ISyE 6644 – Spring 2023

Homework #2 — Due Thursday, February 16

Follow the instructions on the syllabus to submit a single archive containing all Simio files and a single PDF document containing all your derivations and experimental outcomes. Name the archive as `LastName-FirstName.zip`.

1. Parts (steel plates) arrive at a processing center with c identical drills (service capacity of c units) at a rate of $\lambda = 1/2$ per minute. When all drills are busy, the plates are placed in a single FIFO queue. After processing, all plates leave the system. Assume that the travel times from the front door to the drill(s) and from the drill(s) to the exit are zero (hence we can use Connectors).

   Build a Simio model with the interarrival time distribution, the processing time distribution, and the service capacity c as model Referenced Properties. A Server object with a capacity c is equivalent to c parallel "servers" with a single waiting line (queue). Make sure that the Default Units of the first two properties are minutes.

   Create a nice animation using the following instructions:

   - Use "appropriate" symbols for the Server object modeling the drill(s) and Entity modeling the plates. You can download such symbols from the 3D Warehouse.
   - Add a Status Label to display the number of busy drills when the model runs.

   Create four experiments, each consisting of 50 independent replications over 8 days, with a warm-up period of one day, and compute SMORE plots for the following random variables observed during the 7-day data collection window: the average time in system for plates, the average number of plates in the system, and the drill utilization. (This is possible because the three Referenced Properties become experimental controls.) Compare the point estimates and approximate 95% confidence intervals (from the SMORE plots) with the steady-state means obtained from Section 2.3 of the Simio text. For single-server systems, you can use Kingman's formula and Little's law. For Markovian models with service capacity ≥ 2 you can use an online calculator, such as `https://www.supositorio.com/rcalc/rcalclite.htm`. In each case give the notation of the queueing system. How close are the point estimates to the theoretical means from queueing theory? Do the confidence intervals for the three means contain the actual means? How confident are you that the simulation models in the four experiments reached steady state?

   (a) Exponential interarrival times, exponential service times with mean $E(S) = 1.5$ minutes, and $c = 1$.

   (b) Exponential interarrival times, exponential service times with mean $E(S) = 3$ minutes, and service capacity $c = 2$ drills.

   (c) Exponential interarrival times, triangular service times with minimum 0.5 minutes, most likely value 1 minute, and maximum 3 minutes (mean $E(S) = 1.5$), and $c = 1$. The triangular distribution is discussed at `https://en.wikipedia.org/wiki/Triangular_distribution`. To get the expression in Simio, start typing `Random.Tria...`.

   (d) Exponential interarrival times, gamma service times with shape parameter $\alpha = 1/2$ and scale parameter $\beta = 3$ (mean $E(S) = \alpha\beta$ and $\text{Var}(S) = \alpha\beta^2$), and $c = 1$. The gamma distribution is discussed at https://en.wikipedia.org/wiki/Gamma_distribution (I am using α in place of k).

(e) Enhance the model from Problem 1(d) above using Paths to model the movement of parts. Assume that the plates are moved with a speed of 5 m/sec, the distance from the entrance to the drill(s) is 10 meters, and the distance from the drill(s) to the exit is 5 meters. Do the output statistics (SMORE plots) reflect this modeling change?

2. The Weibull distribution is used to model lifetimes of various pieces of equipment or times to complete tasks. The random variable T follows this distribution is its pdf has the form

$$f(t) = \alpha\lambda(\lambda t)^{\alpha-1}e^{-(\lambda t)^{\alpha}}, \quad x > 0,$$

where $\alpha > 0$ is a shape parameter and $\lambda > 0$ is a scale parameter. We write $T \sim \text{Weibull}(\alpha, \lambda)$.

(a) Show that the cdf of T is given by

$$F(t) = 1 - e^{-(\lambda t)^{\alpha}}, \quad t > 0.$$

(b) Recall that the *inverse-transform* method generates realizations of Y by inverting the c.d.f. $F(\cdot)$ from part (a): If U is a uniform$(0, 1)$ random variable, then $F^{-1}(U)$ has the distribution of X. Invert the c.d.f from part (a) to obtain an appropriate formula.

(c) Find the density of the rv $Z = \lambda T$, and name the distribution of Z.

3. Problem 4.7 from Law (2015).

4. Problem 4.10 from Law (2015).

5. Problem 4.11 from Law (2015).

6. Suppose that $\{U_1, U_2, \ldots\}$ are i.i.d. random variables from the uniform$(0, 1)$ distribution. For $n \geq 2$, let $S_n = \sum_{i=1}^{n} U_i$.

(a) Compute the mean and variance of S_n.

(b) Find the limiting distribution of

$$Z_n = \frac{S_n - E(S_n)}{\sqrt{\text{Var}(S_n)}}$$

as $n \to \infty$.

(c) Use the result in part (b) to approximate the probability $P\{Z_{12} > 2\}$.

7. Consider the following i.i.d. sample from a continuous distribution (not highly skewed) with mean μ: 7.3, 6.1, 3.8, 8.4 6.9, 7.1, 5.3, 8.2, 4.9, 5.8.
A review of one-sided CIs and hypothesis tests is given in the following site (I encourage you to browse higher levels of the entire thread):
https://www.itl.nist.gov/div898/handbook/prc/section2/prc221.htm.

(a) Compute the sample mean $\bar{X}_{10}$, the sample standard deviation S_{10}, and an approximate 95% CI for μ.

(b) Test the null hypothesis H_0: $\mu = 6$ at level of significance $\alpha = 0.05$. Relate the outcome of this hypothesis test with the CI from part (a).

(c) An upper one-sided $100(1-\alpha)\%$ CI for μ is a random interval $(-\infty, U]$, with the upper limit U computed from a dataset, such that $P(\mu \leq U) = 1-\alpha$. Compute an (approximate) upper one-sided 95% CI for μ. Then test the hypothesis H_0: $\mu \leq 7$ at level of significance $\alpha = 0.05$.

(d) Compute an (approximate) lower one-sided 95% CI for μ. Then test the hypothesis H_0: $\mu < 5.25$ at level of significance $\alpha = 0.05$.