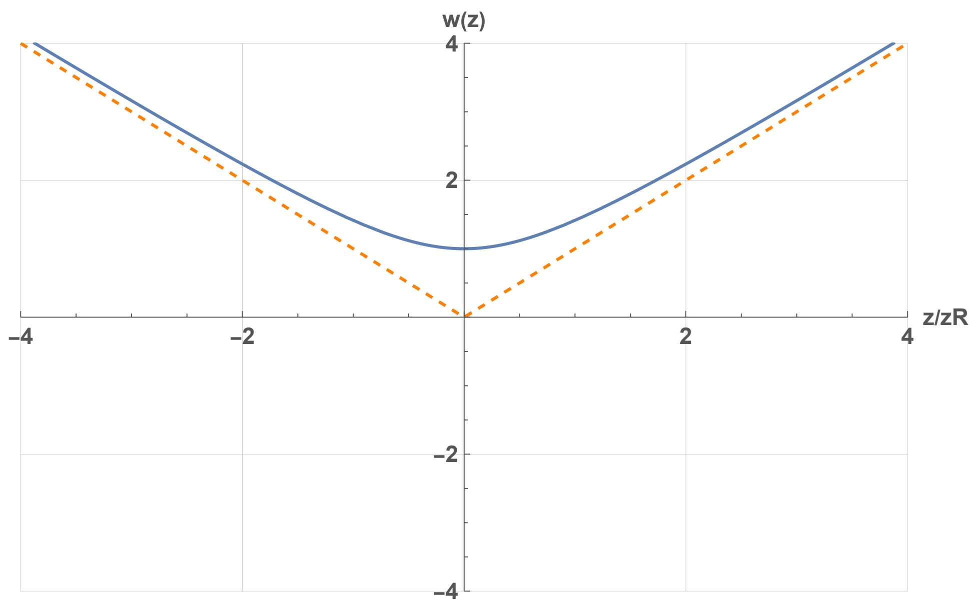
Gaussian beams

Beam spot along propagation

In[ ]:=
```
Show[Plot[w[1, z, 1], {z, -4, 4}, PlotRange → {{0 - 4, 4}, {-4, 4}},
  AxesLabel → {"z/zR", "w(z)"}, GridLines → Automatic,
  LabelStyle → Directive[Bold, Medium], AxesStyle → Directive[Darker@Gray, 12]],
 Plot[Abs[z], {z, -4, 4}, PlotStyle → Directive[{Orange, Dashed}]], ImageSize → 500]
```

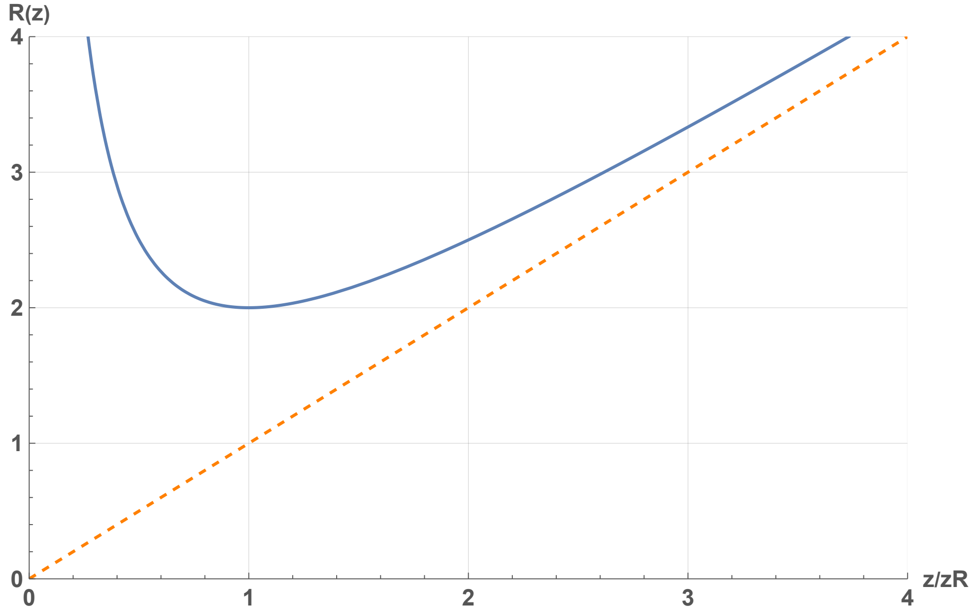
Out[ ]=



Beam’s curvature along propagation

In[∘]:=
```
Show[Plot[R[z, 1], {z, 0, 4}, PlotRange → {{0, 4}, {0, 4}},
  AxesLabel → {"z/zR", "R(z)"}, GridLines → Automatic,
  LabelStyle → Directive[Bold, Medium], AxesStyle → Directive[Darker@Gray, 12]],
 Plot[z, {z, 0, 4}, PlotStyle → Directive[{Orange, Dashed}]], ImageSize → 500]
```

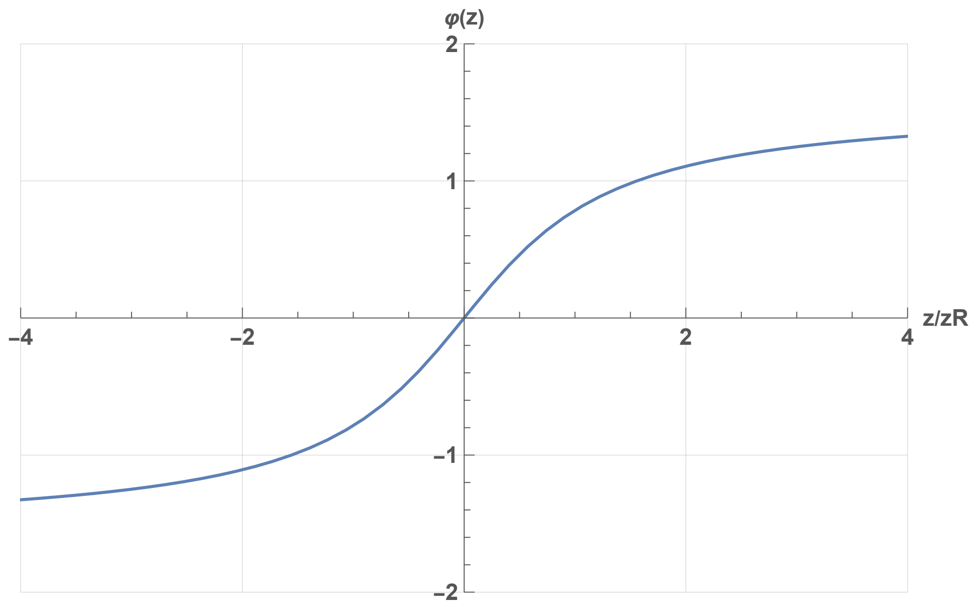
Out[∘]=



Gouy Phase along propagation

In[∘]:=
```
Show[Plot[φ[z, 1], {z, -4, 4},
  PlotRange → {{-4, 4}, {-2, 2}}, AxesLabel → {"z/zR", "φ(z)"},
  GridLines → Automatic, LabelStyle → Directive[Bold, Medium],
  AxesStyle → Directive[Darker@Gray, 12]], ImageSize → 500]
```

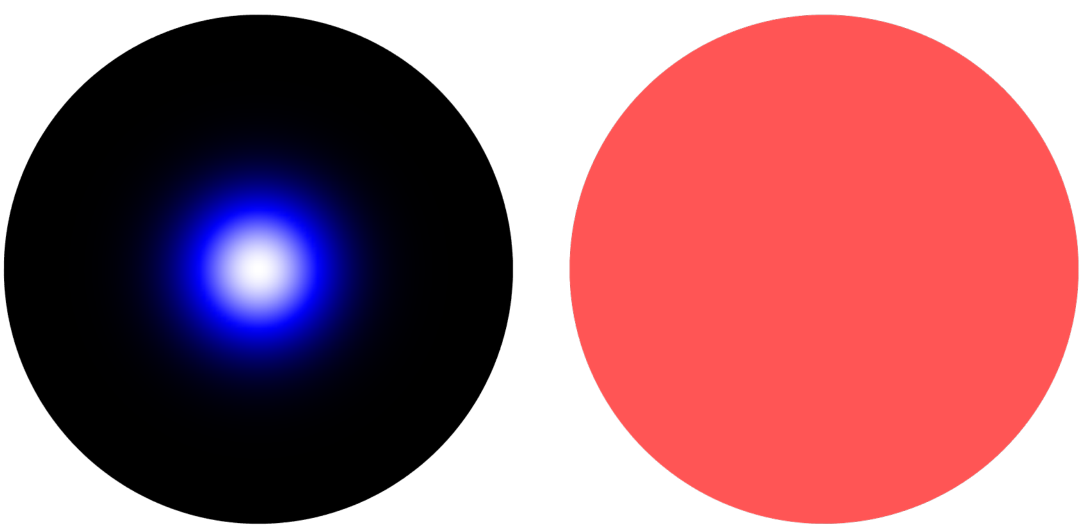
Out[∘]=



Intensity and phase of a Gaussian beam at z=0

In[∘]:=
```
GraphicsRow[{Int[Gaussian[0.8]], Ph[Gaussian[0.8]]}]
```

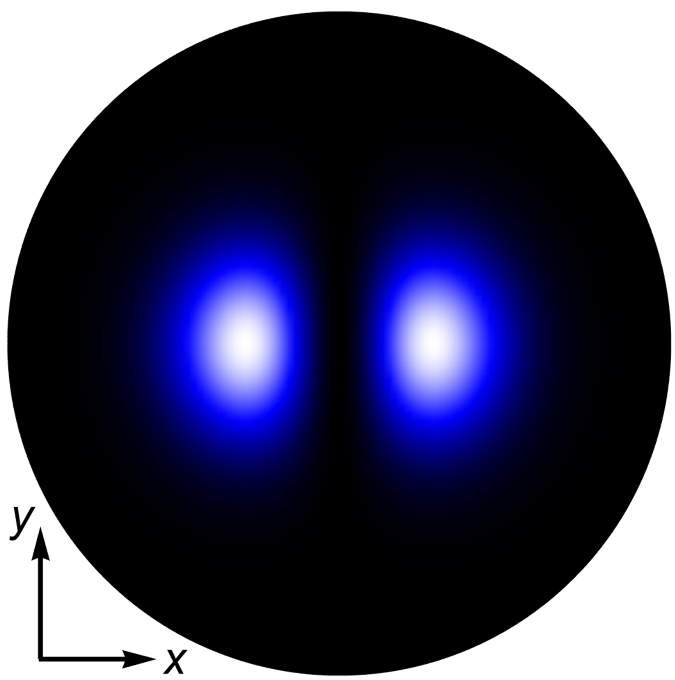
Out[∘]=



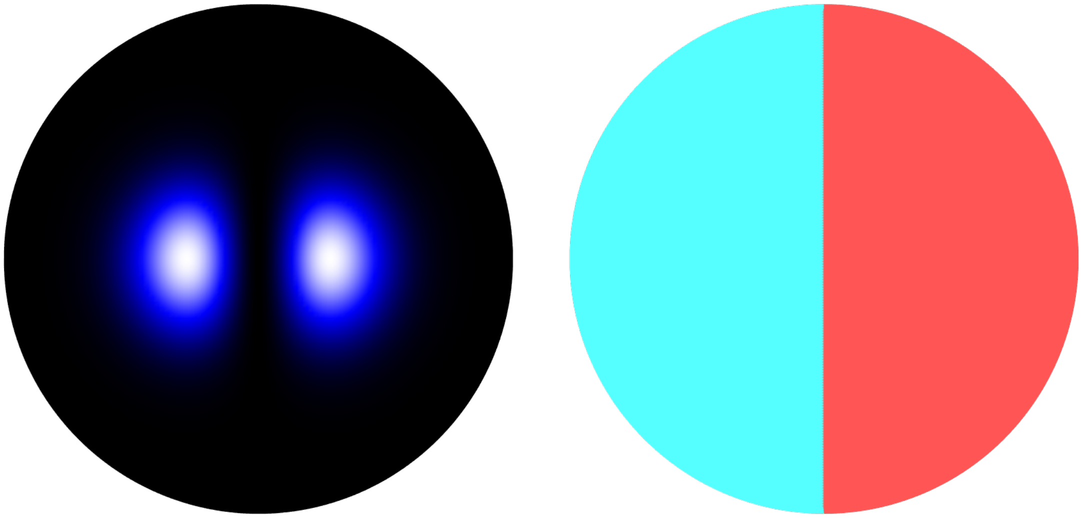
Higher-orders modes

In[ ]:=
```
Show[Int[HG[1, 0, 0.8]], Graphics[
  {{Black, Thickness[0.007], Arrowheads[0.05], Arrow[{{-1.8, -1.9}, {-1.8, -1.1}}]},
   {Black, Thickness[0.007], Arrowheads[0.05],
    Arrow[{{-1.81, -1.9}, {-1.1, -1.9}}]}}], Graphics[
  {{Inset[Style["y", Black], {-1.9, -1.05}, BaseStyle → Directive[Large, Italic]]},
   {Inset[Style["x", Black], {-.98, -1.9},
     BaseStyle → Directive[Large, Italic]]}}], PlotRange → All]
```

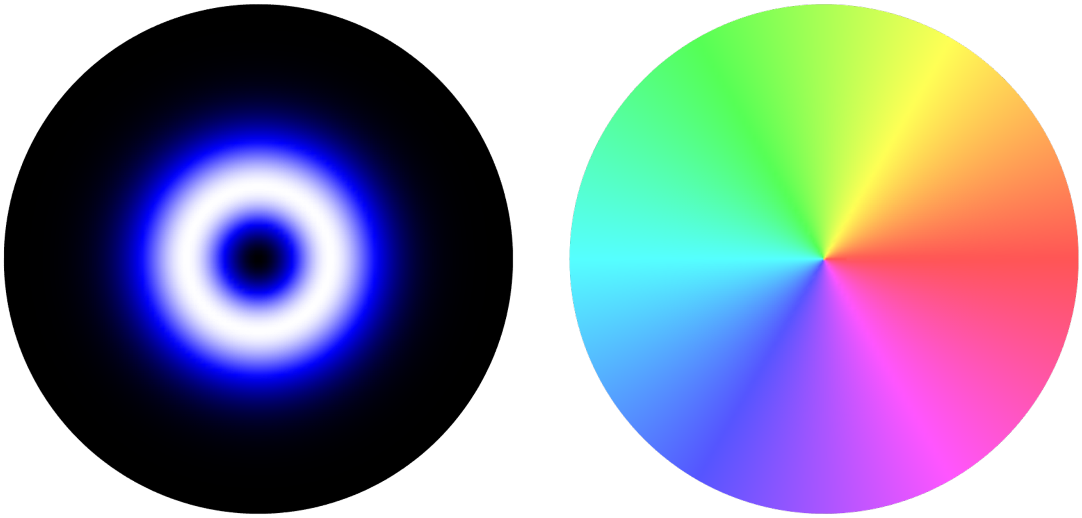
Out[ ]=



In[ ]:=
```
GraphicsRow[{Int[HG[1, 0, 0.8]], Ph[HG[1, 0, 0.8]]}]
```
Out[ ]=



In[ ]:=
```
GraphicsRow[{Int[LG[1, 0, 0.8]], Ph[LG[1, 0, 0.8]]}]
```
Out[ ]=



Functions

```
SetDirectory@NotebookDirectory[];
```

Gaussian Beam parameters

In[ ]:=

```
(*Curvature of the wavefunction*)
```
$$R[z\_, zR\_] := z \left(1 + \frac{zR^2}{z^2}\right);$$

```
(*Beam spot / waist parameter*)
```
$$w[w0\_, z\_, zR\_] := w0\, \text{Sqrt}\left[1 + \frac{z^2}{zR^2}\right]$$

```
(*Gouy Phase*)
φ[z_, zR_] := ArcTan[zR, z]
```

$$zR[w0\_, \lambda\_] := \frac{\pi\, w0^2}{\lambda};$$

Paraxial beams at z=0

```
(* Simple Gaussian Beam *)
```
$$\text{Gaussian}[w0\_] := \frac{e^{-\frac{x^2}{w0^2}-\frac{y^2}{w0^2}} \sqrt{\frac{2}{\pi}}}{w0};$$

```
(*Laguerre-Gaussian Mode*)
```
$$\text{LG}[l\_, P\_:0, w0\_:1] := \text{Sqrt}\left[\frac{2\,\text{Factorial}[P]}{\pi\,\text{Factorial}[P + \text{Abs}[l]]}\right] \frac{1}{w0} \left(\frac{\text{Sqrt}\left[x^2 + y^2\right] \sqrt{2}}{w0}\right)^{\text{Abs}[l]}$$
$$\text{LaguerreL}\left[P, \text{Abs}[l], \frac{2\left(x^2 + y^2\right)}{w0^2}\right] \text{Exp}\left[\frac{-\left(x^2 + y^2\right)}{w0^2}\right] \text{Exp}[i\, l\, \text{ArcTan}[x, y]];$$

```
(*Hermite-Gaussian Mode*)
```
$$\text{HG}[n\_, l\_, w0\_:1] := \frac{(2/\pi)^{(1/4)}\,\text{Sqrt}\left[\frac{1}{2^n\,\text{Factorial}[n]}\right]}{\text{Sqrt}[w0]} \text{HermiteH}\left[n, \frac{\text{Sqrt}[2]\,x}{w0}\right]$$
$$\text{Exp}\left[\frac{-x^2}{w0^2}\right] \frac{(2/\pi)^{1/4}\,\text{Sqrt}\left[\frac{1}{2^l\,\text{Factorial}[l]}\right]}{\text{Sqrt}[w0]} \text{HermiteH}\left[l, \frac{\text{Sqrt}[2]\,y}{w0}\right] \text{Exp}\left[\frac{-y^2}{w0^2}\right];$$

Paraxial beams at z!=0

In[∘]:=

```
(*Laguerre-Gaussian Mode*)
LGz[l_, P_, w0_, λ_] := Module[{zr, A, B},
   zr = zR[w0, λ];
   A = Sqrt[(2 Factorial[P])/(π Factorial[P + Abs[l]])] (1/(Sqrt[w[w0, z, zr]])) ((Sqrt[x^2 + y^2] √2)/(w[w0, z, zR[w0, λ]]))^Abs[l]
     Exp[(-(x^2 + y^2))/(w[w0, z, zr]^2)] LaguerreL[P, Abs[l], (2 (x^2 + y^2))/(w[w0, z, zr]^2)] Exp[(-ⅈ 2 π)/λ ((x^2 + y^2))/(2 R[z, zr])]
     Exp[ⅈ l ArcTan[x, y]] Exp[-ⅈ (2 P + Abs[l] + 1) φ[z, zr]];
   Return[A]];
```

```
(*Hermite-Gaussian Mode*)
HGz[n_, l_, w0_, λ_] := Module[{zr, A, B},
  zr = zR[w0, λ];
  A = (2 / π)^(1/4) Sqrt[1/(2^n Factorial[n])] (2 / π)^(1/4) Sqrt[1/(2^l Factorial[l])]
    1/(w[w0, z, zr]) HermiteH[n, (Sqrt[2] x)/(w[w0, z, zr])] HermiteH[l, (Sqrt[2] y)/(w[w0, z, zr])]
    Exp[(-x^2)/(w[w0, z, zr]^2)] Exp[(-y^2)/(w[w0, z, zr]^2)] Exp[(-ⅈ 2 π)/λ ((x^2 + y^2))/(2 R[z, zr])] ×
    Exp[-ⅈ (n + l + 1) φ[z, zr]];
  Return[A]]
```

Non diffracting beams

Tools

In[ ]:=
```
(*Simple Intensity of the beam*)
Int[ψ_, NN_ : 100] := DensityPlot[Abs[ψ]^2, {x, -2, 2}, {y, -2, 2}, PlotPoints → NN,
   Exclusions → None, RegionFunction → Function[{x, y, f}, x^2 + y^2 < 4],
   ColorFunction → (Blend[{Black, Blue, White}, #] &),
   Axes → None, Frame → False, PlotRange → All];
(*Simple Phase of the beam*)
Ph[ψ_, NN_ : 100] := DensityPlot[Arg[ψ], {x, -2, 2}, {y, -2, 2},
   ColorFunction → (Lighter@Hue[Rescale[#, {0, 2 π}, {0, 1}]] &),
   ColorFunctionScaling → False, PlotPoints → NN, Exclusions → None,
   RegionFunction → Function[{x, y, f}, x^2 + y^2 < 4], Axes → None, Frame → False];
```