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Short communication

Allocation of resources for the Gaussian multiple access channel with practical partial cooperation

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ABSTRACT

A Gaussian multiple access channel, with partial cooperation between sources, is considered. We develop an encoding scheme in which the transmission is carried out over three orthogonal time phases. The first two phases are exploited such that the two sources can practically and partially exchange their messages. In particular, each phase is terminated when each user can generate the other user's codeword. Then, the two users can cooperatively transmit in the third phase. This formulation is used to (i) develop the achievable rate region, and (ii) numerically study the importance of each phase's length and the allocated power to each user in the three characterized transmission phases.

1. Introduction

Increasing and improving achievable rates, of different users in any multiuser channel, is a major problem in today's wireless communication networks. Two main techniques to attain such goals are cooperation and resource allocation. Optimization of resource allocation like power and bandwidth was extensively studied in many cooperative scenarios like relay channel and multiple access channel (MAC). For instance, the authors in [1] investigated the optimum power and time for a relay channel with decode-and-forward strategy. In addition, allocating the right power and time for the multiple parallel relay channel was investigated in [2]. Further, the authors in [3,4] studied the problem of allocating power to the two users forming the cooperative MAC. For instance, the authors in [4] divided the available user power between the cooperative phase and the transmission phase such that the sum rate is maximized.

In the simplest cooperative networks, an intermediate node is employed to help the source–destination pair, as in the relay channel [5]. This method of cooperation was widely investigated in many different scenarios such as the relay broadcast channel [6], multiple access relay channel [7], and cooperative MAC [8,9]. For instance, the author in [8] developed the capacity region of the MAC channel with partial cooperative encoders. In this setting, the encoders are assumed to be connected via links with finite capacities. Further, the authors in [10] considered the case in which each encoder, sharing the MAC, has the ability to learn the other user's codeword before transmission. In particular, they derived how much of the codewords the encoders should learn and they also obtained the associated capacity region. However, the established capacity regions in [8,10] are upper bounds since the sources do not spend power and bandwidth to develop common information. In a recent study, the authors in [9] considered the MAC channel, with partial cooperative sources, in which each source can divide its signal into private and public parts. In their cooperating encoding scheme, each user can fully decode and then forward the public part of the other user.

In this paper, the MAC channel in which two sources want to communicate with a common destination is considered. This communication scenario may model two phones communicating with their respective receiver i.e., base station or access point. In the considered channel model, at first, the encoders can partially and practically exchange their information, and can then cooperatively transmit to their common destination. In particular, the transmission period is divided into three phases. Specifically, one of the two sources can transmit such that the other user can capture its signal at the end of the first phase. Then, in the second phase, the second source can transmit until the first source can get the second source's signal. We remind that the scenarios in [8,10] neither spend power nor bandwidth during the cooperative phases. In the last phase, the two sources can cooperate to transmit their messages. In this case, the achievable rate region is initially obtained. Then, resources such as bandwidth and power are optimized such that the achievable rate region is maximized.

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Specifically, the duration of each phase and the associated power are considered. Our numerical results show that as the number of channel uses allocated to phase 3 increases, so does the achievable rate region. Moreover, under certain conditions, as the duration of either the first or the second phase reduces, the other user may allocate more power for cooperation.

The considered channel model is presented in Section 2. Next, the achievable rate is derived in Section 3. Then, in Section 4, some numerical examples are studied to show the value of our theoretical results. Finally, the letter is concluded in Section 5.

2. Multiple access channel with partial decoding

The Gaussian cooperative MAC, as shown in Fig. 1, is considered. In its simplest form, this channel model is composed of two sources, S_1 and S_2, that want to communicate with a common destination. These two sources are assumed to operate in half-duplex mode since a node cannot simultaneously transmit and receive. Each source, $S_k, k \in \{1, 2\}$ wants to send a message, M_k, which is uniformly selected from the set $M_k = [1, e^{nR_k}]$ to a destination over n uses of the channel such that the probability of error vanishes for sufficiently large n. This communication is performed over three orthogonal time phases. Initially, the two transmitters may partially exchange their information and also transmit to their destination in the first two phases. In addition, suppose that each user knows the other user's code-book. Then, each user may partially listen to the other user until it can generate the other user's entire codeword. For instance, in the first phase, which lasts for l_1 uses of the channel, only the source, S_1, may transmit to both S_2 and the common receiver. Thus, the received signals at the source, S_2, and the destination are respectively described as

$$Y_{12} = g_{12}\sqrt{\mu_1}X_1 + Z_2, \quad (1)$$

$$Y_{D1} = h_1\sqrt{\mu_1}X_1 + Z_D.$$

where X_1 is the transmitted signal by the source S_1. This signal is subjected to average power P_1 and the factor μ_1 determines the power allocated to transmit X_1 in the first phase. Indeed, g_{12} is the channel gain between the two sources, h_1 is the channel gain between S_1 and the common destination. Further, Z_2 and Z_D are the additive white Gaussian noises (AWGN) at the source S_2 and the destination, respectively. These noises are independent and have identical distributions with zero mean and variances normalized to 1 through this letter.

Similarly, during the second phase, which lasts for $(l_2 - l_1)$ uses of the channel, only source S_2 can transmit such that the received signals by the source S_1 and the destination are respectively modelled as

$$Y_{21} = g_{21}\sqrt{\zeta_2}X_2 + Z_1, \quad (2)$$
$$Y_{D2} = h_2\sqrt{\zeta_2}X_2 + Z_D.$$

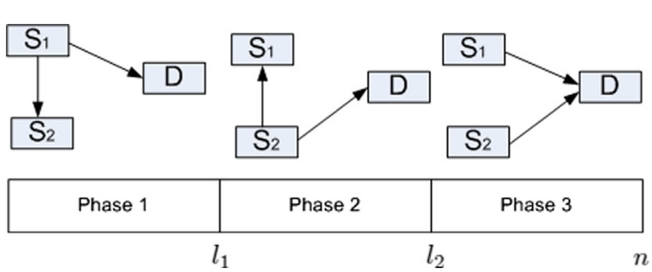

Fig. 1. MAC channel with a three-phase transmission scheme. The duration of first-, second- and third-phases are $l_1, (l_2 - l_1)$, and $(n - l_2)$ uses of the channel, respectively.

where X_2 is the signal generated by S_2. The average transmit power by this source is constrained by P_2 , and the factor ζ_2 determines the allocated power to transmit X_2 in the second phase. Further, g_{21} and h_2 are the channel gains from S_2 to the source S_1 and the destination, respectively. Moreover, Z_1 is an AWGN with zero mean and unit variance.

Finally, in the last phase, $(n - l_2)$ uses of the channel are used such that the two sources can cooperate to send their signals. This can be obtained by employing superposition encoding [11] at each source. Therefore, the sources S_1 and S_2 can generate the following two signals, respectively,

$$X_{13} = \sqrt{\mu_3\alpha}X_1 + \sqrt{\frac{\mu_3\bar{\alpha}P_1}{P_2}}X_2, \quad (3)$$

$$X_{23} = \sqrt{\zeta_3\beta}X_2 + \sqrt{\frac{\zeta_3\bar{\beta}P_2}{P_1}}X_1,$$

where α and β are the power allocation factors used by S_1 and S_2, respectively, in phase 3 to transmit their signals. In addition, $\bar{\alpha} = (1 - \alpha)$, and $\bar{\beta} = (1 - \beta)$ are also used by S_1 and S_2, respectively, to forward the other user's signal. Further, μ_3 and ζ_3 determine the total allocated power to S_1 and S_2 during the third phase, respectively. In this case, the received signal at the destination is expressed as

$$Y_{D3} = h_1X_{13} + h_2X_{23} + Z_D. \quad (4)$$

Remark-1: Note that the sources, S_1, and S_2, do not transmit in the second and the first phases, respectively. Thus, each source should properly allocate its power between the two phases. For instance, the source S_1 may allocate μ_1P_1 in the first phase with transmission period l_1, and μ_3P_1 in the third phase which lasts for $(n - l_2)$ uses. Thus, the average power is given as

$$\left(\frac{l_1}{n}\right)\mu_1P_1 + \left(\frac{n - l_2}{n}\right)\mu_3P_1 = P_1. \quad (5)$$

A similar analysis may be used to understand the power allocation at S_2. In this case, the average power is given by

$$\left(\frac{l_2 - l_1}{n}\right)\zeta_2P_2 + \left(\frac{n - l_2}{n}\right)\zeta_3P_2 = P_2. \quad (6)$$

3. Achievable rate region of MAC with partial cooperation

In this section, we develop a three-phase transmission scheme such that the two users can exchange their signals and also transmit to their destination. In particular, the first two phases enable the users to partially exchange their messages. For example, each user may listen to the other source until it can completely generate its signal. Then, in the last phase, the two users can cooperatively transmit to their receiver. Here, we outline the signalling and the associated achievable rates in the three phases.

Phase 1: In this phase, the source S_1 is only allowed to transmit. This transmission can be captured by both S_2 and the common destination. The source S_2 is required to partially listen to S_1 until it can generate the entire codeword of S_1. Further, the common destination may decode the signal X_1 at the end of phase 3. In this case, the received signals are previously given in (1). At the end of this phase, and depend on the first l_1 symbols, the source S_2 may decode the signal X_1 if

$$R_{S_1S_2} \leqslant \left(\frac{l_1}{n}\right)C\left(|g_{12}|^2\mu_1P_1\right) = I_1 \quad (7)$$

where $C(x) = \frac{1}{2}\log(1 + x)$. This decoding may be achieved at the source S_2 with probability of error upper bounded by $P_{e,S_2} \leqslant e^{-l_1E(G,l_1)}$ in which the Gallager's random coding exponent, $E(G, l_1)$, is expressed by [12]

$$E(m,G) = \max_{r\geqslant 1, 0\leqslant\sigma\leqslant 1} rG(1+\sigma) + \frac{1}{2}\log(1-2rG) + \frac{\sigma}{2} \times \log\left(1-2rG+\frac{G}{1+\sigma}\right) - \frac{n}{l_1}R_{S_1S_2} \tag{8}$$

where r and σ are two free parameters to be optimized. Further, $G = |g_{12}|^2\mu_1 P_1$ is the signal-to-noise ratio over the channel from S_1 to S_2. Noting that S_2 receives a noisy version, Y_{12}, of the transmitted signal X_1. Consequently, an additional noise term with variance

$$\widehat{N}_{S_2} = P_{e,S_2}E\left[(X_1 - Y_{12})^2\right] = 2P_{e,S_2}(\mu_1 P_1). \tag{9}$$

is generated. Therefore, the source S_2 uses the decoded part of X_1, in the first phase to generate the remaining part, i.e., the $(n-l_1)$ symbols. This part is cooperatively transmitted in phase 3 to the destination.

Phase 2: In this phase, only the source S_2 can transmit. This transmission can be observed by both the source S_1 and the destination. The source S_1 may partially listen to S_2 until it can generate the entire codeword of the user S_2. Indeed, the common destination may decode the signal X_2 at the end of phase 3. At S_1, the decoding process can be performed as described in the previous phase with slight modifications. This decoding can be performed with the rate given by

$$R_{S_2S_1} \leqslant \left(\frac{l_2-l_1}{n}\right)C\left(|g_{21}|^2\zeta_2P_2\right) = I_2 \tag{10}$$

and noise variance given by

$$\widehat{N}_{S_2} = 2P_{e,S_1}(\zeta_2P_2). \tag{11}$$

Phase 3: In this phase, the two users can cooperatively transmit to their common destination. These two sources may employ the superposition encoding scheme [11] to generate their signals, as characterized in (3). Thus, the combined received signal is given by

$$Y_{D3} = \left(\sqrt{\mu_3\alpha} + \sqrt{\frac{\zeta_3\bar{\beta}P_2}{P_1}}\right)X_1 + \left(\sqrt{\zeta_3\beta} + \sqrt{\frac{\mu_3\bar{\alpha}P_1}{P_2}}\right)X_2 + Z_D. \tag{12}$$

Now, at the end of phase-3, the destination may decode both X_1 and X_2. For instance, the total achievable rate of the first user, in which the transmission is performed over two orthogonal phases, can be given as [11,13]

$$\begin{aligned} R_{11} &\leqslant \left(\frac{l_1}{n}\right)C\left(|h_1|^2\mu_1P_1\right) = I_3, \\ R_{13} &\leqslant \left(\frac{n-l_2}{n}\right)C\left(\frac{A_{11}}{1+|h_2|^2\widehat{N}_{S_2}}\right) = I_4, \\ R_{11}+R_{13} &\leqslant \left(\frac{n-(l_2-l_1)}{n}\right)C\left(\frac{A_1}{1+|h_2|^2\widehat{N}_{S_2}}\right) = I_5, \end{aligned} \tag{13}$$

where

$$A_{11} = |h_1|^2\mu_3\alpha P_1 + |h_2|^2\zeta_3\bar{\beta}P_2 + 2|h_1||h_2|\sqrt{(\mu_3\alpha P_1)(\zeta_3\bar{\beta}P_2)},$$
$$A_1 = A_{11} + |h_1|^2\mu_1P_1.$$

In a similar manner, the total achievable rate of the second user can be expressed as [11,13]

$$\begin{aligned} R_{22} &\leqslant \left(\frac{l_2-l_1}{n}\right)C\left(|h_2|^2\zeta_2P_2\right) = I_6, \\ R_{23} &\leqslant \left(\frac{n-l_2}{n}\right)C\left(\frac{A_{22}}{1+|h_1|^2\widehat{N}_{S_1}}\right) = I_7, \\ R_{22}+R_{23} &\leqslant \left(\frac{n-l_1}{n}\right)C\left(\frac{A_2}{1+|h_1|^2\widehat{N}_{S_1}}\right) = I_8, \end{aligned} \tag{14}$$

where

$$A_{22} = |h_2|^2\zeta_3\beta P_2 + |h_1|^2\mu_3\bar{\alpha}P_1 + 2|h_1||h_2|\sqrt{(\zeta_3\beta P_2)(\mu_3\bar{\alpha}P_1)},$$
$$A_2 = A_{22} + |h_2|^2\zeta_2P_2.$$

Based on the transmission over these three phases, the achievable rate region is given in the following theorem.

Theorem. *The achievable rate region of the MAC channel, with partial cooperation between the two sources, is given by the convex hull of the rate pair* (R_1, R_2) *satisfying*

$$\begin{aligned} R_1 &= \min\{I_3+I_4, I_5\}, \\ R_2 &= \min\{I_6+I_7, I_8\}, \\ R_1+R_2 &= \min\{I_3+I_4+I_8, I_5+I_6+I_7, I_5+I_8, I_3+I_4+I_6+I_7\}. \end{aligned} \tag{15}$$

Proof. The proof can be easily obtained by developing the three-phase transmission scheme, as illustrated before. Now, by combining the rates in (7), (10), (13), and (14), the achievable rate region as given in the theorem is obtained. □

Remarks-2:

- The term $I_1 > I_3$ and $I_2 > I_6$ should be maintained for beneficial cooperation.
- For user S_1, the achievable rate reaches its maximum point in the case of $I_3 + I_4 = I_5$. In addition, the sum rate, $R_1 + R_2$ reaches its maximum when all terms are nearly equal.
- The developed achievable rate region is equivalent to that for the multiple access relay channel (MARC) in which a common relay is used to help both users.
- Each source can be considered as a half-duplex relay for the other user. Thus, in the case of full decode and forward, the achievable rate, for a given user, is similar to that obtained in [13].

Remarks-3:

- We remind that, in [9], each user needs to fully decode the other user's message before they start cooperatively transmit to their destination. However, in our encoding scheme, each user may get the other user's signal after partially listening to it.
- We recall that the author in [8] derived the achievable rate region in the case that the two transmitters are connected via links with finite capacities. In addition, the authors in [10] considered the case in which each encoder may get the other user's signal before they start transmission to their destination. However, in our formulation, the two users may simultaneously exchange their signals and transmit to their common destination.

4. Numerical results

In this section, three numerical examples are considered to show the value of our theoretical results. In these examples, the symmetrical case in which $P_1 = P_2 = 20, h_1 = h_2 = 2,$ and $g_{12} = g_{21} = 5$ are considered. In addition, the total number of channel uses n is fixed to 60.

Fig. 2 shows the achievable rate region of the MAC channel in three different scenarios. In particular, the achievable rate region in which the two users can fully cooperate, forms the outer bound. On the contrary, the inner bound is obtained in the case of no cooperation. Additionally, this figure shows that the developed

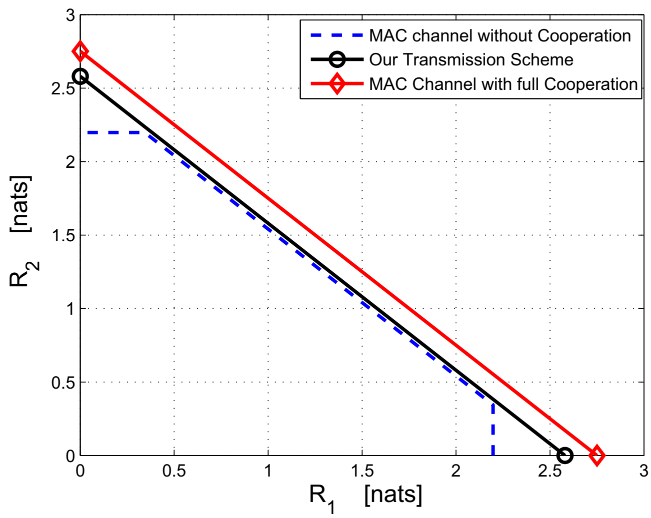

Fig. 2. MAC channel with a three-phase transmission scheme.

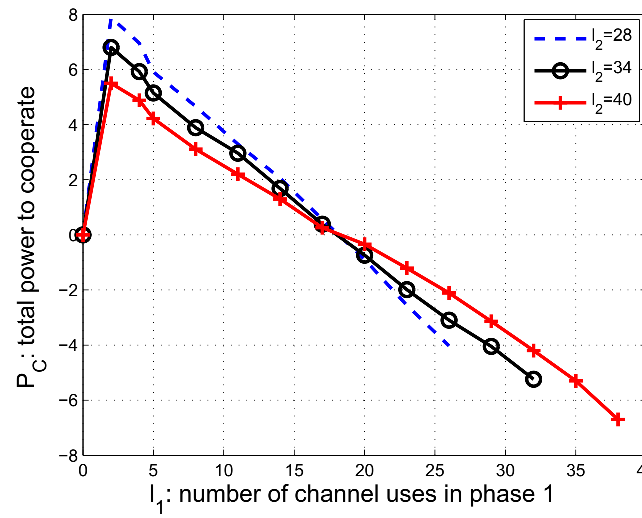

Fig. 4. Total power to cooperate P_C vs. l_1 for different values of l_2, and R_2 is fixed to 1.

achievable rate in the *Theorem* is in between the previous two bounds. This is a reasonable result since our developed achievable rate region is based on partial cooperation between the two users. We note that the point $(R_1 = 2.57, R_2 = 0)$ is achieved when S_2 completely exploits its power in forwarding the first user's signal. In addition, this point is attained in the case of $l_1 = 10, l_2 = l_1, \zeta_2 = 0, \zeta_3 = 1.2, \mu_1 = 2.2$, and $\mu_3 = 0.76$.

In the next two numerical examples, we study the relation between R_1 and l_1, and the associated total power to cooperate, $P_C = \zeta_3\bar{\beta}P_2 - \mu_3\bar{\alpha}P_1$, for different values of l_2 in the case that R_2 is fixed to 1. In particular, Fig. 3 shows that R_1 increases as l_2 decreases. This is because as l_2 decreases, more uses of the channel are used to cooperatively transmit to the destination. In addition, for instance, in the case of $l_2 = 28, R_1$ has a constant value in the range of $5 \leqslant l_1 \leqslant 23$. For small values of l_1, the second user, S_2, is given more uses of the channel, i.e., $(l_2 - l_1)$. Thus, it needs a small amount of its power to keep $R_2 = 1$. Consequently, S_2 can allocate more power to forward the first user's signal, as shown in Fig. 4. On the contrary, as $(l_2 - l_1)$ decreases, more power should be allocated to help the second user have a constant rate. Therefore, as shown in Fig. 4, the source, S_1, may allocate more power to achieve this goal.

Now, we turn to the asymmetric case wherein the two users have i) different power constraints, and/or ii) different channel gains with their destination. In this numerical example, the total available power $P_1 + P_2$ is fixed to 40 whereas the power constraint P_1 is varied. This is to understand where the available power should go as the channel gains h_1 and h_2 may vary. In particular, Fig. 5 shows that when the two users have equal channel gains, i.e., $h_1 = h_2$, it does not matter where the power should go. This is because either transmitting the signal directly, from S_1 to D, or relaying via S_2 to D, have the same effect. Second, for the case that $h_1 \neq h_2$, more power should be allocated to the user with stronger channel gain. For example, in the case of $h_1 = 2$ and $h_2 = 1$, the achievable rate R_1 increases as more power is allocated to S_1. This agrees with that obtained in [14] where the total power should be given to the user with higher channel gain to maximize the sum rate. Further, as shown in Fig. 5, the opposite happens for the case of $h_1 = 2$ and $h_2 = 3$. In this case, the user S_2 may be closer to the destination such that it has a higher channel gain. Thus, relaying

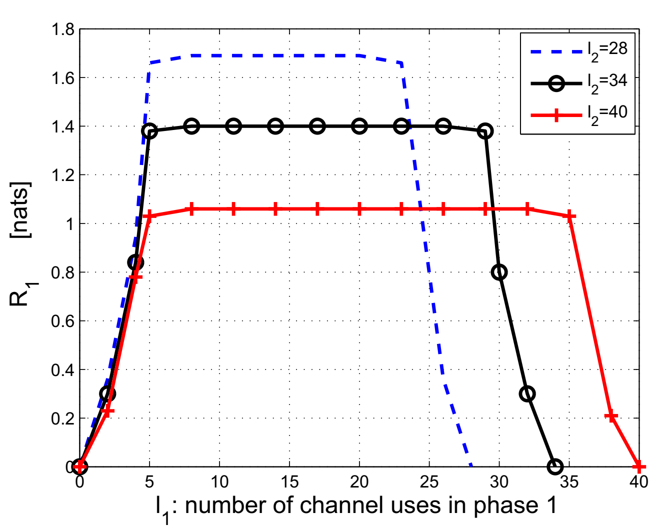

Fig. 3. R_1 vs l_1 for different values of l_2, and R_2 is fixed to 1.

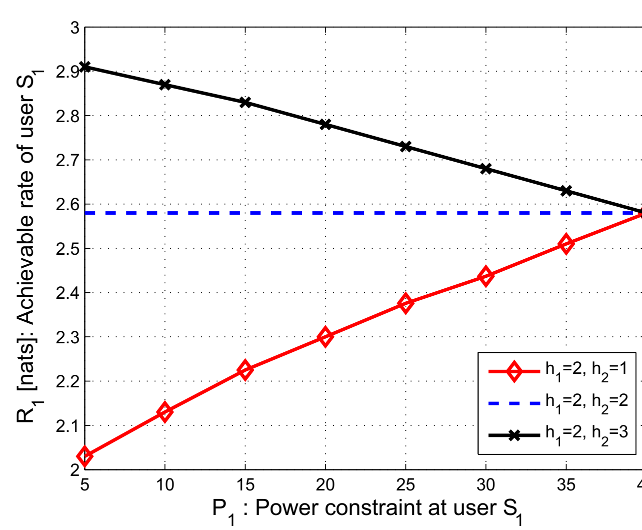

Fig. 5. Achievable rate R_1 vs. P_1 for different values of the channel gains h_1 and h_2.

the signal via S_2 can achieve more rate than directly transmitting to the destination. Consequently, more power should be allocated to S_2.

5. Concluding remarks

We considered the MAC channel in the case that the two users may exchange their messages at first and then cooperatively transmit to their destination. In particular, we developed a three-phase transmission scheme such that the two users are able to exchange their signals and then communicate with their receiver. Finally, some rigorous numerical examples were presented to verify our analytical results.

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