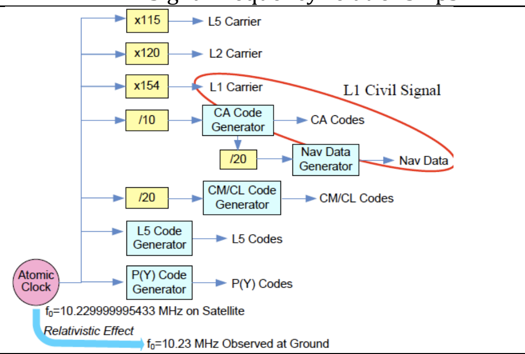
**Euler's Formula:** $e^{jx} = cosx + jsinx$ $\quad log\left(\frac{x}{y}\right) = logx - logy$ $\quad f_d = f\frac{v_{LOS}}{c}$

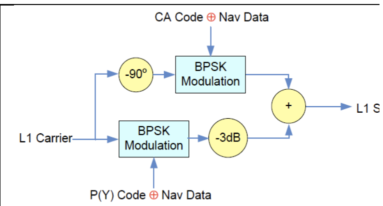
## GPS Signal Structures

| Band | L1 | | L2 | | | L5 |
|---|---|---|---|---|---|---|
| Carrier freq (GHz) | 1.57542 | | 1.2276 | | | 1.17645 |
| Code | CA | P(Y) | P(Y) | CM | CL | CA |
| Code length (chips) | 1023 | 23,017,555.5 | | 10,230 | 767,250 | 10,230 |
| Code period | 1 ms | 1 wk | | 20 ms | 1.5 s | 1 ms |
| Chip rate (MHz) | 1.023 | 10.23 | | 1.023 | | 10.23 |
| Data rate (bps) | 50 | | | 25* | Data less | 50* / Data less |
| Max power at receiver (dBW) | -157.7 | -160.7 | -163.7 | -160 | | -154 |

GPS SV signals

Signal frequency relationships




Signal modulation at L1

PRN Sequence:

1. 50% are 0, 50% are 1
2. 50% of run lengths are 1, 25% are length 2, ...
3. if the sequence is shifted, resulting seq. has equal number of agreements and disagreements as orig.

BPSK (Binary Phase Shift Keying)

- Given carrier: s (t) = Acos(w t +P)
- Its phase modulated version is:
    - Scm(t) = Acos(w t + P + O(t))

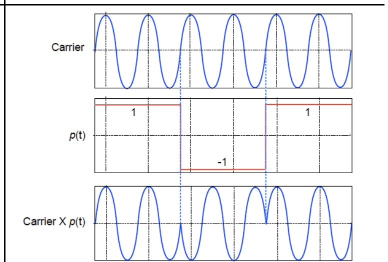
For BPSK, $\theta(t)$ can take 2 values: $0$ and $\pi$

$$\boxed{s_{cm}(t) = \begin{cases} A\cos(\omega_c t + \varphi) & \theta(t) = 0 \\ -A\cos(\omega_c t + \varphi) & \theta(t) = \pi \end{cases}}$$

BPSK modulation ←→Binary Amplitude Shift Keying:

$s_{cm}(t) = p(t)A\cos(\omega_c t + \varphi)$

$p(t)$ takes 2 values: 1 and -1




Signal frequency domain representation:

- DTFS and CTFT

$$x[n] = \sum_{k=\langle N\rangle} X[k]e^{jk\frac{2\pi}{N}n}$$

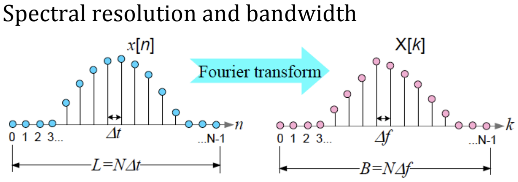
$$X[k] = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{-jk\frac{2\pi}{N}n}$$

Both are periodic with period=N

*Synthesis equation* $x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}df$

*Analysis equation* $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$

Spectral resolution and bandwidth




Spectral resolution: $\Delta f = \frac{1}{L}$ $\quad$ Bandwidth: $B = N\Delta f = \frac{1}{\Delta t} = f_s$

$X[k] \Rightarrow X(k\Delta f) = X(f)$

$x(t) = x(n\Delta t) \Rightarrow x[n]$

Time duration $L$ — Spectral span $B$ — Time step size $\Delta t = \frac{1}{B}$ — Frequency resolution $\Delta f = \frac{1}{L}$

L1 Signal mathematical rep.

$$s_{L1} = s_C + s_P$$

$$s_C = \sqrt{2}A\boxed{C(t)}\boxed{D(t)}\boxed{\sin[2\pi(f_{L1}+f_D)t+\varphi]}$$

$$s_P = AP(t)D(t)\cos[2\pi(f_{L1}+f_D)t+\varphi]$$

$A$: Protected signal amplitude
$f_{L1}$: L1 carrier frequency 1.57542GHz
$f_D$: L1 carrier frequency Doppler shift
$\varphi$: L1 carrier frequency phase

$C(t)$: C
$P(t)$: P c
$D(t)$: Na

## CA Code and Signal Power Spectrum

Correlation

- $r_{xy}(\tau) = \int_T x(t)y(t-\tau)d\tau$
- $z_{xy}[n] = \sum_{m=-\infty}^{\infty} x[m]y[m-n]$

Power spectrum's relation to auto-correlation

- Power: $S(f) = |X(f)|^2$
- Auto: $R(\tau) = \int x(t)x(t+\tau)dt$
- $S(f) = \int R(\tau)e^{-j2\pi f\tau}d\tau$
- $R(\tau) \leftrightarrow S(f)$

Example with square pulse:

- $S_1(f) = A^2T^2sinc(\pi fT)$
- $R_1(\tau) = A^2T\left(1 - \frac{|\tau|}{T}\right), |\tau| \le T$

For a sequence of N pulses:

- $R_2(\tau) = NR_1(\tau)$ (same with S)

## Signal Simulation

CA Code alignment at receiver

- CA=[CASamps(N-n0+2:N) CASamps(1:Ns-n0+1)]
- N=samps in code pd., n0=init. code samp index at rcvr, Ns=total samps rcv'd, icp=N-n0+2

Carrier generation

- $car = cos[2\pi(f_{L1}+f_d)t_s[0:N_s-1]+\varphi]$
- ts=1/fs,

Carrier to noise ratio

- $\frac{C}{N_0} = 10log\left(\frac{P_s}{Pn}\right)\ (dB - Hz)$
- C/N spec. by user, Ps rel. to A, Pn env/device

Ps and Amplitude relationship

- $S(f) = \frac{A^2T}{2}sinc(\pi fT)$ approximate PSD of $AC(t)\cos(2\pi(f_{L1}+f_d)t+\varphi)$ at the baseband
- $P_s = \int_{-\infty}^{\infty} S(f)df = \frac{1}{2}A^2$

Noise power density and amplitude

- $P_n = kT_k;\ \ T_k = T_{ant} + T_{RX}$
- $T_{ant} \sim 100K;\ \ T_{RX} = 290(NF-1)$
- $A = \sqrt{2P_n} * 10^{\frac{C/N_0}{20}}$

Noise generation

- for WGN, normal distrib.: $\sigma^2 = P_n = kT_k$
- $\therefore noise = \sqrt{kT_k} * randn(1, N_s)$

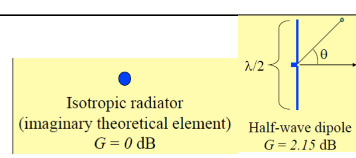
## Antenna

Antenna gain

$$G = 10\log_{10}\left[\frac{\left(\frac{dP}{d\Omega}\right)_{max}}{\frac{P}{4\pi}}\right]$$

Max. power radiated per unit solid angle

Ave. power radiated per unit solid angle




Half-wave dipole:

G = 2.15 dB

$$\frac{dP}{d\Omega} = \frac{\mu_0 cI^2}{8\pi^2}\frac{\cos^2(\frac{\pi}{2}\sin\theta)}{\cos^2\theta} \qquad P = 1.2188\frac{\mu_0 cI^2}{4\pi}$$

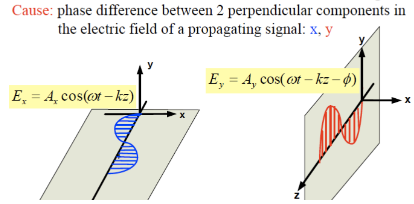
$$G = 10\log_{10}\left[\frac{\left(\frac{dP}{d\Omega}\right)_{max}}{\frac{P}{4\pi}}\right] = 10\log_{10}\left[\frac{\frac{\mu_0 cI^2}{8\pi^2}}{1.2188\frac{\mu_0 cI^2}{4\pi\times 4\pi}}\right] = 10\log_{10}\frac{2}{1.2188} = 2.15dB$$

Noise Figure

- device's contribution to thermal noise at output
- increase in a device noise power from in- to output
    - $NF = 10log_{10}\left(\frac{N_{out}}{N_{in}}\right) - G$
- amount of decrease in the SNR
    - $NF = SNR_{out} - SNR_{in}$
- only applies to bandwidth of interest
- typical range: 0.5 – 4~8 dB

Polarization

Definition: the orientation of the electric field vector in a signal
Cause: phase difference between 2 perpendicular components in the electric field of a propagating signal: x, y



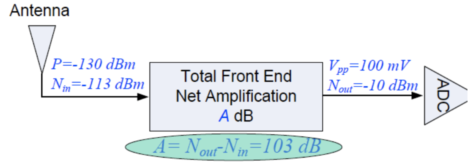

Polarization cont'd

| Ay/Ax | Polarization Pattern | | | | |
|---|---|---|---|---|---|
| ∞ | \| | \| | \| | \| | \| |
| 1 | \ | ↻ | / | ↻ | \ |
| 0 | — | — | — | — | — |
| ϕ | -180° | -90° | 0° | 90° | 180° |

## RX Front End Circuit

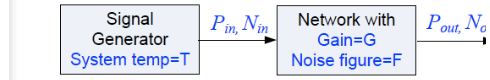
### Nominal GPS SNR at a RX
- L1 CA P @ RX: $P = -160dBW = -130dBm$
- RX input noise P: $N_{in} = kTB$
- If B=2MHz, 10log(B)=63 dB
- Assume kT = -209 dBW = -179 dBm
- $N_{in} = -179 + 63 =$ -116 dBm
- Nominal SNR: $SNR = P - N_{in} = -14\ dB$

### RX Front End Amplification




$$N_{out}=\frac{1}{2}\frac{v_{pp}^2}{R_{ADC}}=\frac{1}{2}\frac{0.1^2}{50}=0.1mW=-10dBm$$

### RX Noise Figure Calc.




Recall noise figure definition: $F=\frac{SNR_{in}}{SNR_{out}}=\frac{P_{in}/N_{in}}{P_{out}/N_{out}}=\frac{1}{G}\frac{N_{out}}{N_{in}}$

$N_{out} = FGN_{in} = GN_{in} + N_T;\ N_T = (F-1)GN_{in}$

For cascaded networks:

$Overall\ gain\ G = G_1 G_2 \dots G_N$

$$F = F_1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1G_2} + \cdots + \frac{F_N-1}{G_1G_2\dots G_{N-1}}$$

### Mixing and Down Conversion Output

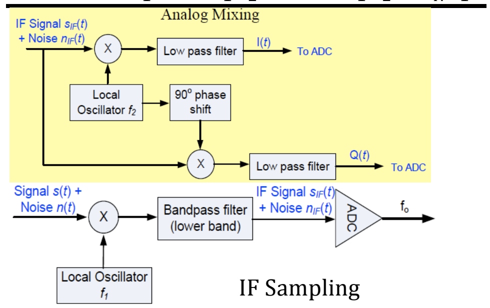
$$Output = s_{IF}(t) + n_{IF}(t) + spurious\quad components$$

$$s_{IF}(t)=s(t)\cos(\omega_1 t)=AC(t)D(t)\cos[(\omega_{L1}+\omega_d)t+\phi]\cos(\omega_1 t)$$

$$s_{IF}(t)=\frac{A}{2}C(t)D(t)\{\underbrace{\cos[(\omega_{L1}-\omega_1+\omega_d)t+\phi]}_{\text{Pass through bandpass filter}}+\underbrace{\cos[(\omega_{L1}+\omega_1+\omega_d)t+\phi]}_{\text{Rejected by bandpass filter}}\}$$

$$s_{IF}(t)=\frac{A}{2}C(t)D(t)\cos[(\omega_{IF}+\omega_d)t+\phi]$$
$$n_{IF}(t)=r(t)\cos[\omega_{IF}t+\phi_n(t)]$$
$$\omega_{IF}=\omega_{L1}-\omega_1$$

### Why use IF samplin to convert to baseband?
- Analog mixing generates I and Q channels
  - Channel imbalance
  - More complex hardware components
  - Sig strength division between 2 ch's
- IF sampling moves ADC to output of IF
  - Single channel
  - Less hardware
  - Signal strength preservation

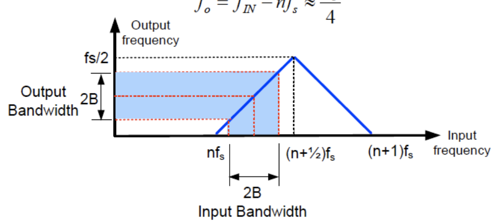


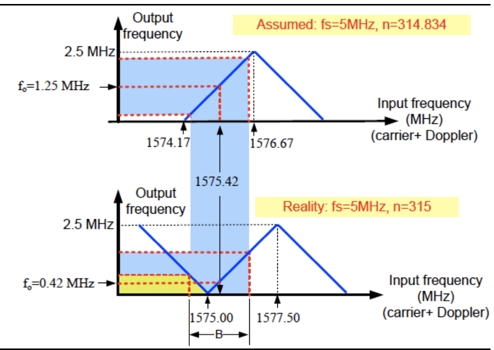

### Sampling Frequency and CA Code Chipping Rate
- Fs should not be int multiples of fc
  - Choose fs=5MHz (separated from 5fc)
- Fc should include Doppler
  - $f_c = 1.023\ MHz + f_d$

note: $f_{in} = f_{L1} + f_d$ for direct sampling
$f_{in} = f_{IF}$ for IF sampling

### Sampling Frequency Selection and Aliasing

$$f_o = f_{IN} - nf_s \approx \frac{f_s}{4}$$

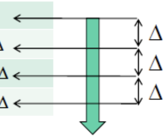



### Sampling Frequency Selection Criteria
1. $f_s > 2B$
2. $f_s \neq m(f_c + \Delta f_d)$
3. select n so fo only contains freq. w/in 1 oct.
4. $f_o = f_{in} - nf_s \approx \frac{f_s}{4}$
5. $f_o = f_{in} - nf_s \approx \frac{3f_s}{4}$ (no band aliasing)

### ADC Output Range ($v_{pp}$) and Step Size ($\Delta$)
- N-bit ADC: $2^N$ levels separated by $\Delta$

| Digital Rep. | Data Range | Output Levels | General Formula |
|---|---|---|---|
| 00 | $x<-0.8$ | -1 | $V_{min}$ |
| 01 | $-0.8<x<0$ | -0.3 | $V_{min}+\Delta$ |
| 10 | $0<x<0.8$ | 0.3 | $V_{min}+2\Delta$ |
| 11 | $x>0.8$ | 1 | $V_{min}+3\Delta$ |

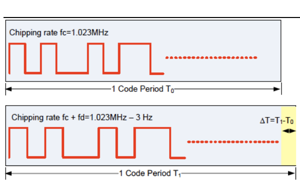



- $\Delta = \frac{v_{pp}}{2^N - 1}$

### Increasing SNR
1. use more bits in ADC
2. higher sampling frequency
   a. get more details about signal
3. longer data
   a. accumulate more coherent energy

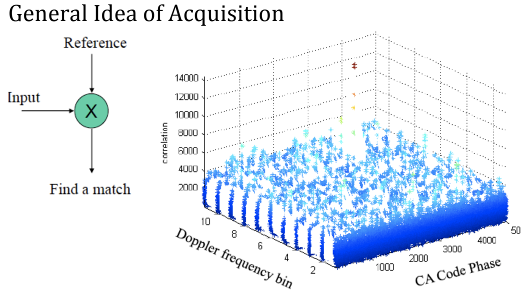
## Signal Acquisition

### Acquisition Basics for Software RX
- performed using a block of data
- performed for each SV in sequence
- time elapse between SV acq. & tracking data
- fast acquisition is key to real time software rx




Doppler effect on CA code

### General Idea of Acquisition




### Acquisition Data Length Selection
- minimum: 1ms to ensure 1 pd CA code included
- max: 10 ms to ensure no nav data transition
- main constraint: nav data bit transition spread carrier causing SNR loss in acquisition
- secondary constraint: Doppler effect on CA code

### Good Search Bin Size?
General consideration
- Large bin size → larger errors, few computations
- Small bin size → smaller errors, more computations
- Compromised approach:
  - Work with large bin size first
  - Find potential cell, zoom into it
  - Sub-divide potential cell into smaller bins for 2nd round search (fine acquisition)
- Code phase bin
  - Hardware-based receiver, typical bin size =1/2 chip
  - Software-based receiver, typical bin size = 1~ 2Δt$_s$
- Doppler frequency bin
  - Determined by data length T: Δf=1/T
  - For nominal satellite signal acquisition:
    - T= 1~10ms → Δf=1kHz ~100Hz