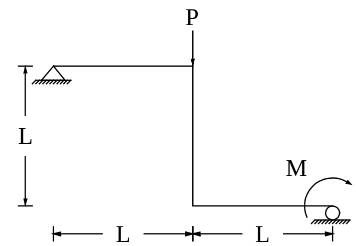
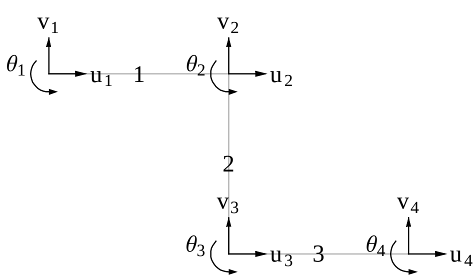
Example 4.10 Three element frame (p. 270)

$$M = 20\,\text{kN} - m;\ P = 10\,\text{kN};\ L = 1\,m;\ E = 210\,\text{GPa};\ A = 4\,\text{x}10^{-2}\,m^2;\ I = 4\,\text{x}10^{-4}\,m^4$$





Use $\text{kN} - m$ units for numerical computations. The computed displacements will be in m and stresses in kN/m^2.

Specified nodal loads

Node	dof	Value
2	v_2	-10
4	θ_4	-20

Global equations at start of the element assembly process

$$\begin{pmatrix} 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \\ u_3 \\ v_3 \\ \theta_3 \\ u_4 \\ v_4 \\ \theta_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -20 \end{pmatrix}$$

Equations for element 1

$E = 2.1\times 10^{8}$; $I = 0.0004$; $A = 0.04$; $q = \{0., 0.\}$

Nodal coordinates

Element node	Global node number	x	y
1	1	0	0
2	2	1	0

Length = 1; Direction cosines: $\ell_s = 1$ $m_s = 0$

Element equations in local coordinates

$$10^{6.} \begin{pmatrix} 8.4 & 0 & 0 & -8.4 & 0 & 0 \\ 0 & 1.008 & 0.504 & 0 & -1.008 & 0.504 \\ 0 & 0.504 & 0.336 & 0 & -0.504 & 0.168 \\ -8.4 & 0 & 0 & 8.4 & 0 & 0 \\ 0 & -1.008 & -0.504 & 0 & 1.008 & -0.504 \\ 0 & 0.504 & 0.168 & 0 & -0.504 & 0.336 \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{pmatrix} = \begin{pmatrix} 0. \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \end{pmatrix}$$

Global to local transformation, $\boldsymbol{T} = \begin{pmatrix} 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \end{pmatrix}$

Element equations in global coordinates

$$10^{6.}\begin{pmatrix} 8.4 & 0 & 0 & -8.4 & 0 & 0 \\ 0 & 1.008 & 0.504 & 0 & -1.008 & 0.504 \\ 0 & 0.504 & 0.336 & 0 & -0.504 & 0.168 \\ -8.4 & 0 & 0 & 8.4 & 0 & 0 \\ 0 & -1.008 & -0.504 & 0 & 1.008 & -0.504 \\ 0 & 0.504 & 0.168 & 0 & -0.504 & 0.336 \end{pmatrix}\begin{pmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} 0. \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \end{pmatrix}$$

The element contributes to {1, 2, 3, 4, 5, 6} global degrees of freedom.

Adding element equations into appropriate locations we have

$$10^{6}\begin{pmatrix} 8.4 & 0 & 0 & -8.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.008 & 0.504 & 0 & -1.008 & 0.504 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.504 & 0.336 & 0 & -0.504 & 0.168 & 0 & 0 & 0 & 0 & 0 & 0 \\ -8.4 & 0 & 0 & 8.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1.008 & -0.504 & 0 & 1.008 & -0.504 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.504 & 0.168 & 0 & -0.504 & 0.336 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \\ u_3 \\ v_3 \\ \theta_3 \\ u_4 \\ v_4 \\ \theta_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -10. \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -20 \end{pmatrix}$$

Equations for element 2

$E = 2.1\times 10^{8}$; $I = 0.0004$; $A = 0.04$; $q = \{0., 0.\}$

Nodal coordinates

Element node	Global node number	x	y
1	2	1	0
2	3	1	−1

Length = 1; Direction cosines: $\ell_s = 0$ $m_s = -1$

Element equations in local coordinates

$$10^{6.}\begin{pmatrix} 8.4 & 0 & 0 & -8.4 & 0 & 0 \\ 0 & 1.008 & 0.504 & 0 & -1.008 & 0.504 \\ 0 & 0.504 & 0.336 & 0 & -0.504 & 0.168 \\ -8.4 & 0 & 0 & 8.4 & 0 & 0 \\ 0 & -1.008 & -0.504 & 0 & 1.008 & -0.504 \\ 0 & 0.504 & 0.168 & 0 & -0.504 & 0.336 \end{pmatrix}\begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{pmatrix} = \begin{pmatrix} 0. \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \end{pmatrix}$$

Global to local transformation, $\boldsymbol{T} = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

Element equations in global coordinates

$$10^{6.}\begin{pmatrix} 1.008 & 0 & 0.504 & -1.008 & 0 & 0.504 \\ 0 & 8.4 & 0 & 0 & -8.4 & 0 \\ 0.504 & 0 & 0.336 & -0.504 & 0 & 0.168 \\ -1.008 & 0 & -0.504 & 1.008 & 0 & -0.504 \\ 0 & -8.4 & 0 & 0 & 8.4 & 0 \\ 0.504 & 0 & 0.168 & -0.504 & 0 & 0.336 \end{pmatrix}\begin{pmatrix} u_2 \\ v_2 \\ \theta_2 \\ u_3 \\ v_3 \\ \theta_3 \end{pmatrix} = \begin{pmatrix} 0. \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \end{pmatrix}$$

The element contributes to {4, 5, 6, 7, 8, 9} global degrees of freedom.

Adding element equations into appropriate locations we have

$$10^{6}\begin{pmatrix} 8.4 & 0 & 0 & -8.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.008 & 0.504 & 0 & -1.008 & 0.504 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.504 & 0.336 & 0 & -0.504 & 0.168 & 0 & 0 & 0 & 0 & 0 & 0 \\ -8.4 & 0 & 0 & 9.408 & 0 & 0.504 & -1.008 & 0 & 0.504 & 0 & 0 & 0 \\ 0 & -1.008 & -0.504 & 0 & 9.408 & -0.504 & 0 & -8.4 & 0 & 0 & 0 & 0 \\ 0 & 0.504 & 0.168 & 0.504 & -0.504 & 0.672 & -0.504 & 0 & 0.168 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.008 & 0 & -0.504 & 1.008 & 0 & -0.504 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -8.4 & 0 & 0 & 8.4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.504 & 0 & 0.168 & -0.504 & 0 & 0.336 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \\ u_3 \\ v_3 \\ \theta_3 \\ u_4 \\ v_4 \\ \theta_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -10. \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -20 \end{pmatrix}$$

Equations for element 3

$E = 2.1\times 10^{8}$; $I = 0.0004$; $A = 0.04$; $q = \{0., 0.\}$

Nodal coordinates

Element node	Global node number	x	y
1	3	1	−1
2	4	2	−1

Length = 1;   Direction cosines: $\ell_s = 1$ $m_s = 0$

Element equations in local coordinates

$$10^{6.}\begin{pmatrix} 8.4 & 0 & 0 & -8.4 & 0 & 0 \\ 0 & 1.008 & 0.504 & 0 & -1.008 & 0.504 \\ 0 & 0.504 & 0.336 & 0 & -0.504 & 0.168 \\ -8.4 & 0 & 0 & 8.4 & 0 & 0 \\ 0 & -1.008 & -0.504 & 0 & 1.008 & -0.504 \\ 0 & 0.504 & 0.168 & 0 & -0.504 & 0.336 \end{pmatrix}\begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{pmatrix} = \begin{pmatrix} 0. \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \end{pmatrix}$$

Global to local transformation, $\boldsymbol{T} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

Element equations in global coordinates

$$10^{6.}\begin{pmatrix} 8.4 & 0 & 0 & -8.4 & 0 & 0 \\ 0 & 1.008 & 0.504 & 0 & -1.008 & 0.504 \\ 0 & 0.504 & 0.336 & 0 & -0.504 & 0.168 \\ -8.4 & 0 & 0 & 8.4 & 0 & 0 \\ 0 & -1.008 & -0.504 & 0 & 1.008 & -0.504 \\ 0 & 0.504 & 0.168 & 0 & -0.504 & 0.336 \end{pmatrix}\begin{pmatrix} u_3 \\ v_3 \\ \theta_3 \\ u_4 \\ v_4 \\ \theta_4 \end{pmatrix} = \begin{pmatrix} 0. \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \end{pmatrix}$$

The element contributes to {7, 8, 9, 10, 11, 12} global degrees of freedom.

Adding element equations into appropriate locations we have

$$10^6\begin{pmatrix} 8.4 & 0 & 0 & -8.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.008 & 0.504 & 0 & -1.008 & 0.504 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.504 & 0.336 & 0 & -0.504 & 0.168 & 0 & 0 & 0 & 0 & 0 & 0 \\ -8.4 & 0 & 0 & 9.408 & 0 & 0.504 & -1.008 & 0 & 0.504 & 0 & 0 & 0 \\ 0 & -1.008 & -0.504 & 0 & 9.408 & -0.504 & 0 & -8.4 & 0 & 0 & 0 & 0 \\ 0 & 0.504 & 0.168 & 0.504 & -0.504 & 0.672 & -0.504 & 0 & 0.168 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.008 & 0 & -0.504 & 9.408 & 0 & -0.504 & -8.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & -8.4 & 0 & 0 & 9.408 & 0.504 & 0 & -1.008 & 0.504 \\ 0 & 0 & 0 & 0.504 & 0 & 0.168 & -0.504 & 0.504 & 0.672 & 0 & -0.504 & 0.168 \\ 0 & 0 & 0 & 0 & 0 & 0 & -8.4 & 0 & 0 & 8.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.008 & -0.504 & 0 & 1.008 & -0.504 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.504 & 0.168 & 0 & -0.504 & 0.336 \end{pmatrix}$$

$$\begin{pmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \\ u_3 \\ v_3 \\ \theta_3 \\ u_4 \\ v_4 \\ \theta_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -10. \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -20. \end{pmatrix}$$

Essential boundary conditions

Node	dof	Value
1	u_1	0
	v_1	0
4	v_4	0

Remove {1, 2, 11} rows and columns.

After adjusting for essential boundary conditions we have

$$10^6\begin{pmatrix} 0.336 & 0 & -0.504 & 0.168 & 0 & 0 & 0 & 0 & 0 \\ 0 & 9.408 & 0 & 0.504 & -1.008 & 0 & 0.504 & 0 & 0 \\ -0.504 & 0 & 9.408 & -0.504 & 0 & -8.4 & 0 & 0 & 0 \\ 0.168 & 0.504 & -0.504 & 0.672 & -0.504 & 0 & 0.168 & 0 & 0 \\ 0 & -1.008 & 0 & -0.504 & 9.408 & 0 & -0.504 & -8.4 & 0 \\ 0 & 0 & -8.4 & 0 & 0 & 9.408 & 0.504 & 0 & 0.504 \\ 0 & 0.504 & 0 & 0.168 & -0.504 & 0.504 & 0.672 & 0 & 0.168 \\ 0 & 0 & 0 & 0 & -8.4 & 0 & 0 & 8.4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.504 & 0.168 & 0 & 0.336 \end{pmatrix}\begin{pmatrix} \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \\ u_3 \\ v_3 \\ \theta_3 \\ u_4 \\ \theta_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -10. \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -20. \end{pmatrix}$$

Solving the final system of global equations we get

$$\{\theta_1 = 0.0000784722,\ u_2 = 0,\ v_2 = 0.0000685516,\ \theta_2 = 0.0000487103,\ u_3 = 0.0000189484,$$
$$v_3 = 0.0000703373,\ \theta_3 = -0.0000108135,\ u_4 = 0.0000189484,\ \theta_4 = -0.000159623\}$$

Complete table of nodal values

	u	v	θ
1	0	0	0.0000784722
2	0	0.0000685516	0.0000487103
3	0.0000189484	0.0000703373	−0.0000108135
4	0.0000189484	0	−0.000159623

Computation of reactions

Equation numbers of dof with specified values: {1, 2, 11}

Extracting equations {1, 2, 11} from the global system we have

$$10^6\begin{pmatrix} 8.4 & 0 & 0 & -8.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.008 & 0.504 & 0 & -1.008 & 0.504 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.008 & -0.504 & 0 & 1.008 & -0.504 \end{pmatrix}\begin{pmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \\ u_3 \\ v_3 \\ \theta_3 \\ u_4 \\ v_4 \\ \theta_4 \end{pmatrix} = \begin{pmatrix} R_1 + 0. \\ R_2 + 0. \\ R_3 + 0. \end{pmatrix}$$

Substituting the nodal values and re−arranging

$$\begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix} = 10^6$$

$$\begin{pmatrix} 8.4 & 0 & 0 & -8.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.008 & 0.504 & 0 & -1.008 & 0.504 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.008 & -0.504 & 0 & 1.008 & -0.504 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0.0000784722 \\ 0 \\ 0.0000685516 \\ 0.0000487103 \\ 0.0000189484 \\ 0.0000703373 \\ -0.0000108135 \\ 0.0000189484 \\ 0 \\ -0.000159623 \end{pmatrix}$$

Carrying out computations, the reactions are as follows.

Label	dof	Reaction
R_1	u_1	0
R_2	v_1	−5.
R_3	v_4	15.

Sum of Reactions

dof: u	0
dof: v	10.
dof: θ	0

Solution for element 1

$E = 2.1\times 10^8$; $I = 0.0004$; $A = 0.04$; $q = \{0., 0.\}$

Length = 1; Direction cosines: $\ell_s = 1$ $m_s = 0$

Nodal values in global coordinates, $\boldsymbol{d}^T = (0 \quad 0 \quad 0.0000784722 \quad 0 \quad 0.0000685516 \quad 0.0000487103)$

Global to local transformation, $\boldsymbol{T} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

Nodal values in local coordinates, $\boldsymbol{d}_\ell^{\mathrm{T}} = \boldsymbol{T}\boldsymbol{d} = (\,0 \quad 0 \quad 0.0000784722 \quad 0 \quad 0.0000685516 \quad 0.0000487103\,)$

Axial displacement interpolation functions, $\boldsymbol{N}_{\mathrm{u}}^{\mathrm{T}} = \{1 - \mathrm{s},\ \mathrm{s}\}$

Axial displacement, $\mathrm{u(s)} = \boldsymbol{N}_{\mathrm{u}}^{\mathrm{T}} \begin{pmatrix} \mathrm{d}_1 \\ \mathrm{d}_4 \end{pmatrix} = 0$

Axial force, EA du(s)/ds = 0

Beam bending interpolation functions, $\boldsymbol{N}_{\mathrm{v}}^{\mathrm{T}} = \{2\,\mathrm{s}^3 - 3\,\mathrm{s}^2 + 1,\ \mathrm{s}^3 - 2\,\mathrm{s}^2 + \mathrm{s},\ 3\,\mathrm{s}^2 - 2\,\mathrm{s}^3,\ \mathrm{s}^3 - \mathrm{s}^2\}$

Transverse displacement, $\mathrm{v(s)} = \boldsymbol{N}_{\mathrm{v}}^{\mathrm{T}} \begin{pmatrix} \mathrm{d}_2 \\ \mathrm{d}_3 \\ \mathrm{d}_5 \\ \mathrm{d}_6 \end{pmatrix} = 0.0000784722\,\mathrm{s} - 9.92063 \times 10^{-6}\,\mathrm{s}^3$

Fixed−end displacement solution, $= 0.\,(1 - \mathrm{s})^2\,\mathrm{s}^2$

Total transverse displacement, $\mathrm{v(s)} = 0.0000784722\,\mathrm{s} - 9.92063 \times 10^{-6}\,\mathrm{s}^3$

Bending moment, $\mathrm{M} = \mathrm{EI}\ \mathrm{d}^2\mathrm{v(s)}/\mathrm{ds}^2 = -5.\,\mathrm{s}$

Shear force, $\mathrm{V(s)} = \mathrm{dM/ds} = -5.$

Solution for element 2

$\mathrm{E} = 2.1 \times 10^8;$ $\mathrm{I} = 0.0004;$ $\mathrm{A} = 0.04;$ $\mathrm{q} = \{0.,\ 0.\}$

Length = 1; Direction cosines: $\ell_{\mathrm{s}} = 0$ $m_{\mathrm{s}} = -1$

Nodal values in global coordinates, $\boldsymbol{d}^{\mathrm{T}} =$
$(\,0 \quad 0.0000685516 \quad 0.0000487103 \quad 0.0000189484 \quad 0.0000703373 \quad -0.0000108135\,)$

Global to local transformation, $\boldsymbol{T} = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

Nodal values in local coordinates, $\boldsymbol{d}_\ell^{\mathrm{T}} = \boldsymbol{T}\boldsymbol{d} =$

$$(\,-0.0000685516 \quad 0 \quad 0.0000487103 \quad -0.0000703373 \quad 0.0000189484 \quad -0.0000108135\,)$$

Axial displacement interpolation functions, $\boldsymbol{N}_u^{\mathrm{T}} = \{1 - s,\ s\}$

Axial displacement, $u(s) = \boldsymbol{N}_u^{\mathrm{T}}\begin{pmatrix} d_1 \\ d_4 \end{pmatrix} = -1.78571\times 10^{-6}\, s - 0.0000685516$

Axial force, EA du(s)/ds = −15.

Beam bending interpolation functions, $\boldsymbol{N}_v^{\mathrm{T}} = \{2 s^3 - 3 s^2 + 1,\ s^3 - 2 s^2 + s,\ 3 s^2 - 2 s^3,\ s^3 - s^2\}$

Transverse displacement, $v(s) = \boldsymbol{N}_v^{\mathrm{T}}\begin{pmatrix} d_2 \\ d_3 \\ d_5 \\ d_6 \end{pmatrix} = 0.0000487103\, s - 0.0000297619\, s^2$

Fixed−end displacement solution, $= 0.\,(1 - s)^2\, s^2$

Total transverse displacement, $v(s) = 0.0000487103\, s - 0.0000297619\, s^2$

Bending moment, $M = EI\, d^2 v(s)/ds^2 = -5.$

Shear force, $V(s) = dM/ds = 0$

Solution for element 3

$E = 2.1\times 10^8$; $\quad I = 0.0004$; $\quad A = 0.04$; $\quad q = \{0.,\ 0.\}$

Length = 1; $\quad$ Direction cosines: $\ell_s = 1 \quad m_s = 0$

Nodal values in global coordinates, $\boldsymbol{d}^{\mathrm{T}} =$

$$(\,0.0000189484 \quad 0.0000703373 \quad -0.0000108135 \quad 0.0000189484 \quad 0 \quad -0.000159623\,)$$

Global to local transformation, $\boldsymbol{T} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

Nodal values in local coordinates, $\boldsymbol{d}_\ell^{\mathrm{T}} = \boldsymbol{T}\boldsymbol{d} =$

$$(\,0.0000189484 \quad 0.0000703373 \quad -0.0000108135 \quad 0.0000189484 \quad 0 \quad -0.000159623\,)$$

Axial displacement interpolation functions, $\boldsymbol{N}_u^{\mathrm{T}} = \{1 - s,\ s\}$

Axial displacement, $u(s) = \boldsymbol{N}_u^{\mathrm{T}}\begin{pmatrix} d_1 \\ d_4 \end{pmatrix} = 0.0000189484$

Axial force, EA du(s)/ds = 0

Beam bending interpolation functions, $\boldsymbol{N}_v^T = \{2s^3 - 3s^2 + 1,\ s^3 - 2s^2 + s,\ 3s^2 - 2s^3,\ s^3 - s^2\}$

Transverse displacement, $v(s) = \boldsymbol{N}_v^T \begin{pmatrix} d_2 \\ d_3 \\ d_5 \\ d_6 \end{pmatrix} =$

$$-0.0000297619\, s^3 - 0.0000297619\, s^2 - 0.0000108135\, s + 0.0000703373$$

Fixed−end displacement solution, $= 0.\,(1 - s)^2 s^2$

Total transverse displacement, $v(s) = -0.0000297619\, s^3 - 0.0000297619\, s^2 - 0.0000108135\, s + 0.0000703373$

Bending moment, $M = EI\, d^2v(s)/ds^2 = -15.\,s - 5.$

Shear force, $V(s) = dM/ds = -15.$

Forces at element ends

	x	y	Axial force	Bending moment	Shear force
1	0	0	0	0	−5.
	1	0	0	−5.	−5.
2	1	0	−15.	−5.	0
	1	−1	−15.	−5.	0
3	1	−1	0	−5.	−15.
	2	−1	0	−20.	−15.