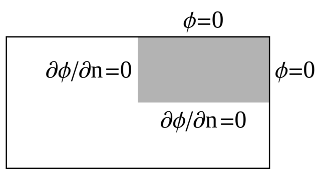
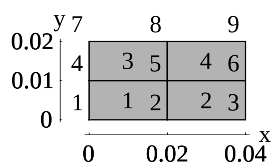
Example 5.4: Torsion of a rectangular shaft (p. 342)

Find stresses developed in a $4\,\text{cm} \times 8\,\text{cm}$ rectangular shaft when it is subjected to a torque of 500 N-m. The shaft is 1 m long and $G = 76.9\,\text{GPa}$. A quarter of the domain needs to be modeled because of symmetry.





The governing differential equation for the problem is

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + 2\,G\,\theta = 0$$

where G is shear modulus and θ is angle of twist per unit length. The boundary condition is $\phi = 0$ on the boundary. As a result of essential boundary condition $\phi = 0$ at nodes {3, 6, 9, 8, 7}. There are no nonzero natural boundary conditions.

Since θ is unknown, we start by arbitrarily assuming Gθ =1. After performing the analysis, we compute the total torque T_a. This torque corresponds to the assumed value of Gθ. Since the relationship between the torque and the angle of twist is linear, the actual value of θ can then be computed using the given value of torque T as follows.

$$\theta = T / (G\, T_a)$$

The actual ϕ values are obtained by multiplying the computed values by the actual Gθ value. The complete finite element solution, using N and m units, is as follows.

Global equations at start of the element assembly process

$$\begin{pmatrix} 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0 \end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\\ \phi_5\\ \phi_6\\ \phi_7\\ \phi_8\\ \phi_9\end{pmatrix}=\begin{pmatrix}0\\0\\0\\0\\0\\0\\0\\0\\0\end{pmatrix}$$

Equations for element 1

Element dimensions: $a = 0.01$; $b = 0.005$

$k_x = 1$; $k_y = 1$; $p = 0$; $q = 2$

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 0.833333 & 0.166667 & -0.416667 & -0.583333\\ 0.166667 & 0.833333 & -0.583333 & -0.416667\\ -0.416667 & -0.583333 & 0.833333 & 0.166667\\ -0.583333 & -0.416667 & 0.166667 & 0.833333 \end{pmatrix}; \quad \boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix}; \quad \boldsymbol{r}_{\mathrm{q}} = \begin{pmatrix}0.0001\\0.0001\\0.0001\\0.0001\end{pmatrix}$$

Complete element equations

$$\begin{pmatrix} 0.833333 & 0.166667 & -0.416667 & -0.583333\\ 0.166667 & 0.833333 & -0.583333 & -0.416667\\ -0.416667 & -0.583333 & 0.833333 & 0.166667\\ -0.583333 & -0.416667 & 0.166667 & 0.833333 \end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2\\ \phi_5\\ \phi_4\end{pmatrix}=\begin{pmatrix}0.0001\\0.0001\\0.0001\\0.0001\end{pmatrix}$$

The element contributes to {1, 2, 5, 4} global degrees of freedom.

Adding element equations into appropriate locations we have

$$\begin{pmatrix} 0.833333 & 0.166667 & 0 & -0.583333 & -0.416667 & 0&0&0&0\\ 0.166667 & 0.833333 & 0 & -0.416667 & -0.583333 & 0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ -0.583333 & -0.416667 & 0 & 0.833333 & 0.166667 & 0&0&0&0\\ -0.416667 & -0.583333 & 0 & 0.166667 & 0.833333 & 0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0 \end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\\ \phi_5\\ \phi_6\\ \phi_7\\ \phi_8\\ \phi_9\end{pmatrix}=\begin{pmatrix}0.0001\\0.0001\\0\\0.0001\\0.0001\\0\\0\\0\\0\end{pmatrix}$$

Equations for element 2

Element dimensions: $a = 0.01$; $b = 0.005$

$k_x = 1$; $k_y = 1$; $p = 0$; $q = 2$

$$\boldsymbol{k}_k = \begin{pmatrix} 0.833333 & 0.166667 & -0.416667 & -0.583333 \\ 0.166667 & 0.833333 & -0.583333 & -0.416667 \\ -0.416667 & -0.583333 & 0.833333 & 0.166667 \\ -0.583333 & -0.416667 & 0.166667 & 0.833333 \end{pmatrix}; \quad \boldsymbol{k}_p = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}; \quad \boldsymbol{r}_q = \begin{pmatrix} 0.0001 \\ 0.0001 \\ 0.0001 \\ 0.0001 \end{pmatrix}$$

Complete element equations

$$\begin{pmatrix} 0.833333 & 0.166667 & -0.416667 & -0.583333 \\ 0.166667 & 0.833333 & -0.583333 & -0.416667 \\ -0.416667 & -0.583333 & 0.833333 & 0.166667 \\ -0.583333 & -0.416667 & 0.166667 & 0.833333 \end{pmatrix} \begin{pmatrix} \phi_2 \\ \phi_3 \\ \phi_6 \\ \phi_5 \end{pmatrix} = \begin{pmatrix} 0.0001 \\ 0.0001 \\ 0.0001 \\ 0.0001 \end{pmatrix}$$

The element contributes to {2, 3, 6, 5} global degrees of freedom.

Adding element equations into appropriate locations we have

$$\begin{pmatrix} 0.833333 & 0.166667 & 0 & -0.583333 & -0.416667 & 0 & 0 & 0 & 0 \\ 0.166667 & 1.66667 & 0.166667 & -0.416667 & -1.16667 & -0.416667 & 0 & 0 & 0 \\ 0 & 0.166667 & 0.833333 & 0 & -0.416667 & -0.583333 & 0 & 0 & 0 \\ -0.583333 & -0.416667 & 0 & 0.833333 & 0.166667 & 0 & 0 & 0 & 0 \\ -0.416667 & -1.16667 & -0.416667 & 0.166667 & 1.66667 & 0.166667 & 0 & 0 & 0 \\ 0 & -0.416667 & -0.583333 & 0 & 0.166667 & 0.833333 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \\ \phi_9 \end{pmatrix} = \begin{pmatrix} 0.0001 \\ 0.0002 \\ 0.0001 \\ 0.0001 \\ 0.0002 \\ 0.0001 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 3

Element dimensions: $a = 0.01$; $b = 0.005$

$k_x = 1$; $k_y = 1$; $p = 0$; $q = 2$

$$\boldsymbol{k}_k = \begin{pmatrix} 0.833333 & 0.166667 & -0.416667 & -0.583333 \\ 0.166667 & 0.833333 & -0.583333 & -0.416667 \\ -0.416667 & -0.583333 & 0.833333 & 0.166667 \\ -0.583333 & -0.416667 & 0.166667 & 0.833333 \end{pmatrix}; \quad \boldsymbol{k}_p = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}; \quad \boldsymbol{r}_q = \begin{pmatrix} 0.0001 \\ 0.0001 \\ 0.0001 \\ 0.0001 \end{pmatrix}$$

Complete element equations

$$\begin{pmatrix} 0.833333 & 0.166667 & -0.416667 & -0.583333 \\ 0.166667 & 0.833333 & -0.583333 & -0.416667 \\ -0.416667 & -0.583333 & 0.833333 & 0.166667 \\ -0.583333 & -0.416667 & 0.166667 & 0.833333 \end{pmatrix} \begin{pmatrix} \phi_4 \\ \phi_5 \\ \phi_8 \\ \phi_7 \end{pmatrix} = \begin{pmatrix} 0.0001 \\ 0.0001 \\ 0.0001 \\ 0.0001 \end{pmatrix}$$

The element contributes to {4, 5, 8, 7} global degrees of freedom.

Adding element equations into appropriate locations we have

$$\begin{pmatrix} 0.833333 & 0.166667 & 0 & -0.583333 & -0.416667 & 0 & 0 & 0 & 0 \\ 0.166667 & 1.66667 & 0.166667 & -0.416667 & -1.16667 & -0.416667 & 0 & 0 & 0 \\ 0 & 0.166667 & 0.833333 & 0 & -0.416667 & -0.583333 & 0 & 0 & 0 \\ -0.583333 & -0.416667 & 0 & 1.66667 & 0.333333 & 0 & -0.583333 & -0.416667 & 0 \\ -0.416667 & -1.16667 & -0.416667 & 0.333333 & 2.5 & 0.166667 & -0.416667 & -0.583333 & 0 \\ 0 & -0.416667 & -0.583333 & 0 & 0.166667 & 0.833333 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.583333 & -0.416667 & 0 & 0.833333 & 0.166667 & 0 \\ 0 & 0 & 0 & -0.416667 & -0.583333 & 0 & 0.166667 & 0.833333 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \\ \phi_9 \end{pmatrix} = \begin{pmatrix} 0.0001 \\ 0.0002 \\ 0.0001 \\ 0.0002 \\ 0.0003 \\ 0.0001 \\ 0.0001 \\ 0.0001 \\ 0 \end{pmatrix}$$

Equations for element 4

Element dimensions: $a = 0.01$; $b = 0.005$

$k_x = 1$; $k_y = 1$; $p = 0$; $q = 2$

$$\boldsymbol{k}_k = \begin{pmatrix} 0.833333 & 0.166667 & -0.416667 & -0.583333 \\ 0.166667 & 0.833333 & -0.583333 & -0.416667 \\ -0.416667 & -0.583333 & 0.833333 & 0.166667 \\ -0.583333 & -0.416667 & 0.166667 & 0.833333 \end{pmatrix}; \quad \boldsymbol{k}_p = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}; \quad \boldsymbol{r}_q = \begin{pmatrix} 0.0001 \\ 0.0001 \\ 0.0001 \\ 0.0001 \end{pmatrix}$$

Complete element equations

$$\begin{pmatrix} 0.833333 & 0.166667 & -0.416667 & -0.583333 \\ 0.166667 & 0.833333 & -0.583333 & -0.416667 \\ -0.416667 & -0.583333 & 0.833333 & 0.166667 \\ -0.583333 & -0.416667 & 0.166667 & 0.833333 \end{pmatrix} \begin{pmatrix} \phi_5 \\ \phi_6 \\ \phi_9 \\ \phi_8 \end{pmatrix} = \begin{pmatrix} 0.0001 \\ 0.0001 \\ 0.0001 \\ 0.0001 \end{pmatrix}$$

The element contributes to {5, 6, 9, 8} global degrees of freedom.

Adding element equations into appropriate locations we have

$$\begin{pmatrix} 0.833333 & 0.166667 & 0 & -0.583333 & -0.416667 & 0 & 0 & 0 & 0 \\ 0.166667 & 1.66667 & 0.166667 & -0.416667 & -1.16667 & -0.416667 & 0 & 0 & 0 \\ 0 & 0.166667 & 0.833333 & 0 & -0.416667 & -0.583333 & 0 & 0 & 0 \\ -0.583333 & -0.416667 & 0 & 1.66667 & 0.333333 & 0 & -0.583333 & -0.416667 & 0 \\ -0.416667 & -1.16667 & -0.416667 & 0.333333 & 3.33333 & 0.333333 & -0.416667 & -1.16667 & -0.416667 \\ 0 & -0.416667 & -0.583333 & 0 & 0.333333 & 1.66667 & 0 & -0.416667 & -0.583333 \\ 0 & 0 & 0 & -0.583333 & -0.416667 & 0 & 0.833333 & 0.166667 & 0 \\ 0 & 0 & 0 & -0.416667 & -1.16667 & -0.416667 & 0.166667 & 1.66667 & 0.166667 \\ 0 & 0 & 0 & 0 & -0.416667 & -0.583333 & 0 & 0.166667 & 0.833333 \end{pmatrix}$$

Essential boundary conditions

Node	dof	Value
3	ϕ_3	0
6	ϕ_6	0
7	ϕ_7	0
8	ϕ_8	0
9	ϕ_9	0

Remove {3, 6, 7, 8, 9} rows and columns.

After adjusting for essential boundary conditions we have

$$\begin{pmatrix} 0.833333 & 0.166667 & -0.583333 & -0.416667 \\ 0.166667 & 1.66667 & -0.416667 & -1.16667 \\ -0.583333 & -0.416667 & 1.66667 & 0.333333 \\ -0.416667 & -1.16667 & 0.333333 & 3.33333 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_4 \\ \phi_5 \end{pmatrix} = \begin{pmatrix} 0.0001 \\ 0.0002 \\ 0.0002 \\ 0.0004 \end{pmatrix}$$

Solving the final system of global equations we get

$$\{\phi_1 = 0.000380919,\ \phi_2 = 0.000331898,\ \phi_4 = 0.000285245,\ \phi_5 = 0.000255255\}$$

Complete table of nodal values

	ϕ
1	0.000380919
2	0.000331898
3	0
4	0.000285245
5	0.000255255
6	0
7	0
8	0
9	0

Solution for element 1

Coordinates of element center

$x_c = 0.01;\qquad y_c = 0.005$

Element dimensions: $a = 0.01;\qquad b = 0.005$

Interpolation functions in local element coordinates

$$\boldsymbol{N}^T = \{5000.\,t\,s - 25.\,s - 50.\,t + 0.25,\; -5000.\,t\,s + 25.\,s - 50.\,t + 0.25,\; 5000.\,t\,s + 25.\,s + 50.\,t + 0.25,\; -5000.\,t\,s - 25.\,s + 50.\,t + 0.25\}$$

Shift for global coordinates: $s = x - 0.01;\qquad t = y - 0.005$

Interpolation functions in global coordinates

$$\boldsymbol{N}^T = \{5000.\,y\,x - 50.\,x - 100.\,y + 1.,\; 50.\,x - 5000.\,x\,y,\; 5000.\,x\,y,\; 100.\,y - 5000.\,x\,y\}$$

Nodal values, $\boldsymbol{d}^T = \{0.000380919,\ 0.000331898,\ 0.000255255,\ 0.000285245\}$

$\phi(x, y) = \boldsymbol{N}^T\boldsymbol{d} = 0.0951558\,y\,x - 0.0024511\,x - 0.00956742\,y + 0.000380919$

$\partial\phi/\partial x = 0.0951558\,y - 0.0024511;\qquad \partial\phi/\partial y = 0.0951558\,x - 0.00956742$

Solution for element 2

Coordinates of element center

$x_c = 0.03;\qquad y_c = 0.005$

Element dimensions: $a = 0.01;\qquad b = 0.005$

Interpolation functions in local element coordinates

$$\boldsymbol{N}^T = \{5000.\,t\,s - 25.\,s - 50.\,t + 0.25,\; -5000.\,t\,s + 25.\,s - 50.\,t + 0.25,\; 5000.\,t\,s + 25.\,s + 50.\,t + 0.25,\; -5000.\,t\,s - 25.\,s + 50.\,t + 0.25\}$$

Shift for global coordinates: $s = x - 0.03$; $t = y - 0.005$

Interpolation functions in global coordinates

$$\boldsymbol{N}^{T} = \{5000.\, y\, x - 50.\, x - 200.\, y + 2.,\ -5000.\, y\, x + 50.\, x + 100.\, y - 1.,\ 5000.\, x\, y - 100.\, y,\ 200.\, y - 5000.\, x\, y\}$$

Nodal values, $\boldsymbol{d}^{T} = \{0.000331898,\ 0,\ 0,\ 0.000255255\}$

$\phi(x, y) = \boldsymbol{N}^{T}\boldsymbol{d} = 0.383215\, y\, x - 0.0165949\, x - 0.0153286\, y + 0.000663795$

$\partial\phi/\partial x = 0.383215\, y - 0.0165949$; $\partial\phi/\partial y = 0.383215\, x - 0.0153286$

Solution for element 3

Coordinates of element center

$x_c = 0.01$; $y_c = 0.015$

Element dimensions: $a = 0.01$; $b = 0.005$

Interpolation functions in local element coordinates

$$\boldsymbol{N}^{T} = \{5000.\, t\, s - 25.\, s - 50.\, t + 0.25,\ -5000.\, t\, s + 25.\, s - 50.\, t + 0.25,\ 5000.\, t\, s + 25.\, s + 50.\, t + 0.25,\ -5000.\, t\, s - 25.\, s + 50.\, t + 0.25\}$$

Shift for global coordinates: $s = x - 0.01$; $t = y - 0.015$

Interpolation functions in global coordinates

$$\boldsymbol{N}^{T} = \{5000.\, y\, x - 100.\, x - 100.\, y + 2.,\ 100.\, x - 5000.\, x\, y,\ 5000.\, x\, y - 50.\, x,\ -5000.\, y\, x + 50.\, x + 100.\, y - 1.\}$$

Nodal values, $\boldsymbol{d}^{T} = \{0.000285245,\ 0.000255255,\ 0,\ 0\}$

$\phi(x, y) = \boldsymbol{N}^{T}\boldsymbol{d} = 0.149954\, y\, x - 0.00299907\, x - 0.0285245\, y + 0.000570491$

$\partial\phi/\partial x = 0.149954\, y - 0.00299907$; $\partial\phi/\partial y = 0.149954\, x - 0.0285245$

Solution for element 4

Coordinates of element center

$x_c = 0.03$; $y_c = 0.015$

Element dimensions: $a = 0.01$; $b = 0.005$

Interpolation functions in local element coordinates

$$\boldsymbol{N}^{T} = \{5000.\, t\, s - 25.\, s - 50.\, t + 0.25,\ -5000.\, t\, s + 25.\, s - 50.\, t + 0.25,\ 5000.\, t\, s + 25.\, s + 50.\, t + 0.25,\ -5000.\, t\, s - 25.\, s + 50.\, t + 0.25\}$$

Shift for global coordinates: $s = x - 0.03$; $t = y - 0.015$

Interpolation functions in global coordinates

$$\boldsymbol{N}^T = \{5000.\,y\,x - 100.\,x - 200.\,y + 4.,\; -5000.\,y\,x + 100.\,x + 100.\,y - 2.,\; 5000.\,y\,x - 50.\,x - 100.\,y + 1.,\; -5000.\,y\,x + 50.\,x + 200.\,y - 2.\}$$

Nodal values, $\boldsymbol{d}^T = \{0.000255255, 0, 0, 0\}$

$\phi(x, y) = \boldsymbol{N}^T\boldsymbol{d} = 1.27627\,y\,x - 0.0255255\,x - 0.0510509\,y + 0.00102102$

$\partial\phi/\partial x = 1.27627\,y - 0.0255255$; $\partial\phi/\partial y = 1.27627\,x - 0.0510509$

Solution summary

Nodal solution

	x–coord	y–coord	ϕ
1	0.	0.	0.000380919
2	0.02	0.	0.000331898
3	0.04	0.	0
4	0.	0.01	0.000285245
5	0.02	0.01	0.000255255
6	0.04	0.01	0
7	0.	0.02	0
8	0.02	0.02	0
9	0.04	0.02	0

Solution at element centroids

	x–coord	y–coord	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.01	0.005	0.000313329	−0.00197532	−0.00861586
2	0.03	0.005	0.000146788	−0.0146788	−0.00383215
3	0.01	0.015	0.000135125	−0.000749769	−0.027025
4	0.03	0.015	0.0000638136	−0.00638136	−0.0127627

	ϕ_a	$\int\int\phi_a\, dA$
1	$0.0951558\,y\,x - 0.0024511\,x - 0.00956742\,y + 0.000380919$	6.26658×10^{-8}
2	$0.383215\,y\,x - 0.0165949\,x - 0.0153286\,y + 0.000663795$	2.93576×10^{-8}
3	$0.149954\,y\,x - 0.00299907\,x - 0.0285245\,y + 0.000570491$	2.7025×10^{-8}
4	$1.27627\,y\,x - 0.0255255\,x - 0.0510509\,y + 0.00102102$	1.27627×10^{-8}

The total torque is given by

$$T = 2\iint_A \phi \, \mathrm{dA}$$

Summing $\iint \phi \, \mathrm{dA}$ contributions from all elements and multiplying by 2 gives the total torque. Since we are modeling a 1/4th of the shape, the torque for the entire section is

$$T_a = 4\times 2\times \sum(\iint \phi_a \, \mathrm{dA}) = 1.05449\times 10^{-6} \text{ N}\cdot\text{m}$$

Since the actual torque is 500 N-m, the actual value of the angle of twist is

$$\mathrm{G}\theta = 500/T_a = 4.74163\times 10^{8}; \qquad \theta = 0.00616597 \text{ rad/m}$$

The ϕ values are simply scaled by this value of $\mathrm{G}\theta$ and thus the solution corresponding to a torque of 500 N-m is as follows.

$$\phi = (500/T_a)\phi_a = 4.74163\times 10^{8}\, \phi_a$$

	ϕ ($\times 10^6$)	$\tau_{yz}=-\partial\phi/\partial x$ (MPa)	$\tau_{xz}=\partial\phi/\partial y$ (MPa)
1	$45.1194\,y\,x - 1.16222\,x - 4.53652\,y + 0.180618$	$1.16222 - 45.1194\,y$	$45.1194\,x - 4.53652$
2	$181.706\,y\,x - 7.86868\,x - 7.26826\,y + 0.314747$	$7.86868 - 181.706\,y$	$181.706\,x - 7.26826$
3	$71.1025\,y\,x - 1.42205\,x - 13.5253\,y + 0.270506$	$1.42205 - 71.1025\,y$	$71.1025\,x - 13.5253$
4	$605.162\,y\,x - 12.1032\,x - 24.2065\,y + 0.484129$	$12.1032 - 605.162\,y$	$605.162\,x - 24.2065$

Stresses at element centroids

	τ_{yz} (MPa)	τ_{xz} (MPa)
1	0.936622	−4.08532
2	6.96015	−1.81706
3	0.355513	−12.8143
4	3.02581	−6.05162

The maximum shear stress occurs at midpoint of the long side (node 7) which from element 3 is

Stresses at node 7: $\tau_{yz} = 2.22045 \times 10^{-16}$ MPa; $\tau_{xz} = -13.5253$ MPa;
$\tau_{max} = 13.5253$ MPa

An exact solution for the problem is available as follows (Roark's Formulas for Stress and Strain, Seventh Edition, p. 401, McGraw-Hill 2002).

$$\tau_{max} = \frac{3T}{8ab^2}\left(1 + 0.6095\, b/a + 0.8865\,(b/a)^2 - 1.8023\,(b/a)^3 + 0.91\,(b/a)^4\right)$$

where $2a$ is the longer dimension of the section and $2b$ is the shorter dimension.

Exact solution: $\tau_{max} = 15.9136$ MPa