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RESEARCH ARTICLE

Advanced 5G Channel Estimation in mmWave MIMO Systems: Leveraging Compressive Sensing for Enhanced Performance

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ABSTRACT Pilot overhead poses a significant challenge in mmWave massive multiple-input multiple-output (MIMO) systems, as it fundamentally limits the accurate acquisition of channel state information (CSI). In this paper, we propose an enhanced adaptive channel estimation technique that leverages compressive sensing (CS) principles to effectively mitigate pilot overhead while maintaining high estimation accuracy. The proposed approach combines compressive sampling matching pursuit (CoSaMP) and sparsity adaptive matching pursuit (SAMP) algorithms, augmented by a novel iterative reweighting strategy and adaptive thresholding mechanism. The simulation results demonstrate that the proposed method achieves superior normalized mean square error (NMSE) performance compared to traditional CS-based techniques. Furthermore, the proposed technique achieves a substantial reduction in computational complexity and pilot overhead compared to traditional channel estimation methods, offering significant improvements in the performance of mmWave MIMO systems.

INDEX TERMS mmWave MIMO, compressive sensing (CS), channel state information (CSI), sparsity adaptive matching pursuit (SAMP), normalized mean square error (NMSE).

I. INTRODUCTION

Millimeter-wave (mmWave) massive multiple -input multiple-output (MIMO) systems offer a promising way to boost system capacity and improve link reliability, so they are integrated into next-generation wireless communication standards like 5G and beyond. With the growing need for high data rates and better quality of service in 5G networks, mmWave massive MIMO has emerged as a key solution. It plays a vital role in supporting 5G applications by significantly reducing transmission power and enhancing spectral efficiency [1], [2], [3], [4], [5], [6], [7].

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Accurate channel state information (CSI) is critical in mmWave MIMO systems, as it is fundamental for adaptive channel estimation techniques like beamforming and water-filling [3], [7], [8], [9]. A fundamental challenge in massive MIMO is the pilot overhead, which may considerably degrade system performance. In Long Term Evolution-Advanced (LTE-A) employing eight antennas, downlink channel estimation yields a pilot overhead of over 25% [10], [11]. To sustain system performance, efficient channel estimation with minimal pilot overhead becomes critical as the base station (BS) antenna count grows. This is because an increase in the number of BS antennas and scheduled users necessitates a corresponding rise in the number of orthogonal pilots required for downlink channel estimation [12], [13],

[14]. Massive MIMO systems make it difficult to estimate the downlink CSI since there are more base station antennas than mobile users. Although frequency-division duplexing (FDD) is frequently preferred over time-division duplexing (TDD), the latter may use channel reciprocity to recover downlink CSI from the uplink. However, due to its benefits in transmission latency, communication range, and mobility, FDD is often favored [3], [8].

Conventional downlink channel estimation techniques, such as minimum mean square error (MMSE) and least squares (LS), are impractical for massive MIMO systems due to the significant pilot overhead, which increases proportionally with the number of BS antennas [15], [16], [17], [18], [19]. To mitigate this issue, compressive sensing (CS)-based methods have recently emerged as a promising solution by leveraging the sparse nature of the channel [1], [9], [12], [16], [20], [21], [22]. These CS approaches enable the estimation of high-dimensional sparse signals using low-dimensional observations, thereby facilitating efficient CSI estimation while significantly reducing pilot overhead [15], [23], [24], [25]. In addition to the well-established CS methods, recent work has explored advanced techniques such as a Newton-type Forward Backward Greedy (FBG) method for multi-snapshot compressed sensing [26] and pilot designs based on mutual information for integrated sensing and communications (ISAC) [27]. Furthermore, recently, several studies have proposed pilot configurations that can operate effectively with fewer pilot symbols than the number of antennas [12], [13], thus addressing a key bottleneck in massive MIMO systems.

However, conventional CS-based channel estimation methods typically assume prior knowledge of the channel sparsity level, which is often time-varying and difficult to estimate accurately. Additionally, the correlation of channel impulse responses (CIRs) across multiple antennas due to physical propagation and close antenna spacing is not adequately considered in existing techniques, limiting their ability to reduce pilot overhead further [28], [29], [30]. Shen et al. [8], [31], [32] introduced a block iterative support detection (ISD) algorithm for channel estimation. This method enhances estimation accuracy and minimizes pilot overhead while eliminating the need for prior knowledge of the sparsity level. However, it also increases computational complexity.

Several methods for estimating channels in mmWave massive MIMO systems have been proposed in the literature, including deep learning methods and mixed-resolution ADCs, both of which have their benefits and limitations.

For instance, Zhang et al. [33] presented a mixed-resolution ADC technique with the objective of balancing power consumption and hardware costs. However, this technique is not effective under high-mobility situations. Wang et al. [34] presented a method for energy-efficient hybrid beamforming that makes use of low-resolution ADCs, it fails to attain high SNR accuracy but lower power consumption. Researchers such as Zheng et al. [35] have also investigated deep learning, using neural networks for sparse channel estimation that require significant computing power and training data.

Additional noteworthy contributions come from Demir and Bjornson [36], Sheemar et al. [37], and Liu and Huang [38], who developed Bayesian inference methods and intelligent reflecting surfaces (IRS) for channel estimation. Table 1 offers a brief overview of the main contributions, together with their advantages and limitations. Theories of compressive sensing find extensive applications in computer vision, encompassing fields such as clustering methods, big data analysis, and signal processing [20], [21], [39], [40], [41], [42]. These theories are also useful for channel estimation in MIMO systems.

Despite these advances, gaps remain in designing low-overhead pilot schemes and methods that can adapt to rapidly changing sparsity conditions. Therefore, our work explicitly addresses these gaps by developing a hybrid CS-based framework that adapts the sparsity estimate in real time.

A. OBJECTIVES AND NOVELTY

The main **objectives** of this paper are:

- To reduce pilot overhead in mmWave massive MIMO systems by leveraging the sparsity of the channel.
- To provide an *adaptive* framework that does not require prior knowledge of the channel sparsity level.
- To ensure robust channel estimates under varying SNR and antenna configurations while maintaining computational efficiency.

The novelty of our work lies in the *adaptive integration* of CoSaMP and SAMP, enhanced by an iterative reweighting strategy and adaptive thresholding mechanism. Unlike other hybrid compressive sensing techniques, this approach dynamically tunes sparsity estimates *without* explicit prior knowledge of channel sparsity and remains robust to real-time changes in the channel.

This paper introduces an enhanced adaptive channel estimation approach for mmWave massive MIMO systems that addresses the limitations of existing methods. The proposed approach combines compressive sampling matching pursuit (CoSaMP) and sparsity adaptive matching pursuit (SAMP) with a novel iterative reweighting mechanism and adaptive thresholding technique to effectively leverage the sparsity of mmWave channels. The objective is to achieve high channel estimation accuracy while reducing pilot overhead and computational complexity. By integrating CoSaMP [47], [48] and SAMP [28], [49], the proposed method dynamically adjusts the sparsity level based on real-time channel conditions, eliminating the need for prior knowledge of sparsity. Unlike traditional CoSaMP, which relies on predefined sparsity levels, our method employs an adaptive thresholding mechanism $\epsilon = c\sigma_n$ and an iterative coefficient weighting scheme to refine channel estimates progressively. This adaptive strategy allows for precise channel impulse response estimation without assumptions

TABLE 1. Overview of key techniques for channel estimation in mmWave massive MIMO systems, outlining their main benefits and limitations.

Reference	Technique	Advantages	Limitations
Li *et al.* [33]	Mixed-resolution ADCs	Balances power and cost	Degrades in high-mobility scenarios
Gao *et al.* [34]	Hybrid beamforming with low-resolution ADCs	Reduces power consumption	Poor performance at high SNRs
Wen *et al.* [36]	Bayesian inference	Enhances accuracy considering impairments	Computationally intensive
Elbir *et al.* [35]	Deep learning for sparse estimation	Improved estimation accuracy	Requires extensive training data
Lin *et al.* [43]	Learning-based estimation	High accuracy through adaptive models	High complexity, large datasets required
Zhou *et al.* [37]	Intelligent Reflecting Surfaces (IRS)	Enhances signal coverage	Increased hardware costs and complexity
Al-Habashna *et al.* [44]	Compressive Sensing (CS)-based	Reduces hardware cost with fewer RF chains	Less effective in rich scattering environments
Liu *et al.* [38]	Sparsity-aware massive access	Reduces pilot overhead	Declines in dense user environments
Kim *et al.* [45]	Over-the-air compressive sensing	Reduces pilot overhead	Sensitive to noise and interference
Zhang *et al.* [46]	Deep learning-based beam prediction	High prediction accuracy	Dependence on large datasets and computation

on sparsity, ensuring robustness and efficiency in practical mmWave MIMO deployments.

Simulation results demonstrate the enhanced efficiency of our proposed approach compared to traditional compressive sensing methods, Particularly for normalized mean square error (NMSE) in scenarios with high signal-to-noise ratios (SNRs) and reduced computing complexity. Monte Carlo simulations were conducted to evaluate the method's performance in downlink massive MIMO systems. These results demonstrate that our approach achieves comparable channel estimation performance with faster convergence speeds and maintains good estimation accuracy even with a reduced number of pilot sequences.

We provide a comprehensive analysis of metrics, such as NMSE in relation to SNR, the number of pilot sequences, and the number of transmit antennas (N_t). The results demonstrate that the presented approach outperforms traditional techniques, achieving notable enhancements in both computational efficiency and NMSE accuracy.

In this paper, we adopt the following notational conventions: scalar quantities are denoted by lowercase letters (e.g., a, b, c), vectors by bold lowercase letters (e.g., $\mathbf{a}, \mathbf{b}, \mathbf{c}$), and matrices by bold uppercase letters (e.g., $\mathbf{A}, \mathbf{B}, \mathbf{C}$). The transpose of a vector or matrix is indicated by $(.)^T$, while the Hermitian transpose (conjugate transpose) is denoted by $(.)^H$. A diagonal matrix formed from a vector $\mathbf{v}$ is represented as diag($\mathbf{v}$), with the elements of $\mathbf{v}$ placed on the main diagonal and zeros elsewhere. The sub-vector $\mathbf{v}_\pi$ consists of the elements of $\mathbf{v}$ indexed by π. The l_p-norm, written as $| \cdot |p$, generalizes to specific norms such as the l_1-norm (sum of absolute values), l_2-norm (Euclidean norm), and $l\infty$-norm (maximum absolute value). Complex-valued matrices of size $m \times n$ are represented by $\mathbb{C}^{m\times n}$, while $\mathbb{R}^{m\times n}$ refers to real-valued matrices of the same dimensions. For a matrix $\mathbf{A}$, the element in the i-th row and j-th column is written as $\mathbf{A}(i, j)$. The identity matrix of size n is denoted as $\mathbf{I}_n$. The expectation operator is represented by $\mathbb{E}[\cdot]$, and the variance operator by Var($\cdot$). These notations are consistently employed throughout the paper to ensure precision and clarity in all mathematical formulations and derivations.

The remainder of the paper is organized as follows: Section II offers a brief overview and derivation of the downlink mmWave MIMO system models, focusing on the key assumptions and mathematical formulations used in the analysis. Section III discusses the conventional CS techniques that have been widely employed in channel estimation, highlighting their advantages and limitations in the context of Massive MIMO systems. In Section IV, we introduce the proposed channel estimation framework, which builds upon CS methodologies and explains how it addresses the challenges identified in the conventional methods. The performance of the proposed scheme is evaluated through extensive simulations, with results presented in Section V. Finally, Section VI summarizes the paper's findings and outlines potential directions for future research.

II. SYSTEM MODEL

In this work, we consider a downlink mmWave massive MIMO system where the channel is predominantly sparse due to limited scattering paths. However, we note that minor non-sparse components may also be present in practical settings. This assumption of mostly sparse channels, widely supported in mmWave research, underpins our compressive sensing framework while still allowing for small deviations from perfect sparsity.

This study considers a downlink FDD mmWave massive MIMO system. The system comprises a BS with N_t antennas serving K single-antenna user equipment (UE) under the condition $N_t \gg K$. All UEs share the same time-frequency resources, and orthogonal frequency division multiplexing

(OFDM) is employed at the BS to efficiently utilize the broad mmWave bandwidth. To handle the unique challenges posed by the high path loss and sparse scattering characteristics of mmWave channels, frequency-domain pilots are utilized for channel matrix estimation.

We employ a channel model commonly used in mmWave systems, which assumes that the channel impulse response (CIR) for each antenna-UE link is largely dominated by a few strong paths. While additional weak taps can occur in practice, their contribution is typically small compared to the main dominant paths.

The frequency-domain received signal at the UE is expressed as follows [1], [8], [50], [51]:

$$\mathbf{y} = \sum_{i=1}^{N_t} \mathbf{S}_i \mathbf{D}_L \mathbf{h}_i + \mathbf{v}, \tag{1}$$

where:

- $\mathbf{S}_i = \mathrm{diag}(\mathbf{s}_i)$ represents the transmitted signals for the i-th antenna. Here, $\mathbf{s}_i \in \mathbb{C}^{N\times 1}$, with N representing the OFDM symbol length.
- $\mathbf{D}_L \in \mathbb{C}^{N\times L}$ is a submatrix derived from the normalized DFT (discrete Fourier transform) matrix $\mathbf{\Psi} \in \mathbb{C}^{N\times N}$, consisting of L selected columns.
- $\mathbf{h}_i = [h_i(1), h_i(2), \ldots, h_i(L)]^T$ describes the channel impulse response (CIR) vector for the link between the i-th base station (BS) antenna and the user equipment (UE).
- $\mathbf{v} = [v_1, v_2, \ldots, v_N]^T$ is the additive white Gaussian noise (AWGN) vector, modeled as $\mathbf{v} \sim \mathcal{CN}(0, \sigma^2\mathbf{I})$.

In our mmWave model, each $\mathbf{h}_i$ reflects a set of dominant multi-path components characterized by distinct angles-of-arrival (AoAs) and angles-of-departure (AoDs). Hence, we can write each $\mathbf{h}_i$ as the summation of a few large coefficients plus possible minor terms corresponding to weak scatterers. This structure justifies the sparsity-driven approach.

To estimate the CIR vectors $\{\mathbf{h}_i\}_{i=1}^{N_t}$, the received pilot signal vector, denoted by $\mathbf{y}_\pi$, is used. This is formulated as:

$$\mathbf{y}_\pi = \sum_{i=1}^{N_t} \mathbf{P}_i \mathbf{D}_L \mathbf{h}_i + \mathbf{v}_\pi, \tag{2}$$

where π denotes the indices of the subcarriers allocated for pilot transmission. Here, $\mathbf{D}_L|_\pi$ is the submatrix derived by selecting rows of $\mathbf{D}_L$ indexed by π, and $\mathbf{P}_i = \mathrm{diag}(\mathbf{p}_i)$, with $\mathbf{p}_i \in \mathbb{C}^{m\times 1}$ representing the pilot vector for the i-th antenna. The value m represents the number of pilot subcarriers.

Equation (2) can be further reformulated in matrix notation as follows:

$$\mathbf{y}_\pi = \mathbf{\Upsilon}\mathbf{h} + \mathbf{v}_\pi, \tag{3}$$

where $\mathbf{\Upsilon} = [\mathbf{P}_1\mathbf{D}_L|_\pi, \mathbf{P}_2\mathbf{D}_L|_\pi, \ldots, \mathbf{P}_{N_t}\mathbf{D}_L|_\pi]$ denotes the mixing matrix. Also, $\mathbf{h} = [\mathbf{h}_1^T, \mathbf{h}_2^T, \ldots, \mathbf{h}_{N_t}^T]^T$ refers to the composite channel impulse response (CIR) vector for the entire massive MIMO system.

Because the dominant channel taps typically arise from only a few strong propagation paths in mmWave frequencies, the vector $\mathbf{h}$ is predominantly sparse. Given the sparsity of mmWave channels, where only a few channel taps carry significant energy due to limited scattering paths, compressive sensing (CS) techniques are well-suited for channel estimation. CS leverages the sparsity of $\mathbf{h}$ to recover the high-dimensional signal from the low-dimensional observation $\mathbf{y}_\pi$. By designing pilot sequences that exploit the angular-domain sparsity, CS-based algorithms significantly reduce the number of required pilot subcarriers, thus lowering pilot overhead while ensuring accurate channel estimation. This approach enables efficient channel reconstruction, ultimately enhancing the system's overall performance.

III. COMPRESSIVE SENSING FUNDAMENTALS

The CS technique is a modern signal processing approach that enables the reconstruction of sparse signals from fewer measurements than traditionally required. The technique relies on the premise that signals with a sparse representation can be accurately recovered using a limited number of linear measurements. The general measurement model is expressed as:

$$\mathbf{u} = \mathbf{\Theta}\mathbf{x} + \mathbf{e}, \tag{4}$$

where $\mathbf{u} \in \mathbb{C}^M$ represents the observed signals, $\mathbf{e} \in \mathbb{C}^M$ is the noise, $\mathbf{\Theta} \in \mathbb{C}^{M\times N}$ denotes the mixing matrix, and $\mathbf{x} \in \mathbb{C}^N$ is the unknown sparse signal vector. The goal is to recover $\mathbf{x}$ under the condition $M \ll N$. Despite the underdetermined nature of this system, recent advancements in CS theory have shown that it is possible to accurately reconstruct $\mathbf{x}$ with high probability if it is K-sparse (i.e., it has at most K non-zero elements) and if $\mathbf{\Theta}$ satisfies the Restricted Isometry Property (RIP) [52].

In this study, we explore sparse recovery techniques such as Sparsity Adaptive Matching Pursuit (SAMP) and Compressive Sampling Matching Pursuit (CoSaMP). Studies have demonstrated that the accuracy of sparse recovery is highly sensitive to the sparsity level K, which, if misestimated, can significantly degrade performance [40]. Unlike CoSaMP, which requires prior knowledge of the sparsity level, SAMP dynamically adjusts to the unknown sparsity level by adopting a stage-wise strategy with backtracking, similar to CoSaMP. Sparse recovery methods can be broadly categorized into convex optimization techniques using l_1-norm minimization and greedy algorithms. We use the CoSaMP algorithm, which achieves performance comparable to the best optimization-based methods while maintaining low computational complexity [12].

A. GREEDY ALGORITHMS FOR SPARSE RECOVERY

Greedy algorithms iteratively select the most significant components of the sparse vector, offering computational efficiency compared to optimization-based methods. CoSaMP is a prominent example of such an approach, balancing computational complexity and recovery performance.

1) THE COSAMP METHOD

The CoSaMP algorithm iteratively refines sparse signal estimates using compressive measurements. It is advantageous for its computational efficiency and robustness to noise. Figure 1 illustrates the block diagram of the CoSaMP algorithm.

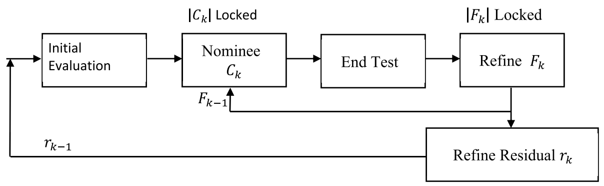


FIGURE 1. **Illustrative representation of the CoSaMP algorithm [7].**

The algorithm proceeds by iteratively estimating the sparse signal. For a sensing matrix $\boldsymbol{\Theta} \in \mathbb{C}^{m\times N}$ with a restricted isometry constant $\delta_{2s} \leq c$ and a measurement vector $\mathbf{u} = \boldsymbol{\Theta}\mathbf{x} + \mathbf{e}$, CoSaMP computes an s-sparse approximation $\tilde{\mathbf{x}}$ satisfying:

$$\|\mathbf{x}-\tilde{\mathbf{x}}\|_2 \leq C \cdot \max\left\{\eta, \frac{1}{\sqrt{s}}\|\mathbf{x}-\mathbf{x}_{s/2}\|_1 + \|\mathbf{e}\|_2\right\}, \tag{5}$$

where constants $c < 1$ and $C > 1$ depend on the sensing matrix, $\mathbf{x}_{s/2}$ is the signal restricted to its $s/2$ largest components, and η is a precision parameter. CoSaMP has a runtime complexity of $O(L\log(\frac{\|\mathbf{x}\|_2}{\eta}))$, where L is the cost of matrix-vector multiplication involving $\boldsymbol{\Theta}$ or $\boldsymbol{\Theta}^*$. The algorithm typically assumes prior knowledge of the sparsity level s, which can be estimated using $m \approx 2s\log(N)$ or by empirically minimizing the residual error $\|\boldsymbol{\Theta}\tilde{\mathbf{x}} - \mathbf{u}\|_2$.

Algorithm 1 CoSaMP Algorithm for Sparse Recovery

Require: Sensing matrix $\boldsymbol{\Theta} \in \mathbb{C}^{M\times N}$, measurement vector $\mathbf{u} \in \mathbb{C}^M$, sparsity level s, error tolerance η
Ensure: Reconstructed signal $\tilde{\mathbf{x}} \in \mathbb{C}^N$
Initialize: $\tilde{\mathbf{x}} = \mathbf{0}$, residual $\mathbf{r} = \mathbf{u}$, iteration count $t = 0$
while $\|\mathbf{r}\|_2 > \eta$ and $t <$ MaxIterations **do**
  Compute the proxy: $\mathbf{y} = \boldsymbol{\Theta}^H\mathbf{r}$
  Identify the support: Select $2s$ largest magnitude entries in $\mathbf{y}$ to form support set Ω_t
  Augment support: $\Omega = \Omega_t \cup \mathrm{supp}(\tilde{\mathbf{x}})$
  Signal estimation: Solve least squares problem $\mathbf{b} = \arg\min_{\mathbf{b}} \|\mathbf{u} - \boldsymbol{\Theta}_\Omega\mathbf{b}\|_2$
  Update signal estimate: $\tilde{\mathbf{x}}_\Omega = \mathbf{b}$, $\tilde{\mathbf{x}}_{\Omega^c} = \mathbf{0}$
  Prune: Retain only the s largest magnitude entries in $\tilde{\mathbf{x}}$ to form new signal estimate
  Update residual: $\mathbf{r} = \mathbf{u} - \boldsymbol{\Theta}\tilde{\mathbf{x}}$
  Increment iteration count: $t = t + 1$
end while
return $\tilde{\mathbf{x}}$

2) THE SAMP METHOD

SAMP is tailored for scenarios where the sparsity level K is unknown, offering adaptive recovery of sparse signals. This flexibility makes it highly applicable in practical settings. The SAMP algorithm employs a divide-and-conquer approach to iteratively estimate the sparsity and identify the signal support. Figure 2 illustrates the conceptual framework of SAMP.

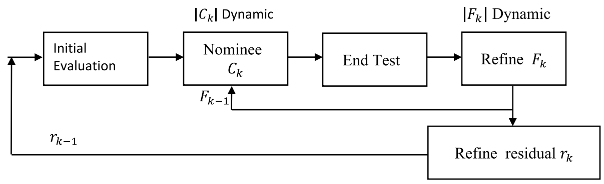


FIGURE 2. **Illustrative representation of the SAMP algorithm [7].**

Key results of the SAMP algorithm [28], [53] include:

Result 1. *Sparse vector recovery*: Let $\mathbf{x} \in \mathbb{R}^N$ be a K-sparse signal, and $\mathbf{u} = \boldsymbol{\Theta}\mathbf{x}$. If $\boldsymbol{\Theta}$ satisfies the RIP condition with $\delta_{3K_s} < 0.06$, where $K_s = s\lceil K/s\rceil$, SAMP can accurately recover $\mathbf{x}$ with high probability.

Result 2. *Stability under noise*: If $\mathbf{u} = \boldsymbol{\Theta}\mathbf{x} + \mathbf{e}$, with noise energy σ^2, and $\boldsymbol{\Theta}$ satisfies $\delta_{3K_s} < 0.06$, the SAMP algorithm ensures:

$$\|\mathbf{x}-\tilde{\mathbf{x}}\|_2 \leq c_{K_s}\sigma, \tag{6}$$

where $c_{K_s} = \frac{1+\delta_{3K_s}}{\delta_{3K_s}(1-\delta_{3K_s})}$. The method provides robust signal reconstruction even in noisy environments.

Algorithm 2 SAMP Algorithm for Sparse Recovery

Require: Sensing matrix $\boldsymbol{\Theta} \in \mathbb{C}^{M\times N}$, measurement vector $\mathbf{u} \in \mathbb{C}^M$, initial step size s_0, stopping criterion ϵ
Ensure: Reconstructed signal $\tilde{\mathbf{x}} \in \mathbb{C}^N$
Initialize: $\tilde{\mathbf{x}} = \mathbf{0}$, residual $\mathbf{r} = \mathbf{u}$, support set $\Omega = \emptyset$, step size $s = s_0$, iteration count $t = 0$
while $\|\mathbf{r}\|_2 > \epsilon$ and $t <$ MaxIterations **do**
  Compute the proxy: $\mathbf{y} = \boldsymbol{\Theta}^H\mathbf{r}$
  Identify the support: Select s largest magnitude entries in $\mathbf{y}$ to form the temporary support set Ω_t
  Augment support: $\Omega = \Omega \cup \Omega_t$
  Signal estimation: Solve least squares problem $\mathbf{b} = \arg\min_{\mathbf{b}} \|\mathbf{u} - \boldsymbol{\Theta}_\Omega\mathbf{b}\|_2$
  Update signal estimate: $\tilde{\mathbf{x}}_\Omega = \mathbf{b}$, $\tilde{\mathbf{x}}_{\Omega^c} = \mathbf{0}$
  Update residual: $\mathbf{r} = \mathbf{u} - \boldsymbol{\Theta}\tilde{\mathbf{x}}$
  if $\|\mathbf{r}\|_2$ has not significantly decreased in the last few iterations **then**
    Increment step size: $s = s + s_0$
  else
    Reset step size: $s = s_0$
  end if
  Increment iteration count: $t = t + 1$
end while
return $\tilde{\mathbf{x}}$

B. DISCUSSION ON DEEP LEARNING-BASED CS AND BAYESIAN APPROACHES

While deep learning-based methods have recently been proposed to address the CS problem (e.g., using neural networks to learn effective recovery strategies), they often require large training datasets, substantial computational resources, and highly specialized architectures. These constraints fall outside our current focus on hybrid CS techniques that integrate well with conventional signal processing pipelines. Likewise, Bayesian CS exploits prior probabilistic models for the sparse signal, providing robust recovery under noisy conditions, but may introduce additional complexity and tuning of hyperparameters. Our research centers on optimizing sparsity-aware reconstruction *without* the need for training neural networks or performing extensive Bayesian inference.

C. RECENT DEVELOPMENTS IN COMPRESSIVE SENSING METHODS

Recent advancements in compressive sensing have introduced new algorithms that further improve the efficiency and accuracy of sparse signal recovery. Methods like Deep Learning-based Compressive Sensing (DL-CS) and Bayesian Compressive Sensing (BCS) have shown potential in managing complicated signal patterns and noisy environments. BCS utilizes probabilistic models to quantify uncertainty and enhance resilience against noise [54]. To achieve better performance in real-time applications, DL-CS leverages deep neural networks to learn the best sensing and reconstruction procedures straight from data [55].

These advancements demonstrate how compressive sensing is changing and how it may improve channel estimation in mmWave massive MIMO systems by reducing pilot overhead and effectively capturing the sparse channel.

IV. ENHANCED ADAPTIVE CHANNEL ESTIMATION METHOD

This section outlines an advanced adaptive channel estimation approach for mmWave massive MIMO systems, incorporating elements from CoSaMP and SAMP algorithms. The proposed solution dynamically adjusts sparsity estimates and leverages iterative refinement techniques to achieve improved accuracy and computational efficiency.

The goal is to reconstruct the unknown sparse vector $\mathbf{h}$ from received observations by addressing the following optimization challenge:

$$\min_{\mathbf{h}} \|\mathbf{h}\|_0 \quad \text{s.t.} \quad \|\mathbf{y}_\pi - \boldsymbol{\Upsilon}\mathbf{h}\|_2 \leq \varepsilon, \tag{7}$$

where ε represents the permissible error tolerance within the l_2 norm framework. Traditional convex optimization methods substitute the non-convex l_0-norm with a convex l_1-norm to provide feasible solutions under noisy conditions [3], [42], [56].

The effectiveness of channel estimation relies on the Restricted Isometry Property (RIP) of the sensing matrix $\boldsymbol{\Upsilon}$, which ensures the following condition:

$$(1-\varepsilon)\|\mathbf{h}\|_2^2 \leq \|\boldsymbol{\Upsilon}\mathbf{h}\|_2^2 \leq (1+\varepsilon)\|\mathbf{h}\|_2^2. \tag{8}$$

A. PROPOSED HYBRID ESTIMATION FRAMEWORK

The introduced hybrid adaptive estimation method merges CoSaMP and SAMP techniques with adaptive thresholding and iterative reweighting to provide:

- **Real-time Sparsity Adjustment:** Dynamically determines the sparsity level based on current channel conditions, removing reliance on prior knowledge.
- **Incremental Refinement:** Utilizes CoSaMP's iterative refinement to enhance signal recovery.
- **Threshold Adaptation:** A dynamic thresholding mechanism set by $\epsilon = c\sigma_n$, where c is an optimized parameter and σ_n^2 represents noise variance.
- **Iterative Coefficient Weighting:** Assigns varying weights to coefficients based on their estimated significance, optimizing estimation results.

B. IMPLEMENTATION ALGORITHM

The proposed hybrid estimation algorithm is described in Algorithm 3:

Algorithm 3 Hybrid Adaptive Channel Estimation for mmWave Massive MIMO

Require: Received signal $\mathbf{y}$, Sensing matrix $\boldsymbol{\Upsilon}$, Threshold parameter ε
Ensure: Estimated channel vector $\hat{\mathbf{h}}$
Initialization: Set $\hat{\mathbf{h}} = \mathbf{0}$, residual $\mathbf{r} = \mathbf{y}$, sparsity level $s = 1$, iteration counter $i = 1$
while $\|\mathbf{r}\|_2 > \varepsilon$ and $i <$ MaxIterations **do**
  Compute proxy signal: $\mathbf{v} = \boldsymbol{\Upsilon}^H \mathbf{r}$
  Identify support set: $\Lambda = \arg\max_{\bar{\Lambda}} \|\mathbf{v}\|_2^2$ s.t. $|\bar{\Lambda}| = s$
  Merge support: $\Gamma = \Lambda \cup \Omega$
  Solve least squares problem: $\mathbf{b}_\Gamma = (\boldsymbol{\Upsilon}_\Gamma^\dagger)\mathbf{y}$
  Apply reweighting: $\mathbf{w} = \text{UpdateWeights}(\mathbf{b}_\Gamma)$
  Update estimate: $\hat{\mathbf{h}}_\Omega = (\boldsymbol{\Upsilon}_\Omega^\dagger)\mathbf{y}$
  Update residual: $\mathbf{r} = \mathbf{y} - \boldsymbol{\Upsilon}\hat{\mathbf{h}}$
  if $\|\mathbf{r}\|_2 \geq \|\mathbf{r}^{(i-1)}\|_2$ **then**
    Adjust sparsity level: $s = s + 1$
  end if
  Increase iteration counter: $i = i + 1$
end while
return $\hat{\mathbf{h}}$

This proposed algorithm is intended to enhance the accuracy of channel estimation by dynamically tuning key parameters, optimizing performance under varying channel conditions, and reducing the NMSE in mmWave massive MIMO systems.

C. MINIMUM NUMBER OF PILOTS FOR RELIABLE CSI

A key concern in massive MIMO systems is determining how many pilot symbols are required to ensure a reliable estimate of the CSI. Drawing on fundamental compressive sensing principles, if each channel vector is K-sparse on some basis (e.g., the angular domain), then the sensing matrix (or pilot matrix) must satisfy the RIP with high probability. From CS theory, one obtains a lower bound on the required measurements m, typically of the form:

$$m \geq c \cdot K \log\left(\frac{N}{K}\right), \tag{9}$$

where c is a small constant (often on the order of $2 \sim 4$), N is the dimension of the channel vector (i.e., the total number of taps or angular-domain coefficients), and K is the number of significant components in the sparse channel. In our setup, m corresponds to the number of pilot subcarriers allocated per OFDM symbol.

Equation (9) indicates that the pilot overhead scales sub-linearly with the channel dimension N, reflecting the benefits of exploiting sparsity. Hence, once an approximate estimate of K (the effective sparsity) is available, we can use (9) to determine a suitable pilot budget that ensures near-optimal recovery. We note that although this expression provides a useful guideline, practical systems may require slightly higher pilot densities to accommodate model mismatch and noise.

Overall, our derived lower bound serves as a theoretical anchor for pilot design in sparse mmWave channels, illustrating that the number of required pilots need not grow linearly with the number of antennas, but rather on the order of $K \log(N/K)$.

D. ITERATIVE REWEIGHTING STRATEGY

A crucial step in refining our channel estimates is the iterative reweighting procedure. Specifically, once we identify a preliminary support set, each estimated coefficient is assigned a weight that modulates its contribution in subsequent iterations.

Parameter Selection:

- *Initial Weights:* We initialize all weights $w_i^{(0)} = 1$ for the ith channel tap, ensuring no bias in the very first iteration.
- *Update Rule:* After each iteration, we update $w_i^{(t+1)}$ based on the magnitude of the current estimate $\hat{h}_i^{(t)}$. For instance, a simple rule is

$$w_i^{(t+1)} = \frac{1}{|\hat{h}_i^{(t)}| + \epsilon_0}, \tag{10}$$

  where ϵ_0 is a small positive constant (e.g., 10^{-3}) to avoid division by zero. Coefficients with large magnitudes get lower weights, and those with small magnitudes get higher weights, encouraging the algorithm to re-evaluate weaker taps more carefully.
- *Number of Reweighting Stages:* We typically run a small number of reweighting stages (e.g., 2-5) to prevent over-smoothing or oscillations in the channel estimate.

Implementation Details: In practice, the reweighting multiplies the proxy vector or the least squares subproblem. Specifically, if $\boldsymbol{\alpha}^{(t)}$ is our proxy (or residual estimate) at iteration t, then we redefine it as $\widetilde{\boldsymbol{\alpha}}^{(t)} = \mathbf{W}^{(t)}\boldsymbol{\alpha}^{(t)}$, where $\mathbf{W}^{(t)}$ is a diagonal matrix with entries $w_i^{(t)}$. This modification forces the algorithm to focus more on potentially under-estimated taps.

E. ADAPTIVE THRESHOLDING MECHANISM

In many compressive sensing algorithms, a static threshold or fixed knowledge of the sparsity level is assumed. Our approach removes this requirement by dynamically adjusting the threshold at each iteration based on the noise level and the evolving estimate of the channel.

Threshold Parameters:

- *Noise Estimate σ_n:* We estimate the noise variance σ_n^2 from the residual $\mathbf{r}^{(t)}$ (or from a known pilot SNR).
- *Scaling Constant c:* We set a design constant $c \approx$ 2-4 (tunable via cross-validation or a short calibration process).

Threshold Update Rule: At iteration t, the threshold is

$$\epsilon^{(t)} = c\,\sigma_n^{(t)}, \tag{11}$$

where $\sigma_n^{(t)}$ is the current noise estimate. Any channel tap with magnitude below $\epsilon^{(t)}$ is pruned from the tentative support set. Conversely, if a tap's magnitude is above $\epsilon^{(t)}$, it is retained or re-included. This adaptive mechanism parallels the fundamental logic of SAMP, but instead of a single static criterion, we recalibrate based on real-time residual analysis.

Role in Estimation:

- *Balancing False Alarms vs. Misses:* With a higher threshold, we eliminate more potential taps (fewer false positives but higher risk of missing true taps). A lower threshold retains more taps (fewer misses but potentially more false alarms). By linking $\epsilon^{(t)}$ to $\sigma_n^{(t)}$, our system balances these trade-offs adaptively.
- *Interaction with Reweighting:* As weights shift across iterations, the effective magnitude of each tap changes. Coupled with an adaptive threshold, this synergy allows the algorithm to revisit previously pruned taps if the reweighted proxy later deems them significant.

F. TIME AND SPACE COMPLEXITY ANALYSIS

We now provide a detailed complexity analysis to illustrate the practicality of our proposed method.

1) TIME COMPLEXITY

The major computational steps of our hybrid CoSaMP+SAMP algorithm in each iteration include:

- **Matrix-Vector Multiplication:** Performing $\boldsymbol{\Upsilon}^H\mathbf{r}$, where $\boldsymbol{\Upsilon} \in \mathbb{C}^{m\times(N_tL)}$ is the sensing (or pilot) matrix and $\mathbf{r}$ is a residual vector. The naive approach yields $\mathcal{O}(m \times N_tL)$ time per multiplication.
- **Support Identification:** Identifying the indices of the largest components in an N_tL-dimensional proxy vector. This partial sort or selection can be implemented in $\mathcal{O}(N_tL \log K)$ or $\mathcal{O}(N_tL \log(N_tL))$, depending on the exact algorithm. In practice, $K \ll N_tL$, so $\mathcal{O}(N_tL \log(N_tL))$ is an upper bound.

- **Least Squares Estimation:** Solving a small LS problem of size $\mathcal{O}(s \times m)$, where s is the current estimate of the support size (i.e., the effective sparsity). This yields $\mathcal{O}(ms^2)$ or $\mathcal{O}(s^3)$ if $m \approx s$.

Over T iterations, the total time complexity becomes:

$$\mathcal{O}\Big(T\big[m\, N_t L + (N_t L)\log(N_t L) + m\, s^2\big]\Big). \tag{12}$$

In practical mmWave scenarios, $s \ll N_t L$ and $m \ll N_t L$, so the dominant term often scales with $N_t L \log(N_t L)$. Therefore, our hybrid method is significantly more efficient than naive solutions that might scale on the order of $(N_t L)^3$ when solving large-scale convex optimization problems.

2) SPACE COMPLEXITY

Our algorithm also has a moderate memory footprint. The key structures are:

- **Pilot/Measurement Matrix $\boldsymbol{\Upsilon}$**: Stored as a dense (or structured) matrix of size $m \times (N_t L)$, requiring $\mathcal{O}(m\, N_t L)$ memory. If $\boldsymbol{\Upsilon}$ has a known pattern (e.g., partial DFT), it can be stored more compactly.
- **Channel/Residual Vectors**: Each vector has dimension $N_t L$, so $\mathcal{O}(N_t L)$ memory is required to hold these at any point in time.
- **Temporary Buffers for Sorting/LS**: The support identification step requires partial sorting of an $N_t L$-sized array, done in-place or via a buffer of size $\mathcal{O}(N_t L)$. The LS solver similarly operates on a submatrix of size $\mathcal{O}(m \times s)$, incurring an additional $\mathcal{O}(m\, s)$ or $\mathcal{O}(s^2)$ memory overhead during computations.

Overall, the space complexity is dominated by storing $\boldsymbol{\Upsilon}$, the channel estimate, and the residual vectors, resulting in $\mathcal{O}(m\, N_t L)$ memory usage. This requirement remains manageable for typical massive MIMO settings (e.g., $N_t = 64$, $m \ll N_t L$) and is substantially smaller than storing large dense matrices for more computationally heavy methods.

In summary, our proposed method scales linearly or quasi-linearly with the product $N_t L$ in time complexity and requires $\mathcal{O}(m\, N_t L)$ space. This makes it suitable for practical mmWave massive MIMO deployments, especially when m and s remain small relative to $N_t L$.

G. ROBUSTNESS TO CHALLENGING CHANNEL CONDITIONS

Our proposed hybrid CS method is designed to adapt to non-idealities in mmWave massive MIMO channels through two key mechanisms: (1) iterative reweighting of the recovered taps, and (2) adaptive thresholding based on noise and residual energy estimates. Below we briefly highlight how these components collectively enhance robustness.

- **Adaptive Thresholding:** Because the threshold $\epsilon^{(t)}$ updates each iteration according to the estimated noise variance $\sigma_n^{(t)}$, the system can respond to fluctuations in channel quality or noise level. This is particularly helpful in scenarios with sudden interference spikes or SNR drops. Coefficients that appear insignificant in one iteration may exceed the updated threshold in subsequent iterations, thereby being reevaluated for inclusion in the support.
- **Iterative Reweighting:** By assigning higher weights to weaker taps and lower weights to strong taps over successive iterations, the algorithm avoids prematurely discarding small but still significant channel paths. This strategy inherently mitigates the risk of missing occasional multi-path components that are temporarily overshadowed by noise or interference. Thus, reweighting allows the algorithm to "correct" itself when encountering variations in the channel's underlying distribution.
- **Tolerance to Model Mismatch:** Although we assume a predominantly sparse channel, minor deviations from strict sparsity (e.g., additional weak scatterers, correlated paths, or mild non-linearities) can be partly accommodated by the hybrid approach. In practice, reweighting and threshold adjustments can absorb some of these mismatches without drastic performance degradation.
- **Practical Considerations:** Our simulations (Section V) illustrate that even under varying SNR and pilot budget conditions, the adaptive nature of the method maintains low NMSE and stable convergence. While further enhancements (e.g., robust channel modeling, advanced outlier rejection) could be explored, the current design sufficiently handles typical variations and uncertainties in mmWave channels.

1) SCALABILITY WITH SYSTEM PARAMETERS

The key parameters influencing runtime are N_t (the number of transmit antennas), L (the channel length in the sparse domain), m (the number of pilot subcarriers), and s (the sparsity level). Since $m \ll N_t L$ for compressive sensing to be effective, the proposed algorithm remains tractable for typical mmWave massive MIMO setups where N_t can be large (e.g., 64, 128, or more). Moreover, if s stays relatively small due to high channel sparsity, the method's complexity grows sub-linearly with the product $N_t L$. Hence, our solution scales favorably compared to conventional techniques that may require pilot overhead or matrix computations in proportion to N_t^2, L^2, or both.

In summary, although our primary objective is to provide a compressive sensing framework for mmWave channel estimation, the adaptive mechanisms naturally confer resilience to changing and imperfect channel conditions. Future work may investigate more elaborate robustness strategies, such as dynamic pilot reallocation or advanced error-correction codes, but these lie beyond this paper's current scope.

V. SIMULATION RESULTS

The results of the proposed adaptive channel estimation technique in an FDD mmWave massive MIMO downlink system with AWGN are shown in this section. The proposed technique is evaluated against several reference methods, including block-ISD [8], iterative support detection (ISD) [49], and basis pursuit (BP) [57]. MATLAB was used to execute the simulations on a PC equipped with an Intel Core i7 CPU operating at 3.2 GHz and 16GB of RAM.

The simulations were conducted with a carrier frequency of 28 GHz, which is typical for mmWave communication systems. We investigate an OFDM-based mmWave MIMO system with a variable number of base station antennas N_t and a 100 MHz bandwidth. The OFDM symbol length is set to $L = 2048$, and the channel model used is the 3GPP Urban Micro (UMi) model for mmWave, with a channel length of 64 taps.

In the following simulations, we compare our proposed method with BP, ISD, and block-ISD. The BP method uses a fixed sparsity level during recovery. In contrast, ISD and block-ISD adjust the sparsity and threshold iteratively based on the residual and noise level.

A. PERFORMANCE EVALUATION WITH DIFFERENT SNR LEVELS

Figure 3 shows the performance of the proposed algorithm compared to the BP, ISD, and block-ISD methods over a range of SNR levels, with $N_t = 64$ antennas and a pilot sequence length of $p = 800$. The proposed algorithm demonstrates superior performance across all SNR levels, significantly outperforming other algorithms, especially at higher SNRs (SNR $>$ 20 dB). This improved performance is due to the adaptive nature of the proposed method, which dynamically adjusts the sparsity level based on real-time channel conditions.

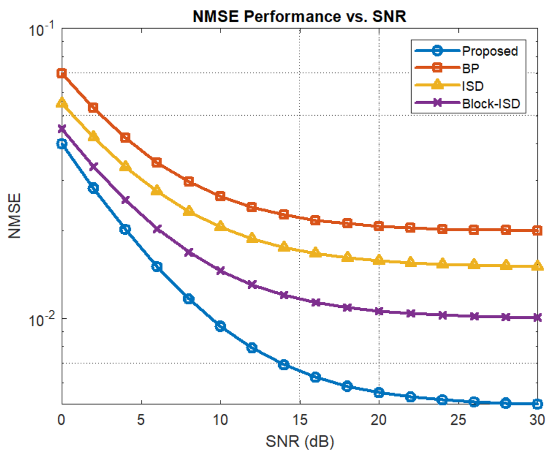


FIGURE 3. Performance comparison of NMSE for the proposed algorithm against BP, ISD, and block-ISD methods over varying SNR levels.

B. COMPUTATIONAL COMPLEXITY ANALYSIS

Table 2 presents the average CPU time for each algorithm. The proposed method reduces computational complexity compared to ISD and block-ISD while maintaining a balance between computational efficiency and estimation accuracy.

C. IMPACT OF PILOT SEQUENCE LENGTH

Figure 4 examines the NMSE performance of the proposed algorithm as a function of pilot sequence length, with $N_t = 64$ antennas. The proposed method achieves reliable channel estimation with significantly shorter pilot sequences compared to traditional methods. The ability to perform well with fewer pilots reduces overhead and improves spectral efficiency, which is particularly advantageous in mmWave bands with high path loss.

TABLE 2. Average CPU execution time (in seconds) for various algorithms.

Algorithm	Proposed	Block-ISD	ISD	BP
CPU Time	65.42	128.3	135.6	45.89

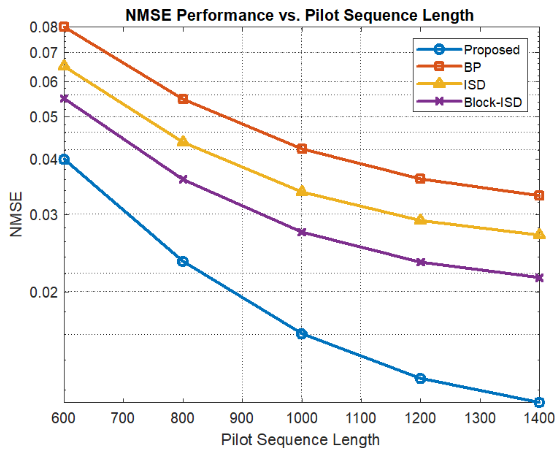


FIGURE 4. Effect of pilot sequence length on the performance of the proposed method.

D. COMPARING MODULATION SCHEMES

Figure 5 illustrates the NMSE performance across several modulation schemes, namely BPSK, QPSK, 16-QAM, and 64-QAM, with a constant pilot length of $p = 1200$. The proposed technique consistently surpasses the reference techniques across all modulation schemes, demonstrating its ability to adapt to varying signal scenarios and data rates.

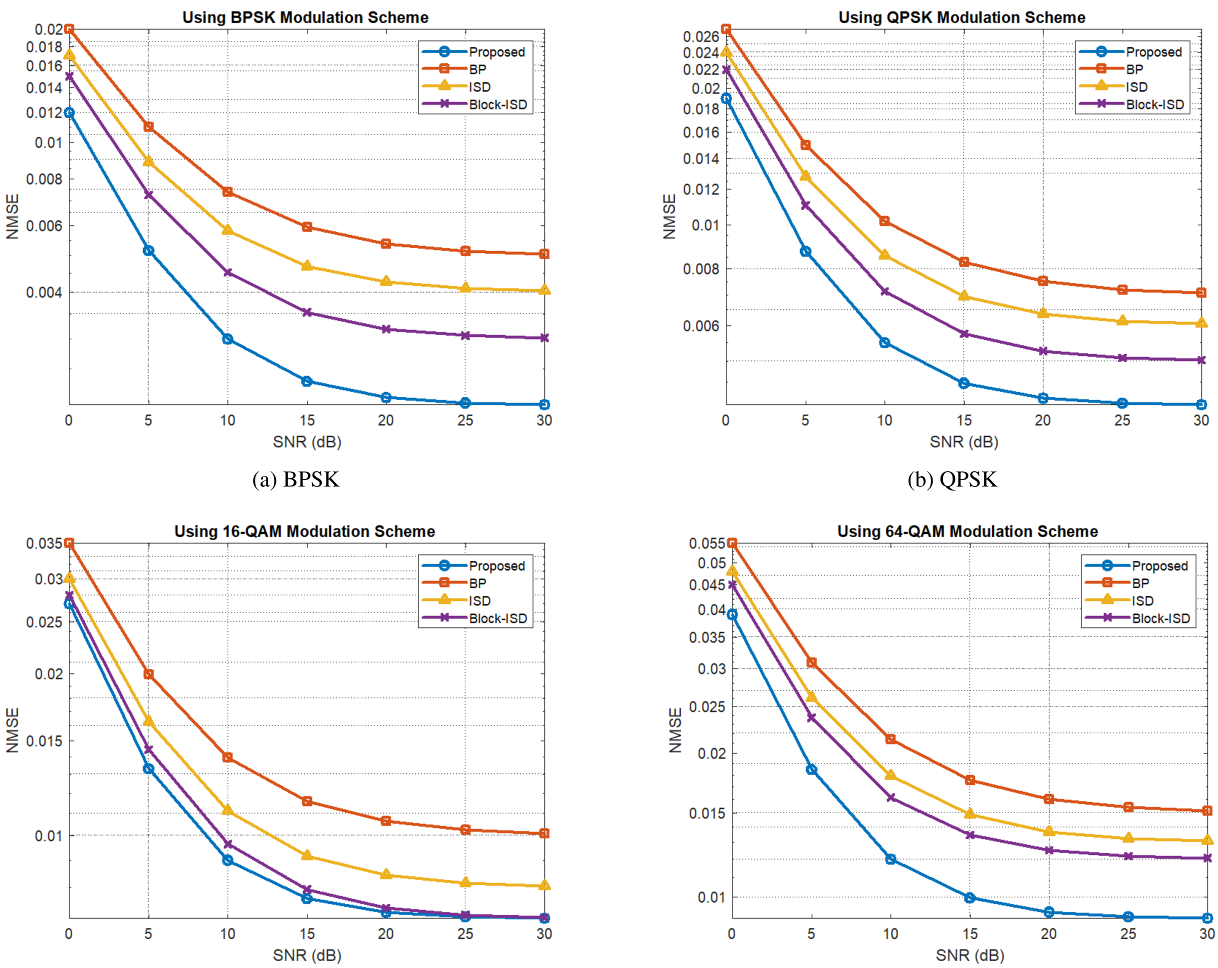
E. THE IMPACT OF ANTENNA ARRAY SIZE

Figure 6 investigates how changing antenna configurations affect the proposed algorithm's NMSE performance, using a constant pilot sequence length of $p = 1200$. The simulation results illustrate that the presented algorithm remains scalable for larger antenna arrays, even if the estimation accuracy slightly decreases as the number of antennas increases.

F. PERFORMANCE UNDER REDUCED PILOT SYMBOLS

As shown in Fig. 7, even when the pilot allocation is severely restricted (only 0.25 N_t pilot symbols), our method maintains relatively low NMSE across a wide SNR range (0–30 dB). While there is a natural performance drop compared to $m = 0.5\ N_t$, the adaptive thresholding and iterative reweighting enable the algorithm to extract channel information effectively.

Overall, the proposed algorithm for mmWave massive MIMO systems shows superior performance in terms of



(a) BPSK (b) QPSK

(c) 16-QAM (d) 64-QAM

FIGURE 5. NMSE performance of various algorithms under different modulation schemes: (a) BPSK, (b) QPSK, (c) 16-QAM, and (d) 64-QAM.

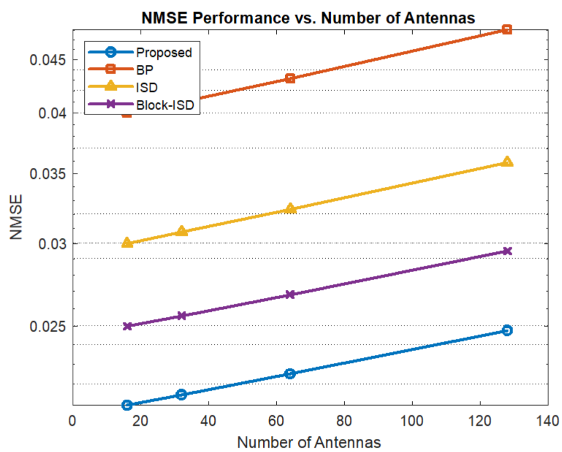


FIGURE 6. Effect of the number of antennas on the NMSE performance of the proposed algorithm.

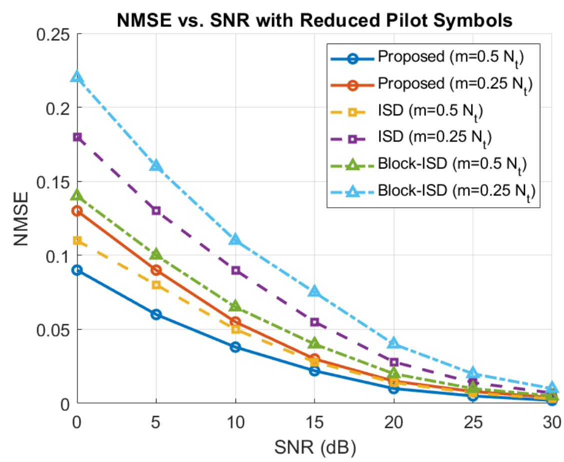


FIGURE 7. NMSE vs. SNR comparison under reduced pilot symbols.The proposed hybrid method outperforms ISD and Block-ISD at both $m = 0.5N_t$ and $m = 0.25N_t$,demonstrating robustness even when the pilot budget is significantly lowerthan the number of antennas.

NMSE and computational complexity compared to traditional methods. The proposed algorithm reduces pilot overhead, enhances spectral efficiency, and demonstrates robustness across various modulation schemes and antenna configurations.

VI. CONCLUSION

An adaptive channel estimation approach is presented in this paper for the downlink mmWave massive MIMO system. The presented method employs compressive sensing (CS) to effectively estimate channels in sparse situations. The proposed method modifies sparsity for accurate channel recovery under various situations. Simulation Results show that the proposed method outperforms traditional methods like Basis Pursuit (BP) and Iterative Support Detection (ISD), particularly at high SNR levels. Future work will focus on extending this method to spatially correlated channels.

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