Receiver Position Estimation

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Position Estimation

> Satellite position in the transmitted time "t - τ".

> Pseudo-range between satellite and user in the received time "t"

$$\rho^{(k)}(t) = r^{(k)}(t, t-\tau) + \underbrace{c\left[\delta t_u(t) - \delta t^{(k)}(t-\tau)\right]}_{\text{Clock Errors}} + I^{(k)}(t) + T^{(k)}(t) + \varepsilon_\rho^{(k)}(t)$$

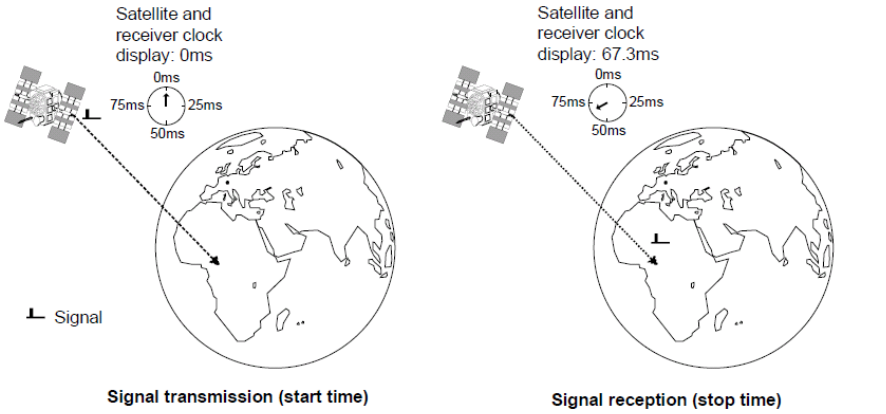
The reason why we call "pseudo-range" is from second term.

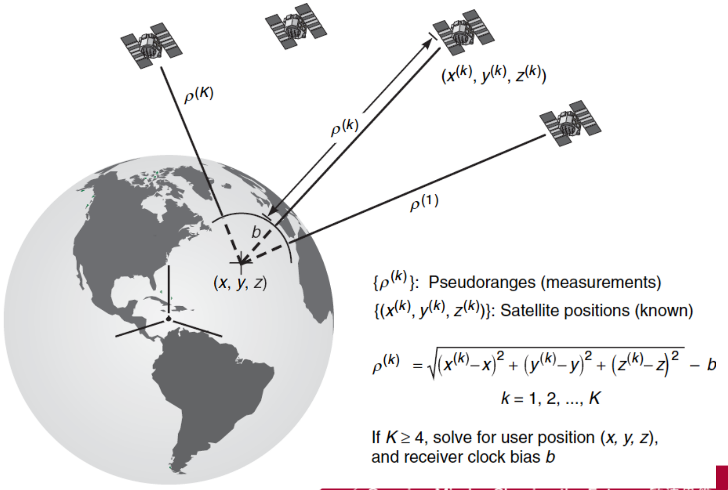
Satellite clock and Receiver clock are not synchronized.

How many unknown parameters do we have ?

x, y, z, receiver clock offset

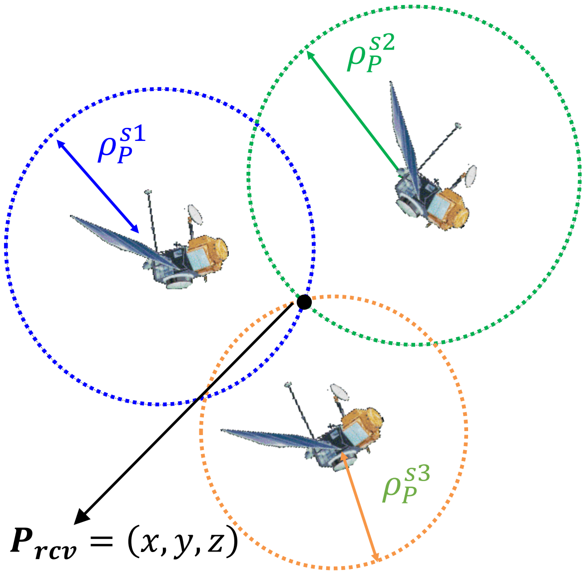
- <u>Satellite clock</u> is corrected using navigation data.
- Fortunately, receiver clock offset is same for all satellites.
- Therefore, unknown variables should be solved are x, y, z and receiver clock offset.





Triangulation Positioning Theory

Goal: Solve x,y,z!



Known Information

$$\rho_P^{s1} = \sqrt{(x^{s1} - x)^2 + (y^{s1} - y)^2 + (z^{s1} - z)^2} + b$$

$$\rho_P^{s2} = \sqrt{(x^{s2} - x)^2 + (y^{s2} - y)^2 + (z^{s2} - z)^2} + b$$

$$\rho_P^{s3} = \sqrt{(x^{s3} - x)^2 + (y^{s3} - y)^2 + (z^{s3} - z)^2} + b$$

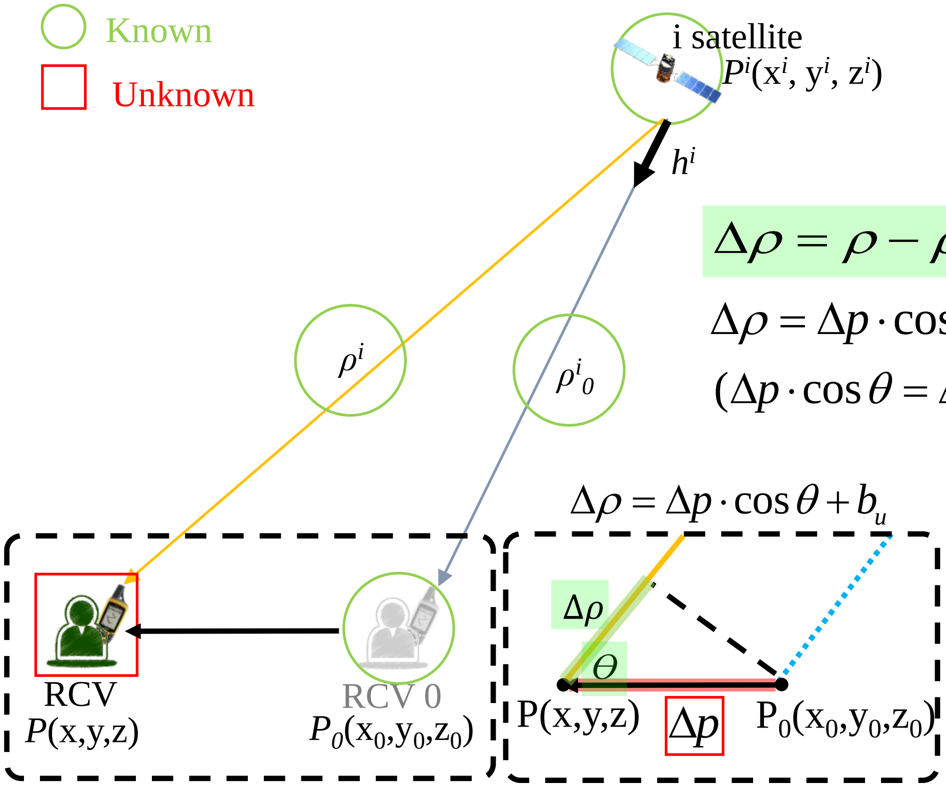
Pseudorange measurement

Satellite Positions

3 equations with 4 unknowns! Therefore, 4 satellites are required

Can we solve? YES! How!? Mathematically, linearize the equation by Taylor Series Expansion at a point we GUESS.

Positioning using Least Square Estimation



$$\Delta\rho = \rho - \rho_0 = (\rho_{true} + b_u) - \rho_0$$

$$\Delta\rho = \Delta p \cdot \cos\theta + b_u$$

$$(\Delta p \cdot \cos\theta = \Delta\bar{p} \times 1 \times \bar{\theta}_{uni})$$

Positioning using Least Square Estimation

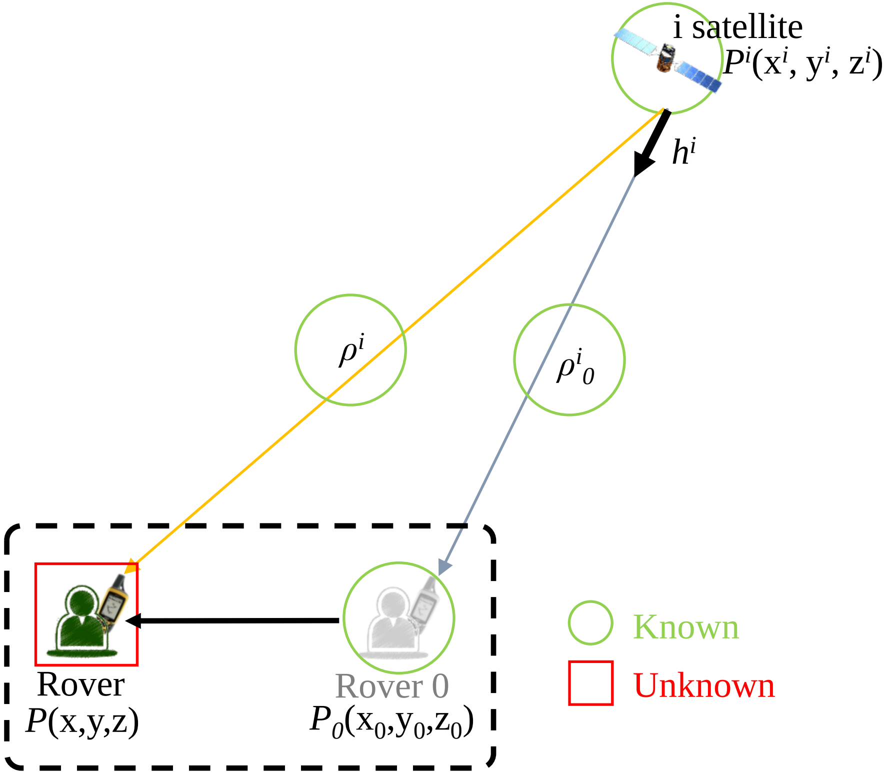
$$\Delta\rho = \rho - \rho_0 = (\rho_{true} + b_u) - \rho_0$$

$$\Delta\rho = \Delta p \cdot \cos\theta + b_u \qquad (\Delta p \cdot \cos\theta = \Delta\bar{p} \times 1 \times \bar{\theta}_{uni})$$

$$\Delta\rho^i = \begin{bmatrix} \dfrac{(x^i - x_0)}{\rho^i{}_0} & \dfrac{(y^i - y_0)}{\rho^i{}_0} & \dfrac{(z^i - z_0)}{\rho^i{}_0} & 1 \end{bmatrix} \cdot \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta b \end{bmatrix}$$

$$\Delta\rho = G\Delta p \qquad (\Delta p = G^{-1}\Delta\rho)$$

$$\begin{bmatrix} \Delta p_x \\ \Delta p_y \\ \Delta p_z \\ \Delta b_u \end{bmatrix} = \begin{bmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \\ b_u - b_{u,0} \end{bmatrix} \begin{bmatrix} x_n \\ y_n \\ z_n \\ b_{u,n} \end{bmatrix} = \begin{bmatrix} x_{n-1} \\ y_{n-1} \\ z_{n-1} \\ b_{u,n-1} \end{bmatrix} + \begin{bmatrix} \Delta p_{x,n-1} \\ \Delta p_{y,n-1} \\ \Delta p_{z,n-1} \\ \Delta b_{u,n-1} \end{bmatrix}$$



$$\Delta\boldsymbol{\rho} = \boldsymbol{\rho}_{meas} - \boldsymbol{\rho}_0 - \boxed{b_u} \text{ where } \boldsymbol{\rho}_0^{(i)} = \left\| \boldsymbol{P}^{(i)} - \boldsymbol{P}_0 \right\|$$

$$\Delta\boldsymbol{\rho} = \boldsymbol{G}\boxed{\Delta\boldsymbol{p}}$$

$\boxed{}$ Unknown

$$\begin{bmatrix} \Delta\rho^1 \\ \Delta\rho^2 \\ \Delta\rho^3 \\ \Delta\rho^4 \end{bmatrix} = \begin{bmatrix} \frac{(x^1 - x_0)}{\rho_0^1} & \frac{(y^1 - y_0)}{\rho_0^1} & \frac{(z^1 - z_0)}{\rho_0^1} & 1 \\ \frac{(x^2 - x_0)}{\rho_0^2} & \frac{(y^2 - y_0)}{\rho_0^2} & \frac{(z^2 - z_0)}{\rho_0^2} & 1 \\ \frac{(x^3 - x_0)}{\rho_0^3} & \frac{(y^3 - y_0)}{\rho_0^3} & \frac{(z^3 - z_0)}{\rho_0^3} & 1 \\ \frac{(x^4 - x_0)}{\rho_0^4} & \frac{(y^4 - y_0)}{\rho_0^4} & \frac{(z^4 - z_0)}{\rho_0^4} & 1 \end{bmatrix} \boxed{\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ b_u \end{bmatrix}}$$

$$\boxed{\Delta\boldsymbol{p}} = \boldsymbol{G}^{-1}\Delta\boldsymbol{\rho} \longrightarrow \boxed{\Delta\boldsymbol{p}} = \left(\boldsymbol{G}^T\boldsymbol{G}\right)^{-1}\boldsymbol{G}^T\Delta\boldsymbol{\rho}$$

Satellite more than 4
then we need pseudo-inverse

$$\boldsymbol{P}_1 = \boldsymbol{P}_0 + \Delta\boldsymbol{p}$$

Flowchart of Iterative LS

Initialize $\boldsymbol{P_0}$

$$\boldsymbol{\rho}_{\boldsymbol{0}}^{(sv)} = \left\|\boldsymbol{P}^{(sv)} - \boldsymbol{P_0}\right\|$$

$$\Delta\boldsymbol{\rho} = \boldsymbol{\rho_{meas}} - \boldsymbol{\rho_0}$$

$i = 0$

$$\boldsymbol{G} = \begin{bmatrix} \frac{(x^1-x_i)}{\rho_i^1} & \frac{(y^1-y_i)}{\rho_i^1} & \frac{(z^1-z_i)}{\rho_i^1} & 1 \\ \frac{(x^2-x_i)}{\rho_i^2} & \frac{(y^2-y_i)}{\rho_0^2} & \frac{(z^2-z_i)}{\rho_0^2} & 1 \\ \frac{(x^3-x_i)}{\rho_i^3} & \frac{(y^3-y_i)}{\rho_0^3} & \frac{(z^3-z_i)}{\rho_0^3} & 1 \\ \frac{(x^4-x_i)}{\rho_i^4} & \frac{(y^4-y_i)}{\rho_i^4} & \frac{(z^4-z_i)}{\rho_i^4} & 1 \end{bmatrix}$$

$$\Delta\boldsymbol{p} = \left(\boldsymbol{G}^T\boldsymbol{G}\right)^{-1}\boldsymbol{G}^T\Delta\boldsymbol{\rho}$$

$$\boldsymbol{P_{i+1}} = \boldsymbol{P_i} + \Delta\boldsymbol{p}$$

$$\|\Delta\boldsymbol{p}\| < 10^{-4}$$

Y

$$\boldsymbol{P_{final}} = \boldsymbol{P_{i+1}}$$

N

$i = i + 1$

$$\boldsymbol{\rho}_{\boldsymbol{i}}^{(sv)} = \left\|\boldsymbol{P}^{(sv)} - \boldsymbol{P_i}\right\|$$

$$\Delta\boldsymbol{\rho} = \boldsymbol{\rho_{meas}} - \boldsymbol{\rho_i} - b_u$$

Example of Iterations in LS method

- 4 unknown variables (x,y,x,clock) are present.
- At least 4 visible satellites are required.
- With true satellite positions and true range between satellites and user antenna, the calculated position is true (only one solution).
- It is impossible in a practical sense.
- Least-Square method (LS method) is mainly used for the estimation of user antenna position.

Example of Iterations in LS method

- The user antenna was located in PolyU campus.
- If we set (0, 0, 0) as an initial x, y, z positions,
- After the first iteration, the estimated position was 22.156, 114.191, 1252955m. (Po Toi Island)
- Secondly, it was 22.304, 114.101, 42298m (close to near sea of Kowloon)
- Thirdly, it was 22.305166, 114.181192, 116m (about 30m away from antenna)
- Fourth, it was 22.305843, 114.181064, 63m (within 2m from antenna)