Computer Implementation 1.3 *(Matlab)* *Triangular plane stress element (p. 21)*

MatlabFiles\Chap1\PlaneStressTriElement.m

```
function k = PlaneStressTriElement(e, nu, h, coord)
% k = PlaneStressTriElement(e, nu, h, coord)
% Generates stiffness matrix for a triangular element for plane stress
% e = Modulus of elasticity
% nu = Poisson's ratio
% h = Thickness
% coord = Coordinates at the element ends

x1=coord(1,1); y1=coord(1,2);
x2=coord(2,1); y2=coord(2,2);
x3=coord(3,1); y3=coord(3,2);
b1 = y2 - y3; b2 = y3 - y1; b3 = y1 - y2;
c1 = x3 - x2; c2 = x1 - x3; c3 = x2 - x1;
f1 = x2*y3 - x3*y2; f2 = x3*y1 - x1*y3; f3 = x1*y2 - x2*y1;
A = (f1 + f2 + f3)/2;
C = e/(1 - nu^2)*[1, nu, 0; nu, 1, 0; 0, 0, (1 - nu)/2];
B = [b1, 0, c1; 0, c1, b1; b2, 0, c2; 0, c2, b2;
    b3, 0, c3; 0, c3, b3]/(2*A);
k = h*A*(B*C*B');
```

MatlabFiles\Chap1\PlaneStressTriLoad.m

```
function rq = PlaneStressTriLoad(side, qn, qt, h, coord)
% rq = PlaneStressTriLoad(side, qn, qt, h, coord)
% Generates equivalent load vector for a triangular element
% side = side over which the load is specified
% qn, qt = load components in the normal and the tangential direction
% h = thickness
% coord = coordinates at the element ends

x1=coord(1,1); y1=coord(1,2);
x2=coord(2,1); y2=coord(2,2);
x3=coord(3,1); y3=coord(3,2);
switch (side)
case 1
    L=sqrt((x2-x1)^2+(y2-y1)^2);
    nx=(y2-y1)/L; ny=-(x2-x1)/L;
    qx = nx*qn - ny*qt;
    qy = ny*qn + nx*qt;
    rq = h*L/2 * [qx; qy; qx; qy; 0; 0];
case 2
    L=sqrt((x2-x3)^2+(y2-y3)^2);
    nx=(y3-y2)/L; ny=-(x3-x2)/L;
    qx = nx*qn - ny*qt;
    qy = ny*qn + nx*qt;
    rq = h*L/2 * [0; 0; qx; qy; qx; qy];
case 3
    L=sqrt((x3-x1)^2+(y3-y1)^2);
    nx=(y1-y3)/L; ny=-(x1-x3)/L;
    qx = nx*qn - ny*qt;
    qy = ny*qn + nx*qt;
    rq = h*L/2 * [qx; qy; 0; 0; qx; qy];
end
```

Using these functions finite element equations for any triangular element for a plane stress problem can easily be written. As an example we use these functions to develop matrices for the element number 2 in the finite element model of the notched beam. The element is connected between nodes 4, 7 and 11. There is an applied load in the negative outer normal direction ($q_n = -50$ and $q_t = 0$) on side 3 of the element. With $E = 3000000$, $\nu = 0.2$ and $h = 4$ the $\boldsymbol{k}$ matrix and the equivalent nodal load vector $\boldsymbol{r}_q$ for the element are computed as follows.

MatlabFiles\Chap1\PlaneStressElementEx1.m

```
% Plane stress element equations
nodes = [0, 5; 0, 22/3; 0, 29/3; 0, 12; 3, 5; 4, 22/3;
    5, 29/3; 6, 0; 6, 5/2; 6, 5; 6, 12; 8, 22/3; 10, 29/3;
    23/2, 17/3; 12, 12; 15, 4; 17, 53/6; 20, 0;45/2, 12;
    24, 8; 80/3, 4; 33, 12; 100/3, 8; 37,0; 40, 12; 121/3, 4;
    131/3, 8; 47, 12; 54, 0; 54, 4; 54, 8; 54, 12];
k = PlaneStressTriElement(3000000, .2, 4, nodes([4 7 11],:))
rq = PlaneStressTriLoad (3, -50, 0, 4, nodes([4 7 11],:))
```

```
>> PlaneStressElementEx1

k =

  1.0e+007 *

    0.2609   -0.0625   -0.1071    0.1250   -0.1538   -0.0625
   -0.0625    0.1419    0.2500   -0.2679   -0.1875    0.1260
   -0.1071    0.2500    0.6429         0   -0.5357   -0.2500
    0.1250   -0.2679         0    1.6071   -0.1250   -1.3393
   -0.1538   -0.1875   -0.5357   -0.1250    0.6895    0.3125
   -0.0625    0.1260   -0.2500   -1.3393    0.3125    1.2133


rq =

     0
  -600
     0
     0
     0
  -600
```