ECE 470/570.S Software Receiver Technologies

Instructor: Jade Morton

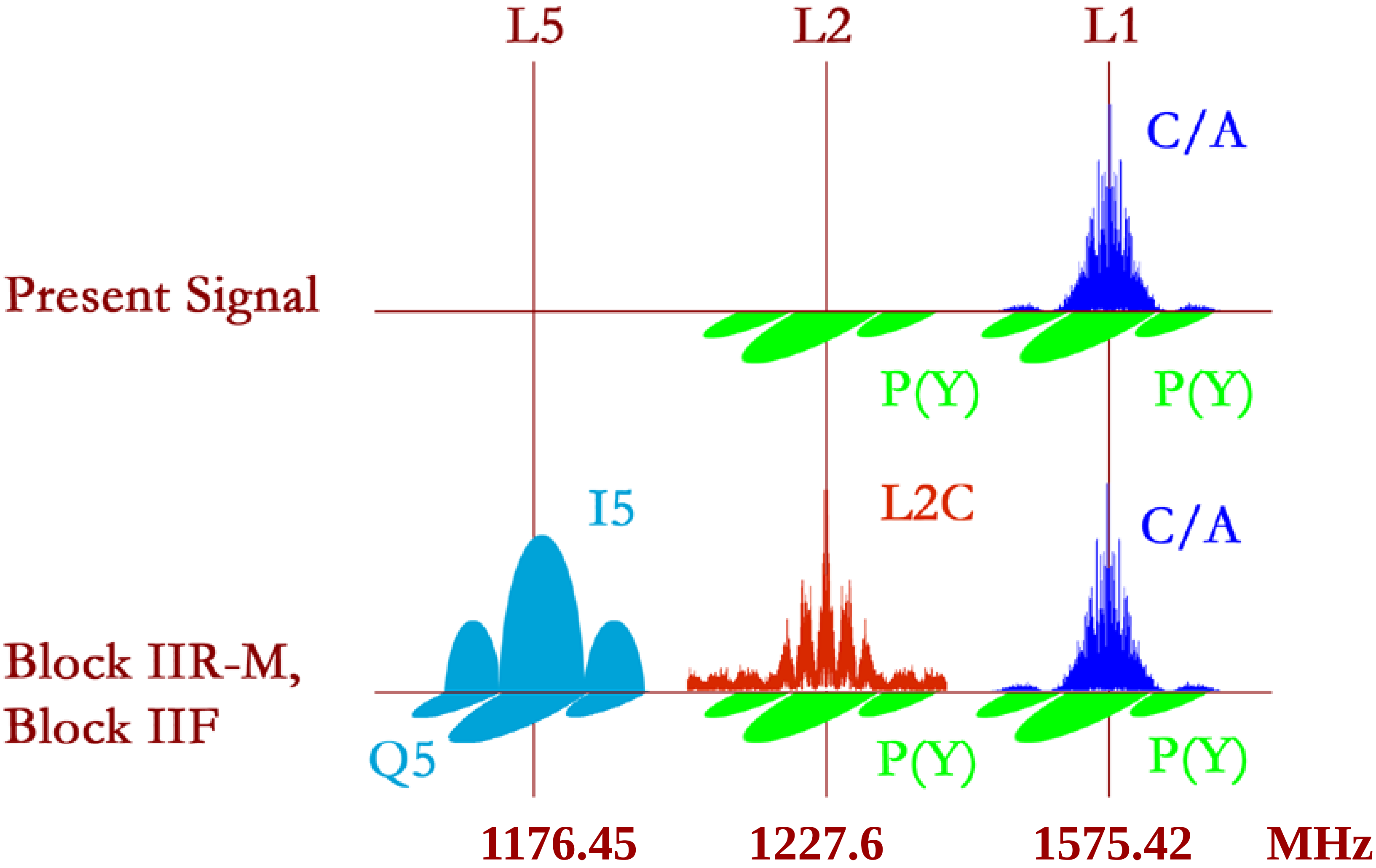
Lecture 2
GPS Signal Structure

Today's Topics

1. Basic composition of GPS signals/modulation scheme
2. Basic signal concepts:
   – PRN code, chipping rate
   – BPSK
   – Time domain vs. frequency domain
   – Spectrum, bandwidth
   – Square pulse properties
   – Spread spectrum
3. GPS signal mathematics representation

PART 1
Basic Composition of GPS Signals/Modulation Scheme

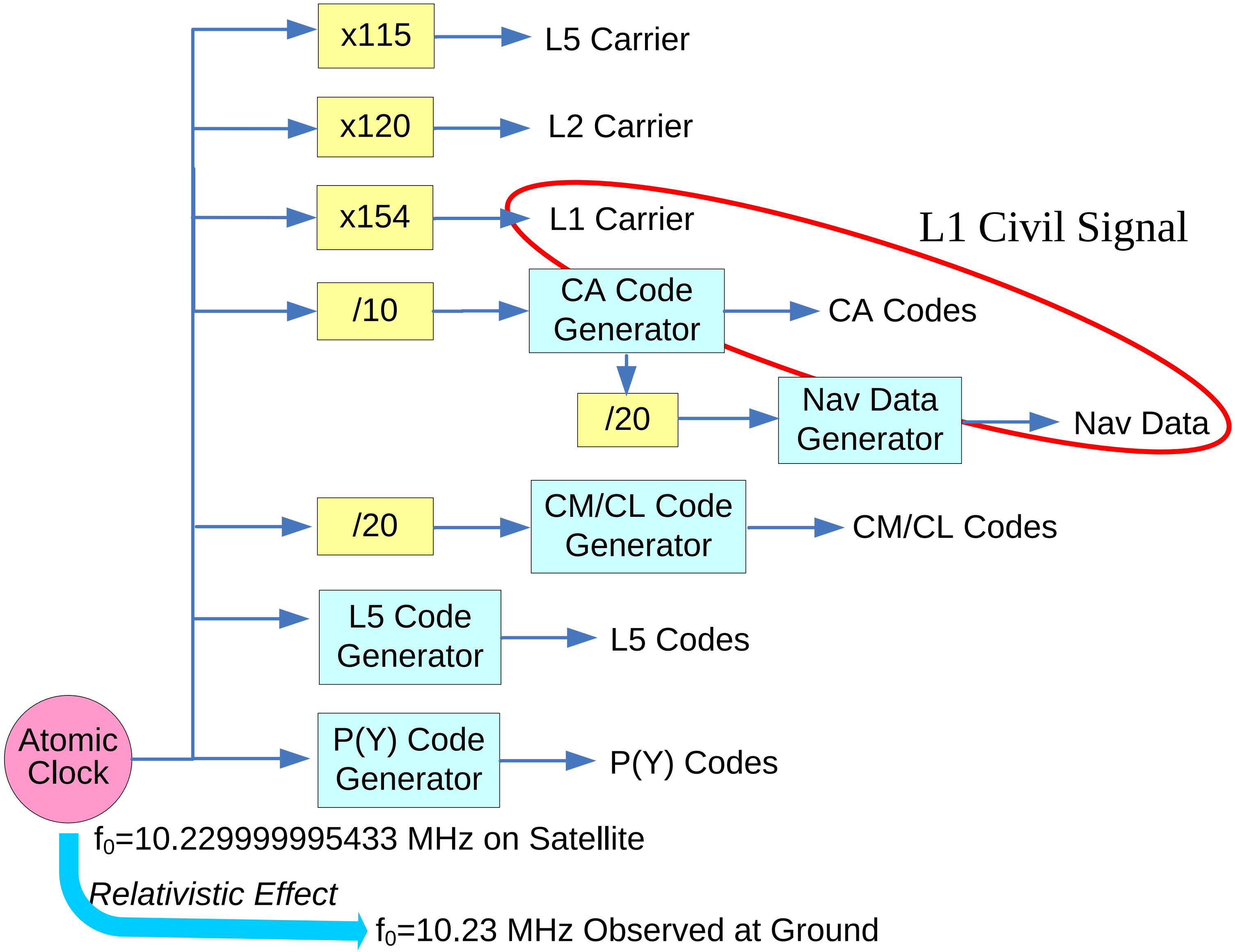
GPS Signals Spectral Allocations (M-code not included)



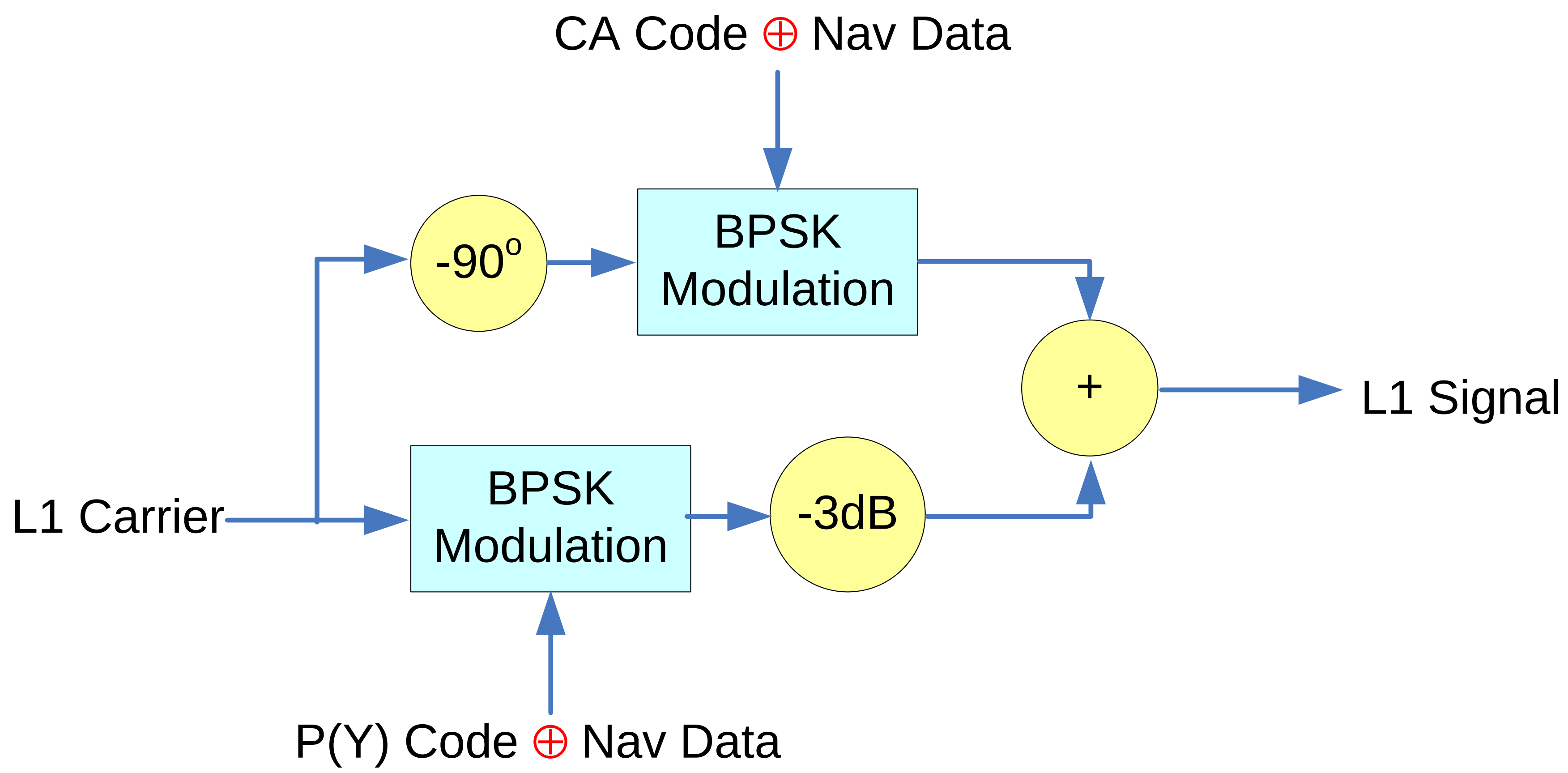
GPS Satellite Signals Summary (not include M-code)

Band	L1		L2			L5	
Carrier freq (GHz)	1.57542		1.2276			1.17645	
Code	CA	P(Y)	P(Y)	CM	CL	CA	
Code length (chips)	1023	23,017,555.5		10,230	767,250	10,230	
Code period	1 ms	1 wk		20 ms	1.5 s	1 ms	
Chip rate (MHz)	1.023	10.23		1.023		10.23	
Data rate (bps)	50			25*	Data less	50*	Data less
Max power at receiver (dBW)	-157.7	-160.7	-163.7	-160		-154	

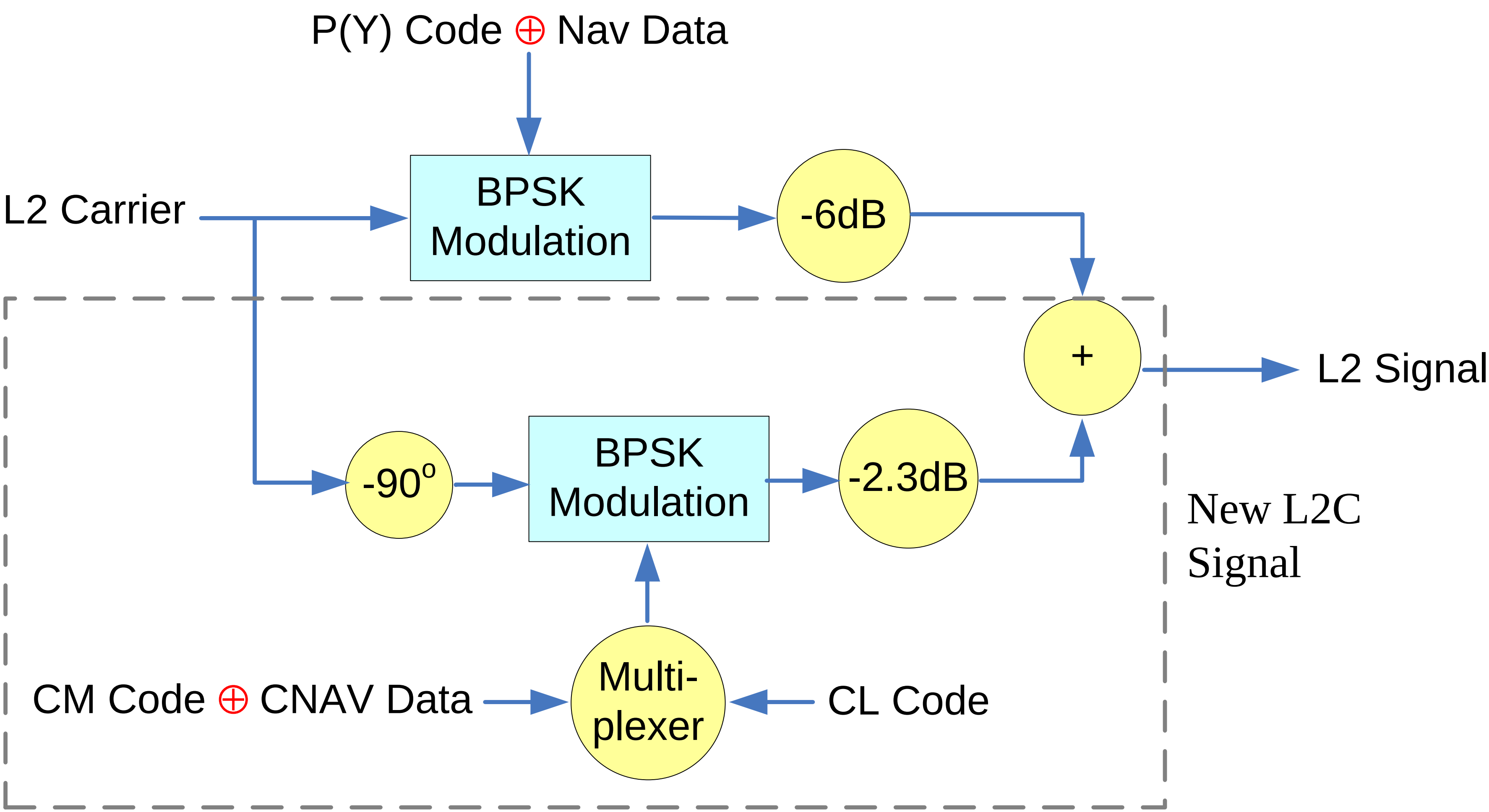
Current GPS Signal Frequency Relationships



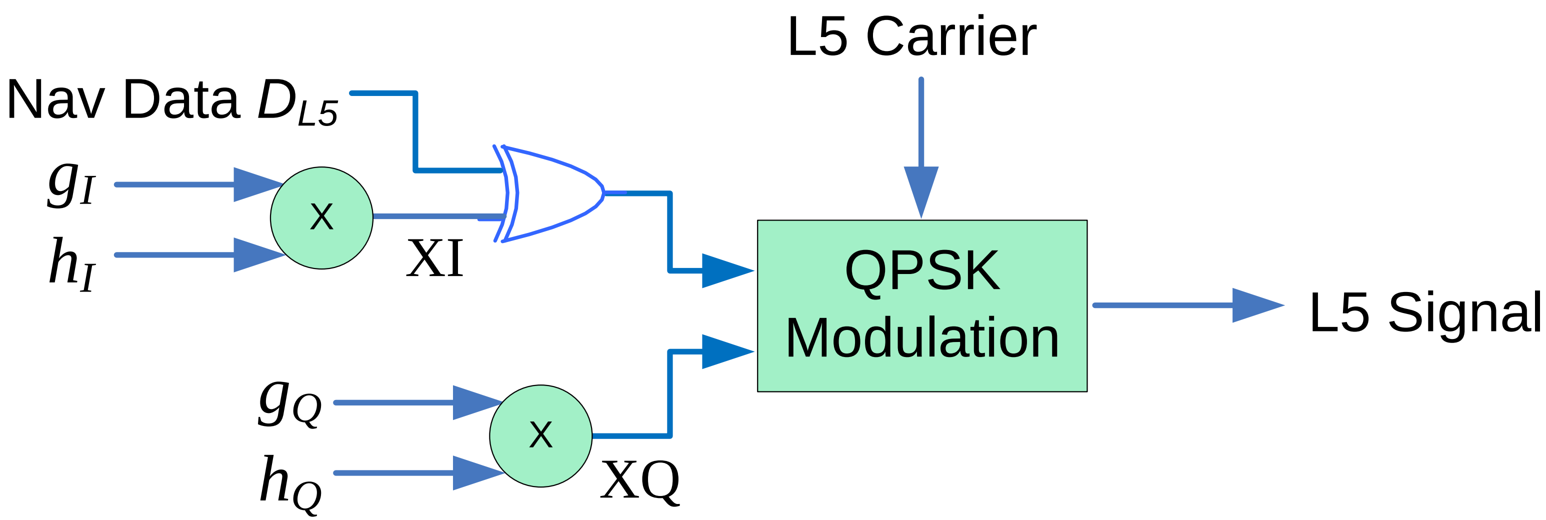
Current GPS Signal Modulation at L1



GPS Signal Modulation Format at L2



GPS Signal Modulation at L5



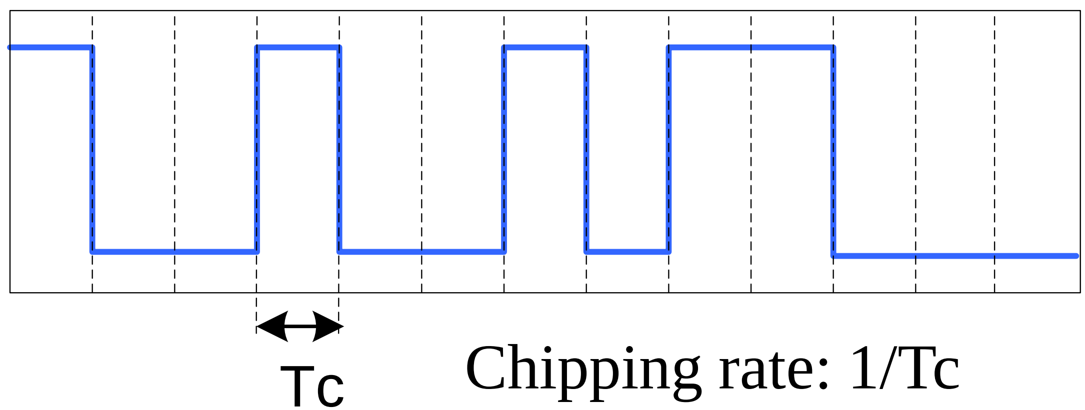
PART 2
Basic Signal Concepts
(A Review)

Pseudorandom (PRN) Sequence and Chipping Rate

Definition:

A binary sequence of "0"s and "1"s satisfying 3 conditions:

1. 50% are "0" and 50% are "1".
2. 50% of all run lengths ("0" and "1") are 1; 25% are of length 2; 12.5% are of length 3; ….
3. If a sequence is shifted, the resulting sequence will have an equal number of agreements and disagreements with the original sequence.



Chipping rate: 1/Tc

BPSK: Binary Phase Shift Keying

Given a carrier: $s_c(t) = A\cos(\omega_c t + \varphi)$

Its phase modulated version is: $s_{cm}(t) = A\cos(\omega_c t + \varphi + \theta(t))$

For BPSK, $\theta(t)$ can take 2 values: *0* and π

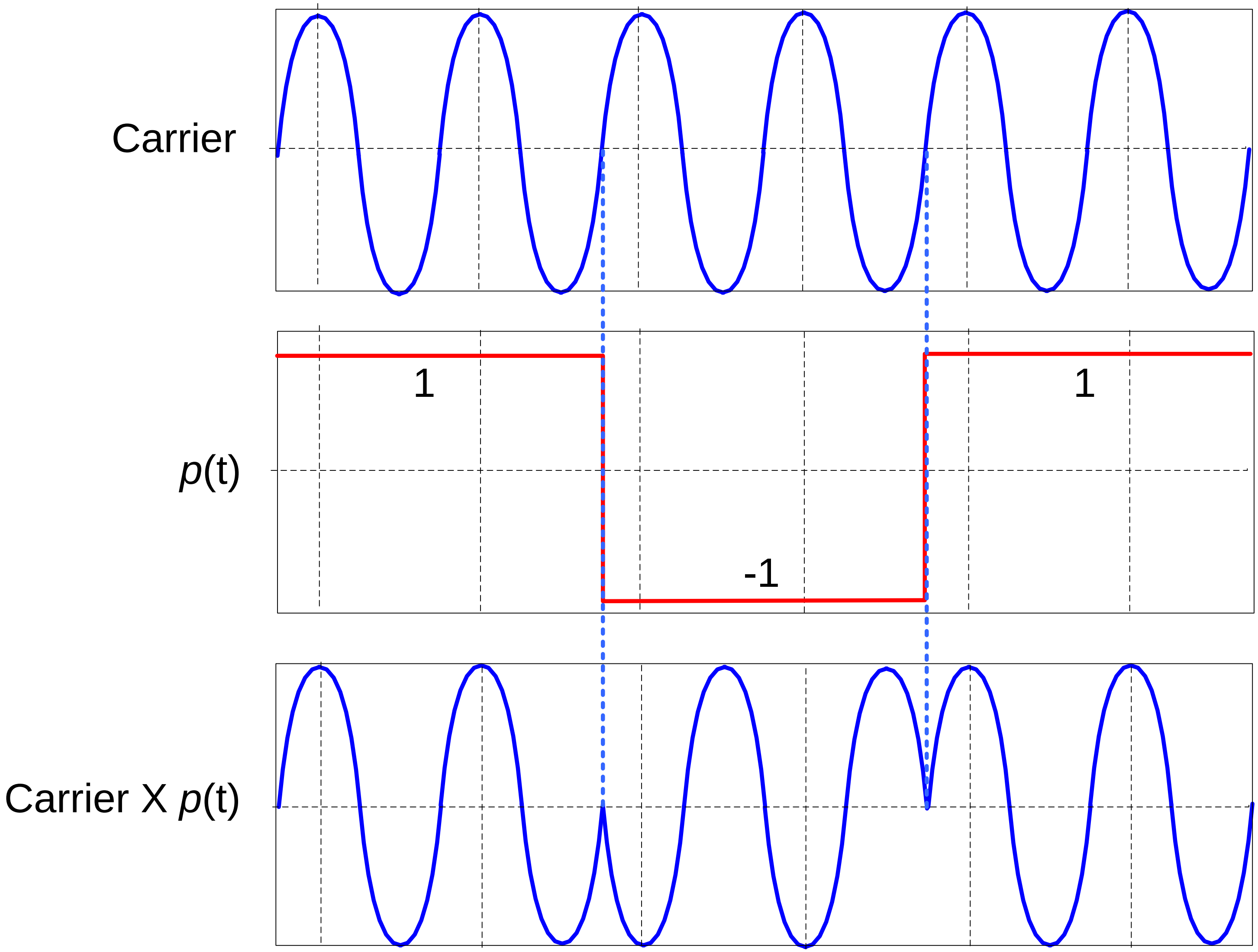
$$\boxed{s_{cm}(t) = \begin{cases} A\cos(\omega_c t + \varphi) & \theta(t) = 0 \\ -A\cos(\omega_c t + \varphi) & \theta(t) = \pi \end{cases}}$$

BPSK modulation ←→Binary Amplitude Shift Keying:

$$s_{cm}(t) = p(t)A\cos(\omega_c t + \varphi)$$

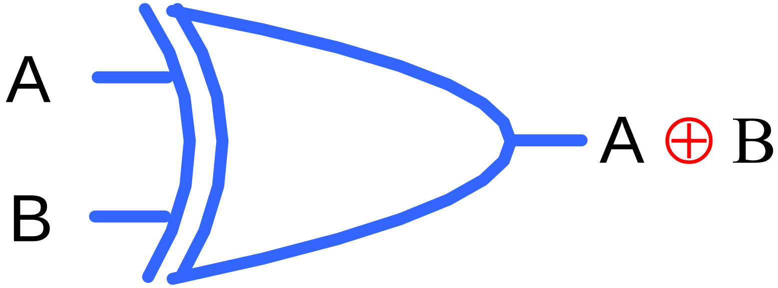
$p(t)$ takes 2 values: 1 and -1

A BPSK Example:



A Basic Modulation Operation: Exclusive-or ⊕

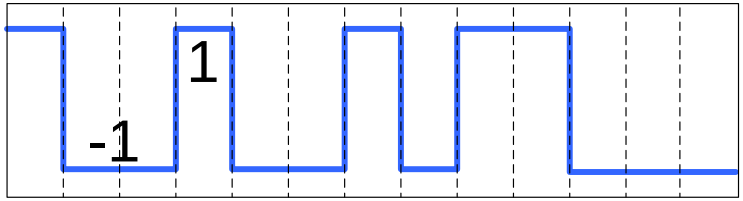
$$A \oplus B = A\overline{B} + \overline{A}B$$



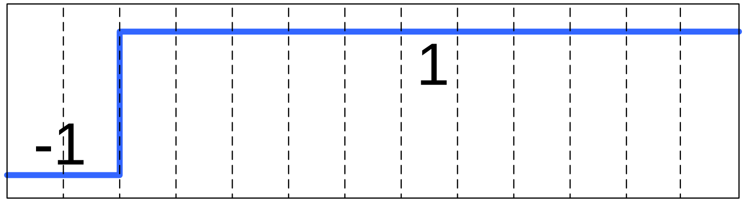
A	B	A ⊕ B
0	0	0
0	1	1
1	0	1
1	1	0

An Exclusive-or ⊕ Operation Example:

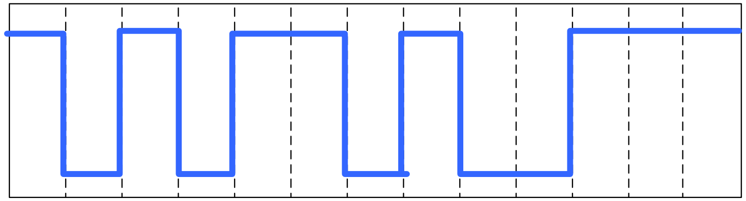
CA Code



Nav Data



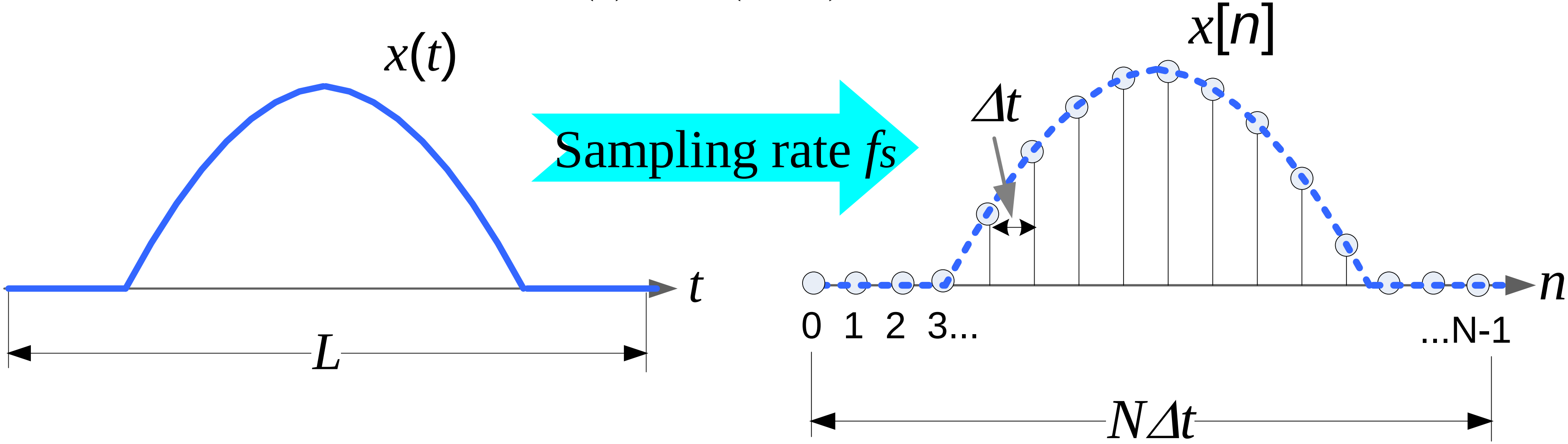
CA Code ⊕ Nav Data
(subsitiute -1 with 0)



CA Code ⊕ Nav Data = - $\underbrace{\text{Nav Data x CA Code}}_{\text{BPSK}}$

Sampling Concept

$$x(t) = x(n\Delta t) \Rightarrow x[n]$$



Sample interval

$$\Delta t = \frac{1}{f_s}$$

Total number of samples

$$N = \frac{L}{\Delta t}$$

Signal Frequency Domain Representation

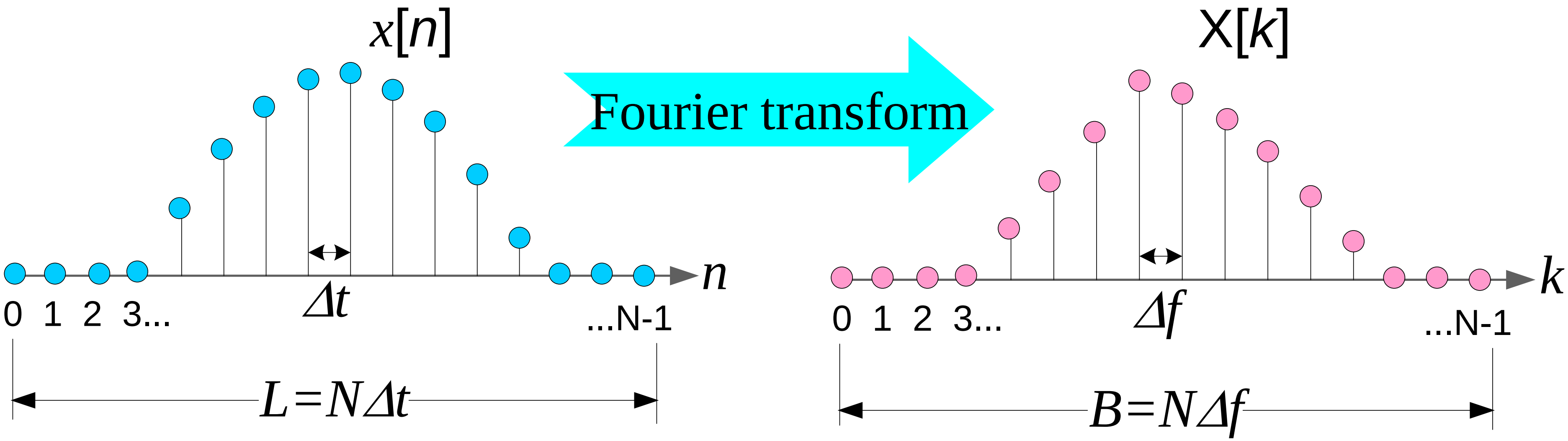
Periodic Signal Discrete Time Fourier Series (DT FS):

$$x[n] = \sum_{k=\langle N \rangle} X[k] e^{jk\frac{2\pi}{N}n}$$

$$X[k] = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\frac{2\pi}{N}n}$$

Both are periodic with period=N

Spectral Resolution and Bandwidth



Spectral resolution: $\Delta f = \dfrac{1}{L}$

Bandwidth: $B = N\Delta f = \dfrac{1}{\Delta t} = f_s$

$$X[k] \Rightarrow X(k\Delta f) = X(f)$$

$$x(t) = x(n\Delta t) \Rightarrow x[n]$$

Time duration L | Spectral span B

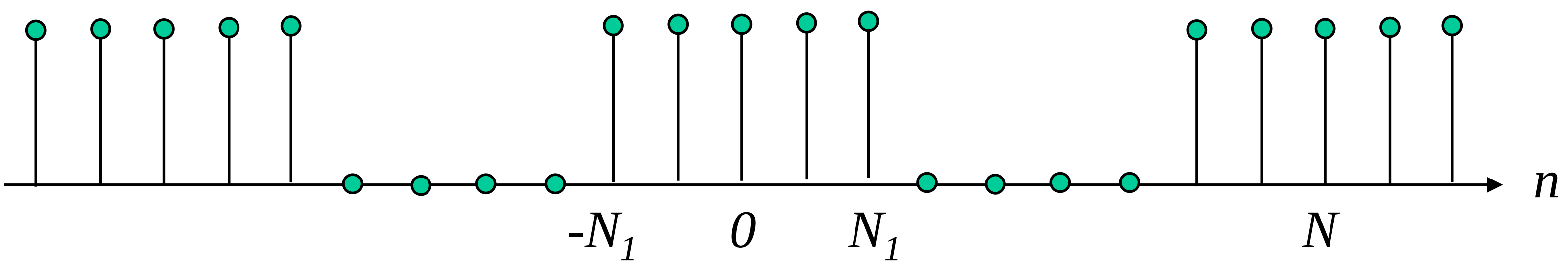
Time step size $\Delta t = \dfrac{1}{B}$ | Frequency resolution $\Delta f = \dfrac{1}{L}$

Examples:

What are Fourier series (spectrum) for these two signals?

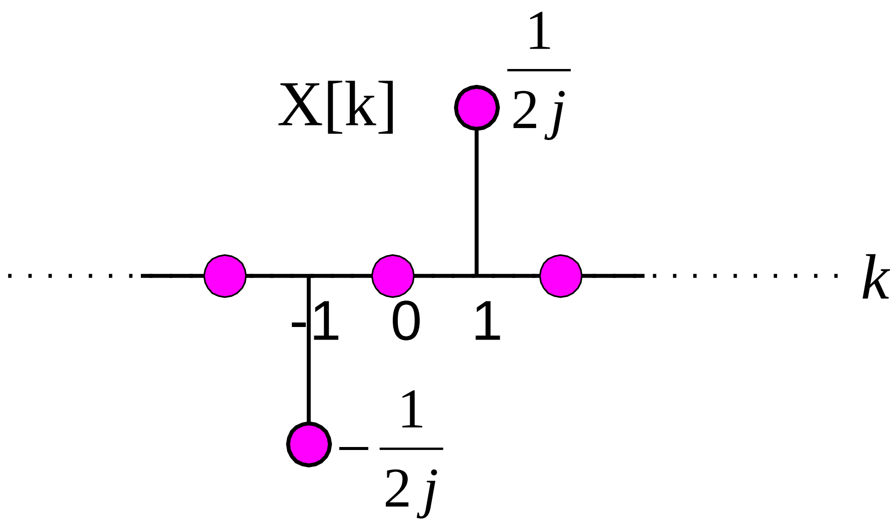
(1). $x[n] = \sin \omega_0 n \qquad \omega_0 = \dfrac{2\pi}{N}$

(2). Periodic square wave

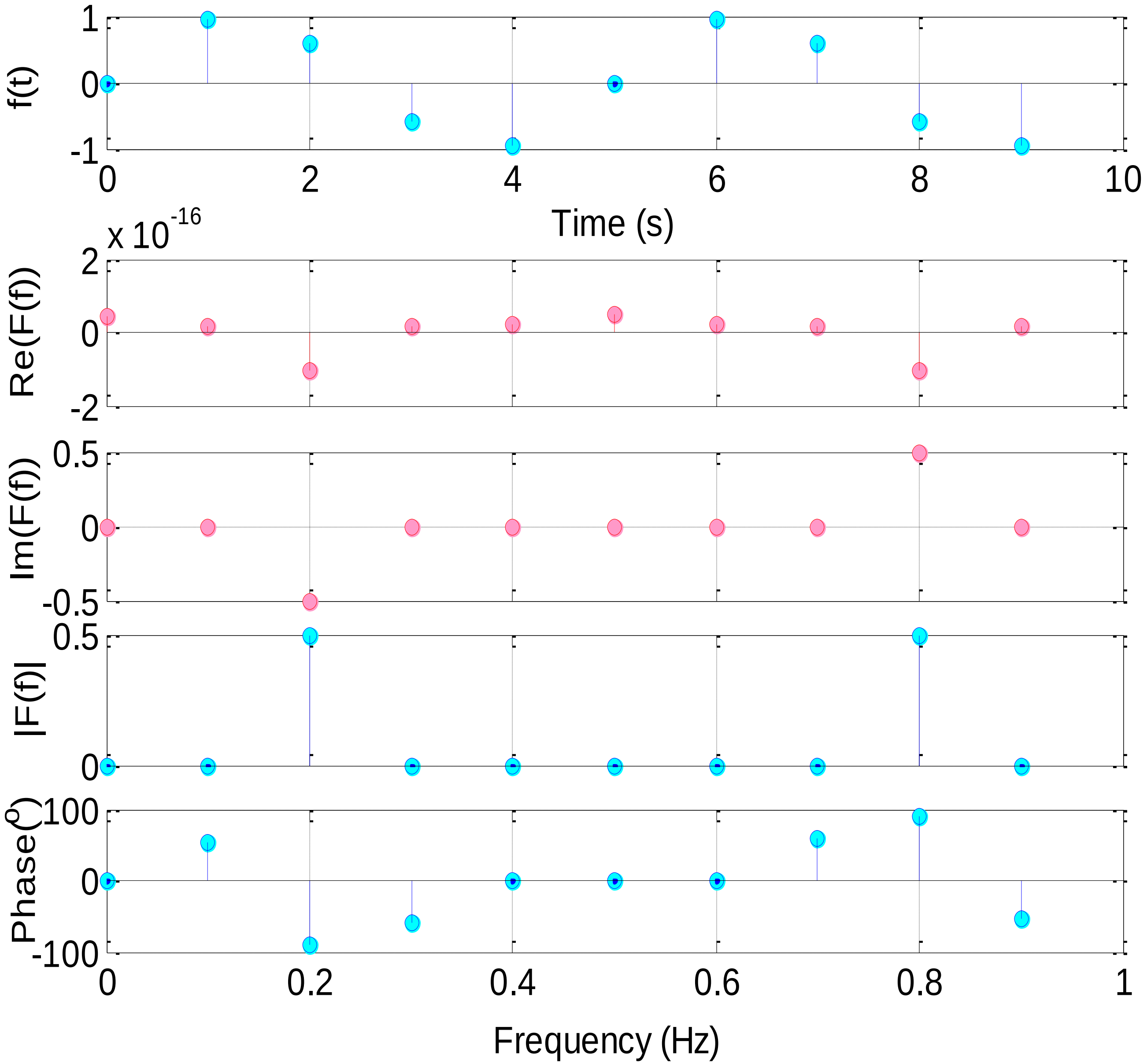


Example (1) Analytical Solution:

$$x[n] = \sin \omega_0 n \overset{\text{DTFS}}{\Longleftrightarrow} \begin{cases} \text{For } k \neq \pm 1 \quad X[k] = 0 \\ X[1] = \dfrac{1}{2j} \quad X[-1] = -\dfrac{1}{2j} \end{cases}$$



Example 1 MatLab Solution



Signal parameters:
a=1;
T=5;
fs=1;
L=10;

Example 1 MatLab Code

```
%---Signal parameters
a=1;                          % Amplitude
T=5;                          % Signal period in second
fs=1;                         % Sampling frequency in Hz
dt=1/fs;                      % In between sample time interval

L=10;                         %Data length in seconds
df=1/L;                       %Spectrum frequency interval
N=L/dt;                       %Total number of samples
n=0:N-1;                      %Sample index vector

t=dt*n;                       %Time vector
f=df*n;                       %Frequency vector

%---Generate time domain carrier signal and spectrum
carrier=a*sin(2*pi*t/T);      %Carrier samples
fCarrier=fft(carrier)/N;      %Carrier spectral samples

% Plot results
figure(1);
subplot(511); stem(t, carrier); grid on; xLabel('Time (s)');yLabel('Carrier');
subplot(512); stem(f, real(fCarrier),'ro');grid on;yLabel('Real');
subplot(513); stem(f, imag(fCarrier),'ro');grid on;yLabel('Imag');
subplot(514); stem(f,abs(fCarrier));grid on;yLabel('Magnitude');
subplot(515); stem(f,angle(fCarrier)*r2d); grid on; xLabel('Frequency (Hz)');
```

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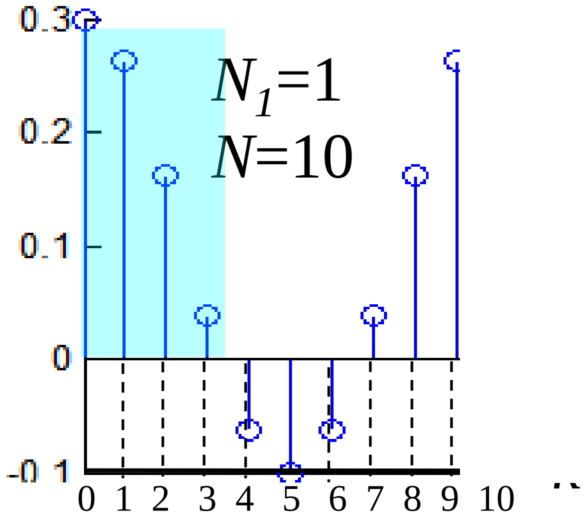
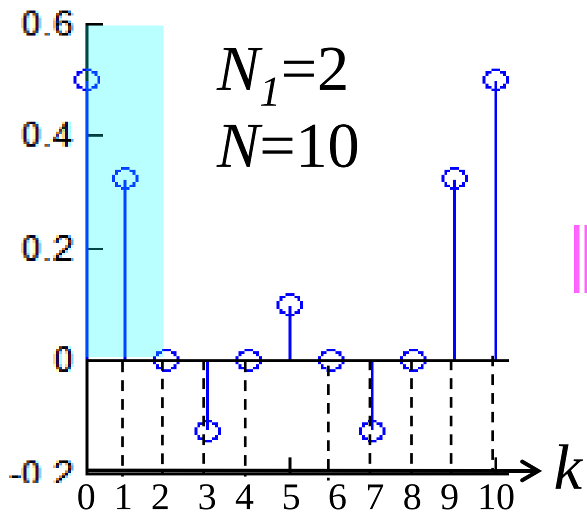
Example (2) Analytical Solution:

$$\Pi(N_1, N) \xleftrightarrow{\text{DTFS}} X[k] = \frac{1}{N} \frac{\sin\left(\frac{2N_1+1}{N} k\pi\right)}{\sin\left(\frac{1}{N} k\pi\right)}$$

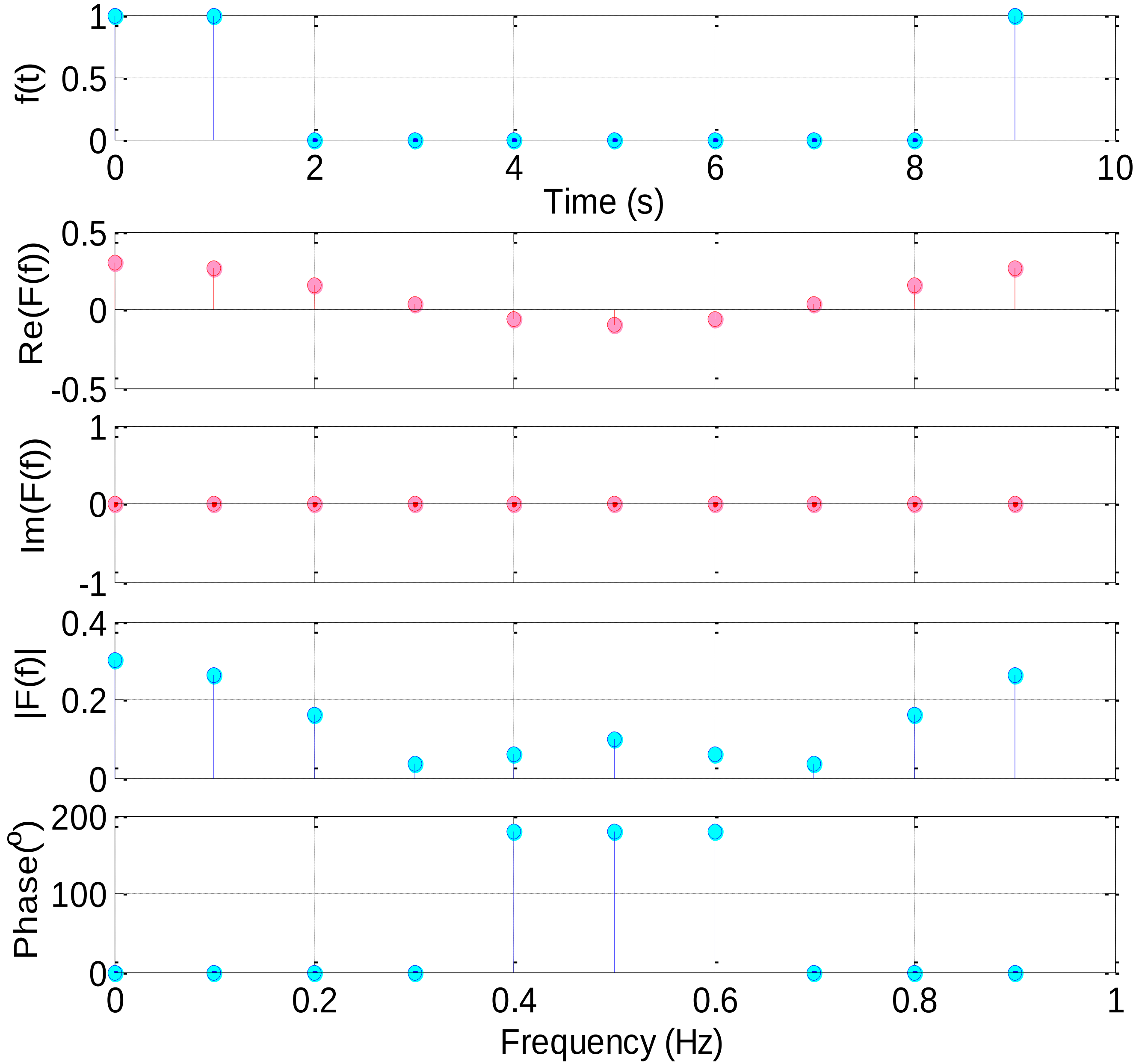




Larger N_1/N
Smaller BW

Example 2 MatLab Solution

Signal parameters:
a=1;
Period=10;
Duty Cycle=30%;
fs=1;
L=10;



Example 2 MatLab Code

```
%---Signal parameters
a=1;                          % Amplitude
TSqr=5;                       % Signal period in second
dutyCycle=30;                 % Duty cycle in percentage
fs=1;                         % Sampling frequency in Hz
dt=1/fs;                      % In between sample time interval

L=10;                         %Data length in seconds
df=1/L;                       %Spectrum frequency interval
N=L/dt;                       %Total number of samples
n=0:N-1;                      %Sample index vector

t=dt*n;                       %Time vector
f=df*n;                       %Frequency vector

%---Generate time domain pulse signal and spectrum
t0 = floor(dutyCycle/100*TSqr/2);          % Number of samples shifted left
code=(square(2*pi*fSqr*(t+t0),dutyCycle)+1)/2;  %Square wave samples
fCode=fft(code)/N;                          %Square wave spectrum

% Plot results
subplot(511);stem(t, code); grid on; xLabel('Time (s)'); yLabel('Code');
subplot(512);stem(f, real(fCode),'ro');grid on; yLabel('Real');
subplot(513);stem(f, imag(fCode),'ro');grid on; yLabel('Imag');
subplot(514);stem(f,abs(fCode));grid on;yLabel('Magnitude');
subplot(515);stem(f,angle(fCode)*r2d); grid on; xLabel('Frequency (Hz)');
```

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Generating Signals in MatLab Code Essentials

1. Basic time domain signal parameters:
   - Period ***T***, amplitude ***a***, phase ***phi***
   - Sampling frequency ***fs*** → time resolution ***dt*** = 1/*fs*
   - Data length ***L***
2. Frequency domain parameters
   - Frequency resolution ***df*** = 1/*L*
   - Spectral coverage ***B*** = 1/*fs*
3. Sample vector
   - Number of samples per period: ***N***=*L*/*dt*
   - Sample index vector: ***n*** = 0:*N*-1
4. Time and frequency vector
   - Time vector: ***t*** = *dt***n*
   - Frequency vector: ***f*** = *df***n*
5. Time signal vector definition
6. Frequency domain transformation
7. Plot results

Things to Watch

- Changing T
- Changing phase
- Changing duty cycle
- Changing fs
- Changing L

Spread Spectrum

A narrow band signal when modulated by a binary sequence, the spectrum of the signal became broadened.
The spectrum spread is proportional to the chipping rate 1/Tc

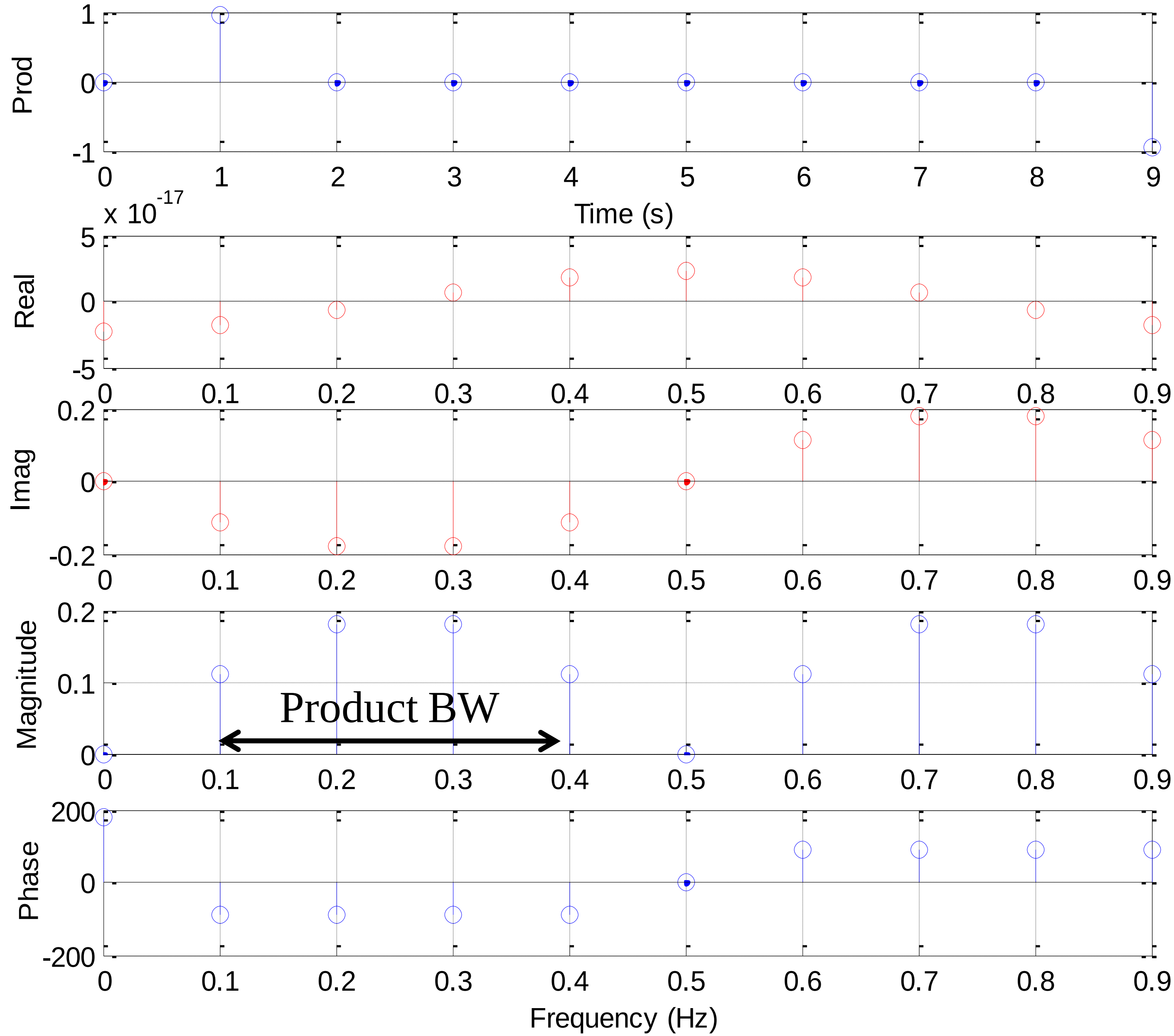
If $\quad x[n] \Leftrightarrow X[k]$

Then based on frequency shift property:

$$x[n]\sin(m\omega_o n) \Leftrightarrow \frac{1}{2j}\left(X[k+m] - X[k-m]\right)$$

BW are determined by N_1/N
Smaller N_1/N → larger BW → more spread

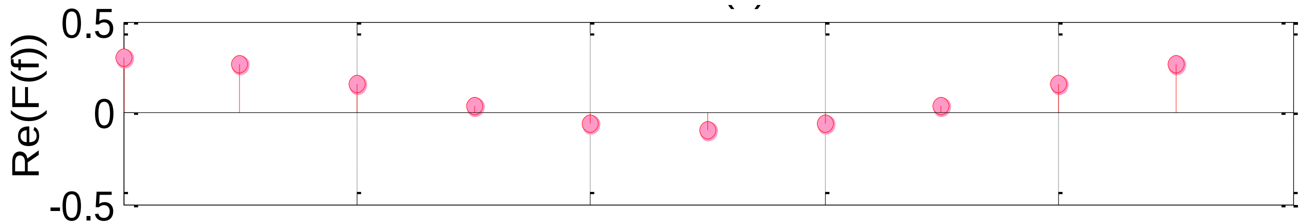
Product of Example 1 and Example 2 signal



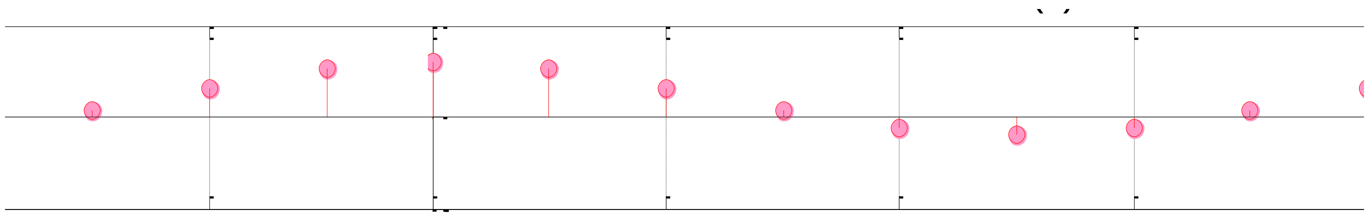
Spread Spectrum Analysis

$$x[n]\sin(m\omega_o n) \Leftrightarrow \frac{1}{2j}\left(X[k+m]-X[k-m]\right)$$

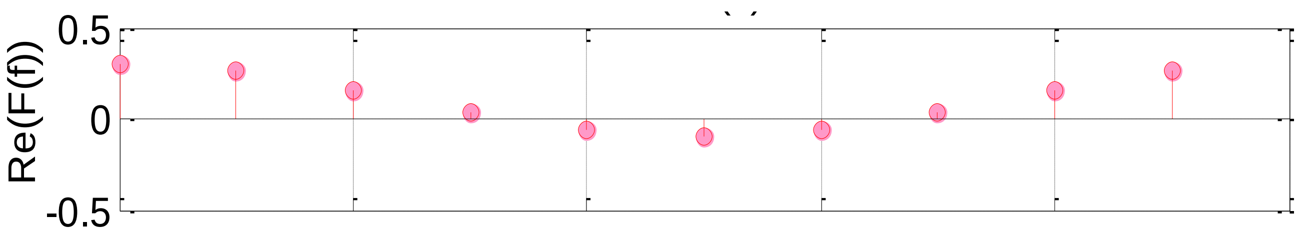
X(k)



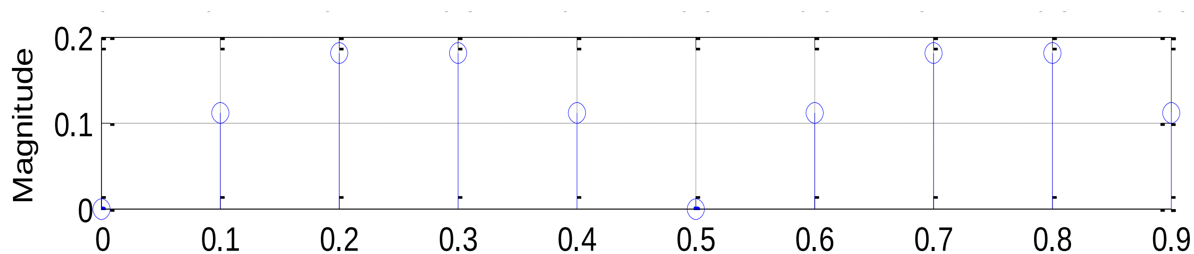
X(k-m)



X(k+m)

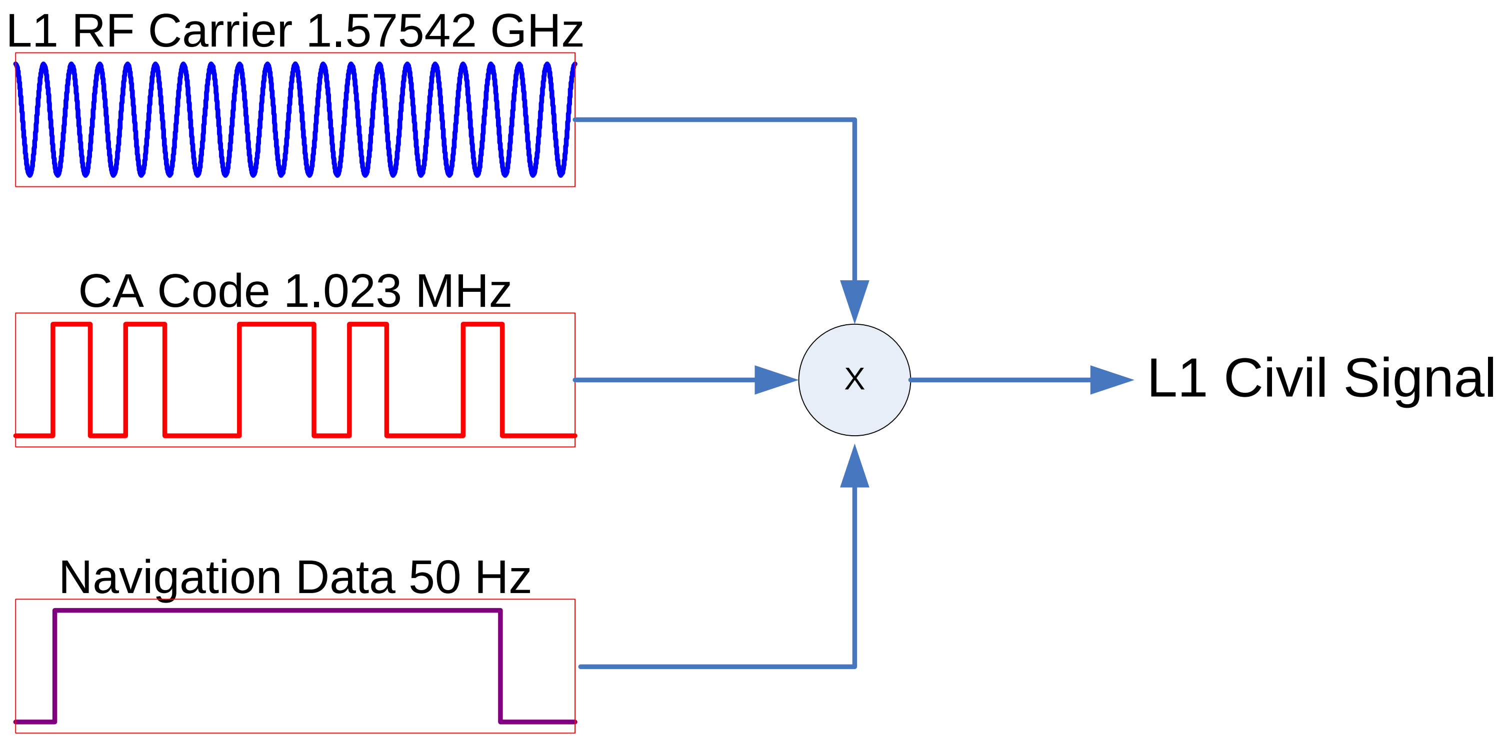


|X(k+m)-X(k-m)|/2



PART 3
GPS Signal Mathematical Representation

GPS L1 Civil Signal Generation:



L1 GPS Signal Mathematical Representation

$$s_{L1} = s_C + s_P \qquad (3.1)$$

$$s_C = \sqrt{2}A\boxed{C(t)}\boxed{D(t)}\boxed{\sin[2\pi(f_{L1} + f_D)t + \varphi]} \qquad (3.2)$$

$$s_P = AP(t)D(t)\cos[2\pi(f_{L1} + f_D)t + \varphi] \qquad (3.3)$$

A: Protected signal amplitude

f_{L1}: L1 carrier frequency 1.57542GHz

f_D: L1 carrier frequency Doppler shift

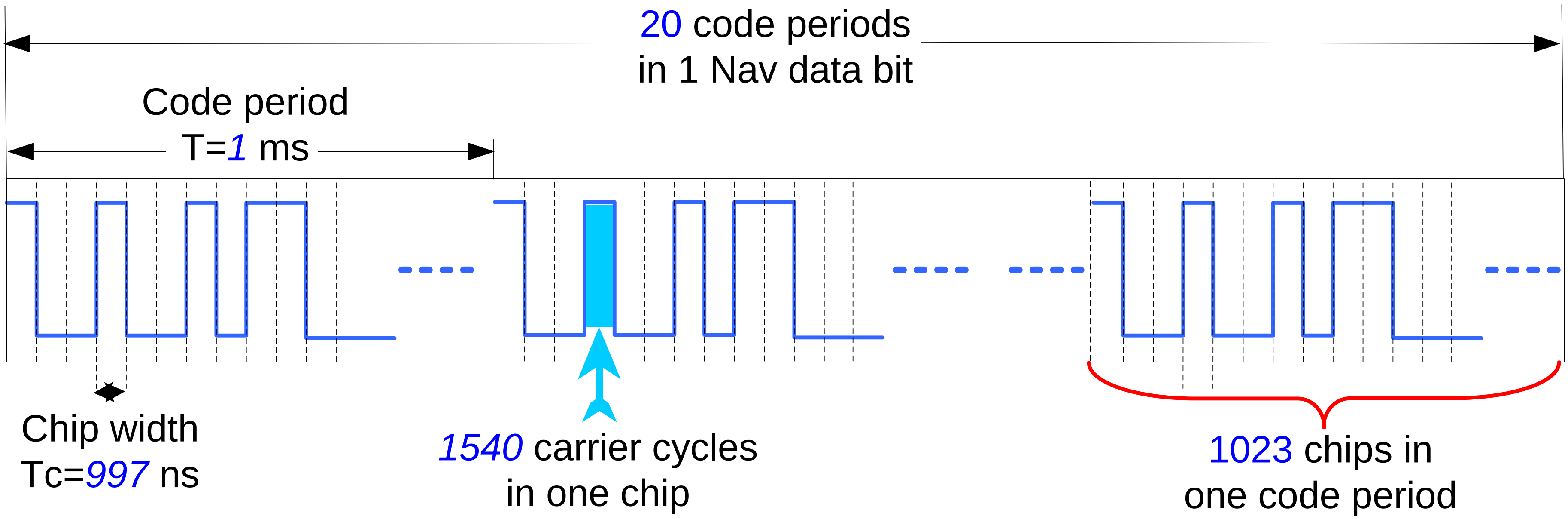
φ: L1 carrier frequency phase

$C(t)$: CA code

$P(t)$: P code

$D(t)$: Nav data

GPS L1 Signal Relative Time Scale



Reading and Assignment 2

- Today's coverage:
  - Misra and Enge, ch. 8.4
- Next lecture Topic: CA Code Generation and Properties
- Assignment #2:
  - A discrete periodic square wave signal has period N and duty cycle is 50%. Derive its frequency domain representation (its Fourier series). Write a MatLab program to verify your results and plot its spectrum. Of course, when you write your Matlab program, you will need to assign specific values to N. You can use N=5 and 6 to test your results.
  - Hint: If you have forgotten about how to do DFT, consult the book Signals and Systems by Oppenheim and Williski, Prentice Hall, 1997. If you have used a different textbook for ECE 306, you can consult that book as well.
  - Due: Friday, Jan. 21, 5PM.