Technische Universität München

Institute of Flight System Dynamics

Design, Implementation and Evaluation of an Incremental Nonlinear Dynamic Inversion Controller for a Nano-Quadrotor

–

Entwurf, Implementierung und Evaluierung eines Inkrementellen Nichtlinearen Dynamischen Inversionsreglers für einen Nano-Quadrotor

Semesterarbeit

Author: Evghenii Volodscoi

Matriculation number: 03663176

Supervisor: Dr. Ewoud Smeur

April 2020

Statutory Declaration

I, Evghenii Volodscoi, declare on oath towards the Institute of Flight System Dynamics of Technische Universitt Mnchen, that I have prepared the present Semester Thesis independently and with the aid of nothing but the resources listed in the bibliography.

This thesis has neither as-is nor similarly been submitted to any other university.

Garching,

Kurzfassung

Deutsche Kurzfassung der Arbeit.

Abstract

English abstract of the thesis.

Table of Contents

List of Figures

List of Tables

Table of Acronyms

Acronym	Description
NDI	Nonlinear Dynamic Inversion

Table of Symbols

Latin Letters

Symbol	Unit	Description
F	N	Force
g	m/s^2	Gravitational acceleration

Greek Letters

Symbol	Unit	Description
α	rad	Angle of attack
ζ	–	Damping of a linear second order system

Indices

Symbol	Unit	Description
m		Variable related to pitch moment
W		Wind

1 Introduction

1.1 Motivation

1.2 Contribution of the Thesis

1.3 Structure of the Thesis

2 Theoretical Background

2.1 General Equations of Motion

General:
- Linear Momentum
- Angular Momentum
- Attitude differential equations
- Position differential equations
- External forces and moments

with some of the used assumptions

2.2 Nonlinear Dynamic Inversion

In this subsection, the Nonlinear Dynamic Inversion (NDI) method is explained. NDI is based on feedback linearization and is also called *Input-Output Linearization*. This method is often involved in tracking control tasks, where objective is to track some desired trajectory. (cite-tag Slotine) To show it in practice, consider following nonlinear system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u} \tag{2.1a}$$

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}) \tag{2.1b}$$

where $\boldsymbol{x}$ is the $n \times 1$ state vector, $\boldsymbol{u}$ the $m \times 1$ input vector, $\boldsymbol{y}$ the $m \times 1$ system output vector, $\boldsymbol{f}(\boldsymbol{x})$ and $\boldsymbol{h}(\boldsymbol{x})$ nonlinear vector fields and $\boldsymbol{G}$ an $m \times n$ matrix. Note that the system presented in Equation (2.1b) is affine in the input, which is not allways the case. To deal with this problem the system can be transformed into nonlinear normal (canonical) form through the state transformation.

The idea behind the input-output linearization method is to find a direct relationship between the desired system output and the control input and then invert it. To generate this relationship the output $\boldsymbol{y}$ is differentiated until the input $\boldsymbol{u}$ appears

$$\begin{aligned}\dot{\boldsymbol{y}} &= \frac{\partial \boldsymbol{y}}{\partial t} = \frac{\partial \boldsymbol{h}(\boldsymbol{x})}{\partial t}\frac{\partial \boldsymbol{x}}{\partial t} = \frac{\partial \boldsymbol{h}(\boldsymbol{x})}{\partial t}\dot{\boldsymbol{x}} = \nabla \boldsymbol{h}(\boldsymbol{x})[\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u}] \\ &= \nabla \boldsymbol{h}(\boldsymbol{x})\boldsymbol{f}(\boldsymbol{x}) + \nabla \boldsymbol{h}(\boldsymbol{x})\boldsymbol{G}(\boldsymbol{x})\boldsymbol{u}] = L_{\boldsymbol{f}}\boldsymbol{h}(\boldsymbol{x}) + L_{\boldsymbol{G}}\boldsymbol{h}(\boldsymbol{x})\boldsymbol{u}\end{aligned} \tag{2.2}$$

In Equation (2.2) $L_{\boldsymbol{f}}\boldsymbol{h}(\boldsymbol{x})$ is called Lie derivative of $\boldsymbol{h}(\boldsymbol{x})$ with respect to $\boldsymbol{f}(\boldsymbol{x})$. The Lie derivative is defined as $L_{\boldsymbol{f}}\boldsymbol{h}(\boldsymbol{x}) = \nabla \boldsymbol{h}(\boldsymbol{x})\boldsymbol{f}(\boldsymbol{x})$ with ∇ being the Nabla operator. Thus, it represents a directional derivative of $\boldsymbol{h}(\boldsymbol{x})$ along the direction of the vector field $\boldsymbol{f}(\boldsymbol{x})$. If the term $L_{\boldsymbol{G}}\boldsymbol{h}(\boldsymbol{x})\boldsymbol{u}$ is not zero, the relationship between input and output is

$$\dot{\boldsymbol{y}} = L_{\boldsymbol{f}}\boldsymbol{h}(\boldsymbol{x}) + L_{\boldsymbol{G}}\boldsymbol{h}(\boldsymbol{x})\boldsymbol{u} \tag{2.3}$$

Now Equation (2.3) can be used to formulate the control law by solving it for $\boldsymbol{u}$ and substituting $\dot{\boldsymbol{y}}$ with $\boldsymbol{\nu}$

$$\boldsymbol{u} = L_{\boldsymbol{G}}\boldsymbol{h}(\boldsymbol{x})^{-1}(\boldsymbol{\nu} - L_{\boldsymbol{f}}\boldsymbol{h}(\boldsymbol{x})) \tag{2.4}$$

The variable $\boldsymbol{\nu}$ is called an *equivalent input*. It represents the desired output of the system.

In the example provided above the input-output relationsip was found after the first differentiation of the output. If this is not the case and the term $L_{\boldsymbol{G}}\boldsymbol{h}(\boldsymbol{x})$ is zero, output $\boldsymbol{y}$ has to be differentiated until the Lie derivative with respect to $\boldsymbol{G}$ is nonzero

$$\frac{\partial^i \boldsymbol{y}}{\partial t} = \frac{\partial^i \boldsymbol{h}(\boldsymbol{x})}{\partial t} = L^i_{\boldsymbol{f}}\boldsymbol{h}(\boldsymbol{x}) + L_{\boldsymbol{G}}L^{i-1}_{\boldsymbol{f}}\boldsymbol{h}(\boldsymbol{x})\boldsymbol{u} \tag{2.5}$$

with i being the *relative degree* of the system. Using Equation (2.5) to formulate the control law, leads to the following expression for $\boldsymbol{u}$

$$\boldsymbol{u} = L_{\boldsymbol{G}}L^{i-1}_{\boldsymbol{f}}\boldsymbol{h}(\boldsymbol{x})^{-1}(\boldsymbol{\nu} - L^i_{\boldsymbol{f}}\boldsymbol{h}(\boldsymbol{x})) \tag{2.6}$$

Equation (2.7) applied to Equation (2.5) yields the simple linear relation

$$\boldsymbol{y}^i = \boldsymbol{\nu} \tag{2.7}$$

Despite wide usage (especially in the past of the flight control design) of the NDI method and its numerrous extensions (cite-tag Horn), it also has drawbacks. The major one is that the control law derived with NDI is dependend on the dynamics model and its uncertainties. As shown in the previous chapter (cite-tag prev-chapter), the equations of motion of an aircraft usually have a complicated nonlinear character. Thus, creating a perfectly aqurate model of the aircraft is not an easy task. To overcome this obstacle an extension of the NDI method called INDI can be incorporated.

2.3 Incremental Nonlinear Dynamic Inversion

- Here only the general principle is provided, next subsections show the full derivation of the two controller loops.

2.3.1 INDI inner loop

- Derivation of the inner INDI loop (detailed equations).

2.3.2 INDI outer loop

- Derivation of the outer INDI loop (detailed equations).

3 Implementation

3.1 Research Quadrotor

- Some facts about Crazyflie hardware (foto, uC frequency, weight, length)

3.2 Simulink Model

3.2.1 Purpose

- Estimation of relevant components (Matrices...)
- Testing of the PD-gains
- Testing the filter

3.2.2 Structure

- Parameters
- Actuator dynamics
- Filter
- Images of the Simulink model

3.2.3 Simulation Results

3.3 Implementation on Hardware

3.3.1 Structure of the Code

3.3.2 Testing with contact Forces and Moments

4 Results

5 Discussion

Appendix