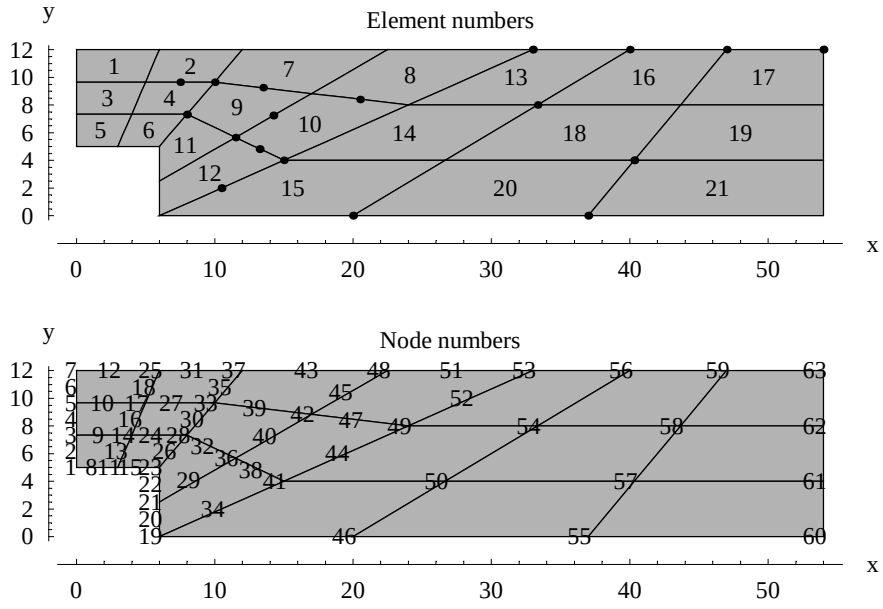
Example 7.8: Notched beam using transition from 8 to 4 node elements (p. 516)

Many practical problems can be analyzed efficiently by using higher order elements in the region of high stress gradients and low order elements elsewhere. Appropriate order elements must be used in the transition region between the high to low-order elements. To demonstrate this, consider analysis of the notched beam of Figure. To capture stress concentration in the vicinity of the notch. we employ 8 node elements. Away from the notch the stresses do not change that rapidly and thus we could use 4 node elements. To maintain the compatibility of the displacement field over the entire mesh it is necessary to use 5 node elements in the transition region from 4 to 8 noded elements. Taking advantage of symmetry the right half of the beam is modelled as shown in Figure. The first 12 elements are the 8 noded elements. The elements 16 through 21 are 4 node elements. The elements 13, 14 and 15 in the transition region must have quadratic displacement field along their left edges and linear along the right sides. Thus 5 node elements are used. Due to symmetry nodes 1 through 7 are restrained in the *x* direction. Both horizontal and vertical displacements are zero at nodes 60 through 63 because of the fixed support.



Equations for element 1

E = 3000000; ν = 0.2; h = 4

Nodal coordinates

Element node	Global node number	x	y
1	7	0.	12.
2	6	0.	10.8333
3	5	0.	9.66667
4	10	2.5	9.66667
5	17	5.	9.66667
6	18	5.5	10.8333
7	25	6.	12.
8	12	3.	12.

Complete element equations for element 1

$$\begin{pmatrix}
1.039\times10^{7} & -3.7758\times10^{6} & -1.0664\times10^{7} & 940803. & 4.69977\times \\
-3.7758\times10^{6} & 1.98446\times10^{7} & 2.60747\times10^{6} & -2.73384\times10^{7} & -337260. \\
-1.0664\times10^{7} & 2.60747\times10^{6} & 2.38523\times10^{7} & -363938. & -9.64706\times \\
940803. & -2.73384\times10^{7} & -363938. & 5.36865\times10^{7} & -728632. \\
4.69977\times10^{6} & -337260. & -9.64706\times10^{6} & -728632. & 9.4665\times1 \\
79407. & 9.64362\times10^{6} & -2.3953\times10^{6} & -2.49316\times10^{7} & 3.28954\times \\
-3.57384\times10^{6} & 1.13964\times10^{6} & 88305.9 & 3.02426\times10^{6} & -4.32916\times \\
1.13964\times10^{6} & -3.9769\times10^{6} & 3.02426\times10^{6} & 220765. & -2.02758\times \\
4.82646\times10^{6} & -1.63571\times10^{6} & -6.15553\times10^{6} & 1.0377\times10^{6} & 4.26742\times \\
-1.63571\times10^{6} & 9.21423\times10^{6} & 1.0377\times10^{6} & -1.45749\times10^{7} & -11955.1 \\
-5.52643\times10^{6} & 725863. & 7.57624\times10^{6} & 363938. & -5.59103\times \\
725863. & -1.31378\times10^{7} & 363938. & 2.48849\times10^{7} & -938035. \\
3.59742\times10^{6} & 3271.92 & -5.13538\times10^{6} & -642807. & 3.99154\times \\
-413395. & 5.64421\times10^{6} & -642807. & -1.21602\times10^{7} & 1.28649\times \\
-3.74931\times10^{6} & 1.27253\times10^{6} & 85141.9 & -3.63133\times10^{6} & -2.85798\times \\
2.93919\times10^{6} & 106407. & -3.63133\times10^{6} & 212855. & -532568.
\end{pmatrix}$$

Equations for element 2

E = 3000000; ν = 0.2; h = 4

Nodal coordinates

Element node	Global node number	x	y
1	25	6.	12.
2	18	5.5	10.8333
3	17	5.	9.66667
4	27	7.5	9.66667
5	33	10.	9.66667
6	35	11.	10.8333
7	37	12.	12.
8	31	9.	12.

Complete element equations for element 2

$$\begin{pmatrix}
1.27621\times10^{7} & -4.52645\times10^{6} & -1.16541\times10^{7} & 857747. & 4.84711\times1\\
-4.52645\times10^{6} & 2.57748\times10^{7} & 2.52441\times10^{6} & -2.98136\times10^{7} & -595112.\\
-1.16541\times10^{7} & 2.52441\times10^{6} & 2.42683\times10^{7} & -1.09182\times10^{6} & -8.72604\times1\\
857747. & -2.98136\times10^{7} & -1.09182\times10^{6} & 5.47263\times10^{7} & -828300.\\
4.84711\times10^{6} & -595112. & -8.72604\times10^{6} & -828300. & 7.82417\times1\\
-178446. & 1.0012\times10^{7} & -2.49497\times10^{6} & -2.2629\times10^{7} & 2.4586\times10\\
-4.39851\times10^{6} & 1.7467\times10^{6} & -1.63984\times10^{6} & 3.02426\times10^{6} & -4.01002\times1\\
1.7467\times10^{6} & -6.03857\times10^{6} & 3.02426\times10^{6} & -4.09961\times10^{6} & -756066.\\
5.86093\times10^{6} & -1.98492\times10^{6} & -6.77698\times10^{6} & 1.13737\times10^{6} & 4.51918\times1\\
-1.98492\times10^{6} & 1.18004\times10^{7} & 1.13737\times10^{6} & -1.61285\times10^{7} & -452532.\\
-5.96494\times10^{6} & 808920. & 7.16031\times10^{6} & 1.09182\times10^{6} & -5.08349\times1\\
808920. & -1.42341\times10^{7} & 1.09182\times10^{6} & 2.38451\times10^{7} & -838367.\\
3.83177\times10^{6} & -406851. & -4.7918\times10^{6} & -559750. & 3.35618\times1\\
-823518. & 6.2301\times10^{6} & -559750. & -1.13012\times10^{7} & 937280.\\
-5.28431\times10^{6} & 2.4333\times10^{6} & 2.16019\times10^{6} & -3.63133\times10^{6} & -2.7271\times10\\
4.09997\times10^{6} & -3.73109\times10^{6} & -3.63133\times10^{6} & 5.40047\times10^{6} & 74499.1
\end{pmatrix}$$

Equations for element 3

E = 3000000; ν = 0.2; h = 4

Nodal coordinates

Element node	Global node number	x	y
1	5	0.	9.66667
2	4	0.	8.5
3	3	0.	7.33333
4	9	2.	7.33333
5	14	4.	7.33333
6	16	4.5	8.5
7	17	5.	9.66667
8	10	2.5	9.66667

Complete element equations for element 3

$$\begin{pmatrix}
9.77365\times10^{6} & -3.82578\times10^{6} & -8.70305\times10^{6} & 965082. & 4.26335\times \\
-3.82578\times10^{6} & 1.70172\times10^{7} & 2.63175\times10^{6} & -2.25702\times10^{7} & -365994. \\
-8.70305\times10^{6} & 2.63175\times10^{6} & 2.06887\times10^{7} & -444997. & -7.65393\times \\
965082. & -2.25702\times10^{7} & -444997. & 4.44535\times10^{7} & -705731. \\
4.26335\times10^{6} & -365994. & -7.65393\times10^{6} & -705731. & 9.00667\times \\
50672.6 & 8.08326\times10^{6} & -2.3724\times10^{6} & -2.01506\times10^{7} & 3.23061\times \\
-3.97388\times10^{6} & 1.20877\times10^{6} & 108452. & 2.95375\times10^{6} & -5.6626\times1 \\
1.20877\times10^{6} & -3.87026\times10^{6} & 2.95375\times10^{6} & 271131. & -1.91727\times \\
4.59147\times10^{6} & -1.67611\times10^{6} & -5.31167\times10^{6} & 1.08531\times10^{6} & 4.27498\times \\
-1.67611\times10^{6} & 7.98938\times10^{6} & 1.08531\times10^{6} & -1.22634\times10^{7} & -63340.7 \\
-4.63029\times10^{6} & 701585. & 5.02558\times10^{6} & 444997. & -4.72703\times \\
701585. & -1.07631\times10^{7} & 444997. & 1.98322\times10^{7} & -960936. \\
3.54408\times10^{6} & -40538.1 & -4.25781\times10^{6} & -602082. & 3.75577\times \\
-457205. & 4.7953\times10^{6} & -602082. & -9.83193\times10^{6} & 1.24885\times \\
-4.86534\times10^{6} & 1.36632\times10^{6} & 103714. & -3.69633\times10^{6} & -3.25722\times \\
3.03298\times10^{6} & -681572. & -3.69633\times10^{6} & 259286. & -466188.
\end{pmatrix}$$

Equations for element 4

E = 3000000; ν = 0.2; h = 4

Nodal coordinates

Element node	Global node number	x	y
1	17	5.	9.66667
2	16	4.5	8.5
3	14	4.	7.33333
4	24	6.	7.33333
5	28	8.	7.33333
6	30	9.	8.5
7	33	10.	9.66667
8	27	7.5	9.66667

Complete element equations for element 4

$$\begin{pmatrix}
1.22193\times10^{7} & -4.73397\times10^{6} & -9.70228\times10^{6} & 865580. & 4.44354\times \\
-4.73397\times10^{6} & 2.31313\times10^{7} & 2.53225\times10^{6} & -2.50683\times10^{7} & -681316. \\
-9.70228\times10^{6} & 2.53225\times10^{6} & 2.11973\times10^{7} & -1.33499\times10^{6} & -6.73892\times \\
865580. & -2.50683\times10^{7} & -1.33499\times10^{6} & 4.57249\times10^{7} & -830109. \\
4.44354\times10^{6} & -681316. & -6.73892\times10^{6} & -830109. & 7.45445\times \\
-264649. & 8.53372\times10^{6} & -2.49678\times10^{6} & -1.78631\times10^{7} & 2.20218\times \\
-4.87677\times10^{6} & 1.95135\times10^{6} & -1.5794\times10^{6} & 2.95375\times10^{6} & -5.49229\times \\
1.95135\times10^{6} & -6.12749\times10^{6} & 2.95375\times10^{6} & -3.94851\times10^{6} & -345495. \\
5.67133\times10^{6} & -2.10337\times10^{6} & -5.96739\times10^{6} & 1.20969\times10^{6} & 4.58546\times \\
-2.10337\times10^{6} & 1.0689\times10^{7} & 1.20969\times10^{6} & -1.39027\times10^{7} & -606689. \\
-5.05962\times10^{6} & 801087. & 4.51701\times10^{6} & 1.33499\times10^{6} & -4.21346\times \\
801087. & -1.18365\times10^{7} & 1.33499\times10^{6} & 1.85608\times10^{7} & -836558. \\
3.8285\times10^{6} & -538281. & -3.94219\times10^{6} & -502580. & 3.16422\times \\
-954947. & 5.50636\times10^{6} & -502580. & -9.04288\times10^{6} & 821587. \\
-6.52398\times10^{6} & 2.77225\times10^{6} & 2.2159\times10^{6} & -3.69633\times10^{6} & -3.20299\times \\
4.43892\times10^{6} & -4.82817\times10^{6} & -3.69633\times10^{6} & 5.53976\times10^{6} & 276396.
\end{pmatrix}$$

Equations for element 5

E = 3000000; ν = 0.2; h = 4

Nodal coordinates

Element node	Global node number	x	y
1	3	0.	7.33333
2	2	0.	6.16667
3	1	0.	5.
4	8	1.5	5.
5	11	3.	5.
6	13	3.5	6.16667
7	14	4.	7.33333
8	9	2.	7.33333

Complete element equations for element 5

$$\begin{pmatrix}
9.48869\times10^{6} & -3.90299\times10^{6} & -6.7153\times10^{6} & 1.00354\times10^{6} & 3.95938\times \\
-3.90299\times10^{6} & 1.4333\times10^{7} & 2.67021\times10^{6} & -1.78006\times10^{7} & -411255. \\
-6.7153\times10^{6} & 2.67021\times10^{6} & 1.7896\times10^{7} & -572609. & -5.60222\times \\
1.00354\times10^{6} & -1.78006\times10^{7} & -572609. & 3.53875\times10^{7} & -670012. \\
3.95938\times10^{6} & -411255. & -5.60222\times10^{6} & -670012. & 9.03321\times \\
5411.26 & 6.58405\times10^{6} & -2.33668\times10^{6} & -1.53554\times10^{7} & 3.13558\times \\
-4.68741\times10^{6} & 1.31838\times10^{6} & 140552. & 2.8414\times10^{6} & -7.7342\times1 \\
1.31838\times10^{6} & -3.90861\times10^{6} & 2.8414\times10^{6} & 351380. & -1.73947\times \\
4.53756\times10^{6} & -1.74046\times10^{6} & -4.53833\times10^{6} & 1.16194\times10^{6} & 4.53432\times \\
-1.74046\times10^{6} & 6.84838\times10^{6} & 1.16194\times10^{6} & -9.99597\times10^{6} & -146104. \\
-3.76089\times10^{6} & 663125. & 2.10397\times10^{6} & 572609. & -3.92159\times \\
663125. & -8.38984\times10^{6} & 572609. & 1.46125\times10^{7} & -996655. \\
3.68225\times10^{6} & -108225. & -3.41738\times10^{6} & -539158. & 3.70029\times \\
-524892. & 4.03518\times10^{6} & -539158. & -7.53106\times10^{6} & 1.18998\times \\
-6.50428\times10^{6} & 1.51122\times10^{6} & 132681. & -3.79772\times10^{6} & -3.96919\times \\
3.17788\times10^{6} & -1.70148\times10^{6} & -3.79772\times10^{6} & 331703. & -362062.
\end{pmatrix}$$

Equations for element 6

E = 3000000; ν = 0.2; h = 4

Nodal coordinates

Element node	Global node number	x	y
1	14	4.	7.33333
2	13	3.5	6.16667
3	11	3.	5.
4	15	4.5	5.
5	23	6.	5.
6	26	7.	6.16667
7	28	8.	7.33333
8	24	6.	7.33333

Complete element equations for element 6

$$\begin{pmatrix}
1.20474\times10^{7} & -5.05264\times10^{6} & -7.72952\times10^{6} & 879575. & 4.19129\times1\\
-5.05264\times10^{6} & 2.07298\times10^{7} & 2.54624\times10^{6} & -2.03362\times10^{7} & -817100.\\
-7.72952\times10^{6} & 2.54624\times10^{6} & 1.85504\times10^{7} & -1.71783\times10^{6} & -4.69594\times1\\
879575. & -2.03362\times10^{7} & -1.71783\times10^{6} & 3.70235\times10^{7} & -835301.\\
4.19129\times10^{6} & -817100. & -4.69594\times10^{6} & -835301. & 7.62699\times1\\
-400433. & 7.16382\times10^{6} & -2.50197\times10^{6} & -1.30897\times10^{7} & 1.78621\times1\\
-5.714\times10^{6} & 2.2747\times10^{6} & -1.48311\times10^{6} & 2.8414\times10^{6} & -7.80448\times1\\
2.2747\times10^{6} & -6.47509\times10^{6} & 2.8414\times10^{6} & -3.70776\times10^{6} & 318772.\\
5.68938\times10^{6} & -2.29093\times10^{6} & -5.24953\times10^{6} & 1.32723\times10^{6} & 4.9394\times10\\
-2.29093\times10^{6} & 9.72794\times10^{6} & 1.32723\times10^{6} & -1.1774\times10^{7} & -854978.\\
-4.17524\times10^{6} & 787092. & 1.44956\times10^{6} & 1.71783\times10^{6} & -3.3993\times10\\
787092. & -9.4257\times10^{6} & 1.71783\times10^{6} & 1.29765\times10^{7} & -831366.\\
4.04403\times10^{6} & -741342. & -3.14471\times10^{6} & -415191. & 3.17758\times1\\
-1.15801\times10^{6} & 4.93963\times10^{6} & -415191. & -6.84938\times10^{6} & 639512.\\
-8.35339\times10^{6} & 3.29398\times10^{6} & 2.30281\times10^{6} & -3.79772\times10^{6} & -4.03553\times1\\
4.96065\times10^{6} & -6.32423\times10^{6} & -3.79772\times10^{6} & 5.75701\times10^{6} & 594254.
\end{pmatrix}$$

Equations for element 7

E = 3000000; ν = 0.2; h = 4

Nodal coordinates

Element node	Global node number	x	y
1	37	12.	12.
2	35	11.	10.8333
3	33	10.	9.66667
4	39	13.5	9.25
5	42	17.	8.83333
6	45	19.75	10.4167
7	48	22.5	12.
8	43	17.25	12.

Complete element equations for element 7

$$\begin{pmatrix}
1.83404\times10^{7} & -4.35842\times10^{6} & -1.81952\times10^{7} & 134448. & 6.56745\times \\
-4.35842\times10^{6} & 4.22068\times10^{7} & 1.80111\times10^{6} & -4.56941\times10^{7} & -543792. \\
-1.81952\times10^{7} & 1.80111\times10^{6} & 3.2475\times10^{7} & 635354. & -1.21258\times \\
134448. & -4.56941\times10^{7} & 635354. & 7.71512\times10^{7} & -1.71559\times \\
6.56745\times10^{6} & -543792. & -1.21258\times10^{7} & -1.71559\times10^{6} & 8.65201\times \\
-127125. & 1.44838\times10^{7} & -3.38226\times10^{6} & -2.92744\times10^{7} & 2.30227\times \\
-6.26209\times10^{6} & 2.32135\times10^{6} & -874506. & 1.74831\times10^{6} & -3.60781\times \\
2.32135\times10^{6} & -1.22346\times10^{7} & 1.74831\times10^{6} & -4.1997\times10^{6} & 588829. \\
7.7732\times10^{6} & -1.77674\times10^{6} & -9.38062\times10^{6} & 696646. & 5.49223\times \\
-1.77674\times10^{6} & 1.78443\times10^{7} & 696646. & -2.2906\times10^{7} & -524631. \\
-7.52043\times10^{6} & 223969. & 1.00103\times10^{7} & 2.54253\times10^{6} & -5.90223\times \\
223969. & -1.85616\times10^{7} & 2.54253\times10^{6} & 2.88728\times10^{7} & -1.26881\times \\
4.30118\times10^{6} & -506274. & -5.82154\times10^{6} & -772169. & 3.3185\times1 \\
-922941. & 8.38971\times10^{6} & -772169. & -1.3595\times10^{7} & 799313. \\
-5.00445\times10^{6} & 2.83879\times10^{6} & 3.91247\times10^{6} & -3.26953\times10^{6} & -2.39436\times \\
4.50546\times10^{6} & -6.43433\times10^{6} & -3.26953\times10^{6} & 9.64515\times10^{6} & 362408.
\end{pmatrix}$$

Equations for element 8

E = 3000000; ν = 0.2; h = 4

Nodal coordinates

Element node	Global node number	x	y
1	48	22.5	12.
2	45	19.75	10.4167
3	42	17.	8.83333
4	47	20.5	8.41667
5	49	24.	8.
6	52	28.5	10.
7	53	33.	12.
8	51	27.75	12.

Complete element equations for element 8

$$\begin{pmatrix}
2.22103\times10^{7} & -5.87167\times10^{6} & -1.54514\times10^{7} & 140088. & 6.43023\times10 \\
-5.87167\times10^{6} & 5.06997\times10^{7} & 1.80675\times10^{6} & -3.90268\times10^{7} & -1.21061\times10 \\
-1.54514\times10^{7} & 1.80675\times10^{6} & 2.73231\times10^{7} & -1.23767\times10^{6} & -6.65328\times10 \\
140088. & -3.90268\times10^{7} & -1.23767\times10^{6} & 6.33113\times10^{7} & -1.35486\times10 \\
6.43023\times10^{6} & -1.21061\times10^{6} & -6.65328\times10^{6} & -1.35486\times10^{6} & 6.4807\times10^{6} \\
-793945. & 1.38134\times10^{7} & -3.02153\times10^{6} & -1.5914\times10^{7} & 27783.7 \\
-9.20805\times10^{6} & 3.46648\times10^{6} & -3.62774\times10^{6} & 1.56405\times10^{6} & -7.51542\times10 \\
3.46648\times10^{6} & -1.86811\times10^{7} & 1.56405\times10^{6} & -1.10325\times10^{7} & 3.13164\times10 \\
9.39696\times10^{6} & -2.44962\times10^{6} & -8.61461\times10^{6} & 951272. & 6.38939\times10 \\
-2.44962\times10^{6} & 2.13828\times10^{7} & 951272. & -2.08821\times10^{7} & -1.45251\times10 \\
-6.91164\times10^{6} & 494365. & 3.46578\times10^{6} & 3.698\times10^{6} & -3.68259\times10 \\
494365. & -1.69369\times10^{7} & 3.698\times10^{6} & 1.36898\times10^{7} & -843880. \\
5.55894\times10^{6} & -1.36086\times10^{6} & -4.12415\times10^{6} & -463024. & 2.6861\times10^{6} \\
-1.77752\times10^{6} & 1.09074\times10^{7} & -463024. & -9.2557\times10^{6} & -48683.2 \\
-1.20253\times10^{7} & 5.12516\times10^{6} & 7.68229\times10^{6} & -3.29785\times10^{6} & -4.13514\times10 \\
6.79183\times10^{6} & -2.21585\times10^{7} & -3.29785\times10^{6} & 1.911\times10^{7} & 1.75113\times10
\end{pmatrix}$$

Equations for element 9

E = 3000000; ν = 0.2; h = 4

Nodal coordinates

Element node	Global node number	x	y
1	33	10.	9.66667
2	30	9.	8.5
3	28	8.	7.33333
4	32	9.75	6.5
5	36	11.5	5.66667
6	40	14.25	7.25
7	42	17.	8.83333
8	39	13.5	9.25

Complete element equations for element 9

$$\begin{pmatrix}
1.29673\times10^{7} & -2.79914\times10^{6} & -1.07172\times10^{7} & -1.92152\times10^{6} & 5.09418\times1\\
-2.79914\times10^{6} & 2.93466\times10^{7} & -254849. & -2.67301\times10^{7} & -335148.\\
-1.07172\times10^{7} & -254849. & 2.08165\times10^{7} & 3.29751\times10^{6} & -7.6598\times10\\
-1.92152\times10^{6} & -2.67301\times10^{7} & 3.29751\times10^{6} & 4.27671\times10^{7} & -2.44916\times1\\
5.09418\times10^{6} & -335148. & -7.6598\times10^{6} & -2.44916\times10^{6} & 9.48832\times1\\
81518.5 & 9.16828\times10^{6} & -4.11583\times10^{6} & -1.35721\times10^{7} & 1.16752\times1\\
-6.08691\times10^{6} & 2.33529\times10^{6} & 1.64368\times10^{6} & 619133. & -7.47384\times1\\
2.33529\times10^{6} & -1.12968\times10^{7} & 619133. & -1.18712\times10^{6} & 2.18657\times1\\
5.47278\times10^{6} & -1.12164\times10^{6} & -5.83714\times10^{6} & -235426. & 4.90524\times1\\
-1.12164\times10^{6} & 1.22482\times10^{7} & -235426. & -1.3698\times10^{7} & -522089.\\
-4.13746\times10^{6} & -620348. & 2.94811\times10^{6} & 4.37522\times10^{6} & -3.74916\times1\\
-620348. & -9.68526\times10^{6} & 4.37522\times10^{6} & 1.06313\times10^{7} & -1.58679\times1\\
3.79255\times10^{6} & -587934. & -3.75163\times10^{6} & -986883. & 3.37383\times1\\
-1.0046\times10^{6} & 6.00011\times10^{6} & -986883. & -6.67113\times10^{6} & 377908.\\
-6.38524\times10^{6} & 3.38376\times10^{6} & 2.55745\times10^{6} & -2.69888\times10^{6} & -3.97876\times1\\
5.05042\times10^{6} & -9.05101\times10^{6} & -2.69888\times10^{6} & 8.46011\times10^{6} & 1.16118\times1
\end{pmatrix}$$

Equations for element 10

E = 3000000; ν = 0.2; h = 4

Nodal coordinates

Element node	Global node number	x	y
1	42	17.	8.83333
2	40	14.25	7.25
3	36	11.5	5.66667
4	38	13.25	4.83333
5	41	15.	4.
6	44	19.5	6.
7	49	24.	8.
8	47	20.5	8.41667

Complete element equations for element 10

$$\begin{pmatrix}
1.70633\times10^{7} & -4.41939\times10^{6} & -8.5983\times10^{6} & -1.59487\times10^{6} & 5.43427\times1\\
-4.41939\times10^{6} & 3.85866\times10^{7} & 71800.2 & -2.21058\times10^{7} & -1.25443\times1\\
-8.5983\times10^{6} & 71800.2 & 1.82312\times10^{7} & 74843.4 & -3.56588\times1\\
-1.59487\times10^{6} & -2.21058\times10^{7} & 74843.4 & 3.69539\times10^{7} & -1.3315\times10\\
5.43427\times10^{6} & -1.25443\times10^{6} & -3.56588\times10^{6} & -1.3315\times10^{6} & 9.1922\times10\\
-837760. & 1.02102\times10^{7} & -2.99816\times10^{6} & -5.11727\times10^{6} & -2.1988\times10\\
-9.20691\times10^{6} & 3.57048\times10^{6} & -314397. & 32975.4 & -1.2271\times10\\
3.57048\times10^{6} & -1.82838\times10^{7} & 32975.4 & -5.49612\times10^{6} & 5.23176\times1\\
7.22123\times10^{6} & -1.85378\times10^{6} & -5.44631\times10^{6} & 201544. & 6.23978\times1\\
-1.85378\times10^{6} & 1.61952\times10^{7} & 201544. & -1.28773\times10^{7} & -1.69683\times1\\
-3.88838\times10^{6} & -194688. & -2.91085\times10^{6} & 5.22171\times10^{6} & -2.31265\times1\\
-194688. & -9.23812\times10^{6} & 5.22171\times10^{6} & -2.32109\times10^{6} & -747393.\\
5.45934\times10^{6} & -1.60145\times10^{6} & -2.79314\times10^{6} & -389086. & 3.55277\times1\\
-2.01812\times10^{6} & 9.74782\times10^{6} & -389086. & -4.57489\times10^{6} & -852728.\\
-1.34845\times10^{7} & 5.68145\times10^{6} & 5.39772\times10^{6} & -2.21562\times10^{6} & -6.26946\times1\\
7.34812\times10^{6} & -2.51121\times10^{7} & -2.21562\times10^{6} & 1.55385\times10^{7} & 2.84991\times1
\end{pmatrix}$$

Equations for element 11

E = 3000000; ν = 0.2; h = 4

Nodal coordinates

Element node	Global node number	x	y
1	28	8.	7.33333
2	26	7.	6.16667
3	23	6.	5.
4	22	6.	3.75
5	21	6.	2.5
6	29	8.75	4.08333
7	36	11.5	5.66667
8	32	9.75	6.5

Complete element equations for element 11

$$\begin{pmatrix}
8.66496\times10^{6} & -1.72313\times10^{6} & -6.04992\times10^{6} & -2.73051\times10^{6} & 6.47312\times \\
-1.72313\times10^{6} & 1.7769\times10^{7} & -1.06384\times10^{6} & -1.09385\times10^{7} & -1.38552\times \\
-6.04992\times10^{6} & -1.06384\times10^{6} & 1.89395\times10^{7} & 1.65324\times10^{6} & -1.12339\times \\
-2.73051\times10^{6} & -1.09385\times10^{7} & 1.65324\times10^{6} & 1.90996\times10^{7} & 334041. \\
6.47312\times10^{6} & -1.38552\times10^{6} & -1.12339\times10^{7} & 334041. & 2.21584\times \\
-968850. & 7.01682\times10^{6} & -1.33263\times10^{6} & -6.51427\times10^{6} & -5.09142\times \\
-6.60142\times10^{6} & 2.63909\times10^{6} & 7.47218\times10^{6} & -1.95601\times10^{6} & -1.61978\times \\
2.63909\times10^{6} & -1.10734\times10^{7} & -1.95601\times10^{6} & 5.44669\times10^{6} & 5.86686\times \\
3.75242\times10^{6} & -726731. & -3.91519\times10^{6} & -442674. & 6.59708\times \\
-726731. & 7.33253\times10^{6} & -442674. & -6.34727\times10^{6} & -1.50819\times \\
-2.42662\times10^{6} & -691724. & -985300. & 4.77381\times10^{6} & -5.50399\times \\
-691724. & -2.93031\times10^{6} & 4.77381\times10^{6} & -3.77413\times10^{6} & -167314. \\
4.74361\times10^{6} & -1.35168\times10^{6} & -4.41258\times10^{6} & 75860.4 & 6.83\times10^{6} \\
-1.76835\times10^{6} & 5.60796\times10^{6} & 75860.4 & -3.41482\times10^{6} & -1.57401\times \\
-8.55615\times10^{6} & 4.30353\times10^{6} & 185258. & -1.70776\times10^{6} & -9.12289\times \\
5.9702\times10^{6} & -1.27841\times10^{7} & -1.70776\times10^{6} & 6.44263\times10^{6} & 3.52555\times
\end{pmatrix}$$

Equations for element 12

E = 3000000; ν = 0.2; h = 4

Nodal coordinates

Element node	Global node number	x	y
1	36	11.5	5.66667
2	29	8.75	4.08333
3	21	6.	2.5
4	20	6.	1.25
5	19	6.	0.
6	34	10.5	2.
7	41	15.	4.
8	38	13.25	4.83333

Complete element equations for element 12

$$\begin{pmatrix}
1.26253\times10^{7} & -3.29451\times10^{6} & -3.60052\times10^{6} & -2.46779\times10^{6} & 6.15795\times10 \\
-3.29451\times10^{6} & 2.76631\times10^{7} & -801123. & -8.24386\times10^{6} & -2.09804\times10 \\
-3.60052\times10^{6} & -801123. & 1.58998\times10^{7} & -1.76369\times10^{6} & -5.28306\times10 \\
-2.46779\times10^{6} & -8.24386\times10^{6} & -1.76369\times10^{6} & 2.1648\times10^{7} & 933051. \\
6.15795\times10^{6} & -2.09804\times10^{6} & -5.28306\times10^{6} & 933051. & 1.88197\times10 \\
-1.68137\times10^{6} & 9.43405\times10^{6} & -733615. & -2.15213\times10^{6} & -7.66094\times10 \\
-9.64983\times10^{6} & 3.87998\times10^{6} & 5.16841\times10^{6} & -2.50502\times10^{6} & -1.99625\times10 \\
3.87998\times10^{6} & -1.86171\times10^{7} & -2.50502\times10^{6} & 3.61981\times10^{6} & 8.70113\times10 \\
5.42307\times10^{6} & -1.43244\times10^{6} & -3.34289\times10^{6} & -54463.3 & 7.48931\times10 \\
-1.43244\times10^{6} & 1.16394\times10^{7} & -54463.3 & -6.64256\times10^{6} & -2.59605\times10 \\
-1.99495\times10^{6} & -364285. & -7.20807\times10^{6} & 5.78588\times10^{6} & -3.41976\times10 \\
-364285. & -3.46269\times10^{6} & 5.78588\times10^{6} & -1.4823\times10^{7} & 505664. \\
6.44681\times10^{6} & -2.33619\times10^{6} & -3.44574\times10^{6} & 589128. & 6.74557\times10 \\
-2.75285\times10^{6} & 1.04853\times10^{7} & 589128. & -3.34137\times10^{6} & -2.73082\times10 \\
-1.54078\times10^{7} & 6.4466\times10^{6} & 1.81207\times10^{6} & -517095. & -1.05472\times10 \\
8.11327\times10^{6} & -2.88982\times10^{7} & -517095. & 9.93509\times10^{6} & 4.94601\times10
\end{pmatrix}$$

Equations for element 13

E = 3000000; ν = 0.2; h = 4

Nodal coordinates

Element node	Global node number	x	y
1	53	33.	12.
2	52	28.5	10.
3	49	24.	8.
4	54	33.3333	8.
5	56	40.	12.

Complete element equations for element 13

$$\begin{pmatrix}
1.42985\times10^{7} & -2.73728\times10^{6} & -6.43346\times10^{6} & -1.7531\times10^{6} & 262557. \\
-2.73728\times10^{6} & 3.42051\times10^{7} & -86431.8 & -1.77275\times10^{7} & 194064. \\
-6.43346\times10^{6} & -86431.8 & 2.51218\times10^{7} & -3.21918\times10^{6} & -8.22788\times10^{6} \\
-1.7531\times10^{6} & -1.77275\times10^{7} & -3.21918\times10^{6} & 5.7051\times10^{7} & 143509. \\
262557. & 194064. & -8.22788\times10^{6} & 143509. & 5.06795\times10^{6} \\
610731. & 1.37557\times10^{6} & -1.52316\times10^{6} & -2.18026\times10^{7} & 1.12769\times10^{6} \\
-3.29583\times10^{6} & 682485. & -1.1833\times10^{7} & 4.79941\times10^{6} & 3.00432\times10^{6} \\
682485. & -8.13682\times10^{6} & 4.79941\times10^{6} & -2.54729\times10^{7} & -782779. \\
-4.83176\times10^{6} & 1.94716\times10^{6} & 1.37258\times10^{6} & 29354.2 & -106953. \\
3.19716\times10^{6} & -9.71638\times10^{6} & 29354.2 & 7.952\times10^{6} & -682485.
\end{pmatrix}$$

Equations for element 14

E = 3000000; ν = 0.2; h = 4

Nodal coordinates

Element node	Global node number	x	y
1	49	24.	8.
2	44	19.5	6.
3	41	15.	4.
4	50	26.6667	4.
5	54	33.3333	8.

Complete element equations for element 14

$$\begin{pmatrix} 1.61265\times10^7 & -2.53394\times10^6 & -9.51594\times10^6 & -1.52696\times10^6 & 782172. \\ -2.53394\times10^6 & 3.91386\times10^7 & 139709. & -2.50295\times10^7 & 103798. \\ -9.51594\times10^6 & 139709. & 2.87426\times10^7 & -2.49705\times10^6 & -1.11128\times10^7 \\ -1.52696\times10^6 & -2.50295\times10^7 & -2.49705\times10^6 & 6.73937\times10^7 & 278434. \\ 782172. & 103798. & -1.11128\times10^7 & 278434. & 6.99479\times10^6 \\ 520464. & 2.51328\times10^6 & -1.38823\times10^6 & -2.87736\times10^7 & 1.28542\times10^6 \\ -3.73254\times10^6 & 659681. & -1.07586\times10^7 & 4.30342\times10^6 & 3.97826\times10^6 \\ 659681. & -9.26936\times10^6 & 4.30342\times10^6 & -2.36733\times10^7 & -1.00797\times10^6 \\ -3.66022\times10^6 & 1.63076\times10^6 & 2.64463\times10^6 & -557851. & -642464. \\ 2.88076\times10^6 & -7.35304\times10^6 & -557851. & 1.00826\times10^7 & -659681. \end{pmatrix}$$

Equations for element 15

E = 3000000; ν = 0.2; h = 4

Nodal coordinates

Element node	Global node number	x	y
1	41	15.	4.
2	34	10.5	2.
3	19	6.	0.
4	46	20.	0.
5	50	26.6667	4.

Complete element equations for element 15

$$\begin{pmatrix} 1.81228\times10^7 & -2.40825\times10^6 & -1.24057\times10^7 & -1.38977\times10^6 & 1.22994\times10^6 \\ -2.40825\times10^6 & 4.43538\times10^7 & 276901. & -3.20088\times10^7 & 46698.2 \\ -1.24057\times10^7 & 276901. & 3.29383\times10^7 & -2.04025\times10^6 & -1.39029\times10^7 \\ -1.38977\times10^6 & -3.20088\times10^7 & -2.04025\times10^6 & 7.86992\times10^7 & 369640. \\ 1.22994\times10^6 & 46698.2 & -1.39029\times10^7 & 369640. & 9.04082\times10^6 \\ 463365. & 3.53065\times10^6 & -1.29703\times10^6 & -3.5586\times10^7 & 1.38812\times10^6 \\ -4.19374\times10^6 & 648185. & -1.00662\times10^7 & 3.98382\times10^6 & 4.78564\times10^6 \\ 648185. & -1.04429\times10^7 & 3.98382\times10^6 & -2.25136\times10^7 & -1.15627\times10^6 \\ -2.75322\times10^6 & 1.43646\times10^6 & 3.4366\times10^6 & -923441. & -1.15348\times10^6 \\ 2.68646\times10^6 & -5.43278\times10^6 & -923441. & 1.14092\times10^7 & -648185. \end{pmatrix}$$

Equations for element 16

E = 3000000; ν = 0.2; h = 4

Nodal coordinates

Element node	Global node number	x	y
1	56	40.	12.
2	54	33.3333	8.
3	58	43.6667	8.
4	59	47.	12.

Complete element equations for element 16

$$\begin{pmatrix}
1.0601\times10^{7} & -3.47572\times10^{6} & -2.00657\times10^{6} & -1.35515\times10^{6} & -6.53509\times10^{6} & \\
-3.47572\times10^{6} & 2.20204\times10^{7} & -105155. & -7.06087\times10^{6} & 2.60515\times10^{6} & \\
-2.00657\times10^{6} & -105155. & 3.0394\times10^{6} & 555105. & -747733. & 6 \\
-1.35515\times10^{6} & -7.06087\times10^{6} & 555105. & 3.90279\times10^{6} & 1.9449\times10^{6} & \\
-6.53509\times10^{6} & 2.60515\times10^{6} & -747733. & 1.9449\times10^{6} & 9.2894\times10^{6} & \\
2.60515\times10^{6} & -1.42933\times10^{7} & 694895. & 1.82638\times10^{6} & -3.1949\times10^{6} & \\
-2.05931\times10^{6} & 975724. & -285093. & -1.14485\times10^{6} & -2.00657\times10^{6} & -1 \\
2.22572\times10^{6} & -666240. & -1.14485\times10^{6} & 1.3317\times10^{6} & -1.35515\times10^{6} &
\end{pmatrix}$$

Equations for element 17

E = 3000000; ν = 0.2; h = 4

Nodal coordinates

Element node	Global node number	x	y
1	59	47.	12.
2	58	43.6667	8.
3	62	54.	8.
4	63	54.	12.

Complete element equations for element 17

$$\begin{pmatrix}
7.33192\times10^{6} & -2.40857\times10^{6} & -2.54743\times10^{6} & -868385. & -3.9109\times10 \\
-2.40857\times10^{6} & 1.38478\times10^{7} & 381615. & -8.41301\times10^{6} & 2.11838\times1 \\
-2.54743\times10^{6} & 381615. & 4.14503\times10^{6} & 1.43503\times10^{6} & 229968. \\
-868385. & -8.41301\times10^{6} & 1.43503\times10^{6} & 6.66687\times10^{6} & 1.06497\times1 \\
-3.9109\times10^{6} & 2.11838\times10^{6} & 229968. & 1.06497\times10^{6} & 6.22837\times1 \\
2.11838\times10^{6} & -7.73283\times10^{6} & -185035. & 4.27063\times10^{6} & -2.31497\times1 \\
-873585. & -91425.4 & -1.82757\times10^{6} & -1.63162\times10^{6} & -2.54743\times1 \\
1.15857\times10^{6} & 2.29806\times10^{6} & -1.63162\times10^{6} & -2.52449\times10^{6} & -868385.
\end{pmatrix}$$

Equations for element 18

E = 3000000; ν = 0.2; h = 4

Nodal coordinates

Element node	Global node number	x	y
1	54	33.3333	8.
2	50	26.6667	4.
3	57	40.3333	4.
4	58	43.6667	8.

Complete element equations for element 18

$$\begin{pmatrix}
1.09361\times10^{7} & -2.99621\times10^{6} & -3.84805\times10^{6} & -1.1492\times10^{6} & -6.77695\times10^{6} \\
-2.99621\times10^{6} & 2.42009\times10^{7} & 100796. & -1.10879\times10^{7} & 2.3992\times10^{6} \\
-3.84805\times10^{6} & 100796. & 3.67169\times10^{6} & 899398. & 703307. \\
-1.1492\times10^{6} & -1.10879\times10^{7} & 899398. & 6.44755\times10^{6} & 1.6006\times10^{6} \\
-6.77695\times10^{6} & 2.3992\times10^{6} & 703307. & 1.6006\times10^{6} & 9.92169\times10^{6} \\
2.3992\times10^{6} & -1.54746\times10^{7} & 350602. & 4.48995\times10^{6} & -2.8506\times10^{6} \\
-311125. & 496214. & -526954. & -1.3508\times10^{6} & -3.84805\times10^{6} \\
1.74621\times10^{6} & 2.36159\times10^{6} & -1.3508\times10^{6} & 150385. & -1.1492\times10^{6}
\end{pmatrix}$$

Equations for element 19

E = 3000000; ν = 0.2; h = 4

Nodal coordinates

Element node	Global node number	x	y
1	58	43.6667	8.
2	57	40.3333	4.
3	61	54.	4.
4	62	54.	8.

Complete element equations for element 19

$$\begin{pmatrix}
8.02226\times10^{6} & -2.24874\times10^{6} & -4.23634\times10^{6} & -799735. & -4.30532\times1\\
-2.24874\times10^{6} & 1.69163\times10^{7} & 450265. & -1.20586\times10^{7} & 2.04973\times1\\
-4.23634\times10^{6} & 450265. & 5.03236\times10^{6} & 1.5498\times10^{6} & 1.42598\times1\\
-799735. & -1.20586\times10^{7} & 1.5498\times10^{6} & 9.84921\times10^{6} & 950201.\\
-4.30532\times10^{6} & 2.04973\times10^{6} & 1.42598\times10^{6} & 950201. & 7.11569\times1\\
2.04973\times10^{6} & -9.29553\times10^{6} & -299799. & 6.29662\times10^{6} & -2.2002\times10\\
519404. & -251262. & -2.22199\times10^{6} & -1.70027\times10^{6} & -4.23634\times1\\
998738. & 4.43791\times10^{6} & -1.70027\times10^{6} & -4.0872\times10^{6} & -799735.
\end{pmatrix}$$

Equations for element 20

E = 3000000; ∨ = 0.2; h = 4

Nodal coordinates

Element node	Global node number	x	y
1	50	26.6667	4.
2	46	20.	0.
3	55	37.	0.
4	57	40.3333	4.

Complete element equations for element 20

$$\begin{pmatrix}
1.17574\times10^{7} & -2.73792\times10^{6} & -5.48961\times10^{6} & -1.03422\times10^{6} & -7.21872\times10^{6} & 2.284\\
-2.73792\times10^{6} & 2.69773\times10^{7} & 215780. & -1.48699\times10^{7} & 2.28422\times10^{6} & -1.690\\
-5.48961\times10^{6} & 215780. & 4.61747\times10^{6} & 1.10104\times10^{6} & 1.84087\times10^{6} & 148960.\\
-1.03422\times10^{6} & -1.48699\times10^{7} & 1.10104\times10^{6} & 9.37658\times10^{6} & 1.39896\times10^{6} & 6.769\\
-7.21872\times10^{6} & 2.28422\times10^{6} & 1.84087\times10^{6} & 1.39896\times10^{6} & 1.08675\times10^{7} & -2.648\\
2.28422\times10^{6} & -1.6901\times10^{7} & 148960. & 6.76925\times10^{6} & -2.64896\times10^{6} & 2.500\\
950927. & 237921. & -968719. & -1.46578\times10^{6} & -5.48961\times10^{6} & 215780.\\
1.48792\times10^{6} & 4.7935\times10^{6} & -1.46578\times10^{6} & -1.27598\times10^{6} & -1.03422\times10^{6} & -1.486
\end{pmatrix}$$

Equations for element 21

E = 3000000; ∨ = 0.2; h = 4

Nodal coordinates

Element node	Global node number	x	y
1	57	40.3333	4.
2	55	37.	0.
3	60	54.	0.
4	61	54.	4.

Complete element equations for element 21

$$\begin{pmatrix} 9.03487\times10^{6} & -2.16264\times10^{6} & -5.79274\times10^{6} & -761407. & -4.83226\times \\ -2.16264\times10^{6} & 2.0171\times10^{7} & 488593. & -1.56277\times10^{7} & 2.01141\times \\ -5.79274\times10^{6} & 488593. & 6.1275\times10^{6} & 1.61701\times10^{6} & 2.41417\times \\ -761407. & -1.56277\times10^{7} & 1.61701\times10^{6} & 1.31517\times10^{7} & 882987. \\ -4.83226\times10^{6} & 2.01141\times10^{6} & 2.41417\times10^{6} & 882987. & 8.21083\times \\ 2.01141\times10^{6} & -1.09348\times10^{7} & -367013. & 8.20251\times10^{6} & -2.13299\times \\ 1.59013\times10^{6} & -337360. & -2.74893\times10^{6} & -1.73859\times10^{6} & -5.79274\times \\ 912640. & 6.3915\times10^{6} & -1.73859\times10^{6} & -5.7265\times10^{6} & -761407. \end{pmatrix}$$

Essential boundary conditions

Node	dof	Value
1	u_1	0
2	u_2	0
3	u_3	0
4	u_4	0
5	u_5	0
6	u_6	0
7	u_7	0
60	u_{60}	0
	v_{60}	0
61	u_{61}	0
	v_{61}	0
62	u_{62}	0
	v_{62}	0
63	u_{63}	0
	v_{63}	0

After adjusting for essential boundary conditions we have

$$\begin{pmatrix} 1.15632\times10^{7} & -1.53554\times10^{7} & 6.58405\times10^{6} & 0 & 0 \\ -1.53554\times10^{7} & 3.53875\times10^{7} & -1.78006\times10^{7} & 0 & 0 \\ 6.58405\times10^{6} & -1.78006\times10^{7} & 2.84509\times10^{7} & -2.01506\times10^{7} & 8.08326\times \\ 0 & 0 & -2.01506\times10^{7} & 4.44535\times10^{7} & -2.25702\times \end{pmatrix}$$

0	0	8.08326×10^{6}	-2.25702×10^{7}	$3.38974\times$
0	0	0	0	$-2.49316\times$
0	0	0	0	$9.64362\times$
$-72806.$	2.8414×10^{6}	1.31838×10^{6}	0	0
-2.5265×10^{6}	351380.	-3.90861×10^{6}	0	0
$-362062.$	-3.79772×10^{6}	1.26062×10^{6}	2.95375×10^{6}	$1.20877\times$
-2.11306×10^{6}	331703.	-3.02185×10^{6}	271131.	$-3.87026\times$
0	0	$-466188.$	-3.69633×10^{6}	1.0054×1
0	0	-2.07862×10^{6}	259286.	$-1.12039\times$
$-562771.$	1.16194×10^{6}	-1.74046×10^{6}	0	0
5.54667×10^{6}	-9.99597×10^{6}	6.84838×10^{6}	0	0
0	0	0	0	$-532568.$
0	0	0	0	$-2.18723\times$
$-996655.$	572609.	663125.	0	0
-8.45411×10^{6}	1.46125×10^{7}	-8.38984×10^{6}	0	0
1.18998×10^{6}	$-539158.$	$-588232.$	1.08531×10^{6}	$-1.67611\times$
4.75521×10^{6}	-7.53106×10^{6}	1.02853×10^{7}	-1.22634×10^{7}	$7.98938\times$
0	0	0	0	0
0	0	0	0	0
0	0	$-960936.$	444997.	701585.
0	0	-1.08018×10^{7}	1.98322×10^{7}	$-1.07631\times$
0	0	1.24885×10^{6}	$-602082.$	$-469160.$
0	0	5.90013×10^{6}	-9.83193×10^{6}	$1.18658\times$
0	0	0	0	$-938035.$
0	0	0	0	$-1.31636\times$
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	1.28649×
0	0	0	0	7.12694×
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
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0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Solving the final system of global equations we get

$$\{v_1 = -0.0433512,\ v_2 = -0.0433977,\ v_3 = -0.0434424,\ v_4 = -0.0434478,\ v_5 = -0.0434335,\ v_6 = -0.043391,\ v_7 = -0.0433234,\ u_8 = 0.000442152,\ v_8 = -0.0432241,\ u_9 = 0.000121906,\ v_9 = -0.043224,\ u_{10} = -0.000450314,\ v_{10} = -0.0430701,\ u_{11} = 0.000898366,\ v_{11} = -0.0428902,\ u_{12} = -0.00128943,\ v_{12} = -0.0428053,\ u_{13} = 0.000645599,\ v_{13} = -0.0427767,\ u_{14} = 0.000207697,\ v_{14} = -0.0425616,\ u_{15} = 0.0014282,\ v_{15} = -0.0423311,\ u_{16} = -0.000289969,\ v_{16} = -0.0423059,\ u_{17} = -0.000891704,\ v_{17} = -0.0420074,\ u_{18} = -0.00160899,\ v_{18} = -0.0416635,\ u_{19} = 0.00701662,\ v_{19} = -0.0418628,\ u_{20} = 0.00578302,\ v_{20} = -0.0418207,\ u_{21} = 0.0045341,\ v_{21} = -0.0417719,\ u_{22} = 0.00321919,\ v_{22} = -0.0416488,\ u_{23} = 0.00192205,\ v_{23} = -0.0414902,\ u_{24} = 0.000271463,\ v_{24} = -0.0414315,\ u_{25} = -0.00248781,\ v_{25} = -0.0412825,\ u_{26} = 0.00107483,\ v_{26} = -0.0407164,\ u_{27} = -0.00130038,\ v_{27} = -0.0402653,\ u_{28} = 0.000258265,\ v_{28} = -0.0398727,\ u_{29} = 0.00299974,\ v_{29} = -0.0391301,\ u_{30} = -0.000631563,\ v_{30} = -0.0390009,\ u_{31} = -0.00338436,\ v_{31} = -0.038948,\ u_{32} = 0.00091481,\ v_{32} = -0.0382586,\ u_{33} = -0.00163959,\ v_{33} = -0.0380827,\ u_{34} = 0.00507175,\ v_{34} = -0.0374624,\ u_{35} = -0.00275544,\ v_{35} = -0.037128,\ u_{36} = 0.00165284,\ v_{36} = -0.0365735,\ u_{37} = -0.00399364,\ v_{37} = -0.0361434,\ u_{38} = 0.00239557,\ v_{38} = -0.0348281,\ u_{39} = -0.00159262,\ v_{39} = -0.0346746,\ u_{40} = 0.000170754,\ v_{40} = -0.0338948,\ u_{41} = 0.00315312,\ v_{41} = -0.033045,\ u_{42} = -0.00142344,\ v_{42} = -0.0311427,\ u_{43} = -0.00453029,\ v_{43} = -0.0309132,\ u_{44} = 0.00116341,\ v_{44} = -0.0285447,\ u_{45} = -0.0030776,\ v_{45} = -0.028365,\ u_{46} = 0.00696755,\ v_{46} = -0.0280676,\ u_{47} = -0.00115034,\ v_{47} = -0.0275859,\ u_{48} = -0.00473821,\ v_{48} = -0.025587,\ u_{49} = -0.000855872,\ v_{49} = -0.0240683,\ u_{50} = 0.00286123,\ v_{50} = -0.021379,\ u_{51} = -0.0047055,\ v_{51} = -0.020438,\ u_{52} = -0.00281548,\ v_{52} = -0.0196773,\ u_{53} = -0.00449338,\ v_{53} = -0.0155429,\ u_{54} = -0.00103931,\ v_{54} = -0.0149814,\ u_{55} = 0.00603824,\ v_{55} = -0.0112323,\ u_{56} = -0.00408667,\ v_{56} = -0.00921432,\ u_{57} = 0.00202799,\ v_{57} = -0.00842634,\ u_{58} = -0.00079608,\ v_{58} = -0.00596352,\ u_{59} = -0.002965,\ v_{59} = -0.00390572\}$$

Solution summary

Nodal solution

	x	y	u	v
1	0.	5.	0	−0.0433512
2	0.	6.16667	0	−0.0433977
3	0.	7.33333	0	−0.0434424
4	0.	8.5	0	−0.0434478

5	0.	9.66667	0	−0.0434335
6	0.	10.8333	0	−0.043391
7	0.	12.	0	−0.0433234
8	1.5	5.	0.000442152	−0.0432241
9	2.	7.33333	0.000121906	−0.043224
10	2.5	9.66667	−0.000450314	−0.0430701
11	3.	5.	0.000898366	−0.0428902
12	3.	12.	−0.00128943	−0.0428053
13	3.5	6.16667	0.000645599	−0.0427767
14	4.	7.33333	0.000207697	−0.0425616
15	4.5	5.	0.0014282	−0.0423311
16	4.5	8.5	−0.000289969	−0.0423059
17	5.	9.66667	−0.000891704	−0.0420074
18	5.5	10.8333	−0.00160899	−0.0416635
19	6.	0.	0.00701662	−0.0418628
20	6.	1.25	0.00578302	−0.0418207
21	6.	2.5	0.0045341	−0.0417719
22	6.	3.75	0.00321919	−0.0416488
23	6.	5.	0.00192205	−0.0414902
24	6.	7.33333	0.000271463	−0.0414315
25	6.	12.	−0.00248781	−0.0412825
26	7.	6.16667	0.00107483	−0.0407164
27	7.5	9.66667	−0.00130038	−0.0402653
28	8.	7.33333	0.000258265	−0.0398727
29	8.75	4.08333	0.00299974	−0.0391301
30	9.	8.5	−0.000631563	−0.0390009
31	9.	12.	−0.00338436	−0.038948
32	9.75	6.5	0.00091481	−0.0382586
33	10.	9.66667	−0.00163959	−0.0380827
34	10.5	2.	0.00507175	−0.0374624
35	11.	10.8333	−0.00275544	−0.037128
36	11.5	5.66667	0.00165284	−0.0365735
37	12.	12.	−0.00399364	−0.0361434
38	13.25	4.83333	0.00239557	−0.0348281
39	13.5	9.25	−0.00159262	−0.0346746
40	14.25	7.25	0.000170754	−0.0338948
41	15.	4.	0.00315312	−0.033045
42	17.	8.83333	−0.00142344	−0.0311427

43	17.25	12.	−0.00453029	−0.0309132
44	19.5	6.	0.00116341	−0.0285447
45	19.75	10.4167	−0.0030776	−0.028365
46	20.	0.	0.00696755	−0.0280676
47	20.5	8.41667	−0.00115034	−0.0275859
48	22.5	12.	−0.00473821	−0.025587
49	24.	8.	−0.000855872	−0.0240683
50	26.6667	4.	0.00286123	−0.021379
51	27.75	12.	−0.0047055	−0.020438
52	28.5	10.	−0.00281548	−0.0196773
53	33.	12.	−0.00449338	−0.0155429
54	33.3333	8.	−0.00103931	−0.0149814
55	37.	0.	0.00603824	−0.0112323
56	40.	12.	−0.00408667	−0.00921432
57	40.3333	4.	0.00202799	−0.00842634
58	43.6667	8.	−0.00079608	−0.00596352
59	47.	12.	−0.002965	−0.00390572
60	54.	0.	0	0
61	54.	4.	0	0
62	54.	8.	0	0
63	54.	12.	0	0

Solution at selected points on elements

	Coord	Disp	Stresses	Principal stresses	Effective Stress
1	2.75 10.8333	−0.000829492 −0.0429533	−885.347 −38.5711 0 21.4363 0 0	0. −38.0288 −885.89	867.501
2	8.25 10.8333	−0.0022714 −0.0396234	−629.864 −22.6481 0 81.8768 0 0	0. −11.8016 −640.711	634.892
3	2.25 8.5	−0.000138187 −0.0431627	−194.132 −4.10036 0 27.8944 0 0	0. −0.0904196 −198.142	198.097

4	6.75 8.5	−0.000458893 −0.0408707	−219.926 39.0197 0 137.002 0 0	98.0482 0. −278.954	338.792
5	1.75 6.16667	0.000328313 −0.0432499	552.704 −3.33214 0 0.825049 0 0	552.705 0. −3.33336	554.379
6	5.25 6.16667	0.000888449 −0.0419242	387.678 98.8279 0 17.6001 0 0	388.747 97.7594 0.	350.253
7	15.375 10.625	−0.00302925 −0.0328015	−256.073 −45.2711 0 57.0159 0 0	0. −30.8382 −270.506	256.481
8	24.125 10.2083	−0.00299674 −0.0239479	−57.3007 −49.9626 0 40.0476 0 0	0. −13.4163 −93.8469	87.91
9	11.625 7.875	−0.000281328 −0.0364965	−162.463 −84.5421 0 153.548 0 0	34.9111 0. −281.916	300.895
10	16.875 6.625	0.00065803 −0.0312194	−115.316 −105.519 0 108.113 0 0	0. −2.19399 −218.641	217.552
11	7.875 5.125	0.00201247 −0.0399499	121.192 20.5462 0 17.235 0 0	124.062 17.6766 0.	116.236

12	9.625 3.04167	0.00403588 −0.0383074	58.0026 32.1149 0 −23.7665 0 0	72.1215 17.9961 0.	65.0189
13	32.5833 10.	−0.00268924 −0.0158876	77.2584 −77.4671 0 39.9309 0 0	86.9558 0. −87.1645	150.793
14	24.75 6.	0.00103718 −0.0233625	−82.1807 −50.2688 0 56.8608 0 0	0. −7.16757 −125.282	121.856
15	16.9167 2.	0.00499307 −0.0310929	−41.851 −25.3267 0 25.5573 0 0	0. −6.7293 −60.4484	57.3805
16	41. 10.	−0.00222177 −0.00851624	211.669 −122.818 0 95.078 0 0	236.806 0. −147.955	336.161
17	49.6667 10.	−0.00094027 −0.00246731	690.569 198.064 0 259.817 0 0	802.29 86.3428 0.	762.792
18	36. 6.	0.000763459 −0.0126876	−99.7889 −130.189 0 131.99 0 0	17.873 0. −247.851	257.254

19	48. 6.	0.000307979 −0.00359747	−124.137 149.26 0 334.945 0 0	374.327 0. −349.204	626.723
20	31. 2.	0.00447376 −0.0172763	−196.713 −121.448 0 35.7738 0 0	0. −107.158 −211.003	182.742
21	46.3333 2.	0.00201656 −0.00491466	−769.69 96.9889 0 311.696 0 0	197.445 0. −870.146	983.842

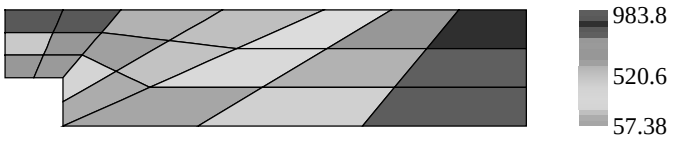
Support reactions

Node	dof	Reaction
1	1	−1492.21
2	1	−3250.09
3	1	−668.521
4	1	1095.16
5	1	1706.8
6	1	5756.08
7	1	2029.83
60	1	−22089.2
60	2	1870.66
61	1	−4913.92
61	2	1954.34
62	1	7693.77
62	2	2332.
63	1	14132.3
63	2	4642.99

Sum of applied loads → (0 −10800.)

Sum of support reactions → (0 10800.)

The effective stresses at element centers are used to create an element stress plot as shown in Figure. There are significant jumps in stresses across element boundaries and thus the model needs to be refined further. Results obtained from refined models using Ansys and Abaqus are presented in Appendix A.


983.8
520.6
57.38