Technische Universität München

Institute of Flight System Dynamics

Design, Implementation and Evaluation of an Incremental Nonlinear Dynamic Inversion Controller for a Nano-Quadrotor

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Entwurf, Implementierung und Evaluierung eines Inkrementellen Nichtlinearen Dynamischen Inversionsreglers für einen Nano-Quadrotor

Semesterarbeit

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Statutory Declaration

I, Evghenii Volodscoi, declare on oath towards the Institute of Flight System Dynamics of Technische Universitt Mnchen, that I have prepared the present Semester Thesis independently and with the aid of nothing but the resources listed in the bibliography.

This thesis has neither as-is nor similarly been submitted to any other university.

Garching,

Kurzfassung

Deutsche Kurzfassung der Arbeit.

Abstract

English abstract of the thesis.

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Latin Letters

Symbol	**Unit**	**Description**
F	N	Force
g	m/s^2	Gravitational acceleration

Greek Letters

Symbol	**Unit**	**Description**
α	rad	Angle of attack
ζ	–	Damping of a linear second order system

Indices

Symbol	**Unit**	**Description**
m		Variable related to pitch moment
W		Wind

1 Introduction

1.1 Motivation

1.2 Contribution of the Thesis

1.3 Structure of the Thesis

2 Theoretical Background

2.1 General Equations of Motion

General:
- Linear Momentum
- Angular Momentum
- Attitude differential equations
- Position differential equations
- External forces and moments

with some of the used assumptions

2.2 Nonlinear Dynamic Inversion

In this subsection, the Nonlinear Dynamic Inversion (NDI) method is explained. The NDI approach is based on feedback linearization and is also called *Input-Output Linearization*. Often, such type of controllers is involved in tracking control tasks, where objective is to track some desired trajectory [1]. To derive it, consider the following nonlinear system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u} \tag{2.1a}$$

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}) \tag{2.1b}$$

where $\boldsymbol{x}$ is the $n \times 1$ state vector, $\boldsymbol{u}$ the $m \times 1$ input vector, $\boldsymbol{y}$ the $m \times 1$ system vector, $\boldsymbol{f}(\boldsymbol{x})$ and $\boldsymbol{h}(\boldsymbol{x})$ nonlinear vector fields and $\boldsymbol{G}$ an $m \times n$ input matrix. Note that the system presented in Equations (2.1) is affine in the input, which is not allways fulfilled. Using a state transformation $\boldsymbol{z} = \boldsymbol{\phi}(\boldsymbol{x})$, the affine system from Equation 2.1a can be transformed into normal (canonical) representation.

The core idea behind the input-output linearization method is to find a direct relationship between the desired system output and the control input. After the relationship is found it is inverted to generate the control law. To derive this relationship the output $\boldsymbol{y}$ is differentiated until the input $\boldsymbol{u}$ appears

$$\begin{aligned} \dot{\boldsymbol{y}} &= \frac{\partial \boldsymbol{y}}{\partial t} = \frac{\partial \boldsymbol{h}(\boldsymbol{x})}{\partial t}\frac{\partial \boldsymbol{x}}{\partial t} = \frac{\partial \boldsymbol{h}(\boldsymbol{x})}{\partial t}\dot{\boldsymbol{x}} = \nabla \boldsymbol{h}(\boldsymbol{x})[\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u}] \\ &= \nabla \boldsymbol{h}(\boldsymbol{x})\boldsymbol{f}(\boldsymbol{x}) + \nabla \boldsymbol{h}(\boldsymbol{x})\boldsymbol{G}(\boldsymbol{x})\boldsymbol{u}] = L_{\boldsymbol{f}}\boldsymbol{h}(\boldsymbol{x}) + L_{\boldsymbol{G}}\boldsymbol{h}(\boldsymbol{x})\boldsymbol{u} \end{aligned} \tag{2.2}$$

In Equation (2.2) $L_{\boldsymbol{f}}\boldsymbol{h}(\boldsymbol{x})$ is called Lie derivative of $\boldsymbol{h}(\boldsymbol{x})$ with respect to $\boldsymbol{f}(\boldsymbol{x})$. The Lie derivative is defined as $L_{\boldsymbol{f}}\boldsymbol{h}(\boldsymbol{x}) = \nabla \boldsymbol{h}(\boldsymbol{x})\boldsymbol{f}(\boldsymbol{x})$ with ∇ being the Nabla operator. Thus, it represents a directional derivative of $\boldsymbol{h}(\boldsymbol{x})$ along the direction of the vector field $\boldsymbol{f}(\boldsymbol{x})$. If the term $L_{\boldsymbol{G}}\boldsymbol{h}(\boldsymbol{x})$ is nonzero, the relationship between input and output is

$$\dot{\boldsymbol{y}} = L_{\boldsymbol{f}}\boldsymbol{h}(\boldsymbol{x}) + L_{\boldsymbol{G}}\boldsymbol{h}(\boldsymbol{x})\boldsymbol{u} \tag{2.3}$$

Now Equation (2.3) can be used to formulate the control law by solving it for $\boldsymbol{u}$ and substituting

$\dot{\boldsymbol{y}}$ with $\boldsymbol{\nu}$

$$\boldsymbol{u} = L_{\boldsymbol{G}}\boldsymbol{h}(\boldsymbol{x})^{-1}(\boldsymbol{\nu} - L_{\boldsymbol{f}}\boldsymbol{h}(\boldsymbol{x})) \tag{2.4}$$

The variable $\boldsymbol{\nu}$ is called an *equivalent input* and represents the desired output of the system.

In the example provided above the input-output relationsip was found after the first differentiation of the output $\boldsymbol{y}$. But if after the first differentiation the term $L_{\boldsymbol{G}}\boldsymbol{h}(\boldsymbol{x})$ is zero, output $\boldsymbol{y}$ has to be differentiated until the Lie derivative with respect to $\boldsymbol{G}$ is nonzero. The i-th derivative of the output is then

$$\frac{\partial^i \boldsymbol{y}}{\partial t} = \frac{\partial^i \boldsymbol{h}(\boldsymbol{x})}{\partial t} = L^i_{\boldsymbol{f}}\boldsymbol{h}(\boldsymbol{x}) + L_{\boldsymbol{G}}L^{i-1}_{\boldsymbol{f}}\boldsymbol{h}(\boldsymbol{x})\boldsymbol{u} \tag{2.5}$$

with i being the *relative degree* of the system. Using Equation (2.5) to formulate the control law, leads to the following expression for the control input $\boldsymbol{u}$

$$\boldsymbol{u} = L_{\boldsymbol{G}}L^{i-1}_{\boldsymbol{f}}\boldsymbol{h}(\boldsymbol{x})^{-1}(\boldsymbol{\nu} - L^i_{\boldsymbol{f}}\boldsymbol{h}(\boldsymbol{x})) \tag{2.6}$$

Thus, Equation (2.7) applied to Equation (2.5) yields the simple linear relation

$$\boldsymbol{y}^i = \boldsymbol{\nu} \tag{2.7}$$

Despite wide usage (especially in the past of the flight control) of the NDI method and its numerrous extensions (cite-tag Horn), it also has drawbacks.

The NDI method was widely adopted for civil and military aircrafts and has numerous of extensions [2]. Nevertheless, it has some drawbacks. The major one is that the control law derived using the NDI approach is dependent on the full system dynamics model [3]. As shown in the previous chapter (cite-tag prev-chapter), the equations of motion of an aircraft usually have a complicated nonlinear character. Thus, describing complex physical phenomena often leads to the inconsistency between the real aircraft and its mathematical representation used in the model. As some of those inconsistencies are inevitable there is a need to build a control law which performance is less dependent on the uncertainties of the model. This method is called Incremental Nonlinear Dynamic Inversion (INDI) and will be discussed in the next subsection.

2.3 Incremental Nonlinear Dynamic Inversion

The INDI is an incremental form of the NDI for which the lack of the accurate system dynamics model does not critically affect the performance of the control algorithm [4]. The incremental form of the system can be obtained by taking a Taylor series expansion of the Equation 2.1a

$$\begin{aligned}\dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x_0}) + \boldsymbol{G}(\boldsymbol{x_0})\boldsymbol{u_0} \\ &+ \frac{\partial}{\partial \boldsymbol{x}}[\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u}]\bigg|_{\boldsymbol{x}=\boldsymbol{x_0},\boldsymbol{u}=\boldsymbol{u_0}}(\boldsymbol{x} - \boldsymbol{x_0}) \\ &+ \frac{\partial}{\partial \boldsymbol{u}}[\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u}]\bigg|_{\boldsymbol{x}=\boldsymbol{x_0},\boldsymbol{u}=\boldsymbol{u_0}}(\boldsymbol{u} - \boldsymbol{u_0})\end{aligned} \tag{2.8}$$

The first term on the right side of the Equation 2.8 is $\dot{\boldsymbol{x}}_0$. Also, evaluating the differentiation of the third term leads to

$$\begin{aligned}\dot{\boldsymbol{x}} &= \dot{\boldsymbol{x}}_0 \\ &+ \frac{\partial}{\partial \boldsymbol{x}}[\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u}]\Big|_{\boldsymbol{x}=\boldsymbol{x}_0, \boldsymbol{u}=\boldsymbol{u}_0}(\boldsymbol{x} - \boldsymbol{x}_0) \\ &+ \boldsymbol{G}(\boldsymbol{x}_0)(\boldsymbol{u} - \boldsymbol{u}_0)\end{aligned} \tag{2.9}$$

The second term of the Equation 2.9 contains partial derivative with respect to the state vector. Considering very small time increments of the controller loop and applying the *principle of time scale separation* the second term vanishes. This is a valid assumption if the dynamics of the actuators is fast compared with the dynamics of the system. Thus, the Equation 2.9 is further simplified to

$$\dot{\boldsymbol{x}} = \dot{\boldsymbol{x}}_0 + \boldsymbol{G}(\boldsymbol{x}_0)(\boldsymbol{u} - \boldsymbol{u}_0) \tag{2.10}$$

Solving Equation 2.10 for $\boldsymbol{u}$ and substituting $\dot{\boldsymbol{y}}$ with $\boldsymbol{\nu}$, the INDI control law can be obtained

$$\boldsymbol{u} = \boldsymbol{u}_0 + \boldsymbol{G}(\boldsymbol{x}_0)^{-1}(\boldsymbol{\nu} - \dot{\boldsymbol{x}}_0) \tag{2.11}$$

where $\dot{\boldsymbol{x}}_0$ is a measurable value from the previous step, $\boldsymbol{u}_0$ the control input from the previous step, $\boldsymbol{\nu}$ the reference value and $\boldsymbol{G}(\boldsymbol{x}_0)$ the control effectiveness matrix. With $\boldsymbol{\Delta u} = \boldsymbol{u} - \boldsymbol{u}_0$ the control law from Equation 2.11 represents an incremental version of the Equation 2.4. This control law results in computing the increment $\boldsymbol{\Delta u}$ of the control input and adding it to the previous value $\boldsymbol{u}_0$, instead if computing complete control input command $\boldsymbol{u}$. As it is less dependent on the model of the system dynamics, it increases the robustness of the system [3].

2.3.1 General INDI

2.3.2 INDI inner loop

- Derivation of the inner INDI loop (detailed equations).

2.3.3 INDI outer loop

- Derivation of the outer INDI loop (detailed equations).

3 Implementation

3.1 Research Quadrotor

- Some facts about Crazyflie hardware (foto, uC frequency, weight, length)

3.2 Simulink Model

3.2.1 Purpose

- Estimation of relevant components (Matrices...)
- Testing of the PD-gains
- Testing the filter

3.2.2 Structure

- Parameters
- Actuator dynamics
- Filter
- Images of the Simulink model

3.2.3 Simulation Results

3.3 Implementation on Hardware

3.3.1 Structure of the Code

3.3.2 Testing with contact Forces and Moments

4 Results

5 Discussion

References

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Appendix