Computer Implementation 8.3 ***(Matlab)*** *Modal analysis of a plane frame (p. 570)*

The following TransientPlaneFrameElement function returns the mass and the stiffness matrix of a plane frame element. To generate the mass matrix the function needs the mass density (ρ).

MatlabFiles\Chap8\TransientPlaneFrameElement.m

```
function [m, k, r] = TransientPlaneFrameElement(modulus, inertia, ...
    A, rho, qs, qt, coord)
% Plane frame element for dynamic analysis
% modulus = modulus of elasticity
% inertia = moment of inertia
% A = area of cross-section
% rho = mass density
% qs = distributed load along the element axis
% qt = distributed load normal to the element axis
% coord = coordinates at the element ends

EI=modulus*inertia; EA = modulus*A;
x1=coord(1,1); y1=coord(1,2);
x2=coord(2,1); y2=coord(2,2);
L=sqrt((x2-x1)^2+(y2-y1)^2);
ls=(x2-x1)/L; ms=(y2-y1)/L;

T = [ls, ms, 0, 0, 0, 0;
    -ms, ls, 0, 0, 0, 0;
    0, 0, 1, 0, 0, 0;
    0, 0, 0, ls, ms, 0;
    0, 0, 0, -ms, ls, 0;
    0, 0, 0, 0, 0, 1];
kl = [EA/L, 0, 0, -(EA/L), 0, 0;
    0, (12*EI)/L^3, (6*EI)/L^2, 0, ...
        -((12*EI)/L^3), (6*EI)/L^2;
      0, (6*EI)/L^2, (4*EI)/L, 0, ...
          -((6*EI)/L^2), (2*EI)/L;
      -(EA/L), 0, 0, EA/L, 0, 0;
      0, -((12*EI)/L^3), -((6*EI)/L^2), 0, ...
          (12*EI)/L^3, -((6*EI)/L^2);
      0, (6*EI)/L^2, (2*EI)/L, 0, ...
          -((6*EI)/L^2), (4*EI)/L];
ml = ((rho*A*L)/420)*[140,0, 0, 70, 0, 0;
    0, 156, 22*L, 0, 54, -13*L;
    0, 22*L, 4*L^2, 0, 13*L, -3*L^2;
    70, 0, 0, 140, 0, 0;
    0, 54, 13*L, 0, 156, -22*L;
    0, -13*L, -3*L^2, 0, -22*L, 4*L^2];
```

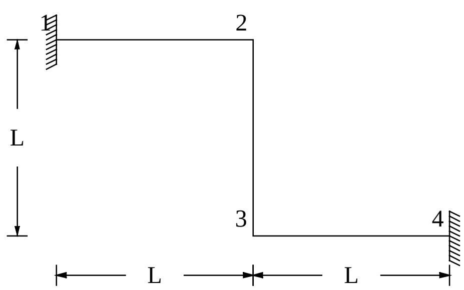
```
rl = [qs*(L/2); qt*(L/2); qt*(L^2/12);
        qs*(L/2); qt*(L/2); -qt*(L^2/12)];
m=T'*ml*T; k=T'*kl*T; r=T'*rl;
```

Using these functions now we consider modal analysis of the plane frame shown. The horizontal members carry a weight 200 N/m in addition to their own weight. The other numerical data is as follows.

Use N-mm units.

$$L = 600\,\text{mm};\quad E = 200\,\text{GPa} = 200,000\,N/\text{mm}^2;\quad \rho = 7840\,\text{kg}\big/ m^3 = 7.84\times 10^{-6}\,\text{kg}/\text{mm}^3;\quad A = 240\,\text{mm}^2;\quad I$$

Additional mass on horizontal members: $\frac{200}{9.81} = 20.3874\,\text{kg}/m = 0.0203874\,\text{kg}/\text{mm}$



MatlabFiles\Chap8\ModalFrameEx.m

```
% Modal analysis of a plane frame
L = 1000; e = 200000; rho = 7.84*10^(-6); a = 240;
inertia = 2000; ma = 0.0203874;
nodes = [0, 0; L, 0; L, -L; 2*L, -L];
conn = [1,2; 2,3; 3,4];
elems = size(conn,1);
lmm=[];
for i=1:elems
    lmm = [lmm; [3*conn(i,1)-2, 3*conn(i,1)-1, 3*conn(i,1),...
            3*conn(i,2)-2, 3*conn(i,2)-1, 3*conn(i,2)]];
end
debc = [1,2,3,10,11,12]; ebcVals=zeros(length(debc),1);
dof=3*size(nodes,1);
M=zeros(dof); K=zeros(dof);

% Generate equations for each element and assemble them.
for i=1:2:elems
    con = conn(i,:);
```

```
    lm = lmm(i,:);
    [m, k, r] = TransientPlaneFrameElement(e, inertia, a, ...
        rho+ma/a, 0, 0, nodes(con,:));
    M(lm, lm) = M(lm, lm) + m;
    K(lm, lm) = K(lm, lm) + k;
end
for i=2
    con = conn(i,:);
    lm = lmm(i,:);
    [m, k, r] = TransientPlaneFrameElement(e, inertia, a, ...
        rho, 0, 0, nodes(con,:));
    M(lm, lm) = M(lm, lm) + m;
    K(lm, lm) = K(lm, lm) + k;
end

% Adjust for essential boundary conditions
dof = length(R);
df = setdiff(1:dof, debc);
Mf = M(df, df);
Kf = K(df, df);

% Compute frequencies and mode shapes
[V, lam] = eig(Kf, Mf);
freq=sqrt(lam)
modeShapes = V
```

```
>> ModalFrameEx

freq =

    0.57558          0          0          0          0          0
          0     4.2955          0          0          0          0
          0          0     4.4396          0          0          0
          0          0          0     76.951          0          0
          0          0          0          0     78.074          0
          0          0          0          0          0     204.62


modeShapes =

  2.4473e-007  -0.00012862   0.00014691      0.24834       0.2519     -0.01503
      0.26046  -0.00030283      0.26486    -0.051657    0.0039949      0.46663
   0.00022612    0.0015179    0.0022738  -0.00040806 -2.4546e-005    0.0025172
  2.4473e-007   0.00012862   0.00014691      0.24834      -0.2519      0.01503
      0.26046   0.00030283      0.26486    -0.051657   -0.0039949     -0.46663
  -0.00022612    0.0015179   -0.0022738   0.00040806 -2.4546e-005    0.0025172
```