Adaptive Control of the Quanser 3DOF Hover

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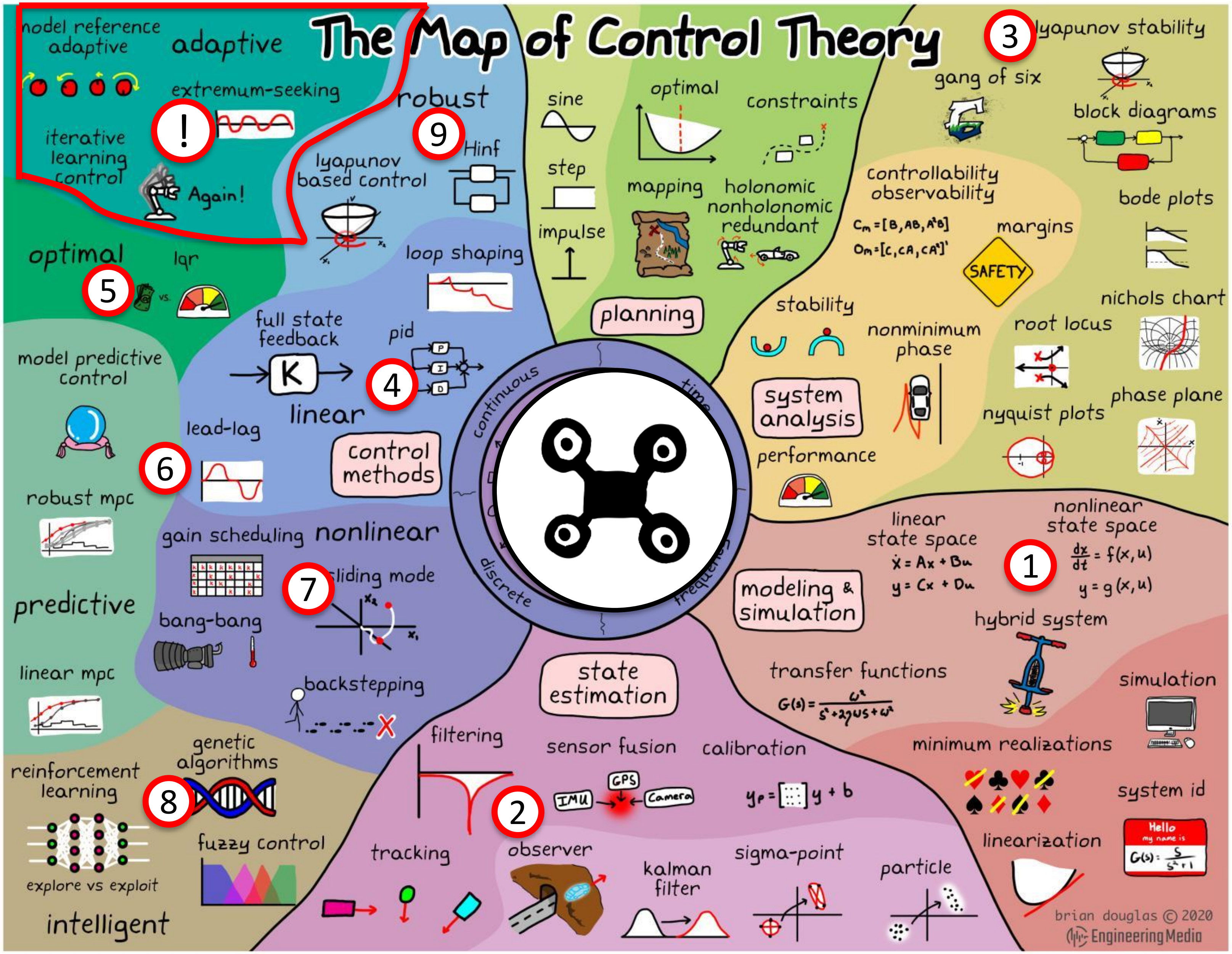

The Map of Control Theory
model reference adaptive
adaptive
extremum-seeking
iterative learning control
Again!
robust
Hinf
lyapunov based control
loop shaping
optimal
lqr
full state feedback
K
pid
P
I
D
linear
lead-lag
control methods
model predictive control
robust mpc
gain scheduling
nonlinear
sliding mode
predictive
bang-bang
backstepping
linear mpc
reinforcement learning
genetic algorithms
fuzzy control
explore vs exploit
intelligent
sine
step
impulse
optimal
constraints
mapping
holonomic
nonholonomic
redundant
planning
lyapunov stability
gang of six
block diagrams
controllability
observability
$C_m = [B, AB, A^2B]$
$O_m = [C, CA, CA^2]'$
margins
SAFETY
bode plots
nichols chart
stability
nonminimum phase
root locus
system analysis
phase plane
nyquist plots
performance
continuous
time
discrete
frequency
linear state space
$\dot{x} = Ax + Bu$
$y = Cx + Du$
nonlinear state space
$\frac{dx}{dt} = f(x,u)$
$y = g(x,u)$
modeling & simulation
hybrid system
transfer functions
$G(s) = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}$
simulation
minimum realizations
system id
Hello my name is
$G(s) = \frac{s}{s^2+1}$
linearization
state estimation
filtering
sensor fusion
IMU
GPS
Camera
calibration
$y_p = [\,]\, y + b$
tracking
observer
kalman filter
sigma-point
particle
brian douglas © 2020
Engineering Media
3
1
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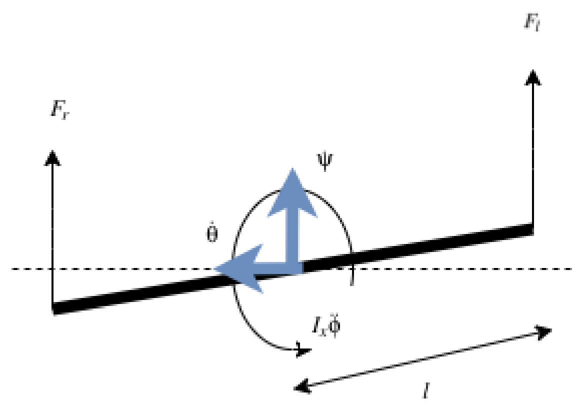

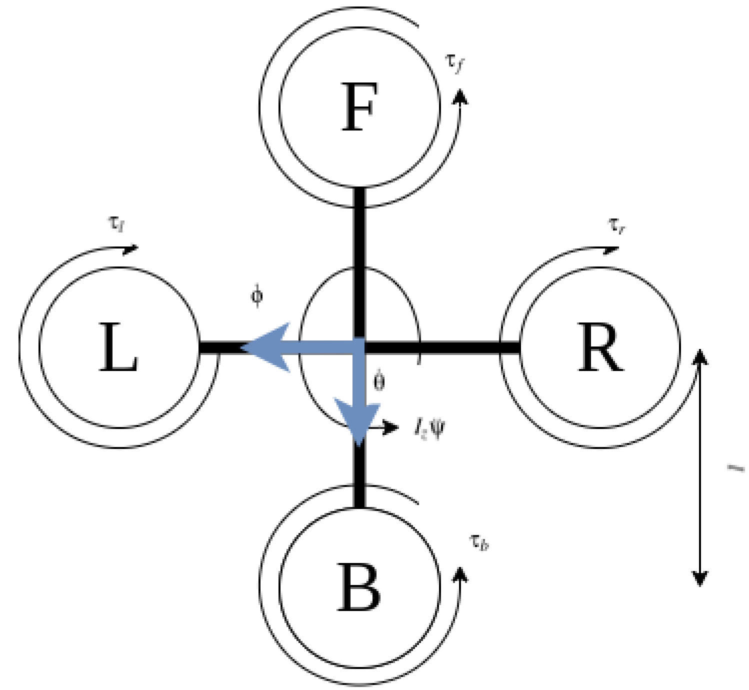

Euler Equation: $$I\frac{d\omega}{dt} + \omega \times I\omega = \tau$$

$$I\begin{bmatrix}\ddot{\phi}\\ \ddot{\theta}\\ \ddot{\psi}\end{bmatrix} + \begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} \times I\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} = \begin{bmatrix}l(F_r - F_l)\\ l(F_f - F_b)\\ \tau_l + \tau_r - \tau_f - \tau_b\end{bmatrix}$$

where

$$F = b\Omega^2$$

$$\tau = d\Omega^2$$

Choosing the moment of inertia matrix to be:

$$I = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}$$

The following three equations are obtained:

$$\begin{cases} \ddot{\phi} = \frac{I_y - I_z}{I_x}\dot{\theta}\dot{\psi} + \frac{\tau_x}{I_x} + \frac{\tau_{wx}}{I_x} \\ \ddot{\theta} = \frac{I_z - I_x}{I_y}\dot{\theta}\dot{\phi} + \frac{\tau_y}{I_y} + \frac{\tau_{wy}}{I_y} \\ \ddot{\psi} = \frac{I_x - I_y}{I_z}\dot{\theta}\dot{\phi} + \frac{\tau_z}{I_z} + \frac{\tau_{wz}}{I_z} \end{cases}$$

System is linearized around the equilibrium point:

$$\phi = \dot{\phi} = \theta = \dot{\theta} = \psi = \dot{\psi} = 0$$

Choosing the states, inputs, and disturbances to be:

$$x^T = [\phi, \quad \dot{\phi}, \quad \theta, \quad \dot{\theta}, \quad \psi, \quad \dot{\psi}]$$

$$u^T = [\tau_x, \quad \tau_y, \quad \tau_z]$$

$$d^T = [\tau_{wx}, \quad \tau_{wy}, \quad \tau_{wz}]$$

The system can be represented in state space as follows:

$$\dot{x} = Ax + Bu + Dd$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{I_x} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{I_y} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{I_z} \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{I_x} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{I_y} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{I_z} \end{bmatrix}$$

CL poles for each decoupled subsystem are chosen based on 20% overshoot and settling time of 1s.

$$\begin{bmatrix}\dot{\phi}\\ \ddot{\phi}\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}\phi\\ \dot{\phi}\end{bmatrix} + \begin{bmatrix}0 & 0 & 0\\ \frac{1}{I_x} & 0 & 0\end{bmatrix} u \longrightarrow K_\phi = \begin{bmatrix}2.527 & 0.442\\ 0 & 0\\ 0 & 0\end{bmatrix}$$

$$\begin{bmatrix}\dot{\theta}\\ \ddot{\theta}\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\end{bmatrix} + \begin{bmatrix}0 & 0 & 0\\ 0 & \frac{1}{I_y} & 0\end{bmatrix} u \longrightarrow K_\theta = \begin{bmatrix}0 & 0\\ 2.527 & 0.442\\ 0 & 0\end{bmatrix}$$

$$\begin{bmatrix}\dot{\psi}\\ \ddot{\psi}\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}\psi\\ \dot{\psi}\end{bmatrix} + \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & \frac{1}{I_z}\end{bmatrix} u \longrightarrow K_\psi = \begin{bmatrix}0 & 0\\ 0 & 0\\ 5.036 & 0.88\end{bmatrix}$$

The gains for each subsystem are concatenated to produce system gain

$$K = \begin{bmatrix}2.527 & 0.442 & 0 & 0 & 0 & 0\\ 0 & 0 & 2.527 & 0.442 & 0 & 0\\ 0 & 0 & 0 & 0 & 5.036 & 0.88\end{bmatrix}$$

Discretized Process/Plant:

$$H_\phi(q) = \frac{B_\phi(q)}{A_\phi(q)} = \frac{\frac{1}{2I_x}h^2 q + \frac{1}{2I_x}h^2}{q^2 - 2q + 1} = \frac{b_{0_\phi} q + b_{1_\phi}}{q^2 + a_1 q + a_2}$$

Desired Process using $\zeta = 0.7$ and $\omega = 1$:

$$G_\phi(q) = \frac{B_m(q)}{A_m(q)} = \frac{0.1761q}{q^2 - 1.3205q + 0.4966} = \frac{b_{m0}q}{q^2 + a_{m1}q + a_{m2}}$$

2DOF Controller:

$$R_\phi \tau_x(t) = T_\phi u_c(t) - S_\phi \phi(t)$$

Diophantine Equation:

1) $B_\phi = B_\phi^+ B_\phi^-$, where $B_\phi^- = b_{0_\phi}$ and $B_\phi^+ = B_\phi / b_{0_\phi}$

2) $deg(A_{0_\phi}) = deg(A_\phi) - deg(B_\phi) - 1 = 0$

3) Let $A_{0_\phi} = 1$

4) $B'_{m_\phi} = \frac{A_m(1) q^{n-d0}}{b_{0_\phi}} = \frac{b_{m0}}{b_{0_\phi}} q$

5) $T_\phi = A_{0_\phi} B'_{m_\phi} = \frac{b_{m0}}{b_{0_\phi}} q$

6) $deg(T_\phi) = deg(R_\phi) = deg(S_\phi) = 1$

7) Let $S_\phi = s_{0_\phi} q + s_{1_\phi}$, and $R'_\phi = 1$
(since R'_ϕ is monic and of degree zero)

8) $R_\phi = R'_\phi B^+ = q + \frac{b_{1_\phi}}{b_{0_\phi}} = q + 1$

Diophantine Equation to find S_ϕ :

$$A_\phi R_\phi + B_\phi S_\phi = A_{0_\phi} A_m \longrightarrow S_\phi = \frac{a_{m1} - a_1}{b_{0_\phi}} q + \frac{a_{m2} - a_2}{b_{0_\phi}}$$

Controller Polynomial are:

$$\begin{cases} T_\phi = \frac{b_{m0}}{b_{0_\phi}} q \\ R_\phi = q + \frac{b_{1_\phi}}{b_{0_\phi}} \\ S_\phi = \frac{a_{m1} - a_1}{b_{0_\phi}} q + \frac{a_{m2} - a_2}{b_{0_\phi}} \end{cases}$$

Controller Design – MRAC Full State Feedback

System

$$\begin{bmatrix}\dot\phi\\ \ddot\phi\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}\phi\\ \dot\phi\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{I_x}\end{bmatrix}\tau_x$$

$$\begin{bmatrix}\dot x_1\\ \dot x_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{I_x}\end{bmatrix}u$$

Reference

$$G_m(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} = \frac{bm0}{s^2+a_{m1}s+a_{m2}}$$

$$\begin{bmatrix}\dot x_{m_1}\\ \dot x_{m_2}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -a_{m_2} & -a_{m_1}\end{bmatrix}\begin{bmatrix}x_{m_1}\\ x_{m_2}\end{bmatrix} + \begin{bmatrix}0\\ b_{m_0}\end{bmatrix}u$$

Control Law

$$u(t) = Mu_c(t) - Lx$$

$$\begin{cases} M = \theta_1 \\ L = (\theta_2 \quad \theta_3)^T \end{cases} \qquad \begin{cases} \theta_1^0 = I_x b_{m0} \\ \theta_2^0 = I_x a_{m2} \\ \theta_3^0 = I_x a_{m1} \end{cases}$$

Error Dynamics

$$\left.\begin{aligned} \tilde\theta &= \theta - \theta^0 \\ \psi &= \begin{bmatrix}0 & 0 & 0\\ \frac{u_c}{I_x} & -\frac{x_1}{I_x} & -\frac{x_2}{I_x}\end{bmatrix}\end{aligned}\right\}$$

$$\dot e = A_m e + \psi\tilde\theta$$

Lyapunov Function

$$V(e,\theta) = \frac{1}{2}e^TPe + \frac{1}{2}\tilde\theta^T\Gamma^{-1}\tilde\theta$$

Barbalat's Lemma

$$\dot V = -\tfrac{1}{2}e^TQe \le 0$$

$$e(t) \to 0 \text{ as } t \to \infty$$

Adaptation Law

$$\frac{d\tilde\theta}{dt} = -\Gamma\Psi^TPe$$

System

$$G(s) = \frac{1}{I_x s^2} = \frac{b_0 B}{A}$$

$$b_0 = \frac{1}{I_x}, \qquad B = 1, \qquad A = s^2$$

Reference

$$G_m(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Control Law

$$Ru = Tu_c - Sy$$

$$\left\{\begin{aligned} A_0(s) &= s + a_0 \\ S(s) &= s_0 s + s_1 \\ R(s) &= R_1 B = R_1 = s + r_1 \\ T(s) &= t_0 A_0 = t_0 s + t_0 a_0 = t_0 s + t_1 \\ Q &= A_0 A_m = (s + a_0)(s^2 + 2\zeta\omega_n s + \omega_n^2) \\ P &= P_1 P_2 = A_0 A_m \end{aligned}\right.$$

Diophantine Equation

$$AR_1 + b_0 S = A_o A_m$$

Controller Design – MRAC Output Feedback

Filtered Error

$$e_f = \frac{Q}{P} e = e$$
$$= y - y_m$$

Error Augmentation

$$\eta = -(\frac{1}{P_1} u(t) + \varphi^T \theta)$$
$$= -\frac{1}{A_m} u - \varphi^T \theta$$

$$\begin{cases} \hat{\theta} = \begin{bmatrix} r'_1 \\ \hat{s_0} \\ \hat{s_1} \\ \hat{t_0} \\ \hat{t_1} \end{bmatrix} \\ \varphi = \frac{1}{P} \begin{bmatrix} u \\ sy \\ y \\ -su_c \\ -u_c \end{bmatrix} \end{cases}$$

Control Input

$$u = -\theta^T (P_1 \varphi)$$

Augmented Error

$$\varepsilon = e_f + \hat{b_0} \eta$$

Adaptation Laws

$$\begin{cases} \dot{\hat{b_0}} = -\gamma_1 \eta \varepsilon \\ \dot{\hat{\theta}} = +\gamma'_2 \varphi \varepsilon \end{cases}$$

Estimated Disturbance

$$\dot{\hat{d}} = +\gamma_d e$$

↓

Updated Control Input

$$u_d(t) = u(t) - \hat{d}$$

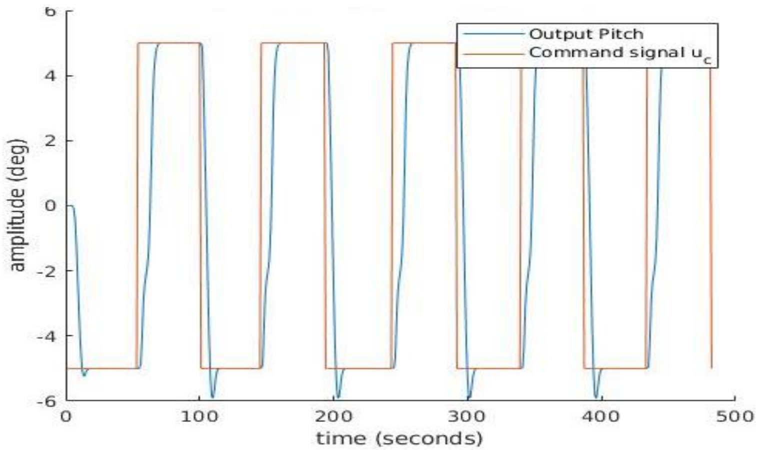

Fig. 1. Plot showing the reference command input uc, and the actual output y for the Pitch/ Roll with Proportional Controller

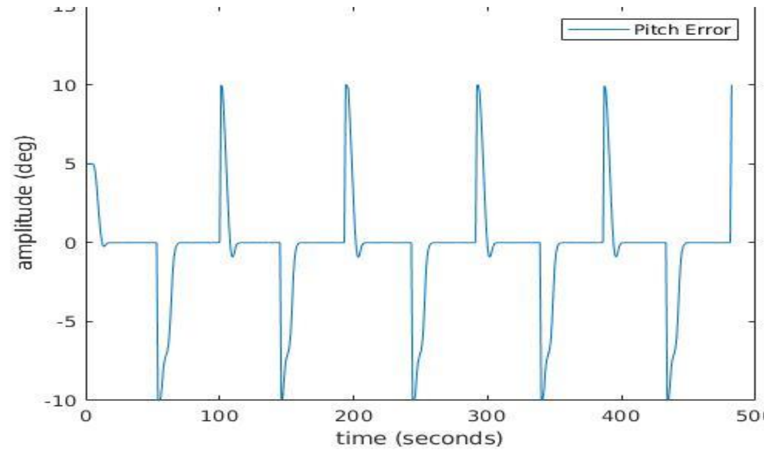

Fig. 3. Plot showing the parameter estimates and their actual values for the Pitch/ Roll with Proportional Controller

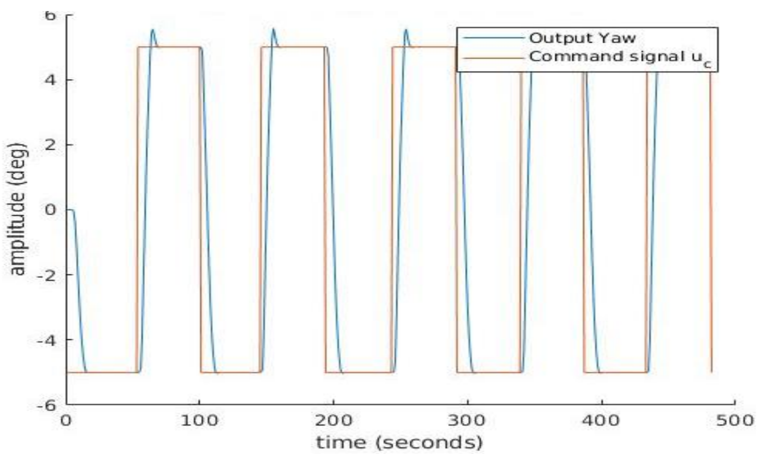

Fig. 2. Plot showing the reference command input uc, and the actual output y for the Yaw with Proportional Controller

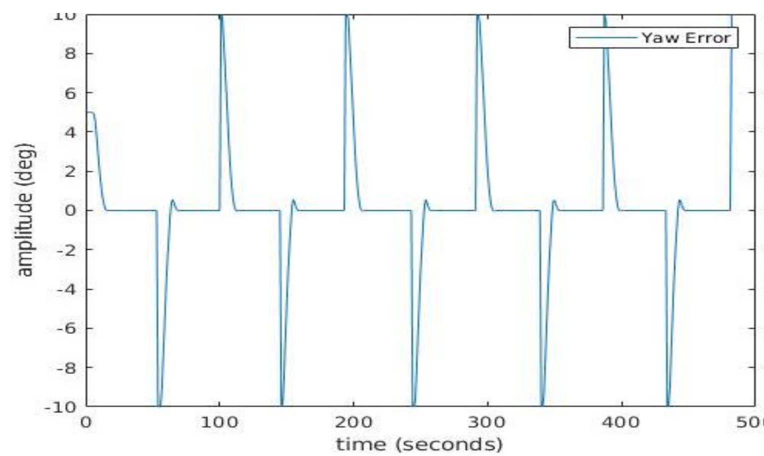

Fig. 4. Plot showing the parameter estimates and their actual values for the Yaw with Proportional Controller

Simulation Results – Indirect Self-Tuning Regulator

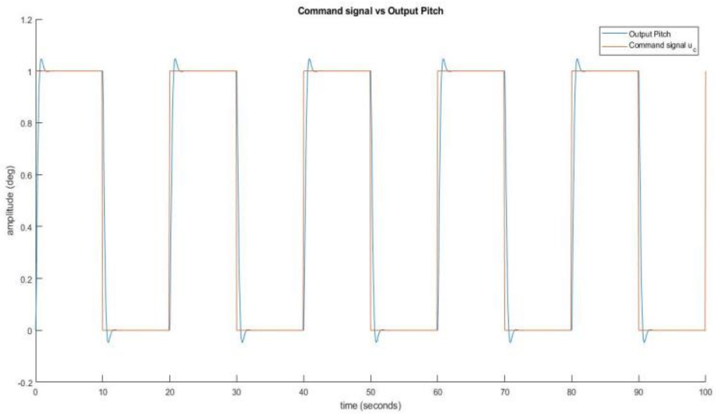

Fig. 1. Plot showing the reference command input uc, and the actual output y for the Pitch/ Roll with ISTR.

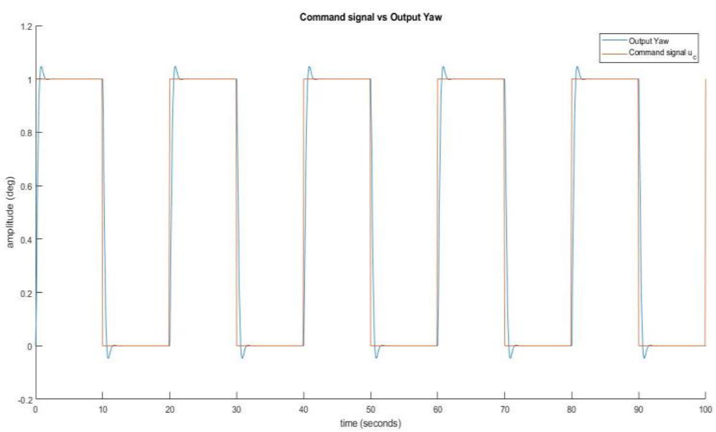

Fig. 2. Plot showing the reference command input uc, and the actual output y for the Yaw with ISTR.

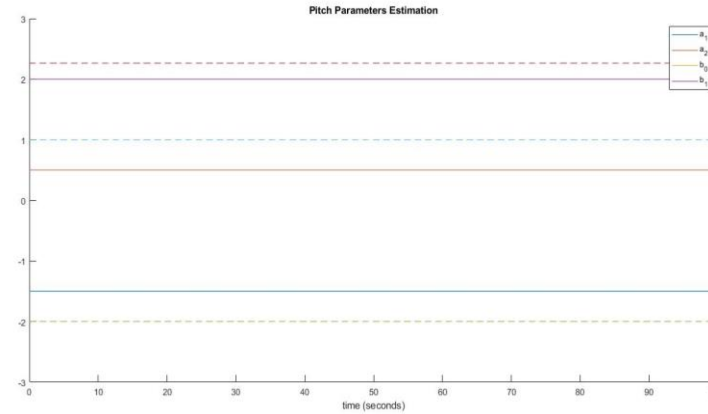

Fig. 3. Plot showing the parameter estimates and their actual values for the Pitch/ Roll with ISTR.

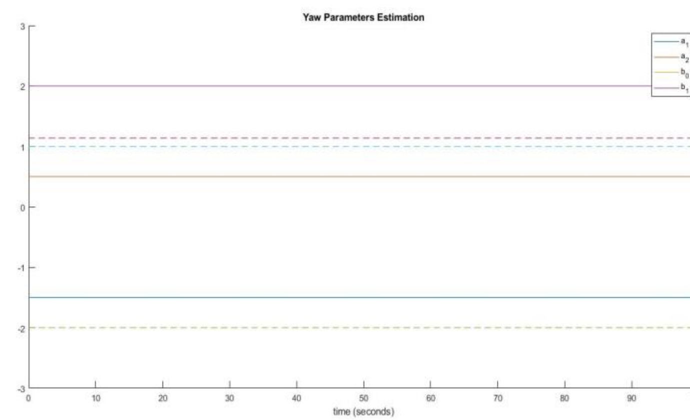

Fig. 4. Plot showing the parameter estimates and their actual values for the Yaw with ISTR.

Simulation Results – MRAC Full State Feedback

Variable	Value
Sampling frequency	$2kHz$
I_{x0}	$0.04kgm^2$
I_{y0}	$0.04kgm^2$
I_{z0}	$0.8kgm^2$
%overshoot	20%
Settling Time	$5s$
Q	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
$\gamma_{pitch}, \gamma_{roll}$	$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$
γ_{yaw}	$\begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$

Controller Initialization for MRAC Full State Feedback.

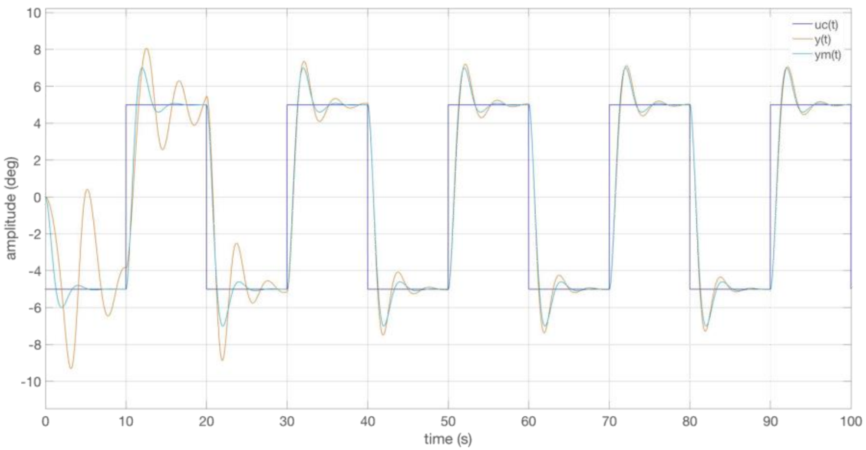

Fig. 1. Plot showing the reference command input uc, the reference output ym, and the actual output y for the Pitch with MRAC Full State Feedback.

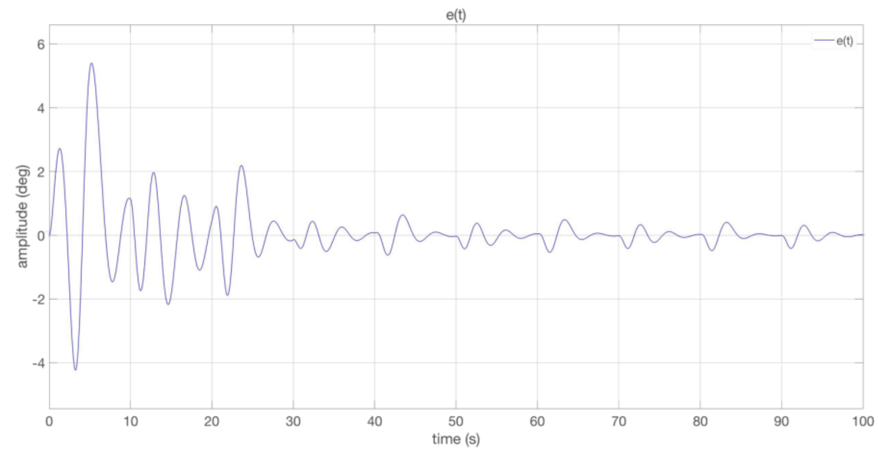

Fig. 3. Plot showing the output tracking error e=y-ym for the Pitch with MRAC Full State Feedback.

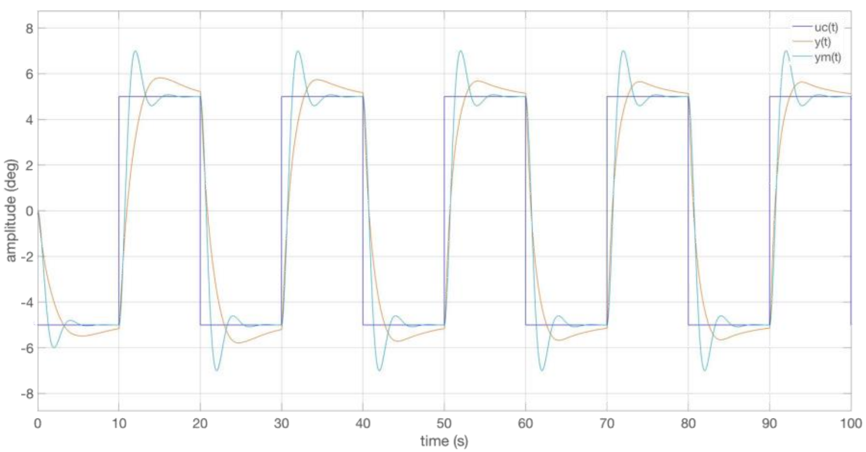

Fig. 2. Plot showing the reference command input uc, the reference output ym, and the actual output y for the Yaw with MRAC Full State Feedback.

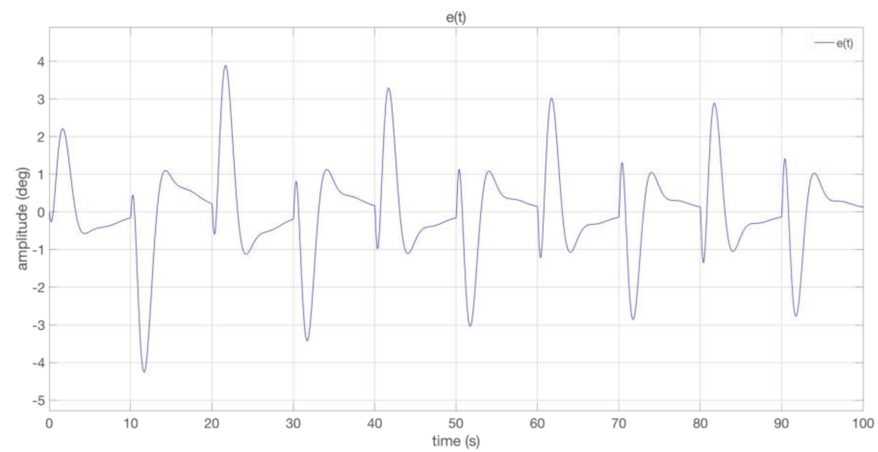

Fig. 4. Plot showing the output tracking error e=y-ym for the Yaw with MRAC Full State Feedback.

Simulation Results – MRAC Full State Feedback

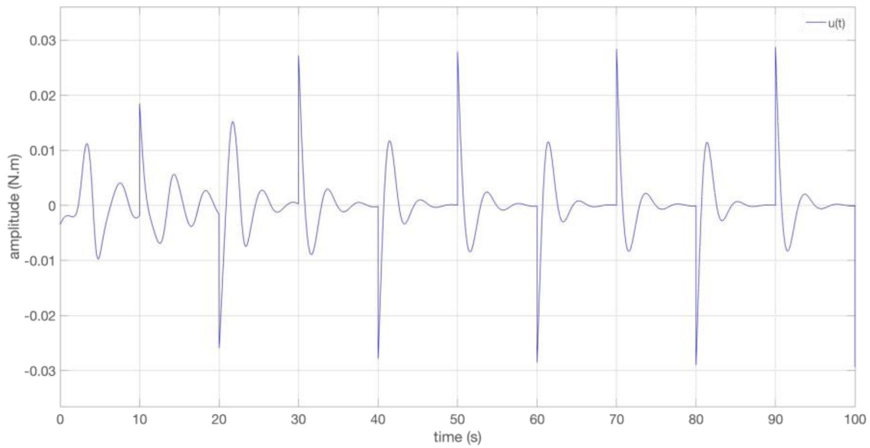

Fig. 1. Plot showing the control input u for the Pitch with MRAC Full State Feedback.

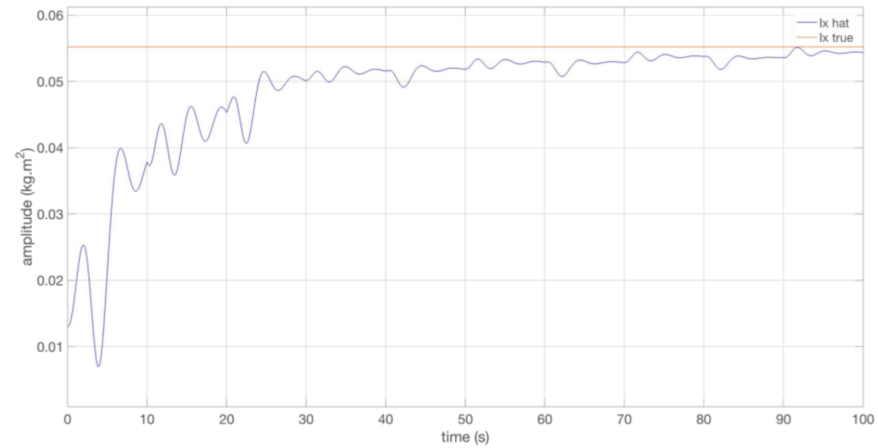

Fig. 3. Plot showing the parameter estimates and their actual values for the Pitch with MRAC Full State Feedback.

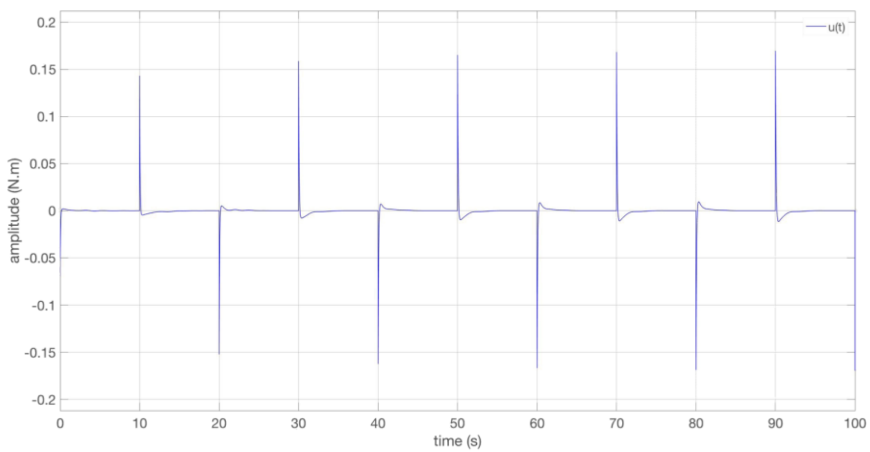

Fig. 2. Plot showing the control input u for the Yaw with MRAC Full State Feedback.

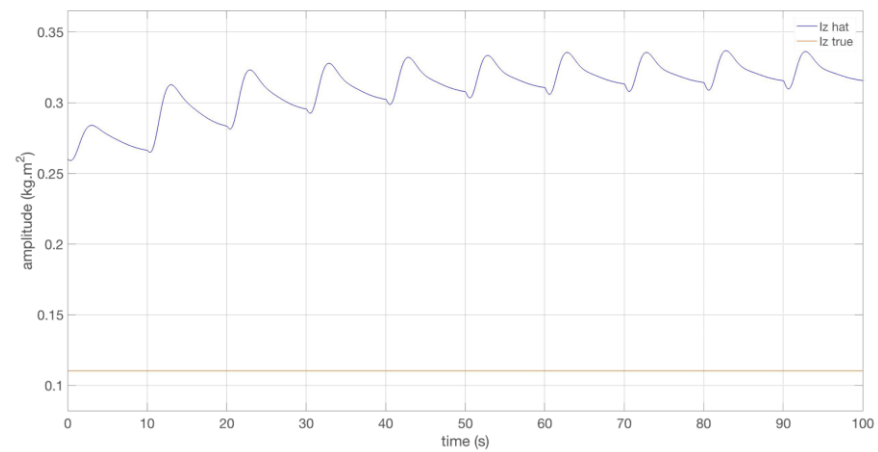

Fig. 4. Plot showing the parameter estimates and their actual values for the Yaw with MRAC Full State Feedback.

Simulation Results – MRAC Output Feedback

Variable	Value
Sampling frequency	$2kHz$
I_{x0}	$0.04kgm^2$
I_{y0}	$0.04kgm^2$
I_{z0}	$0.8kgm^2$
%overshoot	20%
Settling Time	$5s$
a_{0pitch}, a_{0roll}	4
a_{0yaw}	1
γ_{1pitch}, γ_{1roll}, γ_{1yaw}	10
γ_{2pitch}, γ_{2roll}, γ_{2yaw}	10

Controller Initialization for MRAC Output Feedback.

Simulation Results – MRAC Output Feedback

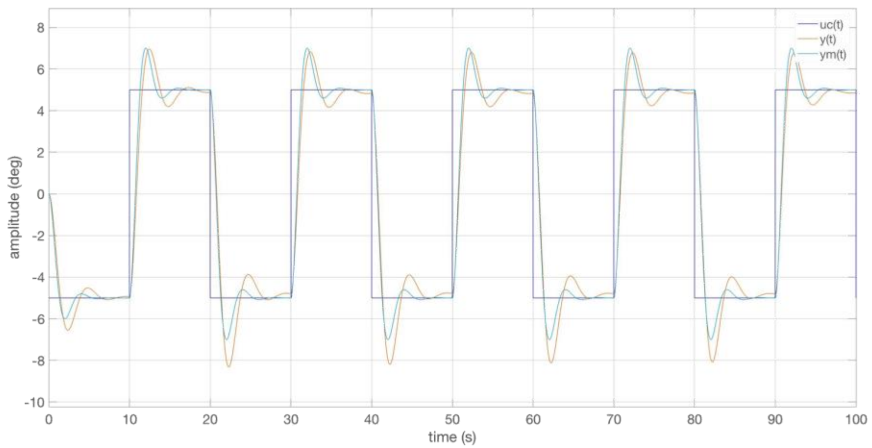

Fig. 1. Plot showing the reference command input uc, the reference output ym, and the actual output y for the Pitch with MRAC Output Feedback.

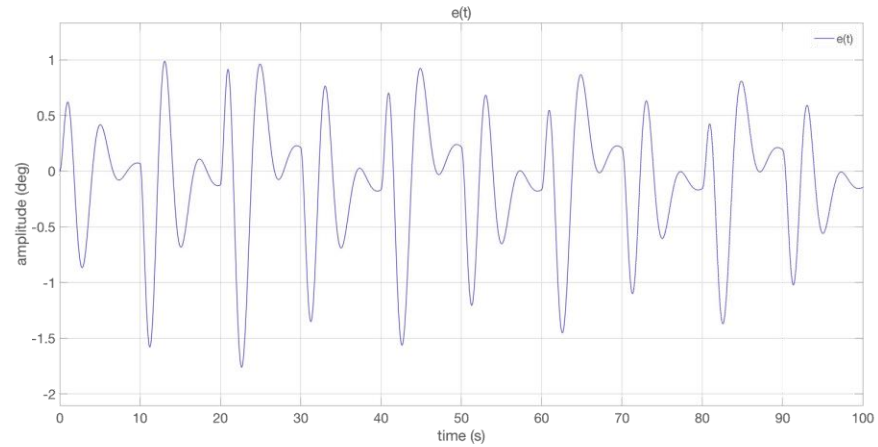

Fig. 3. Plot showing the output tracking error e=y-ym for the Pitch with MRAC Output Feedback.

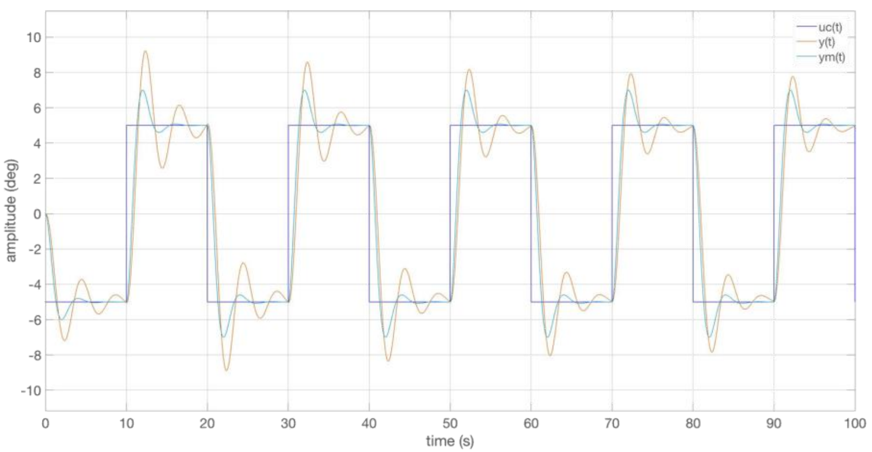

Fig. 2. Plot showing the reference command input uc, the reference output ym, and the actual output y for the Yaw with MRAC Output Feedback.

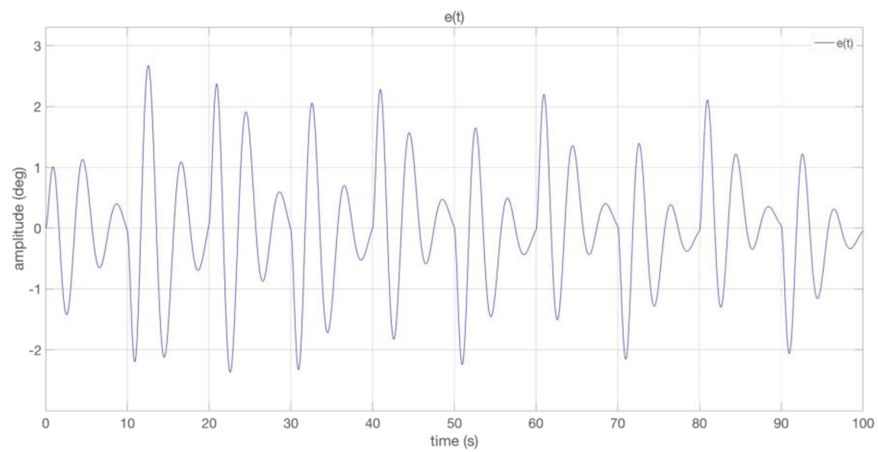

Fig. 4. Plot showing the output tracking error e=y-ym for the Yaw with MRAC Output Feedback.

Simulation Results – MRAC Output Feedback

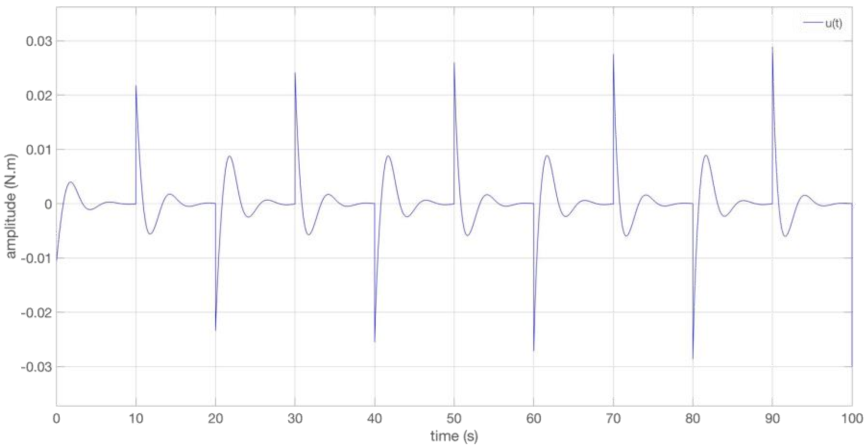

Fig. 1. Plot showing the control input u for the Pitch with MRAC Output Feedback.

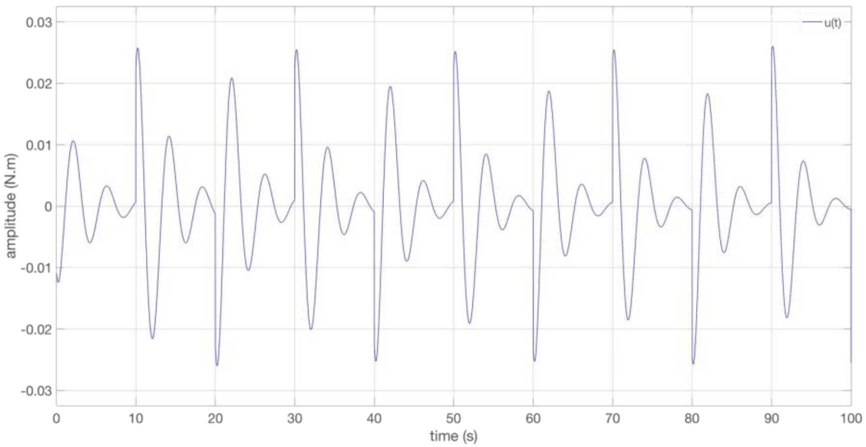

Fig. 2. Plot showing the control input u for the Yaw with MRAC Output Feedback.

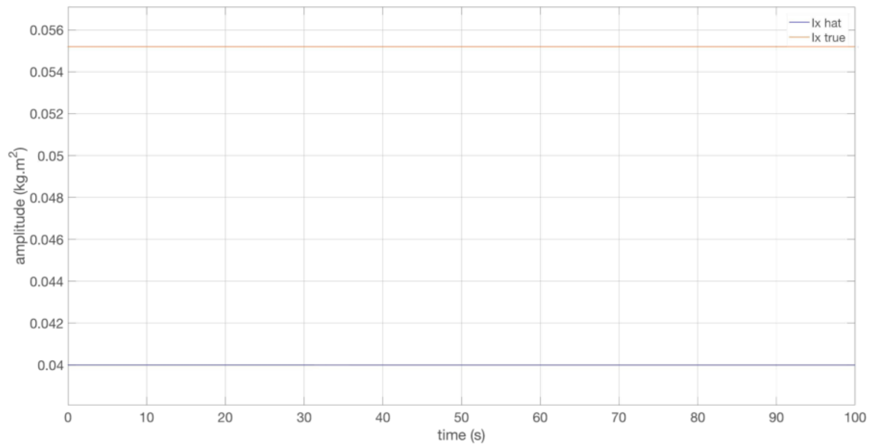

Fig. 3. Plot showing the parameter estimates and their actual values for the Pitch with MRAC Output Feedback.

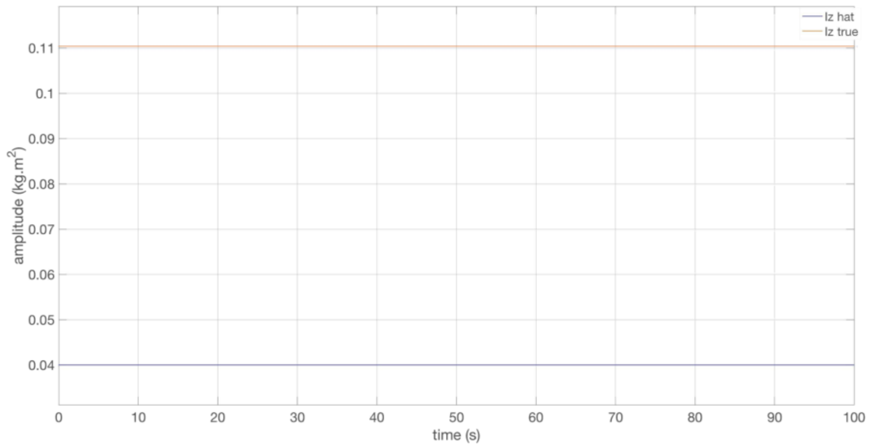

Fig. 4. Plot showing the parameter estimates and their actual values for the Yaw with MRAC Output Feedback.

Simulation Results – MRAC with Disturbance Rejection

Variable	Value
cutoff frequency	$45Hz$
$\hat{d}_{0pitch}, \hat{d}_{0roll}, \hat{d}_{0yaw}$	0
$\gamma_{dpitch}, \gamma_{droll}, \gamma_{dyaw}$	$\begin{bmatrix}5 & 5\end{bmatrix}$

Controller Initialization for MRAC with Disturbance Rejection.

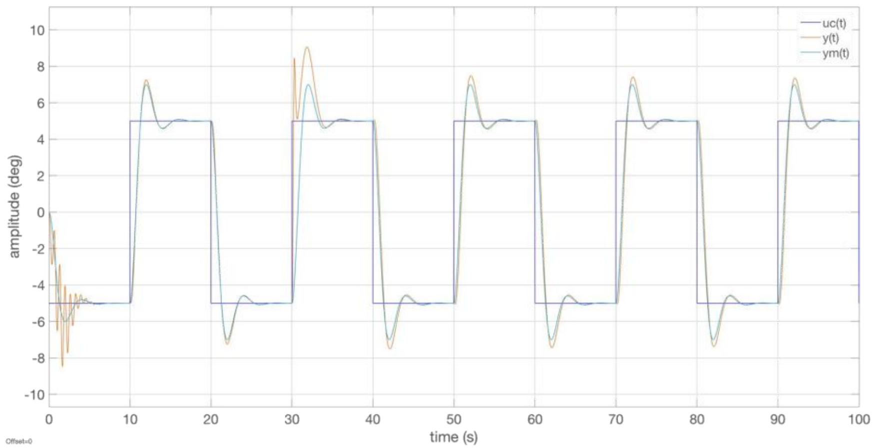

Fig. 1. Plot showing the reference command input uc, the reference output ym, and the actual output y for the Pitch with MRAC Disturbance Rejection.

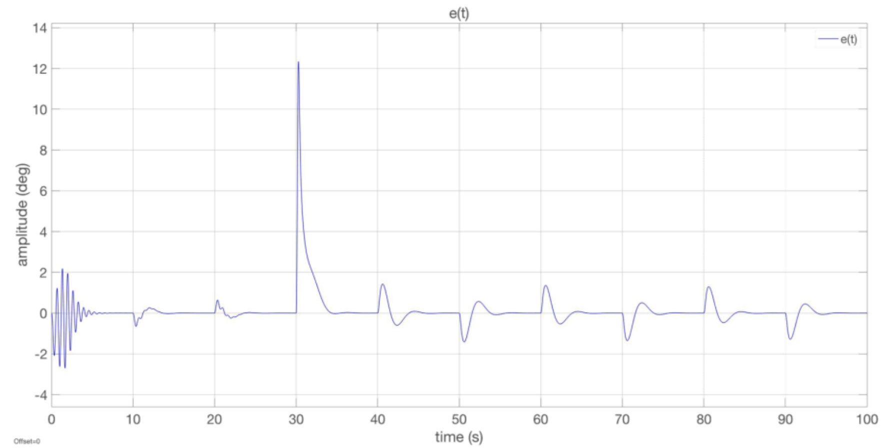

Fig. 3. Plot showing the output tracking error e=y-ym for the Pitch with MRAC Disturbance Rejection.

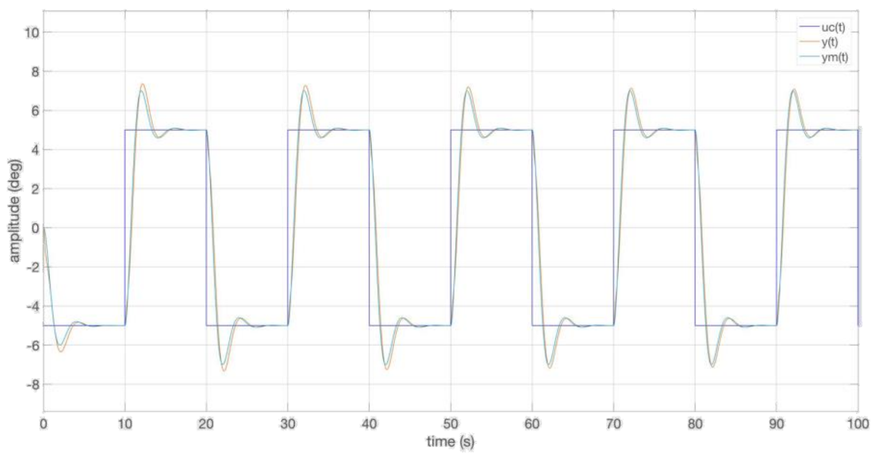

Fig. 2. Plot showing the reference command input uc, the reference output ym, and the actual output y for the Yaw with MRAC Disturbance Rejection.

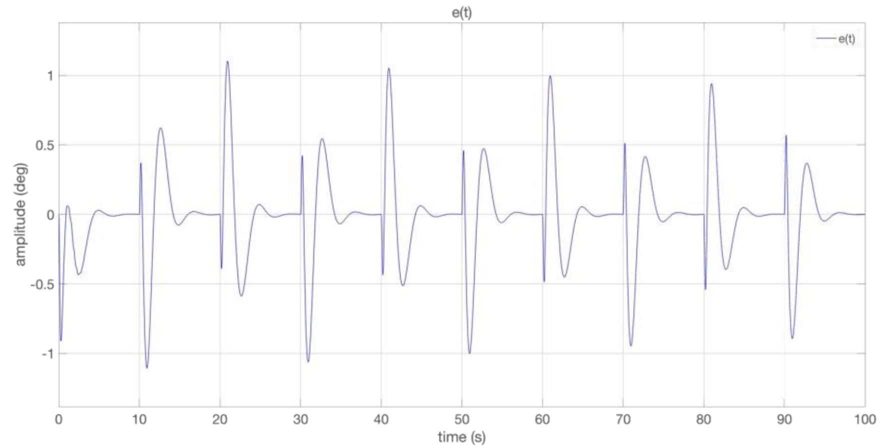

Fig. 4. Plot showing the output tracking error e=y-ym for the Yaw with MRAC Disturbance Rejection.

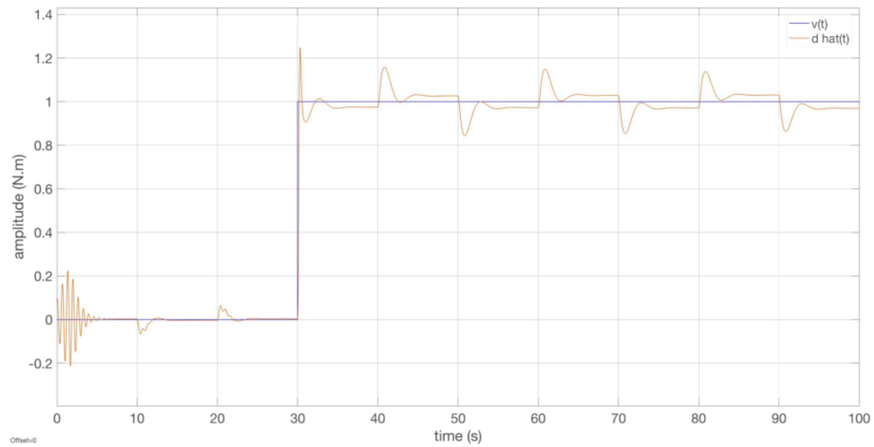

Fig. 1. Plot showing the disturbance estimate and its actual value for the Pitch with MRAC Disturbance Rejection.

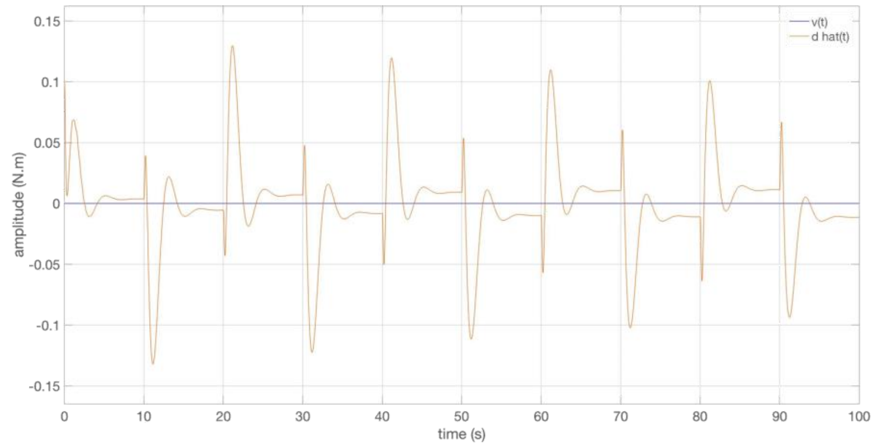

Fig. 2. Plot showing the disturbance estimate and its actual value for the Yaw with MRAC Disturbance Rejection.

Simulation Results – MRAC with Disturbance Rejection

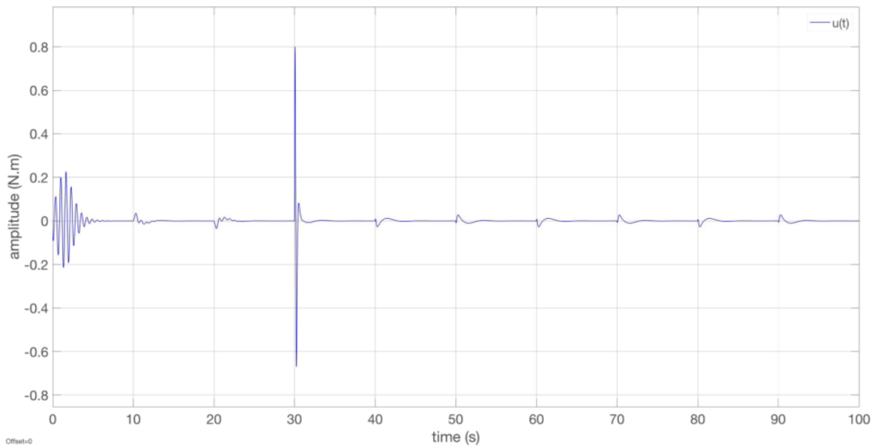

Fig. 1. Plot showing the control input u for the Pitch with MRAC Disturbance Rejection.

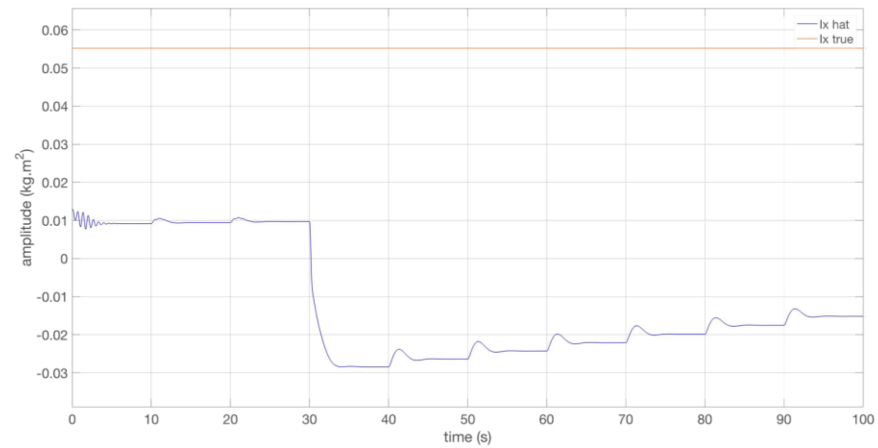

Fig. 3. Plot showing the parameter estimates and their actual values for the Pitch with MRAC Disturbance Rejection.

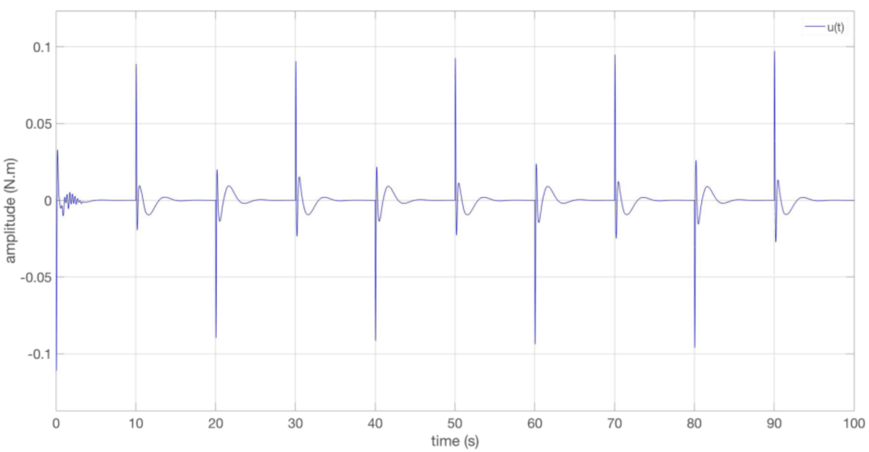

Fig. 2. Plot showing the control input u for the Yaw with MRAC Disturbance Rejection.

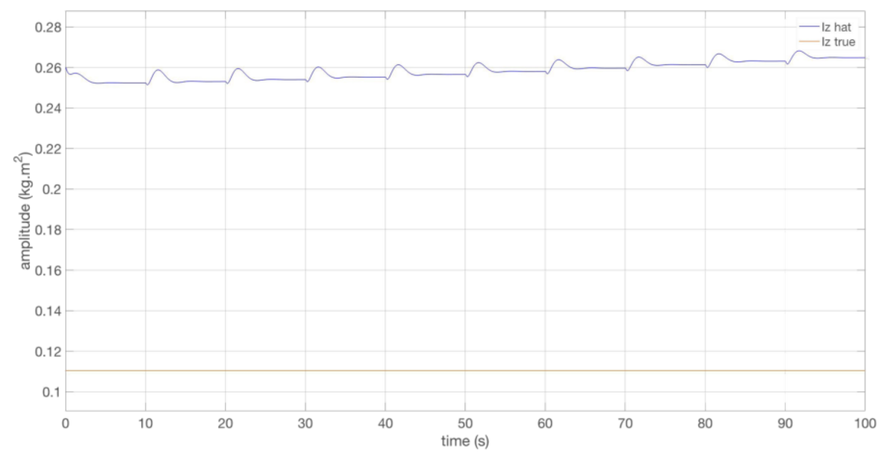

Fig. 4. Plot showing the parameter estimates and their actual values for the Yaw with MRAC Disturbance Rejection.

Controller	Performance
Proportional	• Overshoot • Oscillating error $\pm 20°$
Indirect STR	• Signal tracking • No parameter convergence
MRAC Full State and Output Feedback	• Reference model tracking • $e(t) \rightarrow 0\ as\ t \rightarrow \infty$ • $u(t)$ PE ⇒ parameter convergence
MRAC Disturbance Rejection	• Reference model tracking • Disturbance estimation • Disturbance rejection in $\leq 4s$

Comparison of performance between the proposed controllers