Computer Implementation 1.2 *(Matlab)* *Triangular heat flow element (p. 14)*

The element equations for a triangular heat flow element can be generated conveniently by writing three simple functions. The following *HeatTriElement* function takes thermal conductivities (kx and ky), heat generation (Q) and the element nodal coordinates and returns the $\boldsymbol{k}_k$ matrix and the $\boldsymbol{r}_Q$ vector. A second function called *ConvectionTerm* generates $\boldsymbol{k}_h$ matrix and $\boldsymbol{r}_h$ vector and is needed when there is convection term specified. Three different versions of this function are defined depending on the side on which the convection boundary condition is specified. The third function called *HeatFluxTerm* computes $\boldsymbol{r}_q$ vector when a flux is specified along an element side.

MatlabFiles\Chap1\HeatTriElement.m

```
function[kk,rQ]=HeatTriElement(kx,ky,Q,coord)
%[kk,rQ]=HeatTriElement(kx,ky,Q,coord)
% Generates coefficient matrix of a triangular element for heat flow
% kx,ky=thermal conductivities in the x and y directions
% Q=Heat generation
% coord=coordinates at the element ends

x1=coord(1,1);y1=coord(1,2);
x2=coord(2,1);y2=coord(2,2);
x3=coord(3,1);y3=coord(3,2);
b1=y2-y3;b2=y3-y1;b3=y1-y2;
c1=x3-x2;c2=x1-x3;c3=x2-x1;
f1=x2*y3-x3*y2;f2=x3*y1-x1*y3;f3=x1*y2-x2*y1;
A=(f1+f2+f3)/2;
kxx=1/(4*A)*kx*[b1^2,b1*b2,b1*b3;
        b1*b2,b2^2,b2*b3;b1*b3,b2*b3,b3^2];
kyy=1/(4*A)*ky*[c1^2,c1*c2,c1*c3;
        c1*c2,c2^2,c2*c3;c1*c3,c2*c3,c3^2];
kk=kxx+kyy;
rQ=1/3*Q*A*[1;1;1];
```

MatlabFiles\Chap1\ConvectionTerm.m

```
function [kh, rh] = ConvectionTerm(side, h, Tinf, coord)
% [kh, rh] = ConvectionTerm(side, h, Tinf, coord)
% Generates kh and rh when convection is specified for a triangular element
% side = side over which the convection is specified
% h = convection coefficient
% Tinf = surrounding temperature
% coord = coordinates at the element ends

x1=coord(1,1); y1=coord(1,2);
x2=coord(2,1); y2=coord(2,2);
x3=coord(3,1); y3=coord(3,2);
switch (side)
case 1
    L=sqrt((x2-x1)^2+(y2-y1)^2);
    kh=h*L/6 * [2,1,0; 1,2,0; 0,0,0];
    rh=h*Tinf *L/2 * [1; 1; 0];
case 2
    L=sqrt((x2-x3)^2+(y2-y3)^2);
    kh=h*L/6 * [0,0,0; 0,2,1; 0,1,2];
    rh=h*Tinf *L/2 * [0; 1; 1];
case 3
    L=sqrt((x3-x1)^2+(y3-y1)^2);
    kh=h*L/6 * [2,0,1; 0,0,0; 1,0,2];
```

```
    rh=h*Tinf *L/2 * [1; 0; 1];
end
```

MatlabFiles\Chap1\HeatFluxTerm.m

```
function rq = HeatFluxTerm(side, q, coord)
% rq = HeatFluxTerm(side, q, coord)
% Generates vector resulting from a specified flux for a triangular element
% side = side over which the flux is specified
% q = flux value
% coord = coordinates at the element ends

x1=coord(1,1); y1=coord(1,2);
x2=coord(2,1); y2=coord(2,2);
x3=coord(3,1); y3=coord(3,2);
switch (side)
case 1
    rq = q*sqrt((x2 - x1)^2 + (y2 - y1)^2)/2 * [1; 1; 0];
case 2
    rq = q*sqrt((x2 - x3)^2 + (y2 - y3)^2)/2 * [0; 1; 1];
case 3
    rq = q*sqrt((x3 - x1)^2 + (y3 - y1)^2)/2 * [1; 0; 1];
end
```

Using these functions finite element equations for any triangular element for heat flow problem can easily be written. As an example we use these functions to develop matrices for the element number 20 in the finite element model of the heat flow through the L-shaped solid. The nodal coordinates for the entire model are are first defined. The element is connected between nodes 4, 10 and 5. With $k_x = k_y = 45$ and $Q = 5000000$, the $\boldsymbol{k}_k$ matrix and the $\boldsymbol{r}_Q$ vector for the element are generated by using the HeatTriElement function. There is an applied heat flux on side 3 of the element. With $q = 8000$, the $\boldsymbol{r}_q$ vector for the element is generated using the HeatFluxTerm function. The side 2 of the element is subjected to heat loss by convection. With $h = 55$ and $T_\infty = 20$, the $\boldsymbol{k}_h$ matrix and the $\boldsymbol{r}_h$ vector for the element are are generated by using the ConvectionTerm function. Adding matrices $\boldsymbol{k}_k$ and $\boldsymbol{k}_h$ and vectors $\boldsymbol{r}_Q$, $\boldsymbol{r}_q$, and $\boldsymbol{r}_h$ the complete element equations are as follows.

MatlabFiles\Chap1\HeatElementEx1.m

```
% Generation of element equations for a 2D heat flow problem
nodes = [0., 0.; 0., 0.0075; 0., 0.015; 0., 0.0225;
    0., 0.03; 0.015, 0.; 0.015, 0.0075; 0.015, 0.015;
    0.015, 0.0225; 0.015, 0.03; 0.03, 0.; 0.03, 0.0075;
    0.03, 0.015; 0.03, 0.0225; 0.03, 0.03; 0.045, 0.; 0.045, 0.0075;
    0.045, 0.015; 0.06, 0.; 0.06, 0.0075; 0.06, 0.015];
[kk, rQ] = HeatTriElement(45, 45, 5000000,nodes([4 10 5],:))
rq = HeatFluxTerm(3, 8000, nodes([4 10 5],:))
[kh, rh] = ConvectionTerm(2, 55, 20, nodes([4 10 5],:))
k = kk + kh
r = rq + rQ + rh
```

```
>> HeatElementEx1

kk =

   45.0000         0  -45.0000
         0   11.2500  -11.2500
  -45.0000  -11.2500   56.2500


rQ =
```

```
   93.7500
   93.7500
   93.7500


rq =

    30
     0
    30


kh =

         0         0         0
         0    0.2750    0.1375
         0    0.1375    0.2750


rh =

         0
    8.2500
    8.2500


k =

   45.0000         0  -45.0000
         0   11.5250  -11.1125
  -45.0000  -11.1125   56.5250


r =

  123.7500
  102.0000
  132.0000
```