

## Assessment of Orbit Maintenance Strategies for Small Satellites

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### ABSTRACT

Small satellites are increasingly being deployed in LEO, where orbital decay is prevalent due to drag and solar radiation. In this paper, we are interested in assessing orbit maintenance methods, in terms of the  $\Delta V$  usage, that mitigate the effects of drag and solar radiation pressure. We simulate three different kinds of orbit maintenance strategies, using known parameters from a recently launched small-sat in Singapore – the VELOX-CI. To begin, we first develop an orbital decay model coded decay simulations on Python. Three station-keeping strategies were then proposed; the first is an ‘ideal’ reference strategy, and the latter two are periodic thruster-burn strategies where we vary the frequency of thruster fire by varying the in-track / altitude tolerance band. The total  $\Delta V$  used throughout the lifespan of the mission was then calculated. Results show an intimate connection between all three orbit maintenance strategies. As the threshold tolerance of thruster fire for Methods 2 and 3 tend to zero, the total  $\Delta V$  tends towards the  $\Delta V$  usage of a constant thrust force (Method 1). Conversely, this indicates a potential for minimizing  $\Delta V$  usage in our chosen orbit maintenance strategies, simply by varying the tolerance bands.

### I. INTRODUCTION

Small satellites have limited capacity to carry fuel aboard, yet they need the propellant to maintain orbit against perturbing forces, especially secular variations such as orbit decay due to atmospheric drag and solar radiation pressure in LEO.

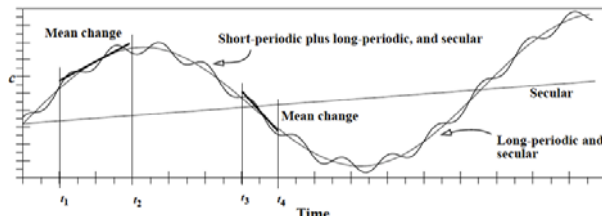
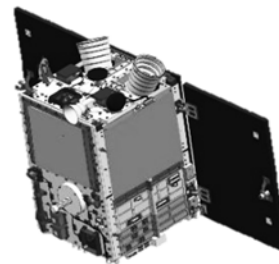


Fig 1: Short-term, long-term, and secular (linear) variations

In order to minimise space usage and maximise fuel efficiency and lifetime, it is important to optimise our orbit maintenance method. Saving on fuel can either free up space used for more sophisticated payloads, or prolong the lifetime of the satellite throughout its mission via station-keeping. In this paper, we are interested in assessing which orbit maintenance method allows for most optimisation of  $\Delta V$  usage. To validate the station-keeping methods, we employ known parameters from a recently launched small-sat in Singapore – the VELOX-CI, as our model of study. First, we develop an orbital decay model for us to simulate decay (efficiently) on Python. We validate this decay against the actual GPS data of the VELOX-CI across 16th December 2015 to 11th November 2016 <sup>[1]</sup> (NTU SARC, 2017).



Circular orbit, near equatorial  
5-year simulation time  
615 X 608 X 848 mm<sup>3</sup>  
Mass = 123 kg  
Semi major axis = 6928.14 km  
Altitude = 545 km  
Inclination = 15 degrees  
RAAN, ARP, True Anomaly = 0

Once we have affirmed that our model is close to the actual decay values, we run simulations for three different station-keeping algorithms, using the current decay model. The independent variable is the distance threshold, or tolerance band (which is an altitudinal tolerance for Method 2, and an in-track tolerance for Method 3), where if the small satellite decays beyond this tolerance, an impulsive thrust will be applied to bring it back to the original altitude. The simulation ran in steps of 60 minutes, calculating and accumulating the total orbit decay for each 60-minute pass. When the total decay exceeds the tolerance band, the thrusters will fire, and the  $\Delta V$  is calculated. The total  $\Delta V$  is calculated through accumulation of the total number of thruster fires in the mission duration. The duration of the simulated mission is 5 Earth years. The simulation experiment is then repeated again for different values of the threshold, and the total  $\Delta V$  usage is plotted, as y-axis, against the threshold variable, as x-axis. This experimental procedure was run for Methods 2 and 3. Method 1 is simply a “reference” strategy to compare results with 2 & 3.

## II. THE ORBITAL DECAY MODEL

First, before we even begin calculating the  $\Delta V$  of each orbit maintenance strategy, it is necessary to develop a simplified, accurate and computationally efficient model for decay, because that is what our orbit maintenance strategy will attempt to oppose. The atmospheric density model used is the NRLMSISE-00. In this section, we will derive our model analytically, and then validate it using actual GPS data from the VELOX-CI, and also compare it with simulations from STK using the High Precision Orbit Propagator (HPOP) module. The pseudocode for the orbital decay simulation is summarised in:

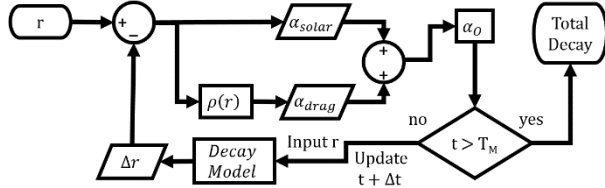


Fig 2: Logic flow chart for orbital decay simulator

We first model atmospheric decay due to the effects of drag and solar radiation pressure. The equation modelling deceleration due to drag [2] is:

$$\alpha_{drag} = -\frac{1}{2} \rho C_d \left( \frac{A}{m} \right) V^2 \quad (Eq 1)$$

$\rho$  = atmospheric density which is a function of altitude  
 $C_d$  = drag coefficient  $\sim 2.2$   
 $A$  = satellite cross-sectional area ( $\sim 0.52\text{m}^2$  for the VELOX-CI)  
 $m$  = the mass of the satellite = 123kg  
 $V$  = linear velocity of the satellite with respect to the Earth

The deceleration from solar radiation pressure [2] is:

$$\alpha_{solar} = \frac{A(1+r)}{m} \times P \approx 2.56 \times 10^{-8} \text{ms}^{-2} \quad (Eq 2)$$

$A$  = cross sectional area ( $\sim 0.52\text{m}^2$  for the VELOX-CI)  
 $r$  = reflection factor ( $0 \leq r \leq 1$ , and  $r = 0.5$  for the VELOX-CI)  
 $m$  = the mass of the satellite = 123kg  
 $P$  = solar radiation flux at 1 AU ( $\sim 5\text{E-6}$ )

In the next step we will derive the rate of orbital decay. We start with the Vis-Visa Equation, which gives the velocity of spacecraft. In the case of a circular orbit, the semi-major axis is equivalent to the radius and so this simplifies the elliptical Vis-Visa equation [2] to the expression below:

$$V^2 = GM_E \cdot \left( \frac{2}{R} - \frac{1}{a} \right) = \frac{GM_E}{R} \quad (Eq 3)$$

Our orbital energy is simply the sum of the kinetic and gravitational potential energies of the satellite. We can substitute the Vis-Visa equation into Equation 4 to simplify the equation so that it varies only with  $R$ .

$$U = KE + GPE = \left( -\frac{GM_E m}{2R} \right) \quad (Eq 4)$$

We may now figure out how orbital energy varies explicitly with radii by differentiation.

$$\frac{dU}{dR} = \frac{GM_E m}{2R^2} \quad (Eq 5)$$

The acceleration from perturbing forces is given as the sum of drag and solar radiation deceleration, (see Eq 1 & 2). It is worth noting that it is a function of the atmospheric density, which is a function of radial distance and so it is not constant throughout the decay process. We have taken it into account.

$$\alpha_o = \alpha_{drag} + \alpha_{solar} \quad (Eq 6)$$

With the negative acceleration and mass, we have force, and therefore find the rate of change of orbital energy. The rate of change of orbital energy is simply the rate at which the satellite traverses an infinitesimal arc-length of the circular trajectory. Using the definition of work done and the arc-length formula, we have:

$$\frac{dU}{dt} = \frac{F \cdot ds}{dt} = \frac{F \cdot R \cdot d\theta}{dt} = F \cdot R \cdot \omega \quad (Eq 7)$$

The angular velocity of the satellite is also its mean motion, found using Kepler's Law of Harmonies:

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{GM_E}{R^3}} \quad (Eq 8)$$

The rate of change of orbital energy can therefore be substituted with identities from Equation 6, 7, 8.

$$\frac{dU}{dt} = m\alpha_o \sqrt{\frac{GM_E}{R}} \quad (Eq 9)$$

Having an equation for the rate of change of orbital energy with respect to both radial distance and time allows us to hence find the rate of change of the radial distance with respect to time.

$$\frac{dR}{dt} = \left( \left( \frac{dU}{dR} \right)^{-1} \cdot \frac{dU}{dt} \right) = 2\alpha_o \sqrt{\frac{R^3}{GM_E}} = \frac{\alpha_o T}{\pi} \quad (Eq 10)$$

Our orbital decay model derivation therefore simplifies itself to the following expression below. Realise that from Equation 12 below, both the period and the negative acceleration due to atmospheric drag and solar radiation are not constants. They are functions of the radial distance from the centre of the Earth,  $R$ .

$$\frac{dR}{dt} = \frac{\alpha_o(R) \cdot T(R)}{\pi} \quad (Eq 11)$$

The decay rate model was tested against STK's HPOP software, and actual GPS data of the VELOX-CI, from 16<sup>th</sup> December 2015 to 11<sup>th</sup> November 2016 (that is the 331 days of recorded decay data in the actual data set of VELOX-CI). A tabulation of comparisons show that our decay rate model overestimates the results from STK by about 63.5%, but it is very close to the actual GPS data, short by about 4.75%. We only simulate the decay for 331 days as that is the GPS data we have on hand.

Derived Model on Python	2.444 km
Decay Data from STK Sim	1.495 km
Decay Data from Actual GPS	2.566 km

Table 1: Comparison of Decay Results



Fig 3: Actual GPS data for VELOX-CI satellite on 16/12/2015



Fig 4: Actual GPS data for VELOX-CI satellite on 11/11/2016

The results are not too far off, and our proposed model is accurate enough. It is impossible (and also unnecessary) to achieve fool-proof accuracy – our project just requires a sufficiently accurate model in our program to model decay behaviour as our primary focus is on ascertaining the best orbit maintenance strategies rather than perfecting an orbital decay model. We will now use this decay model for our station-keeping manoeuvres' simulation on Python.

### III. METHOD OF FORCED KEPLERIAN ORBITS

A Keplerian orbit is simply an idealised trajectory of motion between two bodies (specifically, the spacecraft and planet of interest), and thus it is orbit observed in ideal two-body problems.

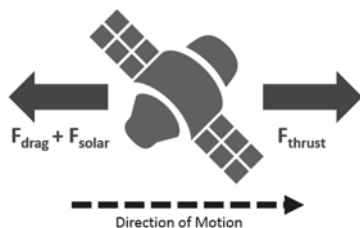


Fig 5: Simple Force Diagram for Forced Keplerian Orbit

In reality, the semi-major axis of the orbit decays primarily due to atmospheric drag and solar radiation pressure. The constant thrust forced Keplerian orbit is an orbital manoeuvre that simple provides an equal but opposite constant thrust force in opposition to orbital decay factors.

$$F_{thrust} = -m(\alpha_{drag} + \alpha_{solar}) = 5.715 \mu N \quad (Eq 12)$$

$$P_{loss} = F_{thrust} \times Velocity = -0.0434W \quad (Eq 13)$$

$$U_{loss} = \int_0^t (P_{loss})dt = -6.84MJ \quad (Eq 14)$$

$$\Delta V_{total} = \int_0^t (\alpha_{drag} + \alpha_{solar})dt = 7.332m/s \quad (Eq 15)$$

In practice, this is not feasible as it is not easy for the satellite engine to calculate out and exert the precise thrust force needed to counteract decay forces in the opposing direction accurately throughout the mission. Furthermore, at the current thrust of 4.768uN, most thrusters including FEEPs and cold gas microthrusters are unable to provide that small resolution of force below 50uN [6]. It is worth taking note of the current  $\Delta V$  value for now, as the relationship with the other orbit maintenance methods will be made clear later on. Therefore, for now we will consider the forced Keplerian orbit as an idealised method for us to benchmark the performance of other orbit maintenance strategies against.

### IV. METHOD OF HOHMANN TRANSFERS

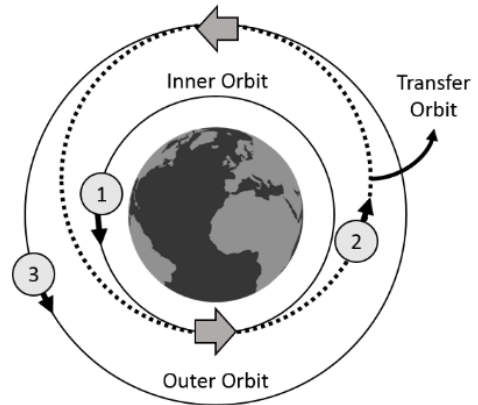


Fig 6: Hohmann Transfer for Maintaining Orbit

A Hohmann Transfer is a fuel efficient orbit manoeuvre that transfers a satellite from one circular orbit to another circular orbit [2]. We will use this method to maintain circular orbit re-iteratively.

The transfer trajectory can be defined simply by the radius of the inner orbit (which in our case is the orbit after the satellite has decayed), and the outer orbit (which in our case is the initial 545km LEO that the simulation

has preset). The semi-major axis of the Hohmann Transfer Orbit is given by:

$$a = \frac{1}{2}(R_{outer} + R_{inner}) \quad (Eq 16)$$

To define the inner orbit, we can define the threshold drop in altitude (as a free parameter) of the satellite before the engines fire.

By varying this threshold drop, we are also varying the radius of the inner orbit before we fire the thrusters. Therefore, the threshold drop in altitude becomes our independent variable.

$$Threshold\ Drop = \Delta a = R_{outer} - R_{inner} \quad (Eq 17)$$

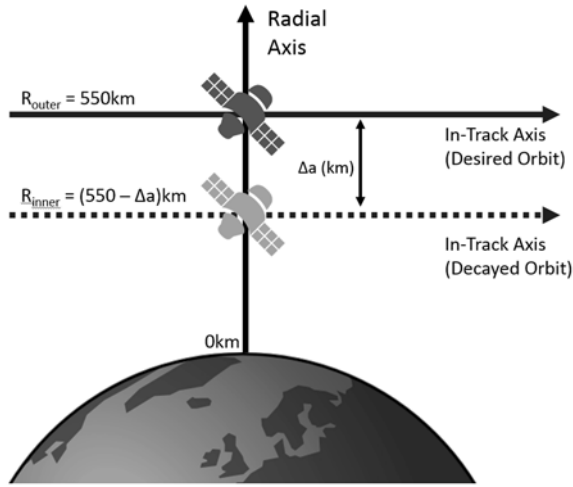


Fig 7: Satellite dips until it crosses boundary of tolerance band  $\Delta a$

To find the required  $\Delta V$ , we need initial and final velocities of the space craft. We use the Vis-Visa Equation, where “r” is the radial position of the satellite and “a” is the instantaneous semi-major axis of orbit:

$$V^2 = GM_E \cdot \left( \frac{2}{r} - \frac{1}{a} \right) \quad (Eq 18)$$

Since the orbit is nearly circular ( $r = a$ ), we can simplify the Vis-Visa expression into:

$$V^2 = \frac{GM_E}{r} \quad (Eq 19)$$

For the initial velocity of our satellite after it has decayed to the point of threshold “ $\Delta a$ ” into the inner orbit:

$$V_{iA} = \sqrt{\frac{GM_E}{R_{inner}}} \quad (Eq 20)$$

The transfer velocity when exiting the inner orbit after the first thrust into the Hohmann Transfer Orbit is:

$$V_{tA} = \sqrt{GM_E \cdot \left( \frac{2}{R_{inner}} - \frac{1}{a} \right)} \quad (Eq 21)$$

The transfer velocity when exiting the Hohmann transfer orbit and entering the required mission orbit, which is the outer orbit B, after the second thruster burn is:

$$V_{tB} = \sqrt{GM_E \cdot \left( \frac{2}{R_{outer}} - \frac{1}{a} \right)} \quad (Eq 22)$$

The final velocity of the satellite in the mission-specific orbit, or the outer orbit, and it is given by:

$$V_{fB} = \sqrt{\frac{GM_E}{R_{outer}}} \quad (Eq 23)$$

The total Delta-V consumed for the entire manoeuvre therefore comprises the scalar sum of two separate burns: one burn for entering the Hohmann Transfer Orbit from the decayed inner orbit at periapsis, and the second burn at the apoapsis performed to “recircularise” the current Hohmann Transfer Orbit into the required mission orbit which is the outer orbit at altitude 545km. The total Delta-V is hence given as:

$$\Delta V_{total} = (|V_{tA} - V_{iA}| + |V_{fB} - V_{tB}|) \quad (Eq 24)$$

The time of transfer of the flight is also accounted for inside our Python code and simulation. By Kepler’s Law of Harmonies, we have the flight duration of the Hohmann Transfer as half the period of the ellipse since the Hohmann trajectory is only half the ellipse:

$$T = \frac{P}{2} = \frac{1}{2} \cdot \sqrt{\frac{4\pi^2 a^3}{GM_E}} \quad (Eq 25)$$

In our Python simulation, each time the threshold drop in altitude exceeds  $\Delta a$ , it will reset the altitude back to 545km and calculate the required  $\Delta V$  per boost. The total  $\Delta V$  is then summed up for the entire duration of the mission for all the boosts. Then, the programme will vary the tolerance band  $\Delta a$  (independent variable), and the algorithm will repeat itself and again record the total  $\Delta V$ . Each solution gives a unique time-of-flight and a unique  $\Delta V$  value for the impulsive manoeuvre used to perform the Hohmann transfer. For example, if we define the threshold drop in altitude “ $\Delta a$ ” as 10km, then the thrusters will only fire when the spacecraft has descended from 545km to 540km, and perform one Hohmann transfer and obtaining the value of one Delta-V for a Hohmann transfer. By varying threshold drop in altitude  $\Delta a$ , we vary the frequency of the thrusters firing, and also the total Delta-V required.

Using Kepler’s Law of Harmonies and our atmospheric decay model, we can figure out the total time of flight for

each Hohmann Transfer, and the period of the satellite at any altitude. We account for all Hohmann transfer flight durations which is the time interval between each transfer as the satellite decays. By defining the total mission duration, we can then easily obtain the total number of Hohmann transfers performed in that mission duration which is depending on our original independent variable “ $\Delta a$ ”. We can then plot out how the total mission required Delta-V Budget varies according to the changes in threshold “ $\Delta a$ ”.

However, one limitation with the Hohmann transfers method, is that the error in the in-track axis accumulates over time as our satellite is always less than or equal to the desired altitude, and thus linear velocity is always greater than or equal to the desired value. As a result, the ground-track of the satellite will constantly change. To correct this, we propose a method where the independent variable is the in-track tolerance band instead.

## V. METHOD OF CYCLICAL DIRECT BURNS

The cyclical direct burn manoeuvres was simulated with the in-track axis tolerance band as the independent variable. This method assumes the position of our actual satellite against some reference imaginary satellite F that is indeed flying in an ideal Keplerian orbit at the mission-specific 545km altitude [3]. Our reference frame is the imaginary satellite F positioned at the origin in Fig 6.

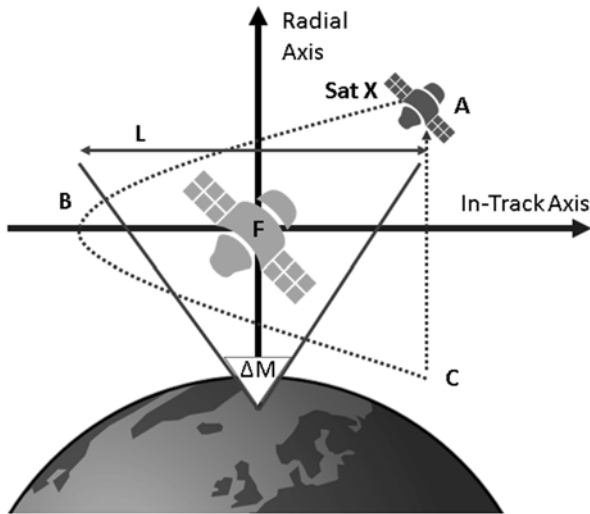


Fig 8: Motion of the original satellite against imaginary satellite F in a reference frame relative to F

Our actual satellite X follows a non-Keplerian perturbed orbit where the rate of decay is finite due to the presence of atmospheric drag and solar radiation pressure from Equations 1 and 2. The reference satellite F follows an ideal Keplerian orbit with no perturbations. Our actual Satellite X will try to follow the ideal reference ghost satellite F using this algorithm [3]:

1. At Point A, our satellite is at a higher altitude, and therefore will lag behind Reference F, according to the Vis-Visa Equation. Simultaneously, the altitude

will also decay due to drag and solar radiation. This causes Satellite X to move from Point A to B.

2. At Point B, our satellite has the same linear velocity as Reference F. However, it will only stay at Point B for an instant, because orbital decay is continuous. It continues to drop in altitude, and this time it will lead ahead of Reference F, according to the Vis-Visa Equation, until it reaches Point C, where Points A and C both have the same In-Track Axis value.
3. At Point C, our satellite performs an impulsive thrust with a Delta-V equivalent to the difference between the velocity at Points A and C, and directing forward in-line with its current trajectory. This increase in velocity results in an increase in altitude back to Point A. The cycle then repeats.

In our previous algorithm on the Hohmann Transfer approach against radial axis deviation, we varied the threshold “ $\Delta a$ ”. For the direct burn approach, since we are more interested in orbit maintenance against the in-track axis deviation, our control variable is now the in-track tolerance L (see Fig 6). The following equations will comprise the mathematics of the algorithm used in one single iteration of this. We assume from our orbital decay model derived, that for a small tolerance band, the decay is roughly constant as long as we do not deviate too far from the orbit altitude we are trying to maintain.

$$\frac{dR}{dt} = \frac{\alpha_o(R) \cdot T(R)}{\pi} \approx -K \quad (\text{Eq 26})$$

The mean motion of can be expressed as a function of its radial distance according to Kepler’s Law of Harmonies.

$$n = \frac{2\pi}{T} = \sqrt{\frac{GM_E}{R^3}} \quad (\text{Eq 27})$$

The rate of change of mean motion with respect to its radial distance is found through differentiation.

$$\frac{dn}{dR} = -\frac{3}{2} \cdot \sqrt{\frac{GM_E}{R^5}} = -\frac{3n}{2R} \quad (\text{Eq 28})$$

With equations 26 and 28, we can get an expression for the rate of change of mean motion with respect to time.

$$\frac{dn}{dt} = \frac{dn}{dR} \cdot \frac{dR}{dt} = \frac{3Kn}{2R} \quad (\text{Eq 29})$$

For a sufficiently small in-track tolerance, the mean motion approximately changes linearly with time.

$$\Delta n = \frac{3Kn}{2R} \cdot \Delta t \quad (\text{Eq 30})$$

The angular displacement experienced by Satellite X from Point A to Point B can be expressed as an integral of the angular velocity or mean motion from Equation 30, with respect to time where:

$$\Delta M = \int_0^t \Delta n dt = \frac{3Kn}{2R} \int_0^t (\tau - \tau_o) dt \quad (Eq 31)$$

Suppose the initial time where the measurement was taken  $\tau_o = 0$ , then this simplifies our expression for  $\Delta M$ :

$$\Delta M = \frac{3Kn}{4R} \cdot t^2 \quad (Eq 32)$$

We have another identity for the angular displacement  $\Delta M$  in the frame of reference about Satellite F. Using the arc-length formula, we approximate  $\Delta M$  in the identity:

$$L = R \cdot \Delta M \quad (Eq 33)$$

$L$  is our pre-defined in-track deviation threshold that we can set as a free parameter (and in fact that is what our algorithm should be updating when looping in sequential iterations), and radial distance  $R$  is approximately constant since our actual Satellite X oscillates about the desired altitude (which in our case is 545km).

This means that  $\Delta M$  is actually already pre-defined. So why did we go through the hassle of deriving the other identity shown in Equation 32? The caveat here is that by equating the equations 32 and 33, we can obtain the time of flight  $T_M$  between each impulsive thrusts; and knowing the time of flight tells us a lot of other information too, such as what the drop in altitude is for the satellite in-between thrusting manoeuvres.

$$T_M = 2t = 4 \cdot \sqrt{\frac{L}{3nK}} \quad (Eq 34)$$

The total drop in altitude from Point A to Point C on Figure 6 is therefore simply the orbit decay rate multiplied by the total time of flight  $T_M$ .

$$\Delta R = \frac{dR}{dt} \cdot T_M = -4 \cdot \sqrt{\frac{LK}{3n}} \quad (Eq 35)$$

Finally, we can obtain the Delta-V for this manoeuvre simply by taking this approximation:

$$\Delta V \approx \frac{1}{2} n \Delta R \quad (Eq 36)$$

Again, given that we have the time of flight between each manoeuvre, and that we can calculate the period of the satellite orbit using Kepler's Law of Harmonies, we may then find out how many manoeuvres are performed in one period, or perhaps how many manoeuvres are required in total for the entire mission duration. With the

total number of manoeuvres and the Delta-V required per manoeuvre, we can calculate out total Delta-V. Finally, this algorithm will be repeated again for various in-track thresholds  $L$ , where we will tabulate out all the possible total Delta-V's that are unique to each in-track threshold that we set. We would then analyse the data.

## VI. RESULTS AND DISCUSSIONS

We have discussed the three orbit maintenance methods used in this paper study. In summary, we have:

1. The Method of Forced Keplerian Orbits
2. The Method of Hohmann Transfers
3. The Method of Cyclical Direct Burns

We will now do a comparison between the results of each method. For the Method of Forced Keplerian Orbits, the total  $\Delta V$  is simply  $= 7.332$  m/s for the full duration of the 5-year mission, and there is no independent variable involved.

$$\Delta V_{total} = \int_0^t (\alpha_{drag} + \alpha_{solar}) dt = 7.332 \text{ m/s} \quad (Eq 37)$$

We now look at the latter two methods, where the  $\Delta V$  is plotted against their tolerance bands.

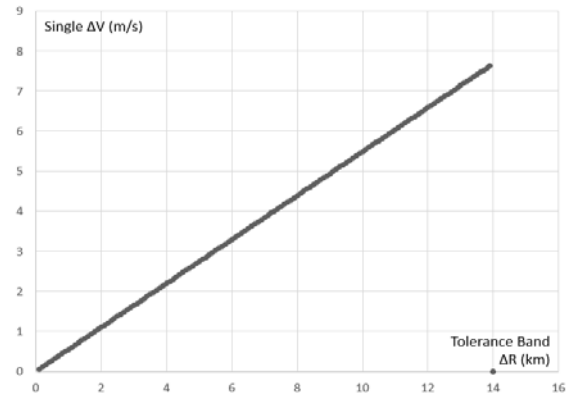


Fig 9: Single  $\Delta V$  of single Hohmann transfer, against radial axis tolerance band (km)

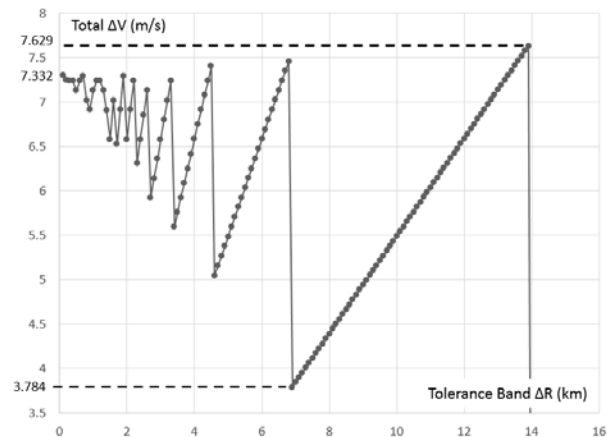
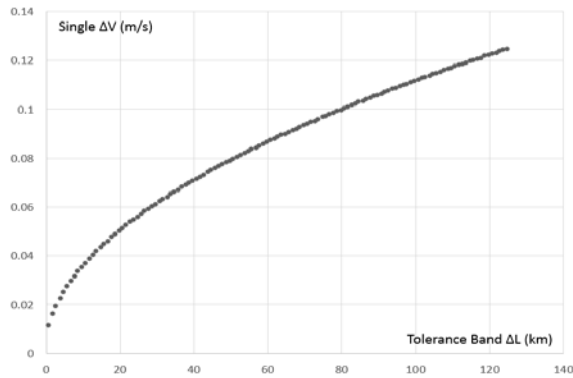
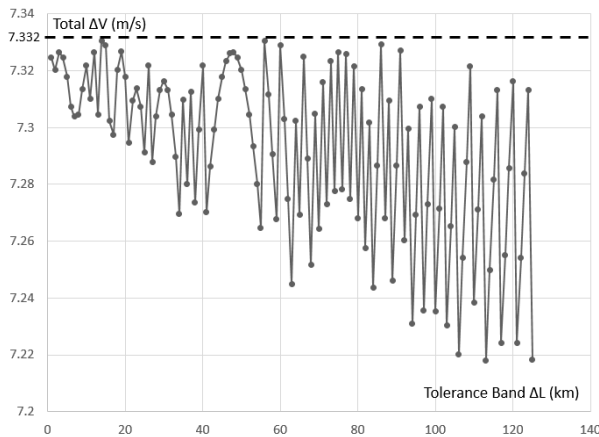


Fig 10: Total  $\Delta V$  of all Hohmann transfers used in 5-year long mission, against radial axis tolerance band (km)





**Fig 11: Single  $\Delta V$  of single direct burn, against the in-track axis tolerance band (km)**



**Fig 12: Total  $\Delta V$  of all cumulative, cyclical direct burns, used in 5-year long mission, against in-track axis tolerance band (km)**

The graphs of interest would be Figures 8 and 10, which comprise the total  $\Delta V$  of Methods 2 and 3 respectively. Interestingly enough, as the tolerance bands (radial axis for Method 2, and in-track axis for Method 3) decreases to zero, the resultant total  $\Delta V \rightarrow 7.332$  m/s, which is exactly the  $\Delta V$  usage of Method 1's Forced Keplerian Orbit, where the total  $\Delta V$  accrues from a constant thrust force. This is understandable since having a tolerance band  $\rightarrow 0$  implies that the frequency of thruster fire is so high that the thruster fire seemingly becomes like a "constant DC" output, instead of periodic thrusts.

For the method of Hohmann transfers, the resultant  $\Delta V$  values diverge from the origin  $\Delta V$  value of 7.332 m/s as the tolerance band increases, oscillating above and below the origin. This implies that for a given mission duration, we may analytically find a global minimum value for total  $\Delta V$ , thus optimising our budget allocation. The cost savings at the global minima of the results from the Hohmann transfer's method is much lower than the global minima of the method of cyclical direct burns.

For the method of cyclical direct burns, likewise, the resultant  $\Delta V$  diverges from the origin value of 7.332 m/s. However, notice that  $\Delta V = 7.332$  m/s forms an asymptotic limit for the  $\Delta V$  usage using this method. Varying the tolerance bands along the in-track axis  $\Delta L$  outputs values that do not exceed this asymptote.

Notably, for Figures 8 and 10, the value of total  $\Delta V$  experiences many jump discontinuities. Total  $\Delta V$  declines for several iterations until at some local minimum point it 'jumps' back to a value at or near 7.332 m/s in the next discrete step. Some manual calculations on hand reveal the reason why – it has to do with the duration of the mission's simulation of 5 years (or for any duration for that matter). Suppose with a fixed simulation duration  $T$ , and for some tolerance band, the time of flight of the space craft before it reaches the tolerance band boundary, fires thrusters, and enters the mission orbit again is  $\tau$ . The total number of thruster burns is given by the rounded floor value of  $(T/\tau)$ . This is an integer number  $N$ . As the tolerance band increases further, it will reach a value where the integer  $N$  increases by 1 to  $(N+1)$ , and thus it fires thrusters once again nearing the end of the simulation without falling much further in the given tolerance band. This final burn results in a discrete 'jump' observed in total  $\Delta V$  value.

## VII. CONCLUSION

Overall, the results indicate that there is potential for minimizing the  $\Delta V$  budget used in orbit maintenance using either Hohmann transfers or cyclical direct burns. The global minima observed in the method of Hohmann transfers is observably lower than that of the method of cyclical direct burns. This is because the Hohmann transfer does not take into account the need to maintain the ground-track of the satellite. However, if there is a need to maintain the said ground track, using the cyclical direct burns option would be preferred, although the cost-savings in  $\Delta V$  would not be as much with the widening of the in-track axis tolerance band. Thus, one important consideration for the mission planner is whether the maintenance of the ground track is essential at the cost of higher  $\Delta V$ . A summary of this study's findings is tabulated in the summary table below.

Strategy	Pros	Cons
Forced Keplerian	Simple method.	Unfeasible, unless high precision-thrusters are available and perturbing forces are well quantified in satellite.
Hohmann Transfers	Delta-V efficient, greater potential for minimisation. Simple algorithm to implement.	Does not compensate for the in-track lead-lag position and timing. Ground track deviates over time. Time of flight between thruster fire must be accurately determined.
Cyclical Direct Burns	Able to perform station-keeping along both radial and in-track axis, as if space craft were following a Keplerian trajectory.	Generally higher in Delta-V usage, and hence less efficient. Time of flight between thruster fire must be accurately determined.

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