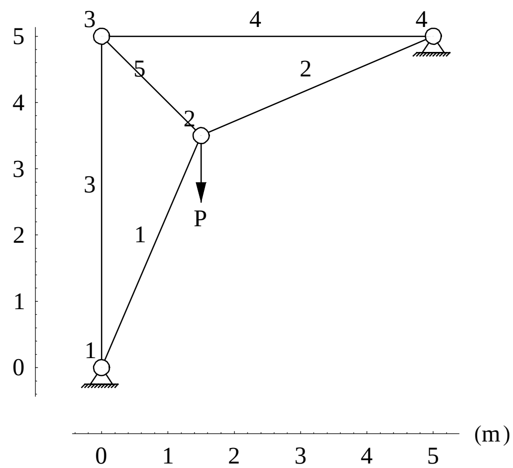
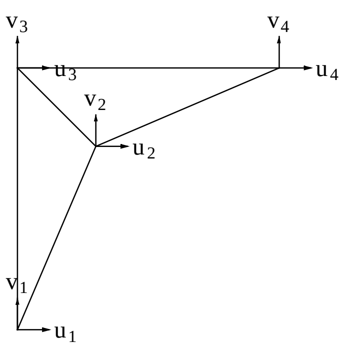
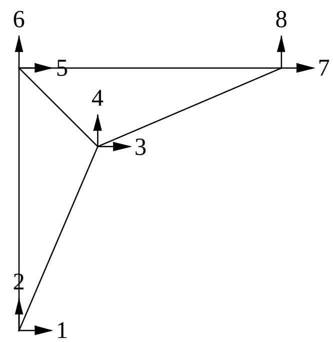
Five-bar truss example: Examples 1.4 p. 25, 1.7 p. 42, and 1.10 p. 50

The area of cross-section for elements 1 and 2 is 40 cm^2, for elements 3 and 4 is 30 cm^2 and for element 5 is 20 cm^2. The first four elements are made of a material with $E = 200$ GPa and the last one with $E = 70$ GPa. The applied load $P = 150$ kN.







Specified nodal loads

Node	dof	Value
2	u_2	0
	v_2	−150000

Global equations at start of the element assembly process

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -150000 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 1

$E = 200000 \qquad A = 4000$

Element node	Global node number	x	y
1	1	0	0
2	2	1500.	3500.

$x_1 = 0 \qquad y_1 = 0 \qquad x_2 = 1500. \qquad y_2 = 3500.$

$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 3807.89$

Direction cosines: $\ell_s = \dfrac{x_2 - x_1}{L} = 0.393919 \qquad m_s = \dfrac{y_2 - y_1}{L} = 0.919145$

Substituting into the truss element equations we get

$$\begin{pmatrix} 32600.2 & 76067.2 & -32600.2 & -76067.2 \\ 76067.2 & 177490. & -76067.2 & -177490. \\ -32600.2 & -76067.2 & 32600.2 & 76067.2 \\ -76067.2 & -177490. & 76067.2 & 177490. \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0. \\ 0. \\ 0. \\ 0. \end{pmatrix}$$

The element contributes to {1, 2, 3, 4} global degrees of freedom.

Locations for element contributions to a global vector: $\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$

and to a global matrix: $\begin{pmatrix} [1,1] & [1,2] & [1,3] & [1,4] \\ [2,1] & [2,2] & [2,3] & [2,4] \\ [3,1] & [3,2] & [3,3] & [3,4] \\ [4,1] & [4,2] & [4,3] & [4,4] \end{pmatrix}$

Adding element equations into appropriate locations we have

$$\begin{pmatrix} 32600.2 & 76067.2 & -32600.2 & -76067.2 & 0 & 0 & 0 & 0 \\ 76067.2 & 177490. & -76067.2 & -177490. & 0 & 0 & 0 & 0 \\ -32600.2 & -76067.2 & 32600.2 & 76067.2 & 0 & 0 & 0 & 0 \\ -76067.2 & -177490. & 76067.2 & 177490. & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -150000. \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 2

$E = 200000 \qquad A = 4000$

Element node	Global node number	x	y
1	2	1500.	3500.
2	4	5000	5000

$x_1 = 1500. \qquad y_1 = 3500. \qquad x_2 = 5000 \qquad y_2 = 5000$

$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 3807.89$

Direction cosines: $\ell_s = \dfrac{x_2 - x_1}{L} = 0.919145 \qquad m_s = \dfrac{y_2 - y_1}{L} = 0.393919$

Substituting into the truss element equations we get

$$\begin{pmatrix} 177490. & 76067.2 & -177490. & -76067.2 \\ 76067.2 & 32600.2 & -76067.2 & -32600.2 \\ -177490. & -76067.2 & 177490. & 76067.2 \\ -76067.2 & -32600.2 & 76067.2 & 32600.2 \end{pmatrix} \begin{pmatrix} u_2 \\ v_2 \\ u_4 \\ v_4 \end{pmatrix} = \begin{pmatrix} 0. \\ 0. \\ 0. \\ 0. \end{pmatrix}$$

The element contributes to {3, 4, 7, 8} global degrees of freedom.

Locations for element contributions to a global vector: $\begin{pmatrix} 3 \\ 4 \\ 7 \\ 8 \end{pmatrix}$

and to a global matrix: $\begin{pmatrix} [3,3] & [3,4] & [3,7] & [3,8] \\ [4,3] & [4,4] & [4,7] & [4,8] \\ [7,3] & [7,4] & [7,7] & [7,8] \\ [8,3] & [8,4] & [8,7] & [8,8] \end{pmatrix}$

Adding element equations into appropriate locations we have

$$\begin{pmatrix} 32600.2 & 76067.2 & -32600.2 & -76067.2 & 0 & 0 & 0 & 0 \\ 76067.2 & 177490. & -76067.2 & -177490. & 0 & 0 & 0 & 0 \\ -32600.2 & -76067.2 & 210090. & 152134. & 0 & 0 & -177490. & -76067.2 \\ -76067.2 & -177490. & 152134. & 210090. & 0 & 0 & -76067.2 & -32600.2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -177490. & -76067.2 & 0 & 0 & 177490. & 76067.2 \\ 0 & 0 & -76067.2 & -32600.2 & 0 & 0 & 76067.2 & 32600.2 \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -150000. \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 3

$E = 200000$ $\quad$ $A = 3000$

Element node	Global node number	x	y
1	1	0	0
2	3	0	5000

$x_1 = 0$ $\quad$ $y_1 = 0$ $\quad$ $x_2 = 0$ $\quad$ $y_2 = 5000$

$$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 5000$$

Direction cosines: $\ell_s = \dfrac{x_2 - x_1}{L} = 0$ $\qquad$ $m_s = \dfrac{y_2 - y_1}{L} = 1$

Substituting into the truss element equations we get

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 120000 & 0 & -120000 \\ 0 & 0 & 0 & 0 \\ 0 & -120000 & 0 & 120000 \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ u_3 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The element contributes to {1, 2, 5, 6} global degrees of freedom.

Locations for element contributions to a global vector: $\begin{pmatrix} 1 \\ 2 \\ 5 \\ 6 \end{pmatrix}$

and to a global matrix: $\begin{pmatrix} [1,1] & [1,2] & [1,5] & [1,6] \\ [2,1] & [2,2] & [2,5] & [2,6] \\ [5,1] & [5,2] & [5,5] & [5,6] \\ [6,1] & [6,2] & [6,5] & [6,6] \end{pmatrix}$

Adding element equations into appropriate locations we have

$$\begin{pmatrix} 32600.2 & 76067.2 & -32600.2 & -76067.2 & 0 & 0 & 0 & 0 \\ 76067.2 & 297490. & -76067.2 & -177490. & 0 & -120000 & 0 & 0 \\ -32600.2 & -76067.2 & 210090. & 152134. & 0 & 0 & -177490. & -76067.2 \\ -76067.2 & -177490. & 152134. & 210090. & 0 & 0 & -76067.2 & -32600.2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -120000 & 0 & 0 & 0 & 120000 & 0 & 0 \\ 0 & 0 & -177490. & -76067.2 & 0 & 0 & 177490. & 76067.2 \\ 0 & 0 & -76067.2 & -32600.2 & 0 & 0 & 76067.2 & 32600.2 \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -150000. \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 4

$E = 200000 \qquad A = 3000$

Element node	Global node number	x	y
1	3	0	5000
2	4	5000	5000

$x_1 = 0 \qquad y_1 = 5000 \qquad x_2 = 5000 \qquad y_2 = 5000$

$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 5000$

Direction cosines: $\ell_s = \dfrac{x_2 - x_1}{L} = 1 \qquad m_s = \dfrac{y_2 - y_1}{L} = 0$

Substituting into the truss element equations we get

$$\begin{pmatrix} 120000 & 0 & -120000 & 0 \\ 0 & 0 & 0 & 0 \\ -120000 & 0 & 120000 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_3 \\ v_3 \\ u_4 \\ v_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The element contributes to {5, 6, 7, 8} global degrees of freedom.

Locations for element contributions to a global vector: $\begin{pmatrix} 5 \\ 6 \\ 7 \\ 8 \end{pmatrix}$

and to a global matrix: $\begin{pmatrix} [5,5] & [5,6] & [5,7] & [5,8] \\ [6,5] & [6,6] & [6,7] & [6,8] \\ [7,5] & [7,6] & [7,7] & [7,8] \\ [8,5] & [8,6] & [8,7] & [8,8] \end{pmatrix}$

Adding element equations into appropriate locations we have

$$\begin{pmatrix} 32600.2 & 76067.2 & -32600.2 & -76067.2 & 0 & 0 & 0 & 0 \\ 76067.2 & 297490. & -76067.2 & -177490. & 0 & -120000 & 0 & 0 \\ -32600.2 & -76067.2 & 210090. & 152134. & 0 & 0 & -177490. & -76067.2 \\ -76067.2 & -177490. & 152134. & 210090. & 0 & 0 & -76067.2 & -32600.2 \\ 0 & 0 & 0 & 0 & 120000 & 0 & -120000 & 0 \\ 0 & -120000 & 0 & 0 & 0 & 120000 & 0 & 0 \\ 0 & 0 & -177490. & -76067.2 & -120000 & 0 & 297490. & 76067.2 \\ 0 & 0 & -76067.2 & -32600.2 & 0 & 0 & 76067.2 & 32600.2 \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -150000. \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 5

$E = 70000 \qquad A = 2000$

Element node	Global node number	x	y
1	2	1500.	3500.
2	3	0	5000

$x_1 = 1500. \qquad y_1 = 3500. \qquad x_2 = 0 \qquad y_2 = 5000$

$$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 2121.32$$

Direction cosines: $\ell_s = \dfrac{x_2 - x_1}{L} = -0.707107 \qquad m_s = \dfrac{y_2 - y_1}{L} = 0.707107$

Substituting into the truss element equations we get

$$\begin{pmatrix} 32998.3 & -32998.3 & -32998.3 & 32998.3 \\ -32998.3 & 32998.3 & 32998.3 & -32998.3 \\ -32998.3 & 32998.3 & 32998.3 & -32998.3 \\ 32998.3 & -32998.3 & -32998.3 & 32998.3 \end{pmatrix} \begin{pmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0. \\ 0. \\ 0. \\ 0. \end{pmatrix}$$

The element contributes to {3, 4, 5, 6} global degrees of freedom.

Locations for element contributions to a global vector: $\begin{pmatrix} 3 \\ 4 \\ 5 \\ 6 \end{pmatrix}$

and to a global matrix: $\begin{pmatrix} [3, 3] & [3, 4] & [3, 5] & [3, 6] \\ [4, 3] & [4, 4] & [4, 5] & [4, 6] \\ [5, 3] & [5, 4] & [5, 5] & [5, 6] \\ [6, 3] & [6, 4] & [6, 5] & [6, 6] \end{pmatrix}$

Adding element equations into appropriate locations we have

$$\begin{pmatrix} 32600.2 & 76067.2 & -32600.2 & -76067.2 & 0 & 0 & 0 & 0 \\ 76067.2 & 297490. & -76067.2 & -177490. & 0 & -120000 & 0 & 0 \\ -32600.2 & -76067.2 & 243089. & 119136. & -32998.3 & 32998.3 & -177490. & -76067.2 \\ -76067.2 & -177490. & 119136. & 243089. & 32998.3 & -32998.3 & -76067.2 & -32600.2 \\ 0 & 0 & -32998.3 & 32998.3 & 152998. & -32998.3 & -120000 & 0 \\ 0 & -120000 & 32998.3 & -32998.3 & -32998.3 & 152998. & 0 & 0 \\ 0 & 0 & -177490. & -76067.2 & -120000 & 0 & 297490. & 76067.2 \\ 0 & 0 & -76067.2 & -32600.2 & 0 & 0 & 76067.2 & 32600.2 \end{pmatrix}$$

$$\begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -150000. \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Essential boundary conditions

Node	dof	Value
1	u_1	0
	v_1	0
4	u_4	0
	v_4	0

Remove {1, 2, 7, 8} rows and columns.

After adjusting for essential boundary conditions we have

$$\begin{pmatrix} 243089. & 119136. & -32998.3 & 32998.3 \\ 119136. & 243089. & 32998.3 & -32998.3 \\ -32998.3 & 32998.3 & 152998. & -32998.3 \\ 32998.3 & -32998.3 & -32998.3 & 152998. \end{pmatrix} \begin{pmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -150000. \\ 0 \\ 0 \end{pmatrix}$$

Solving the final system of global equations we get

$\{u_2 = 0.538954,\ v_2 = -0.953061,\ u_3 = 0.264704,\ v_3 = -0.264704\}$

Complete table of nodal values

	u	v
1	0	0
2	0.538954	−0.953061
3	0.264704	−0.264704
4	0	0

Computation of reactions

Equation numbers of dof with specified values: {1, 2, 7, 8}

Extracting equations {1, 2, 7, 8} from the global system we have

$$\begin{pmatrix} 32600.2 & 76067.2 & -32600.2 & -76067.2 & 0 & 0 & 0 & 0 \\ 76067.2 & 297490. & -76067.2 & -177490. & 0 & -120000 & 0 & 0 \\ 0 & 0 & -177490. & -76067.2 & -120000 & 0 & 297490. & 76067.2 \\ 0 & 0 & -76067.2 & -32600.2 & 0 & 0 & 76067.2 & 32600.2 \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{pmatrix} = \begin{pmatrix} R_1 + 0. \\ R_2 + 0. \\ R_3 + 0. \\ R_4 + 0. \end{pmatrix}$$

Substituting the nodal values and re−arranging

$$\begin{pmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{pmatrix} = \begin{pmatrix} 32600.2 & 76067.2 & -32600.2 & -76067.2 & 0 & 0 & 0 & 0 \\ 76067.2 & 297490. & -76067.2 & -177490. & 0 & -120000 & 0 & 0 \\ 0 & 0 & -177490. & -76067.2 & -120000 & 0 & 297490. & 76067.2 \\ 0 & 0 & -76067.2 & -32600.2 & 0 & 0 & 76067.2 & 32600.2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0.538954 \\ -0.953061 \\ 0.264704 \\ -0.264704 \\ 0 \\ 0 \end{pmatrix}$$

Carrying out computations, the reactions are as follows.

Label	dof	Reaction
R_1	u_1	54926.7
R_2	v_1	159927.
R_3	u_4	−54926.7
R_4	v_4	−9926.67

Sum of Reactions

dof: u	0
dof: v	150000.

Solution for element 1

Nodal coordinates

Element node	Global node number	x	y
1	1	0	0
2	2	1500.	3500.

$x_1 = 0$ $\quad y_1 = 0$ $\quad x_2 = 1500.$ $\quad y_2 = 3500.$

$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 3807.89$

Direction cosines: $\ell_s = \dfrac{x_2 - x_1}{L} = 0.393919$ $\qquad m_s = \dfrac{y_2 - y_1}{L} = 0.919145$

Global to local transformation matrix, $\boldsymbol{T} = \begin{pmatrix} 0.393919 & 0.919145 & 0 & 0 \\ 0 & 0 & 0.393919 & 0.919145 \end{pmatrix}$

Element nodal displacements in global coordinates, $\boldsymbol{d} = \begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0.538954 \\ -0.953061 \end{pmatrix}$

Element nodal displacements in local coordinates, $\boldsymbol{d}_\ell = \boldsymbol{T}\,\boldsymbol{d} = \begin{pmatrix} 0. \\ -0.663697 \end{pmatrix}$

Axial displacements at element ends, $d_1 = 0.$ $\quad d_2 = -0.663697$

$E = 200000 \quad A = 4000$

Axial strain, $\epsilon = (d_2 - d_1)/L = -0.000174295$

Axial stress, $\sigma = E\epsilon = -34.8591$ $\quad$ Axial force $= \sigma A = -139436.$

Solution for element 2

Nodal coordinates

Element node	Global node number	x	y
1	2	1500.	3500.
2	4	5000	5000

$x_1 = 1500. \quad y_1 = 3500. \quad x_2 = 5000 \quad y_2 = 5000$

$$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 3807.89$$

Direction cosines: $\ell_s = \frac{x_2 - x_1}{L} = 0.919145 \quad m_s = \frac{y_2 - y_1}{L} = 0.393919$

Global to local transformation matrix, $\boldsymbol{T} = \begin{pmatrix} 0.919145 & 0.393919 & 0 & 0 \\ 0 & 0 & 0.919145 & 0.393919 \end{pmatrix}$

Element nodal displacements in global coordinates, $\boldsymbol{d} = \begin{pmatrix} u_2 \\ v_2 \\ u_4 \\ v_4 \end{pmatrix} = \begin{pmatrix} 0.538954 \\ -0.953061 \\ 0 \\ 0 \end{pmatrix}$

Element nodal displacements in local coordinates, $\boldsymbol{d}_\ell = \boldsymbol{T}\,\boldsymbol{d} = \begin{pmatrix} 0.119947 \\ 0. \end{pmatrix}$

Axial displacements at element ends, $d_1 = 0.119947$ $\quad d_2 = 0.$

$E = 200000 \quad A = 4000$

Axial strain, $\epsilon = (d_2 - d_1)/L = -0.0000314997$

Axial stress, $\sigma = E\epsilon = -6.29994$ $\quad$ Axial force $= \sigma A = -25199.8$

Solution for element 3

Nodal coordinates

Element node	Global node number	x	y
1	1	0	0
2	3	0	5000

$x_1 = 0 \quad y_1 = 0 \quad x_2 = 0 \quad y_2 = 5000$

$$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 5000$$

Direction cosines: $\ell_s = \frac{x_2 - x_1}{L} = 0 \quad m_s = \frac{y_2 - y_1}{L} = 1$

Global to local transformation matrix, $\boldsymbol{T} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Element nodal displacements in global coordinates, $\boldsymbol{d} = \begin{pmatrix} u_1 \\ v_1 \\ u_3 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0.264704 \\ -0.264704 \end{pmatrix}$

Element nodal displacements in local coordinates, $\boldsymbol{d}_\ell = \boldsymbol{T}\,\boldsymbol{d} = \begin{pmatrix} 0. \\ -0.264704 \end{pmatrix}$

Axial displacements at element ends, $d_1 = 0.$ $\quad d_2 = -0.264704$

$E = 200000 \quad A = 3000$

Axial strain, $\epsilon = (d_2 - d_1)/L = -0.0000529407$

Axial stress, $\sigma = E\epsilon = -10.5881$ $\quad$ Axial force $= \sigma A = -31764.4$

Solution for element 4

Nodal coordinates

Element node	Global node number	x	y
1	3	0	5000
2	4	5000	5000

$x_1 = 0 \quad y_1 = 5000 \quad x_2 = 5000 \quad y_2 = 5000$

$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 5000$

Direction cosines: $\ell_s = \dfrac{x_2 - x_1}{L} = 1 \quad m_s = \dfrac{y_2 - y_1}{L} = 0$

Global to local transformation matrix, $\boldsymbol{T} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

Element nodal displacements in global coordinates, $\boldsymbol{d} = \begin{pmatrix} u_3 \\ v_3 \\ u_4 \\ v_4 \end{pmatrix} = \begin{pmatrix} 0.264704 \\ -0.264704 \\ 0 \\ 0 \end{pmatrix}$

Element nodal displacements in local coordinates, $\boldsymbol{d}_\ell = \boldsymbol{T}\,\boldsymbol{d} = \begin{pmatrix} 0.264704 \\ 0. \end{pmatrix}$

Axial displacements at element ends, $d_1 = 0.264704$ $\quad d_2 = 0.$

$E = 200000 \quad A = 3000$

Axial strain, $\epsilon = (d_2 - d_1)/L = -0.0000529407$

Axial stress, $\sigma = E\epsilon = -10.5881$ $\quad$ Axial force $= \sigma A = -31764.4$

Solution for element 5

Nodal coordinates

Element node	Global node number	x	y
1	2	1500.	3500.
2	3	0	5000

$x_1 = 1500. \quad y_1 = 3500. \quad x_2 = 0 \quad y_2 = 5000$

$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 2121.32$

Direction cosines: $\ell_s = \frac{x_2 - x_1}{L} = -0.707107$ $m_s = \frac{y_2 - y_1}{L} = 0.707107$

Global to local transformation matrix, $\boldsymbol{T} = \begin{pmatrix} -0.707107 & 0.707107 & 0 & 0 \\ 0 & 0 & -0.707107 & 0.707107 \end{pmatrix}$

Element nodal displacements in global coordinates, $\boldsymbol{d} = \begin{pmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0.538954 \\ -0.953061 \\ 0.264704 \\ -0.264704 \end{pmatrix}$

Element nodal displacements in local coordinates, $\boldsymbol{d}_\ell = \boldsymbol{T}\,\boldsymbol{d} = \begin{pmatrix} -1.05501 \\ -0.374347 \end{pmatrix}$

Axial displacements at element ends, $d_1 = -1.05501$ $d_2 = -0.374347$

$E = 70000$ $A = 2000$

Axial strain, $\epsilon = (d_2 - d_1)/L = 0.000320869$

Axial stress, $\sigma = E\epsilon = 22.4608$ Axial force $= \sigma A = 44921.7$

Solution summary

Nodal solution

	x−coord	y−coord	u	v
1	0	0	0	0
2	1500.	3500.	0.538954	−0.953061
3	0	5000	0.264704	−0.264704
4	5000	5000	0	0

Element solution

	Stress	Axial force
1	−34.8591	−139436.
2	−6.29994	−25199.8
3	−10.5881	−31764.4
4	−10.5881	−31764.4
5	22.4608	44921.7

Support reactions

Node	dof	Reaction
1	u_1	54926.7
1	v_1	159927.
4	u_4	−54926.7
4	v_4	−9926.67

Sum of applied loads → (0 −150000.)

Sum of support reactions → (0 150000.)