Computer Implementation 4.4 ***(Matlab)*** *Plane frame (p. 277)*

The analysis of plane frames can be performed conveniently by writing two simple *Matlab* functions, one for defining the element stiffness matrix and the other for computing the element axial force, bending moment, and the shear force.

MatlabFiles\Chap4\PlaneFrameElement.m

```
function [ke, rq] = PlaneFrameElement(modulus, inertia, A, qs, qt, coord)
% [ke, rq] = PlaneFrameElement(modulus, inertia, A, qs, qt, coord)
% Generates equations for a plane frame element
% modulus = modulus of elasticity
% inertia = moment of inertia
% A = area of cross-section
% qs = distributed load along the element axis
% qt = distributed load normal to the element axis
% coord = coordinates at the element ends

EI=modulus*inertia; EA = modulus*A;
x1=coord(1,1); y1=coord(1,2);
x2=coord(2,1); y2=coord(2,2);
L=sqrt((x2-x1)^2+(y2-y1)^2);
ls=(x2-x1)/L; ms=(y2-y1)/L;

ke = [(EA*L^2*ls^2 + 12*EI*ms^2)/L^3, ((-12*EI + EA*L^2)*ls*ms)/L^3, ...
        (-6*EI*ms)/L^2, -((EA*L^2*ls^2 + 12*EI*ms^2)/L^3), ...
        ((12*EI - EA*L^2)*ls*ms)/L^3, (-6*EI*ms)/L^2;
    ((-12*EI + EA*L^2)*ls*ms)/L^3, (12*EI*ls^2 + EA*L^2*ms^2)/L^3, ...
        (6*EI*ls)/L^2, ((12*EI - EA*L^2)*ls*ms)/L^3, ...
        -((12*EI*ls^2 + EA*L^2*ms^2)/L^3), (6*EI*ls)/L^2;
    (-6*EI*ms)/L^2, (6*EI*ls)/L^2, (4*EI)/L, ...
        (6*EI*ms)/L^2, (-6*EI*ls)/L^2,(2*EI)/L;
    -((EA*L^2*ls^2 + 12*EI*ms^2)/L^3), ((12*EI -EA*L^2)*ls*ms)/L^3, ...
        (6*EI*ms)/L^2, (EA*L^2*ls^2 + 12*EI*ms^2)/L^3, ...
        ((-12*EI + EA*L^2)*ls*ms)/L^3, (6*EI*ms)/L^2;
    ((12*EI - EA*L^2)*ls*ms)/L^3, -((12*EI*ls^2 + EA*L^2*ms^2)/L^3),...
        (-6*EI*ls)/L^2, ((-12*EI + EA*L^2)*ls*ms)/L^3, ...
        (12*EI*ls^2 + EA*L^2*ms^2)/L^3, (-6*EI*ls)/L^2;
    (-6*EI*ms)/L^2, (6*EI*ls)/L^2, (2*EI)/L, (6*EI*ms)/L^2, ...
        (-6*EI*ls)/L^2, (4*EI)/L];
rq = [(L*(ls*qs - ms*qt))/2; (L*(ms*qs + ls*qt))/2;(L^2*qt)/12;
    (L*(ls*qs - ms*qt))/2; (L*(ms*qs + ls*qt))/2; -(L^2*qt)/12];
```

MatlabFiles\Chap4\PlaneFrameResults.m

```
function [f, bm, V] = PlaneFrameResults(modulus, inertia, A, ...
    qs, qt, coord, dn)
% [f, bm, V] = PlaneFrameResults(modulus, inertia, A, ...
%    qs, qt, coord, dn)
% Generates frame element results
% modulus = modulus of elasticity
% inertia = moment of inertia
% A = area of cross-section
% qs = distributed load along the element axis
% qt = distributed load normal to the element axis
% coord = coordinates at the element ends
% dn = nodal solution
% [f, bm, V] = [axial force, bending moment, shear]

EI=modulus*inertia; EA = modulus*A;
x1=coord(1,1); y1=coord(1,2);
x2=coord(2,1); y2=coord(2,2);
L=sqrt((x2-x1)^2+(y2-y1)^2);
ls=(x2-x1)/L; ms=(y2-y1)/L;

u = dn([1,4]);
v = dn([2,3,5,6]);

f=[]; bm=[]; V=[];
% Change increment to get results at more points
for s=0:L/2:L
    x = x1 + s*ls; y = y1+ s*ms;
    f = [f; [x,y,EA*(u(2)-u(1))/L]];
    dn2 = [(12*s)/L^3 - 6/L^2, (6*s)/L^2 - 4/L, 6/L^2 - (12*s)/L^3, ...
            (6*s)/L^2 - 2/L];
    bm = [bm; [x, y, EI*dn2*v+(qt*(L^2 - 6*s*L + 6*s^2))/(12)]];
    dn3 = [12/L^3, 6/L^2, -(12/L^3), 6/L^2];
    V = [V; [x, y, EI*dn3*v+((qt*(12*s - 6*L))/(12))]];
end
```

Using these functions now we consider solution of the two element model.

MatlabFiles\Chap4\PlaneFrameEx.m

```
% Plane frame example
e = 30000; a = 100; inertia = 1000; L = 15*12; q = 1/12;
nodes = [0, 0; L/sqrt(2), L/sqrt(2); L + L/sqrt(2), L/sqrt(2)];
conn=[1,2; 2,3];
lmm=[1,2,3,4,5,6; 4,5,6,7,8,9];
n=3*length(nodes);
debc=[1,2,3,7,8,9]; ebcVals=zeros(length(debc),1);
```

```
K=zeros(n); R = zeros(n,1);
% Generate equations for each element and assemble them.
for i=1
   lm=lmm(i,:);
   con=conn(i,:);
   [ke, rq] = PlaneFrameElement(e, inertia, a, 0, -q, nodes(con,:));
   K(lm, lm) = K(lm, lm) + ke;
   R(lm) = R(lm) + rq;
end
for i=2
   lm=lmm(i,:);
   con=conn(i,:);
   [ke, rq] = PlaneFrameElement(e, inertia, a, 0, 0, nodes(con,:));
   K(lm, lm) = K(lm, lm) + ke;
   R(lm) = R(lm) + rq;
end
K
R
% Nodal solution and reactions
d = NodalSoln(K, R, debc, ebcVals)
fa=[]; bma=[]; Va=[];
for i=1
   lm=lmm(i,:);
   con=conn(i,:);
   [f, bm, V]=PlaneFrameResults(e, inertia, a, 0, -q, ...
      nodes(con,:), d(lm));
   fa = [fa; f]; bma = [bma; bm]; Va = [Va; V];
end
for i=2
   lm=lmm(i,:);
   con=conn(i,:);
   [f, bm, V]=PlaneFrameResults(e, inertia, a, 0, 0, ...
      nodes(con,:), d(lm));
   fa = [fa; f]; bma = [bma; bm]; Va = [Va; V];
end
fa
bma
Va
```

```
>> PlaneFrameEx

K =

  Columns 1 through 6

     8364.2      8302.5     -3928.4     -8364.2     -8302.5     -3928.4
     8302.5      8364.2      3928.4     -8302.5     -8364.2      3928.4
```

```
   -3928.4       3928.4  6.6667e+005       3928.4      -3928.4  3.3333e+005
   -8364.2      -8302.5       3928.4        25031       8302.5       3928.4
   -8302.5      -8364.2      -3928.4       8302.5       8425.9       1627.2
   -3928.4       3928.4  3.3333e+005       3928.4       1627.2  1.3333e+006
         0            0            0       -16667            0            0
         0            0            0            0      -61.728      -5555.6
         0            0            0            0       5555.6  3.3333e+005

  Columns 7 through 9

         0            0            0
         0            0            0
         0            0            0
    -16667            0            0
         0      -61.728       5555.6
         0      -5555.6  3.3333e+005
     16667            0            0
         0       61.728      -5555.6
         0      -5555.6  6.6667e+005


R =

    5.3033
   -5.3033
      -225
    5.3033
   -5.3033
       225
         0
         0
         0


d =

          0
          0
          0
 0.00060161
 -0.0012547
 0.00016851
          0
          0
          0
```

fa =

0	0	10.027
63.64	63.64	10.027
127.28	127.28	10.027
127.28	127.28	-10.027
217.28	127.28	-10.027
307.28	127.28	-10.027

bma =

0	0	-288.14
63.64	63.64	140.58
127.28	127.28	-105.69
127.28	127.28	-105.37
217.28	127.28	-28.085
307.28	127.28	49.199

Va =

0	0	8.5136
63.64	63.64	1.0136
127.28	127.28	-6.4864
127.28	127.28	0.85871
217.28	127.28	0.85871
307.28	127.28	0.85871