans. Signal Process.*, 2007, **55**, (7), pp. 3837–3850
21 Gaur, S., Acharya, J., Gao, L.: 'Enhancing ZF-DPC performance with receiver processing', *IEEE Trans. Wirel. Commun.*, 2011, **10**, (12), pp. 4052–4056
22 Tran, L.-N., Juntti, M., Bengtsson, M., Ottersten, B.: 'Weighted sum rate maximization for MIMO broadcast channels using dirty paper coding and zero-forcing methods', *IEEE Trans. Commun.*, 2013, **61**, (6), pp. 2362–2373
23 Zhang, L., Jiang, J., Cui, S.: 'Achievable rate regions for discrete Memoryless interference channel with state information'. IEEE Int. Conf. Communications, pp. 1–5, June
24 Ghasemi-Goojani, S., Behroozi, H.: 'On the sum-capacity and lattice-based transmission strategies for state-dependent Gaussian interference channel'. Int. Symp. Personal Indoor and Mobile Radio Commun. (PIMRC), September 2012, pp. 1944–1948
25 Zhang, L., Cui, S., Jiang, J.: 'Gaussian interference channel with state information'. IEEE Wireless Communication and Networking Conf. (WCNC), 2011, pp. 1960–1965
26 Caire, G., Shamai, S.: 'On the capacity of some channels with channel state information', *IEEE Trans. Inform. Theory*, 1999, **45**, pp. 2007–2019
27 Yu, W., Cioffi, J.: 'Sum capacity of Gaussian vector broadcast channels', *IEEE Trans. Inform. Theory*, 2004, **50**, pp. 1875–1892
28 Hon-Fah Chong, H., Motani, M., Garg: 'A Comparsion of two achievable rate regions for the interference channel'. IEEE Int. Symp. Information Theory, 2007
29 Chong, H.-F., Motani, M., Garg, H., El-Gamal, H.: 'On the Han-Kobayashi region for the interference channel', *IEEE Trans. Inform. Theory*, 2008, **54**, pp. 3188–3195
30 Kramer, G.: 'Review of rate regions for interference channels'. Int. Seminar on Communication, February 2006, pp. 162–165
31 Marler, R., Arora, J.: 'Survey of multi-objective optimization methods for engineering', *Struct. Multidisc. Optim.*, 2004, **26**, pp. 369–395
32 Stanimirovic, I.P.: 'Compendious lexicographic method for multi-objective optimization', *Ser. Math. Inform.*, 2012, **27**, pp. 55–66
33 Minella, G., Ruiz, R., Ciavotta, M.: 'A review and evaluation of multiobjective algorithms for the flowshop scheduling problem', *J. Inform. Comput.*, 2008, **20**, (3), pp. 451–471
34 Chung, G., Sridharan, S., Vishwanath, S., Hwang, C.-S.: 'On the capacity of overlay cognitive radios with partial cognition', *IEEE Trans. Inform. Theory*, 2012, **58**, pp. 2935–2949

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10 Appendix 1: proof of Theorem 1

In this appendix, we prove Theorem 1 in which the interference signal from the primary user is non-causally available at both the cognitive transmitter and the relay.

Random codebook generation: During block index $i \in \{1, 2\}$, source j divides its message $m_j[i] \in \{1, \ldots, 2^{nRj}\}$ into two independent parts m_{j1} and m_{j2}. In this formulation, m_{jj} represents the private signal, for $j \neq k$, $k = 1, 2$, m_{jk} represents the public signal.

1. The first source generates the following signals

   - Generate $2^{nR_{12}}$ codewords u_{12}^n according to $\prod_{k=1}^n p(u_{12,k})$. Label these codewords as $u_{12}^n(m_{12})$, $m_{12} \in \left[1, 2^{nR_{12}}\right]$.
   - For each $u_{12}^n(m_{12})$, independently generate $2^{nR_{11}}$ codewords x_1^n according to $\prod_{k=1}^n p(x_{1,k}|u_{12,k})$. Label these codewords as $x_1^n(m_{11}, m_{12}, m_{22})$, $m_{11} \in [1, 2^{nR_{11}}]$.

2. The second source generates the following signals:

   - Generate $2^{nR_{21}}$ codewords u_{21}^n according to $\prod_{k=1}^n p(u_{21,k})$. Label these codewords as $u_{21}^n(m_{21})$, $m_{21} \in [1, 2^{nR_{21}}]$.
   - For each $u_{21}^n(m_{21})$, independently generate $2^{n(R_{22}+R'_{22})}$ codewords u_{22}^n according to $\prod_{k=1}^n p(u_{22,k}|u_{12,k})$. Label these codewords as $u_{22}^n(m_{21}, m_{22}, v_{22})$, $m_{22} \in [1, 2^{nR_{22}}]$ and $v_{22} \in [1, 2nR'_{22}]$. Here, R'_{22} is the binning rate and v_{22} is the GP random binning codeword.
   - For each pair of $u_{21}^n(.)$ and $u_{22}^n(.)$, generate x_2^n according to $\prod_{k=1}^n p(x_{2,k}|u_{21,k}, u_{22,k})$. Label these codewords as $x_2^n(m_{21}, m_{22}, v_{22})$.

Encoding: We now describe the encoding process at both the first and second transmitters for the ith block. Let m_{11}, m_{12} and m_{21}, m_{22} be the messages to be sent from the primary and cognitive transmitters, respectively.

- The first transmits the signal $x_1^n\left(m_{22}, m_{11}|m_{12}\right)$.
- The second transmitter, using GP random binning scheme, searches for a bin index v_{22} so that $\left(x_1^n(m_{11}|m_{12}), u_{21}^n(m_{21}), u_{22}^n(m_{22}, v_{22}|m_{21})\right) \in A_\epsilon^n(P_{X_1 U_{22}|U_{21}})$ where A_ϵ^n is the set of jointly typical sequences of length n, such that $v_{22[i]}$ exists with high probability if

$$R'_{22} \geq I\left(U_{22}; X_1|U_{12}, U_{21}\right) \tag{28}$$

Briefly, in GP encoding technique, for a given state vector X_1 and the message m_{22}, the cognitive transmitter searches for a sequence U_{22} such that the pair (U_{22}, X_1) is jointly typical. In addition, if the number of sequences in bin m_{22} is larger than $2^{I\left(U_{22};X_1|U_{12},U_{21}\right)}$, then the probability of finding such U_{22} tends to zero for sufficiently large n [19]. Finally, this cognitive transmitter sends $x_2^n(m_{21}, m_{22}, v_{22})$.

Decoding: The decoding procedure at the both destinations are as follows

- The primary destination searches for a unique (m_{11}, m_{12}) for some m_{21} such that $\left(u_{12}^n(m_{12}), x_1(m_{11}|m_{12}), u_{21}^n(m_{21}), y_1\right) \in A_\epsilon^n\left(P_{X_1 Y_1|U_{12}U_{21}}\right)$. The decoding error probability can be arbitrary small if n is sufficiently large and [29]

$$R_{11} \leq I(X_1; Y_1|U_{12}, U_{21}, Q) \tag{29a}$$

$$R_{21} + R_{11} \leq I(U_{21}, X_1; Y_1|U_{12}, Q) \tag{29b}$$

$$R_{12} + R_{11} \leq I(X_1; Y_1|U_{21}, Q) \tag{29c}$$

$$R_{12} + R_{21} + R_{11} \leq I(U_{21}, X_1; Y_1|Q) \tag{29d}$$

Here, after decoding the public messages U_{12} and U_{21}. Then, the private signal U_{11} can be obtained using (29a) with high probability.

- The cognitive destination looks for a unique (m_{21}, m_{22}) for some m_{12} such that $\left(u_{21}^n(m_{21}), u_{22}^n(m_{22}, v_{22}|m_{21}), u_{12}^n(m_{12}), y_2\right) \in A_\epsilon^n\left(P_{U_{22}Y_2|U_{12}U_{21}}\right)$ with arbitrarily small probability of error for sufficiently large n and [29]

$$R_{22} + R'_{22} \leq I(U_{22}; Y_2|U_{12}, U_{21}, Q) \tag{30a}$$

$$R_{12} + R_{22} + R'_{22} \leq I(U_{12}, U_{22}; Y_2|U_{21}, Q) \tag{30b}$$

$$R_{21} + R_{22} + R'_{22} \leq I(U_{22}; Y_2|U_{12}, Q) \tag{30c}$$

$$R_{12} + R_{21} + R_{22} + R'_{22} \leq I(U_{12}, U_{22}; Y_2|Q) \tag{30d}$$