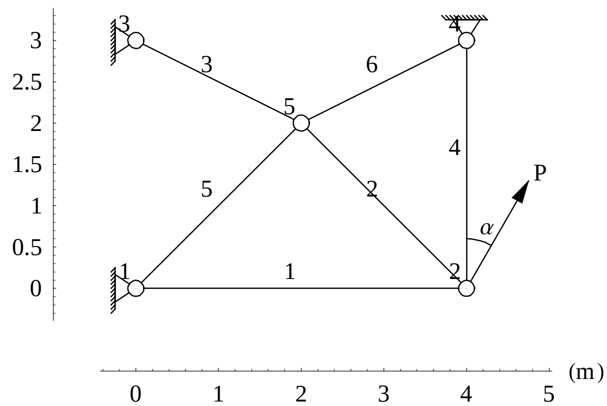
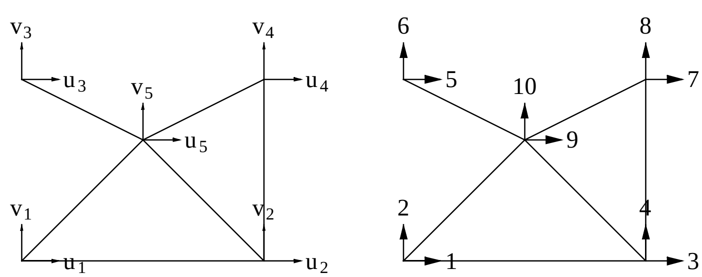
Example 4.1: Six-bar truss (p. 226)

All members have the same cross-sectional area and are of the same material, $E = 200\,\text{GPa}$ and $A = 0.001\,m^2$. The load $P = 20\,\text{kN}$ and acts at an angle $\theta = 30\,°$. The dimensions in meters are shown in the figure.





For numerical calculations the $N - \text{mm}$ units are convenient. The displacements will be in mm and the stresses in MPa. The complete computations are as follows.

Specified nodal loads

Node	dof	Value
2	u_2	10000.
	v_2	17320.5

Global equations at start of the element assembly process

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \\ u_5 \\ v_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 10000. \\ 17320.5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 1

$E = 200000 \qquad A = 1000.$

Element node	Global node number	x	y
1	1	0	0
2	2	4000.	0

$x_1 = 0 \qquad y_1 = 0 \qquad x_2 = 4000. \qquad y_2 = 0$

$$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 4000.$$

Direction cosines: $\ell_s = \dfrac{x_2 - x_1}{L} = 1. \qquad m_s = \dfrac{y_2 - y_1}{L} = 0$

Substituting into the truss element equations we get

$$\begin{pmatrix} 50000. & 0. & -50000. & 0. \\ 0. & 0. & 0. & 0. \\ -50000. & 0. & 50000. & 0. \\ 0. & 0. & 0. & 0. \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0. \\ 0. \\ 0. \\ 0. \end{pmatrix}$$

The element contributes to {1, 2, 3, 4} global degrees of freedom.

Locations for element contributions to a global vector: $\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$

and to a global matrix: $\begin{pmatrix} [1, 1] & [1, 2] & [1, 3] & [1, 4] \\ [2, 1] & [2, 2] & [2, 3] & [2, 4] \\ [3, 1] & [3, 2] & [3, 3] & [3, 4] \\ [4, 1] & [4, 2] & [4, 3] & [4, 4] \end{pmatrix}$

Adding element equations into appropriate locations we have

$$\begin{pmatrix} 50000. & 0 & -50000. & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -50000. & 0 & 50000. & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \\ u_5 \\ v_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 10000. \\ 17320.5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 2

$E = 200000$ $\qquad$ $A = 1000.$

Element node	Global node number	x	y
1	2	4000.	0
2	5	2000.	2000.

$x_1 = 4000.$ $\qquad$ $y_1 = 0$ $\qquad$ $x_2 = 2000.$ $\qquad$ $y_2 = 2000.$

$$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 2828.43$$

Direction cosines: $\ell_s = \dfrac{x_2 - x_1}{L} = -0.707107$ $\qquad$ $m_s = \dfrac{y_2 - y_1}{L} = 0.707107$

Substituting into the truss element equations we get

$$\begin{pmatrix} 35355.3 & -35355.3 & -35355.3 & 35355.3 \\ -35355.3 & 35355.3 & 35355.3 & -35355.3 \\ -35355.3 & 35355.3 & 35355.3 & -35355.3 \\ 35355.3 & -35355.3 & -35355.3 & 35355.3 \end{pmatrix} \begin{pmatrix} u_2 \\ v_2 \\ u_5 \\ v_5 \end{pmatrix} = \begin{pmatrix} 0. \\ 0. \\ 0. \\ 0. \end{pmatrix}$$

The element contributes to {3, 4, 9, 10} global degrees of freedom.

Locations for element contributions to a global vector: $\begin{pmatrix} 3 \\ 4 \\ 9 \\ 10 \end{pmatrix}$

and to a global matrix: $\begin{pmatrix} [3, 3] & [3, 4] & [3, 9] & [3, 10] \\ [4, 3] & [4, 4] & [4, 9] & [4, 10] \\ [9, 3] & [9, 4] & [9, 9] & [9, 10] \\ [10, 3] & [10, 4] & [10, 9] & [10, 10] \end{pmatrix}$

Adding element equations into appropriate locations we have

$$\begin{pmatrix} 50000. & 0 & -50000. & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -50000. & 0 & 85355.3 & -35355.3 & 0 & 0 & 0 & 0 & -35355.3 & 35355.3 \\ 0 & 0 & -35355.3 & 35355.3 & 0 & 0 & 0 & 0 & 35355.3 & -35355.3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -35355.3 & 35355.3 & 0 & 0 & 0 & 0 & 35355.3 & -35355.3 \\ 0 & 0 & 35355.3 & -35355.3 & 0 & 0 & 0 & 0 & -35355.3 & 35355.3 \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \\ u_5 \\ v_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 10000. \\ 17320.5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 3

$E = 200000 \qquad A = 1000.$

Element node	Global node number	x	y
1	5	2000.	2000.
2	3	0	3000.

$x_1 = 2000. \qquad y_1 = 2000. \qquad x_2 = 0 \qquad y_2 = 3000.$

$$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 2236.07$$

Direction cosines: $\ell_s = \dfrac{x_2 - x_1}{L} = -0.894427 \qquad m_s = \dfrac{y_2 - y_1}{L} = 0.447214$

Substituting into the truss element equations we get

$$\begin{pmatrix} 71554.2 & -35777.1 & -71554.2 & 35777.1 \\ -35777.1 & 17888.5 & 35777.1 & -17888.5 \\ -71554.2 & 35777.1 & 71554.2 & -35777.1 \\ 35777.1 & -17888.5 & -35777.1 & 17888.5 \end{pmatrix} \begin{pmatrix} u_5 \\ v_5 \\ u_3 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0. \\ 0. \\ 0. \\ 0. \end{pmatrix}$$

The element contributes to {9, 10, 5, 6} global degrees of freedom.

Locations for element contributions to a global vector: $\begin{pmatrix} 9 \\ 10 \\ 5 \\ 6 \end{pmatrix}$

and to a global matrix: $\begin{pmatrix} [9, 9] & [9, 10] & [9, 5] & [9, 6] \\ [10, 9] & [10, 10] & [10, 5] & [10, 6] \\ [5, 9] & [5, 10] & [5, 5] & [5, 6] \\ [6, 9] & [6, 10] & [6, 5] & [6, 6] \end{pmatrix}$

Adding element equations into appropriate locations we have

$$\begin{pmatrix} 50000. & 0 & -50000. & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -50000. & 0 & 85355.3 & -35355.3 & 0 & 0 & 0 & 0 & -35355.3 & 35355.3 \\ 0 & 0 & -35355.3 & 35355.3 & 0 & 0 & 0 & 0 & 35355.3 & -35355.3 \\ 0 & 0 & 0 & 0 & 71554.2 & -35777.1 & 0 & 0 & -71554.2 & 35777.1 \\ 0 & 0 & 0 & 0 & -35777.1 & 17888.5 & 0 & 0 & 35777.1 & -17888.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -35355.3 & 35355.3 & -71554.2 & 35777.1 & 0 & 0 & 106910. & -71132.4 \\ 0 & 0 & 35355.3 & -35355.3 & 35777.1 & -17888.5 & 0 & 0 & -71132.4 & 53243.9 \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \\ u_5 \\ v_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 10000. \\ 17320.5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 4

$E = 200000 \qquad A = 1000.$

Element node	Global node number	x	y
1	2	4000.	0
2	4	4000.	3000.

$x_1 = 4000. \qquad y_1 = 0 \qquad x_2 = 4000. \qquad y_2 = 3000.$

$$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 3000.$$

Direction cosines: $\ell_s = \dfrac{x_2 - x_1}{L} = 0. \qquad m_s = \dfrac{y_2 - y_1}{L} = 1.$

Substituting into the truss element equations we get

$$\begin{pmatrix} 0. & 0. & 0. & 0. \\ 0. & 66666.7 & 0. & -66666.7 \\ 0. & 0. & 0. & 0. \\ 0. & -66666.7 & 0. & 66666.7 \end{pmatrix} \begin{pmatrix} u_2 \\ v_2 \\ u_4 \\ v_4 \end{pmatrix} = \begin{pmatrix} 0. \\ 0. \\ 0. \\ 0. \end{pmatrix}$$

The element contributes to {3, 4, 7, 8} global degrees of freedom.

Locations for element contributions to a global vector: $\begin{pmatrix} 3 \\ 4 \\ 7 \\ 8 \end{pmatrix}$

and to a global matrix: $\begin{pmatrix} [3,3] & [3,4] & [3,7] & [3,8] \\ [4,3] & [4,4] & [4,7] & [4,8] \\ [7,3] & [7,4] & [7,7] & [7,8] \\ [8,3] & [8,4] & [8,7] & [8,8] \end{pmatrix}$

Adding element equations into appropriate locations we have

$$\begin{pmatrix} 50000. & 0 & -50000. & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -50000. & 0 & 85355.3 & -35355.3 & 0 & 0 & 0 & 0 & -35355.3 & 35355.3 \\ 0 & 0 & -35355.3 & 102022. & 0 & 0 & 0 & -66666.7 & 35355.3 & -35355.3 \\ 0 & 0 & 0 & 0 & 71554.2 & -35777.1 & 0 & 0 & -71554.2 & 35777.1 \\ 0 & 0 & 0 & 0 & -35777.1 & 17888.5 & 0 & 0 & 35777.1 & -17888.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -66666.7 & 0 & 0 & 0 & 66666.7 & 0 & 0 \\ 0 & 0 & -35355.3 & 35355.3 & -71554.2 & 35777.1 & 0 & 0 & 106910. & -71132.4 \\ 0 & 0 & 35355.3 & -35355.3 & 35777.1 & -17888.5 & 0 & 0 & -71132.4 & 53243.9 \end{pmatrix}$$

$$\begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \\ u_5 \\ v_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 10000. \\ 17320.5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 5

$E = 200000$ $A = 1000.$

Element node	Global node number	x	y
1	1	0	0
2	5	2000.	2000.

$x_1 = 0$ $y_1 = 0$ $x_2 = 2000.$ $y_2 = 2000.$

$$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 2828.43$$

Direction cosines: $\ell_s = \dfrac{x_2 - x_1}{L} = 0.707107$ $m_s = \dfrac{y_2 - y_1}{L} = 0.707107$

Substituting into the truss element equations we get

$$\begin{pmatrix} 35355.3 & 35355.3 & -35355.3 & -35355.3 \\ 35355.3 & 35355.3 & -35355.3 & -35355.3 \\ -35355.3 & -35355.3 & 35355.3 & 35355.3 \\ -35355.3 & -35355.3 & 35355.3 & 35355.3 \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ u_5 \\ v_5 \end{pmatrix} = \begin{pmatrix} 0. \\ 0. \\ 0. \\ 0. \end{pmatrix}$$

The element contributes to {1, 2, 9, 10} global degrees of freedom.

Locations for element contributions to a global vector: $\begin{pmatrix} 1 \\ 2 \\ 9 \\ 10 \end{pmatrix}$

and to a global matrix: $\begin{pmatrix} [1, 1] & [1, 2] & [1, 9] & [1, 10] \\ [2, 1] & [2, 2] & [2, 9] & [2, 10] \\ [9, 1] & [9, 2] & [9, 9] & [9, 10] \\ [10, 1] & [10, 2] & [10, 9] & [10, 10] \end{pmatrix}$

Adding element equations into appropriate locations we have

$$\begin{pmatrix} 85355.3 & 35355.3 & -50000. & 0 & 0 & 0 & 0 & 0 & -35355.3 & -35355.3 \\ 35355.3 & 35355.3 & 0 & 0 & 0 & 0 & 0 & 0 & -35355.3 & -35355.3 \\ -50000. & 0 & 85355.3 & -35355.3 & 0 & 0 & 0 & 0 & -35355.3 & 35355.3 \\ 0 & 0 & -35355.3 & 102022. & 0 & 0 & 0 & -66666.7 & 35355.3 & -35355.3 \\ 0 & 0 & 0 & 0 & 71554.2 & -35777.1 & 0 & 0 & -71554.2 & 35777.1 \\ 0 & 0 & 0 & 0 & -35777.1 & 17888.5 & 0 & 0 & 35777.1 & -17888.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -66666.7 & 0 & 0 & 0 & 66666.7 & 0 & 0 \\ -35355.3 & -35355.3 & -35355.3 & 35355.3 & -71554.2 & 35777.1 & 0 & 0 & 142265. & -35777.1 \\ -35355.3 & -35355.3 & 35355.3 & -35355.3 & 35777.1 & -17888.5 & 0 & 0 & -35777.1 & 88599.2 \end{pmatrix}$$

$$\begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \\ u_5 \\ v_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 10000. \\ 17320.5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 6

$E = 200000 \qquad A = 1000.$

Element node	Global node number	x	y
1	5	2000.	2000.
2	4	4000.	3000.

$x_1 = 2000. \qquad y_1 = 2000. \qquad x_2 = 4000. \qquad y_2 = 3000.$

$$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 2236.07$$

Direction cosines: $\ell_s = \frac{x_2 - x_1}{L} = 0.894427 \qquad m_s = \frac{y_2 - y_1}{L} = 0.447214$

Substituting into the truss element equations we get

$$\begin{pmatrix} 71554.2 & 35777.1 & -71554.2 & -35777.1 \\ 35777.1 & 17888.5 & -35777.1 & -17888.5 \\ -71554.2 & -35777.1 & 71554.2 & 35777.1 \\ -35777.1 & -17888.5 & 35777.1 & 17888.5 \end{pmatrix} \begin{pmatrix} u_5 \\ v_5 \\ u_4 \\ v_4 \end{pmatrix} = \begin{pmatrix} 0. \\ 0. \\ 0. \\ 0. \end{pmatrix}$$

The element contributes to {9, 10, 7, 8} global degrees of freedom.

Locations for element contributions to a global vector: $\begin{pmatrix} 9 \\ 10 \\ 7 \\ 8 \end{pmatrix}$

and to a global matrix: $\begin{pmatrix} [9, 9] & [9, 10] & [9, 7] & [9, 8] \\ [10, 9] & [10, 10] & [10, 7] & [10, 8] \\ [7, 9] & [7, 10] & [7, 7] & [7, 8] \\ [8, 9] & [8, 10] & [8, 7] & [8, 8] \end{pmatrix}$

Adding element equations into appropriate locations we have

$$\left(\begin{array}{cccccccccc} 85355.3 & 35355.3 & -50000. & 0 & 0 & 0 & 0 & 0 & -35355.3 & -35355 \\ 35355.3 & 35355.3 & 0 & 0 & 0 & 0 & 0 & 0 & -35355.3 & -35355 \\ -50000. & 0 & 85355.3 & -35355.3 & 0 & 0 & 0 & 0 & -35355.3 & 35355 \\ 0 & 0 & -35355.3 & 102022. & 0 & 0 & 0 & -66666.7 & 35355.3 & -35355 \\ 0 & 0 & 0 & 0 & 71554.2 & -35777.1 & 0 & 0 & -71554.2 & 35777 \\ 0 & 0 & 0 & 0 & -35777.1 & 17888.5 & 0 & 0 & 35777.1 & -17888 \\ 0 & 0 & 0 & 0 & 0 & 0 & 71554.2 & 35777.1 & -71554.2 & -35777 \\ 0 & 0 & 0 & -66666.7 & 0 & 0 & 35777.1 & 84555.2 & -35777.1 & -17888 \\ -35355.3 & -35355.3 & -35355.3 & 35355.3 & -71554.2 & 35777.1 & -71554.2 & -35777.1 & 213819. & 0 \\ -35355.3 & -35355.3 & 35355.3 & -35355.3 & 35777.1 & -17888.5 & -35777.1 & -17888.5 & 0 & 106488 \end{array}\right.$$

Essential boundary conditions

Node	dof	Value
1	u_1	0
	v_1	0
3	u_3	0
	v_3	0
4	u_4	0
	v_4	0

Remove {1, 2, 5, 6, 7, 8} rows and columns.

After adjusting for essential boundary conditions we have

$$\begin{pmatrix} 85355.3 & -35355.3 & -35355.3 & 35355.3 \\ -35355.3 & 102022. & 35355.3 & -35355.3 \\ -35355.3 & 35355.3 & 213819. & 0 \\ 35355.3 & -35355.3 & 0 & 106488. \end{pmatrix} \begin{pmatrix} u_2 \\ v_2 \\ u_5 \\ v_5 \end{pmatrix} = \begin{pmatrix} 10000. \\ 17320.5 \\ 0 \\ 0 \end{pmatrix}$$

Solving the final system of global equations we get

$$\{u_2 = 0.213105,\ v_2 = 0.249979,\ u_5 = -0.00609705,\ v_5 = 0.0122424\}$$

Complete table of nodal values

	u	v
1	0	0
2	0.213105	0.249979
3	0	0
4	0	0
5	−0.00609705	0.0122424

Computation of reactions

Equation numbers of dof with specified values: {1, 2, 5, 6, 7, 8}

Extracting equations {1, 2, 5, 6, 7, 8} from the global system we have

$$\begin{pmatrix} 85355.3 & 35355.3 & -50000. & 0 & 0 & 0 & 0 & 0 & -35355.3 & -35355.3 \\ 35355.3 & 35355.3 & 0 & 0 & 0 & 0 & 0 & 0 & -35355.3 & -35355.3 \\ 0 & 0 & 0 & 0 & 71554.2 & -35777.1 & 0 & 0 & -71554.2 & 35777.1 \\ 0 & 0 & 0 & 0 & -35777.1 & 17888.5 & 0 & 0 & 35777.1 & -17888.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 71554.2 & 35777.1 & -71554.2 & -35777.1 \\ 0 & 0 & 0 & -66666.7 & 0 & 0 & 35777.1 & 84555.2 & -35777.1 & -17888.5 \end{pmatrix}$$

$$\begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \\ u_5 \\ v_5 \end{pmatrix} = \begin{pmatrix} R_1 + 0. \\ R_2 + 0. \\ R_3 + 0. \\ R_4 + 0. \\ R_5 + 0. \\ R_6 + 0. \end{pmatrix}$$

Substituting the nodal values and re−arranging

$$\begin{pmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_6 \end{pmatrix} =$$

$$\begin{pmatrix} 85355.3 & 35355.3 & -50000. & 0 & 0 & 0 & 0 & 0 & -35355.3 & -35355.3 \\ 35355.3 & 35355.3 & 0 & 0 & 0 & 0 & 0 & 0 & -35355.3 & -35355.3 \\ 0 & 0 & 0 & 0 & 71554.2 & -35777.1 & 0 & 0 & -71554.2 & 35777.1 \\ 0 & 0 & 0 & 0 & -35777.1 & 17888.5 & 0 & 0 & 35777.1 & -17888.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 71554.2 & 35777.1 & -71554.2 & -35777.1 \\ 0 & 0 & 0 & -66666.7 & 0 & 0 & 35777.1 & 84555.2 & -35777.1 & -17888.5 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ 0.213105 \\ 0.249979 \\ 0 \\ 0 \\ 0 \\ 0 \\ -0.00609705 \\ 0.0122424 \end{pmatrix}$$

Carrying out computations, the reactions are as follows.

Label	dof	Reaction
R_1	u_1	−10872.5
R_2	v_1	−217.271
R_3	u_3	874.267
R_4	v_3	−437.133
R_5	u_4	−1.72786
R_6	v_4	−16666.1

Sum of Reactions

dof: u	−10000.
dof: v	−17320.5

Solution for element 1

Nodal coordinates

Element node	Global node number	x	y
1	1	0	0
2	2	4000.	0

$x_1 = 0 \qquad y_1 = 0 \qquad x_2 = 4000. \qquad y_2 = 0$

$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 4000.$

Direction cosines: $\ell_s = \dfrac{x_2 - x_1}{L} = 1. \qquad m_s = \dfrac{y_2 - y_1}{L} = 0$

Global to local transformation matrix, $\boldsymbol{T} = \begin{pmatrix} 1. & 0 & 0 & 0 \\ 0 & 0 & 1. & 0 \end{pmatrix}$

Element nodal displacements in global coordinates, $\boldsymbol{d} = \begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0.213105 \\ 0.249979 \end{pmatrix}$

Element nodal displacements in local coordinates, $\boldsymbol{d}_\ell = \boldsymbol{T}\ \boldsymbol{d} = \begin{pmatrix} 0. \\ 0.213105 \end{pmatrix}$

Axial displacements at element ends, $d_1 = 0. \qquad d_2 = 0.213105$

$E = 200000 \qquad A = 1000.$

Axial strain, $\epsilon = (d_2 - d_1)/L = 0.0000532763$

Axial stress, $\sigma = E\epsilon = 10.6553 \qquad$ Axial force $= \sigma A = 10655.3$

Solution for element 2

Nodal coordinates

Element node	Global node number	x	y
1	2	4000.	0
2	5	2000.	2000.

$x_1 = 4000. \qquad y_1 = 0 \qquad x_2 = 2000. \qquad y_2 = 2000.$

$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 2828.43$

Direction cosines: $\ell_s = \dfrac{x_2 - x_1}{L} = -0.707107 \qquad m_s = \dfrac{y_2 - y_1}{L} = 0.707107$

Global to local transformation matrix, $\boldsymbol{T} = \begin{pmatrix} -0.707107 & 0.707107 & 0 & 0 \\ 0 & 0 & -0.707107 & 0.707107 \end{pmatrix}$

Element nodal displacements in global coordinates, $\boldsymbol{d} = \begin{pmatrix} u_2 \\ v_2 \\ u_5 \\ v_5 \end{pmatrix} = \begin{pmatrix} 0.213105 \\ 0.249979 \\ -0.00609705 \\ 0.0122424 \end{pmatrix}$

Element nodal displacements in local coordinates, $\boldsymbol{d}_\ell = \boldsymbol{T}\ \boldsymbol{d} = \begin{pmatrix} 0.0260733 \\ 0.0129679 \end{pmatrix}$

Axial displacements at element ends, $d_1 = 0.0260733$ $\quad$ $d_2 = 0.0129679$

$E = 200000$ $\quad$ $A = 1000.$

Axial strain, $\epsilon = (d_2 - d_1)/L = -4.63345 \times 10^{-6}$

Axial stress, $\sigma = E\epsilon = -0.926689$ $\quad$ Axial force $= \sigma A = -926.689$

Solution for element 3

Nodal coordinates

Element node	Global node number	x	y
1	5	2000.	2000.
2	3	0	3000.

$x_1 = 2000.$ $\quad$ $y_1 = 2000.$ $\quad$ $x_2 = 0$ $\quad$ $y_2 = 3000.$

$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 2236.07$

Direction cosines: $\ell_s = \dfrac{x_2 - x_1}{L} = -0.894427$ $\quad$ $m_s = \dfrac{y_2 - y_1}{L} = 0.447214$

Global to local transformation matrix, $\boldsymbol{T} = \begin{pmatrix} -0.894427 & 0.447214 & 0 & 0 \\ 0 & 0 & -0.894427 & 0.447214 \end{pmatrix}$

Element nodal displacements in global coordinates, $\boldsymbol{d} = \begin{pmatrix} u_5 \\ v_5 \\ u_3 \\ v_3 \end{pmatrix} = \begin{pmatrix} -0.00609705 \\ 0.0122424 \\ 0 \\ 0 \end{pmatrix}$

Element nodal displacements in local coordinates, $\boldsymbol{d}_\ell = \boldsymbol{T}\ \boldsymbol{d} = \begin{pmatrix} 0.0109283 \\ 0. \end{pmatrix}$

Axial displacements at element ends, $d_1 = 0.0109283$ $\quad$ $d_2 = 0.$

$E = 200000$ $\quad$ $A = 1000.$

Axial strain, $\epsilon = (d_2 - d_1)/L = -4.8873 \times 10^{-6}$

Axial stress, $\sigma = E\epsilon = -0.97746$ Axial force $= \sigma A = -977.46$

Solution for element 4

Nodal coordinates

Element node	Global node number	x	y
1	2	4000.	0
2	4	4000.	3000.

$x_1 = 4000.$ $y_1 = 0$ $x_2 = 4000.$ $y_2 = 3000.$

$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 3000.$

Direction cosines: $\ell_s = \dfrac{x_2 - x_1}{L} = 0.$ $m_s = \dfrac{y_2 - y_1}{L} = 1.$

Global to local transformation matrix, $\boldsymbol{T} = \begin{pmatrix} 0. & 1. & 0 & 0 \\ 0 & 0 & 0. & 1. \end{pmatrix}$

Element nodal displacements in global coordinates, $\boldsymbol{d} = \begin{pmatrix} u_2 \\ v_2 \\ u_4 \\ v_4 \end{pmatrix} = \begin{pmatrix} 0.213105 \\ 0.249979 \\ 0 \\ 0 \end{pmatrix}$

Element nodal displacements in local coordinates, $\boldsymbol{d}_\ell = \boldsymbol{T}\ \boldsymbol{d} = \begin{pmatrix} 0.249979 \\ 0. \end{pmatrix}$

Axial displacements at element ends, $d_1 = 0.249979$ $d_2 = 0.$

$E = 200000$ $A = 1000.$

Axial strain, $\epsilon = (d_2 - d_1)/L = -0.0000833262$

Axial stress, $\sigma = E\epsilon = -16.6652$ Axial force $= \sigma A = -16665.2$

Solution for element 5

Nodal coordinates

Element node	Global node number	x	y
1	1	0	0
2	5	2000.	2000.

$x_1 = 0$ $y_1 = 0$ $x_2 = 2000.$ $y_2 = 2000.$

$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 2828.43$

Direction cosines: $\ell_s = \dfrac{x_2 - x_1}{L} = 0.707107$ $m_s = \dfrac{y_2 - y_1}{L} = 0.707107$

Global to local transformation matrix, $\boldsymbol{T} = \begin{pmatrix} 0.707107 & 0.707107 & 0 & 0 \\ 0 & 0 & 0.707107 & 0.707107 \end{pmatrix}$

Element nodal displacements in global coordinates, $\boldsymbol{d} = \begin{pmatrix} u_1 \\ v_1 \\ u_5 \\ v_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -0.00609705 \\ 0.0122424 \end{pmatrix}$

Element nodal displacements in local coordinates, $\boldsymbol{d}_\ell = \boldsymbol{T}\ \boldsymbol{d} = \begin{pmatrix} 0. \\ 0.00434542 \end{pmatrix}$

Axial displacements at element ends, $d_1 = 0.$ $\qquad d_2 = 0.00434542$

$E = 200000 \qquad A = 1000.$

Axial strain, $\epsilon = (d_2 - d_1)/L = 1.53634 \times 10^{-6}$

Axial stress, $\sigma = E\epsilon = 0.307267$ $\qquad$ Axial force $= \sigma A = 307.267$

Solution for element 6

Nodal coordinates

Element node	Global node number	x	y
1	5	2000.	2000.
2	4	4000.	3000.

$x_1 = 2000. \qquad y_1 = 2000. \qquad x_2 = 4000. \qquad y_2 = 3000.$

$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 2236.07$

Direction cosines: $\ell_s = \dfrac{x_2 - x_1}{L} = 0.894427 \qquad m_s = \dfrac{y_2 - y_1}{L} = 0.447214$

Global to local transformation matrix, $\boldsymbol{T} = \begin{pmatrix} 0.894427 & 0.447214 & 0 & 0 \\ 0 & 0 & 0.894427 & 0.447214 \end{pmatrix}$

Element nodal displacements in global coordinates, $\boldsymbol{d} = \begin{pmatrix} u_5 \\ v_5 \\ u_4 \\ v_4 \end{pmatrix} = \begin{pmatrix} -0.00609705 \\ 0.0122424 \\ 0 \\ 0 \end{pmatrix}$

Element nodal displacements in local coordinates, $\boldsymbol{d}_\ell = \boldsymbol{T}\ \boldsymbol{d} = \begin{pmatrix} 0.0000215983 \\ 0. \end{pmatrix}$

Axial displacements at element ends, $d_1 = 0.0000215983$ $\qquad d_2 = 0.$

$E = 200000 \qquad A = 1000.$

Axial strain, $\epsilon = (d_2 - d_1)/L = -9.65904 \times 10^{-9}$

Axial stress, $\sigma = E\epsilon = -0.00193181$ Axial force $= \sigma A = -1.93181$

Solution summary

Nodal solution

	x–coord	y–coord	u	v
1	0	0	0	0
2	4000.	0	0.213105	0.249979
3	0	3000.	0	0
4	4000.	3000.	0	0
5	2000.	2000.	−0.00609705	0.0122424

Element solution

	Stress	Axial force
1	10.6553	10655.3
2	−0.926689	−926.689
3	−0.97746	−977.46
4	−16.6652	−16665.2
5	0.307267	307.267
6	−0.00193181	−1.93181

Support reactions

Node	dof	Reaction
1	u_1	−10872.5
1	v_1	−217.271
3	u_3	874.267
3	v_3	−437.133
4	u_4	−1.72786
4	v_4	−16666.1

Sum of applied loads → (10000. 17320.5)

Sum of support reactions → (−10000. −17320.5)