MCG 4345 – Aerodynamics

Potential Flow Past an Airfoil Using the Vortex Panel Method

Written by: Michael Fiaani

Student ID: 300014312

A Computational Assignment

Presented to: Professor Hamed Sadeghi

Table of Contents:

List of Figures:

List of Tables:

1.0 Introduction

The vortex panel method is a computational tool that characterizes the aerodynamics of an airfoil specified by their NACA number. This numerical method is helpful for modelling an airfoil, assuming the effects of viscosity and compressibility are negligible. Although this may sound inefficient, the vortex panel method still provides accurate approximations to study the aerodynamic values of a given airfoil. This report will concentrate on a 4-digit NACA airfoil, the NACA 2418. Starting off the analysis, it is important to determine the geometry presented by the vortex panel method. To find the contour of the NACA 2418 first need to determine the half-thickness distribution which is represented with eq'n 1 below:

$$\boldsymbol{y_t = \frac{T}{0.2}\left(a_0 x^{\frac{1}{2}} - a_1 x - a_2 x^2 + a_3 x^3 - a_4 x^4\right)} \tag{1}$$

Where the coefficients are listed below:

$$\boldsymbol{a_0 = 0.2969,\ a_1 = -0.126,\ a_2 = -0.3516\ , a_3 = 0.2843, a_4 = -0.1015}$$

The thickness distribution is calculated, the camber line will be found using eq'ns 2 and 3. Where p represents the maximum camber position. Note that finding the maximum camber position will be shown later in the report.

$$\boldsymbol{y_c = \frac{m}{p^2}(2px - x^2)} \text{ for } \boldsymbol{0 \leq x < p} \tag{2}$$

$$\boldsymbol{y_c = \frac{m}{(1-p^2)}((1-2p) + 2px - x^2)} \text{ for } \boldsymbol{p \leq x \leq 1} \tag{3}$$

Equation 2 represents y_c before the maximum camber and equation 3 represents y_c after the maximum camber.

$$\boldsymbol{y_{top} = y_c + y_t} \tag{4}$$

$$\boldsymbol{y_{bot} = y_c - y_t} \tag{5}$$

Using equations 1, 2 and 3 finds the plot for the contour of the NACA 2418. Where equations 4 represents the upper surface and equation 5 represents the bottom surface of the contour plot.

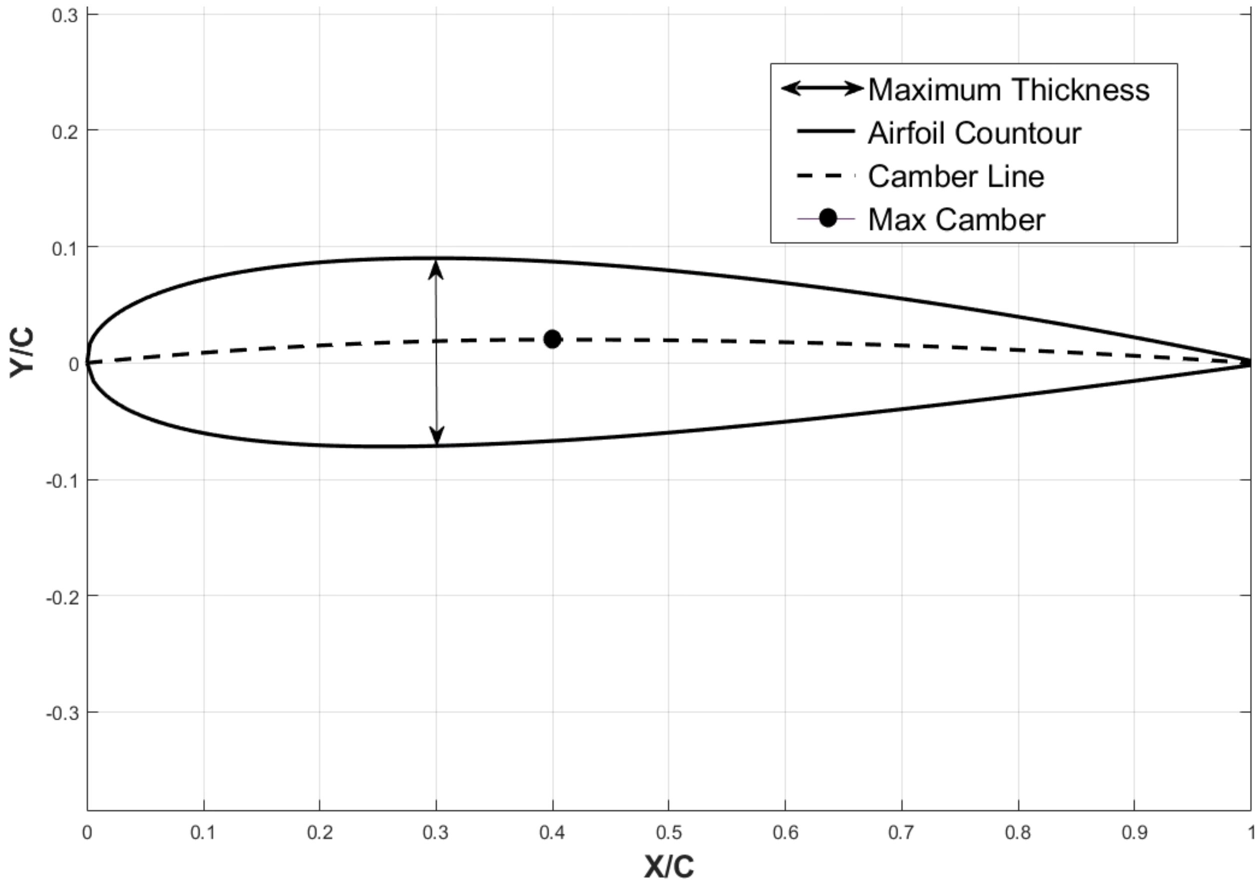

Figure 1: Contour plot of the NACA 2418

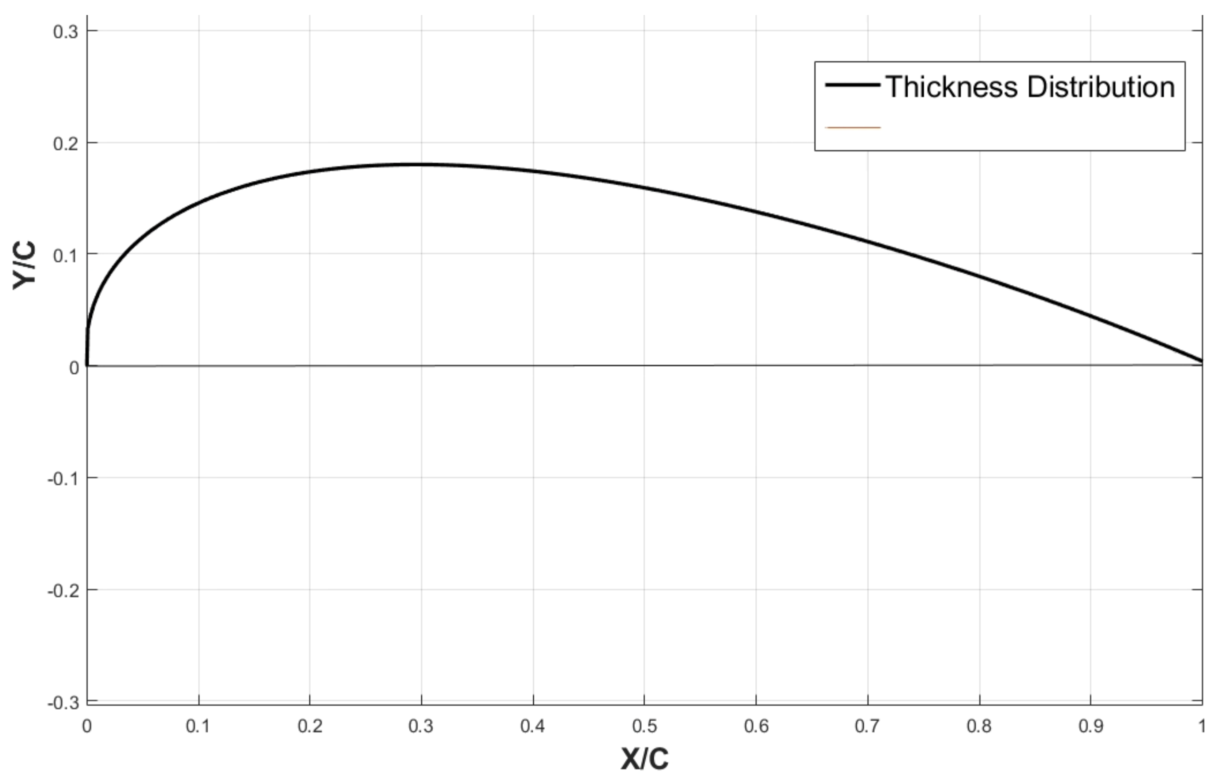

Figure 2: NACA 2418 Thickness Distribution

Figure 1 shows the contour plot of the NACA 2418 where equations 1 to 5 were used to determine the conditions of the upper and lower surfaces. Figure 2 gives the thickness distribution multiplied by a factor of 2 (Since equation 1 represents half-thickness). Now the thickness distribution and the contour plots of the NACA 2418 have been determined, the maximum thickness and camber can be found. The NACA series itself represents characteristics of the airfoil depending on the placement of those values [1]. Represent 2418 as MPXX. Where:

$$\boldsymbol{M = 2, P = 4}, \text{ and } \boldsymbol{XX = 18}$$

The maximum camber and location then can be found with:

$$\boldsymbol{Y_{c,max} = \frac{M}{100} = \frac{2}{100} = 0.02} \qquad (6)$$

$$\boldsymbol{x_{c,max} = \frac{P}{10} = \frac{4}{10} = 0.4} \qquad (7)$$

So, the maximum camber value and location is found with equations 6 and 7. Where the maximum camber value is 0.02 at a 40% camber position. Next, the maximum thickness is found below.

$$\boldsymbol{y_{t,max} = \frac{XX}{100} = \frac{18}{100} = 0.18} \qquad (8)$$

Therefore, the maximum thickness is 0.18. To find the location of the maximum, plug equation 8 into equation 1 and solve for x.

$$\boldsymbol{0.18 = \frac{T}{0.2}\left(a_0 x^{\frac{1}{2}} - a_1 x - a_2 x^2 + a_3 x^3 - a_4 x^4\right)}$$

After solving for x, the maximum thickness location was found to be at x/c=0.3 or 30% of the chord length. The camber and thickness maximum location and value have also been plotted in Figure 1. Next characteristics to define are the radius of curvature at the leading edge, which is represented with the equation below:

$$\boldsymbol{\frac{r_{LE}}{c} = 1.102\left(\frac{t_{max}}{c}\right)^2 = 1.102\left(\frac{0.18}{c}\right)^2 = 0.0356} \qquad (9)$$

Therefore, the radius of curvature at the leading edge was found to be 0.0356 or 3.56% of the chord length. The angle of the trailing edge is found by comparing the point of the trailing edge with the second last points of both the top and bottom surfaces. Therefore, comparing the three points (0,1), (0.955, 0.012) and (0.955, -0.0081). The **angle at the trailing edge** was found to be **20.12º**. Figure 3 shown below plots the Contour of the NACA 2418 with 100 panels.

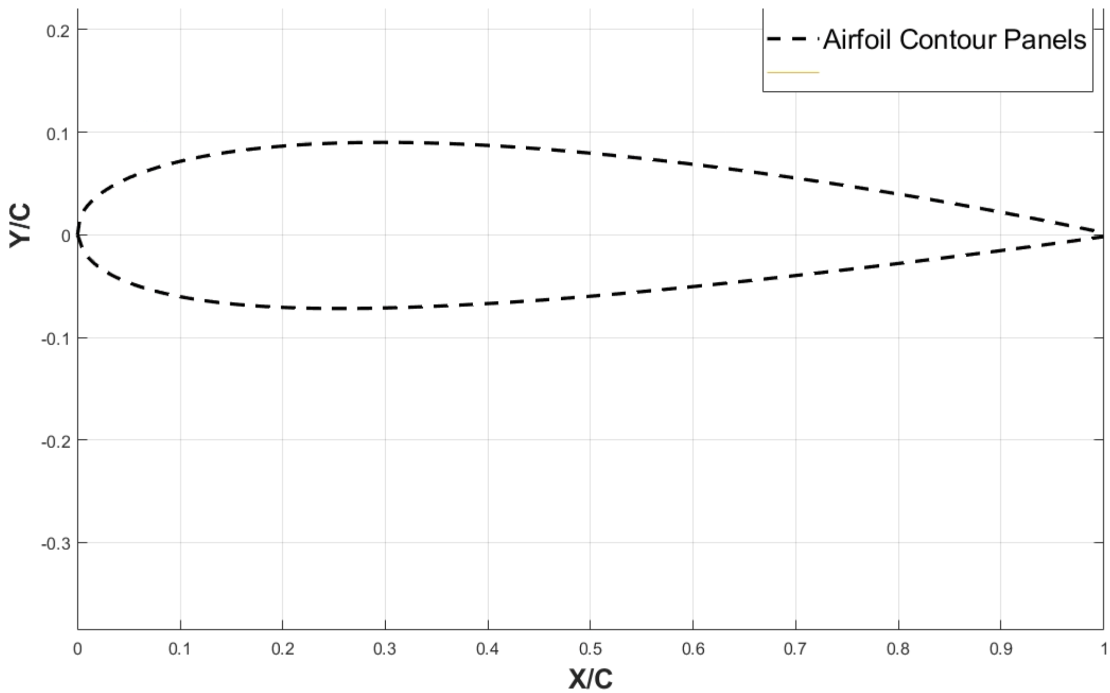

Figure 3: NACA 2418 Airfoil Contour using 100 panels

2.0 Calculations and Discussion

2.1 Potential Flow Coefficient Calculation

The potential flow of the NACA 2418 was calculated using the vortex panel method code. The distribution of surface velocity and the coefficient of pressure for the upper and lower surfaces have been calculated and plotted below. Where the Coefficient of Pressure distribution graphs are shown in Figure 4, 5 and 6 and the Surface Velocity graphs are shown in Figure 7.

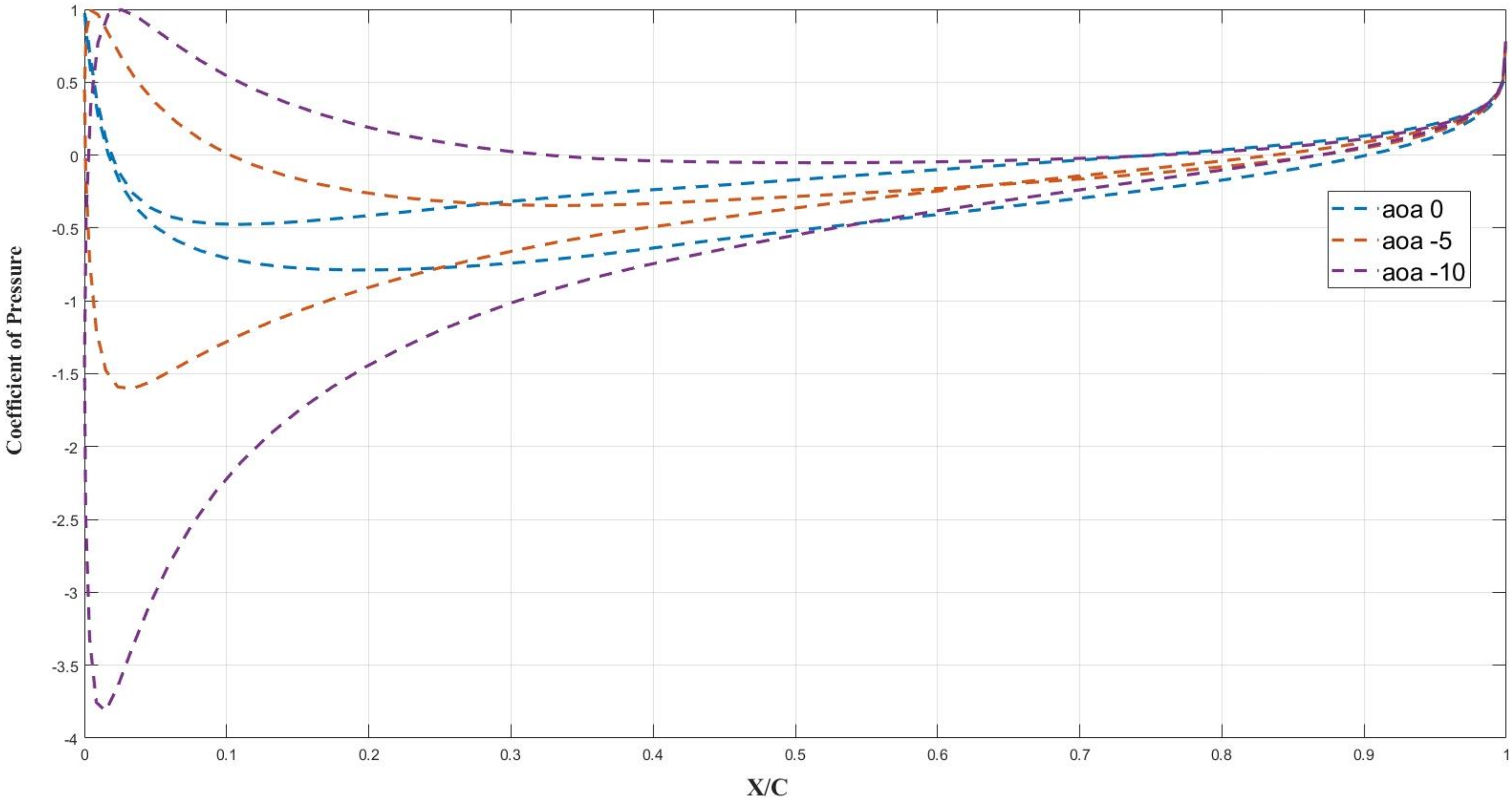

Figure 4: Pressure Coefficient Distribution for Bottom surface (Angle of attack for -10 to 0)

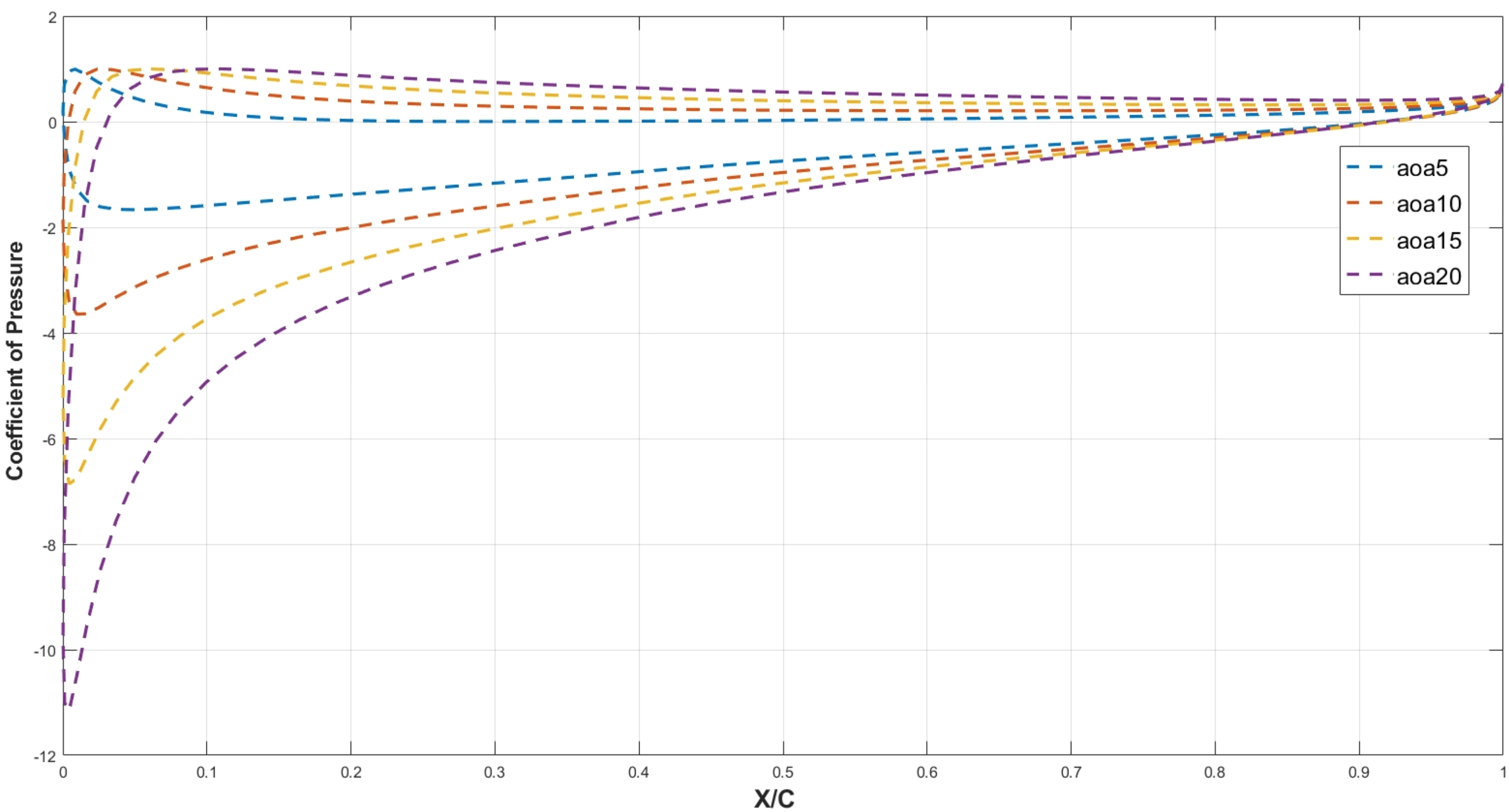

Figure 5: Pressure Coefficient Distribution for Upper surface (Angle of attack for 5 to 20)

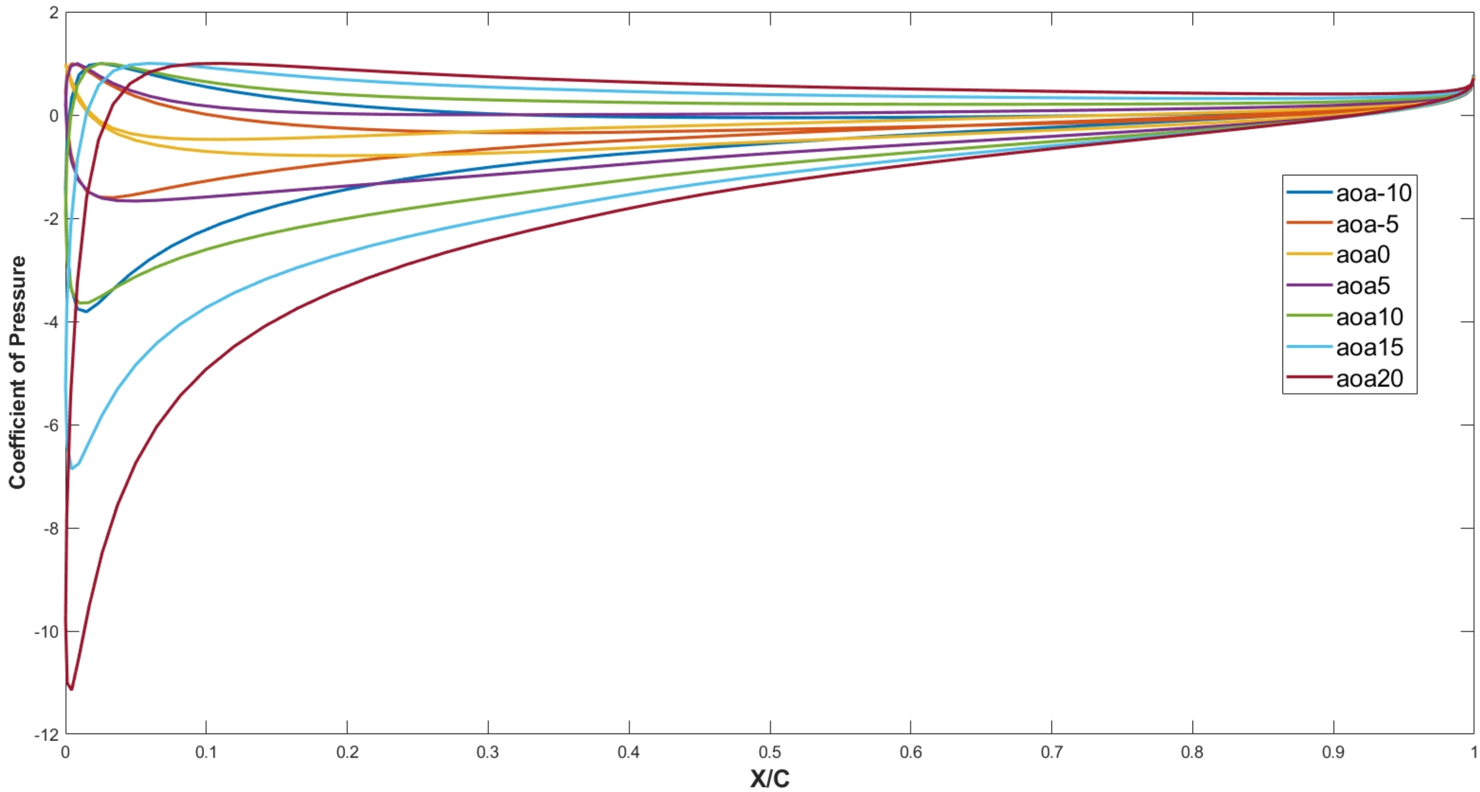

Figure 6: Pressure Coefficient Distribution for Whole surface (Angle of attack for -10 to 20)

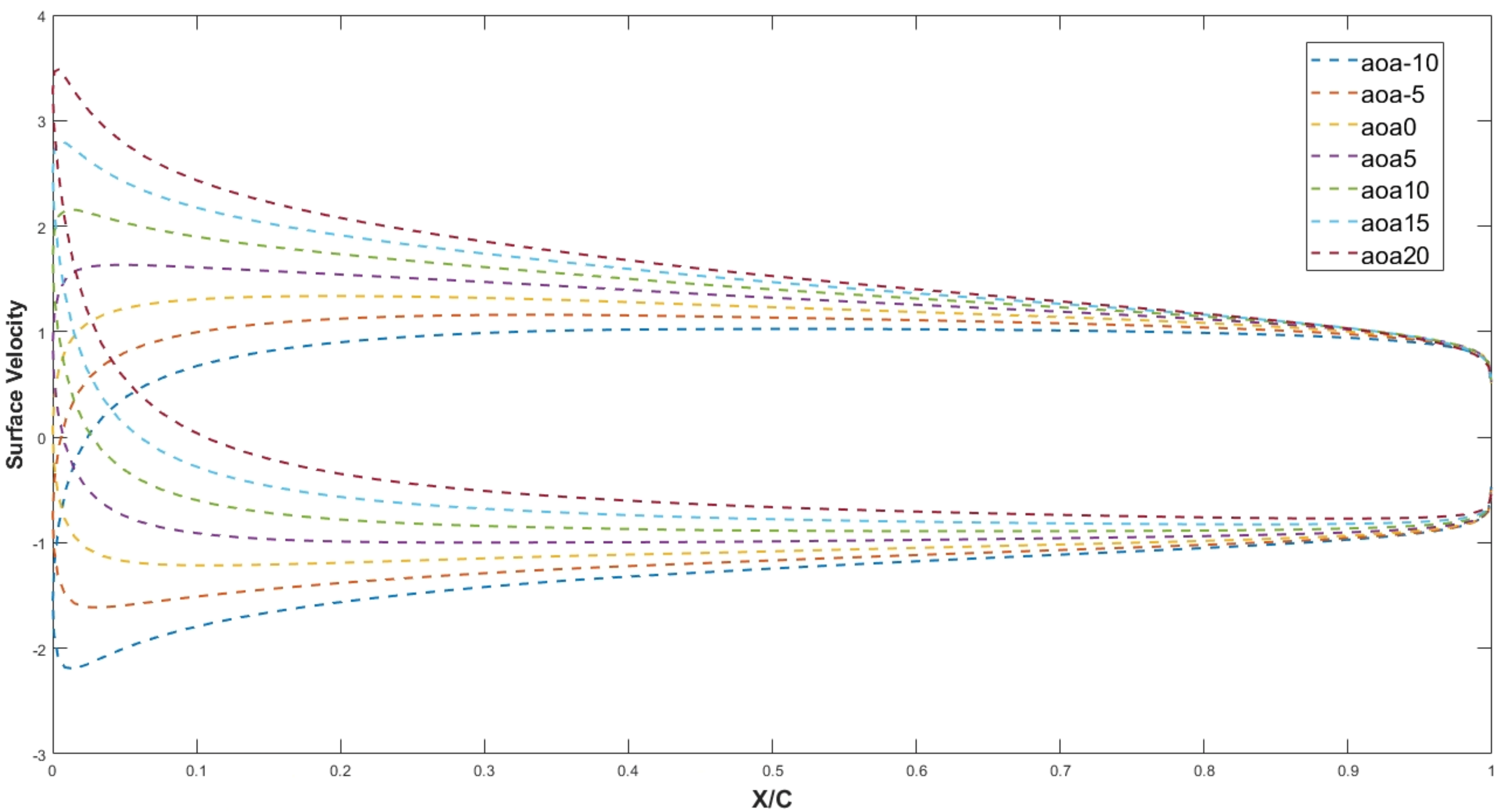

Figure 7: Surface Velocity Distribution of Angles of Attack for -10 to 20

2.2 Potential Flow Coefficient Discussion

The distribution of Surface pressures for Figures 4, 5 and 6 are as to be expected. As the angle of attack is increased, the coefficient of pressure decreases. For the lower surface, seen in Figure 4, the largest angle of attack (-10) moves further upwards. While for the upper surface, seen in Figure 5, the same happens with the angle of attack of (20). Looking at Figure 6, the entire distribution of surface pressure, the increase an angle of attack also shows this move upwards. Another observation to bring up is the point of maximum suction. This is represented as the coefficient near the leading edge of the airfoil. This decreases when the angle of attack is increased, which is also to be expected. Regarding the distribution of surface velocity, as angle of attack is increased, surface velocity increases nearing the leading edge. Overall, the vortex panel method gives accurate descriptions regarding Coefficient of Pressure and Surface Velocity.

2.3 Lift Coefficient Calculation and Discussion

Lift coefficient was calculated using three different numerical methods. Integrating the surface pressure, integrating the intensity of circulation, and using thin airfoil theory. The equations used for these calculations are listed below:

$$\boldsymbol{C_L = \int_{LE}^{TE} \Delta C_p dx} \qquad (10)$$

$$\boldsymbol{C_L = \int_{LE}^{TE} \Gamma dx} \qquad (11)$$

$$\boldsymbol{C_L = 2\pi(\alpha - a_{L0})} \qquad (12)$$

Where, a_{L0} is the angle of zero lift represented by:

$$\boldsymbol{a_{L0} = \int_0^{\pi} \frac{dz}{dx}(1 - cos\theta)d\theta} \qquad (13)$$

These calculations were found, and the results are shown in the figure below. The actual C_L shown in the last column of figure 8 was approximated from a graph with a Reynolds number of 100,000, seen online [2]. This was added to the chart so the numerical methods can be compared with the experimental data.

Angle of Attack	Cp	Circulation	Thin Airfoil	Actual CL
-10	-0.9849	-0.8726	-0.9849	-0.9000
-5	-0.3554	-0.3242	-0.3554	-0.4000
0	0.2767	0.2240	0.2765	0.3100
5	0.9068	0.7723	0.9068	0.5000
10	1.5300	1.3207	1.5100	0.8500
15	2.1516	1.8689	2.1214	1.1750
20	2.7368	2.4170	2.7010	1.4200

Figure 8: Coefficient of Lift Results with Corresponding Methods

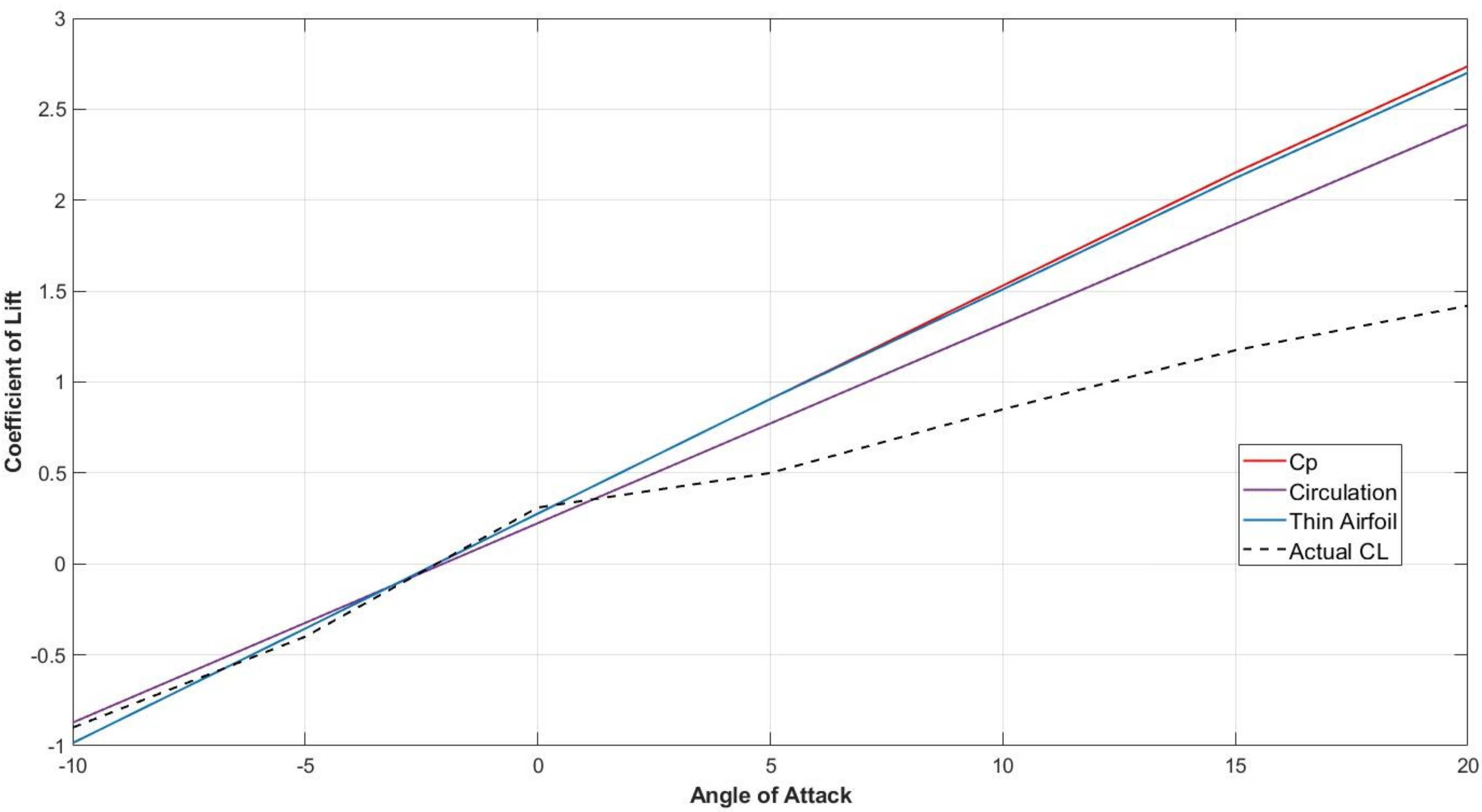

Figure 9: Coefficient of Lift to Angle of Attack

Figure 9, shown above plots the relationship between the 3 different approaches for calculating Coefficient of Lift. Also, the actual Coefficient of Lift is shown to see the comparison between the approximated methods and the experimental data. It is worth noting these methods are very accurate between the angles of attack of -10 and 0. But, when the angle of attack increases, the Coefficient of Lift for the numerical methods starts diverging away from the actual C_L. It is also worth mentioning that the maximum Coefficient of Lift is 1.475 for a NACA 2418 [3]. Therefore, when the angle of attack starts increasing above 0-5, the numerical methods are no longer a valid tool for approximating Coefficient of Lift.

2.4 Angle of Zero Lift Calculation and Discussion

Now that all the Coefficient of Lifts have been calculated using the different methods, the angle of zero lift can be calculated with equation 12 where it can be arranged as:

$$\boldsymbol{a_{L0} = \alpha - \frac{C_L}{2\pi}} \qquad \textbf{(14)}$$

This calculation will be done with an angle of attack at negative 5 degrees to be consistent. The results will be shown in the table below.

Method	Angle of Zero Lift
C_P Distribution	-4.94
Thin Airfoil Theory	-4.94
Actual C_L	-4.93

Table 1: Angle of Zero Lift Results

This gives an error of less than 1%, this shows that solving for the angle of zero lift using numerical methods is sufficient.

2.5 Pitching Moment Calculation and Discussion

The Pitching moment about the leading edge has been calculated using the sum of surface pressure forces, like the calculations made for Coefficient of Lift. This is shown in the formula below:

$$\boldsymbol{C_m = -\int_{LE}^{TE} C_p x dx} \qquad \textbf{(15)}$$

The results of equation 15 are shown in the figure 10 and the plot of Pitching moment Coefficient versus Angle of Attack are displayed in Figure 11:

Angle of Attack	Cm
-10	-0.1995
-5	-0.0398
0	0.1235
5	0.2856
10	0.4415
15	0.5864
20	0.7238

Figure 10: Pitching Moment Coefficient Results

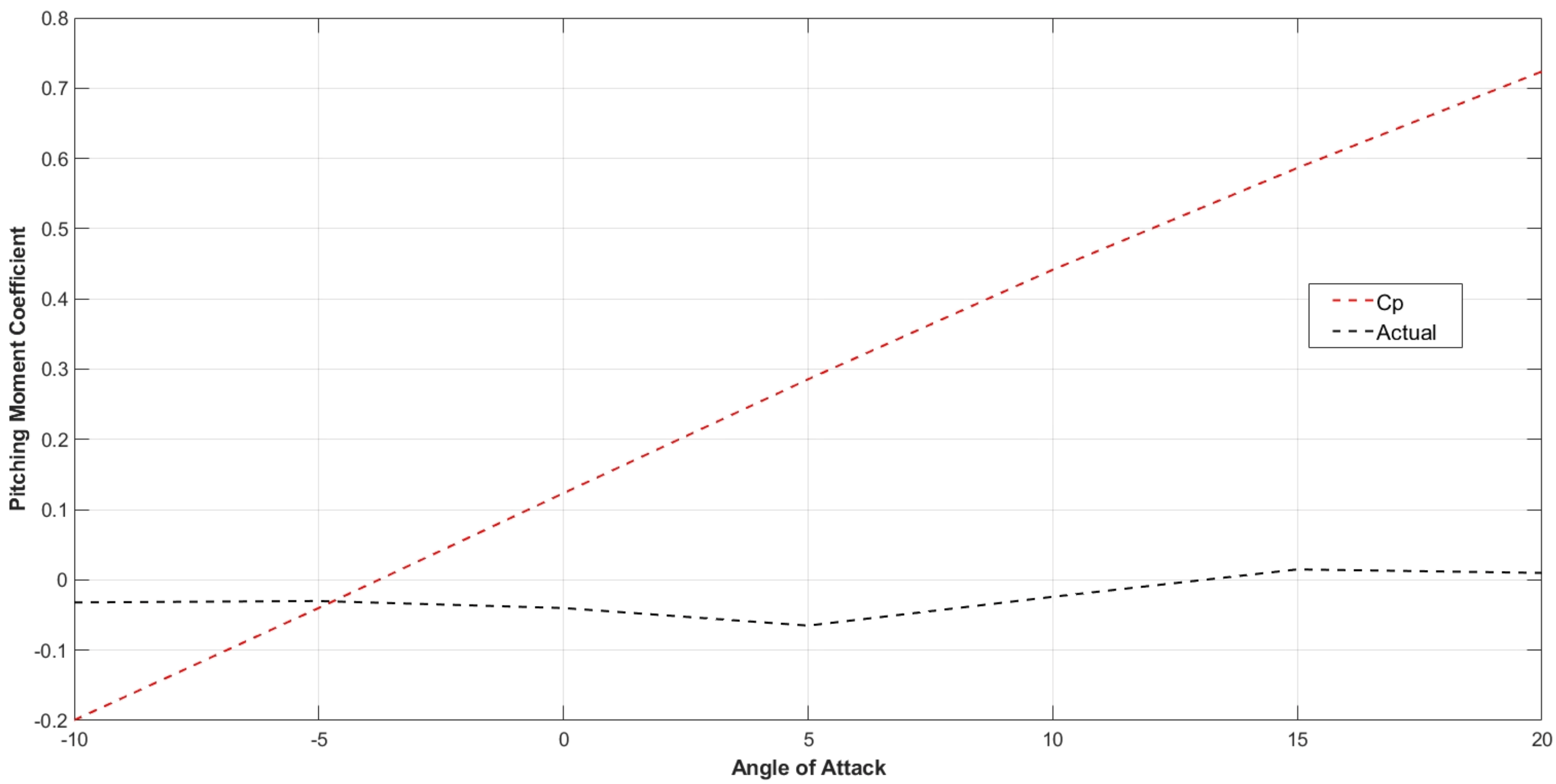

Figure 11: Pitching Moment Coefficient to Angle of Attack

It is worth noting that the actual Pitching Moment Coefficient is approximated from a graph online at a Reynolds number of 100,000 [2]. As seen in Figure 11, the numerical method for solving for Pitching Moment Coefficient are inaccurate for the majority of angle of attacks compared to the experimental results. Since the vortex panel method considers compressibility and viscosity as negligible, not all coefficients can be accurately calculated through these numerical methods.

3.0 Conclusion

This study shows how the vortex panel method in combination with thin airfoil theory is used for modelling NACA airfoils in the aerospace industry. It proved that approximating the airfoil geometry, coefficient of lift (at lower angles of attack) and angle of zero lift are all valid assumptions when solving with these numerical methods. The report also clarified how the vortex panel method failed to approximate the pitching moment coefficient due to the lack of accuracy when neglecting compressibility and viscosity. Overall, these computational tools give accurate approximations for important aerodynamic values important and are proven to simulate the effects on NACA airfoils, while using a lot less computational power.

4.0 References

[1] Airfoil Tools, *NACA 4 digit airfoil generator (NACA 2412 airfoil)*, 2021. [Online]. Available: http://airfoiltools.com/airfoil/naca4digit. [Accessed: 05-Dec-2021].

[2] Airfoil Tools, *NACA 2418 (NACA2418-IL)*, 2021. [Online]. Available: http://airfoiltools.com/airfoil/details?airfoil=naca2418-il&fbclid=IwAR2sZxPO8iaE4vRUUrxA-VDYoW0BHJRk6_ZFSD0ibYN-WYbuhSdUBVRC2l8. [Accessed: 05-Dec-2021].

[3] Nasa, *NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS*, 2021. [Online]. Available: https://ntrs.nasa.gov/api/citations/19930092773/downloads/19930092773.pdf?fbclid=IwAR0Cs6KL_oH36Ha8PviLiQCv9DtS95qvMJpwlBnP9tQXsN9NH7AtRER290g.pdf [Accessed: 07-Dec-2021].