Computer Implementation 3.2 ***(Matlab)*** *Solution of Buckling Problem (p. 207)*

The buckling problem can be implemented easily in *Matlab* by defining simple functions returning element $\boldsymbol{k}$ and $\boldsymbol{k}_p$ matrices as follows.

MatlabFiles\Chap3\BucklingLinElement.m

```
function [ke, kp] = BucklingLinElement(k, coord)
% [ke, kp] = BucklingLinElement(k, coord)
% Generates equations for a linear element for 1D Buckling
% k = bar stiffness (EI)
% coord = coordinates at the element ends

L=coord(2)-coord(1);
ke = k/L*[1, -1; -1, 1];
kp = [L/3, L/6; L/6, L/3];
```

MatlabFiles\Chap3\BucklingQuadElement.m

```
function [ke, kp] = BucklingQuadElement(k, coord)
% [ke, kp] = BucklingQuadElement(k, coord)
% Generates equations for a quadratic element for 1D Buckling
% k = bar stiffness (EI)
% coord = coordinates at the element ends

L=coord(3)-coord(1);
ke = [(7*k)/(3*L), -((8*k)/(3*L)), k/(3*L);
   -((8*k)/(3*L)), (16*k)/(3*L), -((8*k)/(3*L));
   k/(3*L), -((8*k)/(3*L)), (7*k)/(3*L)];
kp = [((2*L)/15), (L/15), -L/30;
   (L/15), ((8*L)/15), (L/15);
   -L/30, (L/15), ((2*L)/15)];
```

Using the BucklingQuadElement function, a solution using 4 quadratic is obtained as follows.

MatlabFiles\Chap3\BucklingEx.m

```
% Solution of Euler buckling using quadratic elements
L = 12*10.; EI = 10^6;
nodes = [0:L/8:L];n=length(nodes);
Ke=zeros(n); Kp=zeros(n);
% Generate equations for each element and assemble them.
for i=1:4
   lm=[2*(i-1)+1,2*(i-1)+2,2*(i-1)+3];
   [ke, kp] = BucklingQuadElement(EI, nodes(lm));
```

```
    Ke(lm, lm) = Ke(lm, lm) + ke;
    Kp(lm, lm) = Kp(lm, lm) + kp;
end
% Adjust for EBC
debc=[1,n];
df = setdiff(1:n, debc);
Kef = Ke(df, df)
Kep = Kp(df, df)
[v,e] = eig(Kef, Kep);
fprintf('Buckling load = %10.6g',e(1,1))
d = zeros(n,1);
d(df) = v(:,1)
plot(nodes,d),title('First buckling mode'), xlabel('x'),ylabel('v')


>> BucklingEx

Kef =

  1.0e+005 *

    1.7778   -0.8889         0         0         0         0         0
   -0.8889    1.5556   -0.8889    0.1111         0         0         0
         0   -0.8889    1.7778   -0.8889         0         0         0
         0    0.1111   -0.8889    1.5556   -0.8889    0.1111         0
         0         0         0   -0.8889    1.7778   -0.8889         0
         0         0         0    0.1111   -0.8889    1.5556   -0.8889
         0         0         0         0         0   -0.8889    1.7778


Kep =

    16     2     0     0     0     0     0
     2     8     2    -1     0     0     0
     0     2    16     2     0     0     0
     0    -1     2     8     2    -1     0
     0     0     0     2    16     2     0
     0     0     0    -1     2     8     2
     0     0     0     0     0     2    16

Buckling load =     685.74
d =

         0
    0.0494
    0.0913
    0.1193
```

0.1292
0.1193
0.0913
0.0494
0