Example 9.8: Seepage through soil (p. 627)

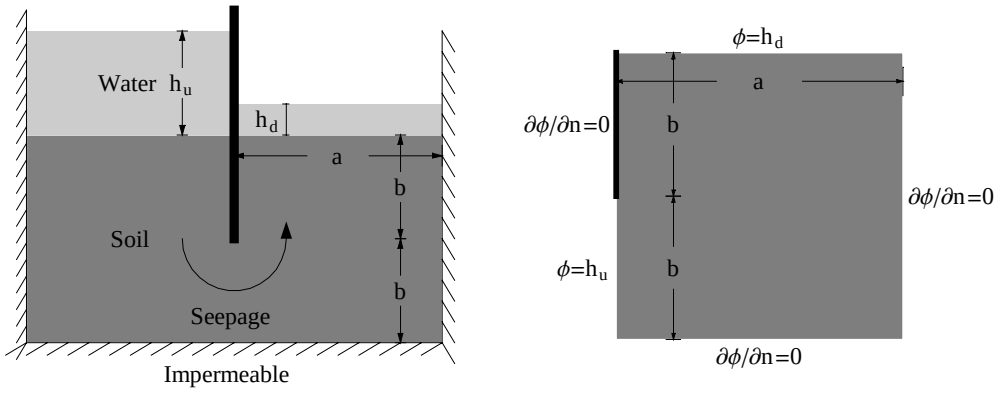
The problem of determining the amount of water that seeps through dams or from underneath sheet piles can be formulated in terms of the following equation

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial \phi}{\partial y}\right) = 0$$

where $\phi(x, y)$ is the hydraulic head (or hydraulic potential) and k_x and k_y are coefficients of permeability in the x and y directions. Typical units for ϕ are meters and those for k_x and k_y are m / day. The fluid velocity components in the x and y directions are related to the hydraulic head as follows.

$$v_x = -k_x \frac{\partial \phi}{\partial x} \text{ and } v_y = -k_y \frac{\partial \phi}{\partial y}$$

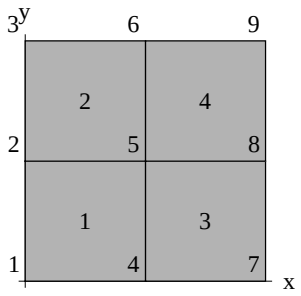
A typical situation is illustrated in Figure. The analytical model consists of the soil on the downstream side. On the impermeable sides the no flow condition is expressed in terms of normal derivative of ϕ being zero. On the top $\phi = h_d$ the hydraulic head on the downstream side. On the left side the boundary condition on the soil below the pile is $\phi = h_u$, the hydraulic head on the upstream side.



As a numerical example, consider the following numerical values.

$$a = b = 10\,m; \quad h_u = 10\,m; \quad h_d = 1\,m; \quad k_x = k_y = 1\,m/s$$

The solution domain is divided into four elements. Solution using second-order ($n = 2$) p formulation is as follows.



Interpolation functions for mapping: $\left\{\frac{1}{4}(1-s)(1-t),\ \frac{1}{4}(s+1)(1-t),\ \frac{1}{4}(s+1)(t+1),\ \frac{1}{4}(1-s)(t+1)\right\}$

Interpolation functions for assumed solution: $\boldsymbol{N}^{\mathrm{T}} = \left\{\frac{1}{4}(1-s)(1-t),\ \frac{1}{4}(s+1)(1-t),\ \frac{1}{4}(s+1)(t+1),\right.$

$$\left.\frac{1}{4}(1-s)(t+1),\ \frac{\left(\frac{3s^2}{2}-\frac{3}{2}\right)(1-t)}{2\sqrt{6}},\ \frac{(s+1)\left(\frac{3t^2}{2}-\frac{3}{2}\right)}{2\sqrt{6}},\ \frac{\left(\frac{3s^2}{2}-\frac{3}{2}\right)(t+1)}{2\sqrt{6}},\ \frac{(1-s)\left(\frac{3t^2}{2}-\frac{3}{2}\right)}{2\sqrt{6}}\right\}$$

$$\partial\boldsymbol{N}^{\mathrm{T}}/\partial s = \left\{\frac{t-1}{4},\ \frac{1-t}{4},\ \frac{t+1}{4},\ \frac{1}{4}(-t-1),\ \frac{1}{2}\sqrt{\frac{3}{2}}\,s(1-t),\ \frac{\frac{3t^2}{2}-\frac{3}{2}}{2\sqrt{6}},\ \frac{1}{2}\sqrt{\frac{3}{2}}\,s(t+1),\ -\frac{\frac{3t^2}{2}-\frac{3}{2}}{2\sqrt{6}}\right\}$$

$$\partial\boldsymbol{N}^{\mathrm{T}}/\partial t = \left\{\frac{s-1}{4},\ \frac{1}{4}(-s-1),\ \frac{s+1}{4},\ \frac{1-s}{4},\ -\frac{\frac{3s^2}{2}-\frac{3}{2}}{2\sqrt{6}},\ \frac{1}{2}\sqrt{\frac{3}{2}}\,(s+1)\,t,\ \frac{\frac{3s^2}{2}-\frac{3}{2}}{2\sqrt{6}},\ \frac{1}{2}\sqrt{\frac{3}{2}}\,(1-s)\,t\right\}$$

Use 2×2 Gauss quadrature for integration.

Global equations at start of the element assembly process

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \\ \phi_9 \\
\delta_1^{\{1,2\}} \\ \delta_1^{\{1,4\}} \\ \delta_1^{\{2,3\}} \\ \delta_1^{\{2,5\}} \\ \delta_1^{\{3,6\}} \\ \delta_1^{\{4,5\}} \\ \delta_1^{\{4,7\}} \\ \delta_1^{\{5,6\}} \\ \delta_1^{\{5,8\}} \\ \delta_1^{\{6,9\}} \\ \delta_1^{\{7,8\}} \\ \delta_1^{\{8,9\}}
\end{pmatrix}
=
\begin{pmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{pmatrix}$$

Equations for element 1

Element coordinates: ({0, 0}   {5, 0}   {5, 5}   {0, 5})

$$x(\mathrm{s},\mathrm{t}) = \frac{5\,\mathrm{s}}{2} + \frac{5}{2}$$

$$y(\mathrm{s},\mathrm{t}) = \frac{5\,\mathrm{t}}{2} + \frac{5}{2}$$

$$\boldsymbol{J} = \begin{pmatrix} \frac{5}{2} & 0 \\ 0 & \frac{5}{2} \end{pmatrix}$$

$$\mathrm{detJ} = \frac{25}{4}$$

Given element data

$k_x = 1$ $k_y = 1$ $p = 0$ $q = 0$

Element data in mapped coordinates

$k_x = 1$ $k_y = 1$ $p = 0$ $q = 0$

Gauss point = {s → −0.57735, t → −0.57735} Weight = 1.

$$\boldsymbol{N}^T = \{0.622008, 0.166667, 0.0446582, 0.166667, -0.321975, -0.086273, -0.086273, -0.321975\}$$

$$\partial \boldsymbol{N}^T/\partial s =$$
$$(\ -0.394338 \quad 0.394338 \quad 0.105662 \quad -0.105662 \quad -0.557678 \quad -0.204124 \quad -0.149429 \quad 0.204124\ )$$

$$\partial \boldsymbol{N}^T/\partial t =$$
$$(\ -0.394338 \quad -0.105662 \quad 0.105662 \quad 0.394338 \quad 0.204124 \quad -0.149429 \quad -0.204124 \quad -0.557678\ )$$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix} \qquad \det J = 6.25$$

$$\boldsymbol{B}^T =$$
$$\begin{pmatrix} -0.157735 & 0.157735 & 0.042265 & -0.042265 & -0.223071 & -0.0816497 & -0.0597717 & 0.0816497 \\ -0.157735 & -0.042265 & 0.042265 & 0.157735 & 0.0816497 & -0.0597717 & -0.0816497 & -0.223071 \end{pmatrix}$$

$k_x = 1.$ $k_y = 1.$ $p = 0.$ $q = 0.$

$$\boldsymbol{k}_k = \begin{pmatrix} 0.311004 & -0.113835 & -0.0833333 & -0.113835 & 0.139419 & 0.139419 & 0.139419 \\ -0.113835 & 0.166667 & 0.0305021 & -0.0833333 & -0.241481 & -0.0647048 & -0.0373573 \\ -0.0833333 & 0.0305021 & 0.0223291 & 0.0305021 & -0.0373573 & -0.0373573 & -0.0373573 \\ -0.113835 & -0.0833333 & 0.0305021 & 0.166667 & 0.139419 & -0.0373573 & -0.0647048 \\ 0.139419 & -0.241481 & -0.0373573 & 0.139419 & 0.352671 & 0.0833333 & 0.0416667 \\ 0.139419 & -0.0647048 & -0.0373573 & -0.0373573 & 0.0833333 & 0.0639958 & 0.0610042 \\ 0.139419 & -0.0373573 & -0.0373573 & -0.0647048 & 0.0416667 & 0.0610042 & 0.0639958 \\ 0.139419 & 0.139419 & -0.0373573 & -0.241481 & -0.227671 & 0.0416667 & 0.0833333 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_q^T = (\ 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0.\ )$$

Gauss point = {s → −0.57735, t → 0.57735} Weight = 1.

$\boldsymbol{N}^{T}$ = {0.166667, 0.0446582, 0.166667, 0.622008, −0.086273, −0.086273, −0.321975, −0.321975}

$\partial \boldsymbol{N}^{T}/\partial s$ =

(−0.105662 0.105662 0.394338 −0.394338 −0.149429 −0.204124 −0.557678 0.204124)

$\partial \boldsymbol{N}^{T}/\partial t$ =

(−0.394338 −0.105662 0.105662 0.394338 0.204124 0.149429 −0.204124 0.557678)

$\boldsymbol{J}^{-T} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix}$ detJ = 6.25

$\boldsymbol{B}^{T}$ =

$$\begin{pmatrix} -0.042265 & 0.042265 & 0.157735 & -0.157735 & -0.0597717 & -0.0816497 & -0.223071 & 0.0816497 \\ -0.157735 & -0.042265 & 0.042265 & 0.157735 & 0.0816497 & 0.0597717 & -0.0816497 & 0.223071 \end{pmatrix}$$

k_x = 1. k_y = 1. p = 0. q = 0.

$$\boldsymbol{k}_k = \begin{pmatrix} 0.166667 & 0.0305021 & -0.0833333 & -0.113835 & -0.0647048 & -0.0373573 & 0.139419 \\ 0.0305021 & 0.0223291 & 0.0305021 & -0.0833333 & -0.0373573 & -0.0373573 & -0.0373573 \\ -0.0833333 & 0.0305021 & 0.166667 & -0.113835 & -0.0373573 & -0.0647048 & -0.241481 \\ -0.113835 & -0.0833333 & -0.113835 & 0.311004 & 0.139419 & 0.139419 & 0.139419 \\ -0.0647048 & -0.0373573 & -0.0373573 & 0.139419 & 0.0639958 & 0.0610042 & 0.0416667 \\ -0.0373573 & -0.0373573 & -0.0647048 & 0.139419 & 0.0610042 & 0.0639958 & 0.0833333 \\ 0.139419 & -0.0373573 & -0.241481 & 0.139419 & 0.0416667 & 0.0833333 & 0.352671 \\ -0.241481 & -0.0373573 & 0.139419 & 0.139419 & 0.0833333 & 0.0416667 & -0.227671 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$\boldsymbol{r}_q^{T}$ = (0. 0. 0. 0. 0. 0. 0. 0.)

Gauss point = {s → 0.57735, t → −0.57735} Weight = 1.

$\boldsymbol{N}^{T}$ = {0.166667, 0.622008, 0.166667, 0.0446582, −0.321975, −0.321975, −0.086273, −0.086273}

$\partial \boldsymbol{N}^{T}/\partial s$ =

(−0.394338 0.394338 0.105662 −0.105662 0.557678 −0.204124 0.149429 0.204124)

$\partial \boldsymbol{N}^{T}/\partial t$ =

(−0.105662 −0.394338 0.394338 0.105662 0.204124 −0.557678 −0.204124 −0.149429)

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix} \qquad \mathrm{detJ} = 6.25$$

$$\boldsymbol{B}^{\mathrm{T}} =$$

$$\begin{pmatrix} -0.157735 & 0.157735 & 0.042265 & -0.042265 & 0.223071 & -0.0816497 & 0.0597717 & 0.0816497 \\ -0.042265 & -0.157735 & 0.157735 & 0.042265 & 0.0816497 & -0.223071 & -0.0816497 & -0.0597717 \end{pmatrix}$$

$\mathrm{k_x} = 1.$ $\mathrm{k_y} = 1.$ $\mathrm{p} = 0.$ $\mathrm{q} = 0.$

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 0.166667 & -0.113835 & -0.0833333 & 0.0305021 & -0.241481 & 0.139419 & -0.0373573 \\ -0.113835 & 0.311004 & -0.113835 & -0.0833333 & 0.139419 & 0.139419 & 0.139419 \\ -0.0833333 & -0.113835 & 0.166667 & 0.0305021 & 0.139419 & -0.241481 & -0.0647048 \\ 0.0305021 & -0.0833333 & 0.0305021 & 0.0223291 & -0.0373573 & -0.0373573 & -0.0373573 \\ -0.241481 & 0.139419 & 0.139419 & -0.0373573 & 0.352671 & -0.227671 & 0.0416667 \\ 0.139419 & 0.139419 & -0.241481 & -0.0373573 & -0.227671 & 0.352671 & 0.0833333 \\ -0.0373573 & 0.139419 & -0.0647048 & -0.0373573 & 0.0416667 & 0.0833333 & 0.0639958 \\ -0.0647048 & 0.139419 & -0.0373573 & -0.0373573 & 0.0833333 & 0.0416667 & 0.0610042 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\,0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0.\,)$$

Gauss point = {s → 0.57735, t → 0.57735} Weight = 1.

$$\boldsymbol{N}^{\mathrm{T}} = \{0.0446582,\ 0.166667,\ 0.622008,\ 0.166667,\ -0.086273,\ -0.321975,\ -0.321975,\ -0.086273\}$$

$$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} =$$

$$(\,-0.105662 \quad 0.105662 \quad 0.394338 \quad -0.394338 \quad 0.149429 \quad -0.204124 \quad 0.557678 \quad 0.204124\,)$$

$$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} =$$

$$(\,-0.105662 \quad -0.394338 \quad 0.394338 \quad 0.105662 \quad 0.204124 \quad 0.557678 \quad -0.204124 \quad 0.149429\,)$$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix} \qquad \mathrm{detJ} = 6.25$$

$$\boldsymbol{B}^{\mathrm{T}} =$$

$$\begin{pmatrix} -0.042265 & 0.042265 & 0.157735 & -0.157735 & 0.0597717 & -0.0816497 & 0.223071 & 0.0816497 \\ -0.042265 & -0.157735 & 0.157735 & 0.042265 & 0.0816497 & 0.223071 & -0.0816497 & 0.0597717 \end{pmatrix}$$

$k_x = 1.$ $k_y = 1.$ $p = 0.$ $q = 0.$

$$\boldsymbol{k}_k = \begin{pmatrix} 0.0223291 & 0.0305021 & -0.0833333 & 0.0305021 & -0.0373573 & -0.0373573 & -0.0373573 \\ 0.0305021 & 0.166667 & -0.113835 & -0.0833333 & -0.0647048 & -0.241481 & 0.139419 \\ -0.0833333 & -0.113835 & 0.311004 & -0.113835 & 0.139419 & 0.139419 & 0.139419 \\ 0.0305021 & -0.0833333 & -0.113835 & 0.166667 & -0.0373573 & 0.139419 & -0.241481 \\ -0.0373573 & -0.0647048 & 0.139419 & -0.0373573 & 0.0639958 & 0.0833333 & 0.0416667 \\ -0.0373573 & -0.241481 & 0.139419 & 0.139419 & 0.0833333 & 0.352671 & -0.227671 \\ -0.0373573 & 0.139419 & 0.139419 & -0.241481 & 0.0416667 & -0.227671 & 0.352671 \\ -0.0373573 & -0.0373573 & 0.139419 & -0.0647048 & 0.0610042 & 0.0416667 & 0.0833333 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_q^T = (\,0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0.\,)$$

Adding contributions from all Gauss points

$$\boldsymbol{k}_k = \begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.2041 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.2041 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.2041 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.2041 \\ -0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.833333 & 1.38778\times10^{-17} & 0.1666 \\ 0.204124 & -0.204124 & -0.204124 & 0.204124 & 1.38778\times10^{-17} & 0.833333 & 6.9388 \\ 0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.166667 & 6.93889\times10^{-18} & 0.8333 \\ -0.204124 & 0.204124 & 0.204124 & -0.204124 & -2.77556\times10^{-17} & 0.166667 & -1.3877 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_q^T = (\,0.\quad 0.\quad 0.\quad 0.\quad 0.\quad 0.\quad 0.\quad 0.\,)$$

$$\begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\ -0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.833333 & 0 & 0.166667 & 0 \\ 0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.833333 & 0 & 0.166667 \\ 0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.166667 & 0 & 0.833333 & 0 \\ -0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.166667 & 0 & 0.833333 \end{pmatrix}$$

$$\begin{pmatrix} \phi_1 \\ \phi_4 \\ \phi_5 \\ \phi_2 \\ \delta_1^{\{1,4\}} \\ \delta_1^{\{4,5\}} \\ \delta_1^{\{2,5\}} \\ \delta_1^{\{1,2\}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 2

Element coordinates: ({0, 5} {5, 5} {5, 10} {0, 10})

$$x(s,t) = \frac{5\,s}{2} + \frac{5}{2}$$

$$y(s,t) = \frac{5\,t}{2} + \frac{15}{2}$$

$$\boldsymbol{J} = \begin{pmatrix} \frac{5}{2} & 0 \\ 0 & \frac{5}{2} \end{pmatrix}$$

$$\det J = \frac{25}{4}$$

Given element data

$k_x = 1$ $k_y = 1$ $p = 0$ $q = 0$

Element data in mapped coordinates

$k_x = 1$ $k_y = 1$ $p = 0$ $q = 0$

Gauss point = {s → −0.57735, t → −0.57735} Weight = 1.

$\boldsymbol{N}^{T}$ = {0.622008, 0.166667, 0.0446582, 0.166667, −0.321975, −0.086273, −0.086273, −0.321975}

$\partial \boldsymbol{N}^{T}/\partial s$ =

(−0.394338 0.394338 0.105662 −0.105662 −0.557678 −0.204124 −0.149429 0.204124)

$\partial \boldsymbol{N}^{T}/\partial t$ =

(−0.394338 −0.105662 0.105662 0.394338 0.204124 −0.149429 −0.204124 −0.557678)

$\boldsymbol{J}^{-T} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix}$ detJ = 6.25

$\boldsymbol{B}^{T}$ =

$$\begin{pmatrix} -0.157735 & 0.157735 & 0.042265 & -0.042265 & -0.223071 & -0.0816497 & -0.0597717 & 0.0816497 \\ -0.157735 & -0.042265 & 0.042265 & 0.157735 & 0.0816497 & -0.0597717 & -0.0816497 & -0.223071 \end{pmatrix}$$

k_x = 1. k_y = 1. p = 0. q = 0.

$$\boldsymbol{k}_k = \begin{pmatrix} 0.311004 & -0.113835 & -0.0833333 & -0.113835 & 0.139419 & 0.139419 & 0.139419 \\ -0.113835 & 0.166667 & 0.0305021 & -0.0833333 & -0.241481 & -0.0647048 & -0.0373573 \\ -0.0833333 & 0.0305021 & 0.0223291 & 0.0305021 & -0.0373573 & -0.0373573 & -0.0373573 \\ -0.113835 & -0.0833333 & 0.0305021 & 0.166667 & 0.139419 & -0.0373573 & -0.0647048 \\ 0.139419 & -0.241481 & -0.0373573 & 0.139419 & 0.352671 & 0.0833333 & 0.0416667 \\ 0.139419 & -0.0647048 & -0.0373573 & -0.0373573 & 0.0833333 & 0.0639958 & 0.0610042 \\ 0.139419 & -0.0373573 & -0.0373573 & -0.0647048 & 0.0416667 & 0.0610042 & 0.0639958 \\ 0.139419 & 0.139419 & -0.0373573 & -0.241481 & -0.227671 & 0.0416667 & 0.0833333 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$\boldsymbol{r}_q^{T}$ = (0. 0. 0. 0. 0. 0. 0. 0.)

Gauss point = {s → −0.57735, t → 0.57735} Weight = 1.

$\boldsymbol{N}^{T}$ = {0.166667, 0.0446582, 0.166667, 0.622008, −0.086273, −0.086273, −0.321975, −0.321975}

$\partial \boldsymbol{N}^{T}/\partial s$ =

(−0.105662 0.105662 0.394338 −0.394338 −0.149429 −0.204124 −0.557678 0.204124)

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} =$

$$\begin{pmatrix} -0.394338 & -0.105662 & 0.105662 & 0.394338 & 0.204124 & 0.149429 & -0.204124 & 0.557678 \end{pmatrix}$$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix} \qquad \det J = 6.25$$

$$\boldsymbol{B}^{\mathrm{T}} = \begin{pmatrix} -0.042265 & 0.042265 & 0.157735 & -0.157735 & -0.0597717 & -0.0816497 & -0.223071 & 0.0816497 \\ -0.157735 & -0.042265 & 0.042265 & 0.157735 & 0.0816497 & 0.0597717 & -0.0816497 & 0.223071 \end{pmatrix}$$

$k_x = 1.$ $\quad k_y = 1.$ $\quad p = 0.$ $\quad q = 0.$

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 0.166667 & 0.0305021 & -0.0833333 & -0.113835 & -0.0647048 & -0.0373573 & 0.139419 \\ 0.0305021 & 0.0223291 & 0.0305021 & -0.0833333 & -0.0373573 & -0.0373573 & -0.0373573 \\ -0.0833333 & 0.0305021 & 0.166667 & -0.113835 & -0.0373573 & -0.0647048 & -0.241481 \\ -0.113835 & -0.0833333 & -0.113835 & 0.311004 & 0.139419 & 0.139419 & 0.139419 \\ -0.0647048 & -0.0373573 & -0.0373573 & 0.139419 & 0.0639958 & 0.0610042 & 0.0416667 \\ -0.0373573 & -0.0373573 & -0.0647048 & 0.139419 & 0.0610042 & 0.0639958 & 0.0833333 \\ 0.139419 & -0.0373573 & -0.241481 & 0.139419 & 0.0416667 & 0.0833333 & 0.352671 \\ -0.241481 & -0.0373573 & 0.139419 & 0.139419 & 0.0833333 & 0.0416667 & -0.227671 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

Gauss point = $\{s \to 0.57735, t \to -0.57735\}$ $\qquad$ Weight = 1.

$\boldsymbol{N}^{\mathrm{T}} = \{0.166667, 0.622008, 0.166667, 0.0446582, -0.321975, -0.321975, -0.086273, -0.086273\}$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} =$

$$\begin{pmatrix} -0.394338 & 0.394338 & 0.105662 & -0.105662 & 0.557678 & -0.204124 & 0.149429 & 0.204124 \end{pmatrix}$$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} =$

$$\begin{pmatrix} -0.105662 & -0.394338 & 0.394338 & 0.105662 & 0.204124 & -0.557678 & -0.204124 & -0.149429 \end{pmatrix}$$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix} \qquad \det J = 6.25$$

$\boldsymbol{B}^{\mathrm{T}} =$

$$\begin{pmatrix} -0.157735 & 0.157735 & 0.042265 & -0.042265 & 0.223071 & -0.0816497 & 0.0597717 & 0.0816497 \\ -0.042265 & -0.157735 & 0.157735 & 0.042265 & 0.0816497 & -0.223071 & -0.0816497 & -0.0597717 \end{pmatrix}$$

$k_x = 1.$ $k_y = 1.$ $p = 0.$ $q = 0.$

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 0.166667 & -0.113835 & -0.0833333 & 0.0305021 & -0.241481 & 0.139419 & -0.0373573 \\ -0.113835 & 0.311004 & -0.113835 & -0.0833333 & 0.139419 & 0.139419 & 0.139419 \\ -0.0833333 & -0.113835 & 0.166667 & 0.0305021 & 0.139419 & -0.241481 & -0.0647048 \\ 0.0305021 & -0.0833333 & 0.0305021 & 0.0223291 & -0.0373573 & -0.0373573 & -0.0373573 \\ -0.241481 & 0.139419 & 0.139419 & -0.0373573 & 0.352671 & -0.227671 & 0.0416667 \\ 0.139419 & 0.139419 & -0.241481 & -0.0373573 & -0.227671 & 0.352671 & 0.0833333 \\ -0.0373573 & 0.139419 & -0.0647048 & -0.0373573 & 0.0416667 & 0.0833333 & 0.0639958 \\ -0.0647048 & 0.139419 & -0.0373573 & -0.0373573 & 0.0833333 & 0.0416667 & 0.0610042 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\ 0.\ \ 0.\ \ 0.\ \ 0.\ \ 0.\ \ 0.\ \ 0.\ \ 0.\ )$

Gauss point = {s → 0.57735, t → 0.57735} Weight = 1.

$\boldsymbol{N}^{\mathrm{T}} = \{0.0446582, 0.166667, 0.622008, 0.166667, -0.086273, -0.321975, -0.321975, -0.086273\}$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} =$

$$(\ -0.105662\ \ 0.105662\ \ 0.394338\ \ -0.394338\ \ 0.149429\ \ -0.204124\ \ 0.557678\ \ 0.204124\ )$$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} =$

$$(\ -0.105662\ \ -0.394338\ \ 0.394338\ \ 0.105662\ \ 0.204124\ \ 0.557678\ \ -0.204124\ \ 0.149429\ )$$

$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix}$ $\mathrm{detJ} = 6.25$

$\boldsymbol{B}^{\mathrm{T}} =$

$$\begin{pmatrix} -0.042265 & 0.042265 & 0.157735 & -0.157735 & 0.0597717 & -0.0816497 & 0.223071 & 0.0816497 \\ -0.042265 & -0.157735 & 0.157735 & 0.042265 & 0.0816497 & 0.223071 & -0.0816497 & 0.0597717 \end{pmatrix}$$

$k_x = 1.$ $k_y = 1.$ $p = 0.$ $q = 0.$

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 0.0223291 & 0.0305021 & -0.0833333 & 0.0305021 & -0.0373573 & -0.0373573 & -0.0373573 \\ 0.0305021 & 0.166667 & -0.113835 & -0.0833333 & -0.0647048 & -0.241481 & 0.139419 \\ -0.0833333 & -0.113835 & 0.311004 & -0.113835 & 0.139419 & 0.139419 & 0.139419 \\ 0.0305021 & -0.0833333 & -0.113835 & 0.166667 & -0.0373573 & 0.139419 & -0.241481 \\ -0.0373573 & -0.0647048 & 0.139419 & -0.0373573 & 0.0639958 & 0.0833333 & 0.0416667 \\ -0.0373573 & -0.241481 & 0.139419 & 0.139419 & 0.0833333 & 0.352671 & -0.227671 \\ -0.0373573 & 0.139419 & 0.139419 & -0.241481 & 0.0416667 & -0.227671 & 0.352671 \\ -0.0373573 & -0.0373573 & 0.139419 & -0.0647048 & 0.0610042 & 0.0416667 & 0.0833333 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

Adding contributions from all Gauss points

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.2041 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.2041 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.2041 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.2041 \\ -0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.833333 & 1.38778\times10^{-17} & 0.1666 \\ 0.204124 & -0.204124 & -0.204124 & 0.204124 & 1.38778\times10^{-17} & 0.833333 & 6.9388 \\ 0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.166667 & 6.93889\times10^{-18} & 0.8333 \\ -0.204124 & 0.204124 & 0.204124 & -0.204124 & -2.77556\times10^{-17} & 0.166667 & -1.3877 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\ -0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.833333 & 0 & 0.166667 & 0 \\ 0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.833333 & 0 & 0.166667 \\ 0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.166667 & 0 & 0.833333 & 0 \\ -0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.166667 & 0 & 0.833333 \end{pmatrix}$$

$$\begin{pmatrix} \phi_2 \\ \phi_5 \\ \phi_6 \\ \phi_3 \\ \delta_1^{\{2,5\}} \\ \delta_1^{\{5,6\}} \\ \delta_1^{\{3,6\}} \\ \delta_1^{\{2,3\}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 3

Element coordinates: ({5, 0} {10, 0} {10, 5} {5, 5})

$$x(s,t) = \frac{5\,s}{2} + \frac{15}{2}$$

$$y(s,t) = \frac{5\,t}{2} + \frac{5}{2}$$

$$\boldsymbol{J} = \begin{pmatrix} \frac{5}{2} & 0 \\ 0 & \frac{5}{2} \end{pmatrix}$$

$$\text{detJ} = \frac{25}{4}$$

Given element data

$k_x = 1$ $k_y = 1$ $p = 0$ $q = 0$

Element data in mapped coordinates

$k_x = 1$ $k_y = 1$ $p = 0$ $q = 0$

Gauss point = {s → −0.57735, t → −0.57735} Weight = 1.

$\boldsymbol{N}^{\mathrm{T}} = \{0.622008, 0.166667, 0.0446582, 0.166667, -0.321975, -0.086273, -0.086273, -0.321975\}$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} =$

$$(\,-0.394338 \quad 0.394338 \quad 0.105662 \quad -0.105662 \quad -0.557678 \quad -0.204124 \quad -0.149429 \quad 0.204124\,)$$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} =$

$$(\,-0.394338 \quad -0.105662 \quad 0.105662 \quad 0.394338 \quad 0.204124 \quad -0.149429 \quad -0.204124 \quad -0.557678\,)$$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix} \qquad \mathrm{detJ} = 6.25$$

$\boldsymbol{B}^{\mathrm{T}} =$

$$\begin{pmatrix} -0.157735 & 0.157735 & 0.042265 & -0.042265 & -0.223071 & -0.0816497 & -0.0597717 & 0.0816497 \\ -0.157735 & -0.042265 & 0.042265 & 0.157735 & 0.0816497 & -0.0597717 & -0.0816497 & -0.223071 \end{pmatrix}$$

$k_x = 1. \qquad k_y = 1. \qquad p = 0. \qquad q = 0.$

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 0.311004 & -0.113835 & -0.0833333 & -0.113835 & 0.139419 & 0.139419 & 0.139419 \\ -0.113835 & 0.166667 & 0.0305021 & -0.0833333 & -0.241481 & -0.0647048 & -0.0373573 \\ -0.0833333 & 0.0305021 & 0.0223291 & 0.0305021 & -0.0373573 & -0.0373573 & -0.0373573 \\ -0.113835 & -0.0833333 & 0.0305021 & 0.166667 & 0.139419 & -0.0373573 & -0.0647048 \\ 0.139419 & -0.241481 & -0.0373573 & 0.139419 & 0.352671 & 0.0833333 & 0.0416667 \\ 0.139419 & -0.0647048 & -0.0373573 & -0.0373573 & 0.0833333 & 0.0639958 & 0.0610042 \\ 0.139419 & -0.0373573 & -0.0373573 & -0.0647048 & 0.0416667 & 0.0610042 & 0.0639958 \\ 0.139419 & 0.139419 & -0.0373573 & -0.241481 & -0.227671 & 0.0416667 & 0.0833333 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\,0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0.\,)$$

Gauss point = $\{s \to -0.57735, t \to 0.57735\}$ Weight = 1.

$\boldsymbol{N}^{\mathrm{T}} = \{0.166667, 0.0446582, 0.166667, 0.622008, -0.086273, -0.086273, -0.321975, -0.321975\}$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} =$

$$(\,-0.105662 \quad 0.105662 \quad 0.394338 \quad -0.394338 \quad -0.149429 \quad -0.204124 \quad -0.557678 \quad 0.204124\,)$$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} =$

$$(\,-0.394338 \quad -0.105662 \quad 0.105662 \quad 0.394338 \quad 0.204124 \quad 0.149429 \quad -0.204124 \quad 0.557678\,)$$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix} \qquad \mathrm{detJ} = 6.25$$

$$\boldsymbol{B}^{\mathrm{T}} =$$

$$\begin{pmatrix} -0.042265 & 0.042265 & 0.157735 & -0.157735 & -0.0597717 & -0.0816497 & -0.223071 & 0.0816497 \\ -0.157735 & -0.042265 & 0.042265 & 0.157735 & 0.0816497 & 0.0597717 & -0.0816497 & 0.223071 \end{pmatrix}$$

$\mathrm{k_x} = 1.$ $\mathrm{k_y} = 1.$ $\mathrm{p} = 0.$ $\mathrm{q} = 0.$

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 0.166667 & 0.0305021 & -0.0833333 & -0.113835 & -0.0647048 & -0.0373573 & 0.139419 \\ 0.0305021 & 0.0223291 & 0.0305021 & -0.0833333 & -0.0373573 & -0.0373573 & -0.0373573 \\ -0.0833333 & 0.0305021 & 0.166667 & -0.113835 & -0.0373573 & -0.0647048 & -0.241481 \\ -0.113835 & -0.0833333 & -0.113835 & 0.311004 & 0.139419 & 0.139419 & 0.139419 \\ -0.0647048 & -0.0373573 & -0.0373573 & 0.139419 & 0.0639958 & 0.0610042 & 0.0416667 \\ -0.0373573 & -0.0373573 & -0.0647048 & 0.139419 & 0.0610042 & 0.0639958 & 0.0833333 \\ 0.139419 & -0.0373573 & -0.241481 & 0.139419 & 0.0416667 & 0.0833333 & 0.352671 \\ -0.241481 & -0.0373573 & 0.139419 & 0.139419 & 0.0833333 & 0.0416667 & -0.227671 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\,0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0.\,)$$

Gauss point = $\{\mathrm{s} \to 0.57735,\ \mathrm{t} \to -0.57735\}$ Weight = 1.

$$\boldsymbol{N}^{\mathrm{T}} = \{0.166667,\ 0.622008,\ 0.166667,\ 0.0446582,\ -0.321975,\ -0.321975,\ -0.086273,\ -0.086273\}$$

$$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} =$$

$$(\,-0.394338 \quad 0.394338 \quad 0.105662 \quad -0.105662 \quad 0.557678 \quad -0.204124 \quad 0.149429 \quad 0.204124\,)$$

$$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} =$$

$$(\,-0.105662 \quad -0.394338 \quad 0.394338 \quad 0.105662 \quad 0.204124 \quad -0.557678 \quad -0.204124 \quad -0.149429\,)$$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix} \qquad \mathrm{detJ} = 6.25$$

$$\boldsymbol{B}^{\mathrm{T}} =$$

$$\begin{pmatrix} -0.157735 & 0.157735 & 0.042265 & -0.042265 & 0.223071 & -0.0816497 & 0.0597717 & 0.0816497 \\ -0.042265 & -0.157735 & 0.157735 & 0.042265 & 0.0816497 & -0.223071 & -0.0816497 & -0.0597717 \end{pmatrix}$$

$k_x = 1.$ $k_y = 1.$ $p = 0.$ $q = 0.$

$$\boldsymbol{k}_k = \begin{pmatrix} 0.166667 & -0.113835 & -0.0833333 & 0.0305021 & -0.241481 & 0.139419 & -0.0373573 \\ -0.113835 & 0.311004 & -0.113835 & -0.0833333 & 0.139419 & 0.139419 & 0.139419 \\ -0.0833333 & -0.113835 & 0.166667 & 0.0305021 & 0.139419 & -0.241481 & -0.0647048 \\ 0.0305021 & -0.0833333 & 0.0305021 & 0.0223291 & -0.0373573 & -0.0373573 & -0.0373573 \\ -0.241481 & 0.139419 & 0.139419 & -0.0373573 & 0.352671 & -0.227671 & 0.0416667 \\ 0.139419 & 0.139419 & -0.241481 & -0.0373573 & -0.227671 & 0.352671 & 0.0833333 \\ -0.0373573 & 0.139419 & -0.0647048 & -0.0373573 & 0.0416667 & 0.0833333 & 0.0639958 \\ -0.0647048 & 0.139419 & -0.0373573 & -0.0373573 & 0.0833333 & 0.0416667 & 0.0610042 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_q^T = (\,0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0.\,)$$

Gauss point = {s → 0.57735, t → 0.57735} Weight = 1.

$$\boldsymbol{N}^T = \{0.0446582,\ 0.166667,\ 0.622008,\ 0.166667,\ -0.086273,\ -0.321975,\ -0.321975,\ -0.086273\}$$

$$\partial \boldsymbol{N}^T/\partial s =$$

$$(\,-0.105662 \quad 0.105662 \quad 0.394338 \quad -0.394338 \quad 0.149429 \quad -0.204124 \quad 0.557678 \quad 0.204124\,)$$

$$\partial \boldsymbol{N}^T/\partial t =$$

$$(\,-0.105662 \quad -0.394338 \quad 0.394338 \quad 0.105662 \quad 0.204124 \quad 0.557678 \quad -0.204124 \quad 0.149429\,)$$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix} \qquad \det J = 6.25$$

$$\boldsymbol{B}^T =$$

$$\begin{pmatrix} -0.042265 & 0.042265 & 0.157735 & -0.157735 & 0.0597717 & -0.0816497 & 0.223071 & 0.0816497 \\ -0.042265 & -0.157735 & 0.157735 & 0.042265 & 0.0816497 & 0.223071 & -0.0816497 & 0.0597717 \end{pmatrix}$$

$k_x = 1.$ $k_y = 1.$ $p = 0.$ $q = 0.$

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 0.0223291 & 0.0305021 & -0.0833333 & 0.0305021 & -0.0373573 & -0.0373573 & -0.0373573 \\ 0.0305021 & 0.166667 & -0.113835 & -0.0833333 & -0.0647048 & -0.241481 & 0.139419 \\ -0.0833333 & -0.113835 & 0.311004 & -0.113835 & 0.139419 & 0.139419 & 0.139419 \\ 0.0305021 & -0.0833333 & -0.113835 & 0.166667 & -0.0373573 & 0.139419 & -0.241481 \\ -0.0373573 & -0.0647048 & 0.139419 & -0.0373573 & 0.0639958 & 0.0833333 & 0.0416667 \\ -0.0373573 & -0.241481 & 0.139419 & 0.139419 & 0.0833333 & 0.352671 & -0.227671 \\ -0.0373573 & 0.139419 & 0.139419 & -0.241481 & 0.0416667 & -0.227671 & 0.352671 \\ -0.0373573 & -0.0373573 & 0.139419 & -0.0647048 & 0.0610042 & 0.0416667 & 0.0833333 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\,0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0.\,)$$

Adding contributions from all Gauss points

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.2041 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.2041 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.2041 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.2041 \\ -0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.833333 & 1.38778\times10^{-17} & 0.1666 \\ 0.204124 & -0.204124 & -0.204124 & 0.204124 & 1.38778\times10^{-17} & 0.833333 & 6.9388 \\ 0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.166667 & 6.93889\times10^{-18} & 0.8333 \\ -0.204124 & 0.204124 & 0.204124 & -0.204124 & -2.77556\times10^{-17} & 0.166667 & -1.3877 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\,0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0.\,)$$

$$\begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\ -0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.833333 & 0 & 0.166667 & 0 \\ 0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.833333 & 0 & 0.166667 \\ 0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.166667 & 0 & 0.833333 & 0 \\ -0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.166667 & 0 & 0.833333 \end{pmatrix}$$

$$\begin{pmatrix} \phi_4 \\ \phi_7 \\ \phi_8 \\ \phi_5 \\ \delta_1^{\{4,7\}} \\ \delta_1^{\{7,8\}} \\ \delta_1^{\{5,8\}} \\ \delta_1^{\{4,5\}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 4

Element coordinates: ({5, 5} {10, 5} {10, 10} {5, 10})

$$x(s,t) = \frac{5\,s}{2} + \frac{15}{2}$$

$$y(s,t) = \frac{5\,t}{2} + \frac{15}{2}$$

$$\boldsymbol{J} = \begin{pmatrix} \frac{5}{2} & 0 \\ 0 & \frac{5}{2} \end{pmatrix}$$

$$\det J = \frac{25}{4}$$

Given element data

$k_x = 1$ $k_y = 1$ $p = 0$ $q = 0$

Element data in mapped coordinates

$k_x = 1$ $k_y = 1$ $p = 0$ $q = 0$

Gauss point = {s → −0.57735, t → −0.57735} Weight = 1.

$\boldsymbol{N}^{T}$ = {0.622008, 0.166667, 0.0446582, 0.166667, −0.321975, −0.086273, −0.086273, −0.321975}

$\partial\boldsymbol{N}^{T}/\partial s$ =

(−0.394338 0.394338 0.105662 −0.105662 −0.557678 −0.204124 −0.149429 0.204124)

$\partial\boldsymbol{N}^{T}/\partial t$ =

(−0.394338 −0.105662 0.105662 0.394338 0.204124 −0.149429 −0.204124 −0.557678)

$\boldsymbol{J}^{-T} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix}$ detJ = 6.25

$\boldsymbol{B}^{T}$ =

$$\begin{pmatrix} -0.157735 & 0.157735 & 0.042265 & -0.042265 & -0.223071 & -0.0816497 & -0.0597717 & 0.0816497 \\ -0.157735 & -0.042265 & 0.042265 & 0.157735 & 0.0816497 & -0.0597717 & -0.0816497 & -0.223071 \end{pmatrix}$$

k_x = 1. k_y = 1. p = 0. q = 0.

$$\boldsymbol{k}_k = \begin{pmatrix} 0.311004 & -0.113835 & -0.0833333 & -0.113835 & 0.139419 & 0.139419 & 0.139419 \\ -0.113835 & 0.166667 & 0.0305021 & -0.0833333 & -0.241481 & -0.0647048 & -0.0373573 \\ -0.0833333 & 0.0305021 & 0.0223291 & 0.0305021 & -0.0373573 & -0.0373573 & -0.0373573 \\ -0.113835 & -0.0833333 & 0.0305021 & 0.166667 & 0.139419 & -0.0373573 & -0.0647048 \\ 0.139419 & -0.241481 & -0.0373573 & 0.139419 & 0.352671 & 0.0833333 & 0.0416667 \\ 0.139419 & -0.0647048 & -0.0373573 & -0.0373573 & 0.0833333 & 0.0639958 & 0.0610042 \\ 0.139419 & -0.0373573 & -0.0373573 & -0.0647048 & 0.0416667 & 0.0610042 & 0.0639958 \\ 0.139419 & 0.139419 & -0.0373573 & -0.241481 & -0.227671 & 0.0416667 & 0.0833333 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$\boldsymbol{r}_q^{T}$ = (0. 0. 0. 0. 0. 0. 0. 0.)

Gauss point = {s → −0.57735, t → 0.57735} Weight = 1.

$\boldsymbol{N}^{T}$ = {0.166667, 0.0446582, 0.166667, 0.622008, −0.086273, −0.086273, −0.321975, −0.321975}

$\partial\boldsymbol{N}^{T}/\partial s$ =

(−0.105662 0.105662 0.394338 −0.394338 −0.149429 −0.204124 −0.557678 0.204124)

$\partial\boldsymbol{N}^{T}/\partial t$ =

(−0.394338 −0.105662 0.105662 0.394338 0.204124 0.149429 −0.204124 0.557678)

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix} \qquad \text{detJ} = 6.25$$

$$\boldsymbol{B}^{T} =$$

$$\begin{pmatrix} -0.042265 & 0.042265 & 0.157735 & -0.157735 & -0.0597717 & -0.0816497 & -0.223071 & 0.0816497 \\ -0.157735 & -0.042265 & 0.042265 & 0.157735 & 0.0816497 & 0.0597717 & -0.0816497 & 0.223071 \end{pmatrix}$$

$k_x = 1.$ $\quad k_y = 1.$ $\quad p = 0.$ $\quad q = 0.$

$$\boldsymbol{k}_k = \begin{pmatrix} 0.166667 & 0.0305021 & -0.0833333 & -0.113835 & -0.0647048 & -0.0373573 & 0.139419 \\ 0.0305021 & 0.0223291 & 0.0305021 & -0.0833333 & -0.0373573 & -0.0373573 & -0.0373573 \\ -0.0833333 & 0.0305021 & 0.166667 & -0.113835 & -0.0373573 & -0.0647048 & -0.241481 \\ -0.113835 & -0.0833333 & -0.113835 & 0.311004 & 0.139419 & 0.139419 & 0.139419 \\ -0.0647048 & -0.0373573 & -0.0373573 & 0.139419 & 0.0639958 & 0.0610042 & 0.0416667 \\ -0.0373573 & -0.0373573 & -0.0647048 & 0.139419 & 0.0610042 & 0.0639958 & 0.0833333 \\ 0.139419 & -0.0373573 & -0.241481 & 0.139419 & 0.0416667 & 0.0833333 & 0.352671 \\ -0.241481 & -0.0373573 & 0.139419 & 0.139419 & 0.0833333 & 0.0416667 & -0.227671 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_q^{T} = (\,0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0.\,)$$

Gauss point = $\{s \to 0.57735,\ t \to -0.57735\}$ $\qquad$ Weight = 1.

$$\boldsymbol{N}^{T} = \{0.166667,\ 0.622008,\ 0.166667,\ 0.0446582,\ -0.321975,\ -0.321975,\ -0.086273,\ -0.086273\}$$

$$\partial \boldsymbol{N}^{T}/\partial s =$$

$$(\,-0.394338 \quad 0.394338 \quad 0.105662 \quad -0.105662 \quad 0.557678 \quad -0.204124 \quad 0.149429 \quad 0.204124\,)$$

$$\partial \boldsymbol{N}^{T}/\partial t =$$

$$(\,-0.105662 \quad -0.394338 \quad 0.394338 \quad 0.105662 \quad 0.204124 \quad -0.557678 \quad -0.204124 \quad -0.149429\,)$$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix} \qquad \text{detJ} = 6.25$$

$$\boldsymbol{B}^{T} =$$

$$\begin{pmatrix} -0.157735 & 0.157735 & 0.042265 & -0.042265 & 0.223071 & -0.0816497 & 0.0597717 & 0.0816497 \\ -0.042265 & -0.157735 & 0.157735 & 0.042265 & 0.0816497 & -0.223071 & -0.0816497 & -0.0597717 \end{pmatrix}$$

$k_x = 1.$ $k_y = 1.$ $p = 0.$ $q = 0.$

$$\boldsymbol{k}_k = \begin{pmatrix} 0.166667 & -0.113835 & -0.0833333 & 0.0305021 & -0.241481 & 0.139419 & -0.0373573 \\ -0.113835 & 0.311004 & -0.113835 & -0.0833333 & 0.139419 & 0.139419 & 0.139419 \\ -0.0833333 & -0.113835 & 0.166667 & 0.0305021 & 0.139419 & -0.241481 & -0.0647048 \\ 0.0305021 & -0.0833333 & 0.0305021 & 0.0223291 & -0.0373573 & -0.0373573 & -0.0373573 \\ -0.241481 & 0.139419 & 0.139419 & -0.0373573 & 0.352671 & -0.227671 & 0.0416667 \\ 0.139419 & 0.139419 & -0.241481 & -0.0373573 & -0.227671 & 0.352671 & 0.0833333 \\ -0.0373573 & 0.139419 & -0.0647048 & -0.0373573 & 0.0416667 & 0.0833333 & 0.0639958 \\ -0.0647048 & 0.139419 & -0.0373573 & -0.0373573 & 0.0833333 & 0.0416667 & 0.0610042 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_q^T = (\, 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \,)$$

Gauss point = {s → 0.57735, t → 0.57735} Weight = 1.

$$\boldsymbol{N}^T = \{0.0446582,\ 0.166667,\ 0.622008,\ 0.166667,\ -0.086273,\ -0.321975,\ -0.321975,\ -0.086273\}$$

$\partial \boldsymbol{N}^T / \partial s =$

$$(\, -0.105662 \quad 0.105662 \quad 0.394338 \quad -0.394338 \quad 0.149429 \quad -0.204124 \quad 0.557678 \quad 0.204124 \,)$$

$\partial \boldsymbol{N}^T / \partial t =$

$$(\, -0.105662 \quad -0.394338 \quad 0.394338 \quad 0.105662 \quad 0.204124 \quad 0.557678 \quad -0.204124 \quad 0.149429 \,)$$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix} \qquad \det J = 6.25$$

$\boldsymbol{B}^T =$

$$\begin{pmatrix} -0.042265 & 0.042265 & 0.157735 & -0.157735 & 0.0597717 & -0.0816497 & 0.223071 & 0.0816497 \\ -0.042265 & -0.157735 & 0.157735 & 0.042265 & 0.0816497 & 0.223071 & -0.0816497 & 0.0597717 \end{pmatrix}$$

$k_x = 1.$ $k_y = 1.$ $p = 0.$ $q = 0.$

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 0.0223291 & 0.0305021 & -0.0833333 & 0.0305021 & -0.0373573 & -0.0373573 & -0.0373573 \\ 0.0305021 & 0.166667 & -0.113835 & -0.0833333 & -0.0647048 & -0.241481 & 0.139419 \\ -0.0833333 & -0.113835 & 0.311004 & -0.113835 & 0.139419 & 0.139419 & 0.139419 \\ 0.0305021 & -0.0833333 & -0.113835 & 0.166667 & -0.0373573 & 0.139419 & -0.241481 \\ -0.0373573 & -0.0647048 & 0.139419 & -0.0373573 & 0.0639958 & 0.0833333 & 0.0416667 \\ -0.0373573 & -0.241481 & 0.139419 & 0.139419 & 0.0833333 & 0.352671 & -0.227671 \\ -0.0373573 & 0.139419 & 0.139419 & -0.241481 & 0.0416667 & -0.227671 & 0.352671 \\ -0.0373573 & -0.0373573 & 0.139419 & -0.0647048 & 0.0610042 & 0.0416667 & 0.0833333 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\,0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0.\,)$$

Adding contributions from all Gauss points

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.2041 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.2041 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.2041 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.2041 \\ -0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.833333 & 1.38778\times10^{-17} & 0.1666 \\ 0.204124 & -0.204124 & -0.204124 & 0.204124 & 1.38778\times10^{-17} & 0.833333 & 6.9388 \\ 0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.166667 & 6.93889\times10^{-18} & 0.8333 \\ -0.204124 & 0.204124 & 0.204124 & -0.204124 & -2.77556\times10^{-17} & 0.166667 & -1.3877 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\,0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0.\,)$$

$$\begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\ -0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.833333 & 0 & 0.166667 & 0 \\ 0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.833333 & 0 & 0.166667 \\ 0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.166667 & 0 & 0.833333 & 0 \\ -0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.166667 & 0 & 0.833333 \end{pmatrix}$$

$$\begin{pmatrix} \phi_5 \\ \phi_8 \\ \phi_9 \\ \phi_6 \\ \delta_1^{\{5,8\}} \\ \delta_1^{\{8,9\}} \\ \delta_1^{\{6,9\}} \\ \delta_1^{\{5,6\}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Global equations after assembling all elements

$$\begin{pmatrix}
0.666667 & -0.166667 & 0 & -0.166667 & -0.333333 & 0 & 0 & 0 & 0 \\
-0.166667 & 1.33333 & -0.166667 & -0.333333 & -0.333333 & -0.333333 & 0 & 0 & 0 \\
0 & -0.166667 & 0.666667 & 0 & -0.333333 & -0.166667 & 0 & 0 & 0 \\
-0.166667 & -0.333333 & 0 & 1.33333 & -0.333333 & 0 & -0.166667 & -0.333333 & 0 \\
-0.333333 & -0.333333 & -0.333333 & -0.333333 & 2.66667 & -0.333333 & -0.333333 & -0.333333 & -0.333333 \\
0 & -0.333333 & -0.166667 & 0 & -0.333333 & 1.33333 & 0 & -0.333333 & -0.166667 \\
0 & 0 & 0 & -0.166667 & -0.333333 & 0 & 0.666667 & -0.166667 & 0 \\
0 & 0 & 0 & -0.333333 & -0.333333 & -0.333333 & -0.166667 & 1.33333 & -0.166667 \\
0 & 0 & 0 & 0 & -0.333333 & -0.166667 & 0 & -0.166667 & 0.666667 \\
-0.204124 & -0.204124 & 0 & 0.204124 & 0.204124 & 0 & 0 & 0 & 0 \\
-0.204124 & 0.204124 & 0 & -0.204124 & 0.204124 & 0 & 0 & 0 & 0 \\
0 & -0.204124 & -0.204124 & 0 & 0.204124 & 0.204124 & 0 & 0 & 0 \\
0.204124 & -0.408248 & 0.204124 & 0.204124 & -0.408248 & 0.204124 & 0 & 0 & 0 \\
0 & 0.204124 & -0.204124 & 0 & 0.204124 & -0.204124 & 0 & 0 & 0 \\
0.204124 & 0.204124 & 0 & -0.408248 & -0.408248 & 0 & 0.204124 & 0.204124 & 0 \\
0 & 0 & 0 & -0.204124 & 0.204124 & 0 & -0.204124 & 0.204124 & 0 \\
0 & 0.204124 & 0.204124 & 0 & -0.408248 & -0.408248 & 0 & 0.204124 & 0.204124 \\
0 & 0 & 0 & 0.204124 & -0.408248 & 0.204124 & 0.204124 & -0.408248 & 0.204124 \\
0 & 0 & 0 & 0 & 0.204124 & -0.204124 & 0 & 0.204124 & -0.204124 \\
0 & 0 & 0 & 0.204124 & 0.204124 & 0 & -0.204124 & -0.204124 & 0 \\
0 & 0 & 0 & 0 & 0.204124 & 0.204124 & 0 & -0.204124 & -0.204124
\end{pmatrix}$$

Essential boundary conditions

On element 1, side 4, specified value = 10

$$\left\{\phi_2,\ \phi_1,\ \delta_1^{\{1,2\}}\right\} = \{10,\ 10,\ 0\}$$

On element 2, side 3, specified value = 1

$$\left\{\phi_6,\ \phi_3,\ \delta_1^{\{3,6\}}\right\} = \{1,\ 1,\ 0\}$$

On element 4, side 3, specified value = 1

$$\left\{\phi_9,\ \phi_6,\ \delta_1^{\{6,9\}}\right\} = \{1,\ 1,\ 0\}$$

Known values from EBC

$$\left\{\phi_1 = 10,\ \phi_2 = 10,\ \phi_3 = 1,\ \phi_6 = 1,\ \phi_9 = 1,\ \delta_1^{\{1,2\}} = 0,\ \delta_1^{\{3,6\}} = 0,\ \delta_1^{\{6,9\}} = 0\right\}$$

Global equations after EBC

$$\begin{pmatrix}
1.33333 & -0.333333 & -0.166667 & -0.333333 & -0.204124 & 0 & 0.204124 & -0.408248 & -0.204124 \\
-0.333333 & 2.66667 & -0.333333 & -0.333333 & 0.204124 & 0.204124 & -0.408248 & -0.408248 & 0.204124 \\
-0.166667 & -0.333333 & 0.666667 & -0.166667 & 0 & 0 & 0 & 0.204124 & -0.204124 \\
-0.333333 & -0.333333 & -0.166667 & 1.33333 & 0 & 0 & 0 & 0.204124 & 0.204124 \\
-0.204124 & 0.204124 & 0 & 0 & 0.833333 & 0 & 0.166667 & 0 & 0 \\
0 & 0.204124 & 0 & 0 & 0 & 0.833333 & 0 & 0 & 0 \\
0.204124 & -0.408248 & 0 & 0 & 0.166667 & 0 & 1.66667 & 0 & 0 \\
-0.408248 & -0.408248 & 0.204124 & 0.204124 & 0 & 0 & 0 & 1.66667 & 0 \\
-0.204124 & 0.204124 & -0.204124 & 0.204124 & 0 & 0 & 0 & 0 & 0.833333 \\
0 & -0.408248 & 0 & 0.204124 & 0 & 0.166667 & 0 & 0 & 0 \\
0.204124 & -0.408248 & 0.204124 & -0.408248 & 0 & 0 & 0 & 0 & 0.166667 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0 & 0 & 0 & 0.166667 & 0 \\
0 & 0.204124 & 0 & -0.204124 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Solving the final system of global equations we get

$$\{\phi_4 = 6.12921,\ \phi_5 = 4.66596,\ \phi_7 = 5.08248,\ \phi_8 = 3.96195,$$
$$\delta_1^{\{1,4\}} = 0.0857292,\ \delta_1^{\{2,3\}} = 1.44833,\ \delta_1^{\{2,5\}} = 1.36347,\ \delta_1^{\{4,5\}} = -0.887807,\ \delta_1^{\{4,7\}} = 0.54473,$$
$$\delta_1^{\{5,6\}} = -0.708827,\ \delta_1^{\{5,8\}} = 0.440829,\ \delta_1^{\{7,8\}} = -0.251281,\ \delta_1^{\{8,9\}} = -0.0306799\}$$

Solution for element 1

DOF values for the element

$$\{\phi_1 = 10,\ \phi_4 = 6.12921,\ \phi_5 = 4.66596,\ \phi_2 = 10,\ \delta_1^{\{1,4\}} = 0.0857292,\ \delta_1^{\{4,5\}} = -0.887807,\ \delta_1^{\{2,5\}} = 1.36347,\ \delta_1^{\{1,2\}} = 0\}$$

$$\boldsymbol{d}^{\mathrm{T}} = (\ 10 \quad 6.12921 \quad 4.66596 \quad 10 \quad 0.0857292 \quad -0.887807 \quad 1.36347 \quad 0\ )$$

Mapping

$$x(s,t) = \frac{5\,s}{2} + \frac{5}{2}$$

$$y(s,t) = \frac{5\,t}{2} + \frac{5}{2}$$

$$\boldsymbol{J} = \begin{pmatrix} \frac{5}{2} & 0 \\ 0 & \frac{5}{2} \end{pmatrix}$$

Element solution at $\{s \to 0,\ t \to 0\}$

Location: {2.5, 2.5}

$\boldsymbol{N}^{\mathrm{T}} = (\,0.25 \quad 0.25 \quad 0.25 \quad 0.25 \quad -0.306186 \quad -0.306186 \quad -0.306186 \quad -0.306186\,)$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} = (\,-0.25 \quad 0.25 \quad 0.25 \quad -0.25 \quad 0. \quad -0.306186 \quad 0. \quad 0.306186\,)$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} = (\,-0.25 \quad -0.25 \quad 0.25 \quad 0.25 \quad 0.306186 \quad 0. \quad -0.306186 \quad 0.\,)$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} \frac{2}{5} & 0 \\ 0 & \frac{2}{5} \end{pmatrix}$$

$\boldsymbol{B}_{\mathrm{x}}^{\mathrm{T}} = \partial \boldsymbol{N}^{\mathrm{T}}/\partial x = (\,-0.1 \quad 0.1 \quad 0.1 \quad -0.1 \quad 0. \quad -0.122474 \quad 0. \quad 0.122474\,)$

$\boldsymbol{B}_{\mathrm{y}}^{\mathrm{T}} = \partial \boldsymbol{N}^{\mathrm{T}}/\partial y = (\,-0.1 \quad -0.1 \quad 0.1 \quad 0.1 \quad 0.122474 \quad 0. \quad -0.122474 \quad 0.\,)$

$\phi = \boldsymbol{N}^{\mathrm{T}}\boldsymbol{d} = 7.5269$

$\partial\phi/\partial x = \boldsymbol{B}_{\mathrm{x}}^{\mathrm{T}}\boldsymbol{d} = -0.811749$

$\partial\phi/\partial y = \boldsymbol{B}_{\mathrm{y}}^{\mathrm{T}}\boldsymbol{d} = -0.302817$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.5269	−0.811749	−0.302817

Solution for element 2

DOF values for the element

$\left\{\phi_2 = 10,\ \phi_5 = 4.66596,\ \phi_6 = 1,\ \phi_3 = 1,\ \delta_1^{\{2,5\}} = 1.36347,\ \delta_1^{\{5,6\}} = -0.708827,\ \delta_1^{\{3,6\}} = 0,\ \delta_1^{\{2,3\}} = 1.44833\right\}$

$\boldsymbol{d}^{\mathrm{T}} = (\,10 \quad 4.66596 \quad 1 \quad 1 \quad 1.36347 \quad -0.708827 \quad 0 \quad 1.44833\,)$

Mapping

$$x(\mathrm{s},\mathrm{t}) = \frac{5\,\mathrm{s}}{2} + \frac{5}{2}$$

$$y(\mathrm{s},\mathrm{t}) = \frac{5\,\mathrm{t}}{2} + \frac{15}{2}$$

$$\boldsymbol{J} = \begin{pmatrix} \frac{5}{2} & 0 \\ 0 & \frac{5}{2} \end{pmatrix}$$

Element solution at $\{\mathrm{s} \to 0,\ \mathrm{t} \to 0\}$

Location: {2.5, 7.5}

$\boldsymbol{N}^{\mathrm{T}} = (\,0.25 \quad 0.25 \quad 0.25 \quad 0.25 \quad -0.306186 \quad -0.306186 \quad -0.306186 \quad -0.306186\,)$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} = (\,-0.25 \quad 0.25 \quad 0.25 \quad -0.25 \quad 0. \quad -0.306186 \quad 0. \quad 0.306186\,)$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} = (\ -0.25\quad -0.25\quad 0.25\quad 0.25\quad 0.306186\quad 0.\quad -0.306186\quad 0.\ )$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} \frac{2}{5} & 0 \\ 0 & \frac{2}{5} \end{pmatrix}$$

$\boldsymbol{B}_{\mathrm{x}}^{\mathrm{T}} = \partial \boldsymbol{N}^{\mathrm{T}}/\partial x = (\ -0.1\quad 0.1\quad 0.1\quad -0.1\quad 0.\quad -0.122474\quad 0.\quad 0.122474\ )$

$\boldsymbol{B}_{\mathrm{y}}^{\mathrm{T}} = \partial \boldsymbol{N}^{\mathrm{T}}/\partial y = (\ -0.1\quad -0.1\quad 0.1\quad 0.1\quad 0.122474\quad 0.\quad -0.122474\quad 0.\ )$

$\phi = \boldsymbol{N}^{\mathrm{T}}\boldsymbol{d} = 3.52259$

$\partial\phi/\partial x = \boldsymbol{B}_{\mathrm{x}}^{\mathrm{T}}\boldsymbol{d} = -0.269207$

$\partial\phi/\partial y = \boldsymbol{B}_{\mathrm{y}}^{\mathrm{T}}\boldsymbol{d} = -1.09961$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	7.5	3.52259	−0.269207	−1.09961

Solution for element 3

DOF values for the element

$\{\phi_4 = 6.12921,\ \phi_7 = 5.08248,\ \phi_8 = 3.96195,\ \phi_5 = 4.66596,$
$\delta_1^{\{4,7\}} = 0.54473,\ \delta_1^{\{7,8\}} = -0.251281,\ \delta_1^{\{5,8\}} = 0.440829,\ \delta_1^{\{4,5\}} = -0.887807\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 6.12921\quad 5.08248\quad 3.96195\quad 4.66596\quad 0.54473\quad -0.251281\quad 0.440829\quad -0.887807\ )$

Mapping

$$x(\mathrm{s},\mathrm{t}) = \frac{5\,\mathrm{s}}{2} + \frac{15}{2}$$

$$y(\mathrm{s},\mathrm{t}) = \frac{5\,\mathrm{t}}{2} + \frac{5}{2}$$

$$\boldsymbol{J} = \begin{pmatrix} \frac{5}{2} & 0 \\ 0 & \frac{5}{2} \end{pmatrix}$$

Element solution at $\{\mathrm{s} \to 0,\ \mathrm{t} \to 0\}$

Location: {7.5, 2.5}

$\boldsymbol{N}^{\mathrm{T}} = (\ 0.25\quad 0.25\quad 0.25\quad 0.25\quad -0.306186\quad -0.306186\quad -0.306186\quad -0.306186\ )$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} = (\ -0.25\quad 0.25\quad 0.25\quad -0.25\quad 0.\quad -0.306186\quad 0.\quad 0.306186\ )$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} = (\ -0.25\quad -0.25\quad 0.25\quad 0.25\quad 0.306186\quad 0.\quad -0.306186\quad 0.\ )$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} \frac{2}{5} & 0 \\ 0 & \frac{2}{5} \end{pmatrix}$$

$\boldsymbol{B}_{\mathrm{x}}^{\mathrm{T}} = \partial \boldsymbol{N}^{\mathrm{T}}/\partial x = (\,-0.1 \quad 0.1 \quad 0.1 \quad -0.1 \quad 0. \quad -0.122474 \quad 0. \quad 0.122474\,)$

$\boldsymbol{B}_{\mathrm{y}}^{\mathrm{T}} = \partial \boldsymbol{N}^{\mathrm{T}}/\partial y = (\,-0.1 \quad -0.1 \quad 0.1 \quad 0.1 \quad 0.122474 \quad 0. \quad -0.122474 \quad 0.\,)$

$\phi = \boldsymbol{N}^{\mathrm{T}}\boldsymbol{d} = 5.00691$

$\partial\phi/\partial x = \boldsymbol{B}_{\mathrm{x}}^{\mathrm{T}}\boldsymbol{d} = -0.253032$

$\partial\phi/\partial y = \boldsymbol{B}_{\mathrm{y}}^{\mathrm{T}}\boldsymbol{d} = -0.245653$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	2.5	5.00691	−0.253032	−0.245653

Solution for element 4

DOF values for the element

$\left\{\phi_5 = 4.66596,\ \phi_8 = 3.96195,\ \phi_9 = 1,\ \phi_6 = 1,\ \delta_1^{\{5,8\}} = 0.440829,\ \delta_1^{\{8,9\}} = -0.0306799,\ \delta_1^{\{6,9\}} = 0,\ \delta_1^{\{5,6\}} = -0.708827\right\}$

$\boldsymbol{d}^{\mathrm{T}} = (\,4.66596 \quad 3.96195 \quad 1 \quad 1 \quad 0.440829 \quad -0.0306799 \quad 0 \quad -0.708827\,)$

Mapping

$x(\mathrm{s},\mathrm{t}) = \dfrac{5\,\mathrm{s}}{2} + \dfrac{15}{2}$

$y(\mathrm{s},\mathrm{t}) = \dfrac{5\,\mathrm{t}}{2} + \dfrac{15}{2}$

$\boldsymbol{J} = \begin{pmatrix} \frac{5}{2} & 0 \\ 0 & \frac{5}{2} \end{pmatrix}$

Element solution at $\{\mathrm{s} \to 0,\ \mathrm{t} \to 0\}$

Location: {7.5, 7.5}

$\boldsymbol{N}^{\mathrm{T}} = (\,0.25 \quad 0.25 \quad 0.25 \quad 0.25 \quad -0.306186 \quad -0.306186 \quad -0.306186 \quad -0.306186\,)$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} = (\,-0.25 \quad 0.25 \quad 0.25 \quad -0.25 \quad 0. \quad -0.306186 \quad 0. \quad 0.306186\,)$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} = (\,-0.25 \quad -0.25 \quad 0.25 \quad 0.25 \quad 0.306186 \quad 0. \quad -0.306186 \quad 0.\,)$

$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} \frac{2}{5} & 0 \\ 0 & \frac{2}{5} \end{pmatrix}$

$\boldsymbol{B}_{\mathrm{x}}^{\mathrm{T}} = \partial \boldsymbol{N}^{\mathrm{T}}/\partial x = (\,-0.1 \quad 0.1 \quad 0.1 \quad -0.1 \quad 0. \quad -0.122474 \quad 0. \quad 0.122474\,)$

$\boldsymbol{B}_{\mathrm{y}}^{\mathrm{T}} = \partial \boldsymbol{N}^{\mathrm{T}}/\partial y = (\,-0.1 \quad -0.1 \quad 0.1 \quad 0.1 \quad 0.122474 \quad 0. \quad -0.122474 \quad 0.\,)$

$\phi = \boldsymbol{N}^{\mathrm{T}}\boldsymbol{d} = 2.74843$

$$\partial\phi/\partial x = \boldsymbol{B}_x^{\mathrm{T}}\boldsymbol{d} = -0.153456$$

$$\partial\phi/\partial y = \boldsymbol{B}_y^{\mathrm{T}}\boldsymbol{d} = -0.608801$$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	7.5	2.74843	−0.153456	−0.608801

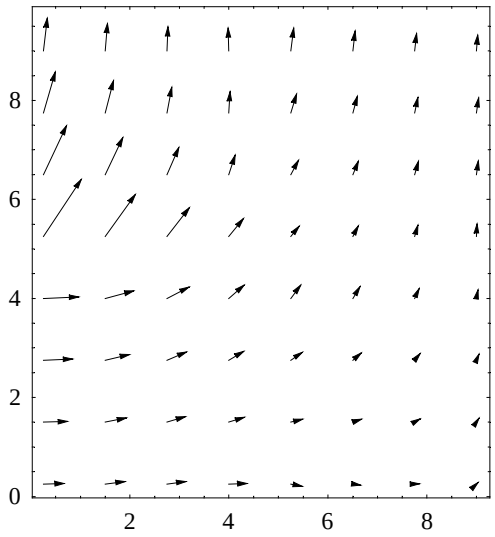
Nodal solution summary

dof	x	y	Value
ϕ_1	0	0	10
ϕ_2	0	5	10
ϕ_3	0	10	1
ϕ_4	5	0	6.12921
ϕ_5	5	5	4.66596
ϕ_6	5	10	1
ϕ_7	10	0	5.08248
ϕ_8	10	5	3.96195
ϕ_9	10	10	1

Element solution summary

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.5269	−0.811749	−0.302817
2	2.5	7.5	3.52259	−0.269207	−1.09961
3	7.5	2.5	5.00691	−0.253032	−0.245653
4	7.5	7.5	2.74843	−0.153456	−0.608801

Computing $\partial\phi/\partial x$ and $\partial\phi/\partial y$ at several points within each element, the velocity field as shown in Figure is obtained.



The $n = 1$ solution corresponds to the conventional rectangular element with bi-linear solution and no p-modes.

Equations for element 1

$$\begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_4 \\ \phi_5 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 2

$$\begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 \end{pmatrix} \begin{pmatrix} \phi_2 \\ \phi_5 \\ \phi_6 \\ \phi_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 3

$$\begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 \end{pmatrix} \begin{pmatrix} \phi_4 \\ \phi_7 \\ \phi_8 \\ \phi_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 4

$$\begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 \end{pmatrix} \begin{pmatrix} \phi_5 \\ \phi_8 \\ \phi_9 \\ \phi_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Global equations after assembling all elements

$$\begin{pmatrix} 0.666667 & -0.166667 & 0 & -0.166667 & -0.333333 & 0 & 0 & 0 & 0 \\ -0.166667 & 1.33333 & -0.166667 & -0.333333 & -0.333333 & -0.333333 & 0 & 0 & 0 \\ 0 & -0.166667 & 0.666667 & 0 & -0.333333 & -0.166667 & 0 & 0 & 0 \\ -0.166667 & -0.333333 & 0 & 1.33333 & -0.333333 & 0 & -0.166667 & -0.333333 & 0 \\ -0.333333 & -0.333333 & -0.333333 & -0.333333 & 2.66667 & -0.333333 & -0.333333 & -0.333333 & -0.333333 \\ 0 & -0.333333 & -0.166667 & 0 & -0.333333 & 1.33333 & 0 & -0.333333 & -0.166667 \\ 0 & 0 & 0 & -0.166667 & -0.333333 & 0 & 0.666667 & -0.166667 & 0 \\ 0 & 0 & 0 & -0.333333 & -0.333333 & -0.333333 & -0.166667 & 1.33333 & -0.166667 \\ 0 & 0 & 0 & 0 & -0.333333 & -0.166667 & 0 & -0.166667 & 0.666667 \end{pmatrix}$$

Essential boundary conditions

On element 1, side 4, specified value = 10

$\{\phi_2, \phi_1\} = \{10, 10\}$

On element 2, side 3, specified value = 1

$\{\phi_6, \phi_3\} = \{1, 1\}$

On element 4, side 3, specified value = 1

$\{\phi_9, \phi_6\} = \{1, 1\}$

Known values from EBC

$\{\phi_1 = 10, \phi_2 = 10, \phi_3 = 1, \phi_6 = 1, \phi_9 = 1\}$

Global equations after EBC

$$\begin{pmatrix} 1.33333 & -0.333333 & -0.166667 & -0.333333 \\ -0.333333 & 2.66667 & -0.333333 & -0.333333 \\ -0.166667 & -0.333333 & 0.666667 & -0.166667 \\ -0.333333 & -0.333333 & -0.166667 & 1.33333 \end{pmatrix} \begin{pmatrix} \phi_4 \\ \phi_5 \\ \phi_7 \\ \phi_8 \end{pmatrix} = \begin{pmatrix} 5. \\ 7.66667 \\ 0 \\ 0.5 \end{pmatrix}$$

Solving the final system of global equations we get

$$\{\phi_4 = 6.52857,\ \phi_5 = 4.79286,\ \phi_7 = 4.98571,\ \phi_8 = 3.82857\}$$

Solution for element 1

DOF values for the element

$$\{\phi_1 = 10,\ \phi_4 = 6.52857,\ \phi_5 = 4.79286,\ \phi_2 = 10\}$$

$$\boldsymbol{d}^{\mathrm{T}} = (\ 10 \quad 6.52857 \quad 4.79286 \quad 10\ )$$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.83036	−0.867857	−0.173571

Solution for element 2

DOF values for the element

$$\{\phi_2 = 10,\ \phi_5 = 4.79286,\ \phi_6 = 1,\ \phi_3 = 1\}$$

$$\boldsymbol{d}^{\mathrm{T}} = (\ 10 \quad 4.79286 \quad 1 \quad 1\ )$$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	7.5	4.19821	−0.520714	−1.27929

Solution for element 3

DOF values for the element

$$\{\phi_4 = 6.52857,\ \phi_7 = 4.98571,\ \phi_8 = 3.82857,\ \phi_5 = 4.79286\}$$

$$\boldsymbol{d}^{\mathrm{T}} = (\ 6.52857 \quad 4.98571 \quad 3.82857 \quad 4.79286\ )$$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	2.5	5.03393	−0.250714	−0.289286

Solution for element 4

DOF values for the element

$$\{\phi_5 = 4.79286,\ \phi_8 = 3.82857,\ \phi_9 = 1,\ \phi_6 = 1\}$$

$$\boldsymbol{d}^{\mathrm{T}} = (\ 4.79286 \quad 3.82857 \quad 1 \quad 1\ )$$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	7.5	2.65536	−0.0964286	−0.662143

Nodal solution summary

dof	x	y	Value
ϕ_1	0	0	10
ϕ_2	0	5	10
ϕ_3	0	10	1
ϕ_4	5	0	6.52857
ϕ_5	5	5	4.79286
ϕ_6	5	10	1
ϕ_7	10	0	4.98571
ϕ_8	10	5	3.82857
ϕ_9	10	10	1

Element solution summary

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.83036	−0.867857	−0.173571
2	2.5	7.5	4.19821	−0.520714	−1.27929
3	7.5	2.5	5.03393	−0.250714	−0.289286
4	7.5	7.5	2.65536	−0.0964286	−0.662143

Solution with $n = 2$

Equations for element 1

$$\begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\ -0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.833333 & 0 & 0.166667 & 0 \\ 0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.833333 & 0 & 0.166667 \\ 0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.166667 & 0 & 0.833333 & 0 \\ -0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.166667 & 0 & 0.833333 \end{pmatrix}$$

$$\begin{pmatrix} \phi_1 \\ \phi_4 \\ \phi_5 \\ \phi_2 \\ \delta_1^{\{1,4\}} \\ \delta_1^{\{4,5\}} \\ \delta_1^{\{2,5\}} \\ \delta_1^{\{1,2\}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 2

$$\begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\ -0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.833333 & 0 & 0.166667 & 0 \\ 0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.833333 & 0 & 0.166667 \\ 0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.166667 & 0 & 0.833333 & 0 \\ -0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.166667 & 0 & 0.833333 \end{pmatrix}$$

$$\begin{pmatrix} \phi_2 \\ \phi_5 \\ \phi_6 \\ \phi_3 \\ \delta_1^{\{2,5\}} \\ \delta_1^{\{5,6\}} \\ \delta_1^{\{3,6\}} \\ \delta_1^{\{2,3\}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 3

$$\begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\ -0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.833333 & 0 & 0.166667 & 0 \\ 0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.833333 & 0 & 0.166667 \\ 0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.166667 & 0 & 0.833333 & 0 \\ -0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.166667 & 0 & 0.833333 \end{pmatrix}$$

$$\begin{pmatrix} \phi_4 \\ \phi_7 \\ \phi_8 \\ \phi_5 \\ \delta_1^{\{4,7\}} \\ \delta_1^{\{7,8\}} \\ \delta_1^{\{5,8\}} \\ \delta_1^{\{4,5\}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 4

$$\begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\ -0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.833333 & 0 & 0.166667 & 0 \\ 0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.833333 & 0 & 0.166667 \\ 0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.166667 & 0 & 0.833333 & 0 \\ -0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.166667 & 0 & 0.833333 \end{pmatrix}$$

$$\begin{pmatrix} \phi_5 \\ \phi_8 \\ \phi_9 \\ \phi_6 \\ \delta_1^{\{5,8\}} \\ \delta_1^{\{8,9\}} \\ \delta_1^{\{6,9\}} \\ \delta_1^{\{5,6\}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Global equations after assembling all elements

$$\begin{pmatrix}
0.666667 & -0.166667 & 0 & -0.166667 & -0.333333 & 0 & 0 & 0 & 0 \\
-0.166667 & 1.33333 & -0.166667 & -0.333333 & -0.333333 & -0.333333 & 0 & 0 & 0 \\
0 & -0.166667 & 0.666667 & 0 & -0.333333 & -0.166667 & 0 & 0 & 0 \\
-0.166667 & -0.333333 & 0 & 1.33333 & -0.333333 & 0 & -0.166667 & -0.333333 & 0 \\
-0.333333 & -0.333333 & -0.333333 & -0.333333 & 2.66667 & -0.333333 & -0.333333 & -0.333333 & -0.333333 \\
0 & -0.333333 & -0.166667 & 0 & -0.333333 & 1.33333 & 0 & -0.333333 & -0.166667 \\
0 & 0 & 0 & -0.166667 & -0.333333 & 0 & 0.666667 & -0.166667 & 0 \\
0 & 0 & 0 & -0.333333 & -0.333333 & -0.333333 & -0.166667 & 1.33333 & -0.166667 \\
0 & 0 & 0 & 0 & -0.333333 & -0.166667 & 0 & -0.166667 & 0.666667 \\
-0.204124 & -0.204124 & 0 & 0.204124 & 0.204124 & 0 & 0 & 0 & 0 \\
-0.204124 & 0.204124 & 0 & -0.204124 & 0.204124 & 0 & 0 & 0 & 0 \\
0 & -0.204124 & -0.204124 & 0 & 0.204124 & 0.204124 & 0 & 0 & 0 \\
0.204124 & -0.408248 & 0.204124 & 0.204124 & -0.408248 & 0.204124 & 0 & 0 & 0 \\
0 & 0.204124 & -0.204124 & 0 & 0.204124 & -0.204124 & 0 & 0 & 0 \\
0.204124 & 0.204124 & 0 & -0.408248 & -0.408248 & 0 & 0.204124 & 0.204124 & 0 \\
0 & 0 & 0 & -0.204124 & 0.204124 & 0 & -0.204124 & 0.204124 & 0 \\
0 & 0.204124 & 0.204124 & 0 & -0.408248 & -0.408248 & 0 & 0.204124 & 0.204124 \\
0 & 0 & 0 & 0.204124 & -0.408248 & 0.204124 & 0.204124 & -0.408248 & 0.204124 \\
0 & 0 & 0 & 0 & 0.204124 & -0.204124 & 0 & 0.204124 & -0.204124 \\
0 & 0 & 0 & 0.204124 & 0.204124 & 0 & -0.204124 & -0.204124 & 0 \\
0 & 0 & 0 & 0 & 0.204124 & 0.204124 & 0 & -0.204124 & -0.204124
\end{pmatrix}$$

Essential boundary conditions

On element 1, side 4, specified value = 10

$$\left\{\phi_2,\ \phi_1,\ \delta_1^{\{1,2\}}\right\} = \{10,\ 10,\ 0\}$$

On element 2, side 3, specified value = 1

$$\left\{\phi_6,\ \phi_3,\ \delta_1^{\{3,6\}}\right\} = \{1,\ 1,\ 0\}$$

On element 4, side 3, specified value = 1

$$\left\{\phi_9,\ \phi_6,\ \delta_1^{\{6,9\}}\right\} = \{1,\ 1,\ 0\}$$

Known values from EBC

$$\left\{\phi_1 = 10,\ \phi_2 = 10,\ \phi_3 = 1,\ \phi_6 = 1,\ \phi_9 = 1,\ \delta_1^{\{1,2\}} = 0,\ \delta_1^{\{3,6\}} = 0,\ \delta_1^{\{6,9\}} = 0\right\}$$

Global equations after EBC

$$\begin{pmatrix}
1.33333 & -0.333333 & -0.166667 & -0.333333 & -0.204124 & 0 & 0.204124 & -0.408248 & -0.204124 \\
-0.333333 & 2.66667 & -0.333333 & -0.333333 & 0.204124 & 0.204124 & -0.408248 & -0.408248 & 0.204124 \\
-0.166667 & -0.333333 & 0.666667 & -0.166667 & 0 & 0 & 0 & 0.204124 & -0.204124 \\
-0.333333 & -0.333333 & -0.166667 & 1.33333 & 0 & 0 & 0 & 0.204124 & 0.204124 \\
-0.204124 & 0.204124 & 0 & 0 & 0.833333 & 0 & 0.166667 & 0 & 0 \\
0 & 0.204124 & 0 & 0 & 0 & 0.833333 & 0 & 0 & 0 \\
0.204124 & -0.408248 & 0 & 0 & 0.166667 & 0 & 1.66667 & 0 & 0 \\
-0.408248 & -0.408248 & 0.204124 & 0.204124 & 0 & 0 & 0 & 1.66667 & 0 \\
-0.204124 & 0.204124 & -0.204124 & 0.204124 & 0 & 0 & 0 & 0 & 0.833333 \\
0 & -0.408248 & 0 & 0.204124 & 0 & 0.166667 & 0 & 0 & 0 \\
0.204124 & -0.408248 & 0.204124 & -0.408248 & 0 & 0 & 0 & 0 & 0.166667 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0 & 0 & 0 & 0.166667 & 0 \\
0 & 0.204124 & 0 & -0.204124 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Solving the final system of global equations we get

$$\{\phi_4 = 6.12921,\ \phi_5 = 4.66596,\ \phi_7 = 5.08248,\ \phi_8 = 3.96195,$$
$$\delta_1^{\{1,4\}} = 0.0857292,\ \delta_1^{\{2,3\}} = 1.44833,\ \delta_1^{\{2,5\}} = 1.36347,\ \delta_1^{\{4,5\}} = -0.887807,\ \delta_1^{\{4,7\}} = 0.54473,$$
$$\delta_1^{\{5,6\}} = -0.708827,\ \delta_1^{\{5,8\}} = 0.440829,\ \delta_1^{\{7,8\}} = -0.251281,\ \delta_1^{\{8,9\}} = -0.0306799\}$$

Solution for element 1

DOF values for the element

$$\{\phi_1 = 10,\ \phi_4 = 6.12921,\ \phi_5 = 4.66596,\ \phi_2 = 10,\ \delta_1^{\{1,4\}} = 0.0857292,\ \delta_1^{\{4,5\}} = -0.887807,\ \delta_1^{\{2,5\}} = 1.36347,\ \delta_1^{\{1,2\}} = 0\}$$

$$\boldsymbol{d}^{\mathrm{T}} = (\ 10 \quad 6.12921 \quad 4.66596 \quad 10 \quad 0.0857292 \quad -0.887807 \quad 1.36347 \quad 0\ )$$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.5269	−0.811749	−0.302817

Solution for element 2

DOF values for the element

$$\{\phi_2 = 10,\ \phi_5 = 4.66596,\ \phi_6 = 1,\ \phi_3 = 1,\ \delta_1^{\{2,5\}} = 1.36347,\ \delta_1^{\{5,6\}} = -0.708827,\ \delta_1^{\{3,6\}} = 0,\ \delta_1^{\{2,3\}} = 1.44833\}$$

$$\boldsymbol{d}^{\mathrm{T}} = (\ 10 \quad 4.66596 \quad 1 \quad 1 \quad 1.36347 \quad -0.708827 \quad 0 \quad 1.44833\ )$$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	7.5	3.52259	−0.269207	−1.09961

Solution for element 3

DOF values for the element

$\{\phi_4 = 6.12921,\ \phi_7 = 5.08248,\ \phi_8 = 3.96195,\ \phi_5 = 4.66596,$
$\delta_1^{\{4,7\}} = 0.54473,\ \delta_1^{\{7,8\}} = -0.251281,\ \delta_1^{\{5,8\}} = 0.440829,\ \delta_1^{\{4,5\}} = -0.887807\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 6.12921 \quad 5.08248 \quad 3.96195 \quad 4.66596 \quad 0.54473 \quad -0.251281 \quad 0.440829 \quad -0.887807\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	2.5	5.00691	−0.253032	−0.245653

Solution for element 4

DOF values for the element

$\{\phi_5 = 4.66596,\ \phi_8 = 3.96195,\ \phi_9 = 1,\ \phi_6 = 1,\ \delta_1^{\{5,8\}} = 0.440829,\ \delta_1^{\{8,9\}} = -0.0306799,\ \delta_1^{\{6,9\}} = 0,\ \delta_1^{\{5,6\}} = -0.708827\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 4.66596 \quad 3.96195 \quad 1 \quad 1 \quad 0.440829 \quad -0.0306799 \quad 0 \quad -0.708827\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	7.5	2.74843	−0.153456	−0.608801

Nodal solution summary

dof	x	y	Value
ϕ_1	0	0	10
ϕ_2	0	5	10
ϕ_3	0	10	1
ϕ_4	5	0	6.12921
ϕ_5	5	5	4.66596
ϕ_6	5	10	1
ϕ_7	10	0	5.08248
ϕ_8	10	5	3.96195
ϕ_9	10	10	1

Element solution summary

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.5269	−0.811749	−0.302817
2	2.5	7.5	3.52259	−0.269207	−1.09961
3	7.5	2.5	5.00691	−0.253032	−0.245653
4	7.5	7.5	2.74843	−0.153456	−0.608801

Solution with $n = 3$

Equations for element 1

$$\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 & 0.0 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 & -0.0 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 & 0.0 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 & -0.0 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 & 0.7 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0.3 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Equations for element 2

$$\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 & 0.0 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 & -0.0 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 & 0.0 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 & -0.0 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 & 0.7 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0.3 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Equations for element 3

$$\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 & 0.0\\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 & -0.0\\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 & 0.0\\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 & -0.0\\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 & 0\\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 & 0\\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 & 0\\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 & 0\\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 & 0.7\\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0\\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0.3\\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Equations for element 4

$$\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 & 0.0\\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 & -0.0\\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 & 0.0\\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 & -0.0\\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 & 0\\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 & 0\\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 & 0\\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 & 0\\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 & 0.7\\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0\\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0.3\\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Global equations after assembling all elements

$$\begin{pmatrix}
0.666667 & -0.166667 & 0 & -0.166667 & -0.333333 & 0 & 0 & 0\\
-0.166667 & 1.33333 & -0.166667 & -0.333333 & -0.333333 & -0.333333 & 0 & 0\\
0 & -0.166667 & 0.666667 & 0 & -0.333333 & -0.166667 & 0 & 0\\
-0.166667 & -0.333333 & 0 & 1.33333 & -0.333333 & 0 & -0.166667 & -0.333333
\end{pmatrix}$$

$$\begin{pmatrix}
-0.333333 & -0.333333 & -0.333333 & -0.333333 & 2.66667 & -0.333333 & -0.333333 & -0.333333 \\
0 & -0.333333 & -0.166667 & 0 & -0.333333 & 1.33333 & 0 & -0.333333 \\
0 & 0 & 0 & -0.166667 & -0.333333 & 0 & 0.666667 & -0.166667 \\
0 & 0 & 0 & -0.333333 & -0.333333 & -0.333333 & -0.166667 & 1.33333 \\
0 & 0 & 0 & 0 & -0.333333 & -0.166667 & 0 & -0.166667 \\
-0.204124 & -0.204124 & 0 & 0.204124 & 0.204124 & 0 & 0 & 0 \\
-0.204124 & 0.204124 & 0 & -0.204124 & 0.204124 & 0 & 0 & 0 \\
0 & -0.204124 & -0.204124 & 0 & 0.204124 & 0.204124 & 0 & 0 \\
0.204124 & -0.408248 & 0.204124 & 0.204124 & -0.408248 & 0.204124 & 0 & 0 \\
0 & 0.204124 & -0.204124 & 0 & 0.204124 & -0.204124 & 0 & 0 \\
0.204124 & 0.204124 & 0 & -0.408248 & -0.408248 & 0 & 0.204124 & 0.204124 \\
0 & 0 & 0 & -0.204124 & 0.204124 & 0 & -0.204124 & 0.204124 \\
0 & 0.204124 & 0.204124 & 0 & -0.408248 & -0.408248 & 0 & 0.204124 \\
0 & 0 & 0 & 0.204124 & -0.408248 & 0.204124 & 0.204124 & -0.408248 \\
0 & 0 & 0 & 0 & 0.204124 & -0.204124 & 0 & 0.204124 \\
0 & 0 & 0 & 0.204124 & 0.204124 & 0 & -0.204124 & -0.204124 \\
0 & 0 & 0 & 0 & 0.204124 & 0.204124 & 0 & -0.204124 \\
0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 & 0 & 0 & 0 \\
0 & 0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 & 0 & 0 \\
-0.0527046 & 0.105409 & -0.0527046 & 0.0527046 & -0.105409 & 0.0527046 & 0 & 0 \\
0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 & -0.0527046 & 0 & 0 \\
-0.0527046 & 0.0527046 & 0 & 0.105409 & -0.105409 & 0 & -0.0527046 & 0.0527046 \\
0 & 0 & 0 & 0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 \\
0 & -0.0527046 & 0.0527046 & 0 & 0.105409 & -0.105409 & 0 & -0.0527046 \\
0 & 0 & 0 & -0.0527046 & 0.105409 & -0.0527046 & 0.0527046 & -0.105409 \\
0 & 0 & 0 & 0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 \\
0 & 0 & 0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 & -0.0527046 \\
0 & 0 & 0 & 0 & -0.0527046 & 0.0527046 & 0 & 0.0527046
\end{pmatrix}$$

Essential boundary conditions

On element 1, side 4, specified value = 10

$$\left\{\phi_2,\ \phi_1,\ \delta_1^{\{1,2\}},\ \delta_2^{\{1,2\}}\right\} = \{10,\ 10,\ 0,\ 0\}$$

On element 2, side 3, specified value = 1

$$\left\{\phi_6,\ \phi_3,\ \delta_1^{\{3,6\}},\ \delta_2^{\{3,6\}}\right\} = \{1,\ 1,\ 0,\ 0\}$$

On element 4, side 3, specified value = 1

$$\left\{\phi_9,\ \phi_6,\ \delta_1^{\{6,9\}},\ \delta_2^{\{6,9\}}\right\} = \{1,\ 1,\ 0,\ 0\}$$

Known values from EBC

$$\left\{\phi_1 = 10,\ \phi_2 = 10,\ \phi_3 = 1,\ \phi_6 = 1,\ \phi_9 = 1,\ \delta_1^{\{1,2\}} = 0,\ \delta_1^{\{3,6\}} = 0,\ \delta_1^{\{6,9\}} = 0,\ \delta_2^{\{1,2\}} = 0,\ \delta_2^{\{3,6\}} = 0,\ \delta_2^{\{6,9\}} = 0\right\}$$

Global equations after EBC

$$\begin{pmatrix}
1.33333 & -0.333333 & -0.166667 & -0.333333 & -0.204124 & 0 & 0.204124 & -0.408248 & -0.204 \\
-0.333333 & 2.66667 & -0.333333 & -0.333333 & 0.204124 & 0.204124 & -0.408248 & -0.408248 & 0.204 \\
-0.166667 & -0.333333 & 0.666667 & -0.166667 & 0 & 0 & 0 & 0.204124 & -0.204 \\
-0.333333 & -0.333333 & -0.166667 & 1.33333 & 0 & 0 & 0 & 0.204124 & 0.204 \\
-0.204124 & 0.204124 & 0 & 0 & 0.866667 & 0 & 0.133333 & 0 & 0 \\
0 & 0.204124 & 0 & 0 & 0 & 0.866667 & 0 & 0 & 0 \\
0.204124 & -0.408248 & 0 & 0 & 0.133333 & 0 & 1.73333 & 0 & 0 \\
-0.408248 & -0.408248 & 0.204124 & 0.204124 & 0 & 0 & 0 & 1.73333 & 0 \\
-0.204124 & 0.204124 & -0.204124 & 0.204124 & 0 & 0 & 0 & 0 & 0.866 \\
0 & -0.408248 & 0 & 0.204124 & 0 & 0.133333 & 0 & 0 & 0 \\
0.204124 & -0.408248 & 0.204124 & -0.408248 & 0 & 0 & 0 & 0 & 0.133 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0 & 0 & 0 & 0.133333 & 0 \\
0 & 0.204124 & 0 & -0.204124 & 0 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.0527046 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.105409 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.105409 & -0.105409 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.105409 & 0 & -0.0527046 & 0 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.105409 & 0.0527046 & -0.105409 & 0 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.0527046 & 0 & 0.0527046 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Solving the final system of global equations we get

$\{\phi_4 = 6.1987,\ \phi_5 = 4.62455,\ \phi_7 = 5.04395,\ \phi_8 = 3.95026,\ \delta_1^{\{1,4\}} = 0.172592,\ \delta_1^{\{2,3\}} = 1.36673,\ \delta_1^{\{2,5\}} = 1.28806,$
$\delta_1^{\{4,5\}} = -0.842123,\ \delta_1^{\{4,7\}} = 0.564241,\ \delta_1^{\{5,6\}} = -0.654287,\ \delta_1^{\{5,8\}} = 0.416699,\ \delta_1^{\{7,8\}} = -0.301232,$
$\delta_1^{\{8,9\}} = -0.0581536,\ \delta_2^{\{1,4\}} = 0.25397,\ \delta_2^{\{2,3\}} = -0.537466,\ \delta_2^{\{2,5\}} = -0.316048,\ \delta_2^{\{4,5\}} = -0.093706,$
$\delta_2^{\{4,7\}} = -0.0363905,\ \delta_2^{\{5,6\}} = 0.309716,\ \delta_2^{\{5,8\}} = 0.000501449,\ \delta_2^{\{7,8\}} = 0.0763353,\ \delta_2^{\{8,9\}} = -0.081967\}$

Solution for element 1

DOF values for the element

$\{\phi_1 = 10,\ \phi_4 = 6.1987,\ \phi_5 = 4.62455,\ \phi_2 = 10,\ \delta_1^{\{1,4\}} = 0.172592,\ \delta_1^{\{4,5\}} = -0.842123,$
$\delta_1^{\{2,5\}} = 1.28806,\ \delta_1^{\{1,2\}} = 0,\ \delta_2^{\{1,4\}} = 0.25397,\ \delta_2^{\{4,5\}} = -0.093706,\ \delta_2^{\{2,5\}} = -0.316048,\ \delta_2^{\{1,2\}} = 0\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 10\quad 6.1987\quad 4.62455\quad 10\quad 0.172592\quad -0.842123\quad 1.28806\quad 0\quad 0.25397\quad -0.093706\quad -0.316048\quad 0\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.51643	−0.804721	−0.279215

Solution for element 2

DOF values for the element

$\{\phi_2 = 10,\ \phi_5 = 4.62455,\ \phi_6 = 1,\ \phi_3 = 1,\ \delta_1^{\{2,5\}} = 1.28806,\ \delta_1^{\{5,6\}} = -0.654287,\ \delta_1^{\{3,6\}} = 0,$
$\delta_1^{\{2,3\}} = 1.36673,\ \delta_2^{\{2,5\}} = -0.316048,\ \delta_2^{\{5,6\}} = 0.309716,\ \delta_2^{\{3,6\}} = 0,\ \delta_2^{\{2,3\}} = -0.537466\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 10\quad 4.62455\quad 1\quad 1\quad 1.28806\quad -0.654287\quad 0\quad 1.36673\quad -0.316048\quad 0.309716\quad 0\quad -0.537466\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	7.5	3.54361	−0.240051	−1.06869

Solution for element 3

DOF values for the element

$\{\phi_4 = 6.1987,\ \phi_7 = 5.04395,\ \phi_8 = 3.95026,\ \phi_5 = 4.62455,\ \delta_1^{\{4,7\}} = 0.564241,\ \delta_1^{\{7,8\}} = -0.301232,\ \delta_1^{\{5,8\}} = 0.416699,$
$\delta_1^{\{4,5\}} = -0.842123,\ \delta_2^{\{4,7\}} = -0.0363905,\ \delta_2^{\{7,8\}} = 0.0763353,\ \delta_2^{\{5,8\}} = 0.000501449,\ \delta_2^{\{4,5\}} = -0.093706\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 6.1987\quad 5.04395\quad 3.95026\quad 4.62455\quad 0.564241\quad -0.301232\quad 0.416699\quad -0.842123\quad -0.0363905\quad 0.0763$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	2.5	5.00409	−0.243475	−0.245966

Solution for element 4

DOF values for the element

$\{\phi_5 = 4.62455,\ \phi_8 = 3.95026,\ \phi_9 = 1,\ \phi_6 = 1,\ \delta_1^{\{5,8\}} = 0.416699,\ \delta_1^{\{8,9\}} = -0.0581536,\ \delta_1^{\{6,9\}} = 0,$
$\delta_1^{\{5,6\}} = -0.654287,\ \delta_2^{\{5,8\}} = 0.000501449,\ \delta_2^{\{8,9\}} = -0.081967,\ \delta_2^{\{6,9\}} = 0,\ \delta_2^{\{5,6\}} = 0.309716\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 4.62455 \quad 3.95026 \quad 1 \quad 1 \quad 0.416699 \quad -0.0581536 \quad 0 \quad -0.654287 \quad 0.000501449 \quad -0.081967 \quad 0 \quad 0.309716$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	7.5	2.73426	−0.140519	−0.642457

Nodal solution summary

dof	x	y	Value
ϕ_1	0	0	10
ϕ_2	0	5	10
ϕ_3	0	10	1
ϕ_4	5	0	6.1987
ϕ_5	5	5	4.62455
ϕ_6	5	10	1
ϕ_7	10	0	5.04395
ϕ_8	10	5	3.95026
ϕ_9	10	10	1

Element solution summary

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.51643	−0.804721	−0.279215
2	2.5	7.5	3.54361	−0.240051	−1.06869
3	7.5	2.5	5.00409	−0.243475	−0.245966
4	7.5	7.5	2.73426	−0.140519	−0.642457

Solution summary with $n = 4$

Equations for element 1

$$\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 \\
0 & 0 & 0 & 0 & -0.408248 & -0.408248 & -0.408248 & -0.408248
\end{pmatrix}$$

Equations for element 2

$$\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 \\
0 & 0 & 0 & 0 & -0.408248 & -0.408248 & -0.408248 & -0.408248
\end{pmatrix}$$

Equations for element 3

$$\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 \\
0 & 0 & 0 & 0 & -0.408248 & -0.408248 & -0.408248 & -0.408248
\end{pmatrix}$$

Equations for element 4

$$\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 \\
0 & 0 & 0 & 0 & -0.408248 & -0.408248 & -0.408248 & -0.408248
\end{pmatrix}$$

Global equations after assembling all elements

$$\begin{pmatrix}
0.666667 & -0.166667 & 0 & -0.166667 & -0.333333 & 0 & 0 & 0 \\
-0.166667 & 1.33333 & -0.166667 & -0.333333 & -0.333333 & -0.333333 & 0 & 0 \\
0 & -0.166667 & 0.666667 & 0 & -0.333333 & -0.166667 & 0 & 0 \\
-0.166667 & -0.333333 & 0 & 1.33333 & -0.333333 & 0 & -0.166667 & -0.333333 \\
-0.333333 & -0.333333 & -0.333333 & -0.333333 & 2.66667 & -0.333333 & -0.333333 & -0.333333 \\
0 & -0.333333 & -0.166667 & 0 & -0.333333 & 1.33333 & 0 & -0.333333 \\
0 & 0 & 0 & -0.166667 & -0.333333 & 0 & 0.666667 & -0.166667 \\
0 & 0 & 0 & -0.333333 & -0.333333 & -0.333333 & -0.166667 & 1.33333 \\
0 & 0 & 0 & 0 & -0.333333 & -0.166667 & 0 & -0.166667 \\
-0.204124 & -0.204124 & 0 & 0.204124 & 0.204124 & 0 & 0 & 0 \\
-0.204124 & 0.204124 & 0 & -0.204124 & 0.204124 & 0 & 0 & 0 \\
0 & -0.204124 & -0.204124 & 0 & 0.204124 & 0.204124 & 0 & 0
\end{pmatrix}$$

$$
\left(\begin{array}{llllllll}
0.204124 & -0.408248 & 0.204124 & 0.204124 & -0.408248 & 0.204124 & 0 & 0 \\
0 & 0.204124 & -0.204124 & 0 & 0.204124 & -0.204124 & 0 & 0 \\
0.204124 & 0.204124 & 0 & -0.408248 & -0.408248 & 0 & 0.204124 & 0.204124 \\
0 & 0 & 0 & -0.204124 & 0.204124 & 0 & -0.204124 & 0.204124 \\
0 & 0.204124 & 0.204124 & 0 & -0.408248 & -0.408248 & 0 & 0.204124 \\
0 & 0 & 0 & 0.204124 & -0.408248 & 0.204124 & 0.204124 & -0.408248 \\
0 & 0 & 0 & 0 & 0.204124 & -0.204124 & 0 & 0.204124 \\
0 & 0 & 0 & 0.204124 & 0.204124 & 0 & -0.204124 & -0.204124 \\
0 & 0 & 0 & 0 & 0.204124 & 0.204124 & 0 & -0.204124 \\
0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 & 0 & 0 & 0 \\
0 & 0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 & 0 & 0 \\
-0.0527046 & 0.105409 & -0.0527046 & 0.0527046 & -0.105409 & 0.0527046 & 0 & 0 \\
0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 & -0.0527046 & 0 & 0 \\
-0.0527046 & 0.0527046 & 0 & 0.105409 & -0.105409 & 0 & -0.0527046 & 0.0527046 \\
0 & 0 & 0 & 0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 \\
0 & -0.0527046 & 0.0527046 & 0 & 0.105409 & -0.105409 & 0 & -0.0527046 \\
0 & 0 & 0 & -0.0527046 & 0.105409 & -0.0527046 & 0.0527046 & -0.105409 \\
0 & 0 & 0 & 0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 \\
0 & 0 & 0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 & -0.0527046 \\
0 & 0 & 0 & 0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right.
$$

Essential boundary conditions

On element 1, side 4, specified value = 10

$$\left\{\phi_2,\ \phi_1,\ \delta_1^{\{1,2\}},\ \delta_2^{\{1,2\}},\ \delta_3^{\{1,2\}}\right\} = \{10,\ 10,\ 0,\ 0,\ 0\}$$

On element 2, side 3, specified value = 1

$$\left\{\phi_6,\ \phi_3,\ \delta_1^{\{3,6\}},\ \delta_2^{\{3,6\}},\ \delta_3^{\{3,6\}}\right\} = \{1,\ 1,\ 0,\ 0,\ 0\}$$

On element 4, side 3, specified value = 1

$$\left\{\phi_9,\ \phi_6,\ \delta_1^{\{6,9\}},\ \delta_2^{\{6,9\}},\ \delta_3^{\{6,9\}}\right\} = \{1,\ 1,\ 0,\ 0,\ 0\}$$

Known values from EBC

$$\left\{\phi_1 = 10,\ \phi_2 = 10,\ \phi_3 = 1,\ \phi_6 = 1,\ \phi_9 = 1,\ \delta_1^{\{1,2\}} = 0,\ \delta_1^{\{3,6\}} = 0,\right.$$
$$\left.\delta_1^{\{6,9\}} = 0,\ \delta_2^{\{1,2\}} = 0,\ \delta_2^{\{3,6\}} = 0,\ \delta_2^{\{6,9\}} = 0,\ \delta_3^{\{1,2\}} = 0,\ \delta_3^{\{3,6\}} = 0,\ \delta_3^{\{6,9\}} = 0\right\}$$

Global equations after EBC

$$\begin{pmatrix}
1.33333 & -0.333333 & -0.166667 & -0.333333 & -0.204124 & 0 & 0.204124 & -0.408248 \\
-0.333333 & 2.66667 & -0.333333 & -0.333333 & 0.204124 & 0.204124 & -0.408248 & -0.408248 \\
-0.166667 & -0.333333 & 0.666667 & -0.166667 & 0 & 0 & 0 & 0.204124 \\
-0.333333 & -0.333333 & -0.166667 & 1.33333 & 0 & 0 & 0 & 0.204124 \\
-0.204124 & 0.204124 & 0 & 0 & 0.866667 & 0 & 0.133333 & 0 \\
0 & 0.204124 & 0 & 0 & 0 & 0.866667 & 0 & 0 \\
0.204124 & -0.408248 & 0 & 0 & 0.133333 & 0 & 1.73333 & 0 \\
-0.408248 & -0.408248 & 0.204124 & 0.204124 & 0 & 0 & 0 & 1.73333 \\
-0.204124 & 0.204124 & -0.204124 & 0.204124 & 0 & 0 & 0 & 0 \\
0 & -0.408248 & 0 & 0.204124 & 0 & 0.133333 & 0 & 0 \\
0.204124 & -0.408248 & 0.204124 & -0.408248 & 0 & 0 & 0 & 0 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0 & 0 & 0 & 0.133333 \\
0 & 0.204124 & 0 & -0.204124 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.0527046 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.105409 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.105409 & -0.105409 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0.0527046 & -0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0.105409 & 0 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.105409 & 0.0527046 & -0.105409 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & -0.0527046 & 0 & 0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0436436 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0436436 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.408248 & 0 & -0.408248 & -0.408248 \\
0 & 0 & 0 & 0 & 0 & -0.408248 & -0.408248 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.408248 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Solving the final system of global equations we get

$$\{\phi_4 = 6.24941,\ \phi_5 = 4.78965,\ \phi_7 = 4.95572,\ \phi_8 = 3.83463,\ \delta_1^{\{1,4\}} = 0.522616,\ \delta_1^{\{2,3\}} = 2.25151,$$
$$\delta_1^{\{2,5\}} = 2.04415,\ \delta_1^{\{4,5\}} = -0.49774,\ \delta_1^{\{4,7\}} = 0.559187,\ \delta_1^{\{5,6\}} = -0.14886,\ \delta_1^{\{5,8\}} = 0.462738,$$
$$\delta_1^{\{7,8\}} = -0.429955,\ \delta_1^{\{8,9\}} = -0.163152,\ \delta_2^{\{1,4\}} = 0.224069,\ \delta_2^{\{2,3\}} = -0.489115,\ \delta_2^{\{2,5\}} = -0.290896,$$
$$\delta_2^{\{4,5\}} = -0.0775514,\ \delta_2^{\{4,7\}} = -0.0172751,\ \delta_2^{\{5,6\}} = 0.261655,\ \delta_2^{\{5,8\}} = -0.0192845,$$
$$\delta_2^{\{7,8\}} = 0.0560094,\ \delta_2^{\{8,9\}} = -0.0341941,\ \delta_3^{\{1,4\}} = -0.0848399,\ \delta_3^{\{2,3\}} = 0.110627,\ \delta_3^{\{2,5\}} = 0.0770081,$$
$$\delta_3^{\{4,5\}} = -0.0107503,\ \delta_3^{\{4,7\}} = 0.000383013,\ \delta_3^{\{5,6\}} = -0.0719367,\ \delta_3^{\{5,8\}} = 0.00577932,$$
$$\delta_3^{\{7,8\}} = 0.00720908,\ \delta_3^{\{8,9\}} = 0.0332466,\ \delta_1^1 = 1.05585,\ \delta_1^2 = 2.11616,\ \delta_1^3 = 0.0480867,\ \delta_1^4 = 0.076917\}$$

Solution for element 1

DOF values for the element

$$\{\phi_1 = 10,\ \phi_4 = 6.24941,\ \phi_5 = 4.78965,\ \phi_2 = 10,\ \delta_1^{\{1,4\}} = 0.522616,\ \delta_1^{\{4,5\}} = -0.49774,$$
$$\delta_1^{\{2,5\}} = 2.04415,\ \delta_1^{\{1,2\}} = 0,\ \delta_2^{\{1,4\}} = 0.224069,\ \delta_2^{\{4,5\}} = -0.0775514,\ \delta_2^{\{2,5\}} = -0.290896,\ \delta_2^{\{1,2\}} = 0,$$
$$\delta_3^{\{1,4\}} = -0.0848399,\ \delta_3^{\{4,5\}} = -0.0107503,\ \delta_3^{\{2,5\}} = 0.0770081,\ \delta_3^{\{1,2\}} = 0,\ \delta_1^1 = 1.05585\}$$

$\boldsymbol{d}^{\mathrm{T}} = (\ 10\quad 6.24941\quad 4.78965\quad 10\quad 0.522616\quad -0.49774\quad 2.04415\quad 0\quad 0.224069\quad -0.0775514\quad -0.290896\quad 0\quad -$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.52003	−0.825071	−0.312494

Solution for element 2

DOF values for the element

$\{\phi_2 = 10,\ \phi_5 = 4.78965,\ \phi_6 = 1,\ \phi_3 = 1,\ \delta_1^{\{2,5\}} = 2.04415,\ \delta_1^{\{5,6\}} = -0.14886,\ \delta_1^{\{3,6\}} = 0,$
$\delta_1^{\{2,3\}} = 2.25151,\ \delta_2^{\{2,5\}} = -0.290896,\ \delta_2^{\{5,6\}} = 0.261655,\ \delta_2^{\{3,6\}} = 0,\ \delta_2^{\{2,3\}} = -0.489115,$
$\delta_3^{\{2,5\}} = 0.0770081,\ \delta_3^{\{5,6\}} = -0.0719367,\ \delta_3^{\{3,6\}} = 0,\ \delta_3^{\{2,3\}} = 0.110627,\ \delta_1^{2} = 2.11616\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 10\quad 4.78965\quad 1\quad 1\quad 2.04415\quad -0.14886\quad 0\quad 2.25151\quad -0.290896\quad 0.261655\quad 0\quad -0.489115\quad 0.0770081$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	7.5	3.73481	−0.189595	−0.996245

Solution for element 3

DOF values for the element

$\{\phi_4 = 6.24941,\ \phi_7 = 4.95572,\ \phi_8 = 3.83463,\ \phi_5 = 4.78965,\ \delta_1^{\{4,7\}} = 0.559187,\ \delta_1^{\{7,8\}} = -0.429955,\ \delta_1^{\{5,8\}} = 0.462738,$
$\delta_1^{\{4,5\}} = -0.49774,\ \delta_2^{\{4,7\}} = -0.0172751,\ \delta_2^{\{7,8\}} = 0.0560094,\ \delta_2^{\{5,8\}} = -0.0192845,\ \delta_2^{\{4,5\}} = -0.0775514,$
$\delta_3^{\{4,7\}} = 0.000383013,\ \delta_3^{\{7,8\}} = 0.00720908,\ \delta_3^{\{5,8\}} = 0.00577932,\ \delta_3^{\{4,5\}} = -0.0107503,\ \delta_1^{3} = 0.0480867\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 6.24941\quad 4.95572\quad 3.83463\quad 4.78965\quad 0.559187\quad -0.429955\quad 0.462738\quad -0.49774\quad -0.0172751\quad 0.0560($

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	2.5	4.94684	−0.226552	−0.242614

Solution for element 4

DOF values for the element

$\{\phi_5 = 4.78965,\ \phi_8 = 3.83463,\ \phi_9 = 1,\ \phi_6 = 1,\ \delta_1^{\{5,8\}} = 0.462738,\ \delta_1^{\{8,9\}} = -0.163152,\ \delta_1^{\{6,9\}} = 0,$
$\delta_1^{\{5,6\}} = -0.14886,\ \delta_2^{\{5,8\}} = -0.0192845,\ \delta_2^{\{8,9\}} = -0.0341941,\ \delta_2^{\{6,9\}} = 0,\ \delta_2^{\{5,6\}} = 0.261655,$
$\delta_3^{\{5,8\}} = 0.00577932,\ \delta_3^{\{8,9\}} = 0.0332466,\ \delta_3^{\{6,9\}} = 0,\ \delta_3^{\{5,6\}} = -0.0719367,\ \delta_1^{4} = 0.076917\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 4.78965\quad 3.83463\quad 1\quad 1\quad 0.462738\quad -0.163152\quad 0\quad -0.14886\quad -0.0192845\quad -0.0341941\quad 0\quad 0.261655$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	7.5	2.63491	−0.0857831	−0.641989

Nodal solution summary

dof	x	y	Value
ϕ_1	0	0	10
ϕ_2	0	5	10
ϕ_3	0	10	1
ϕ_4	5	0	6.24941
ϕ_5	5	5	4.78965
ϕ_6	5	10	1
ϕ_7	10	0	4.95572
ϕ_8	10	5	3.83463
ϕ_9	10	10	1

Element solution summary

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.52003	−0.825071	−0.312494
2	2.5	7.5	3.73481	−0.189595	−0.996245
3	7.5	2.5	4.94684	−0.226552	−0.242614
4	7.5	7.5	2.63491	−0.0857831	−0.641989

Solution summary with $n = 5$

Equations for element 1

$$
\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.408248 & -0.408248 & -0.408248 & -0.408248 \\
0 & 0 & 0 & 0 & 0.105409 & 0 & -0.105409 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.105409 & 0 & 0.105409
\end{pmatrix}
$$

Equations for element 2

$$
\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.408248 & -0.408248 & -0.408248 & -0.408248 \\
0 & 0 & 0 & 0 & 0.105409 & 0 & -0.105409 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.105409 & 0 & 0.105409
\end{pmatrix}
$$

Equations for element 3

$$
\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.408248 & -0.408248 & -0.408248 & -0.408248 \\
0 & 0 & 0 & 0 & 0.105409 & 0 & -0.105409 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.105409 & 0 & 0.105409
\end{pmatrix}
$$

Equations for element 4

$$\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.408248 & -0.408248 & -0.408248 & -0.408248 \\
0 & 0 & 0 & 0 & 0.105409 & 0 & -0.105409 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.105409 & 0 & 0.105409
\end{pmatrix}$$

Global equations after assembling all elements

$$\begin{pmatrix}
0.666667 & -0.166667 & 0 & -0.166667 & -0.333333 & 0 & 0 & 0 \\
-0.166667 & 1.33333 & -0.166667 & -0.333333 & -0.333333 & -0.333333 & 0 & 0 \\
0 & -0.166667 & 0.666667 & 0 & -0.333333 & -0.166667 & 0 & 0
\end{pmatrix}$$

$$
\begin{vmatrix}
-0.166667 & -0.333333 & 0 & 1.33333 & -0.333333 & 0 & -0.166667 & -0.333333 \\
-0.333333 & -0.333333 & -0.333333 & -0.333333 & 2.66667 & -0.333333 & -0.333333 & -0.333333 \\
0 & -0.333333 & -0.166667 & 0 & -0.333333 & 1.33333 & 0 & -0.333333 \\
0 & 0 & 0 & -0.166667 & -0.333333 & 0 & 0.666667 & -0.166667 \\
0 & 0 & 0 & -0.333333 & -0.333333 & -0.333333 & -0.166667 & 1.33333 \\
0 & 0 & 0 & 0 & -0.333333 & -0.166667 & 0 & -0.166667 \\
-0.204124 & -0.204124 & 0 & 0.204124 & 0.204124 & 0 & 0 & 0 \\
-0.204124 & 0.204124 & 0 & -0.204124 & 0.204124 & 0 & 0 & 0 \\
0 & -0.204124 & -0.204124 & 0 & 0.204124 & 0.204124 & 0 & 0 \\
0.204124 & -0.408248 & 0.204124 & 0.204124 & -0.408248 & 0.204124 & 0 & 0 \\
0 & 0.204124 & -0.204124 & 0 & 0.204124 & -0.204124 & 0 & 0 \\
0.204124 & 0.204124 & 0 & -0.408248 & -0.408248 & 0 & 0.204124 & 0.204124 \\
0 & 0 & 0 & -0.204124 & 0.204124 & 0 & -0.204124 & 0.204124 \\
0 & 0.204124 & 0.204124 & 0 & -0.408248 & -0.408248 & 0 & 0.204124 \\
0 & 0 & 0 & 0.204124 & -0.408248 & 0.204124 & 0.204124 & -0.408248 \\
0 & 0 & 0 & 0 & 0.204124 & -0.204124 & 0 & 0.204124 \\
0 & 0 & 0 & 0.204124 & 0.204124 & 0 & -0.204124 & -0.204124 \\
0 & 0 & 0 & 0 & 0.204124 & 0.204124 & 0 & -0.204124 \\
0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 & 0 & 0 & 0 \\
0 & 0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 & 0 & 0 \\
-0.0527046 & 0.105409 & -0.0527046 & 0.0527046 & -0.105409 & 0.0527046 & 0 & 0 \\
0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 & -0.0527046 & 0 & 0 \\
-0.0527046 & 0.0527046 & 0 & 0.105409 & -0.105409 & 0 & -0.0527046 & 0.0527046 \\
0 & 0 & 0 & 0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 \\
0 & -0.0527046 & 0.0527046 & 0 & 0.105409 & -0.105409 & 0 & -0.0527046 \\
0 & 0 & 0 & -0.0527046 & 0.105409 & -0.0527046 & 0.0527046 & -0.105409 \\
0 & 0 & 0 & 0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 \\
0 & 0 & 0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 & -0.0527046 \\
0 & 0 & 0 & 0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{vmatrix}
$$

$$\left(\begin{matrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{matrix}\right.$$

Essential boundary conditions

On element 1, side 4, specified value = 10

$$\left\{\phi_2,\ \phi_1,\ \delta_1^{\{1,2\}},\ \delta_2^{\{1,2\}},\ \delta_3^{\{1,2\}},\ \delta_4^{\{1,2\}}\right\} = \{10,\ 10,\ 0,\ 0,\ 0,\ 0\}$$

On element 2, side 3, specified value = 1

$$\left\{\phi_6,\ \phi_3,\ \delta_1^{\{3,6\}},\ \delta_2^{\{3,6\}},\ \delta_3^{\{3,6\}},\ \delta_4^{\{3,6\}}\right\} = \{1,\ 1,\ 0,\ 0,\ 0,\ 0\}$$

On element 4, side 3, specified value = 1

$$\left\{\phi_9,\ \phi_6,\ \delta_1^{\{6,9\}},\ \delta_2^{\{6,9\}},\ \delta_3^{\{6,9\}},\ \delta_4^{\{6,9\}}\right\} = \{1,\ 1,\ 0,\ 0,\ 0,\ 0\}$$

Known values from EBC

$$\left\{\phi_1 = 10,\ \phi_2 = 10,\ \phi_3 = 1,\ \phi_6 = 1,\ \phi_9 = 1,\ \delta_1^{\{1,2\}} = 0,\ \delta_1^{\{3,6\}} = 0,\ \delta_1^{\{6,9\}} = 0,\ \delta_2^{\{1,2\}} = 0,\right.$$
$$\left.\delta_2^{\{3,6\}} = 0,\ \delta_2^{\{6,9\}} = 0,\ \delta_3^{\{1,2\}} = 0,\ \delta_3^{\{3,6\}} = 0,\ \delta_3^{\{6,9\}} = 0,\ \delta_4^{\{1,2\}} = 0,\ \delta_4^{\{3,6\}} = 0,\ \delta_4^{\{6,9\}} = 0\right\}$$

Global equations after EBC

$$\begin{pmatrix}
1.33333 & -0.333333 & -0.166667 & -0.333333 & -0.204124 & 0 & 0.204124 & -0.408248 \\
-0.333333 & 2.66667 & -0.333333 & -0.333333 & 0.204124 & 0.204124 & -0.408248 & -0.408248 \\
-0.166667 & -0.333333 & 0.666667 & -0.166667 & 0 & 0 & 0 & 0.204124 \\
-0.333333 & -0.333333 & -0.166667 & 1.33333 & 0 & 0 & 0 & 0.204124 \\
-0.204124 & 0.204124 & 0 & 0 & 0.866667 & 0 & 0.133333 & 0 \\
0 & 0.204124 & 0 & 0 & 0 & 0.866667 & 0 & 0 \\
0.204124 & -0.408248 & 0 & 0 & 0.133333 & 0 & 1.73333 & 0 \\
-0.408248 & -0.408248 & 0.204124 & 0.204124 & 0 & 0 & 0 & 1.73333 \\
-0.204124 & 0.204124 & -0.204124 & 0.204124 & 0 & 0 & 0 & 0 \\
0 & -0.408248 & 0 & 0.204124 & 0 & 0.133333 & 0 & 0 \\
0.204124 & -0.408248 & 0.204124 & -0.408248 & 0 & 0 & 0 & 0 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0 & 0 & 0 & 0.133333 \\
0 & 0.204124 & 0 & -0.204124 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.0527046 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.105409 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.105409 & -0.105409 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0.105409 & 0 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.105409 & 0.0527046 & -0.105409 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & -0.0527046 & 0 & 0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0
\end{pmatrix}$$

$$
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0436436 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0436436 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.408248 & 0 & -0.408248 & -0.408248 \\
0 & 0 & 0 & 0 & 0.105409 & 0 & -0.105409 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.105409 \\
0 & 0 & 0 & 0 & 0 & -0.408248 & -0.408248 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.105409 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.105409 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.408248 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.105409 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

Solving the final system of global equations we get

$$\{\phi_4 = 6.13561,\ \phi_5 = 4.75006,\ \phi_7 = 4.92791,\ \phi_8 = 3.79922,\ \delta_1^{\{1,4\}} = 0.407813,\ \delta_1^{\{2,3\}} = 2.51783,$$
$$\delta_1^{\{2,5\}} = 2.2194,\ \delta_1^{\{4,5\}} = -0.615162,\ \delta_1^{\{4,7\}} = 0.478187,\ \delta_1^{\{5,6\}} = -0.219334,\ \delta_1^{\{5,8\}} = 0.443003,$$
$$\delta_1^{\{7,8\}} = -0.444232,\ \delta_1^{\{8,9\}} = -0.177441,\ \delta_2^{\{1,4\}} = -0.0619089,\ \delta_2^{\{2,3\}} = -1.25615,\ \delta_2^{\{2,5\}} = -0.853448,$$
$$\delta_2^{\{4,5\}} = 0.0924994,\ \delta_2^{\{4,7\}} = 0.000292546,\ \delta_2^{\{5,6\}} = -0.0886744,\ \delta_2^{\{5,8\}} = -0.0239389,\ \delta_2^{\{7,8\}} = 0.0401044,$$
$$\delta_2^{\{8,9\}} = 0.0182006,\ \delta_3^{\{1,4\}} = -0.096009,\ \delta_3^{\{2,3\}} = 0.122953,\ \delta_3^{\{2,5\}} = 0.0855238,\ \delta_3^{\{4,5\}} = -0.0152258,$$
$$\delta_3^{\{4,7\}} = -0.00200726,\ \delta_3^{\{5,6\}} = -0.080264,\ \delta_3^{\{5,8\}} = 0.00691246,\ \delta_3^{\{7,8\}} = 0.0122907,\ \delta_3^{\{8,9\}} = 0.0375753,$$
$$\delta_4^{\{1,4\}} = 0.0212248,\ \delta_4^{\{2,3\}} = -0.026227,\ \delta_4^{\{2,5\}} = -0.0181012,\ \delta_4^{\{4,5\}} = 0.00151976,\ \delta_4^{\{4,7\}} = 0.000640333,$$
$$\delta_4^{\{5,6\}} = 0.0161698,\ \delta_4^{\{5,8\}} = -0.000534594,\ \delta_4^{\{7,8\}} = -0.00156006,\ \delta_4^{\{8,9\}} = -0.00611264,$$
$$\delta_1^1 = 1.02677,\ \delta_2^1 = 0.46184,\ \delta_3^1 = -0.885507,\ \delta_1^2 = 2.30553,\ \delta_2^2 = -1.58099,\ \delta_3^2 = -1.28613,$$
$$\delta_1^3 = -0.070527,\ \delta_2^3 = 0.101823,\ \delta_3^3 = 0.016889,\ \delta_1^4 = 0.0235907,\ \delta_2^4 = -0.152386,\ \delta_3^4 = -0.0108173\}$$

Solution for element 1

DOF values for the element

$$\{\phi_1 = 10,\ \phi_4 = 6.13561,\ \phi_5 = 4.75006,\ \phi_2 = 10,\ \delta_1^{\{1,4\}} = 0.407813,\ \delta_1^{\{4,5\}} = -0.615162,$$
$$\delta_1^{\{2,5\}} = 2.2194,\ \delta_1^{\{1,2\}} = 0,\ \delta_2^{\{1,4\}} = -0.0619089,\ \delta_2^{\{4,5\}} = 0.0924994,\ \delta_2^{\{2,5\}} = -0.853448,\ \delta_2^{\{1,2\}} = 0,$$
$$\delta_3^{\{1,4\}} = -0.096009,\ \delta_3^{\{4,5\}} = -0.0152258,\ \delta_3^{\{2,5\}} = 0.0855238,\ \delta_3^{\{1,2\}} = 0,\ \delta_4^{\{1,4\}} = 0.0212248,$$
$$\delta_4^{\{4,5\}} = 0.00151976,\ \delta_4^{\{2,5\}} = -0.0181012,\ \delta_4^{\{1,2\}} = 0,\ \delta_1^1 = 1.02677,\ \delta_2^1 = 0.46184,\ \delta_3^1 = -0.885507\}$$

$$\boldsymbol{d}^{\mathrm{T}} = (\ 10\quad 6.13561\quad 4.75006\quad 10\quad 0.407813\quad -0.615162\quad 2.2194\quad 0\quad -0.0619089\quad 0.0924994\quad -0.853448\quad 0\ \cdots$$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.48739	−0.863053	−0.276887

Solution for element 2

DOF values for the element

$$\{\phi_2 = 10,\ \phi_5 = 4.75006,\ \phi_6 = 1,\ \phi_3 = 1,\ \delta_1^{\{2,5\}} = 2.2194,\ \delta_1^{\{5,6\}} = -0.219334,\ \delta_1^{\{3,6\}} = 0,$$
$$\delta_1^{\{2,3\}} = 2.51783,\ \delta_2^{\{2,5\}} = -0.853448,\ \delta_2^{\{5,6\}} = -0.0886744,\ \delta_2^{\{3,6\}} = 0,\ \delta_2^{\{2,3\}} = -1.25615,$$
$$\delta_3^{\{2,5\}} = 0.0855238,\ \delta_3^{\{5,6\}} = -0.080264,\ \delta_3^{\{3,6\}} = 0,\ \delta_3^{\{2,3\}} = 0.122953,\ \delta_4^{\{2,5\}} = -0.0181012,$$
$$\delta_4^{\{5,6\}} = 0.0161698,\ \delta_4^{\{3,6\}} = 0,\ \delta_4^{\{2,3\}} = -0.026227,\ \delta_1^2 = 2.30553,\ \delta_2^2 = -1.58099,\ \delta_3^2 = -1.28613\}$$

$$\boldsymbol{d}^{\mathrm{T}} = (\ 10\quad 4.75006\quad 1\quad 1\quad 2.2194\quad -0.219334\quad 0\quad 2.51783\quad -0.853448\quad -0.0886744\quad 0\quad -1.25615\quad 0.0855238$$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	7.5	3.68376	−0.316262	−1.10231

Solution for element 3

DOF values for the element

$\{\phi_4 = 6.13561,\ \phi_7 = 4.92791,\ \phi_8 = 3.79922,\ \phi_5 = 4.75006,\ \delta_1^{\{4,7\}} = 0.478187,$
$\delta_1^{\{7,8\}} = -0.444232,\ \delta_1^{\{5,8\}} = 0.443003,\ \delta_1^{\{4,5\}} = -0.615162,\ \delta_2^{\{4,7\}} = 0.000292546,\ \delta_2^{\{7,8\}} = 0.0401044,$
$\delta_2^{\{5,8\}} = -0.0239389,\ \delta_2^{\{4,5\}} = 0.0924994,\ \delta_3^{\{4,7\}} = -0.00200726,\ \delta_3^{\{7,8\}} = 0.0122907,$
$\delta_3^{\{5,8\}} = 0.00691246,\ \delta_3^{\{4,5\}} = -0.0152258,\ \delta_4^{\{4,7\}} = 0.000640333,\ \delta_4^{\{7,8\}} = -0.00156006,$
$\delta_4^{\{5,8\}} = -0.000534594,\ \delta_4^{\{4,5\}} = 0.00151976,\ \delta_1^3 = -0.070527,\ \delta_2^3 = 0.101823,\ \delta_3^3 = 0.016889\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 6.13561\ \ 4.92791\ \ 3.79922\ \ 4.75006\ \ 0.478187\ \ -0.444232\ \ 0.443003\ \ -0.615162\ \ 0.000292546\ \ 0.040$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	2.5	4.9193	−0.228476	−0.247952

Solution for element 4

DOF values for the element

$\{\phi_5 = 4.75006,\ \phi_8 = 3.79922,\ \phi_9 = 1,\ \phi_6 = 1,\ \delta_1^{\{5,8\}} = 0.443003,\ \delta_1^{\{8,9\}} = -0.177441,\ \delta_1^{\{6,9\}} = 0,$
$\delta_1^{\{5,6\}} = -0.219334,\ \delta_2^{\{5,8\}} = -0.0239389,\ \delta_2^{\{8,9\}} = 0.0182006,\ \delta_2^{\{6,9\}} = 0,\ \delta_2^{\{5,6\}} = -0.0886744,$
$\delta_3^{\{5,8\}} = 0.00691246,\ \delta_3^{\{8,9\}} = 0.0375753,\ \delta_3^{\{6,9\}} = 0,\ \delta_3^{\{5,6\}} = -0.080264,\ \delta_4^{\{5,8\}} = -0.000534594,$
$\delta_4^{\{8,9\}} = -0.00611264,\ \delta_4^{\{6,9\}} = 0,\ \delta_4^{\{5,6\}} = 0.0161698,\ \delta_1^4 = 0.0235907,\ \delta_2^4 = -0.152386,\ \delta_3^4 = -0.0108173\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 4.75006\ \ 3.79922\ \ 1\ \ 1\ \ 0.443003\ \ -0.177441\ \ 0\ \ -0.219334\ \ -0.0239389\ \ 0.0182006\ \ 0\ \ -0.088674$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	7.5	2.62783	−0.0930979	−0.617761

Nodal solution summary

dof	x	y	Value
ϕ_1	0	0	10
ϕ_2	0	5	10
ϕ_3	0	10	1
ϕ_4	5	0	6.13561
ϕ_5	5	5	4.75006
ϕ_6	5	10	1
ϕ_7	10	0	4.92791
ϕ_8	10	5	3.79922
ϕ_9	10	10	1

Element solution summary

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.48739	−0.863053	−0.276887
2	2.5	7.5	3.68376	−0.316262	−1.10231
3	7.5	2.5	4.9193	−0.228476	−0.247952
4	7.5	7.5	2.62783	−0.0930979	−0.617761

Solution summary with $n = 6$

Equations for element 1

$$
\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.408248 & -0.408248 & -0.408248 & -0.408248 \\
0 & 0 & 0 & 0 & 0.105409 & 0 & -0.105409 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.105409 & 0 & 0.105409 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

Equations for element 2

$$
\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.408248 & -0.408248 & -0.408248 & -0.408248 \\
0 & 0 & 0 & 0 & 0.105409 & 0 & -0.105409 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.105409 & 0 & 0.105409 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

Equations for element 3

$$
\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.408248 & -0.408248 & -0.408248 & -0.408248 \\
0 & 0 & 0 & 0 & 0.105409 & 0 & -0.105409 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.105409 & 0 & 0.105409 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

Equations for element 4

$$
\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.408248 & -0.408248 & -0.408248 & -0.408248 \\
0 & 0 & 0 & 0 & 0.105409 & 0 & -0.105409 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.105409 & 0 & 0.105409 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

Global equations after assembling all elements

$$
\begin{pmatrix}
0.666667 & -0.166667 & 0 & -0.166667 & -0.333333 & 0 & 0 & 0 \\
-0.166667 & 1.33333 & -0.166667 & -0.333333 & -0.333333 & -0.333333 & 0 & 0 \\
0 & -0.166667 & 0.666667 & 0 & -0.333333 & -0.166667 & 0 & 0 \\
-0.166667 & -0.333333 & 0 & 1.33333 & -0.333333 & 0 & -0.166667 & -0.333333 \\
-0.333333 & -0.333333 & -0.333333 & -0.333333 & 2.66667 & -0.333333 & -0.333333 & -0.333333 \\
0 & -0.333333 & -0.166667 & 0 & -0.333333 & 1.33333 & 0 & -0.333333 \\
0 & 0 & 0 & -0.166667 & -0.333333 & 0 & 0.666667 & -0.166667 \\
0 & 0 & 0 & -0.333333 & -0.333333 & -0.333333 & -0.166667 & 1.33333 \\
0 & 0 & 0 & 0 & -0.333333 & -0.166667 & 0 & -0.166667 \\
-0.204124 & -0.204124 & 0 & 0.204124 & 0.204124 & 0 & 0 & 0 \\
-0.204124 & 0.204124 & 0 & -0.204124 & 0.204124 & 0 & 0 & 0 \\
0 & -0.204124 & -0.204124 & 0 & 0.204124 & 0.204124 & 0 & 0 \\
0.204124 & -0.408248 & 0.204124 & 0.204124 & -0.408248 & 0.204124 & 0 & 0 \\
0 & 0.204124 & -0.204124 & 0 & 0.204124 & -0.204124 & 0 & 0 \\
0.204124 & 0.204124 & 0 & -0.408248 & -0.408248 & 0 & 0.204124 & 0.204124 \\
0 & 0 & 0 & -0.204124 & 0.204124 & 0 & -0.204124 & 0.204124 \\
0 & 0.204124 & 0.204124 & 0 & -0.408248 & -0.408248 & 0 & 0.204124 \\
0 & 0 & 0 & 0.204124 & -0.408248 & 0.204124 & 0.204124 & -0.408248 \\
0 & 0 & 0 & 0 & 0.204124 & -0.204124 & 0 & 0.204124 \\
0 & 0 & 0 & 0.204124 & 0.204124 & 0 & -0.204124 & -0.204124 \\
0 & 0 & 0 & 0 & 0.204124 & 0.204124 & 0 & -0.204124 \\
0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 & 0 & 0 & 0 \\
0 & 0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 & 0 & 0 \\
-0.0527046 & 0.105409 & -0.0527046 & 0.0527046 & -0.105409 & 0.0527046 & 0 & 0 \\
0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 & -0.0527046 & 0 & 0 \\
-0.0527046 & 0.0527046 & 0 & 0.105409 & -0.105409 & 0 & -0.0527046 & 0.0527046 \\
0 & 0 & 0 & 0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 \\
0 & -0.0527046 & 0.0527046 & 0 & 0.105409 & -0.105409 & 0 & -0.0527046 \\
0 & 0 & 0 & -0.0527046 & 0.105409 & -0.0527046 & 0.0527046 & -0.105409 \\
0 & 0 & 0 & 0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 \\
0 & 0 & 0 & 0.0527046 & 0.0527046 & 0 & 0.0527046 & 0.0527046
\end{pmatrix}
$$

0	0	0	−0.0527046	0.0527046	0	0.0527046	−0.0527046
0	0	0	0	−0.0527046	0.0527046	0	0.0527046
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

$$\left(\begin{matrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{matrix}\right.$$

Essential boundary conditions

On element 1, side 4, specified value = 10

$$\left\{\phi_2,\ \phi_1,\ \delta_1^{\{1,2\}},\ \delta_2^{\{1,2\}},\ \delta_3^{\{1,2\}},\ \delta_4^{\{1,2\}},\ \delta_5^{\{1,2\}}\right\} = \{10,\ 10,\ 0,\ 0,\ 0,\ 0,\ 0\}$$

On element 2, side 3, specified value = 1

$$\left\{\phi_6,\ \phi_3,\ \delta_1^{\{3,6\}},\ \delta_2^{\{3,6\}},\ \delta_3^{\{3,6\}},\ \delta_4^{\{3,6\}},\ \delta_5^{\{3,6\}}\right\} = \{1,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0\}$$

On element 4, side 3, specified value = 1

$$\left\{\phi_9,\ \phi_6,\ \delta_1^{\{6,9\}},\ \delta_2^{\{6,9\}},\ \delta_3^{\{6,9\}},\ \delta_4^{\{6,9\}},\ \delta_5^{\{6,9\}}\right\} = \{1,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0\}$$

Known values from EBC

$$\left\{\phi_1 = 10,\ \phi_2 = 10,\ \phi_3 = 1,\ \phi_6 = 1,\ \phi_9 = 1,\ \delta_1^{\{1,2\}} = 0,\ \delta_1^{\{3,6\}} = 0,\ \delta_1^{\{6,9\}} = 0,\ \delta_2^{\{1,2\}} = 0,\ \delta_2^{\{3,6\}} = 0,\ \delta_2^{\{6,9\}} = 0,\right.$$
$$\left.\delta_3^{\{1,2\}} = 0,\ \delta_3^{\{3,6\}} = 0,\ \delta_3^{\{6,9\}} = 0,\ \delta_4^{\{1,2\}} = 0,\ \delta_4^{\{3,6\}} = 0,\ \delta_4^{\{6,9\}} = 0,\ \delta_5^{\{1,2\}} = 0,\ \delta_5^{\{3,6\}} = 0,\ \delta_5^{\{6,9\}} = 0\right\}$$

Global equations after EBC

$$\begin{pmatrix}
1.33333 & -0.333333 & -0.166667 & -0.333333 & -0.204124 & 0 & 0.204124 & -0.408248 \\
-0.333333 & 2.66667 & -0.333333 & -0.333333 & 0.204124 & 0.204124 & -0.408248 & -0.408248 \\
-0.166667 & -0.333333 & 0.666667 & -0.166667 & 0 & 0 & 0 & 0.204124 \\
-0.333333 & -0.333333 & -0.166667 & 1.33333 & 0 & 0 & 0 & 0.204124 \\
-0.204124 & 0.204124 & 0 & 0 & 0.866667 & 0 & 0.133333 & 0 \\
0 & 0.204124 & 0 & 0 & 0 & 0.866667 & 0 & 0 \\
0.204124 & -0.408248 & 0 & 0 & 0.133333 & 0 & 1.73333 & 0 \\
-0.408248 & -0.408248 & 0.204124 & 0.204124 & 0 & 0 & 0 & 1.73333 \\
-0.204124 & 0.204124 & -0.204124 & 0.204124 & 0 & 0 & 0 & 0 \\
0 & -0.408248 & 0 & 0.204124 & 0 & 0.133333 & 0 & 0 \\
0.204124 & -0.408248 & 0.204124 & -0.408248 & 0 & 0 & 0 & 0 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0 & 0 & 0 & 0.133333 \\
0 & 0.204124 & 0 & -0.204124 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.0527046 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.105409 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.105409 & -0.105409 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0.105409 & 0 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.105409 & 0.0527046 & -0.105409 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & -0.0527046 & 0 & 0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & 0.0218218 & 0
\end{pmatrix}$$

$$
\left|\begin{array}{cccccccc}
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0436436 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0436436 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.408248 & 0 & -0.408248 & -0.408248 \\
0 & 0 & 0 & 0 & 0.105409 & 0 & -0.105409 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.105409 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.408248 & -0.408248 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.105409 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.105409 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right.
$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.408248 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.105409 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Solving the final system of global equations we get

$$\{\phi_4 = 6.18542,\ \phi_5 = 4.69347,\ \phi_7 = 4.91823,\ \phi_8 = 3.78303,\ \delta_1^{\{1,4\}} = 0.500969,\ \delta_1^{\{2,3\}} = 2.6612,\ \delta_1^{\{2,5\}} = 2.27686,$$
$$\delta_1^{\{4,5\}} = -0.578562,\ \delta_1^{\{4,7\}} = 0.548653,\ \delta_1^{\{5,6\}} = -0.225498,\ \delta_1^{\{5,8\}} = 0.396936,\ \delta_1^{\{7,8\}} = -0.441505,$$
$$\delta_1^{\{8,9\}} = -0.179644,\ \delta_2^{\{1,4\}} = 0.129744,\ \delta_2^{\{2,3\}} = -1.72136,\ \delta_2^{\{2,5\}} = -1.20961,\ \delta_2^{\{4,5\}} = -0.0572092,$$
$$\delta_2^{\{4,7\}} = -0.0365082,\ \delta_2^{\{5,6\}} = 0.130747,\ \delta_2^{\{5,8\}} = -0.0044276,\ \delta_2^{\{7,8\}} = 0.0249143,\ \delta_2^{\{8,9\}} = 0.0385166,$$
$$\delta_3^{\{1,4\}} = 0.102927,\ \delta_3^{\{2,3\}} = 0.541385,\ \delta_3^{\{2,5\}} = 0.384263,\ \delta_3^{\{4,5\}} = 0.0486485,\ \delta_3^{\{4,7\}} = 0.00763707,$$
$$\delta_3^{\{5,6\}} = 0.114595,\ \delta_3^{\{5,8\}} = 0.00731432,\ \delta_3^{\{7,8\}} = 0.0150098,\ \delta_3^{\{8,9\}} = 0.0270124,\ \delta_4^{\{1,4\}} = 0.0341926,$$
$$\delta_4^{\{2,3\}} = -0.0425956,\ \delta_4^{\{2,5\}} = -0.0280251,\ \delta_4^{\{4,5\}} = -0.00156893,\ \delta_4^{\{4,7\}} = -0.00068026,\ \delta_4^{\{5,6\}} = 0.0288099,$$
$$\delta_4^{\{5,8\}} = 0.000376932,\ \delta_4^{\{7,8\}} = 0.0020261,\ \delta_4^{\{8,9\}} = -0.0150241,\ \delta_5^{\{1,4\}} = -0.00475221,\ \delta_5^{\{2,3\}} = 0.00555068,$$
$$\delta_5^{\{2,5\}} = 0.00429799,\ \delta_5^{\{4,5\}} = 0.000528368,\ \delta_5^{\{4,7\}} = -0.0000148321,\ \delta_5^{\{5,6\}} = -0.00312662,$$
$$\delta_5^{\{5,8\}} = 0.0000365732,\ \delta_5^{\{7,8\}} = -0.000575281,\ \delta_5^{\{8,9\}} = 0.000702557,\ \delta_1^1 = 1.1616,\ \delta_2^1 = 0.330829,$$
$$\delta_3^1 = 0.158754,\ \delta_4^1 = -1.01333,\ \delta_5^1 = -0.772856,\ \delta_6^1 = 0.56158,\ \delta_1^2 = 2.4836,\ \delta_2^2 = -1.79584,$$
$$\delta_3^2 = 0.846441,\ \delta_4^2 = -1.61156,\ \delta_5^2 = 1.69435,\ \delta_6^2 = 0.596853,\ \delta_1^3 = -0.0344364,\ \delta_2^3 = -0.0589146,$$
$$\delta_3^3 = 0.0550923,\ \delta_4^3 = -0.00457352,\ \delta_5^3 = 0.0632004,\ \delta_6^3 = 0.0103521,\ \delta_1^4 = 0.00331013,$$
$$\delta_2^4 = 0.0550457,\ \delta_3^4 = 0.130399,\ \delta_4^4 = 0.00610997,\ \delta_5^4 = -0.0485898,\ \delta_6^4 = 0.00704368\}$$

Solution for element 1

DOF values for the element

$\{\phi_1 = 10,\ \phi_4 = 6.18542,\ \phi_5 = 4.69347,\ \phi_2 = 10,\ \delta_1^{\{1,4\}} = 0.500969,\ \delta_1^{\{4,5\}} = -0.578562,\ \delta_1^{\{2,5\}} = 2.27686,$
$\delta_1^{\{1,2\}} = 0,\ \delta_2^{\{1,4\}} = 0.129744,\ \delta_2^{\{4,5\}} = -0.0572092,\ \delta_2^{\{2,5\}} = -1.20961,\ \delta_2^{\{1,2\}} = 0,\ \delta_3^{\{1,4\}} = 0.102927,$
$\delta_3^{\{4,5\}} = 0.0486485,\ \delta_3^{\{2,5\}} = 0.384263,\ \delta_3^{\{1,2\}} = 0,\ \delta_4^{\{1,4\}} = 0.0341926,\ \delta_4^{\{4,5\}} = -0.00156893,$
$\delta_4^{\{2,5\}} = -0.0280251,\ \delta_4^{\{1,2\}} = 0,\ \delta_5^{\{1,4\}} = -0.00475221,\ \delta_5^{\{4,5\}} = 0.000528368,\ \delta_5^{\{2,5\}} = 0.00429799,$
$\delta_5^{\{1,2\}} = 0,\ \delta_1^1 = 1.1616,\ \delta_2^1 = 0.330829,\ \delta_3^1 = 0.158754,\ \delta_4^1 = -1.01333,\ \delta_5^1 = -0.772856,\ \delta_6^1 = 0.56158\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 10\quad 6.18542\quad 4.69347\quad 10\quad 0.500969\quad -0.578562\quad 2.27686\quad 0\quad 0.129744\quad -0.0572092\quad -1.20961\quad 0\quad 0$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.44143	−0.8635	−0.280942

Solution for element 2

DOF values for the element

$\{\phi_2 = 10,\ \phi_5 = 4.69347,\ \phi_6 = 1,\ \phi_3 = 1,\ \delta_1^{\{2,5\}} = 2.27686,\ \delta_1^{\{5,6\}} = -0.225498,\ \delta_1^{\{3,6\}} = 0,$
$\delta_1^{\{2,3\}} = 2.6612,\ \delta_2^{\{2,5\}} = -1.20961,\ \delta_2^{\{5,6\}} = 0.130747,\ \delta_2^{\{3,6\}} = 0,\ \delta_2^{\{2,3\}} = -1.72136,\ \delta_3^{\{2,5\}} = 0.384263,$
$\delta_3^{\{5,6\}} = 0.114595,\ \delta_3^{\{3,6\}} = 0,\ \delta_3^{\{2,3\}} = 0.541385,\ \delta_4^{\{2,5\}} = -0.0280251,\ \delta_4^{\{5,6\}} = 0.0288099,\ \delta_4^{\{3,6\}} = 0,$
$\delta_4^{\{2,3\}} = -0.0425956,\ \delta_5^{\{2,5\}} = 0.00429799,\ \delta_5^{\{5,6\}} = -0.00312662,\ \delta_5^{\{3,6\}} = 0,\ \delta_5^{\{2,3\}} = 0.00555068,$
$\delta_1^2 = 2.4836,\ \delta_2^2 = -1.79584,\ \delta_3^2 = 0.846441,\ \delta_4^2 = -1.61156,\ \delta_5^2 = 1.69435,\ \delta_6^2 = 0.596853\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 10\quad 4.69347\quad 1\quad 1\quad 2.27686\quad -0.225498\quad 0\quad 2.6612\quad -1.20961\quad 0.130747\quad 0\quad -1.72136\quad 0.384263\quad 0.1$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	7.5	3.57625	−0.322093	−1.10679

Solution for element 3

DOF values for the element

$\{\phi_4 = 6.18542,\ \phi_7 = 4.91823,\ \phi_8 = 3.78303,\ \phi_5 = 4.69347,\ \delta_1^{\{4,7\}} = 0.548653,\ \delta_1^{\{7,8\}} = -0.441505,$
$\delta_1^{\{5,8\}} = 0.396936,\ \delta_1^{\{4,5\}} = -0.578562,\ \delta_2^{\{4,7\}} = -0.0365082,\ \delta_2^{\{7,8\}} = 0.0249143,\ \delta_2^{\{5,8\}} = -0.0044276,$
$\delta_2^{\{4,5\}} = -0.0572092,\ \delta_3^{\{4,7\}} = 0.00763707,\ \delta_3^{\{7,8\}} = 0.0150098,\ \delta_3^{\{5,8\}} = 0.00731432,\ \delta_3^{\{4,5\}} = 0.0486485,$
$\delta_4^{\{4,7\}} = -0.00068026,\ \delta_4^{\{7,8\}} = 0.0020261,\ \delta_4^{\{5,8\}} = 0.000376932,\ \delta_4^{\{4,5\}} = -0.00156893,$
$\delta_5^{\{4,7\}} = -0.0000148321,\ \delta_5^{\{7,8\}} = -0.000575281,\ \delta_5^{\{5,8\}} = 0.0000365732,\ \delta_5^{\{4,5\}} = 0.000528368,$
$\delta_1^3 = -0.0344364,\ \delta_2^3 = -0.0589146,\ \delta_3^3 = 0.0550923,\ \delta_4^3 = -0.00457352,\ \delta_5^3 = 0.0632004,\ \delta_6^3 = 0.0103521\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 6.18542\quad 4.91823\quad 3.78303\quad 4.69347\quad 0.548653\quad -0.441505\quad 0.396936\quad -0.578562\quad -0.0365082\quad 0.024$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	2.5	4.90475	−0.23055	−0.25038

Solution for element 4

DOF values for the element

$\{\phi_5 = 4.69347,\ \phi_8 = 3.78303,\ \phi_9 = 1,\ \phi_6 = 1,\ \delta_1^{\{5,8\}} = 0.396936,\ \delta_1^{\{8,9\}} = -0.179644,\ \delta_1^{\{6,9\}} = 0,\ \delta_1^{\{5,6\}} = -0.225498,$
$\delta_2^{\{5,8\}} = -0.0044276,\ \delta_2^{\{8,9\}} = 0.0385166,\ \delta_2^{\{6,9\}} = 0,\ \delta_2^{\{5,6\}} = 0.130747,\ \delta_3^{\{5,8\}} = 0.00731432,$
$\delta_3^{\{8,9\}} = 0.0270124,\ \delta_3^{\{6,9\}} = 0,\ \delta_3^{\{5,6\}} = 0.114595,\ \delta_4^{\{5,8\}} = 0.000376932,\ \delta_4^{\{8,9\}} = -0.0150241,\ \delta_4^{\{6,9\}} = 0,$
$\delta_4^{\{5,6\}} = 0.0288099,\ \delta_5^{\{5,8\}} = 0.0000365732,\ \delta_5^{\{8,9\}} = 0.000702557,\ \delta_5^{\{6,9\}} = 0,\ \delta_5^{\{5,6\}} = -0.00312662,$
$\delta_1^4 = 0.00331013,\ \delta_2^4 = 0.0550457,\ \delta_3^4 = 0.130399,\ \delta_4^4 = 0.00610997,\ \delta_5^4 = -0.0485898,\ \delta_6^4 = 0.00704368\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 4.69347\quad 3.78303\quad 1\quad 1\quad 0.396936\quad -0.179644\quad 0\quad -0.225498\quad -0.0044276\quad 0.0385166\quad 0\quad 0.130747\quad ($

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	7.5	2.62078	−0.0989251	−0.613286

Nodal solution summary

dof	x	y	Value
ϕ_1	0	0	10
ϕ_2	0	5	10
ϕ_3	0	10	1
ϕ_4	5	0	6.18542
ϕ_5	5	5	4.69347
ϕ_6	5	10	1
ϕ_7	10	0	4.91823
ϕ_8	10	5	3.78303
ϕ_9	10	10	1

Element solution summary

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.44143	−0.8635	−0.280942
2	2.5	7.5	3.57625	−0.322093	−1.10679
3	7.5	2.5	4.90475	−0.23055	−0.25038
4	7.5	7.5	2.62078	−0.0989251	−0.613286