GMPE 340

Solution

Problem 3.1

a) $$G(s) = \frac{Ts}{(1+Ts)^2}$$

$$E[x^2] = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \frac{Ts}{(1+Ts)^2} \cdot \frac{T(-s)}{(1+T(-s))^2} \cdot A \, ds$$

$$= \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \left[\frac{\sqrt{A}\,Ts}{(1+2Ts+T^2s^2)}\right] \cdot \left[\frac{(-s)}{\text{---} \;''\; \text{---}}\right] ds$$

integration table:

$n = 2$, $c_1 = \sqrt{A}\,T$ $\quad d_2 = T^2$

$c_0 = 0$ $\quad d_1 = 2T$

$d_0 = 1$

$$E[x^2] = \frac{c_1^2 d_0 + c_0^2 d_2}{2 d_0 d_1 d_2} = \frac{AT^2}{2 \cdot 2T \cdot T^2} = \underline{\underline{\frac{A}{4T}}}$$

b) $$G(s) = \frac{\omega_0^2}{s^2 + 2\xi\omega_0 s + \omega_0^2}$$

integration table:

$n = 2$, $c_1 = 0$, $d_2 = 1$

$c_0 = \sqrt{A}\,\omega_0^2$, $d_1 = 2\xi\omega_0$

$d_0 = \omega_0^2$

$$E[x^2] = \frac{c_1^2 d_0 + c_0^2 d_2}{2 d_0 d_1 d_2} = \frac{A\omega_0^4}{2\cdot\omega_0^2\cdot 2\xi\omega_0} = \underline{\underline{\frac{A\omega_0}{4\xi}}}$$

c) $$G(s) = \frac{s+1}{(s+2)^2} = \frac{s+1}{s^2 + 4s + 4}$$

integration table:

$n = 2$, $c_1 = \sqrt{A}$, $d_2 = 1$

$c_0 = \sqrt{A}$, $d_1 = 4$

$d_0 = 4$

$$E[x^2] = \frac{c_1^2 d_0 + c_0^2 d_2}{2 d_0 d_1 d_2} = \frac{(4+1)\cdot A}{2\cdot 4\cdot 4\cdot 1} = \underline{\underline{\frac{5}{32}A}}