EE-655 MODEM DESIGN, FALL 2015 MEDIUM LENGTH OFDM FINAL EXAM

Problem 1. For the first half of this exam we are going to examine an OFDM Modem whose modulator is implemented by a 128 point output IFFT.

A. Implement an OFDM Modem as a 128 point transform with 54 occupied frequency bins. In particular, bins –27-to-1 and +1 to +27, (bin 0 empty). Each bin is modulated 16-QAM. The time series formed at the output of the transform is separated by a guard band of 32 zero valued samples located at the beginning of the OFDM symbol. Form an OFDM packet composed of 50 OFDM symbol.

B. Plot a power spectrum of the time series formed by a windowed 2048 point FFT. Comment on the side lobe structure of the spectrum.

C. Demodulate the OFDM packet and display the (time) overlaid real part of the successive transforms as well as the (time and frequency) overlaid constellation diagram when there is no channel distortion.

D. Pass the OFDM packet through a channel with impulse response

[1 0 j 0.2 0 0 0 0 0 0 0.1]

Now demodulate the received OFDM packet and display the (time) overlaid real part of the successive transforms as well as the (time and frequency) overlaid constellation diagram.

E. Replace the 32-sample guard band with a 32 sample cyclic prefix taken from the last 32 samples of each OFDM symbol. Pass this new packet through the same channel. Now demodulate the received OFDM packet and display the (time) overlaid real part of the successive transforms as well as the (time and frequency) overlaid constellation diagram.

F. Insert 50 samples of delay and a single OFDM symbol length preamble at the beginning of the OFDM packet formed by zero packing the spectrum 1-to-4. Only insert random QPSK data in bins ±4, ±8, ±12,…., ±24. This symbol has an appended cyclic prefix. Add white noise with standard deviation 0.01 to the received signal. Spin the data being delivered through the channel at 1 degree per sample. Use a 32 sample delay line and form the cross correlation between input and output of the delay line and form the autocorrelation at the output of the delay line. Verify that the sample by sample ratio of the cross to the auto correlation identifies the start of the OFDM frame. Present a figure showing magnitude of cross, auto, and ratio. Also show the angle obtained at the output of the 32 sample averager processing the cross correlation. Verify this is a reasonable estimate of the input rotation rate. Present a figure showing output of 32-point averager and the estimated rate of rotation. Use this estimated rotation rate to de-spin subsequent signal samples following this short preamble. For this problem, we will not use the estimated rotation rate to guide the de-spinning. This was included to illustrate the process. For the remainder of this problem, remove the input spin but not the input noise.

G. Insert a second preamble following the short preamble formed by QPSK modulation in all the bins (-27 through +27 but not bin 0). This symbol has an appended cyclic prefix as do all data symbols. Pass the ofdm packet with two preambles through the channel and add the noise described earlier. When the receiver receives this preamble symbol it demodulates it and uses the known spectral values to measure and correct the distortion caused by the channel. The relationship between channel and received signal at each frequency is of the form $F_{rcvd}(K)=F_{xmtd}(k)*\ F_{channel}(k)$. From this relationship we can estimate the channel and remove it in subsequent observations of the received signal. From the channel estimate use the inverse of the channel at each frequency to demodulate the data segment of the symbol frames and plot the (time) overlaid real part of the successive transforms as well as the (time and frequency) overlaid constellation diagram.

Problem 2. For the second half of this exam we are going to examine a shaped SC-OFDM Modem whose modulator is implemented by a 128 point output IFFT.
We are going to examine a SC-OFDM Modem whose modulator is implemented by a 64 point input FFT followed by a 128 point output IFFT. The spectral shaping between the input and output transforms has the following form:

```
cp0=(floor(4*rand(1,64))-1.5)/0.5+j*(floor(4*rand(1,64))-1.5)/0.5;
fcp0=fft(cp0);

fcp1=zeros(1,128);
fcp1(65+(-32:31))=fftshift(fcp0);
%fcp1(65-32)=sqrt(0.5)*fcp1(65-32);
%fcp1(65+32)=fcp1(65-32);

cp1=2*ifft(fftshift(fcp1));
cp1a=[cp1(97:128) cp1];
```

A. Implement an OFDM Modem as a 128 point transform with 65 occupied frequency bins as shown above. Each of the 64 time samples are modulated 16-QAM constellation points. The interpolated time series formed at the output of the second transform is separated by a guard band of 32 zero valued samples located at the beginning of the OFDM symbol. Form an OFDM packet composed of 50 OFDM symbol.

B. Plot a power spectrum of the time series formed by a windowed 2048 point FFT. Comment on the side lobe structure of the spectrum.

C. Demodulate the OFDM packet and display the (time) overlaid real part of the successive transforms as well as the (time and frequency) overlaid constellation diagram when there is no channel distortion.

D. Pass the OFDM packet through a channel with impulse response

[1 0 j 0.2 0 0 0 0 0 0 0.1]

Now demodulate the received OFDM packet and display the (time) overlaid real part of the successive transforms as well as the (time and frequency) overlaid constellation diagram.

E. Replace the 32-sample guard band with a 32 sample cyclic prefix taken from the last 32 samples of each OFDM symbol. Pass this new packet through the same channel. Now demodulate the received OFDM packet and display the (time) overlaid real part of the successive transforms as well as the (time and frequency) overlaid constellation diagram.

F. Insert 50 samples of delay and a single OFDM symbol length preamble at the beginning of the OFDM packet formed with 4-repeated time segments. Do this with a random qpsk time sequence of length 16, repeated 4-times. Previously we obtained the periodicity by zero packing the spectrum but here we simply repeat the segment. The modulation process interpolates these segments from 16 samples per segment to 32 samples per segment. This symbol has an appended cyclic prefix. Add white noise with standard deviation 0.01 to the received signal. Spin the data being delivered through the channel at 1 degree per sample. Use a 32 sample delay line and form the cross correlation between input and output of the delay line and form the autocorrelation at the output of the delay line. Verify that the sample by sample ratio of the cross to the auto correlation identifies the start of the OFDM frame. Present a figure showing magnitude of cross, auto, and ratio. Also show the angle obtained at the output of the 32 sample averager processing the cross correlation. Verify this is a reasonable estimate of the input rotation rate. Present a figure showing output of 32-point averager and the estimated rate of rotation. Use this estimated rotation rate to de-spin subsequent signal samples following this short preamble. For this problem, we will not use the estimated rotation rate to guide the de-spinning. This was included to illustrate the process. For the remainder of this problem, remove the input spin but not the input noise.

G. Form the long preamble used to estimate the channel frequency response. Since SC-OFDM forms the series in the time domain we have to form a signal with the cascade transforms with approximately constant spectral amplitude. We do this with a linear FM sweep known as CAZAC (constant amplitude, zero auto correlation) as shown below.

```
NN=64;
rr=(0:NN-1).*(1:NN)/2;
prb0=exp(j*2*pi*rr/NN);
fprb0=fft(prb0);
fprb1=zeros(1,128);
fprb1(65+(-32:31))=fprb0;
%fprb1(65-32)=sqrt(1/2)*fprb1(65-32);
%fprb1(65+32)=fprb1(65-32);
prb1=ifft(fftshift(fprb1));
prb1a=[prb1(97:128) prb1];
```

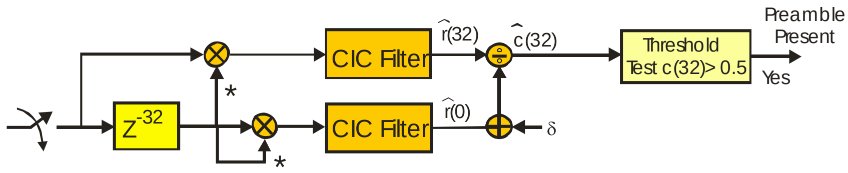
On three subplots plot the real and imaginary parts of the time series and the magnitude of the probe's frequency response.

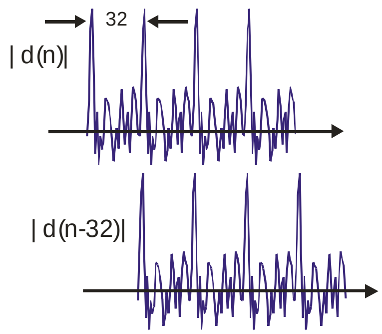
H. Pass the long preamble through the channel as shown below

```
xprb=filter(cc,1,prb1a);
xprb=xprb+0.01*(randn(1,160)+j*randn(1,160));
```

On one of two subplots plot the FFT of the long preamble, the FFT of the channel response to the long preamble, and the FFT of the Channel. All responses are for series of length 128 without their cyclic prefix. On the second subplot plot the estimated channel frequency response and the actual frequency response.

I. Pass the 50 SC-OFDM symbols through the channel and repeat demodulate and channel equalize using the inverse channel of the channel estimate formed in the previous section.





$$\hat{r}(0) = \sum_{k=n}^{31} d(n+k)d^{*}(n+k)$$

$$\hat{r}(32) = \sum_{k=n}^{31} d(n+k)d^{*}(n-32+k)$$

$$\hat{c}(32) = \frac{\hat{r}(32)}{\hat{r}(0)+\delta}$$