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A novel multiple access diamond channel model

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Summary

This paper introduces a new multiuser communication model, which we call multiple access diamond channel. In this channel model, many sources transmit their signals to a destination in the case of no direct links between the end terminals. Therefore, we employ two parallel relays to extend transmission to the destination. Further, we specifically characterize two different transmission schemes along with their respective achievable rate regions. Then, we develop an outer bound to the transmission in each scheme. Moreover, we present many numerical examples to mainly compare between the two different transmission schemes and to distinguish the case for which we attain the capacity region.

KEYWORDS

channel capacity, diamond channel, multiple access channel, relay channel

1 | INTRODUCTION

A multiple access channel (MAC) technique models a communication system in which many users intend to transmit their messages to a common destination.[1-4] For example, Abu Al Haija and Vu[1] and Al-qudah and Bataineh[2] investigated the effect of cooperation between the two users on the achievable rate region. In another multiuser communication network, the basic model of the relay channel consists of a source, a destination, and an intermediate device, that is, the relay.[5-9] The relay either significantly improves the source's signal at its destination or extends the source's signal into its receiver, as in the case of no direct link between the source and the destination. In forwarding the source's signal into its destination, the relay may employ decode-and-forward (DF), amplify-and-forward, or compress-and-forward.

As an extension of the MAC technique, multiple access relay channel (MARC), in which a relay extends the two users' signals' to their destination, was studied in many different scenarios.[10-14] For instance, Sankar et al[11] derived the sum capacity of the degraded Gaussian MARC. Taghavi and Hodtani[13] considered the effect of the relay-source feedback on the achievable rate region. Moreover, Lee et al[12] employed lattice coding to study the achievable rate region of the multiple-input multiple-output MARC. Additionally, to the best of our knowledge, Mohamad et al[15] is the only research work that studied the case in which many relays are used to help the two sources. Specifically, they investigated the benefits of joint network channel coding and decoding for half-duplex slow fading multiple-access multiple-relay channels. In addition, the emphasis was on the most spectrally efficient nonorthogonal multiple access where the relays perform joint network channel coding in combination with selective relaying.

In this paper, we propose the multiple access diamond channel (MADC) in which there is no direct connection between the senders and the destination. In particular, as shown in Figure 1, the transmission from the two sources to the destination is operated through two half-duplex parallel relays. Specifically, the channel from each source to the destination forms a diamond channel (DC). Practically, this setup may model a 5G communication scenario in which

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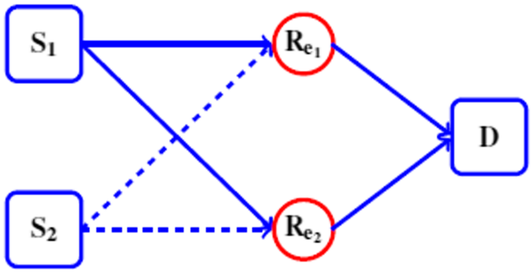

FIGURE 1 Multiple access diamond channel in which the transmission from the two sources to the destination is made possible via two half-duplex relays. The channel from either the source S_1 or the source S_2 via the two relays, R_{e_1} and R_{e_2}, to the destination D may form a diamond channel. The different line formats distinguish between the two intersected diamond channels

many users at the edge of a given cell want to transmit to the base station when there are no direct links (i.e., very weak [nonuseful] links) between the senders and the destination. Therefore, we use two parallel relays to extend the signals from the users to the base station.

The DC, in which two parallel relays extend the transmission from a source into its destination, was introduced by Schein.[16] Then, the DC channel model was extensively studied in many different cases.[17-24] For instance, Xue and Sandhu[17] and Bagheri et al[18] considered the case in which the source–destination pair is aided by two half-duplex relays. In addition, they derived both outer bounds (cut-set upper bound) and inner bounds (achievable rates). Besides, Bagheri et al[18] determined the cases in which the capacity is achieved. Li et al[19] studied the state-dependent degraded broadcast DC in which the source to relays cut is modeled with two noiseless, finite-capacity digital links with a degraded broadcasting structure. In a different scenario, Saeedi Bidokhti and Kramer[21] investigated the case in which the broadcast component from the source to the relays is orthogonal and modeled by two independent bit-pipes. Similarly, Kang et al[24] focused on the particular case of DC where two separate links with finite capacities model the source-relays channel and the link from the two relay nodes to the destination node is a Gaussian MAC. The source-relays channel was considered previously,[20] where the authors proved that transmitting uncorrelated signals achieves the optimal encoding scheme at the source.

The main contribution of this paper resides in characterizing two different transmission schemes such that the two sources, with the help of the two parallel half-duplex relays, can transmit to the destination. Specifically, in each transmission scheme, the transmission is split into two orthogonal transmission phases. In the first phase, the two sources may simultaneously or subsequently transmit to the two relays. Afterwards, in the second phase, the two relays can forward the sources' signals to their destination. Based on these transmission schemes, the achievable rate region is established in two different cases. Then, we derive an outer bound for each transmission scheme. Further, we present many numerical examples to specifically compare between the two transmission schemes and also to determine under which conditions the capacity of the MADC is achieved.

The rest of the paper is organized as follows. In Section 2, we present the MADC model. To derive the achievable rate regions, we establish many transmission schemes in Section 3. Then, we develop an outer bound to the transmission rates over the MADC in Section 4. Further, many numerical examples are presented in Section 5. Finally, we conclude our paper in Section 6.

2 | SYSTEM MODEL

In this paper, we consider the MADC communication model, as illustrated in Figure 1. This channel model is composed of two sources S_1 and S_2 that want to transmit two different signals $X_1 \in \mathcal{X}_1^n$ and $X_2 \in \mathcal{X}_2^n$, respectively, where $\mathcal{X}_1^n$ and $\mathcal{X}_2^n$ are the two input channel alphabets. These two signals are initially transmitted to two parallel half-duplex relays in which the received signals are Y_{r_1} and Y_{r_2}. Then, these two relays can immediately employ DF such that the two relays R_{e_1} and R_{e_2} transmit the signals X_{r_1} and X_{r_2}, respectively. Further, the output alphabet at the destination is represented by Y.

In this MADC channel model, the two transmitters wish to send the messages $M_1 \in \{1, \ldots, 2^{nR_1}\}$ and $M_2 \in \{1, \ldots, 2^{nR_2}\}$ to the receiver over n uses of the channel. The transmission from the two sources to the destination is made possible with the achievable rates R_1 and R_2. The encoding and decoding at the two transmitters, the two relays, and the destination will be shown in Section 3. The achievable rates R_1 and R_2 are said to be achievable if there exists a sequence of $(2^{nR_1}, 2^{nR_2}, n)$ code with a probability of error, P_e^n, that goes to 0 for sufficiently large n. Specifically, P_e, is defined as

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$$P_e = \sum_{m_1, m_2} \frac{1}{2^{n(R_1+R_2)}} P[g(Y) \neq (m_1, m_2) | (m_1, m_2) \text{ was sent}], \tag{1}$$

where $M_1 = m_1$ and $M_2 = m_2$ are the two transmitted messages.

Remark I. In general, the system model may contain many parallel relays to help the transmission from the two sources to the destination. However, this selection may not be optimum. Therefore, a relay selection technique may be exploited to choose the best two relays.[7,25-27] For example, the authors in previous studies[25,26] studied the selection of the best two relays such that the bit error is minimized. In our case, two relays may be selected from many so that we maximize the achievable rate region.

3 | TRANSMISSION SCHEMES OVER MADC

In this section, we propose two different transmission schemes. Based on these schemes, we characterize the achievable rate region of both the discrete memoryless and the Gaussian MADC. The transmission from the two sources, via the two parallel relays, to the destination is operated over two orthogonal transmission phases. Specifically, in the first phase, the two sources can transmit their signals to the two parallel half-duplex relays. Then, in the second phase, the two relays forward the users signals into their destination. Therefore, the achievable rate of user i is $R_i = R_{i1} + R_{i2}$, where R_{i1} and R_{i2} are the achievable subrates in the first and the second phases, respectively.

3.1 | Sources-to-relays transmission schemes

In this section, we present two different transmission schemes such that the two sources can transmit their signals to the relays. These transmission schemes refer to the cases in which the two sources can either subsequently or simultaneously transmit to the two parallel relays. In particular, in the first transmission scheme, the duration of the first phase is divided into two orthogonal subphases such that only one source can transmit at each subphase. In the second transmission scheme, the two sources can simultaneously transmit to the two relays to form what is known as the interference channel (IC).

3.1.1 | Subsequent transmission scheme

In the first transmission mode, the period of the first phase, τ_1, is divided into two subphases with durations, τ_{11} and τ_{12}. In the first subphase, as shown in Figure 2, the first source, S_1, can transmit to the two relays, whereas the second source, S_2, can transmit in the second subphase. It is worth noting that the transmission from a given source to the two relays is equivalent to the transmission over parallel channels. Therefore, the achievable subrates of the first and second users, R_{11} and R_{21}, are given by[28]

$$\begin{aligned} R_{11} &\leq \tau_{11} I(X_1; Y_{r_1}, Y_{r_2}), \\ R_{21} &\leq \tau_{12} I(X_2; Y_{r_1}, Y_{r_2}), \end{aligned} \tag{2}$$

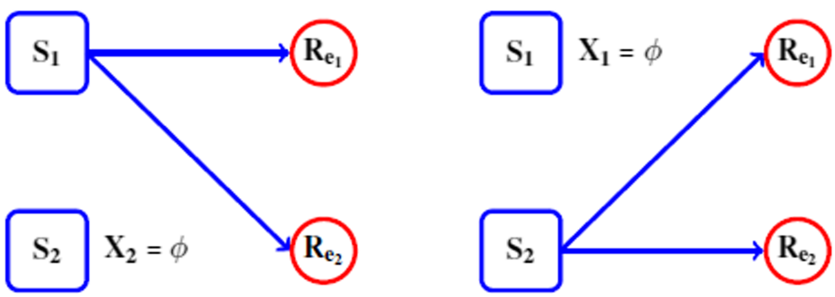

FIGURE 2 Transmission in the first phase may be split into two subphases (subsequent transmission) in which only one source can transmit in each subphase

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where $I(.;.)$ represents the mutual information.

In the case of additive Gaussian channel, the received signals at the two relays over the two orthogonal subphases are expressed by

$$\begin{aligned} Y_{r_{11}} &= h_{11}X_1 + Z_1, \\ Y_{r_{21}} &= h_{12}X_1 + Z_2, \end{aligned} \tag{3}$$

and

$$\begin{aligned} Y_{r_{12}} &= h_{21}X_2 + Z_1, \\ Y_{r_{22}} &= h_{22}X_2 + Z_2, \end{aligned} \tag{4}$$

where, for, $i, j \in \{1, 2\}$, $Y_{r_{ij}}$ is the received signal at the relay R_{e_i} in subphase j, and h_{i1} and h_{i2} are the channel gains from S_i to the two relays in the subphase i. One can notice that the transmission of the signal X_i is limited by an average power P_i. Further, the signals Z_1 and Z_2 are independent and identically distributed additive white Gaussian noise (AWGN) with mean 0 and variance normalized to 1.

To this extent, we are now ready to extend the achievable subrates R_{11} and R_{12} from (2) to the case of AWGN channel. In particular, the achievable subrate R_{11}, in the case of AWGN, is derived as follows:

$$\begin{aligned} R_{11} &\le \tau_{11} I(X_1; Y_{r_1}, Y_{r_2}) \\ &= \tau_{11}\{h(Y_{r_1}, Y_{r_2}) - h(Z_1, Z_2)\} \\ &= \tau_{11}\{h(Y_{r_1}, Y_{r_2}) - \log_2(2\pi e)^2\} \\ &\le \tau_{11} C\left(\frac{\tau_1}{\tau_{11}}\left(|h_{11}|^2 P_1 + |h_{12}|^2 P_1\right)\right), \end{aligned} \tag{5}$$

where $C(x) = \log_2(1+x)$. In this subphase, the transmission from the source, S_1, to the two relays forms a two-look channel, as was previously shown in Cover and Thomas.[28] Similarly, the achievable subrate of the second user, in the first phase, is given by

$$R_{21} \le \tau_{12} C\left(\frac{\tau_1}{\tau_{12}}\left(|h_{21}|^2 P_2 + |h_{22}|^2 P_2\right)\right). \tag{6}$$

3.1.2 | Simultaneous transmission scheme

In the second transmission mode, as presented in Figure 3, the two sources can simultaneously transmit to the two relays to form what is widely known as the IC channel. Specifically, based on whether the IC is in weak, strong, or very strong interference, the achievable rate region is obtained. In the weak interference regime, the best known achievable

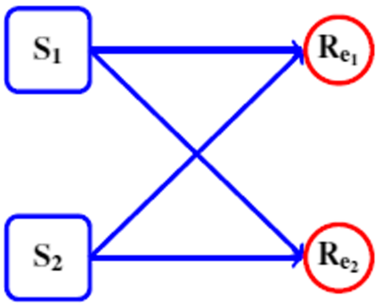

FIGURE 3 The transmission over the MADC in the first phase may be modeled by the IC. In this case, the two sources can simultaneously transmit to the two relays

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rate region is obtained in the case of employing Gelfand–Pinsker (GP) encoding scheme.[29] In particular, GP encoding is normally employed at a given sender in the case that the interference that affect the receiver is available in full at the transmitter. Thus, the transmitter can design its signal such that the effect of the interference is completely canceled at the receiver. For example, in preparing the signal X_2 at the source S_2, the signal X_1 is considered as an available interference. Therefore, the achievable rate region is given by[30,31]

$$\begin{aligned} R_{11} &\leq \tau_1 I(X_1;Y_{r_1}), \\ R_{21} &\leq \tau_1 \{I(X_2;Y_{r_2}) - I(X_2;X_1)\}. \end{aligned} \tag{7}$$

In the case of AWGN, the received signals Y_{r_1} and Y_{r_2} at the two relays are given by

$$\begin{aligned} Y_{r_1} &= X_1 + h_{21}X_2 + Z_1, \\ Y_{r_2} &= h_{12}X_1 + X_2 + Z_2, \end{aligned} \tag{8}$$

where h_{12} and h_{21} are the crossover channel gains. Additionally, in this transmission scheme, the achievable subrates, R_{11} and R_{12}, in the case of AWGN, are expressed by

$$\begin{aligned} R_{11} &\leq \tau_1 I(X_1;Y_{r_1}) \\ &= \tau_1 C\left(\frac{P_1}{1+|h_{21}|^2 P_2}\right), \\ R_{21} &\leq \tau_1 \{I(X_2;Y_{r_2}) - I(X_2;X_1)\} \\ &\overset{(a)}{=} \tau_1 I(X_2;Y_{r_2}|X_1) \\ &= \tau_1 C(P_2), \end{aligned} \tag{9}$$

where the result in step (a) is achieved by employing dirty paper coding (DPC).[32] Specifically, DPC is employed at one of the two transmitters to remove the effect of the other user's signal at its destination. Employing DPC requires the availability of the other user's signal at the desired transmitter before starting the transmission. This availability of the other user's signal can be accomplished by a cooperation between the two senders.

To finish with this phase, the cases of strong IC and very strong IC are summarized in the following remark.

Remark II.

- Under the condition of very strong IC regime, that is, $I(X_2;Y_{r_1}) \geq I(X_2;Y_{r_2}|X_1)$ and $I(X_1;Y_{r_2}) \geq I(X_1;Y_{r_1}|X_2)$, the achievable rate region, in the first phase, is given by[33]

$$\begin{aligned} R_{12} &\leq \tau_1 I(X_1;Y_{r_1}|X_2), \\ R_{22} &\leq \tau_1 I(X_2;Y_{r_2}|X_1). \end{aligned} \tag{10}$$

  For example, a given receiver may initially estimate the nonrequired signal better than the desired receiver.

- The case of strong interference occurs when $I(X_2;Y_{r_1}|X_1) \geq I(X_2;Y_{r_2}|X_1)$ and $I(X_1;Y_{r_2}|X_2) \geq I(X_1;Y_{r_1}|X_2)$, the capacity region of the IC is expressed by[34]

$$\begin{aligned} R_{12} &\leq \tau_1 I(X_1;Y_{r_1}|X_2), \\ R_{22} &\leq \tau_1 I(X_2;Y_2|X_1), \\ R_{12}+R_{22} &\leq \tau_1 I(X_1,X_2;Y_{r_1}), \\ R_{12}+R_{22} &\leq \tau_1 I(X_1,X_2;Y_{r_2}). \end{aligned} \tag{11}$$

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3.2 | Relays-to-destination transmission schemes

In the first phase, the relays receive different signals from many users sharing the communication model. Now, in order to forward the signals in the second phase, the relays may employ DF encoding scheme. In this section, we present a transmission mode such that the relays can forward the users' signals into their common destination. It is worth noting that the second phase may last for τ_2 such that $\tau_1+\tau_2=1$. In particular, the two relays may simultaneously transmit to the destination to form the MAC. Keep in mind that each relay has the messages of the two users. Therefore, the transmission from the two relays to the destination is equivalent to the transmission over a MAC with full cooperation. It is worth mentioning that transmission over a MAC with full cooperation achieves the outer bound and thus the capacity.[2,35] This means that there is no transmission scheme that can achieve better than the MAC with full cooperation. In this transmission scheme, as illustrated in Figure 4, the two relays simultaneously forward to the destination such that the achievable rate region is given by[28,36]

$$\begin{aligned} R_{12} &\le \tau_2 I(X_{r_1};Y|X_{r_2}),\\ R_{22} &\le \tau_2 I(X_{r_2};Y|X_{r_1}),\\ R_{12}+R_{22} &\le \tau_2 I(X_{r_1},X_{r_2};Y). \end{aligned} \tag{12}$$

Furthermore, in the case of AWGN channel, the received signal, Y, at the destination is given by

$$Y=h_1X_{r_1}+h_2X_{r_2}+Z, \tag{13}$$

where h_1 and h_2 are the channel gains from the relays R_{e_1} and R_{e_2} to the destination, respectively. Moreover, the signal Z is the AWGN with a mean of 0 and a variance normalized to 1. To forward the signals of the two users, each relay employs superposition encoding. Specifically, the first and second relays forward

$$\begin{aligned} X_{r_1} &= \sqrt{\alpha P_{r_1}}V_1(m_1)+\sqrt{\bar{\alpha}P_{r_1}}V_2(m_2),\\ X_{r_2} &= \sqrt{\beta P_{r_2}}U_1(m_1)+\sqrt{\bar{\beta}P_{r_2}}U_2(m_2), \end{aligned} \tag{14}$$

where V_1, V_2, U_1, and U_2 are independent random variables with zero mean and unit variance, that is, $\mathcal{N}(0,1)$. Further, α, $\bar{\alpha}=1-\alpha$, β, and $\bar{\beta}=1-\beta$ are the power allocation factors at the two relays. These power allocations are used to forward the two users' messages. To this extent, we substitute the received signal from (13) into (12) such that the achievable rate region of the Gaussian MAC with full cooperation is given by

$$\begin{aligned} R_{12} &\le \tau_2 C\left(\left(|h_1|\sqrt{\alpha P_{r_1}}+|h_2|\sqrt{\beta P_{r_2}}\right)^2\right),\\ R_{22} &\le \tau_2 C\left(\left(|h_1|\sqrt{\bar{\alpha} P_{r_1}}+|h_2|\sqrt{\bar{\beta} P_{r_2}}\right)^2\right),\\ R_{12}+R_{22} &\le \tau_2 C\left(\left(|h_1|\sqrt{\alpha P_{r_1}}+|h_2|\sqrt{\beta P_{r_2}}\right)^2+\left(|h_1|\sqrt{\bar{\alpha} P_{r_1}}+|h_2|\sqrt{\bar{\beta} P_{r_2}}\right)^2\right). \end{aligned} \tag{15}$$

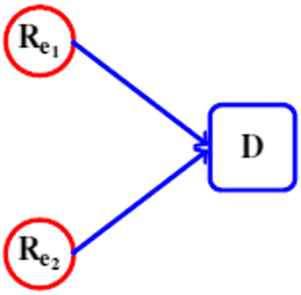

FIGURE 4 Transmission in the second phase may be modeled as a multiple access channel

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Remark III. In this remark, the cases in which the sources' signals are not completely available at the two relays are outlined. We note that the achievable rate region, when the two signals from the different sources are available at the relays in full, encompasses the achievable rate region in all other cases. These comparisons were completely presented in Al-qudah and Bataineh.[2] Here, the other cases are that

- one relay has the two messages, whereas the other relay has only one message. This is referred to as the MAC with unidirectional cooperation. We refer the reader to Maric et al[37] for a comprehensive analysis.
- each relay has only one user's message. Here, the encoding scheme and the achievable capacity were exhaustively investigated in Cover and Thomas.[28]

In summary, two transmission modes are presented such that the two sources can transmit to the two half-duplex relays in the first phase. Further, forwarding the signals from the relays to the destination is presented in the second phase. We summarize these findings in the following two theorems.

Theorem 1. The achievable rate region of the discrete MADC, in the cases that (i) the two sources can subsequently transmit in the first phase and (ii) the two parallel relays operate in half-duplex mode, is given by

$$\begin{aligned} R_1 &= \min\{R_{11}, R_{12}\}, \\ R_2 &= \min\{R_{21}, R_{22}\}, \\ R_1 + R_2 &\leq \tau_2 I(X_{r_1}, X_{r_2}; Y), \end{aligned} \tag{16}$$

where

$$\begin{aligned} R_{11} &\leq \tau_{11} I(X_1; Y_{r_1}, Y_{r_2}), \\ R_{21} &\leq \tau_{12} I(X_2; Y_{r_1}, Y_{r_2}), \\ R_{12} &\leq \tau_2 I(X_{r_1}; Y|X_{r_2}), \\ R_{22} &\leq \tau_2 I(X_{r_2}; Y|X_{r_1}). \end{aligned} \tag{17}$$

Proof. The proof can be simply obtained by following the transmission in the two phases, as shown before. Specifically, the subrates are shown in (2) and (12).

It is worth noting that the achievable subrates $R_{11}, R_{12}, R_{21}, R_{22}$, and also the sum rate $R_1 + R_2$ are evaluated in the case of AWGN in (5), (6), and (15).

Theorem 2. The achievable rate region of the discrete MADC, in the cases that (i) the two sources can simultaneously transmit in the first phase in which weak IC is obtained and (ii) the two parallel relays operate in half-duplex mode, is given by

$$\begin{aligned} R_1 &= \min\{R_{11}, R_{12}\}, \\ R_2 &= \min\{R_{21}, R_{22}\}, \\ R_1 + R_2 &\leq \tau_2 I(X_{r_1}, X_{r_2}; Y), \end{aligned} \tag{18}$$

where

$$\begin{aligned} R_{11} &\leq \tau_1 I(X_1; Y_{r_1}), \\ R_{21} &\leq \tau_1 \{I(X_2; Y_{r_2}) - I(X_2; X_1)\}, \\ R_{12} &\leq \tau_2 I(X_{r_1}; Y|X_{r_2}), \\ R_{22} &\leq \tau_2 I(X_{r_2}; Y|X_{r_1}). \end{aligned} \tag{19}$$

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Proof. The proof may be attained by following the transmission in the two phases, as shown before. Specifically, the subrates $R_{11}, R_{12}, R_{21}, R_{22}$, and the sum rate $R_1 + R_2$ are shown in (7) and (12).

Further, we note that the achievable subrates $R_{11}, R_{12}, R_{21}, R_{22}$, and also the sum rate $R_1 + R_2$, in Theorem 2, are also extended to the case of additive Gaussian channel in (9) and (15).

In this section, two different transmission schemes have been developed. In the next section, outer bounds to the achievable rate regions will be derived. After that, many numerical examples will be also presented to first compare between the two transmission schemes and to show the case in which the capacity is achieved.

4 | OUTER BOUNDS ON THE TRANSMISSION OVER MADC

In this section, we present an outer bound on the transmission over the MADC. The derivation of the outer bound is based on the set of steps that were presented in Khojastepour et al.[38] In particular, these steps were first shown[38] to compute outer bounds on the transmission over general multiterminal networks with intermediate half-duplex nodes. Then, these steps were exploited by Xue and Sandhu[17] and Bagheri et al[18] to derive outer bounds on the transmission over DC with half-duplex relays. To this end, we are now ready to derive the outer bound on the transmission over MADC.

Theorem 3. An outer bound region, on the achievable transmission region that is specified in Theorem 1, over the discrete memoryless MADC is given by

$$\begin{aligned} R_{o_1} &\le \min\{\tau_{11} I(X_1; Y_{r_1}, Y_{r_2}), \tau_2 I(X_{r_1}; Y | X_{r_2})\}, \\ R_{o_2} &\le \min\{\tau_{12} I(X_2; Y_{r_1}, Y_{r_2}), \tau_2 I(X_{r_2}; Y | X_{r_1})\}, \\ R_{o_1} + R_{o_2} &\le \tau_2 I(X_{r_1}, X_{r_2}; Y). \end{aligned} \tag{20}$$

As an outline of the proof, we remind that the transmission is split into two orthogonal transmission phases. In the first phase, the transmission of a signal from a source to many receivers is outer bounded by the capacity of the channel with two independent looks.[28] Further, the transmission in the second phase is outer bounded by the capacity of the MAC.[28] By carefully combining the outer bounds over the different transmission phases, the overall outer bound is achieved.

Proof. To derive the outer bound on the transmission over the MADC, we follow the presented steps in Khojastepour et al.[38] In the first step, as previously shown, the duration of the two transmission phases τ_1 and τ_2 sums to 1, that is, $\tau_1 + \tau_2 = 1$. Then, the cut j is used with the sending mode i to derive the rate $R_{o_{ij}}$. Consequently, each rate $R_{u_{ij}}$ is allocated a time interval τ_1 or τ_2 to compute $\sum_i \tau_i R_{o_{ij}}$. Finally, the supremum over all input distributions and scheduling transmission phases is evaluated.

Remark IV. We note that the achievable rate region that is specified in Theorem 1 matches the outer bound, as presented in Theorem 3. However, this may not achieve the capacity of the MADC since the achievable rate region of this transmission scheme may be less than the achievable rate region of the second transmission scheme, as specified in Theorem 2. We will carefully study this issue in the numerical examples.

Theorem 4. An outer bound region, on the transmission region that is specified in Theorem 2, over the discrete memoryless MADC is given by

$$\begin{aligned} R_{o_1} &\le \min\{\tau_1 I(X_1; Y_{r_1}), \tau_2 I(X_{r_1}; Y | X_{r_2})\}, \\ R_{o_{12}} &\le \min\{\tau_1 I(X_2; Y_{r_2}), \tau_2 I(X_{r_2}; Y | X_{r_1})\}, \\ R_{o_1} + R_{o_2} &\le \tau_2 I(X_{r_1}, X_{r_2}; Y). \end{aligned} \tag{21}$$

As an outline of the proof, the transmission from the sources to the common destination is divided into two orthogonal phases. In the first phase, the transmission is outer bounded by the upper bound of the transmission over the IC.[31]

Further, the transmission, in the second phase, is outer bounded by the capacity of the transmission over the MAC. Hence, by combining the outer bounds over the two orthogonal transmission phases, the outer bound can be attained.

Proof. We followed the same steps used to derive the outer bound, as presented in Theorem 3, to establish the outer bound in Theorem 4.

5 | NUMERICAL RESULTS

In this section, we introduce many numerical examples to improve our understanding of the theoretical results that we derived in this paper. In these numerical examples, we set the average transmit power at the two sources and the two relays to 10. The channel gains from the two sources to the two relays h_{11} and h_{22} are normalized to 1, whereas the crossover channel gains, h_{12} and h_{21}, are changed from an example to another. Further, the channel gains, h_1 and h_2, from the two relays to the destination are also normalized to 1.

In Figures 5 and 6, the achievable rate region is shown to compare the two different transmission schemes that we propose. Specifically, in Figure 5, the comparison is made in the case that the crossover channels are weak, that is, h_{12} and h_{21} are less than 1. In the case of subsequent transmission, this figure clearly shows that as the crossover channels increase, so does the achievable rate region. For example, as the channel gain h_{12} becomes stronger, due to the movement of either the transmission nodes or the relays, the first user's signal can be decoded by both of the two relays. As a result, the two relays can cooperatively transmit this signal in the second phase. In the other transmission scheme, the simultaneous transmission, as the crossover channel gains increase, so does the interference. Thus, the achievable rate region reduces. The nonsimilarity is due to the fact of employing DPC at one of the two sources, in the first phase, such that one of the two users can transmit over an interference-free channel.

In Figure 6, we consider the cases in which the crossover channels are in both the strong interference, that is, h_{12} and $h_{21} \geq 1$, and the very strong interference, that is, h_{12} and $h_{21} \gg 1$. In the case of subsequent transmission and similar to the case of weak crossover channels, this figure clearly shows that as the crossover channels increase, so does the achievable rate region. In the other transmission scheme, in which simultaneous transmission is employed in the first phase, the achievable rate region is limited by the sum rate in the first phase, $R_{12} + R_{21}$, in the case of strong interference, whereas the sum rate does not limit the achievable rate region in the case of the very strong interference.

To fully complete the comparisons, Figure 7 compares between the two transmission schemes in the case that the two users have equal achievable rates. Here, one can notice that for small crossover channel gains like $h_{12} < \nu$, using simultaneous transmission in the first phase is preferred. It is worth noting that the very strong IC is equivalent to the case of no interference IC, that is, h_{12} and $h_{21} = 0$. Further, as the IC is in weak interference regime, the achievable rate region reduces. Then, as the IC is in strong interference regime, that is, the crossover channel is larger than 1, the relays (receivers in the first phase) start partially removing the interference from the other user, and so the achievable rate increases. In the case of the very strong interference regime, the relays can fully remove the effect of interference from the other source. Hence, the achievable rate reaches its maximum point. On the contrary, for $h_{12} > \nu$, subsequent

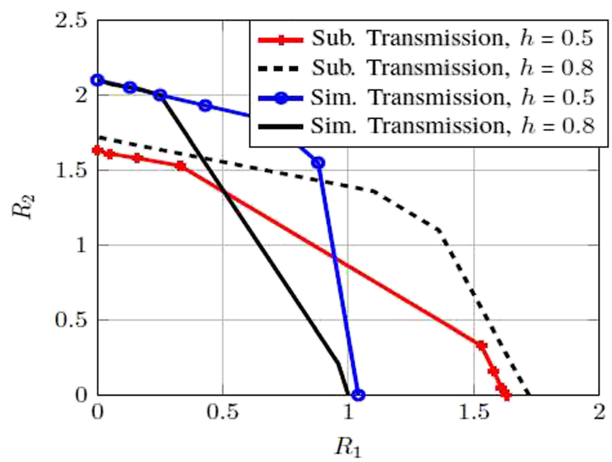

FIGURE 5 The achievable rate region of the Gaussian MADC in the case that the crossover channels between the two sources and the two relays are weak

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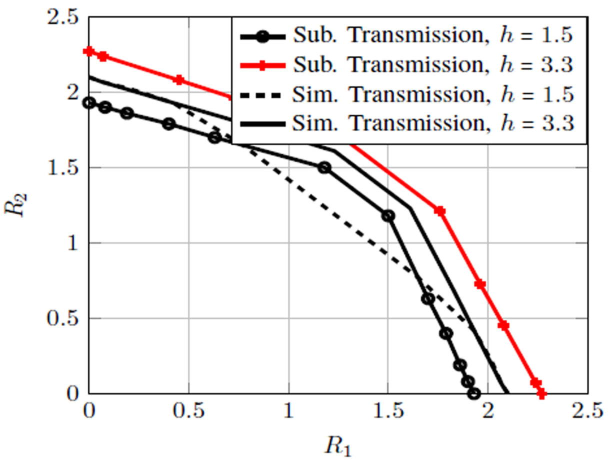

FIGURE 6 The achievable rate region of the Gaussian MADC in the case that the crossover channels between the two sources and the two relays are in strong and very strong cases

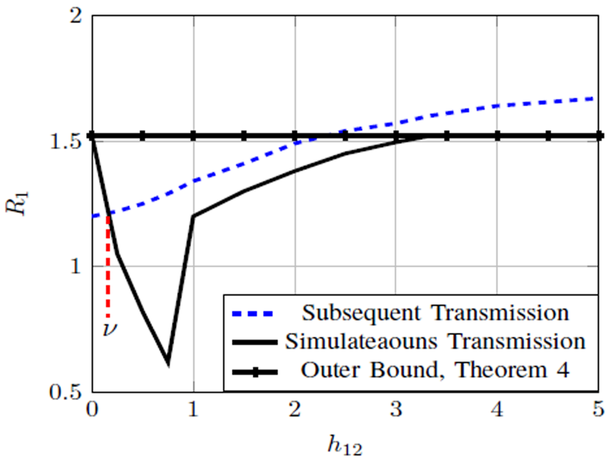

FIGURE 7 The achievable rates and the outer bounds of the user S_1 versus the crossover channel gain h_{12}

transmission should be used in the first phase. In this case, as the crossover channel gains, that is, h_{12} and h_{21}, become stronger, the achievable rate increases. Further, in this transmission scheme, the outer bound matches the achievable rate, as stated in Remark 4. Thus, in the case of $h_{12} > \nu$, the capacity of the MADC is attained.

6 | CONCLUSIONS AND FUTURE WORKS

In this paper, a new multiuser communication model has been developed. This model shows how many sources can transmit their signals to a destination when no direct links are available. To reach the destination, many parallel relays have been employed such that the signals from the different sources are extended to the destination. Based on whether the sources can simultaneously or subsequently transmit to the relays, two different transmission schemes and their respective achievable rate regions have been proposed. In each transmission scheme, the transmission from the sources to the destination is divided into two orthogonal (on time) phases. Then, for each transmission scheme, an outer bound for the achievable rate region is established. Further, many numerical examples have been investigated to mainly compare between the two different transmission schemes and the cases in which the capacity region is achieved.

Future work may concentrate on developing the achievable rate region and also the outer bounds in the case that the relays operate in full-duplex mode. Furthermore, the cases in which (i) more than two sources want to transmit their signals and/or (ii) more than two parallel relays are employed may also be studied.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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