CECL HW6–Markov Chain Model

Group 3

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1 Economic Indicators

Default: A loan is default if payment has not been made for 90 days of due date. There are many stages of Default loan. If payment is not been made for 30 days, then loan become delinquent. Usually default is followed by foreclosure, but some lenders might wait till 180 days before foreclosing the loan.

Prepayment: Prepayment state is when borrower pays off the loan in full or partial amount before the maturity of loan term to get the benefits of lower interest rate. Some lender might impose prepayment penalty on mortgage contract.

2 Methodology

2.1 Markov Chain Model and Transition Probability

A Markov chain is a stochastic process that has the Markov property, which requires the future events depend only on the present but not the past.

$$P(Y_t = y_t | Y_{t-1} = y_{t-1}, Y_{t-2} = y_{t-2}, ..., Y_0 = y_0) = P(Y_t = Y_t | Y_{t-1} = y_{t-1}) \tag{1}$$

Markov Property is a very strong assumption and hardly hold in real life economic data. Here, we include additional covariates and relax the Markov assumption to conditional Markov assumption. This is called conditional Markov chain(CMC).

$$P(Y_t = y_t | Y_{t-1} = y_{t-1}, Y_{t-2} = y_{t-2}, ..., Y_0 = y_0, X_{t-1}) = P(Y_t = Y_t | Y_{t-1} = y_{t-1}, X_{t-1}) \tag{2}$$

In this report, we use a conditional Markov chain to model mortgage data, and investigate the relationship between the conditional transition probability and economic covariates.

We define the conditional Markov chain to be stationary given the set of economic covariates. That is, conditional on covariates, the probability moving from the state j to the state k is homogeneous over time. The probability is called

$$P(\boldsymbol{X}) = \left(p_{jk}(\boldsymbol{X})\right) = \begin{bmatrix} p_{11} & \cdots & p_{1S} \\ \vdots & \ddots & \vdots \\ p_{S1} & \cdots & p_{SS} \end{bmatrix},$$

Figure 1: Probability Transition matrix

the transition probability. Putting all these one-step transition probabilities into a matrix, we have the transition matrix.

With one-step transition probability, we use multinomial logistic regression to capture the transition probability. Then, we could populate the one-step transition matrix. The transition matrix, conditional on covariates, satisfies the following two properties:

Sum of all probabilities in a row is 1

the probability of transition from j to k in m steps is $(\mathrm{P}^m)_{jk}$

Hence one-step transition matrix is essential in conditional Markov chain model, and can be used to characterize the chain together with the state space and initial state.

2.2 Multinomial Logistic Regression

Logistic regression is a special case of the generalized linear model (GLM). GLM is a powerful statistical model to deal with non-normal data, especially popular for binary outcome data. It extends classical linear regression by allowing the linear combination of predictors to be related to the dependent variable, Y, via a link function, g, and specifying the distribution of Y via variance function. Let X be matrix of covariates. We model:

E(Y)=$\mu = g^{-1}(X\beta)$

Var(Y)=Var(μ)

2.3 Different states of Markov Chain

Current: This is the state when borrower is making on time payments. This is a stable state of the loan, but loan can move to delinquent stage if borrower misses the payment. This is end state and loan can not move lower than this state

Delinquency states (various buckets): Delinquency is a stage when borrower is not making payments within 30 days of due date. 1st stage is delinquent when payment is late by 30 days but no later than 60 days, payment made within 60 days to 90 days, 90 days to 120 days, 120 days to 180 days

Transition model.PNG

$$P(\boldsymbol{X}_{it}) = (p_{jk}), \qquad j, k \in \mathcal{S}$$

where

$$p_{jk} := \mathbb{P}(D_{i(t)} = k | (D_{i(t-1)} = j, \boldsymbol{X}_{it})$$

$$= \begin{cases} \dfrac{\exp(\boldsymbol{X_{it}\beta_{jk}})}{\sum_{k=j-1}^{k=j+1} \exp(\boldsymbol{X_{it}\beta_{jk}})} & if\ j = 0,1,2,3,4,5,6\ and\ k = j-1, j, j+1 \\ 1 & if\ j = k = -1\ or\ 7 \\ 0 & otherwise \end{cases}$$

Figure 2: Mortgage Transition Model

Default: After loan payment is not made within 180 days of due date then borrower is declared default, and loan is foreclosed if borrower declares he is not able to make payments. There is very high correlation between delinquencies and default. If borrower is having high delinquencies that means, there are high chances that borrower is going to default. On the contrary, if borrower is having less delinquencies then they are more likely to make on time payments. Once loan has moved to default state, it cannot move to other states.

Default and Prepaid are terminal actions: Once the mortgage loan is prepaid or default, there is no further action to do and these mortgage loans are treated as censored, that is once these happens – decision is irreversible, So these two states are called as absorbing states.

The state space of this discrete Markov chain $S = (1, 0, 1, 2, 3, 4, 5, 6, 7)$

where 1 represents defined prepaid and 7 represents defined default, and the other states represent the corresponding delinquency status.

Denote the delinquency status of the i-th loan at time t as D_{it}. Let $X_{it} = (1, X_{1it}, ..., X_{pit)}$ be the covariates for i-th loan at time t. Let $\beta_d d*$ be the coefficient vector for transition from state d to d^*. Mortgage transition model would be :

Thus, our approach is that we will combine the conditional Markov chain model and multinomial logistic regression to build our mortgage loan given the selected covariates.

step 1: model the transition of loan delinquency as a conditional Markov chain.

step 2: we model the three events(moving to one delinquency higher or one delinquency lower, or stay at the current delinquency.) by a multinomial logistic regression and estimate the conditional transition probability via selected covariates. The estimated transition probability is then used to form the transition matrix and predict future delinquency status.

step 3: The output one-step transition matrix will only have non-zero elements on the main and secondary diagnols and 0 anywhere else. In model fitting, we will treat staying in the same state as a baseline outcome.

3 Data Scope

Now, we start to realize our algorithm. This is a brief explanation of our data-scope.

- We set our study time-window in the four quarters of Year 2015.

- in the logistic regression, we select the following X variables as factors which have instrumental effect on the loan performance.

X_{1it} Unpaid Principle Balance on a specific loan i at time t

X_{2it} Loan Age, the time since the first payment expressed by month for loan i at time t

X_{3t} Interest Rate

X_{4it} Current LTV, loan-to-value ratio for loan i at time t

X_{5i} Credit Score of the borrowwer for loan i at original time

- List of States For the transition matrix,the state space of this discrete Markov chain is -1,0,1,2,3,4,5,6,7, where -1 represents defined prepaid and 7 represents defined default, and other states represent the corresponding delinquency status.

4 Transition Matrix

Before we generate the transition matrix, we define two absorbing states: default and prepaid. Once the loan is default or prepaid, there is no further action to do and these loans are treated as censored, that is default or prepaid decision is irreversible. We can also built multistep transition matrix by fixing the covariates as the baselien data defined before, showing the probability of transitions in nean future, say after six months or one year.

step.jpg

j \ k	-1	0	1	2	3	4	5	6	7
-1	1								
0	0.012	0.984	0.004						
1		0.369	0.381	0.250					
2			0.230	0.258	0.512				
3				0.185	0.213	0.602			
4					0.137	0.171	0.692		
5						0.118	0.131	0.751	
6							0.112	0.111	0.777
7									1

Figure 3: One-Step Transition Matrix

step.jpg

k / j	-1	0	1	2	3	4	5	6	7
-1	1								
0	0.069	0.918	0.008	0.003	0.002	0*	0*	0*	0*
1	0.032	0.629	0.034	0.041	0.054	0.063	0.065	0.051	0.031
2	0.007	0.215	0.037	0.051	0.077	0.107	0.132	0.140	0.234
3	0.001	0.048	0.018	0.028	0.048	0.075	0.113	0.130	0.539
4	0*	0.006	0.005	0.009	0.017	0.032	0.054	0.076	0.800
5	0*	0*	0*	0.002	0.004	0.009	0.019	0.026	0.938
6	0*	0*	0*	0*	0.001	0.002	0.004	0.006	0.986
7									1

Figure 4: Multi-Step Transition Matrix