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Parallel Relay Network with Orthogonal Components: Capacity and Power Allocation

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Abstract The authors consider a relay network in which the source-destination pair is augmented by a set of parallel relays. In this relay network, the source transmits to the destination and the relays over multiple orthogonal channel components. Practically, orthogonal channel components is motivated by the fact that a relay cannot receive and then transmit by using the same frequency band. For this relay network, lower bound on the capacity is shown to match the upper bound, and thus establish the capacity of the discrete memoryless physically degraded relay channel. Then, the capacity result is extended to the case of additive white Gaussian noise channel. Finally, rigorous simulation results are carried out to verify the analytical results.

Keywords Relay network · Orthogonal transmission bands · Decode-and-forward relaying · Gaussian channel

1 Introduction

Nowadays, relaying is an important issue in extending and/or improving the reliability of estimating a signal at its desired destination. The relaying is employed by utilizing another user's resources like antennas and power. Practically, the relay node is a fundamental component of many current communication networks like 4G. In its simplest form, the classical relay channel is composed of a source-destination pair aided by a third node, the relay, as depicted in Fig. 1. This three nodes network was first studied by Meulen [1] and then extensively investigated by Cover and Gamal [2]. Then, this relay channel was investigated in many different scenarios like faded relay channel [3, 4], relay channel with

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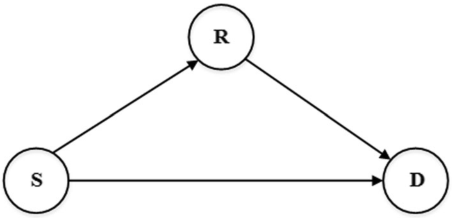
Fig. 1 The basic three nodes relay channel



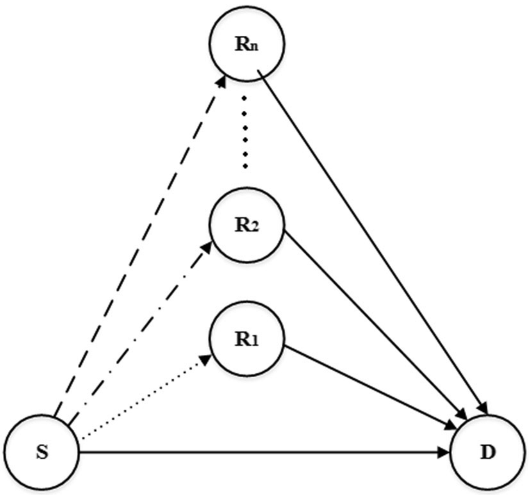
multiple transmit and multiple receive antennas [5, 6], and relay network with orthogonal channel components [7–9]. The capacity of this three nodes network is known in a few cases like degraded relay channel [2, 10], semi-deterministic relay channel [11], and relay with orthogonal channel components [7, 12].

In practical wireless communications scenarios, to avoid self-interference, a relay can not receive and then transmit in the same frequency band. Thus, many practical solutions were proposed such as using a half-duplex relay in the sense that this node may receive in a time slot and then transmit in another time period [9, 13, 14]. Another practical solution was proposed by Zahedi [15] such that the relay can receive and then transmit over orthogonal frequency bands. Other solutions include the scenario studied in [16] where the relay receives using in-band frequency and then relays its signal using out-of-band frequency. Practically, the source may exploit any entity that uses other standards like a 4G base station augmented by components from other networks like Wi-Fi, Wi-MAX, and Bluetooth.

The relay channel, in which the source-destination pair is augmented by multiple parallel relays was studied in many different scenarios [17–22]. For example, the authors in [17–19] studied the value of cooperation and resource allocation. Further, bounds on the achievable rates of a relay channel with multiple relays in which a noncausal interference available only to the source was studied in [21]. Indeed, the authors in [22] developed the capacity of the parallel relay network with infinite number of parallel relays.

In this paper, the authors consider the multiple parallel orthogonal rely network, as shown in Fig. 2. For simplicity, the case of a source-destination pair which is aided by two

Fig. 2 A parallel relay network model with n-Relays in parallel. The source transmits to relay nodes and the destination by using orthogonal frequency bands



parallel relays is initially considered. Then, the obtained results are extended to the case of n parallel relays. In particular, we first generalize the capacity of the relay channel with orthogonal component, which was attained in [15], to the case of multiple orthogonal parallel relay network. Specifically, the capacity of the discrete memoryless relay network, in which multiple parallel relays with orthogonal components are used to improve the rate, is obtained at first and then is extended to the case of Gaussian additive noise channel. Further, rigourous numerical examples are investigated to show the value of optimizing the available power among the relays. Moreover, another numerical example is investigated to show the value of optimizing the number of used parallel relays.

The remainder of this paper is outlined as follows. In Sect. 2, the communication model which we study in this paper is introduced. Then, the capacity of the memoryless discrete parallel relay channel, with orthogonal components, is investigated in Sect. 3. Next, in Sect. 4, the capacity results are extended to the Gaussian parallel relay network, and the optimal power allocation among the source and relay nodes is also studied. Afterwards, numerical examples that validate our theoretical results are presented in Sect. 5. Finally, the paper is concluded in Sect. 6.

2 Channel Model and Preliminaries

A relay network consisting of a source, a destination and multiple parallel relays as depicted in Fig. 3 is considered. The source in this relay network transmits to the destination and the relays over orthogonal channel components. The discrete memoryless relay channel with orthogonal channel components consists of a finite channel input alphabet $\mathcal{X}$, and a finite channel output alphabet $\mathcal{Y}_D$. In this scenario, $X \in \mathcal{X}$ is the input signal associated with the source. Further, $Y_D \in \mathcal{Y}_D$ represents the output alphabet at the destination. In this channel model, the source has a message $W \in \{1, \ldots, 2^{nR}\}$ to be communicated to the receiver over n channel uses. Further, the following definitions describe the relay channel under consideration.

Definition 1 A $(2^{nR}, n)$ code, $R = R_D + R_{R_1} + R_{R_2}$, for the relay channel with orthogonal components consists of the following:

- A message set $W = (W_D, W_{R_1}, W_{R_2}) \in [1:2^{nR_D}] \times [1:2^{nR_{R_1}}] \times [1:2^{nR_{R_2}}]$,
- An encoding function $W = (W_D, W_{R_1}, W_{R_2}) \longrightarrow \mathcal{X}_D^n \times \mathcal{X}_{R_1}^n \times \mathcal{X}_{R_2}^n$,
- A set of deterministic relay functions $f_1, f_2, \ldots, f_n$ such that

$$x_{1i} = f_i(y_{11}, \ldots, y_{1i}) \quad \text{for } 1 \le i \le n,$$

  and

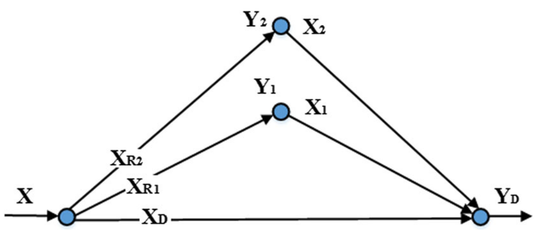


Fig. 3 The general memoryless parallel relay network model with orthogonal components

$$x_{2i} = f_i(y_{21}, \ldots, y_{2i}) \quad \text{for } 1 \le i \le n.$$

- For transmitted messages like $W_D = w_D, W_{R_1} = w_{R_1}$ $W_{R_1} = w_{R_1}$ and $W_{R_2} = w_{R_2}$, the destination maps y_D^n to $(w_D, w_{R_1}, w_{R_2}) \in W = W_D \times W_{R_1} \times W_{R_2}$.

Definition 2 A discrete memoryless relay channel with two parallel relays is said to have orthogonal components if the sender alphabet $\mathcal{X} = \mathcal{X}_D \times \mathcal{X}_{R_1} \times \mathcal{X}_{R_2}$ and the channel transition probability distribution are expressed as

$$p(y_D, y_1, y_2 | x, x_1, x_2) = p(y_1 | x_{R_1}, x_1) p(y_2 | x_{R_2}, x_2) p(y_D | x_D, x_1, x_2) \tag{1}$$

for all

$$(x_D, x_{R_1}, x_{R_2}, x_1, x_2, y_1, y_2, y_D) \in \mathcal{X}_D \times \mathcal{X}_{R_1} \times \mathcal{X}_{R_2} \times \mathcal{X}_1 \mathcal{X}_2 \times \mathcal{Y}_1 \times \mathcal{Y}_2 \times \mathcal{Y}_D. \tag{2}$$

Moreover, a rate R is said to be achievable if, for any $\epsilon > 0$, there exists a sequence of $(2^{nR}, n, P_e)$ code with $P_{e,i}^n$ that goes to zero for sufficiently large n. In this definition, the average error probability is defined as

$$P_{e,i} = \sum_w \frac{1}{2^{nR}} P\left[g_i(Y_{D,i}) \ne w_i \mid w \quad was \quad sent\right], \tag{3}$$

where $W = w$ is the transmitted messages.

3 Capacity of the Parallel Relay Network

In this section, the capacity of the physically degraded parallel relay network with orthogonal channel components is developed. The derivation has two steps in which the achievable rate is derived at first. Then, the converse, which meets the achievable rate, is also developed. In brief, the achievable part is obtained by employing a set of random encoding techniques like block Markov encoding, superposition regular encoding scheme [23, 24] and binning [24] at the source, and decode-and-forward at the parallel relays. Furthermore, the converse part is derived by using the "max-flow min-cut".

Theorem 1 *The capacity of the physically degraded relay network with parallel relay networks and orthogonal channel components is given by*

$$R = max\ min\{I(X_D, X_1, X_2; Y_D), I(X_{R_1}; Y_1 | X_1) + I(X_{R_2}; Y_2 | X_2) + I(X_D; Y_D | X_1, X_2)\},$$

where the maximum is taken over all joint probability mass functions of the form $p(x_D, x_{R_1}, x_{R_2}, x_1, x_2) = p(x_1) p(x_2) p(x_{R_1} | x_1) p(x_{R_2} | x_2) p(x_D | x_1, x_2)$.

Proof The achievability part (lower bound) is introduced in "Appendix 1", and the converse part (upper bound) is given in "Appendix 2".

In summary, the source divides its signal into three parts such that one part is directly transmitted to the destination, the second part is transmitted via one of the relays and the last portion is transmitted via the other relay. In this scheme, each relay employs a backward decoding and then forward encoding. □

Lemma *The result in Theorem* 1 *can be easily generalized to the case of n-parallel relays such that the capacity is given as*

$$R = max\ min\{I(X_D, X_1, X_2, \ldots, X_n; Y), \\ I(X_{R_1}; Y_1|X_1) + I(X_{R_2}; Y_2|X_2) + \cdots + I(X_{R_n}; Y_n|X_n) + I(X_D; Y|X_1, X_2, \ldots, X_n)\}.$$

4 Gaussian Relay Network with Orthogonal Components

In this section, we first extend the capacity of the discrete memoryless parallel relay network to the additive Gaussian channel. Then, the power allocation, at the source and the relays, that can maximize the capacity is investigated.

4.1 Capacity of the Gaussian Relay Network with Orthogonal Components

In this subsection, the capacity of the degraded relay network with orthogonal channel components is extended to the Gaussian case. In this model, Y_i, $i \in \{1, 2, D\}$ is the received signal at the relay R_i, $i \in \{1, 2\}$, and the destination D, respectively

$$Y_i = h_{Si} X_{R_i} + Z_i, \tag{4a}$$

$$Y_D = h_{SD} X_D + h_{1D} X_1 + h_{2D} X_2 + Z_D, \tag{4b}$$

where X_{R_i}, $i \in \{1, 2\}$, and X_D are the transmitted signals by the sender, X_1 and X_2 are the transmitted signals by the relays R_1, and R_2, respectively. Further, h_{S1}, h_{S2}, h_{SD}, h_{1D} and h_{2D} are the channel gains of the links as shown in Fig. 4. Moreover, Z_i, $i \in \{1, 2, D\}$ is assumed to be independent and identically distributed additive white Gaussian noise process with variance N_i, $i \in \{1, 2, D\}$. Moreover, the power constraints at the source and the relays are limited by

$$\frac{1}{n} \sum_{t=1}^{n} E[X_{j,t}^2] \le P_j, \quad j \in \{S, 1, 2\}. \tag{5}$$

In particular, the source uses its power, P_S, to transmit X_D, X_{R_1} and X_{R_2} such that $E[X_{R_1}^2] \le \alpha_1 P_S$, $E[X_{R_2}^2] \le \alpha_2 P_S$ and $E[X_D^2] \le \alpha_3 P_S$. Further, the summation of the power allocation factors α_1, α_2 and α_3 should be less than or equal to 1.

Theorem 2 *The capacity of the AWGN parallel relay channel with orthogonal components subjected to the average power constraints P_S, P_1 and P_2 is given by*

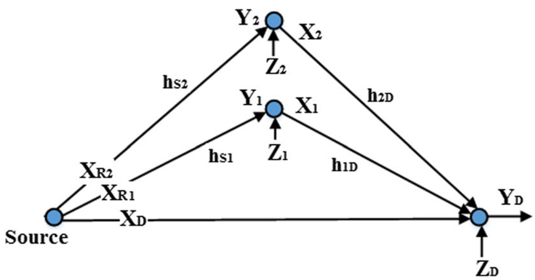


Fig. 4 A Gaussian parallel relay network with orthogonal channel components

$$R = \max_{\alpha_1,\alpha_2,\alpha_3} min\left\{C\left(\frac{A_1}{N_D}\right), C\left(\frac{|h_{S1}|^2\alpha_1 P_S)}{N_1}\right) + C\left(\frac{|h_{S2}|^2\alpha_2 P_S}{N_2}\right) + C\left(\frac{|h_{SD}|^2\,\rho\,\alpha_3 P_S}{N_D}\right)\right\}, \tag{6}$$

where

$$\begin{aligned} A_1 &= |h_{SD}|^2\alpha_3 P_S + |h_{1D}|^2 P_1 + |h_{2D}|^2 P_2 + 2\rho_1|h_{SD}||h_{1D}|\sqrt{\alpha_3 P_S P_1} + 2\rho_2|h_{SD}||h_{2D}|\sqrt{\alpha_3 P_S P_2} \\ &\quad + 2\rho_3|h_{1D}||h_{2D}|\sqrt{P_1 P_2}, \\ \rho &= 1 - \rho_1^2 - \rho_2^2 - \rho_3^2 + 2\rho_1\rho_2\rho_3, \\ C(x) &= \frac{1}{2}\log_2(1+x). \end{aligned}$$

Proof First, consider the multiple access term

$$\begin{aligned} I(X_D, X_1, X_2; Y) &= h(Y_D) - h(Y_D|X_D, X_1, X_2), \\ &= h(Y_D) - h(Z_D), \\ &= C\left(\frac{A_1}{N_D}\right). \end{aligned} \tag{7}$$

Then, the achievable rate at the first relay, R_1, is given as

$$\begin{aligned} I(X_{R_1}; Y_1|X_1) &= h(Y_1|X_1) - h(Y_1|X_1, X_{R_1}), \\ &= h(Y_1|X_1) - h(Z_1), \\ &= C\left(\frac{|h_{S1}|^2\alpha_1 P_S)}{N_1}\right). \end{aligned} \tag{8}$$

Similarly, we may obtain the

$$I(X_{R_2}; Y_2|X_2) = C\left(\frac{|h_{S2}|^2\alpha_2 P_S}{N_2}\right). \tag{9}$$

Next, the rate of estimating the signal X_D at the destination may be obtained as

$$\begin{aligned} I(X_D; Y_D|X_1, X_2) &= h(Y_D|X_1, X_2) - h(Y|X_1, X_2, X_D), \\ &= h(Y_D, X_1, X_2) - h(X_1, X_2) - h(Z_D), \\ &= C\left(\frac{|h_{SD}|^2\,\rho\,\alpha_3 P_S}{N_D}\right), \end{aligned} \tag{10}$$

where $\rho = 1 - \rho_1^2 - \rho_2^2 - \rho_3^2 + 2\rho_1\rho_2\rho_3$.

4.2 Optimal Power Allocation

Due to orthogonality of the channel components, the channel from the source to the destination and the relays is equivalent to the parallel Gaussian channel. For instance, the capacity over any link is not affected by any signal transmitted to any other relay. In this channel model, the optimum power is obtained by employing the water-filling criterion [24]. In this optimization problem, for a given link, the power allotted is inversely proportional to the noise level.

Mathematically, the data rate of the broadcast term, R_{BC}, from the source to the destination and relay nodes is given by

$$\begin{aligned} R_{BC} &= I(X_{R_1};Y_1|X_1) + I(X_{R_2};Y_2|X_2) + I(X_D;Y_D|X_1,X_2), \\ &= C\left(\frac{|h_{S1}|^2\alpha_1 P_S)}{N_1}\right) + C\left(\frac{|h_{S2}|^2\alpha_2 P_S}{N_2}\right) + C\left(\frac{|h_{SD}|^2\,\rho\,\alpha_3 P}{N_D}\right). \end{aligned} \tag{11}$$

Thus, the problem of optimizing the power at the source is reduced to selecting α_1, α_2 and α_3 such that R_{BC} is maximized.

On the other side, the channel from the source and the relay nodes to the destination forms a multiple access channel (MAC) in which multiple sources transmit to a destination. In the MAC channel, the authors in [25, 26] studied the power optimization that maximized the sum rate at the destination. Their results showed that water-filling criterion is again the optimal solution. In this channel model, the MAC, the available power is allocated as i) when one of the two users has a poor channel with the destination, the optimum power is allocated to the user which has the good channel gain with destination, and ii) when the two users have more or less equally channel gains, both users can transmit their signals.

Analytically, the data rate of the MAC term, R_{MAC}, from the source and the relay nodes to the destination is given by

$$\begin{aligned} R_{MAC} &= I(X_D, X_2, X_3; Y), \\ &= C\left(\frac{A_1}{N_D}\right), \end{aligned} \tag{12}$$

where A_1 is given in the previous subsection. Therefore, the problem of maximizing R_{MAC} is reduced to the problem of selecting $\alpha_3 P_S$ at the source, P_1 and P_2 at the relay nodes to maximize R_{MAC}. In particular, if the total power allocated to the relay nodes is limited to P_T, then this power may be divided between the two relay nodes such that R_{MAC} is maximized. Further, in general, we note that the capacity of the relay network is maximized when $R_{BC} \cong R_{MAC}$.

Thus, two important optimization problems may arise. In the first one, by assuming that the total allocated power to all relay nodes is constrained by P_T, then the problem is how to allocate this total power between the relay nodes such that the rate is maximized. Analytically, this optimization problem may be expressed as

$$\begin{aligned} &\text{maximize} && R \\ &\text{subject to} && P_1 + P_2 \le P_T \\ & && 0 \le \alpha_3 P_S \le P_S \end{aligned} \tag{13}$$

Then, the second optimization problem deals with how many relay nodes may be employed such that the rate is maximized, given that N relay nodes can be used. In addition, assume that the following channel gains i) the channel gain, h_{Si} between the source, S, and the relay, i, and ii) the channel gain, h_{iD}, between the relay, i, and the destination, D, are globally known. Then, the optimization problem is reduced to find which relay nodes may be used such that the rate is maximized. Mathematically, this optimization problem may be described as

$$\begin{aligned} &\text{maximize} && R \\ &\text{subject to} && N \\ & && \textstyle\sum_{i=1}^{N} P_i = P_T \end{aligned} \tag{14}$$

We note that these optimization problems are numerically studied in the next section.

5 Numerical Examples

In this section, we numerically evaluate the achievable rate of the relay network with orthogonal channel components. Unless otherwise specified, we initially set the channel gains as follows: $h_{S1} = h_{S2} = 1.8$, and $h_{1S} = h_{2S} = 1$. Further, we set the power constraints at the source and the relay nodes to $P_S = 20$, $P_1 = P_2 = 10$, respectively. In addition, the noise variances N_1, N_2 and N_D are normalized to 1. Finally, we note that the achievable rates here are given in bits per second/channel use (bps).

We first study the importance of allocating the right power to each relay. In this problem, the total allocated power to the relays is kept constant, i.e., $P_T = 20 = P_1 + P_2$ while the power allocated to each relay node is varied to maximize the rate of this relay network. In particular, Figs. 5 and 6 clearly show, based on the channel gain h_{2D}, i) how the achievable rate, and ii) how the power are allocated between the relay nodes, respectively. For instance, in the case that the second relay node has a poor channel with the destination, most of the available power is allocated to the first relay node and thus the achievable rate is significantly improved. However, when both relays have approximately the same channel gain with the destination, the total available power is divided between the two relays. Therefore, marginal improvement is achieved.

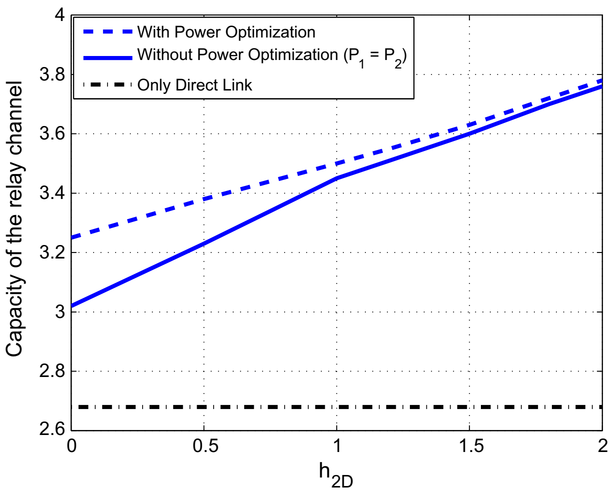
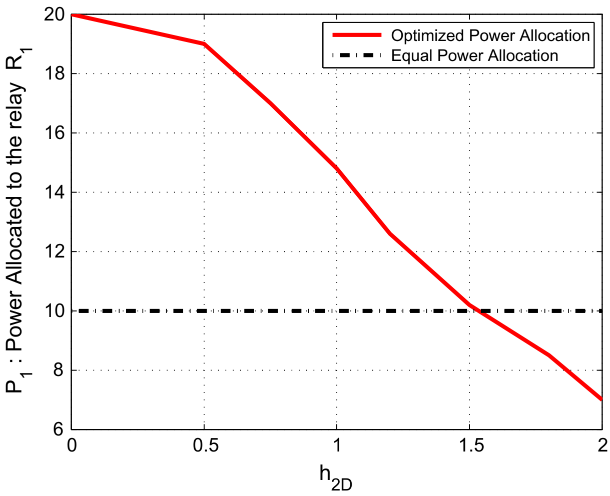


Fig. 5 Achievable capacity of the relay network with/without power optimization between the two relay nodes

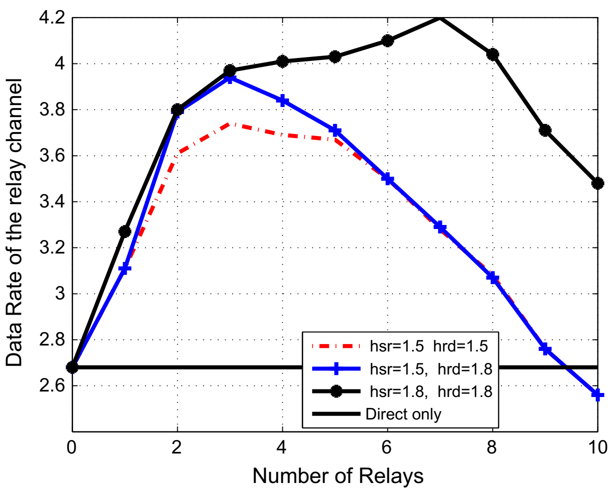
Fig. 6 Power allocated to the first relay versus the channel gain h_{2D}



Now, the achievable rate versus the number of the used relays is studied. In this problem, the total power allocated to all relays is limited to P_T. From the previous numerical example, we learned that for good channel gains between the relay nodes and the destination, power optimizations among the relays may marginally improve the achievable rate. Thus, in this numerical example, the total power is fairly divided among all relay nodes. Further, based on i) the channel gain between the source and relay i, h_{Si}, and ii) the channel gain between the relay i and the destination, h_{iD}, the optimal number of relays that maximize the achievable rate is selected. In addition, for any numerical result, we initially fix h_{Si} and h_{iD} for all relays. Figure 7 shows that the achievable rate is varying with the number of the used relays. Moreover, for any values of h_{Si} and h_{iD}, the optimum number of used relays should be determined.

Finally, the achievable rate is investigated in the case that the power for each relay i is constrained by P_i. In this example, we constraint the source power P_S to 20, and the used

Fig. 7 How the achievable capacity can vary with the number of the employed relays when the total power allocated to all relay nodes is constrained by P_T



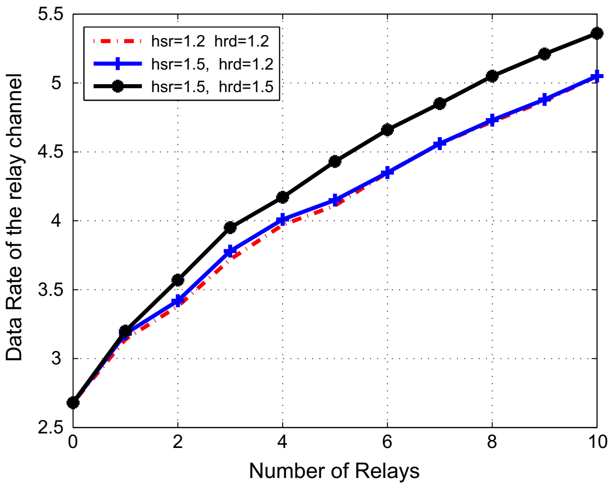


Fig. 8 Achievable capacity of the parallel relay network as a function of the number of employed relay nodes

power for each relay is limited to 10. In addition, for each case, we fix the gain for all channels, as shown in Fig. 8. This figure shows that the achievable rate increases with the number of used relays. Further, the channel gain between the relays and the destination may determine the achievable rate.

6 Concluding Remarks

In this paper, the parallel relay network with orthogonal channel components is investigated. In this channel model, due to practical considerations, these relays cannot receive and transmit over the same frequency band. Thus, the source can use different frequency bands in order to transmit to its destination and the relays. The channel capacity is originally developed in the memoryless case and then is extended to the additive Gaussian channel. Furthermore, rigourous numerical examples are studied to show the benefits of orthogonal channel components and power allocations among the relays.

Appendix 1: Achievability Part

The source uses superposition block Markov encoding and binning scheme. Each relay can employ the decode-and-forward encoding scheme in order to get the signal and then forward it to the destination. In particular, regular encoding with backward decoding is employed by the relays and the destination. In this encoding technique, the source divides its signal into B equally sized blocks. Then, the transmission to the destination and the two relays happens over $B+1$ blocks. Further, for a given block index b, where $b \in [1, B]$, the source divides its signal w into tuple $(w_D, w_{R_1}, w_{R_2}) \in [1, 2^{nR_D}] \times [1, 2^{nR_{R_1}}] \times [1, 2^{nR_{R_2}}]$ such that w_D is transmitted directly to the destination, and w_{R_1} and w_{R_2} are the transmitted signals to the destination via the relays R_1 and R_2, respectively. We note that the average rate triple $\left(R_D \frac{B}{B+1}, R_{R_1} \frac{B}{B+1}, R_{R_2} \frac{B}{B+1}\right)$ approaches (R_D, R_{R_1}, R_{R_2}) as $B \longrightarrow \infty$. Indeed, the channels to both relays and the destination are assumed to be orthogonal.

Random Codebook Generation: First, fix a choice of $p(x_1)p(x_2)p(x_{R_1}|x_1)$ $p(x_{R_2}|x_2)p(x_D|x_1,x_2)$.

- Generate $2^{nR_{R_1}}$ independent identically distributed n-sequences x_1^n, each drawn according to $P(x_1^n)=\prod_{t=1}^n P(x_{1,t})$. Index them as $x_1^n(i)$, $i\in[1:2^{nR_{R_1}}]$.
- Generate $2^{nR_{R_2}}$ independent identically distributed n-sequences x_2^n, each drawn according to $P(x_2^n)=\prod_{t=1}^n P(x_{2,t})$. Index them as $x_2^n(j), j\in[1:2^{nR_{R_2}}]$.
- For each $x_1^n(i)$, generate $2^{nR_{R_1}}$ conditional independent n-sequences $x_{R_1}^n$, drawn according to $P(x_{R_1}^n|x_1^n(i))=\prod_{t=1}^n P(x_{R_1,t}|x_{1,t}(i))$. Index them as $x_{R_1}^n(i,k)$, where $i,k\in[1:2^{nR_{R_1}}]$.
- For each $x_2^n(j)$, generate $2^{nR_{R_2}}$ conditional independent n-sequences $x_{R_2}^n$, drawn according to $P(x_{R_2}^n|x_2^n(j))=\prod_{t=1}^n P(x_{R_2,t}|x_{2,t}(j))$. Index them as $x_{R_2}^n(j,l)$, where $j,l\in[1:2^{nR_{R_1}}]$.
- For each $x_1^n(i)$ and $x_2^n(j)$, generate 2^{nR_D} conditional independent n-sequences x_D^n, drawn according to $P(x_D^n|x_1^n(i),x_2^n(j))=\prod_{t=1}^n P(x_{D,t}|x_{1,t}(i),x_{2,t}(j))$. Index them as $x_D^n(u,v)$, $u,v\in[1:2^{nR_D}]$.

Encoding: We now describe the encoding at the source and the two relays. The encoding is performed in $B+1$ blocks and can be explained as follows:

- Source terminal: In the block b, where $b\in[1,B]$, the message is split into B equally sized blocks $w_{R_1,b}, w_{R_2,b}$, and $w_{D,b}$ for $b=1,2,\ldots,B$. In block $b=1,2,\ldots,B+1$, the sender transmits $x_{R_1,b}^n(w_{R_1,b-1},w_{R_1,b})$, $x_{R_2,b}^n(w_{R_2,b-1},w_{R_2,b})$ and $x_{D,b}^n(w_{D,b-1},w_{D,b})$ where $w_{R_1,0}=w_{R_1,B+1}=w_{R_2,0}=w_{R_2,B+1}=w_{D,0}=w_{D,B+1}=1$.
- Relay R_1 : After the transmission of block b is completed, relay R_1 has received $y_{1,b}^n$. The relay tries to find $(w_{R_1,b})$ such that

$$\left(x_{R_1,b}^n(\hat{w}_{R_1,b-1},\hat{w}_{R_1,b}),x_{1,b}^n(\hat{w}_{R_1,b-1}),y_{1,b}^n\right)\in A_\epsilon^n(X_{R_1},X_1,Y_1) \tag{15}$$

  where A_ϵ^n denotes the strongly jointly $\epsilon-$typical set of length n. Moreover, $\hat{w}_{R_1,b-1}$ are the relay terminals estimate of $w_{R_1,b-1}$.
- Relay R_2: This relay performs the same encoding and decoding as does the relay R_1. Then, the relay R_2 transmits $x_{2,b}^n(\hat{w}_{R_2,b-1})$.

Decoding:

- The Relay R_1 can reliably decode $w_{R_1,b}$ for sufficiently large n and if the past estimate $(\hat{w}_{R_1,b-1})$ is a correct estimate of $(w_{R_1,b-1})$ and

$$R_{R_1}\leq I(X_{R_1};Y_1|X_1). \tag{16}$$

- The Relay R_2 can reliably decode $w_{R_2,b}$ for sufficiently large n and if the past estimate $(\hat{w}_{R_2,b-1})$ is a correct estimate of $(w_{R_2,b-1})$ and

$$R_{R_2}\leq I(X_{R_2};Y_2|X_2). \tag{17}$$

- Using backward decoding, the destination knows $w_{D,b-2}$, $w_{R_1,b-2}$ and $w_{R_2,b-2}$ and searches for a unique $\left(w_{D,b-1},w_{R_2,b-1},w_{D,b-1}\right)$ such that

$$\left(x_1^n\left(\hat{w}_{R_1,b-2},\hat{w}_{R_1,b-1}\right),x_2^n\left(\hat{w}_{R_2,b-2},\hat{w}_{R_2,b-1}\right),x_D^n\left(\hat{w}_{D,b-1}\right),y_{D,b-1}^n\right)\in A_\epsilon^n(X_1,X_2,X_D,Y_D).$$

Then, the destination can decode reliably as long as n is large and

$$R\leq(I(X_D,X_1,X_2;Y_D). \tag{18}$$

We remind that the transmission from the relays and the source to the destination forms a MAC channel in which the sum rate has been considered.

Appendix 2: Converse Part

The best known upper bound on the capacity of the relay channel is the max-flow min-cut upper bound (also referred to as the cut-set bound). This upper bound is tight in all the cases the capacity of the relay channel is known. Although this bound is not believed to be the capacity of the relay channel in general, there is not a single example proving sub-optimality of this upper bound. In this appendix, we present an upper bound on the capacity of the discrete memoryless two-parallel relay channel with orthogonal components using the cut-set bound. The transmission have two steps, which are: i) the broadcast term from the source to the relay nodes and the destination, and ii) the MAC term from the relay nodes and the source to the destination.

First, the broadcast term may be obtained as

$$\begin{aligned}
nR &= H(W),\\
&= I(W;Y^n)+H(W|Y^n),\\
&\leq I(W;Y^n)+n\epsilon_3,\\
&= I(W_{R_1},W_{R_2},W_D;Y^n)+n\epsilon_3,\\
&=^a I(W_{R_1};Y^n)+I(W_{R_2};Y^n|W_{R_1}),\\
&\quad + I(W_D;Y^n|W_{R_1},W_{R_2})+n\epsilon_3,\\
&=^b I(W_{R_1};Y^n)+I(W_{R_2};Y^n)+I(W_D|Y^n)+n\epsilon_3,\\
&= I(W_{R_1};Y_1^n)+I(W_{R_2};Y_2^n)+I(W_D;Y^n)+n\epsilon_3,\\
&= R_{R_1}+R_{R_2}+R_D,
\end{aligned} \tag{19}$$

where a: follows from chain rule. b: follows from the fact that W_{R_1}, W_{R_2},and W_D are independent. In addition, R_{R_1}, R_{R_2}, and R_D are to be derived.

Then, the signal X_{R_1} can be estimated at the first relay, R_1, with rate bounded as

$$\begin{aligned}
nR_{R_1} &= H(W_{R_1}),\\
&= I\left(W_{R_1};Y_1^n\right)+H\left(W_{R_1}|Y_1^n\right),\\
&\leq I\left(W_{R_1};Y_1^n\right)+n\epsilon_1,\\
&= \sum_{i=1}^n I\left(W_{R_1};Y_{1,i}|Y_{1,i-1}\right)+n\epsilon_1,
\end{aligned}$$

$$
\begin{aligned}
&=^a \sum_{i=1}^{n} I\big(W_{R_1}; Y_{1,i}|Y_{1,i-1}, X_{1,i}\big) + n\epsilon_1,\\
&= \sum_{i=1}^{n} H\big(Y_{1,i}|Y_{1,i-1}, X_{1,i}\big) - H\big(Y_{1,i}|Y_{1,i-1}, X_{1,i}, W_{R_1}\big) + n\epsilon_1,\\
&\leq^b \sum_{i=1}^{n} H\big(Y_{1,i}|X_{1,i}\big) - H\big(Y_{1,i}|W_{R_1}, Y_{1,i-1}, X_{R_1,i}, X_{1,i}\big) + n\epsilon_1,\\
&=^c \sum_{i=1}^{n} H\big(Y_{1,i}|X_{1,i}\big) - H\big(Y_{1,i}|X_{R_1,i}, X_{1,i}\big) + n\epsilon_1,\\
&= \sum_{i=1}^{n} I\big(X_{R_1,i}; Y_{1,i}|X_{1,i}\big) + n\epsilon_1,
\end{aligned}
\tag{20}
$$

where a: follows from the fact that $X_{1,i}$ is function of $f_i(Y_{1,i-1})$. b: follows from the fact that conditioning reduces entropy. c: follows from a Markov chain $(W_{R_1}, Y_{1,i-1}) \longrightarrow (X_{R_1,i}, X_{1,i}) \longrightarrow Y_{1,i}$.

Therefore, $R_{R_1} \leq I(X_{R_1}; Y_1|X_1)$ is obtained. In a similar manner, we may easily show that the achievable rate at the second relay, R_2 is bounded as $R_{R_2} \leq I(X_{R_2}; Y_2|X_2)$. Next, the signal X_D may be estimated at the destination with rate bounded as

$$
\begin{aligned}
nR_D &= H(W_{R_D}),\\
&= I(W_D; Y_D^n) + H(W_D|Y_D^n),\\
&\leq I(W_D; Y_D^n) + n\epsilon_2,\\
&= \sum_{i=1}^{n} I(W_D; Y_{D,i}|Y_{D,i-1}) + n\epsilon_2,\\
&= \sum_{i=1}^{n} I(W_D; Y_{D,i}|Y_{D,i-1}, X_{1,i}, X_{2,i}, X_{D,i}) + n\epsilon_2,\\
&= \sum_{i=1}^{n} H(Y_{D,i}|Y_{D,i-1}, X_{1,i}, X_{2,i}, X_{D,i})\\
&\quad - H(Y_{D,i}|Y_{D,i-1}, X_{1,i}, X_{2,i}, X_{D,i}, W_D) + n\epsilon_2,\\
&= \sum_{i=1}^{n} H(Y_{D,i}|X_{1,i}, X_{2,i}) - H(Y_{D,i}|X_{D,i}) + n\epsilon_2,\\
&= \sum_{i=1}^{n} I(X_{D,i}; Y_{D,i}|X_{1,i}, X_{2,i}) + n\epsilon_2.
\end{aligned}
\tag{21}
$$

Thus, $R_D \leq I(X_D; Y_D|X_1, X_2)$ is obtained. Therefore, the broadcast term can be bounded as

$$
\begin{aligned}
R &\leq R_{R_1} + R_{R_2} + R_D,\\
&\leq I(X_{R_1}; Y_1|X_1) + I(X_{R_2}; Y_2|X_2), + I(X_D; Y_D|X_1, X_2).
\end{aligned}
\tag{22}
$$

Now, the multiple access term can be bounded as

$$
\begin{aligned}
nR &= H(W),\\
&= I\big(W;Y_D^n\big) + H\big(W|Y_D^n\big),\\
&\leq^a I\big(W;Y_D^n\big) + n\epsilon_4,\\
&= \sum_{i=1}^{n} I\big(W;Y_{D,i}|Y_{D,i-1}\big) + n\epsilon_4,\\
&= \sum_{i=1}^{n} H\big(Y_{D,i}|Y_{D,i-1}\big) - H\big(Y_{D,i}|W,Y_{D,i-1}\big) + n\epsilon_4,\\
&\leq^b \sum_{i=1}^{n} H(Y_{D,i}) - H\big(Y_{D,i}|W,Y_{D,i-1}\big) + n\epsilon_4,\\
&\leq^c \sum_{i=1}^{n} H(Y_{D,i}) - H\big(Y_{D,i}|X_{1,i},X_{2,i},X_{D,i},W,Y_{D,i-1}\big) + n\epsilon_4,\\
&=^d \sum_{i=1}^{n} \big(H(Y_{D,i}) - H\big(Y_{D,i}|X_{D,i},X_{1,i},X_{2,i}\big)\big) + n\epsilon_4,\\
&= I(X_{D,i},X_{1,i},X_{2,i};Y_i) + n\epsilon_4,
\end{aligned}
\tag{23}
$$

where a: follows from Fano's inequality. b: follows from the fact that conditioning reduces entropy. c: follows from $(X_{1,i}, X_{2,i}, X_{D,i})$ are function of W_i. d: follows from the fact that $(W, Y_{D,i-1}) \longrightarrow (X_{1,i}, X_{2,i}, X_{D,i}) \longrightarrow Y_{D,i}$ form a Markov chain.

Finally, the total upper bound at the destination is attained by taking the minimum of the broadcast term and the multiple access term. Therefore, the following upper bound is achieved

$$R \leq min\{I(X_D, X_1, X_2; Y_D), I(X_{R_1}; Y_1|X_1) + I(X_{R_2}; Y_2|X_2) + I(X_D; Y_D|X_1, X_2)\}$$

References

1. Meulen, E. C. V. D. (1971). Three-terminal communication channels. *Advances in Applied Probability*, *3*, 120–154.
2. Cover, T., & Gamal, A. (1979). Capacity theorems for the relay channel. *IEEE Transactions on Information Theory*, *25*, 572–584.
3. Host-Madsen, A., & Zhang, J. (2005). Capacity bounds and power allocation for wireless relay channels. *IEEE Transactions on Information Theory*, *51*, 2020–2040.
4. Nabar, R., Bolcskei, H., & Kneubuhler, F. (2004). Fading relay channels: Performance limits and space–time signal design. *IEEE Journal on Selected Areas in Communications*, *22*, 1099–1109.
5. Simoens, S., Munoz-Medina, O., Vidal, J., & del Coso, A. (2009). On the Gaussian MIMO relay channel with full channel state information. *IEEE Transactions on Signal Processing*, *57*, 3588–3599.
6. Wen, C.-K., Wong, K.-K., & Ng, C. (2011). On the asymptotic properties of amplify-and-forward MIMO Relay channels. *IEEE Transactions on Communications*, *59*, 590–602.
7. El Gamal, A., & Zahedi, S. (2005). Capacity of a class of relay channels with orthogonal components. *IEEE Transactions on Information Theory*, *51*, 1815–1817.
8. Osmani-Bojd, M., Sahebalam, A., & Hodtani, G. (2011). Capacity region of multiple access relay channels with orthogonal components. In *2011 International conference on ICT convergence (ICTC)* (pp. 74–79).
9. Pham, T., Nguyen, H., & Tuan, H. (2010). Power allocation in orthogonal wireless relay networks with partial channel state information. *IEEE Transactions on Signal Processing*, *58*, 869–878.

10. Kramer, G., Gastpar, M., & Gupta, P. (2005). Cooperative strategies and capacity theorems for relay networks. *IEEE Transactions on Information Theory*, *51*, 3037–3063.
11. Gamal, A., & Aref, M. (1982). The capacity of the semideterministic relay channel (Corresp.). *IEEE Transactions on Information Theory*, *28*, 536–536.
12. Liang, Y., Member, S., Veeravalli, V. V., & Member, S. (2005). Gaussian orthogonal relay channels: Optimal resource allocation and capacity. *IEEE Transactions on Information Theory*, *51*, 3284–3289.
13. Simeone, O., Erkip, E., & Shamai, S. (2011). On codebook information for interference relay channels with out-of-band relaying. *IEEE Transactions on Information Theory*, *57*, 2880–2888.
14. Sahin, O., Simeone, O., & Erkip, E. (2011). Interference channel with an out-of-band relay. *IEEE Transactions on Information Theory*, *57*, 2746–2764.
15. Zahedi, S. (2005). On reliable communication over relay channels. Ph.D. thesis, Stanford University
16. Liang, Y., & Veeravalli, V. (2005). Gaussian orthogonal relay channels: Optimal resource allocation and capacity. *IEEE Transactions on Information Theory*, *51*, 3284–3289.
17. Bakanoglu, K., Tomasin, S., & Erkip, E. (2011). Resource allocation for the parallel relay channel with multiple relays. *IEEE Transactions on Wireless Communications*, *10*, 792–802.
18. Maric, I., & Yates, R. (2004). Forwarding strategies for Gaussian parallel-relay networks. In *Proceedings of international symposium on information theory, 2004* (p. 269).
19. Ghelber, B., & Dabora, R. (2012). The value of cooperation between relays in the multiple access channel with multiple relays. *Transactions on Emerging Telecommunications Technologies*, *23*, 341–359.
20. He, X., & Yener, A. (2007). On the equivocation region of relay channels with orthogonal components. In *Systems and computers conference record of the forty-first asilomar conference on signals, 2007* (pp. 883–887).
21. Sonee, A., & Hodtani, G. (2013). On the capacity of special classes of Gaussian relay networks with orthogonal components and noncausal state information at source. Available at *arXiv*
22. Gastpar, M., & Vetterli, M. (2005). On the capacity of large Gaussian relay networks. *IEEE Transactions on Information Theory*, *51*, 765–779.
23. Han, T., & Kobayashi, K. (1981). A new achievable rate region for the interference channel. *IEEE Transactions on Information Theory*, *27*, 49–60.
24. Cover, T. M., & Thomas, J. A. (2006). *Elements of information theory* (2nd ed.). New York: Wiley.
25. Park, T., Jang, J., Shin, O.-S., & Lee, K. B. (2005). Transmit power allocation for a downlink two-user interference channel. *IEEE Communications Letters*, *9*, 13–15.
26. Liu, N., & Ulukus, S. (2006). Capacity region and optimum power control strategies for fading Gaussian multiple access channels with common data. *IEEE Transactions on Communications*, *54*, 1815–1826.

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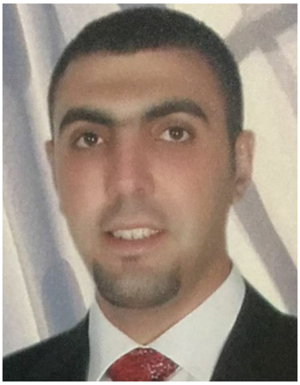

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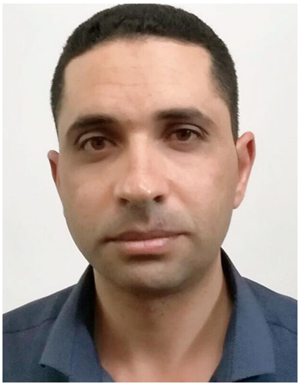

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