$\begin{Bmatrix} X_W \\ Y_W \\ Z_W \end{Bmatrix}$ represent the world coordinates of the point P of the calibration pattern where both the webcams are focused. As the calibration pattern is rotated, the orientation of the point under consideration changes and which is captured by both the webcams. The WCS is consequently converted to the coordinate systems for the respective webcams as shown below. This is used for a single camera calibration.

$$\begin{Bmatrix} X_{L1} \\ Y_{L1} \\ Z_{L1} \end{Bmatrix} = [R_{0-1}] \begin{Bmatrix} X_W \\ Y_W \\ Z_W \end{Bmatrix} + \{t_{0-1}\}$$

Here $[R_{0-1}]$ and $\{t_{0-1}\}$ are 3×3 and 3×1 matrix respectively.

A similar relationship holds true for a stereo system. If the MC is assumed to be Webcam 1, only the orientation $[R_{0-1}]$ and the position vector $\{t_{0-1}\}$ are considered to be extrinsic parameters. Thus, the intrinsic parameters include six independent terms in $[R_{1-2}]$, vector $\{t_{0-1}\}$, the internal parameters for camera 1 and the internal parameters for camera 2. The same relationship is valid as the above one in case of conversion of WCS and coordinates of MC. We find the coordinates of the other webcam with respect to the MC as follows.

$$\begin{Bmatrix} X_{L2} \\ Y_{L2} \\ Z_{L2} \end{Bmatrix} = [R_{1-2}] \begin{Bmatrix} X_W \\ Y_W \\ Z_W \end{Bmatrix} + \{t_{1-2}\}$$

Now the above equations can be rewritten to obtain optimal estimates for extrinsic and intrinsic camera parameters using Levenberg-Marquardt algorithm. For Webcam 1, we use the following equation.

$$\begin{Bmatrix} X_{L1} \\ Y_{L1} \\ Z_{L1} \\ 1 \end{Bmatrix} = \begin{bmatrix} \{[R_{0-1}]\}_{3\times3} & \{\{t_{0-1}\}\}_{3\times1} \\ \{O^T\}_{1\times3} & \{1\}_{1\times1} \end{bmatrix} \begin{Bmatrix} X_W \\ Y_W \\ Z_W \\ 1 \end{Bmatrix}$$

Calculations of Webcam 2 is done with respected to the MC (Webcam 1) as follows:

$$\begin{Bmatrix} X_{L2} \\ Y_{L2} \\ Z_{L2} \\ 1 \end{Bmatrix} = \begin{bmatrix} \{[R_{1-2}]\}_{3\times3} & \{\{t_{1-2}\}\}_{3\times1} \\ \{O^T\}_{1\times3} & \{1\}_{1\times1} \end{bmatrix} \begin{Bmatrix} X_{L1} \\ Y_{L1} \\ Z_{L1} \\ 1 \end{Bmatrix}$$

$$\begin{Bmatrix} X_{L2} \\ Y_{L2} \\ Z_{L2} \\ 1 \end{Bmatrix} = \begin{bmatrix} \{[R_{1-2}]\}_{3\times3} & \{\{t_{1-2}\}\}_{3\times1} \\ \{O^T\}_{1\times3} & \{1\}_{1\times1} \end{bmatrix} \begin{bmatrix} \{[R_{0-1}]\}_{3\times3} & \{\{t_{0-1}\}\}_{3\times1} \\ \{O^T\}_{1\times3} & \{1\}_{1\times1} \end{bmatrix} \begin{Bmatrix} X_W \\ Y_W \\ Z_W \\ 1 \end{Bmatrix}$$

$$\begin{Bmatrix} X_{L2} \\ Y_{L2} \\ Z_{L2} \\ 1 \end{Bmatrix} = \begin{bmatrix} \{[R_{1-2}][R_{0-1}]\}_{3\times3} & \{[R_{0-1}]\{t_{1-2}\} + \{t_{0-1}\}\}_{3\times1} \\ \{O^T\}_{1\times3} & \{1\}_{1\times1} \end{bmatrix} \begin{Bmatrix} X_W \\ Y_W \\ Z_W \\ 1 \end{Bmatrix}$$

$$\begin{Bmatrix} X_{L2} \\ Y_{L2} \\ Z_{L2} \\ 1 \end{Bmatrix} = \begin{bmatrix} \{[R]\}_{3\times3} & \{\{t\}\}_{3\times1} \\ \{O^T\}_{1\times3} & \{1\}_{1\times1} \end{bmatrix} \begin{Bmatrix} X_W \\ Y_W \\ Z_W \\ 1 \end{Bmatrix}$$

Let us say $\{x_s \quad y_s\}$ be the pixel coordinates of the point P as captured by Webcam 2, i.e., 3D to 2D conversion. These coordinates can be linked to the WCS using the following equation.

$$\alpha \begin{Bmatrix} x_s \\ y_s \\ 1 \end{Bmatrix} = \left[\{[A][R]\}_{3\times 3} \quad \{[A]\{t\}\}_{3\times 1} \right] \begin{Bmatrix} X_W \\ Y_W \\ Z_W \\ 1 \end{Bmatrix} = [\{P\}_{3\times 4}] \begin{Bmatrix} X_W \\ Y_W \\ Z_W \\ 1 \end{Bmatrix}$$

Here $A = \begin{bmatrix} f_x & f_s & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}$. The values of individual elements of the matrix A are listed below which are all webcam parameters.

Entity	**Values**
f_x	fS_x
f_y	$fS_y/\sin\theta$
f_s	$-fS_x \cot\theta$
c_x	$-S_x(c_1 - c_2 \cot\theta)$
c_y	$-S_y c_2/\sin\theta$

But the first 3D to 2D conversion takes place at the metrics level which is then converted to pixel level using S_x and S_y along the horizontal and vertical directions respectively. θ represents the non-orthogonality or skew. We can find out the value of α which is:

$$\alpha = R_{31}X_W + R_{32}Y_W + R_{33}Z_W + t_z$$