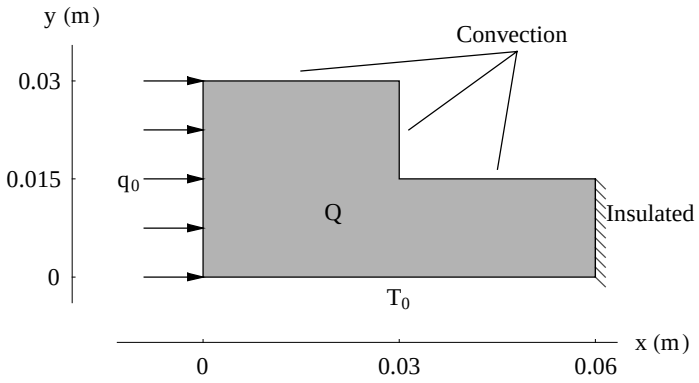
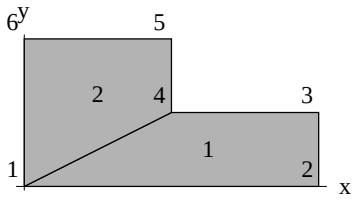
CHAPTER NINE

P—Formulation

Example 9.7: Heat flow in an L-shaped body (p. 624)

Consider two dimensional heat flow over an L-shaped body with thermal conductivity $k = 45\ W/m.°C$ shown in Figure. The bottom is maintained at $T_0 = 110\,°C$. Convection heat loss takes place on the top where the ambient air temperature is $20\,°C$ and the convection heat transfer coefficient is $h = 55\ W/m^2.°C$. The right side is insulated. The left side is subjected to heat flux at a uniform rate of $q_L = 8000\ W/m^2$. Heat is generated in the body at a rate of $Q = 5 \times 10^6\ W/m^3$. Determine temperature distribution in the body.





With $n = 2$ the finite element solution is as follows.

Interpolation functions for mapping: $\left\{\frac{1}{4}(1-s)(1-t),\ \frac{1}{4}(s+1)(1-t),\ \frac{1}{4}(s+1)(t+1),\ \frac{1}{4}(1-s)(t+1)\right\}$

Interpolation functions for assumed solution: $\boldsymbol{N}^{\mathrm{T}} = \left\{\frac{1}{4}(1-s)(1-t),\ \frac{1}{4}(s+1)(1-t),\ \frac{1}{4}(s+1)(t+1),\right.$

$\left.\frac{1}{4}(1-s)(t+1),\ \frac{\left(\frac{3s^2}{2}-\frac{3}{2}\right)(1-t)}{2\sqrt{6}},\ \frac{(s+1)\left(\frac{3t^2}{2}-\frac{3}{2}\right)}{2\sqrt{6}},\ \frac{\left(\frac{3s^2}{2}-\frac{3}{2}\right)(t+1)}{2\sqrt{6}},\ \frac{(1-s)\left(\frac{3t^2}{2}-\frac{3}{2}\right)}{2\sqrt{6}}\right\}$

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial s = \left\{\frac{t-1}{4},\ \frac{1-t}{4},\ \frac{t+1}{4},\ \frac{1}{4}(-t-1),\ \frac{1}{2}\sqrt{\frac{3}{2}}\,s(1-t),\ \frac{\frac{3t^2}{2}-\frac{3}{2}}{2\sqrt{6}},\ \frac{1}{2}\sqrt{\frac{3}{2}}\,s(t+1),\ -\frac{\frac{3t^2}{2}-\frac{3}{2}}{2\sqrt{6}}\right\}$

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial t = \left\{\frac{s-1}{4},\ \frac{1}{4}(-s-1),\ \frac{s+1}{4},\ \frac{1-s}{4},\ -\frac{\frac{3s^2}{2}-\frac{3}{2}}{2\sqrt{6}},\ \frac{1}{2}\sqrt{\frac{3}{2}}\,(s+1)\,t,\ \frac{\frac{3s^2}{2}-\frac{3}{2}}{2\sqrt{6}},\ \frac{1}{2}\sqrt{\frac{3}{2}}\,(1-s)\,t\right\}$

Use 2×2 Gauss quadrature for integration.

Global equations at start of the element assembly process

$$\begin{pmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0 \end{pmatrix} \begin{pmatrix} T_1\\ T_2\\ T_3\\ T_4\\ T_5\\ T_6\\ \delta_1^{\{1,2\}}\\ \delta_1^{\{1,4\}}\\ \delta_1^{\{1,6\}}\\ \delta_1^{\{2,3\}}\\ \delta_1^{\{3,4\}}\\ \delta_1^{\{4,5\}}\\ \delta_1^{\{5,6\}} \end{pmatrix} = \begin{pmatrix} 0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0 \end{pmatrix}$$

Equations for element 1

Element coordinates: ({0, 0}   {0.06, 0}   {0.06, 0.015}   {0.03, 0.015})

$x(s,t) = -0.0075\, t\, s + 0.0225\, s + 0.0075\, t + 0.0375$

$y(s,t) = 0.0075\, t + 0.0075$

$$\boldsymbol{J} = \begin{pmatrix} 0.0225 - 0.0075\, t & 0.0075 - 0.0075\, s \\ 0 & 0.0075 \end{pmatrix}$$

$\det J = 0.00016875 - 0.00005625\, t$

Given element data

$k_x = 45$ $k_y = 45$ $p = 0$ $q = 5000000$

Element data in mapped coordinates

$k_x = 45$ $k_y = 45$ $p = 0$ $q = 5000000$

Gauss point = $\{s \to -0.57735,\ t \to -0.57735\}$ Weight = 1.

$\boldsymbol{N}^T = \{0.622008, 0.166667, 0.0446582, 0.166667, -0.321975, -0.086273, -0.086273, -0.321975\}$

$\partial \boldsymbol{N}^T / \partial s =$

$$\begin{pmatrix} -0.394338 & 0.394338 & 0.105662 & -0.105662 & -0.557678 & -0.204124 & -0.149429 & 0.204124 \end{pmatrix}$$

$\partial \boldsymbol{N}^T / \partial t =$

$$\begin{pmatrix} -0.394338 & -0.105662 & 0.105662 & 0.394338 & 0.204124 & -0.149429 & -0.204124 & -0.557678 \end{pmatrix}$$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 37.2715 & 0. \\ -58.7903 & 133.333 \end{pmatrix} \qquad \mathrm{detJ} = 0.000201226$$

$$\boldsymbol{B}^{\mathrm{T}} = \begin{pmatrix} -14.6976 & 14.6976 & 3.9382 & -3.9382 & -20.7855 & -7.60802 & -5.56946 & 7.60802 \\ -29.3951 & -37.2715 & 7.8764 & 58.7903 & 60.0026 & -7.92339 & -18.4316 & -86.3575 \end{pmatrix}$$

$\mathrm{k_x} = 45. \qquad \mathrm{k_y} = 45. \qquad \mathrm{p} = 0. \qquad \mathrm{q} = 5. \times 10^{6}$

$\boldsymbol{k}_{\mathrm{k}} =$

$$\begin{pmatrix} 9.78042 & 7.96477 & -2.62065 & -15.1245 & -13.205 & 3.12157 & 5.64731 & 21.9739 \\ 7.96477 & 14.5352 & -2.13415 & -20.3658 & -23.0172 & 1.6616 & 5.47942 & 30.1582 \\ -2.62065 & -2.13415 & 0.702202 & 4.05261 & 3.53828 & -0.836423 & -1.51319 & -5.88789 \\ -15.1245 & -20.3658 & 4.05261 & 31.4378 & 32.6839 & -3.94675 & -9.61354 & -46.2442 \\ -13.205 & -23.0172 & 3.53828 & 32.6839 & 36.5136 & -2.87309 & -8.96622 & -48.3529 \\ 3.12157 & 1.6616 & -0.836423 & -3.94675 & -2.87309 & 1.09261 & 1.70611 & 5.67181 \\ 5.64731 & 5.47942 & -1.51319 & -9.61354 & -8.96622 & 1.70611 & 3.35713 & 14.0295 \\ 21.9739 & 30.1582 & -5.88789 & -46.2442 & -48.3529 & 5.67181 & 14.0295 & 68.0541 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\ 625.821 \quad 167.688 \quad 44.9319 \quad 167.688 \quad -323.949 \quad -86.8018 \quad -86.8018 \quad -323.949\ )$$

Gauss point = $\{\mathrm{s} \to -0.57735, \mathrm{t} \to 0.57735\}$ Weight = 1.

$$\boldsymbol{N}^{\mathrm{T}} = \{0.166667, 0.0446582, 0.166667, 0.622008, -0.086273, -0.086273, -0.321975, -0.321975\}$$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} =$

$$(\ -0.105662 \quad 0.105662 \quad 0.394338 \quad -0.394338 \quad -0.149429 \quad -0.204124 \quad -0.557678 \quad 0.204124\ )$$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} =$

$$(\ -0.394338 \quad -0.105662 \quad 0.105662 \quad 0.394338 \quad 0.204124 \quad 0.149429 \quad -0.204124 \quad 0.557678\ )$$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 55.0362 & 0. \\ -86.8113 & 133.333 \end{pmatrix} \qquad \mathrm{detJ} = 0.000136274$$

$$\boldsymbol{B}^{\mathrm{T}} = \begin{pmatrix} -5.81525 & 5.81525 & 21.7028 & -21.7028 & -8.22401 & -11.2342 & -30.6924 & 11.2342 \\ -43.4056 & -23.261 & -20.1446 & 86.8113 & 40.1887 & 37.6442 & 21.1962 & 56.6367 \end{pmatrix}$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^6$

$\boldsymbol{k}_k =$

$$\begin{pmatrix} 11.761 & 5.98419 & 4.58811 & -22.3333 & -10.4041 & -9.61942 & -4.54742 & -15.4761 \\ 5.98419 & 3.52543 & 3.64746 & -13.1571 & -6.02597 & -5.77035 & -4.11804 & -7.67828 \\ 4.58811 & 3.64746 & 5.37694 & -13.6125 & -6.05918 & -6.14547 & -6.70326 & -5.50139 \\ -22.3333 & -13.1571 & -13.6125 & 49.1029 & 22.4892 & 21.5352 & 15.3687 & 28.6557 \\ -10.4041 & -6.02597 & -6.05918 & 22.4892 & 10.3193 & 9.84399 & 6.77169 & 13.3916 \\ -9.61942 & -5.77035 & -6.14547 & 21.5352 & 9.84399 & 9.46398 & 7.00752 & 12.3005 \\ -4.54742 & -4.11804 & -6.70326 & 15.3687 & 6.77169 & 7.00752 & 8.53193 & 5.24729 \\ -15.4761 & -7.67828 & -5.50139 & 28.6557 & 13.3916 & 12.3005 & 5.24729 & 20.4447 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$\boldsymbol{r}_q^T = (\ 113.562 \quad 30.4288 \quad 113.562 \quad 423.818 \quad -58.7839 \quad -58.7839 \quad -219.384 \quad -219.384\ )$

Gauss point = $\{s \to 0.57735, t \to -0.57735\}$ Weight = 1.

$\boldsymbol{N}^T = \{0.166667, 0.622008, 0.166667, 0.0446582, -0.321975, -0.321975, -0.086273, -0.086273\}$

$\partial \boldsymbol{N}^T / \partial s =$

$(\ -0.394338 \quad 0.394338 \quad 0.105662 \quad -0.105662 \quad 0.557678 \quad -0.204124 \quad 0.149429 \quad 0.204124\ )$

$\partial \boldsymbol{N}^T / \partial t =$

$(\ -0.105662 \quad -0.394338 \quad 0.394338 \quad 0.105662 \quad 0.204124 \quad -0.557678 \quad -0.204124 \quad -0.149429\ )$

$\boldsymbol{J}^{-T} = \begin{pmatrix} 37.2715 & 0. \\ -15.7528 & 133.333 \end{pmatrix}$ detJ = 0.000201226

$\boldsymbol{B}^T = \begin{pmatrix} -14.6976 & 14.6976 & 3.9382 & -3.9382 & 20.7855 & -7.60802 & 5.56946 & 7.60802 \\ -7.8764 & -58.7903 & 50.9139 & 15.7528 & 18.4316 & -71.1415 & -29.5705 & -23.1394 \end{pmatrix}$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^6$

$$\boldsymbol{k}_k =$$

$$\begin{pmatrix} 2.51785 & 2.23696 & -4.15542 & -0.599393 & -4.0809 & 6.08651 & 1.3678 & 0.63781 \\ 2.23696 & 33.2534 & -26.5802 & -8.91023 & -7.04583 & 36.86 & 16.4832 & 13.3309 \\ -4.15542 & -26.5802 & 23.6134 & 7.12213 & 9.2388 & -33.0699 & -13.4344 & -10.3967 \\ -0.599393 & -8.91023 & 7.12213 & 2.38749 & 1.88792 & -9.87661 & -4.41667 & -3.57202 \\ -4.0809 & -7.04583 & 9.2388 & 1.88792 & 6.98841 & -13.3055 & -3.88708 & -2.43004 \\ 6.08651 & 36.86 & -33.0699 & -9.87661 & -13.3055 & 46.3533 & 18.6656 & 14.3822 \\ 1.3678 & 16.4832 & -13.4344 & -4.41667 & -3.88708 & 18.6656 & 8.19884 & 6.57963 \\ 0.63781 & 13.3309 & -10.3967 & -3.57202 & -2.43004 & 14.3822 & 6.57963 & 5.37257 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_q^T = (\ 167.688 \quad 625.821 \quad 167.688 \quad 44.9319 \quad -323.949 \quad -323.949 \quad -86.8018 \quad -86.8018\ )$$

Gauss point = {s → 0.57735, t → 0.57735} Weight = 1.

$$\boldsymbol{N}^T = \{0.0446582, 0.166667, 0.622008, 0.166667, -0.086273, -0.321975, -0.321975, -0.086273\}$$

$$\partial \boldsymbol{N}^T/\partial s =$$

$$(\ -0.105662 \quad 0.105662 \quad 0.394338 \quad -0.394338 \quad 0.149429 \quad -0.204124 \quad 0.557678 \quad 0.204124\ )$$

$$\partial \boldsymbol{N}^T/\partial t =$$

$$(\ -0.105662 \quad -0.394338 \quad 0.394338 \quad 0.105662 \quad 0.204124 \quad 0.557678 \quad -0.204124 \quad 0.149429\ )$$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 55.0362 & 0. \\ -23.261 & 133.333 \end{pmatrix} \qquad \det J = 0.000136274$$

$$\boldsymbol{B}^T = \begin{pmatrix} -5.81525 & 5.81525 & 21.7028 & -21.7028 & 8.22401 & -11.2342 & 30.6924 & 11.2342 \\ -11.6305 & -55.0362 & 43.4056 & 23.261 & 23.7407 & 79.1051 & -40.1887 & 15.1758 \end{pmatrix}$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^6$

$\boldsymbol{k}_k =$

$$\begin{pmatrix} 1.03689 & 3.71792 & -3.86973 & -0.88508 & -1.98651 & -5.24132 & 1.77182 & -1.48299 \\ 3.71792 & 18.7821 & -13.8755 & -8.62454 & -7.7192 & -27.0986 & 14.6582 & -4.7212 \\ -3.86973 & -13.8755 & 14.442 & 3.30316 & 7.41377 & 19.5609 & -6.61252 & 5.5346 \\ -0.88508 & -8.62454 & 3.30316 & 6.20646 & 2.29195 & 12.779 & -9.81751 & 0.669587 \\ -1.98651 & -7.7192 & 7.41377 & 2.29195 & 3.87106 & 10.95 & -4.30301 & 2.77594 \\ -5.24132 & -27.0986 & 19.5609 & 12.779 & 10.95 & 39.1478 & -21.61 & 6.5878 \\ 1.77182 & 14.6582 & -6.61252 & -9.81751 & -4.30301 & -21.61 & 15.6813 & -1.62561 \\ -1.48299 & -4.7212 & 5.5346 & 0.669587 & 2.77594 & 6.5878 & -1.62561 & 2.18625 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$\boldsymbol{r}_q^T = (\ 30.4288 \quad 113.562 \quad 423.818 \quad 113.562 \quad -58.7839 \quad -219.384 \quad -219.384 \quad -58.7839\ )$

Adding contributions from all Gauss points

$\boldsymbol{k}_k =$

$$\begin{pmatrix} 25.0962 & 19.9038 & -6.05769 & -38.9423 & -29.6765 & -5.65267 & 4.2395 & 5.65267 \\ 19.9038 & 70.0962 & -38.9423 & -51.0577 & -43.8082 & 5.65267 & 32.5028 & 31.0897 \\ -6.05769 & -38.9423 & 44.1346 & 0.865385 & 14.1317 & -20.4909 & -28.2633 & -16.2514 \\ -38.9423 & -51.0577 & 0.865385 & 89.1346 & 59.353 & 20.4909 & -8.479 & -20.4909 \\ -29.6765 & -43.8082 & 14.1317 & 59.353 & 57.6923 & 4.61538 & -10.3846 & -34.6154 \\ -5.65267 & 5.65267 & -20.4909 & 20.4909 & 4.61538 & 96.0577 & 5.76923 & 38.9423 \\ 4.2395 & 32.5028 & -28.2633 & -8.479 & -10.3846 & 5.76923 & 35.7692 & 24.2308 \\ 5.65267 & 31.0897 & -16.2514 & -20.4909 & -34.6154 & 38.9423 & 24.2308 & 96.0577 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\ 937.5 \quad 937.5 \quad 750. \quad 750. \quad -765.466 \quad -688.919 \quad -612.372 \quad -688.919\ )$$

Natural boundary conditions

Specified NBC values for side 3: $\alpha = -55$ $\beta = 1100$

Interpolation functions for mapping: $\left\{0,\ 0,\ \frac{1-\mathrm{a}}{2},\ \frac{\mathrm{a}+1}{2}\right\}$

Interpolation functions for solution: $\left\{0,\ 0,\ \frac{1-\mathrm{a}}{2},\ \frac{\mathrm{a}+1}{2},\ 0,\ 0,\ \frac{\frac{3\,\mathrm{a}^2}{2}-\frac{3}{2}}{\sqrt{6}},\ 0\right\}$

$x(\mathrm{a}) = 0.045 - 0.015\,\mathrm{a}$ $y(\mathrm{a}) = 0.015$

$\mathrm{J_c} = 0.015$

Value in mapped coordinate: $\alpha(\mathrm{a}) = -55$ $\beta(\mathrm{a}) = 1100$

Gauss point = -0.57735 Weight = 1.

$\boldsymbol{N}_{\mathrm{c}}^{\mathrm{T}} = \{0.,\ 0.,\ 0.788675,\ 0.211325,\ 0.,\ 0.,\ -0.408248,\ 0.\}$

$\mathrm{J_c} = 0.015$ $\alpha = -55.$ $\beta = 1100.$

$$\boldsymbol{k}_{\alpha} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & -0.513157 & -0.1375 & 0. & 0. & 0.26563 & 0. \\ 0. & 0. & -0.1375 & -0.036843 & 0. & 0. & 0.0711752 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0.26563 & 0.0711752 & 0. & 0. & -0.1375 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix} \qquad \boldsymbol{r}_{\beta} = \begin{pmatrix} 0. \\ 0. \\ -13.0131 \\ -3.48686 \\ 0. \\ 0. \\ 6.7361 \\ 0. \end{pmatrix}$$

Gauss point = 0.57735 Weight = 1.

$\boldsymbol{N}_{\mathrm{c}}^{\mathrm{T}} = \{0.,\ 0.,\ 0.211325,\ 0.788675,\ 0.,\ 0.,\ -0.408248,\ 0.\}$

$\mathrm{J_c} = 0.015$ $\alpha = -55.$ $\beta = 1100.$

$$\boldsymbol{k}_{\alpha} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & -0.036843 & -0.1375 & 0. & 0. & 0.0711752 & 0. \\ 0. & 0. & -0.1375 & -0.513157 & 0. & 0. & 0.26563 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0.0711752 & 0.26563 & 0. & 0. & -0.1375 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix} \qquad \boldsymbol{r}_{\beta} = \begin{pmatrix} 0. \\ 0. \\ -3.48686 \\ -13.0131 \\ 0. \\ 0. \\ 6.7361 \\ 0. \end{pmatrix}$$

Adding contributions from all Gauss points

$$\boldsymbol{k}_{\alpha} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & -0.55 & -0.275 & 0. & 0. & 0.336805 & 0. \\ 0. & 0. & -0.275 & -0.55 & 0. & 0. & 0.336805 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0.336805 & 0.336805 & 0. & 0. & -0.275 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix} \qquad \boldsymbol{r}_{\beta} = \begin{pmatrix} 0. \\ 0. \\ -16.5 \\ -16.5 \\ 0. \\ 0. \\ 13.4722 \\ 0. \end{pmatrix}$$

$$\begin{pmatrix} 25.0962 & 19.9038 & -6.05769 & -38.9423 & -29.6765 & -5.65267 & 4.2395 & 5.65267 \\ 19.9038 & 70.0962 & -38.9423 & -51.0577 & -43.8082 & 5.65267 & 32.5028 & 31.0897 \\ -6.05769 & -38.9423 & 43.5846 & 0.590385 & 14.1317 & -20.4909 & -27.9265 & -16.2514 \\ -38.9423 & -51.0577 & 0.590385 & 88.5846 & 59.353 & 20.4909 & -8.1422 & -20.4909 \\ -29.6765 & -43.8082 & 14.1317 & 59.353 & 57.6923 & 4.61538 & -10.3846 & -34.6154 \\ -5.65267 & 5.65267 & -20.4909 & 20.4909 & 4.61538 & 96.0577 & 5.76923 & 38.9423 \\ 4.2395 & 32.5028 & -27.9265 & -8.1422 & -10.3846 & 5.76923 & 35.4942 & 24.2308 \\ 5.65267 & 31.0897 & -16.2514 & -20.4909 & -34.6154 & 38.9423 & 24.2308 & 96.0577 \end{pmatrix}$$

$$\begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ \delta_1^{\{1,2\}} \\ \delta_1^{\{2,3\}} \\ \delta_1^{\{3,4\}} \\ \delta_1^{\{1,4\}} \end{pmatrix} = \begin{pmatrix} 937.5 \\ 937.5 \\ 733.5 \\ 733.5 \\ -765.466 \\ -688.919 \\ -598.9 \\ -688.919 \end{pmatrix}$$

Equations for element 2

Element coordinates: ({0, 0} {0.03, 0.015} {0.03, 0.03} {0, 0.03})

$x(s,t) = 0.015\,s + 0.015$

$y(s,t) = -0.00375\,t\,s + 0.00375\,s + 0.01125\,t + 0.01875$

$$\boldsymbol{J} = \begin{pmatrix} 0.015 & 0 \\ 0.00375 - 0.00375\,t & 0.01125 - 0.00375\,s \end{pmatrix}$$

$\det J = 0.00016875 - 0.00005625\,s$

Given element data

$k_x = 45 \qquad k_y = 45 \qquad p = 0 \qquad q = 5000000$

Element data in mapped coordinates

$k_x = 45 \qquad k_y = 45 \qquad p = 0 \qquad q = 5000000$

Gauss point = $\{s \to -0.57735,\ t \to -0.57735\}$ Weight = 1.

$$\boldsymbol{N}^T = \{0.622008,\ 0.166667,\ 0.0446582,\ 0.166667,\ -0.321975,\ -0.086273,\ -0.086273,\ -0.321975\}$$

$\partial \boldsymbol{N}^T/\partial s =$

$$(\ -0.394338 \quad 0.394338 \quad 0.105662 \quad -0.105662 \quad -0.557678 \quad -0.204124 \quad -0.149429 \quad 0.204124\ )$$

$\partial \boldsymbol{N}^T/\partial t =$

$$(\ -0.394338 \quad -0.105662 \quad 0.105662 \quad 0.394338 \quad 0.204124 \quad -0.149429 \quad -0.204124 \quad -0.557678\ )$$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 66.6667 & -29.3951 \\ 0. & 74.5431 \end{pmatrix} \qquad \det J = 0.000201226$$

$$\boldsymbol{B}^T = \begin{pmatrix} -14.6976 & 29.3951 & 3.9382 & -18.6358 & -43.1788 & -9.21578 & -3.96169 & 30.0013 \\ -29.3951 & -7.8764 & 7.8764 & 29.3951 & 15.216 & -11.1389 & -15.216 & -41.571 \end{pmatrix}$$

$k_x = 45. \qquad k_y = 45. \qquad p = 0. \qquad q = 5.\times 10^6$

$\boldsymbol{k}_k =$

$$\begin{pmatrix} 9.78042 & -1.81564 & -2.62065 & -5.34412 & 1.69644 & 4.19145 & 4.57743 & 7.07244 \\ -1.81564 & 8.3861 & 0.4865 & -7.05695 & -12.5785 & -1.65859 & 0.0307255 & 10.9506 \\ -2.62065 & 0.4865 & 0.702202 & 1.43195 & -0.45456 & -1.1231 & -1.22652 & -1.89505 \\ -5.34412 & -7.05695 & 1.43195 & 10.9691 & 11.3366 & -1.40977 & -3.38164 & -16.128 \\ 1.69644 & -12.5785 & -0.45456 & 11.3366 & 18.979 & 2.06853 & -0.547538 & -17.458 \\ 4.19145 & -1.65859 & -1.1231 & -1.40977 & 2.06853 & 1.89259 & 1.86537 & 1.68943 \\ 4.57743 & 0.0307255 & -1.22652 & -3.38164 & -0.547538 & 1.86537 & 2.23864 & 4.65155 \\ 7.07244 & 10.9506 & -1.89505 & -16.128 & -17.458 & 1.68943 & 4.65155 & 23.799 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_q^T = (\,625.821 \quad 167.688 \quad 44.9319 \quad 167.688 \quad -323.949 \quad -86.8018 \quad -86.8018 \quad -323.949\,)$$

Gauss point = $\{s \to -0.57735,\ t \to 0.57735\}$ Weight = 1.

$$\boldsymbol{N}^T = \{0.166667,\ 0.0446582,\ 0.166667,\ 0.622008,\ -0.086273,\ -0.086273,\ -0.321975,\ -0.321975\}$$

$$\partial \boldsymbol{N}^T/\partial s =$$
$$(\,-0.105662 \quad 0.105662 \quad 0.394338 \quad -0.394338 \quad -0.149429 \quad -0.204124 \quad -0.557678 \quad 0.204124\,)$$

$$\partial \boldsymbol{N}^T/\partial t =$$
$$(\,-0.394338 \quad -0.105662 \quad 0.105662 \quad 0.394338 \quad 0.204124 \quad 0.149429 \quad -0.204124 \quad 0.557678\,)$$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 66.6667 & -7.8764 \\ 0. & 74.5431 \end{pmatrix} \qquad \det J = 0.000201226$$

$$\boldsymbol{B}^T = \begin{pmatrix} -3.9382 & 7.8764 & 25.4569 & -29.3951 & -11.5697 & -14.7852 & -35.5707 & 9.21578 \\ -29.3951 & -7.8764 & 7.8764 & 29.3951 & 15.216 & 11.1389 & -15.216 & 41.571 \end{pmatrix}$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^6$

$$\boldsymbol{k}_k = \begin{pmatrix} 7.96477 & 1.81564 & -3.00435 & -6.77607 & -3.63758 & -2.43767 & 5.31866 & -11.3939 \\ 1.81564 & 1.12352 & 1.25388 & -4.19305 & -1.91042 & -1.84897 & -1.45174 & -2.30764 \\ -3.00435 & 1.25388 & 6.43001 & -4.67955 & -1.58177 & -2.6138 & -9.28489 & 5.08933 \\ -6.77607 & -4.19305 & -4.67955 & 15.6487 & 7.12977 & 6.90044 & 5.41797 & 8.61224 \\ -3.63758 & -1.91042 & -1.58177 & 7.12977 & 3.30863 & 3.08375 & 1.63007 & 4.76231 \\ -2.43767 & -1.84897 & -2.6138 & 6.90044 & 3.08375 & 3.10301 & 3.22755 & 2.95921 \\ 5.31866 & -1.45174 & -9.28489 & 5.41797 & 1.63007 & 3.22755 & 13.5538 & -8.6962 \\ -11.3939 & -2.30764 & 5.08933 & 8.61224 & 4.76231 & 2.95921 & -8.6962 & 16.4177 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_q^T = (\ 167.688 \quad 44.9319 \quad 167.688 \quad 625.821 \quad -86.8018 \quad -86.8018 \quad -323.949 \quad -323.949\ )$$

Gauss point = {s → 0.57735, t → −0.57735} Weight = 1.

$$\boldsymbol{N}^T = \{0.166667,\ 0.622008,\ 0.166667,\ 0.0446582,\ -0.321975,\ -0.321975,\ -0.086273,\ -0.086273\}$$

$$\partial \boldsymbol{N}^T/\partial s =$$

$$(\ -0.394338 \quad 0.394338 \quad 0.105662 \quad -0.105662 \quad 0.557678 \quad -0.204124 \quad 0.149429 \quad 0.204124\ )$$

$$\partial \boldsymbol{N}^T/\partial t =$$

$$(\ -0.105662 \quad -0.394338 \quad 0.394338 \quad 0.105662 \quad 0.204124 \quad -0.557678 \quad -0.204124 \quad -0.149429\ )$$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 66.6667 & -43.4056 \\ 0. & 110.072 \end{pmatrix} \qquad \det J = 0.000136274$$

$$\boldsymbol{B}^T = \begin{pmatrix} -21.7028 & 43.4056 & -10.0723 & -11.6305 & 28.3184 & 10.5981 & 18.8221 & 20.0943 \\ -11.6305 & -43.4056 & 43.4056 & 11.6305 & 22.4684 & -61.3849 & -22.4684 & -16.448 \end{pmatrix}$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^6$

$$\boldsymbol{k}_k =$$

$$\begin{pmatrix} 3.71792 & -2.68103 & -1.75527 & 0.718379 & -5.37136 & 2.96761 & -0.902516 & -1.50123 \\ -2.68103 & 23.1072 & -14.2347 & -6.19157 & 1.55713 & 19.1603 & 10.9906 & 9.72677 \\ -1.75527 & -14.2347 & 12.1758 & 3.81416 & 4.23146 & -16.9939 & -7.14318 & -5.61926 \\ 0.718379 & -6.19157 & 3.81416 & 1.65903 & -0.417231 & -5.13398 & -2.94493 & -2.60628 \\ -5.37136 & 1.55713 & 4.23146 & -0.417231 & 8.01348 & -6.6174 & 0.172815 & 1.22327 \\ 2.96761 & 19.1603 & -16.9939 & -5.13398 & -6.6174 & 23.796 & 9.6811 & 7.49752 \\ -0.902516 & 10.9906 & -7.14318 & -2.94493 & 0.172815 & 9.6811 & 5.26829 & 4.58563 \\ -1.50123 & 9.72677 & -5.61926 & -2.60628 & 1.22327 & 7.49752 & 4.58563 & 4.13516 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_q^T = (\ 113.562 \quad 423.818 \quad 113.562 \quad 30.4288 \quad -219.384 \quad -219.384 \quad -58.7839 \quad -58.7839\ )$$

Gauss point = {s → 0.57735, t → 0.57735} Weight = 1.

$$\boldsymbol{N}^T = \{0.0446582, 0.166667, 0.622008, 0.166667, -0.086273, -0.321975, -0.321975, -0.086273\}$$

$$\partial \boldsymbol{N}^T/\partial s =$$

$$(\ -0.105662 \quad 0.105662 \quad 0.394338 \quad -0.394338 \quad 0.149429 \quad -0.204124 \quad 0.557678 \quad 0.204124\ )$$

$$\partial \boldsymbol{N}^T/\partial t =$$

$$(\ -0.105662 \quad -0.394338 \quad 0.394338 \quad 0.105662 \quad 0.204124 \quad 0.557678 \quad -0.204124 \quad 0.149429\ )$$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 66.6667 & -11.6305 \\ 0. & 110.072 \end{pmatrix} \qquad \det J = 0.000136274$$

$$\boldsymbol{B}^T = \begin{pmatrix} -5.81525 & 11.6305 & 21.7028 & -27.5181 & 7.58788 & -20.0943 & 39.5526 & 11.8703 \\ -11.6305 & -43.4056 & 43.4056 & 11.6305 & 22.4684 & 61.3849 & -22.4684 & 16.448 \end{pmatrix}$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^6$

$$\boldsymbol{k}_k =$$

$$\begin{pmatrix} 1.03689 & 2.68103 & -3.86973 & 0.151811 & -1.87309 & -3.66151 & 0.192009 & -1.59642 \\ 2.68103 & 12.3831 & -10.0057 & -5.05843 & -5.43941 & -17.7725 & 8.80157 & -3.53148 \\ -3.86973 & -10.0057 & 14.442 & -0.566568 & 6.99046 & 13.665 & -0.716586 & 5.95791 \\ 0.151811 & -5.05843 & -0.566568 & 5.47319 & 0.322041 & 7.76902 & -8.27699 & -0.830011 \\ -1.87309 & -5.43941 & 6.99046 & 0.322041 & 3.44886 & 7.52282 & -1.25535 & 2.81861 \\ -3.66151 & -17.7725 & 13.665 & 7.76902 & 7.52282 & 25.5834 & -13.3317 & 4.72884 \\ 0.192009 & 8.80157 & -0.716586 & -8.27699 & -1.25535 & -13.3317 & 12.6892 & 0.612873 \\ -1.59642 & -3.53148 & 5.95791 & -0.830011 & 2.81861 & 4.72884 & 0.612873 & 2.5231 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\,30.4288 \quad 113.562 \quad 423.818 \quad 113.562 \quad -58.7839 \quad -219.384 \quad -219.384 \quad -58.7839\,)$$

Adding contributions from all Gauss points

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 22.5 & 1.9984\times10^{-15} & -11.25 & -11.25 & -9.18559 \\ 6.66134\times10^{-16} & 45. & -22.5 & -22.5 & -18.3712 \\ -11.25 & -22.5 & 33.75 & 1.55431\times10^{-15} & 9.18559 \\ -11.25 & -22.5 & 1.9984\times10^{-15} & 33.75 & 18.3712 \\ -9.18559 & -18.3712 & 9.18559 & 18.3712 & 33.75 \\ 1.05988 & -2.11975 & -7.06584 & 8.12571 & 6.05769 \\ 9.18559 & 18.3712 & -18.3712 & -9.18559 & -6.66134\times10^{-16} \\ -7.41913 & 14.8383 & 3.53292 & -10.952 & -8.65385 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\,937.5 \quad 750. \quad 750. \quad 937.5 \quad -688.919 \quad -612.372 \quad -688.919 \quad -765.466\,)$$

Natural boundary conditions

Specified NBC values for side 2: $\alpha = -55$ $\beta = 1100$

Interpolation functions for mapping: $\left\{0,\ \frac{1-\mathrm{a}}{2},\ \frac{\mathrm{a}+1}{2},\ 0\right\}$

Interpolation functions for solution: $\left\{0,\ \frac{1-\mathrm{a}}{2},\ \frac{\mathrm{a}+1}{2},\ 0,\ 0,\ \frac{\frac{3\,\mathrm{a}^2}{2}-\frac{3}{2}}{\sqrt{6}},\ 0,\ 0\right\}$

$x(\mathrm{a}) = 0.03$ $y(\mathrm{a}) = 0.0075\,\mathrm{a} + 0.0225$

$J_c = 0.0075$

Value in mapped coordinate: $\alpha(a) = -55$ $\beta(a) = 1100$

Gauss point = -0.57735 Weight = 1.

$\boldsymbol{N}_c^T = \{0., 0.788675, 0.211325, 0., 0., -0.408248, 0., 0.\}$

$J_c = 0.0075$ $\alpha = -55.$ $\beta = 1100.$

$$\boldsymbol{k}_\alpha = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & -0.256578 & -0.06875 & 0. & 0. & 0.132815 & 0. & 0. \\ 0. & -0.06875 & -0.0184215 & 0. & 0. & 0.0355876 & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0.132815 & 0.0355876 & 0. & 0. & -0.06875 & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix} \qquad \boldsymbol{r}_\beta = \begin{pmatrix} 0. \\ -6.50657 \\ -1.74343 \\ 0. \\ 0. \\ 3.36805 \\ 0. \\ 0. \end{pmatrix}$$

Gauss point = 0.57735 Weight = 1.

$\boldsymbol{N}_c^T = \{0., 0.211325, 0.788675, 0., 0., -0.408248, 0., 0.\}$

$J_c = 0.0075$ $\alpha = -55.$ $\beta = 1100.$

$$\boldsymbol{k}_\alpha = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & -0.0184215 & -0.06875 & 0. & 0. & 0.0355876 & 0. & 0. \\ 0. & -0.06875 & -0.256578 & 0. & 0. & 0.132815 & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0.0355876 & 0.132815 & 0. & 0. & -0.06875 & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix} \qquad \boldsymbol{r}_\beta = \begin{pmatrix} 0. \\ -1.74343 \\ -6.50657 \\ 0. \\ 0. \\ 3.36805 \\ 0. \\ 0. \end{pmatrix}$$

Adding contributions from all Gauss points

$$\boldsymbol{k}_\alpha = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & -0.275 & -0.1375 & 0. & 0. & 0.168402 & 0. & 0. \\ 0. & -0.1375 & -0.275 & 0. & 0. & 0.168402 & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0.168402 & 0.168402 & 0. & 0. & -0.1375 & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix} \qquad \boldsymbol{r}_\beta = \begin{pmatrix} 0. \\ -8.25 \\ -8.25 \\ 0. \\ 0. \\ 6.7361 \\ 0. \\ 0. \end{pmatrix}$$

Specified NBC values for side 3: $\alpha = -55$ $\beta = 1100$

Interpolation functions for mapping: $\left\{0,\ 0,\ \frac{1-a}{2},\ \frac{a+1}{2}\right\}$

Interpolation functions for solution: $\left\{0,\ 0,\ \frac{1-a}{2},\ \frac{a+1}{2},\ 0,\ 0,\ \frac{\frac{3\,a^2}{2}-\frac{3}{2}}{\sqrt{6}},\ 0\right\}$

$x(a) = 0.015 - 0.015\,a$ $\quad$ $y(a) = 0.03$

$J_c = 0.015$

Value in mapped coordinate: $\alpha(a) = -55$ $\quad$ $\beta(a) = 1100$

Gauss point = -0.57735 $\quad$ Weight = 1.

$\boldsymbol{N}_c^T = \{0.,\ 0.,\ 0.788675,\ 0.211325,\ 0.,\ 0.,\ -0.408248,\ 0.\}$

$J_c = 0.015$ $\quad$ $\alpha = -55.$ $\quad$ $\beta = 1100.$

$$\boldsymbol{k}_\alpha = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & -0.513157 & -0.1375 & 0. & 0. & 0.26563 & 0. \\ 0. & 0. & -0.1375 & -0.036843 & 0. & 0. & 0.0711752 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0.26563 & 0.0711752 & 0. & 0. & -0.1375 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix} \qquad \boldsymbol{r}_\beta = \begin{pmatrix} 0. \\ 0. \\ -13.0131 \\ -3.48686 \\ 0. \\ 0. \\ 6.7361 \\ 0. \end{pmatrix}$$

Gauss point = 0.57735 $\quad$ Weight = 1.

$\boldsymbol{N}_c^T = \{0.,\ 0.,\ 0.211325,\ 0.788675,\ 0.,\ 0.,\ -0.408248,\ 0.\}$

$J_c = 0.015$ $\quad$ $\alpha = -55.$ $\quad$ $\beta = 1100.$

$$\boldsymbol{k}_\alpha = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & -0.036843 & -0.1375 & 0. & 0. & 0.0711752 & 0. \\ 0. & 0. & -0.1375 & -0.513157 & 0. & 0. & 0.26563 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0.0711752 & 0.26563 & 0. & 0. & -0.1375 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix} \qquad \boldsymbol{r}_\beta = \begin{pmatrix} 0. \\ 0. \\ -3.48686 \\ -13.0131 \\ 0. \\ 0. \\ 6.7361 \\ 0. \end{pmatrix}$$

Adding contributions from all Gauss points

$$\boldsymbol{k}_{\alpha} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & -0.55 & -0.275 & 0. & 0. & 0.336805 & 0. \\ 0. & 0. & -0.275 & -0.55 & 0. & 0. & 0.336805 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0.336805 & 0.336805 & 0. & 0. & -0.275 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix} \qquad \boldsymbol{r}_{\beta} = \begin{pmatrix} 0. \\ 0. \\ -16.5 \\ -16.5 \\ 0. \\ 0. \\ 13.4722 \\ 0. \end{pmatrix}$$

Specified NBC values for side 4: $\alpha = 0$ $\beta = 8000$

Interpolation functions for mapping: $\left\{\frac{a+1}{2}, 0, 0, \frac{1-a}{2}\right\}$

Interpolation functions for solution: $\left\{\frac{a+1}{2}, 0, 0, \frac{1-a}{2}, 0, 0, 0, \frac{\frac{3a^2}{2} - \frac{3}{2}}{\sqrt{6}}\right\}$

$x(a) = 0$ $y(a) = 0.015 - 0.015\,a$

$J_c = 0.015$

Value in mapped coordinate: $\alpha(a) = 0$ $\beta(a) = 8000$

Gauss point = −0.57735 Weight = 1.

$\boldsymbol{N}_c^T = \{0.211325, 0., 0., 0.788675, 0., 0., 0., -0.408248\}$

$J_c = 0.015$ $\alpha = 0.$ $\beta = 8000.$

$$\boldsymbol{k}_{\alpha} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix} \qquad \boldsymbol{r}_{\beta} = \begin{pmatrix} -25.359 \\ 0. \\ 0. \\ -94.641 \\ 0. \\ 0. \\ 0. \\ 48.9898 \end{pmatrix}$$

Gauss point = 0.57735 Weight = 1.

$\boldsymbol{N}_c^T = \{0.788675, 0., 0., 0.211325, 0., 0., 0., -0.408248\}$

$J_c = 0.015$ $\alpha = 0.$ $\beta = 8000.$

$$\boldsymbol{k}_\alpha = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix} \qquad \boldsymbol{r}_\beta = \begin{pmatrix} -94.641 \\ 0. \\ 0. \\ -25.359 \\ 0. \\ 0. \\ 0. \\ 48.9898 \end{pmatrix}$$

Adding contributions from all Gauss points

$$\boldsymbol{k}_\alpha = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix} \qquad \boldsymbol{r}_\beta = \begin{pmatrix} -120. \\ 0. \\ 0. \\ -120. \\ 0. \\ 0. \\ 0. \\ 97.9796 \end{pmatrix}$$

$$\begin{pmatrix} 22.5 & 0 & -11.25 & -11.25 & -9.18559 & 1.05988 & 9.18559 & -7.41913 \\ 0 & 44.725 & -22.6375 & -22.5 & -18.3712 & -1.95135 & 18.3712 & 14.8383 \\ -11.25 & -22.6375 & 32.925 & -0.275 & 9.18559 & -6.89743 & -18.0344 & 3.53292 \\ -11.25 & -22.5 & -0.275 & 33.2 & 18.3712 & 8.12571 & -8.84878 & -10.952 \\ -9.18559 & -18.3712 & 9.18559 & 18.3712 & 33.75 & 6.05769 & 0 & -8.65385 \\ 1.05988 & -1.95135 & -6.89743 & 8.12571 & 6.05769 & 54.2375 & 1.44231 & 16.875 \\ 9.18559 & 18.3712 & -18.0344 & -8.84878 & 0 & 1.44231 & 33.475 & 1.15385 \\ -7.41913 & 14.8383 & 3.53292 & -10.952 & -8.65385 & 16.875 & 1.15385 & 46.875 \end{pmatrix}$$

$$\begin{pmatrix} \mathrm{T}_1 \\ \mathrm{T}_4 \\ \mathrm{T}_5 \\ \mathrm{T}_6 \\ \delta_1^{\{1,4\}} \\ \delta_1^{\{4,5\}} \\ \delta_1^{\{5,6\}} \\ \delta_1^{\{1,6\}} \end{pmatrix} = \begin{pmatrix} 817.5 \\ 741.75 \\ 725.25 \\ 801. \\ -688.919 \\ -605.636 \\ -675.447 \\ -667.486 \end{pmatrix}$$

Global equations after assembling all elements

$$\begin{pmatrix}
47.5962 & 19.9038 & -6.05769 & -38.9423 & -11.25 & -11.25 & -29.6765 & -3.53292 & -7.419 \\
19.9038 & 70.0962 & -38.9423 & -51.0577 & 0 & 0 & -43.8082 & 31.0897 & 0 \\
-6.05769 & -38.9423 & 43.5846 & 0.590385 & 0 & 0 & 14.1317 & -16.2514 & 0 \\
-38.9423 & -51.0577 & 0.590385 & 133.31 & -22.6375 & -22.5 & 59.353 & -38.8621 & 14.838 \\
-11.25 & 0 & 0 & -22.6375 & 32.925 & -0.275 & 0 & 9.18559 & 3.532 \\
-11.25 & 0 & 0 & -22.5 & -0.275 & 33.2 & 0 & 18.3712 & -10.952 \\
-29.6765 & -43.8082 & 14.1317 & 59.353 & 0 & 0 & 57.6923 & -34.6154 & 0 \\
-3.53292 & 31.0897 & -16.2514 & -38.8621 & 9.18559 & 18.3712 & -34.6154 & 129.808 & -8.653 \\
-7.41913 & 0 & 0 & 14.8383 & 3.53292 & -10.952 & 0 & -8.65385 & 46.875 \\
-5.65267 & 5.65267 & -20.4909 & 20.4909 & 0 & 0 & 4.61538 & 38.9423 & 0 \\
4.2395 & 32.5028 & -27.9265 & -8.1422 & 0 & 0 & -10.3846 & 24.2308 & 0 \\
1.05988 & 0 & 0 & -1.95135 & -6.89743 & 8.12571 & 0 & 6.05769 & 16.875 \\
9.18559 & 0 & 0 & 18.3712 & -18.0344 & -8.84878 & 0 & 0 & 1.153
\end{pmatrix}$$

Essential boundary conditions

On element 1, side 1, specified value = 110

$$\left\{T_1,\ T_2,\ \delta_1^{\{1,2\}}\right\} = \{110,\ 110,\ 0\}$$

Known values from EBC

$$\left\{T_1 = 110,\ T_2 = 110,\ \delta_1^{\{1,2\}} = 0\right\}$$

Global equations after EBC

$$\begin{pmatrix}
43.5846 & 0.590385 & 0 & 0 & -16.2514 & 0 & -20.4909 & -27.9265 & 0 \\
0.590385 & 133.31 & -22.6375 & -22.5 & -38.8621 & 14.8383 & 20.4909 & -8.1422 & -1.951 \\
0 & -22.6375 & 32.925 & -0.275 & 9.18559 & 3.53292 & 0 & 0 & -6.897 \\
0 & -22.5 & -0.275 & 33.2 & 18.3712 & -10.952 & 0 & 0 & 8.125 \\
-16.2514 & -38.8621 & 9.18559 & 18.3712 & 129.808 & -8.65385 & 38.9423 & 24.2308 & 6.057 \\
0 & 14.8383 & 3.53292 & -10.952 & -8.65385 & 46.875 & 0 & 0 & 16.875 \\
-20.4909 & 20.4909 & 0 & 0 & 38.9423 & 0 & 96.0577 & 5.76923 & 0 \\
-27.9265 & -8.1422 & 0 & 0 & 24.2308 & 0 & 5.76923 & 35.4942 & 0 \\
0 & -1.95135 & -6.89743 & 8.12571 & 6.05769 & 16.875 & 0 & 0 & 54.237 \\
0 & 18.3712 & -18.0344 & -8.84878 & 0 & 1.15385 & 0 & 0 & 1.442
\end{pmatrix}$$

Solving the final system of global equations we get

$\{T_3 = 129.195,\ T_4 = 138.48,\ T_5 = 162.431,\ T_6 = 160.634,\ \delta_1^{\{1,4\}} = -12.0338,$
$\delta_1^{\{1,6\}} = -15.6763,\ \delta_1^{\{2,3\}} = -4.97674,\ \delta_1^{\{3,4\}} = 11.6987,\ \delta_1^{\{4,5\}} = -5.63838,\ \delta_1^{\{5,6\}} = 4.39332\}$

Solution for element 1

DOF values for the element

$\{T_1 = 110,\ T_2 = 110,\ T_3 = 129.195,\ T_4 = 138.48,\ \delta_1^{\{1,2\}} = 0,\ \delta_1^{\{2,3\}} = -4.97674,\ \delta_1^{\{3,4\}} = 11.6987,\ \delta_1^{\{1,4\}} = -12.0338\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 110\quad 110\quad 129.195\quad 138.48\quad 0\quad -4.97674\quad 11.6987\quad -12.0338\ )$

Mapping

$x(s,t) = -0.0075\,t\,s + 0.0225\,s + 0.0075\,t + 0.0375$

$y(s,t) = 0.0075\,t + 0.0075$

$$\boldsymbol{J} = \begin{pmatrix} 0.0225 - 0.0075\,t & 0.0075 - 0.0075\,s \\ 0 & 0.0075 \end{pmatrix}$$

Element solution at $\{s \to 0,\ t \to 0\}$

Location: {0.0375, 0.0075}

$\boldsymbol{N}^{\mathrm{T}} = (\ 0.25\quad 0.25\quad 0.25\quad 0.25\quad -0.306186\quad -0.306186\quad -0.306186\quad -0.306186\ )$

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial s = (\ -0.25\quad 0.25\quad 0.25\quad -0.25\quad 0.\quad -0.306186\quad 0.\quad 0.306186\ )$

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial t = (\ -0.25\quad -0.25\quad 0.25\quad 0.25\quad 0.306186\quad 0.\quad -0.306186\quad 0.\ )$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 44.4444 & 0. \\ -44.4444 & 133.333 \end{pmatrix}$$

$\boldsymbol{B}_x^{\mathrm{T}} = \partial\boldsymbol{N}^{\mathrm{T}}/\partial x = (\ -11.1111\quad 11.1111\quad 11.1111\quad -11.1111\quad 0.\quad -13.6083\quad 0.\quad 13.6083\ )$

$\boldsymbol{B}_y^{\mathrm{T}} = \partial\boldsymbol{N}^{\mathrm{T}}/\partial y = (\ -22.2222\quad -44.4444\quad 22.2222\quad 44.4444\quad 40.8248\quad 13.6083\quad -40.8248\quad -13.6083\ )$

$T = \boldsymbol{N}^{\mathrm{T}}\boldsymbol{d} = 123.545$

$\partial T/\partial x = \boldsymbol{B}_x^{\mathrm{T}}\boldsymbol{d} = -199.205$

$\partial T/\partial y = \boldsymbol{B}_y^{\mathrm{T}}\boldsymbol{d} = 1310.77$

	x	y	T	$\partial T/\partial x$	$\partial T/\partial y$
1	0.0375	0.0075	123.545	−199.205	1310.77

Solution for element 2

DOF values for the element

$\{T_1 = 110,\ T_4 = 138.48,\ T_5 = 162.431,\ T_6 = 160.634,$
$\delta_1^{\{1,4\}} = -12.0338,\ \delta_1^{\{4,5\}} = -5.63838,\ \delta_1^{\{5,6\}} = 4.39332,\ \delta_1^{\{1,6\}} = -15.6763\}$

$\boldsymbol{d}^T = (\ 110 \quad 138.48 \quad 162.431 \quad 160.634 \quad -12.0338 \quad -5.63838 \quad 4.39332 \quad -15.6763\ )$

Mapping

$x(s,t) = 0.015\,s + 0.015$

$y(s,t) = -0.00375\,t\,s + 0.00375\,s + 0.01125\,t + 0.01875$

$$\boldsymbol{J} = \begin{pmatrix} 0.015 & 0 \\ 0.00375 - 0.00375\,t & 0.01125 - 0.00375\,s \end{pmatrix}$$

Element solution at $\{s \to 0,\ t \to 0\}$

Location: {0.015, 0.01875}

$\boldsymbol{N}^T = (\ 0.25 \quad 0.25 \quad 0.25 \quad 0.25 \quad -0.306186 \quad -0.306186 \quad -0.306186 \quad -0.306186\ )$

$\partial \boldsymbol{N}^T/\partial s = (\ -0.25 \quad 0.25 \quad 0.25 \quad -0.25 \quad 0. \quad -0.306186 \quad 0. \quad 0.306186\ )$

$\partial \boldsymbol{N}^T/\partial t = (\ -0.25 \quad -0.25 \quad 0.25 \quad 0.25 \quad 0.306186 \quad 0. \quad -0.306186 \quad 0.\ )$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 66.6667 & -22.2222 \\ 0. & 88.8889 \end{pmatrix}$$

$\boldsymbol{B}_x^T = \partial \boldsymbol{N}^T/\partial x = (\ -11.1111 \quad 22.2222 \quad 11.1111 \quad -22.2222 \quad -6.80414 \quad -20.4124 \quad 6.80414 \quad 20.4124\ )$

$\boldsymbol{B}_y^T = \partial \boldsymbol{N}^T/\partial y = (\ -22.2222 \quad -22.2222 \quad 22.2222 \quad 22.2222 \quad 27.2166 \quad 0. \quad -27.2166 \quad 0.\ )$

$T = \boldsymbol{N}^T\boldsymbol{d} = 151.752$

$\partial T/\partial x = \boldsymbol{B}_x^T\boldsymbol{d} = -2.87239$

$\partial T/\partial y = \boldsymbol{B}_y^T\boldsymbol{d} = 1210.35$

	x	y	T	$\partial T/\partial x$	$\partial T/\partial y$
1	0.015	0.01875	151.752	−2.87239	1210.35

Nodal solution summary

dof	x	y	Value
T_1	0	0	110
T_2	0.06	0	110
T_3	0.06	0.015	129.195
T_4	0.03	0.015	138.48
T_5	0.03	0.03	162.431
T_6	0	0.03	160.634

Element solution summary

	x	y	T	$\partial T/\partial x$	$\partial T/\partial y$
1	0.0375	0.0075	123.545	−199.205	1310.77
2	0.015	0.01875	151.752	−2.87239	1210.35

With $n = 3$ the following finite element solution is obtained.

Interpolation functions for mapping: $\left\{\frac{1}{4}(1-s)(1-t),\ \frac{1}{4}(s+1)(1-t),\ \frac{1}{4}(s+1)(t+1),\ \frac{1}{4}(1-s)(t+1)\right\}$

Interpolation functions for assumed solution: $\boldsymbol{N}^{\mathrm{T}} = \left\{\frac{1}{4}(1-s)(1-t),\ \frac{1}{4}(s+1)(1-t),\ \frac{1}{4}(s+1)(t+1),\right.$

$$\frac{1}{4}(1-s)(t+1),\ \frac{\left(\frac{3s^2}{2}-\frac{3}{2}\right)(1-t)}{2\sqrt{6}},\ \frac{(s+1)\left(\frac{3t^2}{2}-\frac{3}{2}\right)}{2\sqrt{6}},\ \frac{\left(\frac{3s^2}{2}-\frac{3}{2}\right)(t+1)}{2\sqrt{6}},\ \frac{(1-s)\left(\frac{3t^2}{2}-\frac{3}{2}\right)}{2\sqrt{6}},$$

$$\left.\frac{\left(\frac{5s^3}{2}-\frac{5s}{2}\right)(1-t)}{2\sqrt{10}},\ \frac{(s+1)\left(\frac{5t^3}{2}-\frac{5t}{2}\right)}{2\sqrt{10}},\ \frac{\left(\frac{5s^3}{2}-\frac{5s}{2}\right)(t+1)}{2\sqrt{10}},\ \frac{(1-s)\left(\frac{5t^3}{2}-\frac{5t}{2}\right)}{2\sqrt{10}}\right\}$$

$$\partial\boldsymbol{N}^{\mathrm{T}}/\partial s = \left\{\frac{t-1}{4},\ \frac{1-t}{4},\ \frac{t+1}{4},\ \frac{1}{4}(-t-1),\ \frac{1}{2}\sqrt{\frac{3}{2}}\,s(1-t),\ \frac{\frac{3t^2}{2}-\frac{3}{2}}{2\sqrt{6}},\ \frac{1}{2}\sqrt{\frac{3}{2}}\,s(t+1),\right.$$

$$\left.-\frac{\frac{3t^2}{2}-\frac{3}{2}}{2\sqrt{6}},\ \frac{\left(\frac{15s^2}{2}-\frac{5}{2}\right)(1-t)}{2\sqrt{10}},\ \frac{\frac{5t^3}{2}-\frac{5t}{2}}{2\sqrt{10}},\ \frac{\left(\frac{15s^2}{2}-\frac{5}{2}\right)(t+1)}{2\sqrt{10}},\ -\frac{\frac{5t^3}{2}-\frac{5t}{2}}{2\sqrt{10}}\right\}$$

$$\partial\boldsymbol{N}^{\mathrm{T}}/\partial t = \left\{\frac{s-1}{4},\ \frac{1}{4}(-s-1),\ \frac{s+1}{4},\ \frac{1-s}{4},\ -\frac{\frac{3s^2}{2}-\frac{3}{2}}{2\sqrt{6}},\ \frac{1}{2}\sqrt{\frac{3}{2}}\,(s+1)\,t,\ \frac{\frac{3s^2}{2}-\frac{3}{2}}{2\sqrt{6}},\right.$$

$$\left.\frac{1}{2}\sqrt{\frac{3}{2}}\,(1-s)\,t,\ -\frac{\frac{5s^3}{2}-\frac{5s}{2}}{2\sqrt{10}},\ \frac{(s+1)\left(\frac{15t^2}{2}-\frac{5}{2}\right)}{2\sqrt{10}},\ \frac{\frac{5s^3}{2}-\frac{5s}{2}}{2\sqrt{10}},\ \frac{(1-s)\left(\frac{15t^2}{2}-\frac{5}{2}\right)}{2\sqrt{10}}\right\}$$

Use 3×3 Gauss quadrature for integration.

Global equations at start of the element assembly process

$$\begin{pmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ \delta_1^{\{1,2\}} \\ \delta_1^{\{1,4\}} \\ \delta_1^{\{1,6\}} \\ \delta_1^{\{2,3\}} \\ \delta_1^{\{3,4\}} \\ \delta_1^{\{4,5\}} \\ \delta_1^{\{5,6\}} \\ \delta_2^{\{1,2\}} \\ \delta_2^{\{1,4\}} \\ \delta_2^{\{1,6\}} \\ \delta_2^{\{2,3\}} \\ \delta_2^{\{3,4\}} \\ \delta_2^{\{4,5\}} \\ \delta_2^{\{5,6\}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 1

Element coordinates: ({0, 0}   {0.06, 0}   {0.06, 0.015}   {0.03, 0.015})

$x(s,t) = -0.0075\,t\,s + 0.0225\,s + 0.0075\,t + 0.0375$

$y(s,t) = 0.0075\,t + 0.0075$

$$\boldsymbol{J} = \begin{pmatrix} 0.0225 - 0.0075\,t & 0.0075 - 0.0075\,s \\ 0 & 0.0075 \end{pmatrix}$$

$\det J = 0.00016875 - 0.00005625\,t$

Given element data

$k_x = 45$ $k_y = 45$ $p = 0$ $q = 5000000$

Element data in mapped coordinates

$k_x = 45$ $k_y = 45$ $p = 0$ $q = 5000000$

Gauss point = $\{s \to -0.774597,\ t \to -0.774597\}$ Weight = 0.308642

$$\boldsymbol{N}^{\mathrm{T}} = \{0.787298,\ 0.1,\ 0.0127017,\ 0.1,\ -0.217343,\ -0.0276062,\ -0.0276062,\ -0.217343,\ 0.217343,\ 0.0276062,\ 0.0276062,\ 0.217343\}$$

$$\partial\boldsymbol{N}^{\mathrm{T}}/\partial s = \left(\begin{matrix} -0.443649 & 0.443649 & 0.0563508 & -0.0563508 & -0.841765 & -0.122474 & -0.106918 & 0.12 \end{matrix}\right.$$

$$\partial\boldsymbol{N}^{\mathrm{T}}/\partial t = \left(\begin{matrix} -0.443649 & -0.0563508 & 0.0563508 & 0.443649 & 0.122474 & -0.106918 & -0.122474 & -0.841 \end{matrix}\right.$$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 35.3239 & 0. \\ -62.6856 & 133.333 \end{pmatrix} \qquad \det J = 0.000212321$$

$$\boldsymbol{B}^{\mathrm{T}} = \left(\begin{matrix} -15.6714 & 15.6714 & 1.99053 & -1.99053 & -29.7344 & -4.32627 & -3.77676 & 4.32627 & 19 \\ -31.3428 & -35.3239 & 3.98106 & 62.6856 & 69.0965 & -6.57837 & -9.6277 & -119.913 & -51 \end{matrix}\right.$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5.\times 10^6$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{matrix} 3.62115 & 2.54065 & -0.459946 & -5.70185 & -5.01225 & 0.80795 & 1.0644 \\ 2.54065 & 4.4038 & -0.322704 & -6.62174 & -8.57168 & 0.485315 & 0.828348 \\ -0.459946 & -0.322704 & 0.0584208 & 0.72423 & 0.636639 & -0.102623 & -0.135196 \\ -5.70185 & -6.62174 & 0.72423 & 11.5994 & 12.9473 & -1.19064 & -1.75755 \\ -5.01225 & -8.57168 & 0.636639 & 12.9473 & 16.6863 & -0.961057 & -1.63057 \\ 0.80795 & 0.485315 & -0.102623 & -1.19064 & -0.961057 & 0.182807 & 0.234951 \\ 1.0644 & 0.828348 & -0.135196 & -1.75755 & -1.63057 & 0.234951 & 0.315405 \\ 10.8832 & 12.6908 & -1.38235 & -22.1917 & -24.8126 & 2.27099 & 3.35628 \\ 3.84461 & 6.28146 & -0.488329 & -9.63774 & -12.2333 & 0.7463 & 1.24159 \\ -0.368745 & 0.00967665 & 0.0468368 & 0.312232 & -0.00718859 & -0.0906248 & -0.100038 \\ -1.21271 & -1.11924 & 0.154034 & 2.17792 & 2.19617 & -0.262228 & -0.364812 \\ -7.42538 & -8.79378 & 0.943146 & 15.276 & 17.1896 & -1.54524 & -2.29411 \end{matrix}\right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = \left(\begin{matrix} 257.963 & 32.7656 & 4.16178 & 32.7656 & -71.2137 & -9.04532 & -9.04532 & -71.2137 & 71.2137 & 9. \end{matrix}\right.$$

Gauss point = $\{s \to -0.774597,\ t \to 0.\}$ Weight = 0.493827

$\boldsymbol{N}^{\mathrm{T}} = \{0.443649,\ 0.0563508,\ 0.0563508,\ 0.443649,$
$-0.122474,\ -0.0690154,\ -0.122474,\ -0.543357,\ 0.122474,\ 0.,\ 0.122474,\ 0.\}$

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial s =$
$\begin{pmatrix} -0.25 & 0.25 & 0.25 & -0.25 & -0.474342 & -0.306186 & -0.474342 & 0.306186 & 0.316228 & 0. & 0.316228 & 0. \end{pmatrix}$

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial t = \left(\begin{matrix} -0.443649 & -0.0563508 & 0.0563508 & 0.443649 & 0.122474 & 0. & -0.122474 & 0. & -0.122474 \end{matrix} \right.$

$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 44.4444 & 0. \\ -78.871 & 133.333 \end{pmatrix}$ detJ = 0.00016875

$\boldsymbol{B}^{\mathrm{T}} = \left(\begin{matrix} -11.1111 & 11.1111 & 11.1111 & -11.1111 & -21.0819 & -13.6083 & -21.0819 & 13.6083 & 14. \\ -39.4355 & -27.2312 & -12.2043 & 78.871 & 53.7417 & 24.1492 & 21.0819 & -24.1492 & -41 \end{matrix} \right.$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^6$

$$\boldsymbol{k}_k = \left(\begin{matrix}
6.2948 & 3.56407 & 1.34185 & -11.2007 & -7.06908 & -3.00425 & -2.23924 & 3.00425 \\
3.56407 & 3.24373 & 1.70923 & -8.51702 & -6.36635 & -3.03305 & -3.03122 & 3.03305 \\
1.34185 & 1.70923 & 1.02151 & -4.07258 & -3.33796 & -1.67223 & -1.84324 & 1.67223 \\
-11.2007 & -8.51702 & -4.07258 & 23.7903 & 16.7734 & 7.70953 & 7.11371 & -7.70953 \\
-7.06908 & -6.36635 & -3.33796 & 16.7734 & 12.4973 & 5.94265 & 5.91532 & -5.94265 \\
-3.00425 & -3.03305 & -1.67223 & 7.70953 & 5.94265 & 2.88138 & 2.98499 & -2.88138 \\
-2.23924 & -3.03122 & -1.84324 & 7.11371 & 5.91532 & 2.98499 & 3.33333 & -2.98499 \\
3.00425 & 3.03305 & 1.67223 & -7.70953 & -5.94265 & -2.88138 & -2.98499 & 2.88138 \\
5.51769 & 4.80009 & 2.47443 & -12.7922 & -9.42854 & -4.45471 & -4.37388 & 4.45471 \\
1.75682 & 1.21313 & 0.543692 & -3.51364 & -2.39415 & -1.07583 & -0.939181 & 1.07583 \\
0.687852 & 1.46496 & 0.979711 & -3.13252 & -2.84655 & -1.49705 & -1.79189 & 1.49705 \\
13.8314 & 9.55094 & 4.28048 & -27.6628 & -18.8491 & -8.46998 & -7.39415 & 8.46998
\end{matrix} \right.$$

$$\boldsymbol{k}_p = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\ 184.854 \quad 23.4795 \quad 23.4795 \quad 184.854 \quad -51.031 \quad -28.7564 \quad -51.031 \quad -226.399 \quad 51.031 \quad 0. \quad 51$$

Gauss point = $\{s \to -0.774597,\ t \to 0.774597\}$ Weight = 0.308642

$$\boldsymbol{N}^{\mathrm{T}} = \{0.1,\ 0.0127017,\ 0.1,\ 0.787298,\ -0.0276062,\ -0.0276062,\ -0.217343,\ -0.217343,\ 0.0276062,\ -0.0276062,\ 0.217343,\ -0.217343\}$$

$$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} = (\ -0.0563508 \quad 0.0563508 \quad 0.443649 \quad -0.443649 \quad -0.106918 \quad -0.122474 \quad -0.841765 \quad 0.12$$

$$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} = (\ -0.443649 \quad -0.0563508 \quad 0.0563508 \quad 0.443649 \quad 0.122474 \quad 0.106918 \quad -0.122474 \quad 0.84176$$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 59.9142 & 0. \\ -106.324 & 133.333 \end{pmatrix} \qquad \det J = 0.000125179$$

$$\boldsymbol{B}^{\mathrm{T}} = \left(\begin{array}{lllllllll} -3.37622 & 3.37622 & 26.5809 & -26.5809 & -6.40592 & -7.33796 & -50.4337 & 7.33796 & \\ -53.1618 & -13.5049 & -39.6569 & 106.324 & 27.6979 & 27.2777 & 73.1696 & 99.2134 & -2 \end{array} \right.$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^{6}$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{array}{lllllllll}
4.9334 & 1.2284 & 3.50934 & -9.67113 & -2.52243 & -2.47812 & -6.4668 & -9.21306 & 2 \\
1.2284 & 0.336906 & 1.08715 & -2.65245 & -0.687934 & -0.68354 & -2.01403 & -2.28641 & 0 \\
3.50934 & 1.08715 & 3.96264 & -8.55913 & -2.20573 & -2.21984 & -7.37557 & -6.50139 & 1 \\
-9.67113 & -2.65245 & -8.55913 & 20.8827 & 5.41609 & 5.3815 & 15.8564 & 18.0009 & -4 \\
-2.52243 & -0.687934 & -2.20573 & 5.41609 & 1.40515 & 1.39529 & 4.08521 & 4.69594 & -1 \\
-2.47812 & -0.68354 & -2.21984 & 5.3815 & 1.39529 & 1.38726 & 4.11348 & 4.61157 & -1 \\
-6.4668 & -2.01403 & -7.37557 & 15.8564 & 4.08521 & 4.11348 & 13.7303 & 11.9777 & -3 \\
-9.21306 & -2.28641 & -6.50139 & 18.0009 & 4.69594 & 4.61157 & 11.9777 & 17.2071 & -4 \\
2.18473 & 0.586429 & 1.84579 & -4.61695 & -1.19889 & -1.18834 & -3.41593 & -4.06955 & 1 \\
-2.03891 & -0.571967 & -1.89221 & 4.50309 & 1.16646 & 1.1619 & 3.50898 & 3.7919 & -0 \\
3.80809 & 1.21488 & 4.54174 & -9.56471 & -2.46135 & -2.48417 & -8.46107 & -7.04624 & 2 \\
-5.75521 & -1.408 & -3.92195 & 11.0852 & 2.89436 & 2.83732 & 7.21851 & 10.7539 & -2
\end{array} \right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$$\boldsymbol{r}_q^T = (\ 19.3177 \quad 2.45367 \quad 19.3177 \quad 152.088 \quad -5.33288 \quad -5.33288 \quad -41.9857 \quad -41.9857 \quad 5.33288 \quad -5$$

Gauss point = {s → 0., t → −0.774597} Weight = 0.493827

$$\boldsymbol{N}^T = \{0.443649,\ 0.443649,\ 0.0563508,\ 0.0563508,$$
$$-0.543357,\ -0.122474,\ -0.0690154,\ -0.122474,\ 0.,\ 0.122474,\ 0.,\ 0.122474\}$$

$$\partial\boldsymbol{N}^T/\partial s = (\ -0.443649 \quad 0.443649 \quad 0.0563508 \quad -0.0563508 \quad 0. \quad -0.122474 \quad 0. \quad 0.122474 \quad -0.701471$$

$$\partial\boldsymbol{N}^T/\partial t =$$
$$(\ -0.25 \quad -0.25 \quad 0.25 \quad 0.25 \quad 0.306186 \quad -0.474342 \quad -0.306186 \quad -0.474342 \quad 0. \quad 0.316228 \quad 0. \quad 0.316228\ )$$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 35.3239 & 0. \\ -35.3239 & 133.333 \end{pmatrix} \qquad \det J = 0.000212321$$

$$\boldsymbol{B}^T = \begin{pmatrix} -15.6714 & 15.6714 & 1.99053 & -1.99053 & 0. & -4.32627 & 0. & 4.32627 & -24. \\ -17.6619 & -49.0047 & 31.3428 & 35.3239 & 40.8248 & -58.9193 & -40.8248 & -67.5718 & 24. \end{pmatrix}$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^6$

$$\boldsymbol{k}_k = \begin{pmatrix}
2.6306 & 2.92496 & -2.75908 & -2.79647 & -3.40207 & 5.22983 & 3.40207 & \\
2.92496 & 12.4895 & -7.09979 & -8.31464 & -9.43937 & 13.3032 & 9.43937 & 1 \\
-2.75908 & -7.09979 & 4.65376 & 5.20511 & 6.0373 & -8.75379 & -6.0373 & - \\
-2.79647 & -8.31464 & 5.20511 & 5.90601 & 6.80414 & -9.77925 & -6.80414 & -1 \\
-3.40207 & -9.43937 & 6.0373 & 6.80414 & 7.86374 & -11.3491 & -7.86374 & -1 \\
5.22983 & 13.3032 & -8.75379 & -9.77925 & -11.3491 & 16.4676 & 11.3491 & 1 \\
3.40207 & 9.43937 & -6.0373 & -6.80414 & -7.86374 & 11.3491 & 7.86374 & 1 \\
5.31109 & 15.9436 & -9.9521 & -11.3026 & -13.0158 & 18.6964 & 13.0158 & 2 \\
-0.232716 & -7.56141 & 3.43163 & 4.36249 & 4.77291 & -6.38257 & -4.77291 & - \\
-3.47301 & -8.42874 & 5.63615 & 6.26561 & 7.28831 & -10.607 & -7.28831 & -1 \\
-0.0295589 & -0.960424 & 0.435874 & 0.554109 & 0.606238 & -0.810692 & -0.606238 & - \\
-3.55427 & -11.0691 & 6.83445 & 7.78896 & 8.95497 & -12.8357 & -8.95497 & -1
\end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = \left(\begin{matrix} 232.583 & 232.583 & 29.5419 & 29.5419 & -284.855 & -64.2072 & -36.1813 & -64.2072 & 0. & 64.2072 \end{matrix} \right.$$

Gauss point = $\{s \to 0., t \to 0.\}$ Weight = 0.790123

$$\boldsymbol{N}^{\mathrm{T}} = \{0.25, 0.25, 0.25, 0.25, -0.306186, -0.306186, -0.306186, -0.306186, 0., 0., 0., 0.\}$$

$$\partial\boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} =$$

$$\begin{pmatrix} -0.25 & 0.25 & 0.25 & -0.25 & 0. & -0.306186 & 0. & 0.306186 & -0.395285 & 0. & -0.395285 & 0. \end{pmatrix}$$

$$\partial\boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} = \begin{pmatrix} -0.25 & -0.25 & 0.25 & 0.25 & 0.306186 & 0. & -0.306186 & 0. & 0. & -0.395285 & 0. & -0.395285 \end{pmatrix}$$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 44.4444 & 0. \\ -44.4444 & 133.333 \end{pmatrix} \qquad \det J = 0.00016875$$

$$\boldsymbol{B}^{\mathrm{T}} = \left(\begin{matrix} -11.1111 & 11.1111 & 11.1111 & -11.1111 & 0. & -13.6083 & 0. & 13.6083 & -17.568 \\ -22.2222 & -44.4444 & 22.2222 & 44.4444 & 40.8248 & 13.6083 & -40.8248 & -13.6083 & 17.568 \end{matrix} \right.$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^6$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{matrix} 3.7037 & 5.18519 & -3.7037 & -5.18519 & -5.44331 & -0.907218 & 5.44331 & 0.9072 \\ 5.18519 & 12.5926 & -5.18519 & -12.5926 & -10.8866 & -4.53609 & 10.8866 & 4.5360 \\ -3.7037 & -5.18519 & 3.7037 & 5.18519 & 5.44331 & 0.907218 & -5.44331 & -0.9072 \\ -5.18519 & -12.5926 & 5.18519 & 12.5926 & 10.8866 & 4.53609 & -10.8866 & -4.5360 \\ -5.44331 & -10.8866 & 5.44331 & 10.8866 & 10. & 3.33333 & -10. & -3.3333 \\ -0.907218 & -4.53609 & 0.907218 & 4.53609 & 3.33333 & 2.22222 & -3.33333 & -2.2222 \\ 5.44331 & 10.8866 & -5.44331 & -10.8866 & -10. & -3.33333 & 10. & 3.3333 \\ 0.907218 & 4.53609 & -0.907218 & -4.53609 & -3.33333 & -2.22222 & 3.33333 & 2.2222 \\ -1.17121 & -5.85607 & 1.17121 & 5.85607 & 4.30331 & 2.86888 & -4.30331 & -2.8688 \\ 7.02728 & 14.0546 & -7.02728 & -14.0546 & -12.9099 & -4.30331 & 12.9099 & 4.3033 \\ -1.17121 & -5.85607 & 1.17121 & 5.85607 & 4.30331 & 2.86888 & -4.30331 & -2.8688 \\ 7.02728 & 14.0546 & -7.02728 & -14.0546 & -12.9099 & -4.30331 & 12.9099 & 4.3033 \end{matrix} \right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} =$

$$\begin{pmatrix} 166.667 & 166.667 & 166.667 & 166.667 & -204.124 & -204.124 & -204.124 & -204.124 & 0. & 0. & 0. & 0. \end{pmatrix}$$

Gauss point = $\{s \to 0., t \to 0.774597\}$ Weight = 0.493827

$\boldsymbol{N}^{\mathrm{T}} = \{0.0563508, 0.0563508, 0.443649, 0.443649, -0.0690154, -0.122474, -0.543357, -0.122474, 0., -0.122474, 0., -0.122474\}$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial s = (-0.0563508 \quad 0.0563508 \quad 0.443649 \quad -0.443649 \quad 0. \quad -0.122474 \quad 0. \quad 0.122474 \quad -0.089098$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial t =$

$$\begin{pmatrix} -0.25 & -0.25 & 0.25 & 0.25 & 0.306186 & 0.474342 & -0.306186 & 0.474342 & 0. & 0.316228 & 0. & 0.316228 \end{pmatrix}$$

$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 59.9142 & 0. \\ -59.9142 & 133.333 \end{pmatrix}$ detJ = 0.000125179

$\boldsymbol{B}^{\mathrm{T}} = \Bigg(\begin{matrix} -3.37622 & 3.37622 & 26.5809 & -26.5809 & 0. & -7.33796 & 0. & 7.33796 & -5.3 \\ -29.9571 & -36.7096 & 6.75243 & 59.9142 & 40.8248 & 70.5835 & -40.8248 & 55.9076 & 5.3 \end{matrix}$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^6$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{array}{llllllll} 2.52814 & 3.02742 & -0.812345 & -4.74321 & -3.40207 & -5.81304 & 3.40207 & -4.7278 \\ 3.02742 & 3.78038 & -0.439895 & -6.3679 & -4.16891 & -7.27669 & 4.16891 & -5.6402 \\ -0.812345 & -0.439895 & 2.09227 & -0.840027 & 0.766838 & 0.783233 & -0.766838 & 1.5927 \\ -4.74321 & -6.3679 & -0.840027 & 11.9511 & 6.80414 & 12.3065 & -6.80414 & 8.7753 \\ -3.40207 & -4.16891 & 0.766838 & 6.80414 & 4.63626 & 8.01579 & -4.63626 & 6.3491 \\ -5.81304 & -7.27669 & 0.783233 & 12.3065 & 8.01579 & 14.0086 & -8.01579 & 10.8274 \\ 3.40207 & 4.16891 & -0.766838 & -6.80414 & -4.63626 & -8.01579 & 4.63626 & -6.3491 \\ -4.72788 & -5.6402 & 1.59273 & 8.77535 & 6.34913 & 10.8274 & -6.34913 & 8.8446 \\ -0.39472 & -0.595264 & -0.294448 & 1.28443 & 0.606238 & 1.15711 & -0.606238 & 0.7212 \\ -4.05622 & -5.12388 & 0.38724 & 8.79286 & 5.62164 & 9.86924 & -5.62164 & 7.5487 \\ -3.10762 & -4.6865 & -2.31818 & 10.1123 & 4.77291 & 9.10994 & -4.77291 & 5.6783 \\ -2.97106 & -3.48738 & 1.19673 & 5.26171 & 3.95497 & 6.68811 & -3.95497 & 5.5659 \end{array}\right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\ 17.4171 \quad 17.4171 \quad 137.125 \quad 137.125 \quad -21.3315 \quad -37.8549 \quad -167.943 \quad -37.8549 \quad 0. \quad -37.854$$

Gauss point = $\{s \to 0.774597,\ t \to -0.774597\}$ Weight = 0.308642

$$\boldsymbol{N}^{\mathrm{T}} = \{0.1,\ 0.787298,\ 0.1,\ 0.0127017,\ -0.217343,\ -0.217343,\ -0.0276062,\ -0.0276062,\ -0.217343,\ 0.217343,\ -0.0276062,\ 0.0276062\}$$

$$\partial \boldsymbol{N}^{\mathrm{T}}/\partial s = (\ -0.443649 \quad 0.443649 \quad 0.0563508 \quad -0.0563508 \quad 0.841765 \quad -0.122474 \quad 0.106918 \quad 0.12247$$

$$\partial \boldsymbol{N}^{\mathrm{T}}/\partial t = (\ -0.0563508 \quad -0.443649 \quad 0.443649 \quad 0.0563508 \quad 0.122474 \quad -0.841765 \quad -0.122474 \quad -0.10$$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 35.3239 & 0. \\ -7.96212 & 133.333 \end{pmatrix} \qquad \det J = 0.000212321$$

$$\boldsymbol{B}^{\mathrm{T}} = \left(\begin{array}{lllllllll} -15.6714 & 15.6714 & 1.99053 & -1.99053 & 29.7344 & -4.32627 & 3.77676 & 4.32627 & 1 \\ -3.98106 & -62.6856 & 58.7046 & 7.96212 & 9.6277 & -111.26 & -17.1812 & -15.2309 & 1 \end{array}\right.$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^6$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{array}{lllllll}
0.770966 & 0.0116842 & -0.781166 & -0.00148408 & -1.48716 & 1.5061 & 0.0271661 \\
0.0116842 & 12.3119 & -10.7598 & -1.56382 & -0.405588 & 20.3669 & 3.35055 \\
-0.781166 & -10.7598 & 10.1743 & 1.36667 & 1.84123 & -19.2861 & -2.95214 \\
-0.00148408 & -1.56382 & 1.36667 & 0.198631 & 0.0515164 & -2.58694 & -0.425576 \\
-1.48716 & -0.405588 & 1.84123 & 0.0515164 & 2.88057 & -3.53815 & -0.156634 \\
1.5061 & 20.3669 & -19.2861 & -2.58694 & -3.53815 & 36.5592 & 5.5889 \\
0.0271661 & 3.35055 & -2.95214 & -0.425576 & -0.156634 & 5.5889 & 0.912563 \\
-0.0211247 & 3.01542 & -2.61129 & -0.383009 & -0.0530791 & 4.942 & 0.819869 \\
-1.05534 & -1.27661 & 2.1698 & 0.162151 & 2.07492 & -4.14469 & -0.380212 \\
-1.06689 & -13.4512 & 12.8096 & 1.70853 & 2.47599 & -24.2845 & -3.6934 \\
0.0820139 & 3.23992 & -2.91041 & -0.411524 & -0.258964 & 5.51186 & 0.884165 \\
0.0769111 & -2.13701 & 1.78867 & 0.271436 & -0.0818334 & -3.38292 & -0.57911
\end{array}\right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\ 32.7656 \quad 257.963 \quad 32.7656 \quad 4.16178 \quad -71.2137 \quad -71.2137 \quad -9.04532 \quad -9.04532 \quad -71.2137$$

Gauss point = {s → 0.774597, t → 0.}   Weight = 0.493827

$$\boldsymbol{N}^{\mathrm{T}} = \{0.0563508, 0.443649, 0.443649, 0.0563508, \\ -0.122474, -0.543357, -0.122474, -0.0690154, -0.122474, 0., -0.122474, 0.\}$$

$$\partial\boldsymbol{N}^{\mathrm{T}}/\partial\mathrm{s} = \\ (\ -0.25 \quad 0.25 \quad 0.25 \quad -0.25 \quad 0.474342 \quad -0.306186 \quad 0.474342 \quad 0.306186 \quad 0.316228 \quad 0. \quad 0.316228 \quad 0.\ )$$

$$\partial\boldsymbol{N}^{\mathrm{T}}/\partial\mathrm{t} = (\ -0.0563508 \quad -0.443649 \quad 0.443649 \quad 0.0563508 \quad 0.122474 \quad 0. \quad -0.122474 \quad 0. \quad 0.122474 \quad -$$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 44.4444 & 0. \\ -10.0179 & 133.333 \end{pmatrix} \qquad \det\mathrm{J} = 0.00016875$$

$$\boldsymbol{B}^{\mathrm{T}} = \left(\begin{array}{lllllllll}
-11.1111 & 11.1111 & 11.1111 & -11.1111 & 21.0819 & -13.6083 & 21.0819 & 13.6083 & 14.05 \\
-5.00896 & -61.6577 & 56.6487 & 10.0179 & 11.578 & 3.06735 & -21.0819 & -3.06735 & 13.16
\end{array}\right.$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5.\times 10^6$

$$\boldsymbol{k}_\mathrm{k} = \left(\begin{matrix} 0.557049 & 0.695191 & -1.52703 & 0.27479 & -1.09589 & 0.509396 & -0.482417 & - \\ 0.695191 & 14.7192 & -12.6352 & -2.77927 & -1.79862 & -1.27623 & 5.75288 & \\ -1.52703 & -12.6352 & 12.497 & 1.66517 & 3.33796 & 0.0845943 & -3.60007 & - \\ 0.27479 & -2.77927 & 1.66517 & 0.839309 & -0.443457 & 0.682243 & -1.6704 & - \\ -1.09589 & -1.79862 & 3.33796 & -0.443457 & 2.16936 & -0.942652 & 0.751344 & \\ 0.509396 & -1.27623 & 0.0845943 & 0.682243 & -0.942652 & 0.729727 & -1.31832 & - \\ -0.482417 & 5.75288 & -3.60007 & -1.6704 & 0.751344 & -1.31832 & 3.33333 & \\ -0.509396 & 1.27623 & -0.0845943 & -0.682243 & 0.942652 & -0.729727 & 1.31832 & \\ -0.832837 & -2.45766 & 3.38164 & -0.0911478 & 1.68257 & -0.565823 & 0.0705648 & \\ 1.75682 & 21.6255 & -19.8687 & -3.51364 & -4.06082 & -1.07583 & 7.39415 & \\ -0.219366 & 5.09384 & -3.55638 & -1.31809 & 0.264561 & -0.941495 & 2.65255 & \\ 0.223146 & 2.7468 & -2.52366 & -0.446291 & -0.515792 & -0.136648 & 0.939181 & \end{matrix}\right.$$

$$\boldsymbol{k}_\mathrm{p} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_\mathrm{q}^\mathrm{T} = (\ 23.4795 \quad 184.854 \quad 184.854 \quad 23.4795 \quad -51.031 \quad -226.399 \quad -51.031 \quad -28.7564 \quad -51.031 \quad 0.$$

Gauss point = {s → 0.774597, t → 0.774597} Weight = 0.308642

$$\boldsymbol{N}^\mathrm{T} = \{0.0127017, 0.1, 0.787298, 0.1, -0.0276062, -0.217343, -0.217343, -0.0276062, -0.0276062, -0.217343, -0.217343, -0.0276062\}$$

$$\partial\boldsymbol{N}^\mathrm{T}/\partial s = (\ -0.0563508 \quad 0.0563508 \quad 0.443649 \quad -0.443649 \quad 0.106918 \quad -0.122474 \quad 0.841765 \quad 0.12247$$

$$\partial\boldsymbol{N}^\mathrm{T}/\partial t = (\ -0.0563508 \quad -0.443649 \quad 0.443649 \quad 0.0563508 \quad 0.122474 \quad 0.841765 \quad -0.122474 \quad 0.10691$$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 59.9142 & 0. \\ -13.5049 & 133.333 \end{pmatrix} \qquad \det J = 0.000125179$$

$$\boldsymbol{B}^\mathrm{T} = \left(\begin{matrix} -3.37622 & 3.37622 & 26.5809 & -26.5809 & 6.40592 & -7.33796 & 50.4337 & 7.33796 & 4.27 \\ -6.75243 & -59.9142 & 53.1618 & 13.5049 & 14.886 & 113.889 & -27.6979 & 12.6018 & 15.36 \end{matrix}\right.$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5.\times 10^6$

$$\boldsymbol{k}_\mathrm{k} = \left(\begin{matrix}
0.0990899 & 0.683561 & -0.780133 & -0.00251721 & -0.21236 & -1.29396 & 0.0291264 \\
0.683561 & 6.26088 & -5.38166 & -1.56278 & -1.51302 & -11.9065 & 3.18123 \\
-0.780133 & -5.38166 & 6.14198 & 0.019818 & 1.67191 & 10.1873 & -0.229311 \\
-0.00251721 & -1.56278 & 0.019818 & 1.54548 & 0.0534767 & 3.01318 & -2.98105 \\
-0.21236 & -1.51302 & 1.67191 & 0.0534767 & 0.456606 & 2.86582 & -0.155146 \\
-1.29396 & -11.9065 & 10.1873 & 3.01318 & 2.86582 & 22.6446 & -6.12781 \\
0.0291264 & 3.18123 & -0.229311 & -2.98105 & -0.155146 & -6.12781 & 5.75602 \\
-0.191015 & -1.26961 & 1.50385 & -0.0432296 & 0.407868 & 2.40163 & 0.0365786 \\
-0.205477 & -1.57569 & 1.61771 & 0.163457 & 0.445281 & 2.98836 & -0.365555 \\
-0.854755 & -8.00949 & 6.72947 & 2.13477 & 1.89757 & 15.2368 & -4.32623 \\
0.0833207 & 2.68783 & -0.655983 & -2.11517 & -0.244308 & -5.16302 & 4.09947 \\
-0.135228 & -0.774618 & 1.06465 & -0.154802 & 0.284885 & 1.46071 & 0.265409
\end{matrix}\right.$$

$$\boldsymbol{k}_\mathrm{p} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$$\boldsymbol{r}_\mathrm{q}^\mathrm{T} = \left(\begin{matrix} 2.45367 & 19.3177 & 152.088 & 19.3177 & -5.33288 & -41.9857 & -41.9857 & -5.33288 & -5.33288 \end{matrix}\right.$$

Adding contributions from all Gauss points

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{array}{cccccccc}
25.1389 & 19.8611 & -5.97222 & -39.0278 & -29.6466 & -5.44331 & 4.17968 & 5. \\
19.8611 & 70.1389 & -39.0278 & -50.9722 & -43.8381 & 5.44331 & 32.5627 & 31 \\
-5.97222 & -39.0278 & 44.3056 & 0.694444 & 14.1915 & -20.0722 & -28.383 & -16 \\
-39.0278 & -50.9722 & 0.694444 & 89.3056 & 59.2932 & 20.0722 & -8.35937 & -20 \\
-29.6466 & -43.8381 & 14.1915 & 59.2932 & 58.5952 & 4.7619 & -13.6905 & -34 \\
-5.44331 & 5.44331 & -20.0722 & 20.0722 & 4.7619 & 97.0833 & 5.47619 & 37 \\
4.17968 & 32.5627 & -28.383 & -8.35937 & -13.6905 & 5.47619 & 49.881 & 24 \\
5.44331 & 31.299 & -16.6701 & -20.0722 & -34.7619 & 37.9167 & 24.5238 & 97 \\
7.65472 & -7.65472 & 15.3094 & -15.3094 & -8.97549 & -8.97549 & -16.9059 & 8 \\
-1.31762 & 1.31762 & -2.63523 & 2.63523 & -0.922139 & -15.1692 & 1.84428 & 3. \\
-1.07919 & 1.07919 & -2.15838 & 2.15838 & 6.33202 & 6.33202 & -12.664 & -6. \\
1.31762 & -1.31762 & 2.63523 & -2.63523 & 0.922139 & -19.6877 & -1.84428 & -15
\end{array}\right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} =$

$$(\ 937.5 \quad 937.5 \quad 750. \quad 750. \quad -765.466 \quad -688.919 \quad -612.372 \quad -688.919 \quad 0. \quad 59.2927 \quad 0. \quad 59.2927\ )$$

Natural boundary conditions

Specified NBC values for side 3: $\alpha = -55$ $\qquad \beta = 1100$

Interpolation functions for mapping: $\left\{0, 0, \frac{1-\mathrm{a}}{2}, \frac{\mathrm{a}+1}{2}\right\}$

Interpolation functions for solution: $\left\{0, 0, \frac{1-\mathrm{a}}{2}, \frac{\mathrm{a}+1}{2}, 0, 0, \frac{\frac{3\,\mathrm{a}^2}{2} - \frac{3}{2}}{\sqrt{6}}, 0, 0, 0, \frac{\frac{5\,\mathrm{a}}{2} - \frac{5\,\mathrm{a}^3}{2}}{\sqrt{10}}, 0\right\}$

$x(\mathrm{a}) = 0.045 - 0.015\,\mathrm{a}$ $\qquad y(\mathrm{a}) = 0.015$

$\mathrm{J_c} = 0.015$

Value in mapped coordinate: $\alpha(\mathrm{a}) = -55$ $\qquad \beta(\mathrm{a}) = 1100$

Gauss point = −0.774597 Weight = 0.555556

$\boldsymbol{N}_c^T$ = {0., 0., 0.887298, 0.112702, 0., 0., −0.244949, 0., 0., 0., −0.244949, 0.}

J_c = 0.015 α = −55. β = 1100.

$$\boldsymbol{k}_\alpha = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & -0.360845 & -0.0458333 & 0. & 0. & 0.0996155 & 0. & 0. & 0. & 0.0996155 & 0. \\ 0. & 0. & -0.0458333 & -0.0058216 & 0. & 0. & 0.0126528 & 0. & 0. & 0. & 0.0126528 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0.0996155 & 0.0126528 & 0. & 0. & -0.0275 & 0. & 0. & 0. & -0.0275 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0.0996155 & 0.0126528 & 0. & 0. & -0.0275 & 0. & 0. & 0. & -0.0275 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_\beta = \begin{pmatrix} 0. \\ 0. \\ -8.13357 \\ -1.0331 \\ 0. \\ 0. \\ 2.24537 \\ 0. \\ 0. \\ 0. \\ 2.24537 \\ 0. \end{pmatrix}$$

Gauss point = 0. Weight = 0.888889

$\boldsymbol{N}_c^T$ = {0., 0., 0.5, 0.5, 0., 0., −0.612372, 0., 0., 0., 0., 0.}

J_c = 0.015 α = −55. β = 1100.

$\boldsymbol{k}_{\alpha} =$

$$\begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & -0.183333 & -0.183333 & 0. & 0. & 0.224537 & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & -0.183333 & -0.183333 & 0. & 0. & 0.224537 & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0.224537 & 0.224537 & 0. & 0. & -0.275 & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix} \qquad \boldsymbol{r}_{\beta} = \begin{pmatrix} 0. \\ 0. \\ -7.33333 \\ -7.33333 \\ 0. \\ 0. \\ 8.98146 \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \end{pmatrix}$$

Gauss point = 0.774597 Weight = 0.555556

$\boldsymbol{N}_{c}^{T} = \{0., 0., 0.112702, 0.887298, 0., 0., -0.244949, 0., 0., 0., 0.244949, 0.\}$

$J_{c} = 0.015$ $\alpha = -55.$ $\beta = 1100.$

$$\boldsymbol{k}_{\alpha} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & -0.0058216 & -0.0458333 & 0. & 0. & 0.0126528 & 0. & 0. & 0. & -0.0126528 & 0. \\ 0. & 0. & -0.0458333 & -0.360845 & 0. & 0. & 0.0996155 & 0. & 0. & 0. & -0.0996155 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0.0126528 & 0.0996155 & 0. & 0. & -0.0275 & 0. & 0. & 0. & 0.0275 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & -0.0126528 & -0.0996155 & 0. & 0. & 0.0275 & 0. & 0. & 0. & -0.0275 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\beta} = \begin{pmatrix} 0. \\ 0. \\ -1.0331 \\ -8.13357 \\ 0. \\ 0. \\ 2.24537 \\ 0. \\ 0. \\ 0. \\ -2.24537 \\ 0. \end{pmatrix}$$

Adding contributions from all Gauss points

$$\boldsymbol{k}_{\alpha} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & -0.55 & -0.275 & 0. & 0. & 0.336805 & 0. & 0. & 0. & 0.0869626 & 0. \\ 0. & 0. & -0.275 & -0.55 & 0. & 0. & 0.336805 & 0. & 0. & 0. & -0.0869626 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0.336805 & 0.336805 & 0. & 0. & -0.33 & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0.0869626 & -0.0869626 & 0. & 0. & 0. & 0. & 0. & 0. & -0.055 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\beta} = \begin{pmatrix} 0. \\ 0. \\ -16.5 \\ -16.5 \\ 0. \\ 0. \\ 13.4722 \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \end{pmatrix}$$

$$\begin{pmatrix}
25.1389 & 19.8611 & -5.97222 & -39.0278 & -29.6466 & -5.44331 & 4.17968 & 5.44331 & 7.6 \\
19.8611 & 70.1389 & -39.0278 & -50.9722 & -43.8381 & 5.44331 & 32.5627 & 31.299 & -7.6 \\
-5.97222 & -39.0278 & 43.7556 & 0.419444 & 14.1915 & -20.0722 & -28.0462 & -16.6701 & 15.3 \\
-39.0278 & -50.9722 & 0.419444 & 88.7556 & 59.2932 & 20.0722 & -8.02256 & -20.0722 & -15.3 \\
-29.6466 & -43.8381 & 14.1915 & 59.2932 & 58.5952 & 4.7619 & -13.6905 & -34.7619 & -8.9 \\
-5.44331 & 5.44331 & -20.0722 & 20.0722 & 4.7619 & 97.0833 & 5.47619 & 37.9167 & -8.9 \\
4.17968 & 32.5627 & -28.0462 & -8.02256 & -13.6905 & 5.47619 & 49.551 & 24.5238 & -16.9 \\
5.44331 & 31.299 & -16.6701 & -20.0722 & -34.7619 & 37.9167 & 24.5238 & 97.0833 & 8.9 \\
7.65472 & -7.65472 & 15.3094 & -15.3094 & -8.97549 & -8.97549 & -16.9059 & 8.97549 & 30.1 \\
-1.31762 & 1.31762 & -2.63523 & 2.63523 & -0.922139 & -15.1692 & 1.84428 & 3.55023 & 0.2 \\
-1.07919 & 1.07919 & -2.07142 & 2.07142 & 6.33202 & 6.33202 & -12.664 & -6.33202 & 7.1 \\
1.31762 & -1.31762 & 2.63523 & -2.63523 & 0.922139 & -19.6877 & -1.84428 & -15.1692 & -0.2
\end{pmatrix}$$

Equations for element 2

Element coordinates: ({0, 0} {0.03, 0.015} {0.03, 0.03} {0, 0.03})

$x(s,t) = 0.015\,s + 0.015$

$y(s,t) = -0.00375\,t\,s + 0.00375\,s + 0.01125\,t + 0.01875$

$$\boldsymbol{J} = \begin{pmatrix} 0.015 & 0 \\ 0.00375 - 0.00375\,t & 0.01125 - 0.00375\,s \end{pmatrix}$$

$\det J = 0.00016875 - 0.00005625\,s$

Given element data

$k_x = 45$ $k_y = 45$ $p = 0$ $q = 5000000$

Element data in mapped coordinates

$k_x = 45$ $k_y = 45$ $p = 0$ $q = 5000000$

Gauss point = {s → −0.774597, t → −0.774597} Weight = 0.308642

$\boldsymbol{N}^T$ = {0.787298, 0.1, 0.0127017, 0.1, −0.217343,
−0.0276062, −0.0276062, −0.217343, 0.217343, 0.0276062, 0.0276062, 0.217343}

$\partial\boldsymbol{N}^T/\partial s$ = (−0.443649 0.443649 0.0563508 −0.0563508 −0.841765 −0.122474 −0.106918 0.12

$\partial\boldsymbol{N}^T/\partial t$ = (−0.443649 −0.0563508 0.0563508 0.443649 0.122474 −0.106918 −0.122474 −0.841

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 66.6667 & -31.3428 \\ 0. & 70.6477 \end{pmatrix}$$ $\det J = 0.000212321$

$$\boldsymbol{B}^{\mathrm{T}} = \begin{pmatrix} -15.6714 & 31.3428 & 1.99053 & -17.6619 & -59.9564 & -4.81385 & -3.28919 & 34.5482 & 41.2 \\ -31.3428 & -3.98106 & 3.98106 & 31.3428 & 8.65254 & -7.55353 & -8.65254 & -59.4688 & -8.6 \end{pmatrix}$$

$k_x = 45.$ $\qquad$ $k_y = 45.$ $\qquad$ $p = 0.$ $\qquad$ $q = 5.\times 10^6$

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 3.62115 & -1.0805 & -0.459946 & -2.0807 & 1.97106 & 0.920614 & 0.951733 & 3.8 \\ -1.0805 & 2.94366 & 0.137242 & -2.00039 & -5.64316 & -0.356253 & -0.20243 & 3.8 \\ -0.459946 & 0.137242 & 0.0584208 & 0.264284 & -0.250358 & -0.116933 & -0.120886 & -0.4 \\ -2.0807 & -2.00039 & 0.264284 & 3.81681 & 3.92246 & -0.447428 & -0.628416 & -7.2 \\ 1.97106 & -5.64316 & -0.250358 & 3.92246 & 10.8214 & 0.658383 & 0.360772 & -7.6 \\ 0.920614 & -0.356253 & -0.116933 & -0.447428 & 0.658383 & 0.236587 & 0.239424 & 0.8 \\ 0.951733 & -0.20243 & -0.120886 & -0.628416 & 0.360772 & 0.239424 & 0.252678 & 1.1 \\ 3.89992 & 3.89134 & -0.495355 & -7.2959 & -7.6257 & 0.834212 & 1.18228 & 13.9 \\ -1.1066 & 3.91423 & 0.140557 & -2.94819 & -7.51409 & -0.392842 & -0.179334 & 5.7 \\ -0.73952 & 0.489056 & 0.0939313 & 0.156532 & -0.920126 & -0.19636 & -0.186015 & -0.2 \\ -0.841931 & -0.0171721 & 0.106939 & 0.752164 & 0.0593108 & -0.205696 & -0.229632 & -1.4 \\ -2.47417 & -2.84578 & 0.314261 & 5.00569 & 5.565 & -0.517506 & -0.761786 & -9.5 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\ 257.963 \quad 32.7656 \quad 4.16178 \quad 32.7656 \quad -71.2137 \quad -9.04532 \quad -9.04532 \quad -71.2137 \quad 71.2137 \quad 9.$$

Gauss point = $\{s \to -0.774597,\ t \to 0.\}$ $\qquad$ Weight = 0.493827

$$\boldsymbol{N}^{\mathrm{T}} = \{0.443649,\ 0.0563508,\ 0.0563508,\ 0.443649,\ -0.122474,\ -0.0690154,\ -0.122474,\ -0.543357,\ 0.122474,\ 0.,\ 0.122474,\ 0.\}$$

$$\partial\boldsymbol{N}^{\mathrm{T}}/\partial s = (\ -0.25 \quad 0.25 \quad 0.25 \quad -0.25 \quad -0.474342 \quad -0.306186 \quad -0.474342 \quad 0.306186 \quad 0.316228 \quad 0. \quad 0.316228 \quad 0.\ )$$

$$\partial\boldsymbol{N}^{\mathrm{T}}/\partial t = (\ -0.443649 \quad -0.0563508 \quad 0.0563508 \quad 0.443649 \quad 0.122474 \quad 0. \quad -0.122474 \quad 0. \quad -0.122474$$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 66.6667 & -17.6619 \\ 0. & 70.6477 \end{pmatrix} \qquad \mathrm{detJ} = 0.000212321$$

$$\boldsymbol{B}^{\mathrm{T}} = \begin{pmatrix} -8.83097 & 17.6619 & 15.6714 & -24.5024 & -33.7859 & -20.4124 & -29.4596 & 20.4124 & 23. \\ -31.3428 & -3.98106 & 3.98106 & 31.3428 & 8.65254 & 0. & -8.65254 & 0. & -8. \end{pmatrix}$$

$k_x = 45. \qquad k_y = 45. \qquad p = 0. \qquad q = 5. \times 10^6$

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 5.00303 & -0.147183 & -1.24171 & -3.61414 & 0.128182 & 0.850517 & 2.50705 & -0.8 \\ -0.147183 & 1.54661 & 1.23117 & -2.6306 & -2.97802 & -1.70103 & -2.29244 & 1.7 \\ -1.24171 & 1.23117 & 1.23355 & -1.22301 & -2.33566 & -1.50933 & -2.34082 & 1.5 \\ -3.61414 & -2.6306 & -1.22301 & 7.46774 & 5.18549 & 2.35984 & 2.12621 & -2.3 \\ 0.128182 & -2.97802 & -2.33566 & 5.18549 & 5.73906 & 3.25395 & 4.34293 & -3.2 \\ 0.850517 & -1.70103 & -1.50933 & 2.35984 & 3.25395 & 1.96594 & 2.83728 & -1.9 \\ 2.50705 & -2.29244 & -2.34082 & 2.12621 & 4.34293 & 2.83728 & 4.44806 & -2.8 \\ -0.850517 & 1.70103 & 1.50933 & -2.35984 & -3.25395 & -1.96594 & -2.83728 & 1.9 \\ 0.311023 & 2.09961 & 1.55624 & -3.96688 & -4.05873 & -2.23874 & -2.87776 & 2.2 \\ 0.865297 & 0.249373 & -0.00187723 & -1.11279 & -0.507832 & -0.15156 & 0.0382421 & 0.1 \\ -2.06784 & 1.41403 & 1.5614 & -0.907594 & -2.6626 & -1.82208 & -2.9829 & 1.8 \\ 6.81247 & 1.96331 & -0.0147794 & -8.761 & -3.99816 & -1.19323 & 0.301079 & 1.1 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (232.583 \quad 29.5419 \quad 29.5419 \quad 232.583 \quad -64.2072 \quad -36.1813 \quad -64.2072 \quad -284.855 \quad 64.2072 \quad 0.$$

Gauss point = $\{s \to -0.774597,\ t \to 0.774597\}$ Weight = 0.308642

$\boldsymbol{N}^{\mathrm{T}} = \{0.1, 0.0127017, 0.1, 0.787298, -0.0276062,$
$-0.0276062, -0.217343, -0.217343, 0.0276062, -0.0276062, 0.217343, -0.217343\}$

$$\partial \boldsymbol{N}^{\mathrm{T}}/\partial s = (-0.0563508 \quad 0.0563508 \quad 0.443649 \quad -0.443649 \quad -0.106918 \quad -0.122474 \quad -0.841765 \quad 0.12$$

$$\partial \boldsymbol{N}^{\mathrm{T}}/\partial t = (-0.443649 \quad -0.0563508 \quad 0.0563508 \quad 0.443649 \quad 0.122474 \quad 0.106918 \quad -0.122474 \quad 0.84176$$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 66.6667 & -3.98106 \\ 0. & 70.6477 \end{pmatrix} \qquad \mathrm{detJ} = 0.000212321$$

$$\boldsymbol{B}^{\mathrm{T}} = \begin{pmatrix} -1.99053 & 3.98106 & 29.3523 & -31.3428 & -7.61546 & -8.59061 & -55.6301 & 4.81385 & 5. \\ -31.3428 & -3.98106 & 3.98106 & 31.3428 & 8.65254 & 7.55353 & -8.65254 & 59.4688 & -8. \end{pmatrix}$$

$\mathrm{k_x} = 45. \qquad \mathrm{k_y} = 45. \qquad \mathrm{p} = 0. \qquad \mathrm{q} = 5. \times 10^6$

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 2.9086 & 0.344589 & -0.540251 & -2.71294 & -0.755026 & -0.647723 & 1.12627 & -5.524 \\ 0.344589 & 0.0934733 & 0.297852 & -0.735914 & -0.190982 & -0.189528 & -0.551505 & -0.641 \\ -0.540251 & 0.297852 & 2.58738 & -2.34498 & -0.557593 & -0.654901 & -4.91675 & 1.114 \\ -2.71294 & -0.735914 & -2.34498 & 5.79384 & 1.5036 & 1.49215 & 4.34199 & 5.051 \\ -0.755026 & -0.190982 & -0.557593 & 1.5036 & 0.391796 & 0.385654 & 1.02852 & 1.409 \\ -0.647723 & -0.189528 & -0.654901 & 1.49215 & 0.385654 & 0.385877 & 1.21654 & 1.202 \\ 1.12627 & -0.551505 & -4.91675 & 4.34199 & 1.02852 & 1.21654 & 9.34677 & -2.307 \\ -5.52477 & -0.641636 & 1.11482 & 5.05159 & 1.40927 & 1.2027 & -2.30708 & 10.497 \\ 0.768973 & 0.163089 & 0.351937 & -1.284 & -0.338439 & -0.325464 & -0.638754 & -1.443 \\ -0.41584 & -0.158304 & -0.672179 & 1.24632 & 0.318223 & 0.326199 & 1.25751 & 0.763 \\ -1.01647 & 0.331902 & 3.29763 & -2.61306 & -0.608442 & -0.742665 & -6.27811 & 2.041 \\ -3.69916 & -0.395806 & 0.978793 & 3.11617 & 0.878393 & 0.73285 & -1.98453 & 7.036 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (32.7656 \quad 4.16178 \quad 32.7656 \quad 257.963 \quad -9.04532 \quad -9.04532 \quad -71.2137 \quad -71.2137 \quad 9.04532 \quad -$

Gauss point = $\{s \to 0., t \to -0.774597\}$ Weight = 0.493827

$\boldsymbol{N}^{\mathrm{T}} = \{0.443649, 0.443649, 0.0563508, 0.0563508,$
$-0.543357, -0.122474, -0.0690154, -0.122474, 0., 0.122474, 0., 0.122474\}$

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial s = (-0.443649 \quad 0.443649 \quad 0.0563508 \quad -0.0563508 \quad 0. \quad -0.122474 \quad 0. \quad 0.122474 \quad -0.701471$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} =$

$$(\,-0.25 \quad -0.25 \quad 0.25 \quad 0.25 \quad 0.306186 \quad -0.474342 \quad -0.306186 \quad -0.474342 \quad 0. \quad 0.316228 \quad 0. \quad 0.316228\,)$$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 66.6667 & -39.4355 \\ 0. & 88.8889 \end{pmatrix} \qquad \mathrm{detJ} = 0.00016875$$

$$\boldsymbol{B}^{\mathrm{T}} = \left(\begin{array}{ccccccccc} -19.7177 & 39.4355 & -6.10215 & -13.6156 & -12.0746 & 10.5409 & 12.0746 & 26.8709 & -46. \\ -22.2222 & -22.2222 & 22.2222 & 22.2222 & 27.2166 & -42.1637 & -27.2166 & -42.1637 & 0. \end{array} \right.$$

$\mathrm{k_x} = 45. \qquad \mathrm{k_y} = 45. \qquad \mathrm{p} = 0. \qquad \mathrm{q} = 5.\times 10^{6}$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{array}{cccccccc} 3.30981 & -1.06407 & -1.40065 & -0.845094 & -1.37523 & 2.73423 & 1.37523 & 1.52677 \\ -1.06407 & 7.68369 & -2.75426 & -3.86537 & -4.05367 & 5.07247 & 4.05367 & 7.48739 \\ -1.40065 & -2.75426 & 1.99149 & 2.16342 & 2.54435 & -3.75485 & -2.54435 & -4.12853 \\ -0.845094 & -3.86537 & 2.16342 & 2.54704 & 2.88456 & -4.05185 & -2.88456 & -4.88563 \\ -1.37523 & -4.05367 & 2.54435 & 2.88456 & 3.32451 & -4.78061 & -3.32451 & -5.52002 \\ 2.73423 & 5.07247 & -3.75485 & -4.05185 & -4.78061 & 7.08333 & 4.78061 & 7.72883 \\ 1.37523 & 4.05367 & -2.54435 & -2.88456 & -3.32451 & 4.78061 & 3.32451 & 5.52002 \\ 1.52677 & 7.48739 & -4.12853 & -4.88563 & -5.52002 & 7.72883 & 5.52002 & 9.37433 \\ 3.45786 & -6.91571 & 1.07012 & 2.38774 & 2.1175 & -1.84854 & -2.1175 & -4.71228 \\ -2.02406 & -2.97916 & 2.44095 & 2.56227 & 3.06383 & -4.61464 & -3.06383 & -4.8783 \\ 0.439205 & -0.87841 & 0.135923 & 0.303282 & 0.268957 & -0.234795 & -0.268957 & -0.59853 \\ -0.816603 & -5.39408 & 2.81463 & 3.39605 & 3.80325 & -5.26014 & -3.80325 & -6.5238 \end{array} \right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\,184.854 \quad 184.854 \quad 23.4795 \quad 23.4795 \quad -226.399 \quad -51.031 \quad -28.7564 \quad -51.031 \quad 0. \quad 51.031 \quad 0.$$

Gauss point = $\{\mathrm{s} \to 0., \mathrm{t} \to 0.\}$ Weight = 0.790123

$\boldsymbol{N}^{\mathrm{T}} = \{0.25, 0.25, 0.25, 0.25, -0.306186, -0.306186, -0.306186, -0.306186, 0., 0., 0., 0.\}$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} =$

$(-0.25 \quad 0.25 \quad 0.25 \quad -0.25 \quad 0. \quad -0.306186 \quad 0. \quad 0.306186 \quad -0.395285 \quad 0. \quad -0.395285 \quad 0.)$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} = (-0.25 \quad -0.25 \quad 0.25 \quad 0.25 \quad 0.306186 \quad 0. \quad -0.306186 \quad 0. \quad 0. \quad -0.395285 \quad 0. \quad -0.395285)$

$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 66.6667 & -22.2222 \\ 0. & 88.8889 \end{pmatrix}$ $\qquad$ $\det\mathrm{J} = 0.00016875$

$$\boldsymbol{B}^{\mathrm{T}} = \left(\begin{matrix} -11.1111 & 22.2222 & 11.1111 & -22.2222 & -6.80414 & -20.4124 & 6.80414 & 20.4124 & -26.35 \\ -22.2222 & -22.2222 & 22.2222 & 22.2222 & 27.2166 & 0. & -27.2166 & 0. & 0. \end{matrix} \right.$$

$\mathrm{k_x} = 45.$ $\qquad$ $\mathrm{k_y} = 45.$ $\qquad$ $\mathrm{p} = 0.$ $\qquad$ $\mathrm{q} = 5.\times 10^{6}$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{matrix} 3.7037 & 1.48148 & -3.7037 & -1.48148 & -3.17526 & 1.36083 & 3.17526 & -1.36083 \\ 1.48148 & 5.92593 & -1.48148 & -5.92593 & -4.53609 & -2.72166 & 4.53609 & 2.72166 \\ -3.7037 & -1.48148 & 3.7037 & 1.48148 & 3.17526 & -1.36083 & -3.17526 & 1.36083 \\ -1.48148 & -5.92593 & 1.48148 & 5.92593 & 4.53609 & 2.72166 & -4.53609 & -2.72166 \\ -3.17526 & -4.53609 & 3.17526 & 4.53609 & 4.72222 & 0.833333 & -4.72222 & -0.833333 \\ 1.36083 & -2.72166 & -1.36083 & 2.72166 & 0.833333 & 2.5 & -0.833333 & -2.5 \\ 3.17526 & 4.53609 & -3.17526 & -4.53609 & -4.72222 & -0.833333 & 4.72222 & 0.833333 \\ -1.36083 & 2.72166 & 1.36083 & -2.72166 & -0.833333 & -2.5 & 0.833333 & 2.5 \\ 1.75682 & -3.51364 & -1.75682 & 3.51364 & 1.07583 & 3.22749 & -1.07583 & -3.22749 \\ 4.09925 & 5.85607 & -4.09925 & -5.85607 & -6.09636 & -1.07583 & 6.09636 & 1.07583 \\ 1.75682 & -3.51364 & -1.75682 & 3.51364 & 1.07583 & 3.22749 & -1.07583 & -3.22749 \\ 4.09925 & 5.85607 & -4.09925 & -5.85607 & -6.09636 & -1.07583 & 6.09636 & 1.07583 \end{matrix} \right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} =$

$(166.667 \quad 166.667 \quad 166.667 \quad 166.667 \quad -204.124 \quad -204.124 \quad -204.124 \quad -204.124 \quad 0. \quad 0. \quad 0. \quad 0.)$

Gauss point = $\{\mathrm{s} \to 0., \mathrm{t} \to 0.774597\}$ $\qquad$ Weight = 0.493827

$$\boldsymbol{N}^{\mathrm{T}} = \{0.0563508,\ 0.0563508,\ 0.443649,\ 0.443649,$$
$$-0.0690154,\ -0.122474,\ -0.543357,\ -0.122474,\ 0.,\ -0.122474,\ 0.,\ -0.122474\}$$

$$\partial\boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} = (\ -0.0563508 \quad 0.0563508 \quad 0.443649 \quad -0.443649 \quad 0. \quad -0.122474 \quad 0. \quad 0.122474 \quad -0.089098$$

$$\partial\boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} =$$
$$(\ -0.25 \quad -0.25 \quad 0.25 \quad 0.25 \quad 0.306186 \quad 0.474342 \quad -0.306186 \quad 0.474342 \quad 0. \quad 0.316228 \quad 0. \quad 0.316228\ )$$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 66.6667 & -5.00896 \\ 0. & 88.8889 \end{pmatrix} \qquad \mathrm{detJ} = 0.00016875$$

$$\boldsymbol{B}^{\mathrm{T}} = \Bigg(\begin{matrix} -2.50448 & 5.00896 & 28.3244 & -30.8289 & -1.53368 & -10.5409 & 1.53368 & 5.78901 & -5. \\ -22.2222 & -22.2222 & 22.2222 & 22.2222 & 27.2166 & 42.1637 & -27.2166 & 42.1637 & 0. \end{matrix}$$

$$\mathrm{k_x} = 45. \qquad \mathrm{k_y} = 45. \qquad \mathrm{p} = 0. \qquad \mathrm{q} = 5. \times 10^{6}$$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{matrix} 1.87537 & 1.80481 & -2.11787 & -1.56231 & -2.25364 & -3.41464 & 2.25364 & -3.568 \\ 1.80481 & 1.94594 & -1.31982 & -2.43093 & -2.29685 & -3.71164 & 2.29685 & -3.404 \\ -2.11787 & -1.31982 & 4.86036 & -1.42268 & 2.10514 & 2.39402 & -2.10514 & 4.128 \\ -1.56231 & -2.43093 & -1.42268 & 5.41592 & 2.44535 & 4.73226 & -2.44535 & 2.844 \\ -2.25364 & -2.29685 & 2.10514 & 2.44535 & 2.7866 & 4.36394 & -2.7866 & 4.270 \\ -3.41464 & -3.71164 & 2.39402 & 4.73226 & 4.36394 & 7.08333 & -4.36394 & 6.437 \\ 2.25364 & 2.29685 & -2.10514 & -2.44535 & -2.7866 & -4.36394 & 2.7866 & -4.270 \\ -3.56801 & -3.4049 & 4.12853 & 2.84439 & 4.27002 & 6.43784 & -4.27002 & 6.792 \\ 0.0557864 & -0.111573 & -0.630915 & 0.686701 & 0.034162 & 0.234795 & -0.034162 & -0.128 \\ -2.25087 & -2.52555 & 1.30693 & 3.46949 & 2.92495 & 4.82981 & -2.92495 & 4.232 \\ 0.439205 & -0.87841 & -4.96718 & 5.40639 & 0.268957 & 1.84854 & -0.268957 & -1.015 \\ -2.40424 & -2.21881 & 3.04144 & 1.58161 & 2.83103 & 4.18431 & -2.83103 & 4.587 \end{matrix} \right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\ 23.4795 \quad 23.4795 \quad 184.854 \quad 184.854 \quad -28.7564 \quad -51.031 \quad -226.399 \quad -51.031 \quad 0. \quad -51.031$$

Gauss point = $\{s \to 0.774597,\ t \to -0.774597\}$ Weight = 0.308642

$$\boldsymbol{N}^{\mathrm{T}} = \{0.1,\ 0.787298,\ 0.1,\ 0.0127017,\ -0.217343,\ -0.217343,\ -0.0276062,\ -0.0276062,\ -0.217343,\ 0.217343,\ -0.0276062,\ 0.0276062\}$$

$$\partial\boldsymbol{N}^{\mathrm{T}}/\partial s = \left(-0.443649 \quad 0.443649 \quad 0.0563508 \quad -0.0563508 \quad 0.841765 \quad -0.122474 \quad 0.106918 \quad 0.12247 \right.$$

$$\partial\boldsymbol{N}^{\mathrm{T}}/\partial t = \left(-0.0563508 \quad -0.443649 \quad 0.443649 \quad 0.0563508 \quad 0.122474 \quad -0.841765 \quad -0.122474 \quad -0.106 \right.$$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 66.6667 & -53.1618 \\ 0. & 119.828 \end{pmatrix} \qquad \det J = 0.000125179$$

$$\boldsymbol{B}^{\mathrm{T}} = \left(\begin{array}{ccccccccc} -26.5809 & 53.1618 & -19.8285 & -6.75243 & 49.6067 & 36.5848 & 13.6388 & 13.8489 & 30. \\ -6.75243 & -53.1618 & 53.1618 & 6.75243 & 14.6759 & -100.867 & -14.6759 & -12.8118 & 14. \end{array} \right.$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5.\times 10^6$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{array}{cccccccc} 1.30767 & -1.83268 & 0.292235 & 0.232781 & -2.46479 & -0.50655 & -0.458006 & -0.489 \\ -1.83268 & 9.82716 & -6.74626 & -1.24821 & 3.22854 & 12.7043 & 2.61705 & 2.464 \\ 0.292235 & -6.74626 & 5.59714 & 0.856888 & -0.353676 & -10.5841 & -1.82663 & -1.661 \\ 0.232781 & -1.24821 & 0.856888 & 0.158544 & -0.410079 & -1.61366 & -0.332409 & -0.312 \\ -2.46479 & 3.22854 & -0.353676 & -0.410079 & 4.65285 & 0.581608 & 0.801832 & 0.867 \\ -0.50655 & 12.7043 & -10.5841 & -1.61366 & 0.581608 & 20.0159 & 3.4412 & 3.127 \\ -0.458006 & 2.61705 & -1.82663 & -0.332409 & 0.801832 & 3.4412 & 0.697874 & 0.655 \\ -0.489599 & 2.46417 & -1.66158 & -0.312991 & 0.867515 & 3.12766 & 0.655293 & 0.618 \\ -1.60033 & 1.49962 & 0.291185 & -0.190476 & 3.03954 & -0.608202 & 0.35827 & 0.417 \\ 0.211923 & -8.21797 & 6.96223 & 1.04382 & -0.153007 & -13.1708 & -2.2296 & -2.019 \\ -0.348205 & 2.39744 & -1.74472 & -0.304515 & 0.596915 & 3.29007 & 0.641534 & 0.598 \\ 0.452176 & -1.89434 & 1.20155 & 0.240612 & -0.813075 & -2.25822 & -0.501399 & -0.478 \end{array} \right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = \left(19.3177 \quad 152.088 \quad 19.3177 \quad 2.45367 \quad -41.9857 \quad -41.9857 \quad -5.33288 \quad -5.33288 \quad -41.9857 \right.$$

Gauss point = $\{s \to 0.774597,\ t \to 0.\}$ Weight = 0.493827

$\boldsymbol{N}^{\mathrm{T}} = \{0.0563508,\ 0.443649,\ 0.443649,\ 0.0563508,\ -0.122474,\ -0.543357,\ -0.122474,\ -0.0690154,\ -0.122474,\ 0.,\ -0.122474,\ 0.\}$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial s =$

$$\begin{pmatrix} -0.25 & 0.25 & 0.25 & -0.25 & 0.474342 & -0.306186 & 0.474342 & 0.306186 & 0.316228 & 0. & 0.316228 & 0. \end{pmatrix}$$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial t = (-0.0563508 \quad -0.443649 \quad 0.443649 \quad 0.0563508 \quad 0.122474 \quad 0. \quad -0.122474 \quad 0. \quad 0.122474 \quad -$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 66.6667 & -29.9571 \\ 0. & 119.828 \end{pmatrix} \qquad \mathrm{detJ} = 0.000125179$$

$$\boldsymbol{B}^{\mathrm{T}} = \left(\begin{matrix} -14.9786 & 29.9571 & 3.37622 & -18.3548 & 27.9538 & -20.4124 & 35.2918 & 20.4124 & 17.41 \\ -6.75243 & -53.1618 & 53.1618 & 6.75243 & 14.6759 & 0. & -14.6759 & 0. & 14.67 \end{matrix} \right.$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^6$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{matrix} 0.750942 & -0.249643 & -1.13925 & 0.637947 & -1.44041 & 0.850517 & -1.19482 & -0.850 \\ -0.249643 & 10.3582 & -7.58038 & -2.52814 & 0.159161 & -1.70103 & 5.1113 & 1.701 \\ -1.13925 & -7.58038 & 7.89344 & 0.826186 & 2.43286 & -0.191709 & -1.83887 & 0.191 \\ 0.637947 & -2.52814 & 0.826186 & 1.064 & -1.15161 & 1.04223 & -2.07761 & -1.042 \\ -1.44041 & 0.159161 & 2.43286 & -1.15161 & 2.77285 & -1.58728 & 2.14517 & 1.587 \\ 0.850517 & -1.70103 & -0.191709 & 1.04223 & -1.58728 & 1.15906 & -2.00395 & -1.159 \\ -1.19482 & 5.1113 & -1.83887 & -2.07761 & 2.14517 & -2.00395 & 4.06384 & 2.003 \\ -0.850517 & 1.70103 & 0.191709 & -1.04223 & 1.58728 & -1.15906 & 2.00395 & 1.159 \\ -1.0012 & -0.719249 & 2.33386 & -0.613408 & 1.95318 & -0.988743 & 1.11033 & 0.988 \\ 0.703293 & 14.1817 & -12.2331 & -2.65182 & -1.79751 & -1.19323 & 5.49459 & 1.193 \\ -0.755618 & 4.23289 & -1.93787 & -1.53941 & 1.3255 & -1.40541 & 3.02901 & 1.405 \\ 0.08933 & 1.80131 & -1.55381 & -0.336826 & -0.228314 & -0.15156 & 0.697905 & 0.151 \end{matrix} \right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\, 17.4171 \quad 137.125 \quad 137.125 \quad 17.4171 \quad -37.8549 \quad -167.943 \quad -37.8549 \quad -21.3315 \quad -37.8549$$

Gauss point = {s → 0.774597, t → 0.774597}   Weight = 0.308642

$$\boldsymbol{N}^{\mathrm{T}} = \{0.0127017, 0.1, 0.787298, 0.1, -0.0276062, -0.217343, -0.217343, -0.0276062, -0.0276062, -0.217343, -0.217343, -0.0276062\}$$

$$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} = (\, -0.0563508 \quad 0.0563508 \quad 0.443649 \quad -0.443649 \quad 0.106918 \quad -0.122474 \quad 0.841765 \quad 0.12247$$

$$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} = (\, -0.0563508 \quad -0.443649 \quad 0.443649 \quad 0.0563508 \quad 0.122474 \quad 0.841765 \quad -0.122474 \quad 0.10691$$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 66.6667 & -6.75243 \\ 0. & 119.828 \end{pmatrix} \qquad \mathrm{detJ} = 0.000125179$$

$$\boldsymbol{B}^{\mathrm{T}} = \left(\begin{array}{ccccccccc} -3.37622 & 6.75243 & 26.5809 & -29.9571 & 6.30088 & -13.8489 & 56.9447 & 7.44301 & 3.9 \\ -6.75243 & -53.1618 & 53.1618 & 6.75243 & 14.6759 & 100.867 & -14.6759 & 12.8118 & 14.6 \end{array} \right.$$

$\mathrm{k_x} = 45.$ $\mathrm{k_y} = 45.$ $\mathrm{p} = 0.$ $\mathrm{q} = 5.\times 10^{6}$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{array}{cccccccc} 0.0990899 & 0.584471 & -0.780133 & 0.0965727 & -0.209277 & -1.10287 & -0.161966 & -0.1 \\ 0.584471 & 4.99285 & -4.60153 & -0.975796 & -1.28248 & -9.48545 & 2.02497 & -1.0 \\ -0.780133 & -4.60153 & 6.14198 & -0.760315 & 1.64764 & 8.68286 & 1.27516 & 1.5 \\ 0.0965727 & -0.975796 & -0.760315 & 1.63954 & -0.155879 & 1.90546 & -3.13816 & -0.2 \\ -0.209277 & -1.28248 & 1.64764 & -0.155879 & 0.443488 & 2.42197 & 0.249347 & 0.4 \\ -1.10287 & -9.48545 & 8.68286 & 1.90546 & 2.42197 & 18.0223 & -3.94478 & 2.0 \\ -0.161966 & 2.02497 & 1.27516 & -3.13816 & 0.249347 & -3.94478 & 6.0122 & 0.4 \\ -0.194097 & -1.09678 & 1.52813 & -0.237249 & 0.408437 & 2.06757 & 0.409985 & 0.3 \\ -0.195331 & -1.31037 & 1.53783 & -0.032131 & 0.41746 & 2.47918 & 0.0141179 & 0.3 \\ -0.719269 & -6.35558 & 5.66279 & 1.41206 & 1.58483 & 12.0804 & -2.89931 & 1.3 \\ -0.0521652 & 1.80536 & 0.410695 & -2.1639 & 0.0444298 & -3.49439 & 4.16025 & 0.1 \\ -0.145374 & -0.699235 & 1.14453 & -0.299919 & 0.302106 & 1.31285 & 0.542778 & 0.2 \end{array} \right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (2.45367 \quad 19.3177 \quad 152.088 \quad 19.3177 \quad -5.33288 \quad -41.9857 \quad -41.9857 \quad -5.33288 \quad -5.33288$

Adding contributions from all Gauss points

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{matrix}
22.5794 & -0.15873 & -11.0913 & -11.3294 & -9.57439 & 1.04492 & 9.57439 \\
-0.15873 & 45.3175 & -22.8175 & -22.3413 & -17.5936 & -2.08984 & 17.5936 \\
-11.0913 & -22.8175 & 34.0675 & -0.15873 & 8.40797 & -7.09574 & -17.5936 \\
-11.3294 & -22.3413 & -0.15873 & 33.8294 & 18.76 & 8.14067 & -9.57439 \\
-9.57439 & -17.5936 & 8.40797 & 18.76 & 35.6548 & 6.13095 & -1.90476 \\
1.04492 & -2.08984 & -7.09574 & 8.14067 & 6.13095 & 58.4524 & 1.36905 \\
9.57439 & 17.5936 & -17.5936 & -9.57439 & -1.90476 & 1.36905 & 35.6548 \\
-7.41165 & 14.8233 & 3.54787 & -10.9595 & -8.69048 & 15.7738 & 1.19048 \\
2.447 & -4.894 & 4.894 & -2.447 & -3.27359 & -0.461069 & -5.44062 \\
-0.269797 & 0.539595 & -0.539595 & 0.269797 & -1.58301 & -3.16601 & 1.58301 \\
-2.447 & 4.894 & -4.894 & 2.447 & 0.368856 & 0.461069 & -3.27359 \\
1.91368 & -3.82736 & 3.82736 & -1.91368 & 2.24387 & -4.22647 & -2.24387
\end{matrix}\right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} =$

$(937.5 \quad 750. \quad 750. \quad 937.5 \quad -688.919 \quad -612.372 \quad -688.919 \quad -765.466 \quad 59.2927 \quad 0. \quad 59.2927 \quad 0.)$

Natural boundary conditions

Specified NBC values for side 2: $\alpha = -55$ $\qquad \beta = 1100$

Interpolation functions for mapping: $\left\{0, \frac{1-\mathrm{a}}{2}, \frac{\mathrm{a}+1}{2}, 0\right\}$

Interpolation functions for solution: $\left\{0, \frac{1-\mathrm{a}}{2}, \frac{\mathrm{a}+1}{2}, 0, 0, \frac{\frac{3\,\mathrm{a}^2}{2} - \frac{3}{2}}{\sqrt{6}}, 0, 0, 0, \frac{\frac{5\,\mathrm{a}^3}{2} - \frac{5\,\mathrm{a}}{2}}{\sqrt{10}}, 0, 0\right\}$

$x(\mathrm{a}) = 0.03 \qquad y(\mathrm{a}) = 0.0075\,\mathrm{a} + 0.0225$

$J_c = 0.0075$

Value in mapped coordinate: $\alpha(a) = -55$ $\beta(a) = 1100$

Gauss point = -0.774597 Weight = 0.555556

$\boldsymbol{N}_c^T = \{0., 0.887298, 0.112702, 0., 0., -0.244949, 0., 0., 0., 0.244949, 0., 0.\}$

$J_c = 0.0075$ $\alpha = -55.$ $\beta = 1100.$

$\boldsymbol{k}_\alpha =$

$$\begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & -0.180423 & -0.0229167 & 0. & 0. & 0.0498077 & 0. & 0. & 0. & -0.0498077 & 0. & 0. \\
0. & -0.0229167 & -0.0029108 & 0. & 0. & 0.00632641 & 0. & 0. & 0. & -0.00632641 & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0.0498077 & 0.00632641 & 0. & 0. & -0.01375 & 0. & 0. & 0. & 0.01375 & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & -0.0498077 & -0.00632641 & 0. & 0. & 0.01375 & 0. & 0. & 0. & -0.01375 & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$$\boldsymbol{r}_\beta = \begin{pmatrix}
0. \\ -4.06678 \\ -0.516549 \\ 0. \\ 0. \\ 1.12268 \\ 0. \\ 0. \\ 0. \\ -1.12268 \\ 0. \\ 0.
\end{pmatrix}$$

Gauss point = 0. Weight = 0.888889

$\boldsymbol{N}_c^T = \{0., 0.5, 0.5, 0., 0., -0.612372, 0., 0., 0., 0., 0., 0.\}$

$\mathrm{J_c} = 0.0075$ $\alpha = -55.$ $\beta = 1100.$

$\boldsymbol{k}_\alpha =$

$$\begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & -0.0916667 & -0.0916667 & 0. & 0. & 0.112268 & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & -0.0916667 & -0.0916667 & 0. & 0. & 0.112268 & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0.112268 & 0.112268 & 0. & 0. & -0.1375 & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}
\qquad
\boldsymbol{r}_\beta = \begin{pmatrix}
0. \\ -3.66667 \\ -3.66667 \\ 0. \\ 0. \\ 4.49073 \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \\ 0.
\end{pmatrix}$$

Gauss point = 0.774597 Weight = 0.555556

$\boldsymbol{N}_\mathrm{c}^\mathrm{T} = \{0., 0.112702, 0.887298, 0., 0., -0.244949, 0., 0., 0., -0.244949, 0., 0.\}$

$\mathrm{J_c} = 0.0075$ $\alpha = -55.$ $\beta = 1100.$

$\boldsymbol{k}_{\alpha} =$

$$\begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & -0.0029108 & -0.0229167 & 0. & 0. & 0.00632641 & 0. & 0. & 0. & 0.00632641 & 0. & 0. \\
0. & -0.0229167 & -0.180423 & 0. & 0. & 0.0498077 & 0. & 0. & 0. & 0.0498077 & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0.00632641 & 0.0498077 & 0. & 0. & -0.01375 & 0. & 0. & 0. & -0.01375 & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0.00632641 & 0.0498077 & 0. & 0. & -0.01375 & 0. & 0. & 0. & -0.01375 & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$$\boldsymbol{r}_{\beta} = \begin{pmatrix}
0. \\
-0.516549 \\
-4.06678 \\
0. \\
0. \\
1.12268 \\
0. \\
0. \\
0. \\
1.12268 \\
0. \\
0.
\end{pmatrix}$$

Adding contributions from all Gauss points

$$\boldsymbol{k}_{\alpha} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & -0.275 & -0.1375 & 0. & 0. & 0.168402 & 0. & 0. & 0. & -0.0434813 & 0. & 0. \\ 0. & -0.1375 & -0.275 & 0. & 0. & 0.168402 & 0. & 0. & 0. & 0.0434813 & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0.168402 & 0.168402 & 0. & 0. & -0.165 & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & -0.0434813 & 0.0434813 & 0. & 0. & 0. & 0. & 0. & 0. & -0.0275 & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\beta} = \begin{pmatrix} 0. \\ -8.25 \\ -8.25 \\ 0. \\ 0. \\ 6.7361 \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \end{pmatrix}$$

Specified NBC values for side 3: $\alpha = -55$ $\beta = 1100$

Interpolation functions for mapping: $\left\{0,\ 0,\ \frac{1-a}{2},\ \frac{a+1}{2}\right\}$

Interpolation functions for solution: $\left\{0,\ 0,\ \frac{1-a}{2},\ \frac{a+1}{2},\ 0,\ 0,\ \frac{\frac{3a^2}{2}-\frac{3}{2}}{\sqrt{6}},\ 0,\ 0,\ 0,\ \frac{\frac{5a}{2}-\frac{5a^3}{2}}{\sqrt{10}},\ 0\right\}$

$x(a) = 0.015 - 0.015\,a$ $y(a) = 0.03$

$J_c = 0.015$

Value in mapped coordinate: $\alpha(a) = -55$ $\beta(a) = 1100$

Gauss point = −0.774597 Weight = 0.555556

$\boldsymbol{N}_c^T = \{0., 0., 0.887298, 0.112702, 0., 0., -0.244949, 0., 0., 0., -0.244949, 0.\}$

$J_c = 0.015 \qquad \alpha = -55. \qquad \beta = 1100.$

$$\boldsymbol{k}_\alpha = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & -0.360845 & -0.0458333 & 0. & 0. & 0.0996155 & 0. & 0. & 0. & 0.0996155 & 0. \\ 0. & 0. & -0.0458333 & -0.0058216 & 0. & 0. & 0.0126528 & 0. & 0. & 0. & 0.0126528 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0.0996155 & 0.0126528 & 0. & 0. & -0.0275 & 0. & 0. & 0. & -0.0275 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0.0996155 & 0.0126528 & 0. & 0. & -0.0275 & 0. & 0. & 0. & -0.0275 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_\beta = \begin{pmatrix} 0. \\ 0. \\ -8.13357 \\ -1.0331 \\ 0. \\ 0. \\ 2.24537 \\ 0. \\ 0. \\ 0. \\ 2.24537 \\ 0. \end{pmatrix}$$

Gauss point = 0. Weight = 0.888889

$\boldsymbol{N}_c^T = \{0., 0., 0.5, 0.5, 0., 0., -0.612372, 0., 0., 0., 0., 0.\}$

$J_c = 0.015 \qquad \alpha = -55. \qquad \beta = 1100.$

$\boldsymbol{k}_{\alpha} =$

$$\begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & -0.183333 & -0.183333 & 0. & 0. & 0.224537 & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & -0.183333 & -0.183333 & 0. & 0. & 0.224537 & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0.224537 & 0.224537 & 0. & 0. & -0.275 & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix} \qquad \boldsymbol{r}_{\beta} = \begin{pmatrix} 0. \\ 0. \\ -7.33333 \\ -7.33333 \\ 0. \\ 0. \\ 8.98146 \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \end{pmatrix}$$

Gauss point = 0.774597 Weight = 0.555556

$\boldsymbol{N}_{c}^{T} = \{0., 0., 0.112702, 0.887298, 0., 0., -0.244949, 0., 0., 0., 0.244949, 0.\}$

$J_{c} = 0.015$ $\alpha = -55.$ $\beta = 1100.$

$$\boldsymbol{k}_{\alpha} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & -0.0058216 & -0.0458333 & 0. & 0. & 0.0126528 & 0. & 0. & 0. & -0.0126528 & 0. \\ 0. & 0. & -0.0458333 & -0.360845 & 0. & 0. & 0.0996155 & 0. & 0. & 0. & -0.0996155 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0.0126528 & 0.0996155 & 0. & 0. & -0.0275 & 0. & 0. & 0. & 0.0275 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & -0.0126528 & -0.0996155 & 0. & 0. & 0.0275 & 0. & 0. & 0. & -0.0275 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\beta} = \begin{pmatrix} 0. \\ 0. \\ -1.0331 \\ -8.13357 \\ 0. \\ 0. \\ 2.24537 \\ 0. \\ 0. \\ 0. \\ -2.24537 \\ 0. \end{pmatrix}$$

Adding contributions from all Gauss points

$$\boldsymbol{k}_{\alpha} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & -0.55 & -0.275 & 0. & 0. & 0.336805 & 0. & 0. & 0. & 0.0869626 & 0. \\ 0. & 0. & -0.275 & -0.55 & 0. & 0. & 0.336805 & 0. & 0. & 0. & -0.0869626 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0.336805 & 0.336805 & 0. & 0. & -0.33 & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0.0869626 & -0.0869626 & 0. & 0. & 0. & 0. & 0. & 0. & -0.055 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\beta} = \begin{pmatrix} 0. \\ 0. \\ -16.5 \\ -16.5 \\ 0. \\ 0. \\ 13.4722 \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \end{pmatrix}$$

Specified NBC values for side 4: $\alpha = 0$ $\beta = 8000$

Interpolation functions for mapping: $\left\{\frac{a+1}{2}, 0, 0, \frac{1-a}{2}\right\}$

Interpolation functions for solution: $\left\{\frac{a+1}{2}, 0, 0, \frac{1-a}{2}, 0, 0, 0, \frac{\frac{3a^2}{2} - \frac{3}{2}}{\sqrt{6}}, 0, 0, 0, \frac{\frac{5a}{2} - \frac{5a^3}{2}}{\sqrt{10}}\right\}$

$x(a) = 0$ $y(a) = 0.015 - 0.015\,a$

$J_c = 0.015$

Value in mapped coordinate: $\alpha(a) = 0$ $\beta(a) = 8000$

Gauss point = −0.774597 Weight = 0.555556

$\boldsymbol{N}_{\mathrm{c}}^{\mathrm{T}} = \{0.112702, 0., 0., 0.887298, 0., 0., 0., -0.244949, 0., 0., 0., -0.244949\}$

$\mathrm{J}_{\mathrm{c}} = 0.015 \qquad \alpha = 0. \qquad \beta = 8000.$

$$\boldsymbol{k}_\alpha = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix} \qquad \boldsymbol{r}_\beta = \begin{pmatrix} -7.51344 \\ 0. \\ 0. \\ -59.1532 \\ 0. \\ 0. \\ 0. \\ 16.3299 \\ 0. \\ 0. \\ 0. \\ 16.3299 \end{pmatrix}$$

Gauss point = 0. Weight = 0.888889

$\boldsymbol{N}_{\mathrm{c}}^{\mathrm{T}} = \{0.5, 0., 0., 0.5, 0., 0., 0., -0.612372, 0., 0., 0., 0.\}$

$\mathrm{J}_{\mathrm{c}} = 0.015 \qquad \alpha = 0. \qquad \beta = 8000.$

$$\boldsymbol{k}_\alpha = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix} \qquad \boldsymbol{r}_\beta = \begin{pmatrix} -53.3333 \\ 0. \\ 0. \\ -53.3333 \\ 0. \\ 0. \\ 0. \\ 65.3197 \\ 0. \\ 0. \\ 0. \\ 0. \end{pmatrix}$$

Gauss point = 0.774597 Weight = 0.555556

$\boldsymbol{N}_{\mathrm{c}}^{\mathrm{T}} = \{0.887298, 0., 0., 0.112702, 0., 0., 0., -0.244949, 0., 0., 0., 0.244949\}$

$\mathrm{J}_{\mathrm{c}} = 0.015 \qquad \alpha = 0. \qquad \beta = 8000.$

$$\boldsymbol{k}_\alpha = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix} \qquad \boldsymbol{r}_\beta = \begin{pmatrix} -59.1532 \\ 0. \\ 0. \\ -7.51344 \\ 0. \\ 0. \\ 0. \\ 16.3299 \\ 0. \\ 0. \\ 0. \\ -16.3299 \end{pmatrix}$$

Adding contributions from all Gauss points

$$\boldsymbol{k}_\alpha = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix} \qquad \boldsymbol{r}_\beta = \begin{pmatrix} -120. \\ 0. \\ 0. \\ -120. \\ 0. \\ 0. \\ 0. \\ 97.9796 \\ 0. \\ 0. \\ 0. \\ 0. \end{pmatrix}$$

$$\begin{pmatrix}
22.5794 & -0.15873 & -11.0913 & -11.3294 & -9.57439 & 1.04492 & 9.57439 & -7.41165 & 2 \\
-0.15873 & 45.0425 & -22.955 & -22.3413 & -17.5936 & -1.92144 & 17.5936 & 14.8233 & -4 \\
-11.0913 & -22.955 & 33.2425 & -0.43373 & 8.40797 & -6.92734 & -17.2568 & 3.54787 & 4 \\
-11.3294 & -22.3413 & -0.43373 & 33.2794 & 18.76 & 8.14067 & -9.23759 & -10.9595 & -2 \\
-9.57439 & -17.5936 & 8.40797 & 18.76 & 35.6548 & 6.13095 & -1.90476 & -8.69048 & -3 \\
1.04492 & -1.92144 & -6.92734 & 8.14067 & 6.13095 & 58.2874 & 1.36905 & 15.7738 & -0 \\
9.57439 & 17.5936 & -17.2568 & -9.23759 & -1.90476 & 1.36905 & 35.3248 & 1.19048 & -5 \\
-7.41165 & 14.8233 & 3.54787 & -10.9595 & -8.69048 & 15.7738 & 1.19048 & 47.2381 & 0 \\
2.447 & -4.894 & 4.894 & -2.447 & -3.27359 & -0.461069 & -5.44062 & 0.230535 & 24 \\
-0.269797 & 0.496114 & -0.496114 & 0.269797 & -1.58301 & -3.16601 & 1.58301 & 1.58301 & 0 \\
-2.447 & 4.894 & -4.80704 & 2.36004 & 0.368856 & 0.461069 & -3.27359 & -0.230535 & 8 \\
1.91368 & -3.82736 & 3.82736 & -1.91368 & 2.24387 & -4.22647 & -2.24387 & -2.24387 & -0
\end{pmatrix}$$

Global equations after assembling all elements

$$\begin{pmatrix}
47.7183 & 19.8611 & -5.97222 & -39.1865 & -11.0913 & -11.3294 & -29.6466 & -4.13108 \\
19.8611 & 70.1389 & -39.0278 & -50.9722 & 0 & 0 & -43.8381 & 31.299 \\
-5.97222 & -39.0278 & 43.7556 & 0.419444 & 0 & 0 & 14.1915 & -16.6701 \\
-39.1865 & -50.9722 & 0.419444 & 133.798 & -22.955 & -22.3413 & 59.2932 & -37.6658 \\
-11.0913 & 0 & 0 & -22.955 & 33.2425 & -0.43373 & 0 & 8.40797 \\
-11.3294 & 0 & 0 & -22.3413 & -0.43373 & 33.2794 & 0 & 18.76 \\
-29.6466 & -43.8381 & 14.1915 & 59.2932 & 0 & 0 & 58.5952 & -34.7619 \\
-4.13108 & 31.299 & -16.6701 & -37.6658 & 8.40797 & 18.76 & -34.7619 & 132.738 \\
-7.41165 & 0 & 0 & 14.8233 & 3.54787 & -10.9595 & 0 & -8.69048 \\
-5.44331 & 5.44331 & -20.0722 & 20.0722 & 0 & 0 & 4.7619 & 37.9167 \\
4.17968 & 32.5627 & -28.0462 & -8.02256 & 0 & 0 & -13.6905 & 24.5238 \\
1.04492 & 0 & 0 & -1.92144 & -6.92734 & 8.14067 & 0 & 6.13095 \\
9.57439 & 0 & 0 & 17.5936 & -17.2568 & -9.23759 & 0 & -1.90476 \\
7.65472 & -7.65472 & 15.3094 & -15.3094 & 0 & 0 & -8.97549 & 8.97549 \\
3.76462 & -1.31762 & 2.63523 & -7.52923 & 4.894 & -2.447 & 0.922139 & -18.4428 \\
1.91368 & 0 & 0 & -3.82736 & 3.82736 & -1.91368 & 0 & 2.24387 \\
-1.31762 & 1.31762 & -2.63523 & 2.63523 & 0 & 0 & -0.922139 & 3.55023 \\
-1.07919 & 1.07919 & -2.07142 & 2.07142 & 0 & 0 & 6.33202 & -6.33202 \\
-0.269797 & 0 & 0 & 0.496114 & -0.496114 & 0.269797 & 0 & -1.58301 \\
-2.447 & 0 & 0 & 4.894 & -4.80704 & 2.36004 & 0 & 0.368856
\end{pmatrix} -$$

Essential boundary conditions

On element 1, side 1, specified value = 110

$$\left\{T_1,\ T_2,\ \delta_1^{\{1,2\}},\ \delta_2^{\{1,2\}}\right\} = \{110,\ 110,\ 0,\ 0\}$$

Known values from EBC

$$\left\{T_1 = 110,\ T_2 = 110,\ \delta_1^{\{1,2\}} = 0,\ \delta_2^{\{1,2\}} = 0\right\}$$

Global equations after EBC

$$\begin{pmatrix}
43.7556 & 0.419444 & 0 & 0 & -16.6701 & 0 & -20.0722 & -28.0462 & 0 \\
0.419444 & 133.798 & -22.955 & -22.3413 & -37.6658 & 14.8233 & 20.0722 & -8.02256 & -1 \\
0 & -22.955 & 33.2425 & -0.43373 & 8.40797 & 3.54787 & 0 & 0 & -6 \\
0 & -22.3413 & -0.43373 & 33.2794 & 18.76 & -10.9595 & 0 & 0 & 8 \\
-16.6701 & -37.6658 & 8.40797 & 18.76 & 132.738 & -8.69048 & 37.9167 & 24.5238 & 6 \\
0 & 14.8233 & 3.54787 & -10.9595 & -8.69048 & 47.2381 & 0 & 0 & 15 \\
-20.0722 & 20.0722 & 0 & 0 & 37.9167 & 0 & 97.0833 & 5.47619 & 0 \\
-28.0462 & -8.02256 & 0 & 0 & 24.5238 & 0 & 5.47619 & 49.551 & 0 \\
0 & -1.92144 & -6.92734 & 8.14067 & 6.13095 & 15.7738 & 0 & 0 & 58 \\
0 & 17.5936 & -17.2568 & -9.23759 & -1.90476 & 1.19048 & 0 & 0 & 1 \\
2.63523 & -7.52923 & 4.894 & -2.447 & -18.4428 & 0.230535 & -19.6877 & -1.84428 & -0 \\
0 & -3.82736 & 3.82736 & -1.91368 & 2.24387 & -2.24387 & 0 & 0 & -4 \\
-2.63523 & 2.63523 & 0 & 0 & 3.55023 & 0 & -15.1692 & 1.84428 & 0 \\
-2.07142 & 2.07142 & 0 & 0 & -6.33202 & 0 & 6.33202 & -12.664 & 0 \\
0 & 0.496114 & -0.496114 & 0.269797 & -1.58301 & 1.58301 & 0 & 0 & -3 \\
0 & 4.894 & -4.80704 & 2.36004 & 0.368856 & -0.230535 & 0 & 0 & 0
\end{pmatrix}$$

Solving the final system of global equations we get

$$\{T_3 = 125.645,\ T_4 = 138.336,\ T_5 = 161.689,\ T_6 = 159.31,\ \delta_1^{\{1,4\}} = -10.8975,\ \delta_1^{\{1,6\}} = -15.6733,$$
$$\delta_1^{\{2,3\}} = -6.12049,\ \delta_1^{\{3,4\}} = 6.06916,\ \delta_1^{\{4,5\}} = -5.59893,\ \delta_1^{\{5,6\}} = 3.005,\ \delta_2^{\{1,4\}} = -1.5215,$$
$$\delta_2^{\{1,6\}} = -0.51993,\ \delta_2^{\{2,3\}} = 0.300845,\ \delta_2^{\{3,4\}} = 0.807652,\ \delta_2^{\{4,5\}} = -0.0985235,\ \delta_2^{\{5,6\}} = 3.19562\}$$

Solution for element 1

DOF values for the element

$$\{T_1 = 110,\ T_2 = 110,\ T_3 = 125.645,\ T_4 = 138.336,\ \delta_1^{\{1,2\}} = 0,\ \delta_1^{\{2,3\}} = -6.12049,$$
$$\delta_1^{\{3,4\}} = 6.06916,\ \delta_1^{\{1,4\}} = -10.8975,\ \delta_2^{\{1,2\}} = 0,\ \delta_2^{\{2,3\}} = 0.300845,\ \delta_2^{\{3,4\}} = 0.807652,\ \delta_2^{\{1,4\}} = -1.5215\}$$

$$\boldsymbol{d}^{\mathrm{T}} = (\ 110\quad 110\quad 125.645\quad 138.336\quad 0\quad -6.12049\quad 6.06916\quad -10.8975\quad 0\quad 0.300845\quad 0.807652\quad -1.5215\ )$$

Mapping

$$x(s,t) = -0.0075\,t\,s + 0.0225\,s + 0.0075\,t + 0.0375$$

$$y(s,t) = 0.0075\,t + 0.0075$$

$$\boldsymbol{J} = \begin{pmatrix} 0.0225 - 0.0075\,t & 0.0075 - 0.0075\,s \\ 0 & 0.0075 \end{pmatrix}$$

Element solution at {s → 0, t → 0}

Location: {0.0375, 0.0075}

$\boldsymbol{N}^{\mathrm{T}} = (\ 0.25\ \ 0.25\ \ 0.25\ \ 0.25\ \ -0.306186\ \ -0.306186\ \ -0.306186\ \ -0.306186\ \ 0.\ \ 0.\ \ 0.\ \ 0.\ )$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} =$

$(\ -0.25\ \ 0.25\ \ 0.25\ \ -0.25\ \ 0.\ \ -0.306186\ \ 0.\ \ 0.306186\ \ -0.395285\ \ 0.\ \ -0.395285\ \ 0.\ )$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} = (\ -0.25\ \ -0.25\ \ 0.25\ \ 0.25\ \ 0.306186\ \ 0.\ \ -0.306186\ \ 0.\ \ 0.\ \ -0.395285\ \ 0.\ \ -0.395285\ )$

$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 44.4444 & 0. \\ -44.4444 & 133.333 \end{pmatrix}$

$\boldsymbol{B}_{\mathrm{x}}^{\mathrm{T}} = \partial \boldsymbol{N}^{\mathrm{T}}/\partial x =$

$(\ -11.1111\ \ 11.1111\ \ 11.1111\ \ -11.1111\ \ 0.\ \ -13.6083\ \ 0.\ \ 13.6083\ \ -17.5682\ \ 0.\ \ -17.5682\ \ 0.\ )$

$\boldsymbol{B}_{\mathrm{y}}^{\mathrm{T}} = \partial \boldsymbol{N}^{\mathrm{T}}/\partial y = (\ -22.2222\ \ -44.4444\ \ 22.2222\ \ 44.4444\ \ 40.8248\ \ 13.6083\ \ -40.8248\ \ -13.6083\ \ 1$

$\mathrm{T} = \boldsymbol{N}^{\mathrm{T}}\boldsymbol{d} = 124.347$

$\partial \mathrm{T}/\partial x = \boldsymbol{B}_{\mathrm{x}}^{\mathrm{T}}\boldsymbol{d} = -220.205$

$\partial \mathrm{T}/\partial y = \boldsymbol{B}_{\mathrm{y}}^{\mathrm{T}}\boldsymbol{d} = 1502.78$

	x	y	T	$\partial \mathrm{T}/\partial x$	$\partial \mathrm{T}/\partial y$
1	0.0375	0.0075	124.347	−220.205	1502.78

Solution for element 2

DOF values for the element

$\{\mathrm{T}_1 = 110,\ \mathrm{T}_4 = 138.336,\ \mathrm{T}_5 = 161.689,\ \mathrm{T}_6 = 159.31,\ \delta_1^{\{1,4\}} = -10.8975,\ \delta_1^{\{4,5\}} = -5.59893,\ \delta_1^{\{5,6\}} = 3.005,$
$\delta_1^{\{1,6\}} = -15.6733,\ \delta_2^{\{1,4\}} = -1.5215,\ \delta_2^{\{4,5\}} = -0.0985235,\ \delta_2^{\{5,6\}} = 3.19562,\ \delta_2^{\{1,6\}} = -0.51993\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 110\ \ 138.336\ \ 161.689\ \ 159.31\ \ -10.8975\ \ -5.59893\ \ 3.005\ \ -15.6733\ \ -1.5215\ \ -0.0985235\ \ 3.195($

Mapping

$x(\mathrm{s},\mathrm{t}) = 0.015\,\mathrm{s} + 0.015$

$y(\mathrm{s},\mathrm{t}) = -0.00375\,\mathrm{t}\,\mathrm{s} + 0.00375\,\mathrm{s} + 0.01125\,\mathrm{t} + 0.01875$

$\boldsymbol{J} = \begin{pmatrix} 0.015 & 0 \\ 0.00375 - 0.00375\,\mathrm{t} & 0.01125 - 0.00375\,\mathrm{s} \end{pmatrix}$

Element solution at {s → 0, t → 0}

Location: {0.015, 0.01875}

$\boldsymbol{N}^{\mathrm{T}} = (\ 0.25\ \ 0.25\ \ 0.25\ \ 0.25\ \ -0.306186\ \ -0.306186\ \ -0.306186\ \ -0.306186\ \ 0.\ \ 0.\ \ 0.\ \ 0.\ )$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} =$

$(\ -0.25 \quad 0.25 \quad 0.25 \quad -0.25 \quad 0. \quad -0.306186 \quad 0. \quad 0.306186 \quad -0.395285 \quad 0. \quad -0.395285 \quad 0.\ )$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} = (\ -0.25 \quad -0.25 \quad 0.25 \quad 0.25 \quad 0.306186 \quad 0. \quad -0.306186 \quad 0. \quad 0. \quad -0.395285 \quad 0. \quad -0.395285\ )$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 66.6667 & -22.2222 \\ 0. & 88.8889 \end{pmatrix}$$

$\boldsymbol{B}_{\mathrm{x}}^{\mathrm{T}} = \partial \boldsymbol{N}^{\mathrm{T}}/\partial x = (\ -11.1111 \quad 22.2222 \quad 11.1111 \quad -22.2222 \quad -6.80414 \quad -20.4124 \quad 6.80414 \quad 20.4124 \quad -$

$\boldsymbol{B}_{\mathrm{y}}^{\mathrm{T}} = \partial \boldsymbol{N}^{\mathrm{T}}/\partial y =$

$(\ -22.2222 \quad -22.2222 \quad 22.2222 \quad 22.2222 \quad 27.2166 \quad 0. \quad -27.2166 \quad 0. \quad 0. \quad -35.1364 \quad 0. \quad -35.1364\ )$

$\mathrm{T} = \boldsymbol{N}^{\mathrm{T}}\boldsymbol{d} = 151.263$

$\partial \mathrm{T}/\partial x = \boldsymbol{B}_{\mathrm{x}}^{\mathrm{T}}\boldsymbol{d} = -52.3684$

$\partial \mathrm{T}/\partial y = \boldsymbol{B}_{\mathrm{y}}^{\mathrm{T}}\boldsymbol{d} = 1258.08$

	x	y	T	$\partial \mathrm{T}/\partial x$	$\partial \mathrm{T}/\partial y$
1	0.015	0.01875	151.263	−52.3684	1258.08

Nodal solution summary

dof	x	y	Value
T_1	0	0	110
T_2	0.06	0	110
T_3	0.06	0.015	125.645
T_4	0.03	0.015	138.336
T_5	0.03	0.03	161.689
T_6	0	0.03	159.31

Element solution summary

	x	y	T	$\partial \mathrm{T}/\partial x$	$\partial \mathrm{T}/\partial y$
1	0.0375	0.0075	124.347	−220.205	1502.78
2	0.015	0.01875	151.263	−52.3684	1258.08

With $n = 4$ the following finite element solution is obtained.

Interpolation functions for mapping: $\left\{\frac{1}{4}(1-s)(1-t),\ \frac{1}{4}(s+1)(1-t),\ \frac{1}{4}(s+1)(t+1),\ \frac{1}{4}(1-s)(t+1)\right\}$

Interpolation functions for assumed solution: $\boldsymbol{N}^{\mathrm{T}} =$

$$\left\{\frac{1}{4}(1-s)(1-t),\ \frac{1}{4}(s+1)(1-t),\ \frac{1}{4}(s+1)(t+1),\ \frac{1}{4}(1-s)(t+1),\ \frac{\left(\frac{3s^2}{2}-\frac{3}{2}\right)(1-t)}{2\sqrt{6}},\right.$$
$$\frac{(s+1)\left(\frac{3t^2}{2}-\frac{3}{2}\right)}{2\sqrt{6}},\ \frac{\left(\frac{3s^2}{2}-\frac{3}{2}\right)(t+1)}{2\sqrt{6}},\ \frac{(1-s)\left(\frac{3t^2}{2}-\frac{3}{2}\right)}{2\sqrt{6}},\ \frac{\left(\frac{5s^3}{2}-\frac{5s}{2}\right)(1-t)}{2\sqrt{10}},\ \frac{(s+1)\left(\frac{5t^3}{2}-\frac{5t}{2}\right)}{2\sqrt{10}},$$
$$\frac{\left(\frac{5s^3}{2}-\frac{5s}{2}\right)(t+1)}{2\sqrt{10}},\ \frac{(1-s)\left(\frac{5t^3}{2}-\frac{5t}{2}\right)}{2\sqrt{10}},\ \frac{\left(\frac{35s^4}{8}-\frac{21s^2}{4}+\frac{7}{8}\right)(1-t)}{2\sqrt{14}},\ \frac{(s+1)\left(\frac{35t^4}{8}-\frac{21t^2}{4}+\frac{7}{8}\right)}{2\sqrt{14}},$$
$$\left.\frac{\left(\frac{35s^4}{8}-\frac{21s^2}{4}+\frac{7}{8}\right)(t+1)}{2\sqrt{14}},\ \frac{(1-s)\left(\frac{35t^4}{8}-\frac{21t^2}{4}+\frac{7}{8}\right)}{2\sqrt{14}},\ \frac{1}{6}\left(\frac{3s^2}{2}-\frac{3}{2}\right)\left(\frac{3t^2}{2}-\frac{3}{2}\right)\right\}$$

$$\partial\boldsymbol{N}^{\mathrm{T}}/\partial s = \left\{\frac{t-1}{4},\ \frac{1-t}{4},\ \frac{t+1}{4},\ \frac{1}{4}(-t-1),\ \frac{1}{2}\sqrt{\frac{3}{2}}\,s(1-t),\ \frac{\frac{3t^2}{2}-\frac{3}{2}}{2\sqrt{6}},\ \frac{1}{2}\sqrt{\frac{3}{2}}\,s(t+1),\ -\frac{\frac{3t^2}{2}-\frac{3}{2}}{2\sqrt{6}},\right.$$
$$\frac{\left(\frac{15s^2}{2}-\frac{5}{2}\right)(1-t)}{2\sqrt{10}},\ \frac{\frac{5t^3}{2}-\frac{5t}{2}}{2\sqrt{10}},\ \frac{\left(\frac{15s^2}{2}-\frac{5}{2}\right)(t+1)}{2\sqrt{10}},\ -\frac{\frac{5t^3}{2}-\frac{5t}{2}}{2\sqrt{10}},\ \frac{\left(\frac{35s^3}{2}-\frac{21s}{2}\right)(1-t)}{2\sqrt{14}},$$
$$\left.\frac{\frac{35t^4}{8}-\frac{21t^2}{4}+\frac{7}{8}}{2\sqrt{14}},\ \frac{\left(\frac{35s^3}{2}-\frac{21s}{2}\right)(t+1)}{2\sqrt{14}},\ -\frac{\frac{35t^4}{8}-\frac{21t^2}{4}+\frac{7}{8}}{2\sqrt{14}},\ \frac{1}{2}s\left(\frac{3t^2}{2}-\frac{3}{2}\right)\right\}$$

$$\partial\boldsymbol{N}^{\mathrm{T}}/\partial t = \left\{\frac{s-1}{4},\ \frac{1}{4}(-s-1),\ \frac{s+1}{4},\ \frac{1-s}{4},\ -\frac{\frac{3s^2}{2}-\frac{3}{2}}{2\sqrt{6}},\ \frac{1}{2}\sqrt{\frac{3}{2}}\,(s+1)\,t,\ \frac{\frac{3s^2}{2}-\frac{3}{2}}{2\sqrt{6}},\ \frac{1}{2}\sqrt{\frac{3}{2}}\,(1-s)\,t,\right.$$
$$-\frac{\frac{5s^3}{2}-\frac{5s}{2}}{2\sqrt{10}},\ \frac{(s+1)\left(\frac{15t^2}{2}-\frac{5}{2}\right)}{2\sqrt{10}},\ \frac{\frac{5s^3}{2}-\frac{5s}{2}}{2\sqrt{10}},\ \frac{(1-s)\left(\frac{15t^2}{2}-\frac{5}{2}\right)}{2\sqrt{10}},\ -\frac{\frac{35s^4}{8}-\frac{21s^2}{4}+\frac{7}{8}}{2\sqrt{14}},$$
$$\left.\frac{(s+1)\left(\frac{35t^3}{2}-\frac{21t}{2}\right)}{2\sqrt{14}},\ \frac{\frac{35s^4}{8}-\frac{21s^2}{4}+\frac{7}{8}}{2\sqrt{14}},\ \frac{(1-s)\left(\frac{35t^3}{2}-\frac{21t}{2}\right)}{2\sqrt{14}},\ \frac{1}{2}\left(\frac{3s^2}{2}-\frac{3}{2}\right)t\right\}$$

Use 4×4 Gauss quadrature for integration.

Global equations at start of the element assembly process

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ \delta_1^{\{1,2\}} \\ \delta_1^{\{1,4\}} \\ \delta_1^{\{1,6\}} \\ \delta_1^{\{2,3\}} \\ \delta_1^{\{3,4\}} \\ \delta_1^{\{4,5\}} \\ \delta_1^{\{5,6\}} \\ \delta_2^{\{1,2\}} \\ \delta_2^{\{1,4\}} \\ \delta_2^{\{1,6\}} \\ \delta_2^{\{2,3\}} \\ \delta_2^{\{3,4\}} \\ \delta_2^{\{4,5\}} \\ \delta_2^{\{5,6\}} \\ \delta_3^{\{1,2\}} \\ \delta_3^{\{1,4\}} \\ \delta_3^{\{1,6\}} \\ \delta_3^{\{2,3\}} \\ \delta_3^{\{3,4\}} \\ \delta_3^{\{4,5\}} \\ \delta_3^{\{5,6\}} \\ \delta_1^1 \\ \delta_1^2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 1

Element coordinates: ({0, 0}   {0.06, 0}   {0.06, 0.015}   {0.03, 0.015})

$x(\mathrm{s},\mathrm{t}) = -0.0075\,\mathrm{t}\,\mathrm{s} + 0.0225\,\mathrm{s} + 0.0075\,\mathrm{t} + 0.0375$

$y(s,t) = 0.0075\,t + 0.0075$

$$\boldsymbol{J} = \begin{pmatrix} 0.0225 - 0.0075\,t & 0.0075 - 0.0075\,s \\ 0 & 0.0075 \end{pmatrix}$$

$\det J = 0.00016875 - 0.00005625\,t$

Given element data

$k_x = 45 \qquad k_y = 45 \qquad p = 0 \qquad q = 5000000$

Element data in mapped coordinates

$k_x = 45 \qquad k_y = 45 \qquad p = 0 \qquad q = 5000000$

Gauss point = $\{s \to -0.861136,\ t \to -0.861136\}$ Weight = 0.121003

$\boldsymbol{N}^T = \{0.865957,\ 0.0646111,\ 0.00482078,\ 0.0646111,\ -0.147276,\ -0.0109886,\ -0.0109886,\ -0.147276,$
$0.16373,\ 0.0122162,\ 0.0122162,\ 0.16373,\ -0.15229,\ -0.0113627,\ -0.0113627,\ -0.15229,\ 0.0250475\}$

$\partial\boldsymbol{N}^T/\partial s = (\ -0.465284 \quad 0.465284 \quad 0.0347159 \quad -0.0347159 \quad -0.981444 \quad -0.0791321 \quad -0.0732278 \quad 0.0$

$\partial\boldsymbol{N}^T/\partial t = (\ -0.465284 \quad -0.0347159 \quad 0.0347159 \quad 0.465284 \quad 0.0791321 \quad -0.0732278 \quad -0.0791321 \quad -0.$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 34.5321 & 0. \\ -64.269 & 133.333 \end{pmatrix} \qquad \det J = 0.000217189$$

$$\boldsymbol{B}^T = \begin{pmatrix} -16.0673 & 16.0673 & 1.19882 & -1.19882 & -33.8914 & -2.7326 & -2.52871 & 2.7326 & 31. \\ -32.1345 & -34.5321 & 2.39763 & 64.269 & 73.6274 & -4.67797 & -5.84466 & -135.945 & -69. \end{pmatrix}$$

$k_x = 45. \qquad k_y = 45. \qquad p = 0. \qquad q = 5.\times 10^6$

$$\boldsymbol{k}_{\mathrm{k}}=\begin{pmatrix}
1.52651 & 1.00702 & -0.113897 & -2.41964 & -2.15408 & 0.229701 & 0.27016\\
1.00702 & 1.71554 & -0.0751364 & -2.64743 & -3.65082 & 0.139118 & 0.19063\\
-0.113897 & -0.0751364 & 0.00849808 & 0.180535 & 0.160721 & -0.0171385 & -0.02015\\
-2.41964 & -2.64743 & 0.180535 & 4.88653 & 5.64418 & -0.35168 & -0.44064\\
-2.15408 & -3.65082 & 0.160721 & 5.64418 & 7.76939 & -0.297803 & -0.40756\\
0.229701 & 0.139118 & -0.0171385 & -0.35168 & -0.297803 & 0.0347106 & 0.04050\\
0.270164 & 0.190638 & -0.0201576 & -0.440644 & -0.407562 & 0.0405061 & 0.04796\\
5.1144 & 5.60371 & -0.381597 & -10.3365 & -11.9467 & 0.743254 & 0.93148\\
2.05511 & 3.43491 & -0.153337 & -5.33669 & -7.31024 & 0.284689 & 0.38826\\
-0.183481 & -0.0774147 & 0.0136899 & 0.247206 & 0.166375 & -0.0281243 & -0.03195\\
-0.325688 & -0.258479 & 0.0243003 & 0.559866 & 0.552118 & -0.0484924 & -0.05815\\
-4.72238 & -5.19448 & 0.352347 & 9.56451 & 11.0741 & -0.686044 & -0.86032\\
-1.36231 & -2.18617 & 0.101645 & 3.44684 & 4.6533 & -0.189789 & -0.25630\\
0.0544176 & -0.0529113 & -0.00406023 & 0.00255384 & 0.11159 & 0.00923716 & 0.00858\\
0.343912 & 0.315685 & -0.02566 & -0.633936 & -0.673677 & 0.0507011 & 0.06191\\
2.83447 & 3.15734 & -0.211486 & -5.78032 & -6.73068 & 0.411311 & 0.51685\\
-0.547623 & -0.361261 & 0.0408594 & 0.868025 & 0.772757 & -0.0824032 & -0.09691
\end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}}=\begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}}=(\;113.789\quad 8.49006\quad 0.633463\quad 8.49006\quad -19.3524\quad -1.44393\quad -1.44393\quad -19.3524\quad 21.5145\quad 1$$

Gauss point = {s → −0.861136, t → −0.339981} Weight = 0.226852

$\boldsymbol{N}^{T}$ = {0.623472, 0.0465187, 0.0229132, 0.307096, −0.106035, −0.0376036, −0.0522287, −0.503986, 0.117882, 0.0165047, 0.0580638, 0.221207, −0.109646, 0.00606089, −0.0540071, 0.0812318, 0.0857143}

$\partial \boldsymbol{N}^{T}/\partial s$ = (−0.334995 0.334995 0.165005 −0.165005 −0.70662 −0.270795 −0.348052 0.27079

$\partial \boldsymbol{N}^{T}/\partial t$ = (−0.465284 −0.0347159 0.0347159 0.465284 0.0791321 −0.0289107 −0.0791321 −0.

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 39.9204 & 0. \\ -74.2973 & 133.333 \end{pmatrix} \qquad \det J = 0.000187874$$

$$\boldsymbol{B}^{T} = \begin{pmatrix} -13.3731 & 13.3731 & 6.58705 & -6.58705 & -28.2086 & -10.8102 & -13.8944 & 10.8102 & 25. \\ -37.1486 & -29.518 & -7.63061 & 74.2973 & 63.0509 & 16.2646 & 15.3084 & -71.7832 & -59. \end{pmatrix}$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^{6}$

$$\boldsymbol{k}_{k} = \begin{pmatrix} 2.98971 & 1.76006 & 0.37471 & -5.12449 & -3.76867 & -0.881534 & -0.734306 & \\ 1.76006 & 2.01407 & 0.60093 & -4.37506 & -4.29294 & -1.19803 & -1.223 & \\ 0.37471 & 0.60093 & 0.194886 & -1.17053 & -1.27909 & -0.374593 & -0.399562 & \\ -5.12449 & -4.37506 & -1.17053 & 10.6701 & 9.34069 & 2.45416 & 2.35687 & - \\ -3.76867 & -4.29294 & -1.27909 & 9.34069 & 9.15048 & 2.55162 & 2.60284 & - \\ -0.881534 & -1.19803 & -0.374593 & 2.45416 & 2.55162 & 0.731475 & 0.76559 & - \\ -0.734306 & -1.223 & -0.399562 & 2.35687 & 2.60284 & 0.76559 & 0.819701 & - \\ 4.83705 & 4.34106 & 1.18709 & -10.3652 & -9.26516 & -2.4633 & -2.3956 & \\ 3.60525 & 4.05661 & 1.20411 & -8.86597 & -8.64726 & -2.40613 & -2.44941 & \\ 0.848083 & 0.892271 & 0.259141 & -1.99949 & -1.90266 & -0.522964 & -0.526067 & \\ 0.528461 & 1.00699 & 0.336883 & -1.87234 & -2.14223 & -0.639048 & -0.69247 & \\ 4.05777 & 3.00589 & 0.74856 & -7.81222 & -6.42386 & -1.62494 & -1.49556 & \\ -2.4082 & -2.61541 & -0.76764 & 5.79125 & 5.57612 & 1.54175 & 1.5599 & - \\ -0.321699 & -0.175422 & -0.0348883 & 0.53201 & 0.375896 & 0.0851577 & 0.0677213 & - \\ -0.0259925 & -0.366366 & -0.139796 & 0.532155 & 0.777425 & 0.251446 & 0.290245 & - \\ -6.99554 & -5.6388 & -1.46813 & 14.1025 & 12.0434 & 3.11851 & 2.9476 & - \\ 1.81275 & 2.48995 & 0.780552 & -5.08325 & -5.30297 & -1.52248 & -1.59564 & \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = ($ 132.86 9.91303 4.88274 65.4415 −22.5959 −8.01324 −11.1298 −107.398 25.1204 3.5

Gauss point = {s → −0.861136, t → 0.339981}   Weight = 0.226852

$\boldsymbol{N}^{\mathrm{T}} =$

{0.307096, 0.0229132, 0.0465187, 0.623472, −0.0522287, −0.0376036, −0.106035, −0.503986, 0.0580638, −0.0165047, 0.117882, −0.221207, −0.0540071, 0.00606089, −0.109646, 0.0812318, 0.0857143}

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial s = ($ −0.165005 0.165005 0.334995 −0.334995 −0.348052 −0.270795 −0.70662 0.27079

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial t = ($ −0.465284 −0.0347159 0.0347159 0.465284 0.0791321 0.0289107 −0.0791321 0.387

$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 50.125 & 0. \\ -93.2894 & 133.333 \end{pmatrix}$ detJ = 0.000149626

$\boldsymbol{B}^{\mathrm{T}} = \Big($ −8.27086 8.27086 16.7916 −16.7916 −17.4461 −13.5736 −35.4193 13.5736 16
−46.6447 −20.022 −26.6227 93.2894 43.0205 29.1171 55.3692 26.4016 −41

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^{6}$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{array}{cccccccc}
3.42776 & 1.32201 & 1.68465 & -6.43442 & -2.84466 & -1.90301 & -3.49741 & -2.0525 \\
1.32201 & 0.716804 & 1.02631 & -3.06513 & -1.53606 & -1.06194 & -2.14077 & -0.6359 \\
1.68465 & 1.02631 & 1.51327 & -4.22423 & -2.19686 & -1.53216 & -3.15999 & -0.7254 \\
-6.43442 & -3.06513 & -4.22423 & 13.7238 & 6.57759 & 4.49712 & 8.79818 & 3.4139 \\
-2.84466 & -1.53606 & -2.19686 & 6.57759 & 3.29181 & 2.27501 & 4.58221 & 1.3731 \\
-1.90301 & -1.06194 & -1.53216 & 4.49712 & 2.27501 & 1.57638 & 3.19685 & 0.8927 \\
-3.49741 & -2.14077 & -3.15999 & 8.79818 & 4.58221 & 3.19685 & 6.59894 & 1.4985 \\
-2.0525 & -0.635942 & -0.725469 & 3.41391 & 1.37317 & 0.892777 & 1.49852 & 1.3461 \\
2.75702 & 1.47261 & 2.09983 & -6.32945 & -3.15618 & -2.17936 & -4.37931 & -1.3429 \\
-0.374094 & -0.268148 & -0.409275 & 1.05152 & 0.573201 & 0.404021 & 0.855718 & 0.1308 \\
3.06498 & 1.90271 & 2.81774 & -7.78543 & -4.07213 & -2.84379 & -5.88493 & -1.2932 \\
5.27995 & 2.37396 & 3.20932 & -10.8632 & -5.09788 & -3.46641 & -6.67918 & -2.9076 \\
-1.90871 & -0.989587 & -1.39929 & 4.29759 & 2.1216 & 1.46138 & 2.91736 & 0.9522 \\
0.770509 & 0.370239 & 0.511659 & -1.65241 & -0.794432 & -0.543587 & -1.06579 & -0.4064 \\
-1.51966 & -0.998018 & -1.49649 & 4.01417 & 2.13489 & 1.49658 & 3.12688 & 0.6000 \\
6.54673 & 2.77065 & 3.6647 & -12.9821 & -5.95427 & -4.02406 & -7.62008 & -3.7352 \\
4.06082 & 2.26005 & 3.25852 & -9.5794 & -4.84187 & -3.35429 & -6.79871 & -1.9096
\end{array}\right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = \left(\begin{array}{cccccccccc} 52.1188 & 3.8887 & 7.89491 & 105.812 & -8.86397 & -6.38189 & -17.9958 & -85.5339 & 9.85428 & -2. \end{array}\right.$$

Gauss point = {s → −0.861136, t → 0.861136}   Weight = 0.121003

$\boldsymbol{N}^{T}$ = {0.0646111, 0.00482078, 0.0646111, 0.865957, −0.0109886, −0.0109886, −0.147276, −0.147276, 0.0122162, −0.0122162, 0.16373, −0.16373, −0.0113627, −0.0113627, −0.15229, −0.15229, 0.0250475}

$\partial \boldsymbol{N}^{T}/\partial s$ = (−0.0347159 0.0347159 0.465284 −0.465284 −0.0732278 −0.0791321 −0.981444 0.0

$\partial \boldsymbol{N}^{T}/\partial t$ = (−0.465284 −0.0347159 0.0347159 0.465284 0.0791321 0.0732278 −0.0791321 0.981

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 62.3384 & 0. \\ -116.02 & 133.333 \end{pmatrix} \qquad \det J = 0.000120311$$

$$\boldsymbol{B}^{T} = \begin{pmatrix} -2.16413 & 2.16413 & 29.0051 & -29.0051 & -4.56491 & -4.93297 & -61.1817 & 4.93297 \\ -58.0101 & -8.65654 & -49.3536 & 116.02 & 19.0469 & 18.9446 & 103.316 & 121.678 & -1 \end{pmatrix}$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^{6}$

$$\boldsymbol{k}_k = \begin{pmatrix}
2.20763 & 0.325906 & 1.83446 & -4.368 & -0.717366 & -0.71296 & -3.8396 & -4.631 \\
0.325906 & 0.0521593 & 0.321005 & -0.699071 & -0.114486 & -0.114428 & -0.672646 & -0.683 \\
1.83446 & 0.321005 & 2.14684 & -4.30231 & -0.702564 & -0.706252 & -4.50298 & -3.840 \\
-4.368 & -0.699071 & -4.30231 & 9.36938 & 1.53442 & 1.53364 & 9.01522 & 9.154 \\
-0.717366 & -0.114486 & -0.702564 & 1.53442 & 0.251314 & 0.251139 & 1.47213 & 1.503 \\
-0.71296 & -0.114428 & -0.706252 & 1.53364 & 0.251139 & 0.25106 & 1.47996 & 1.494 \\
-3.8396 & -0.672646 & -4.50298 & 9.01522 & 1.47213 & 1.47996 & 9.44505 & 8.037 \\
-4.63114 & -0.683042 & -3.84037 & 9.15455 & 1.50352 & 1.49419 & 8.03792 & 9.715 \\
0.736217 & 0.116689 & 0.711037 & -1.56394 & -0.25621 & -0.255912 & -1.48975 & -1.543 \\
-0.720731 & -0.116486 & -0.724001 & 1.56122 & 0.255595 & 0.255634 & 1.51728 & 1.510 \\
3.44706 & 0.605895 & 4.06763 & -8.12058 & -1.3259 & -1.33323 & -8.53216 & -7.215 \\
-4.18513 & -0.615589 & -3.44978 & 8.2505 & 1.35518 & 1.34649 & 7.22009 & 8.779 \\
-0.585658 & -0.0914152 & -0.548129 & 1.2252 & 0.200821 & 0.200378 & 1.1482 & 1.227 \\
-0.554132 & -0.0910011 & -0.57452 & 1.21965 & 0.199571 & 0.199811 & 1.20424 & 1.161 \\
-1.87772 & -0.334089 & -2.26586 & 4.47767 & 0.730827 & 0.73541 & 4.75335 & 3.930 \\
-2.33475 & -0.340092 & -1.88327 & 4.55812 & 0.748956 & 0.743625 & 3.94088 & 4.898 \\
1.56698 & 0.250786 & 1.54342 & -3.36119 & -0.550459 & -0.550181 & -3.23414 & -3.284
\end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\; 4.70304 \quad 0.350905 \quad 4.70304 \quad 63.033 \quad -0.799858 \quad -0.799858 \quad -10.7202 \quad -10.7202 \quad 0.88922$

Gauss point = $\{s \to -0.339981,\ t \to -0.861136\}$ Weight = 0.226852

$\boldsymbol{N}^{\mathrm{T}} = \{0.623472, 0.307096, 0.0229132, 0.0465187, -0.503986, -0.0522287, -0.0376036, -0.106035, 0.221207, 0.0580638, 0.0165047, 0.117882, 0.0812318, -0.0540071, 0.00606089, -0.109646, 0.0857143\}$

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial s = (\; -0.465284 \quad 0.465284 \quad 0.0347159 \quad -0.0347159 \quad -0.387479 \quad -0.0791321 \quad -0.0289107$

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial t = (\; -0.334995 \quad -0.165005 \quad 0.165005 \quad 0.334995 \quad 0.270795 \quad -0.348052 \quad -0.270795 \quad -0.70662$

$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 34.5321 & 0. \\ -46.2724 & 133.333 \end{pmatrix}$ detJ = 0.000217189

$\boldsymbol{B}^{\mathrm{T}} = \begin{pmatrix} -16.0673 & 16.0673 & 1.19882 & -1.19882 & -13.3805 & -2.7326 & -0.99835 & 2.7326 & -16 \\ -23.1362 & -43.5305 & 20.3942 & 46.2724 & 54.0356 & -42.7453 & -34.7682 & -97.8777 & 6 \end{pmatrix}$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^6$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{array}{cccccccc}
1.75917 & 1.66058 & -1.08885 & -2.33089 & -2.29516 & 2.29001 & 1.81904 & 4.9 \\
1.66058 & 4.77362 & -1.9256 & -4.5086 & -5.6918 & 4.02813 & 3.32002 & 9.5 \\
-1.08885 & -1.9256 & 0.925349 & 2.08911 & 2.40775 & -1.94007 & -1.57476 & -4.4 \\
-2.33089 & -4.5086 & 2.08911 & 4.75038 & 5.5792 & -4.37807 & -3.5643 & -10.0 \\
-2.29516 & -5.6918 & 2.40775 & 5.5792 & 6.87065 & -5.04 & -4.13577 & -11.8 \\
2.29001 & 4.02813 & -1.94007 & -4.37807 & -5.04 & 4.06762 & 3.30111 & 9.2 \\
1.81904 & 3.32002 & -1.57476 & -3.5643 & -4.13577 & 3.30111 & 2.68235 & 7.5 \\
4.9234 & 9.54381 & -4.41846 & -10.0488 & -11.8072 & 9.25952 & 7.53894 & 21.2 \\
0.263401 & -1.20789 & 0.244822 & 0.699663 & 1.25786 & -0.505041 & -0.455838 & -1.4 \\
-2.08469 & -3.61049 & 1.75031 & 3.94487 & 4.52601 & -3.67003 & -2.97689 & -8.3 \\
-0.853912 & -1.73372 & 0.788302 & 1.79933 & 2.1341 & -1.65164 & -1.34677 & -3.8 \\
-4.53719 & -8.8485 & 4.08679 & 9.2989 & 10.9397 & -8.56422 & -6.97423 & -19.6 \\
1.11813 & 4.64453 & -1.69709 & -4.06557 & -5.40515 & 3.54493 & 2.95057 & 8.6 \\
1.19327 & 1.95508 & -0.970633 & -2.17772 & -2.46802 & 2.03578 & 1.64819 & 4.6 \\
-0.237365 & -0.257026 & 0.15615 & 0.338242 & 0.345932 & -0.328183 & -0.261925 & -0.7 \\
2.70612 & 5.38157 & -2.46663 & -5.62107 & -6.63918 & 5.16854 & 4.21167 & 11.8 \\
-3.83136 & -6.97506 & 3.31188 & 7.49454 & 8.69147 & -6.94264 & -5.64084 & -15.8
\end{array}\right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\,153.591 \quad 75.6527 \quad 5.64463 \quad 11.4598 \quad -124.156 \quad -12.8665 \quad -9.26359 \quad -26.1217 \quad 54.4939 \quad 14$$

Gauss point = $\{s \to -0.339981,\ t \to -0.339981\}$ Weight = 0.425293

$\boldsymbol{N}^{T}$ = {0.448887, 0.221103, 0.108906, 0.221103, −0.36286, −0.17873, −0.17873, −0.36286, 0.159264, 0.078447, 0.078447, 0.159264, 0.0584853, 0.0288074, 0.0288074, 0.0584853, 0.29332}

$\partial \boldsymbol{N}^{T}/\partial s$ = (−0.334995 0.334995 0.165005 −0.165005 −0.278977 −0.270795 −0.137413 0.27079

$\partial \boldsymbol{N}^{T}/\partial t$ = (−0.334995 −0.165005 0.165005 0.334995 0.270795 −0.137413 −0.270795 −0.27897

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 39.9204 & 0. \\ -53.4926 & 133.333 \end{pmatrix} \qquad \det J = 0.000187874$$

$$\boldsymbol{B}^{T} = \begin{pmatrix} -13.3731 & 13.3731 & 6.58705 & -6.58705 & -11.1369 & -10.8102 & -5.48557 & 10.8102 & -1 \\ -26.7463 & -39.9204 & 13.1741 & 53.4926 & 51.0292 & -3.83617 & -28.7554 & -51.6825 & \end{pmatrix}$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^{6}$

$$\boldsymbol{k}_k = \begin{pmatrix} 3.21517 & 3.19603 & -1.58366 & -4.82755 & -4.37188 & 0.888717 & 3.02912 \\ 3.19603 & 6.37307 & -1.57423 & -7.99487 & -7.86006 & 0.0308299 & 3.86369 \\ -1.58366 & -1.57423 & 0.780045 & 2.37785 & 2.1534 & -0.437745 & -1.49202 \\ -4.82755 & -7.99487 & 2.37785 & 10.4446 & 10.0785 & -0.481802 & -5.40079 \\ -4.37188 & -7.86006 & 2.1534 & 10.0785 & 9.80875 & -0.270977 & -5.05636 \\ 0.888717 & 0.0308299 & -0.437745 & -0.481802 & -0.270977 & 0.473096 & 0.609848 \\ 3.02912 & 3.86369 & -1.49202 & -5.40079 & -5.05636 & 0.609848 & 3.08128 \\ 4.45041 & 7.93813 & -2.19209 & -10.1964 & -9.91554 & 0.292684 & 5.13035 \\ 0.408243 & -1.04614 & -0.201084 & 0.838985 & 1.04138 & 0.500175 & -0.00271117 \\ 2.56856 & 4.4024 & -1.26517 & -5.7058 & -5.52584 & 0.216703 & 2.91321 \\ -2.0736 & -3.91038 & 1.02137 & 4.96261 & 4.85281 & -0.079888 & -2.44689 \\ 4.05332 & 5.48115 & -1.9965 & -7.53797 & -7.10805 & 0.733061 & 4.2061 \\ 2.22386 & 5.78843 & -1.09538 & -6.91691 & -6.95789 & -0.339728 & 3.04961 \\ -3.11867 & -4.44597 & 1.53613 & 6.02852 & 5.7205 & -0.503013 & -3.29724 \\ 0.26007 & 1.60439 & -0.128099 & -1.73636 & -1.83347 & -0.287143 & 0.60405 \\ -6.75808 & -10.2957 & 3.32875 & 13.725 & 13.1234 & -0.91359 & -7.32145 \\ -2.1644 & -2.15151 & 1.06609 & 3.24982 & 2.94307 & -0.598268 & -2.03915 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (179.334 \quad 88.3324 \quad 43.5089 \quad 88.3324 \quad -144.965 \quad -71.4039 \quad -71.4039 \quad -144.965 \quad 63.6273 \quad 31$

Gauss point = $\{s \to -0.339981,\ t \to 0.339981\}$ Weight = 0.425293

$\boldsymbol{N}^{\mathrm{T}} = \{0.221103, 0.108906, 0.221103, 0.448887, -0.17873, -0.17873, -0.36286, -0.36286, 0.078447, -0.078447, 0.159264, -0.159264, 0.0288074, 0.0288074, 0.0584853, 0.0584853, 0.29332\}$

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial s = (-0.165005 \quad 0.165005 \quad 0.334995 \quad -0.334995 \quad -0.137413 \quad -0.270795 \quad -0.278977 \quad 0.2707$

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial t = (-0.334995 \quad -0.165005 \quad 0.165005 \quad 0.334995 \quad 0.270795 \quad 0.137413 \quad -0.270795 \quad 0.278977$

$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 50.125 & 0. \\ -67.1665 & 133.333 \end{pmatrix}$ detJ = 0.000149626

$\boldsymbol{B}^{\mathrm{T}} = \Big(\begin{matrix} -8.27086 & 8.27086 & 16.7916 & -16.7916 & -6.8878 & -13.5736 & -13.9837 & 13.5736 & -8. \\ -33.5832 & -33.0834 & -0.499824 & 67.1665 & 45.3355 & 36.51 & -17.3681 & 19.0086 & -4. \end{matrix}$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^6$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{matrix}
3.42553 & 2.98568 & -0.349629 & -6.06158 & -4.1967 & -3.18962 & 2.00145 & -2.14 \\
2.98568 & 3.33011 & 0.445048 & -6.76083 & -4.45808 & -3.78032 & 1.3142 & -1.47 \\
-0.349629 & 0.445048 & 0.808125 & -0.903543 & -0.396081 & -0.704929 & -0.647536 & 0.62 \\
-6.06158 & -6.76083 & -0.903543 & 13.726 & 9.05086 & 7.67487 & -2.66811 & 3.00 \\
-4.1967 & -4.45808 & -0.396081 & 9.05086 & 6.02139 & 5.00751 & -1.97894 & 2.20 \\
-3.18962 & -3.78032 & -0.704929 & 7.67487 & 5.00751 & 4.34469 & -1.27228 & 1.45 \\
2.00145 & 1.3142 & -0.647536 & -2.66811 & -1.97894 & -1.27228 & 1.42375 & -1.48 \\
-2.1495 & -1.47934 & 0.625466 & 3.00337 & 2.20001 & 1.45975 & -1.48892 & 1.56 \\
0.625507 & 0.214558 & -0.404467 & -0.435598 & -0.402779 & -0.128018 & 0.560931 & -0.57 \\
1.55867 & 1.25536 & -0.265369 & -2.54866 & -1.79613 & -1.30954 & 0.971678 & -1.03 \\
-3.34818 & -4.11377 & -0.889886 & 8.35184 & 5.41645 & 4.76066 & -1.24951 & 1.45 \\
5.06322 & 5.26797 & 0.363924 & -10.6951 & -7.14305 & -5.88945 & 2.45293 & -2.71 \\
1.89981 & 2.47059 & 0.645435 & -5.01583 & -3.22333 & -2.8887 & 0.628362 & -0.75 \\
3.48953 & 3.54045 & 0.157903 & -7.18789 & -4.82377 & -3.93501 & 1.74385 & -1.91 \\
2.16115 & 3.34521 & 1.28513 & -6.79149 & -4.25472 & -4.02103 & 0.398668 & -0.56 \\
6.38722 & 6.1893 & -0.0109061 & -12.5656 & -8.5093 & -6.8025 & 3.36406 & -3.67 \\
4.08054 & 4.55127 & 0.608249 & -9.24006 & -6.09287 & -5.16659 & 1.79613 & -2.02
\end{matrix}\right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\; 70.3495 \quad 34.6512 \quad 70.3495 \quad 142.825 \quad -56.8673 \quad -56.8673 \quad -115.453 \quad -115.453 \quad 24.9599 \quad -$$

Gauss point = {s → −0.339981, t → 0.861136}   Weight = 0.226852

$$\boldsymbol{N}^{\mathrm{T}} =$$

{0.0465187, 0.0229132, 0.307096, 0.623472, −0.0376036, −0.0522287, −0.503986, −0.106035, 0.0165047, −0.0580638, 0.221207, −0.117882, 0.00606089, −0.0540071, 0.0812318, −0.109646, 0.0857143}

$$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} = (\ -0.0347159 \quad 0.0347159 \quad 0.465284 \quad -0.465284 \quad -0.0289107 \quad -0.0791321 \quad -0.387479 \quad 0.0$$

$$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} = (\ -0.334995 \quad -0.165005 \quad 0.165005 \quad 0.334995 \quad 0.270795 \quad 0.348052 \quad -0.270795 \quad 0.70662 \quad -$$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 62.3384 & 0. \\ -83.5323 & 133.333 \end{pmatrix} \qquad \mathrm{detJ} = 0.000120311$$

$$\boldsymbol{B}^{\mathrm{T}} = \begin{pmatrix} -2.16413 & 2.16413 & 29.0051 & -29.0051 & -1.80225 & -4.93297 & -24.1548 & 4.93297 & - \\ -41.7661 & -24.9005 & -16.8656 & 83.5323 & 38.521 & 53.017 & -3.73898 & 87.606 & - \end{pmatrix}$$

$$\mathrm{k_x} = 45. \qquad \mathrm{k_y} = 45. \qquad \mathrm{p} = 0. \qquad \mathrm{q} = 5. \times 10^{6}$$

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 2.14819 & 1.27155 & 0.788047 & -4.20779 & -1.97119 & -2.70646 & 0.255997 & -4. \\ 1.27155 & 0.767266 & 0.592881 & -2.6317 & -1.18285 & -1.63449 & 0.0501442 & -2. \\ 0.788047 & 0.592881 & 1.38261 & -2.76354 & -0.862123 & -1.27392 & -0.783026 & -1. \\ -4.20779 & -2.6317 & -2.76354 & 9.60302 & 4.01616 & 5.61486 & 0.476885 & 8. \\ -1.97119 & -1.18285 & -0.862123 & 4.01616 & 1.82644 & 2.51918 & -0.123427 & 4. \\ -2.70646 & -1.63449 & -1.27392 & 5.61486 & 2.51918 & 3.48204 & -0.0971173 & 5. \\ 0.255997 & 0.0501442 & -0.783026 & 0.476885 & -0.123427 & -0.0971173 & 0.733757 & -0. \\ -4.50696 & -2.66607 & -1.63893 & 8.81197 & 4.13377 & 5.6745 & -0.548641 & 9. \\ 0.665211 & 0.387108 & 0.186593 & -1.23891 & -0.603098 & -0.82332 & 0.125331 & -1. \\ -2.54766 & -1.54216 & -1.23002 & 5.31984 & 2.3753 & 3.28567 & -0.0666831 & 5. \\ -2.79249 & -1.79195 & -2.227 & 6.81144 & 2.71527 & 3.8273 & 0.631635 & 5. \\ -4.07422 & -2.40574 & -1.44397 & 7.92393 & 3.73208 & 5.12 & -0.52612 & 8. \\ 0.518819 & 0.323459 & 0.331849 & -1.17413 & -0.49406 & -0.690024 & -0.0516673 & -1. \\ -1.6239 & -0.989791 & -0.842933 & 3.45662 & 1.52152 & 2.10945 & 0.00473852 & 3. \\ 2.65408 & 1.77191 & 2.71148 & -7.13747 & -2.65631 & -3.79051 & -1.07737 & -5. \\ -2.2755 & -1.33499 & -0.731682 & 4.34217 & 2.0749 & 2.84035 & -0.353819 & 4. \\ 4.17817 & 2.50841 & 1.83794 & -8.52452 & -3.8727 & -5.34242 & 0.253143 & -8. \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (6.34812 \quad 3.12682 \quad 41.9076 \quad 85.0814 \quad -5.13154 \quad -7.12733 \quad -68.776 \quad -14.47 \quad 2.2523 \quad -7.923$$

Gauss point = {s → 0.339981, t → −0.861136}   Weight = 0.226852

$\boldsymbol{N}^{\mathrm{T}} =$

$\{0.307096, 0.623472, 0.0465187, 0.0229132, -0.503986, -0.106035, -0.0376036, -0.0522287, -0.221207, 0.117882, -0.0165047, 0.0580638, 0.0812318, -0.109646, 0.00606089, -0.0540071, 0.0857143\}$

$$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} = (-0.465284 \quad 0.465284 \quad 0.0347159 \quad -0.0347159 \quad 0.387479 \quad -0.0791321 \quad 0.0289107 \quad 0.079$$

$$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} = (-0.165005 \quad -0.334995 \quad 0.334995 \quad 0.165005 \quad 0.270795 \quad -0.70662 \quad -0.270795 \quad -0.34805$$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 34.5321 & 0. \\ -22.7919 & 133.333 \end{pmatrix} \qquad \mathrm{detJ} = 0.000217189$$

$$\boldsymbol{B}^{\mathrm{T}} = \begin{pmatrix} -16.0673 & 16.0673 & 1.19882 & -1.19882 & 13.3805 & -2.7326 & 0.99835 & 2.7326 & -16. \\ -11.3959 & -55.2707 & 43.8748 & 22.7919 & 27.2746 & -92.4125 & -36.7649 & -48.2105 & 26. \end{pmatrix}$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5.\times 10^{6}$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{matrix}
0.860303 & 0.824121 & -1.15126 & -0.533162 & -1.16579 & 2.43227 & 0.893351 & 1.1 \\
0.824121 & 7.3454 & -5.33383 & -2.83569 & -2.86565 & 11.2271 & 4.54084 & 6.0 \\
-1.15126 & -5.33383 & 4.27117 & 2.21393 & 2.68874 & -8.99682 & -3.57371 & -4.6 \\
-0.533162 & -2.83569 & 2.21393 & 1.15492 & 1.3427 & -4.66259 & -1.86049 & -2.4 \\
-1.16579 & -2.86565 & 2.68874 & 1.3427 & 2.04629 & -5.6694 & -2.19362 & -2.8 \\
2.43227 & 11.2271 & -8.99682 & -4.66259 & -5.6694 & 18.9511 & 7.52676 & 9.8 \\
0.893351 & 4.54084 & -3.57371 & -1.86049 & -2.19362 & 7.52676 & 2.99903 & 3.9 \\
1.12076 & 6.00519 & -4.68248 & -2.44347 & -2.8343 & 9.86134 & 3.93582 & 5.1 \\
-0.0859756 & -3.8754 & 2.56296 & 1.39842 & 1.12835 & -5.39066 & -2.22133 & -2.9 \\
-2.24284 & -10.2448 & 8.22646 & 4.26117 & 5.19906 & -17.3286 & -6.87988 & -9.0 \\
0.423867 & 1.79773 & -1.46537 & -0.756224 & -0.945632 & 3.08705 & 1.22241 & 1.5 \\
-1.01882 & -5.57441 & 4.33108 & 2.26216 & 2.6073 & -9.12105 & -3.64272 & -4.7 \\
0.616021 & -2.1706 & 0.957315 & 0.597268 & -0.0983026 & -2.00498 & -0.912101 & -1.2 \\
1.34037 & 5.91201 & -4.78046 & -2.47192 & -3.05092 & 10.0703 & 3.99325 & 5.2 \\
-0.112046 & -0.928302 & 0.679767 & 0.360581 & 0.370838 & -1.43093 & -0.577813 & -0.7 \\
0.580305 & 3.40336 & -2.61422 & -1.36944 & -1.54597 & 5.50498 & 2.20314 & 2.8 \\
-1.88117 & -9.59798 & 7.54863 & 3.93052 & 4.62883 & -15.8985 & -6.33549 & -8.3
\end{matrix}\right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = \left(\begin{matrix} 75.6527 & 153.591 & 11.4598 & 5.64463 & -124.156 & -26.1217 & -9.26359 & -12.8665 & -54.4939 \end{matrix}\right.$$

Gauss point = $\{s \to 0.339981, t \to -0.339981\}$ Weight = 0.425293

$\boldsymbol{N}^{T}$ = {0.221103, 0.448887, 0.221103, 0.108906, −0.36286, −0.36286, −0.17873, −0.17873, −0.159264, 0.159264, −0.078447, 0.078447, 0.0584853, 0.0584853, 0.0288074, 0.0288074, 0.29332}

$\partial \boldsymbol{N}^{T}/\partial s$ = (−0.334995 0.334995 0.165005 −0.165005 0.278977 −0.270795 0.137413 0.270795

$\partial \boldsymbol{N}^{T}/\partial t$ = (−0.165005 −0.334995 0.334995 0.165005 0.270795 −0.278977 −0.270795 −0.13741

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 39.9204 & 0. \\ -26.3482 & 133.333 \end{pmatrix} \qquad \det J = 0.000187874$$

$$\boldsymbol{B}^{T} = \begin{pmatrix} -13.3731 & 13.3731 & 6.58705 & -6.58705 & 11.1369 & -10.8102 & 5.48557 & 10.8102 & -13.8 \\ -13.1741 & -53.4926 & 40.3185 & 26.3482 & 28.7554 & -30.062 & -39.7266 & -25.4567 & 24.9 \end{pmatrix}$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^{6}$

$$\boldsymbol{k}_k = \begin{pmatrix} 1.26707 & 1.89082 & -2.22655 & -0.931341 & -1.89761 & 1.94379 & 1.61802 & 0.6860 \\ 1.89082 & 10.9316 & -7.43797 & -5.38445 & -4.99521 & 5.26221 & 7.90463 & 5.4160 \\ -2.22655 & -7.43797 & 6.00089 & 3.66363 & 4.43238 & -4.61406 & -5.62915 & -3.4343 \\ -0.931341 & -5.38445 & 3.66363 & 2.65215 & 2.46043 & -2.59195 & -3.89349 & -2.6677 \\ -1.89761 & -4.99521 & 4.43238 & 2.46043 & 3.41905 & -3.54106 & -3.88776 & -2.1991 \\ 1.94379 & 5.26221 & -4.61406 & -2.59195 & -3.54106 & 3.66959 & 4.08083 & 2.3314 \\ 1.61802 & 7.90463 & -5.62915 & -3.89349 & -3.88776 & 4.08083 & 5.78273 & 3.8494 \\ 0.686041 & 5.41604 & -3.43436 & -2.66771 & -2.19914 & 2.33143 & 3.84944 & 2.7502 \\ -0.518339 & -5.46564 & 3.29183 & 2.69215 & 2.02798 & -2.16148 & -3.83828 & -2.8218 \\ 2.10548 & 9.70367 & -7.02953 & -4.77963 & -4.90367 & 5.14068 & 7.13064 & 4.6937 \\ 0.865102 & 1.85722 & -1.80753 & -0.914788 & -1.44666 & 1.49202 & 1.48804 & 0.7750 \\ 1.15619 & 3.5401 & -2.95258 & -1.7437 & -2.21565 & 2.30212 & 2.70492 & 1.6088 \\ 0.622185 & -2.48664 & 0.639641 & 1.22482 & -0.020772 & -0.0399649 & -1.51737 & -1.5127 \\ -3.28873 & -12.9297 & 9.84979 & 6.36863 & 7.06529 & -7.3811 & -9.63017 & -6.1252 \\ -0.104977 & -2.89544 & 1.57425 & 1.42617 & 0.887829 & -0.958551 & -1.9881 & -1.5401 \\ -1.57615 & -6.82381 & 5.03883 & 3.36113 & 3.5534 & -3.72007 & -5.03989 & -3.2752 \\ -1.27287 & -7.35895 & 5.0071 & 3.62471 & 3.36268 & -3.54243 & -5.32125 & -3.6459 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = ($ 88.3324   179.334   88.3324   43.5089   −144.965   −144.965   −71.4039   −71.4039   −63.6273

Gauss point = {s → 0.339981, t → 0.339981}   Weight = 0.425293

$\boldsymbol{N}^{\mathrm{T}}$ = {0.108906, 0.221103, 0.448887, 0.221103, −0.17873, −0.36286, −0.36286, −0.17873, −0.078447, −0.159264, −0.159264, −0.078447, 0.0288074, 0.0584853, 0.0584853, 0.0288074, 0.29332}

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} = ($ −0.165005   0.165005   0.334995   −0.334995   0.137413   −0.270795   0.278977   0.270795

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} = ($ −0.165005   −0.334995   0.334995   0.165005   0.270795   0.278977   −0.270795   0.137413

$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 50.125 & 0. \\ -33.0834 & 133.333 \end{pmatrix}$ detJ = 0.000149626

$\boldsymbol{B}^{\mathrm{T}} = \Big(\begin{matrix} -8.27086 & 8.27086 & 16.7916 & -16.7916 & 6.8878 & -13.5736 & 13.9837 & 13.5736 & -8.54 \\ -16.5417 & -50.125 & 33.5832 & 33.0834 & 31.5599 & 46.1558 & -45.3355 & 9.36286 & 21.48 \end{matrix}$

$\mathrm{k_x} = 45.$ $\mathrm{k_y} = 45.$ $\mathrm{p} = 0.$ $\mathrm{q} = 5.\times 10^{6}$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{array}{cccccccc}
0.979443 & 2.17845 & -1.98848 & -1.16941 & -1.65808 & -1.86485 & 1.81628 & -0.76 \\
2.17845 & 7.39065 & -4.42273 & -5.14637 & -4.36687 & -6.94652 & 6.8385 & -1.02 \\
-1.98848 & -4.42273 & 4.03705 & 2.37416 & 3.36625 & 3.78604 & -3.68744 & 1.55 \\
-1.16941 & -5.14637 & 2.37416 & 3.94163 & 2.65869 & 5.02533 & -4.96734 & 0.23 \\
-1.65808 & -4.36687 & 3.36625 & 2.65869 & 2.98805 & 3.90357 & -3.82135 & 1.11 \\
-1.86485 & -6.94652 & 3.78604 & 5.02533 & 3.90357 & 6.62803 & -6.53556 & 0.70 \\
1.81628 & 6.8385 & -3.68744 & -4.96734 & -3.82135 & -6.53556 & 6.44549 & -0.67 \\
-0.764983 & -1.02243 & 1.55308 & 0.234335 & 1.11388 & 0.709903 & -0.671969 & 0.77 \\
-0.815419 & -3.28631 & 1.65548 & 2.44625 & 1.77326 & 3.17182 & -3.13139 & 0.24 \\
2.14013 & 5.91638 & -4.34492 & -3.71158 & -3.93145 & -5.34625 & 5.24012 & -1.36 \\
0.619207 & 0.220858 & -1.25712 & 0.417058 & -0.739763 & 0.0925124 & -0.123218 & -0.79 \\
1.12154 & 3.96717 & -2.27697 & -2.81175 & -2.29148 & -3.75467 & 3.69906 & -0.48 \\
0.179082 & -0.673564 & -0.363574 & 0.858056 & -0.0172251 & 0.837602 & -0.846483 & -0.42 \\
3.276 & 10.1358 & -6.651 & -6.76084 & -6.306 & -9.37055 & 9.2081 & -1.79 \\
-0.471738 & -3.89866 & 0.957731 & 3.41266 & 1.55872 & 4.03126 & -4.00787 & -0.39 \\
1.58887 & 4.6058 & -3.22575 & -2.96892 & -2.9757 & -4.20377 & 4.12498 & -0.95 \\
1.33861 & 2.9773 & -2.71767 & -1.59824 & -2.2661 & -2.5487 & 2.48232 & -1.04
\end{array}\right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \left(\begin{array}{ccccccccccccccccc}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{array}\right)$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\, 34.6512 \quad 70.3495 \quad 142.825 \quad 70.3495 \quad -56.8673 \quad -115.453 \quad -115.453 \quad -56.8673 \quad -24.9599$$

Gauss point = {s → 0.339981, t → 0.861136} Weight = 0.226852

$$\boldsymbol{N}^{\mathrm{T}} =$$

{0.0229132, 0.0465187, 0.623472, 0.307096, −0.0376036, −0.106035, −0.503986, −0.0522287, −0.0165047, −0.117882, −0.221207, −0.0580638, 0.00606089, −0.109646, 0.0812318, −0.0540071, 0.0857143}

$$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} = (\ -0.0347159\ \ 0.0347159\ \ 0.465284\ \ -0.465284\ \ 0.0289107\ \ -0.0791321\ \ 0.387479\ \ 0.079$$

$$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} = (\ -0.165005\ \ -0.334995\ \ 0.334995\ \ 0.165005\ \ 0.270795\ \ 0.70662\ \ -0.270795\ \ 0.348052\ \ 0$$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 62.3384 & 0. \\ -41.1445 & 133.333 \end{pmatrix} \qquad \mathrm{detJ} = 0.000120311$$

$$\boldsymbol{B}^{\mathrm{T}} = \begin{pmatrix} -2.16413 & 2.16413 & 29.0051 & -29.0051 & 1.80225 & -4.93297 & 24.1548 & 4.93297 & -2.2 \\ -20.5723 & -46.0944 & 25.5221 & 41.1445 & 34.9165 & 97.4719 & -52.0487 & 43.1511 & 17.3 \end{pmatrix}$$

$$\mathrm{k_x} = 45. \qquad \mathrm{k_y} = 45. \qquad \mathrm{p} = 0. \qquad \mathrm{q} = 5. \times 10^{6}$$

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 0.525538 & 1.15889 & -0.721945 & -0.962478 & -0.887002 & -2.44965 & 1.25088 & -1.10 \\ 1.15889 & 2.61525 & -1.36777 & -2.40637 & -1.9719 & -5.53119 & 3.01078 & -2.42 \\ -0.721945 & -1.36777 & 1.83327 & 0.256446 & 1.15868 & 2.87959 & -0.771025 & 1.52 \\ -0.962478 & -2.40637 & 0.256446 & 3.1124 & 1.70022 & 5.10125 & -3.49063 & 2.00 \\ -0.887002 & -1.9719 & 1.15868 & 1.70022 & 1.50133 & 4.16903 & -2.17857 & 1.86 \\ -2.44965 & -5.53119 & 2.87959 & 5.10125 & 4.16903 & 11.6985 & -6.37722 & 5.13 \\ 1.25088 & 3.01078 & -0.771025 & -3.49063 & -2.17857 & -6.37722 & 4.04379 & -2.61 \\ -1.10338 & -2.42976 & 1.52833 & 2.00481 & 1.86139 & 5.13584 & -2.61208 & 2.31 \\ -0.431741 & -0.986616 & 0.463365 & 0.954991 & 0.737913 & 2.08729 & -1.17367 & 0.90 \\ -2.26216 & -5.11586 & 2.62918 & 4.74884 & 3.85208 & 10.8205 & -5.92293 & 4.74 \\ -0.0195576 & -0.301861 & -0.944161 & 1.26558 & 0.10203 & 0.651442 & -1.13969 & 0.02 \\ -0.999503 & -2.19226 & 1.41727 & 1.77449 & 1.68382 & 4.63338 & -2.32921 & 2.09 \\ 0.100301 & 0.19602 & -0.232208 & -0.0641138 & -0.162577 & -0.413046 & 0.132441 & -0.21 \\ -1.35834 & -3.08745 & 1.52028 & 2.92551 & 2.31718 & 6.53101 & -3.62229 & 2.84 \\ -0.773428 & -2.11782 & -0.484914 & 3.37617 & 1.41538 & 4.49796 & -3.58289 & 1.60 \\ -0.562336 & -1.21604 & 0.862531 & 0.915843 & 0.942709 & 2.56922 & -1.2371 & 1.18 \\ 1.8667 & 4.14715 & -2.44864 & -3.56521 & -3.15883 & -8.76784 & 4.57332 & -3.91 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\, 3.12682 \quad 6.34812 \quad 85.0814 \quad 41.9076 \quad -5.13154 \quad -14.47 \quad -68.776 \quad -7.12733 \quad -2.2523 \quad -16.0$

Gauss point = $\{s \to 0.861136,\ t \to -0.861136\}$ Weight = 0.121003

$\boldsymbol{N}^{\mathrm{T}} = \{0.0646111, 0.865957, 0.0646111, 0.00482078, -0.147276, -0.147276, -0.0109886, -0.0109886,$
$-0.16373, 0.16373, -0.0122162, 0.0122162, -0.15229, -0.15229, -0.0113627, -0.0113627, 0.0250475\}$

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial s = (\, -0.465284 \quad 0.465284 \quad 0.0347159 \quad -0.0347159 \quad 0.981444 \quad -0.0791321 \quad 0.0732278 \quad 0.079$

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial t = (\, -0.0347159 \quad -0.465284 \quad 0.465284 \quad 0.0347159 \quad 0.0791321 \quad -0.981444 \quad -0.0791321 \quad -0.0$

$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 34.5321 & 0. \\ -4.79526 & 133.333 \end{pmatrix}$ detJ = 0.000217189

$\boldsymbol{B}^{\mathrm{T}} = \Big(\begin{matrix} -16.0673 & 16.0673 & 1.19882 & -1.19882 & 33.8914 & -2.7326 & 2.52871 & 2.7326 & 31 \\ -2.39763 & -64.269 & 61.8714 & 4.79526 & 5.84466 & -130.48 & -10.9021 & -10.1432 & 7 \end{matrix}$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^6$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{array}{lllllll}
0.312101 & -0.123068 & -0.198215 & 0.00918237 & -0.66056 & 0.421898 & -0.017136 \\
-0.123068 & 5.19014 & -4.67982 & -0.387248 & 0.199758 & 9.86533 & 0.876674 \\
-0.198215 & -4.67982 & 4.52886 & 0.349172 & 0.475706 & -9.55115 & -0.794127 \\
0.00918237 & -0.387248 & 0.349172 & 0.0288935 & -0.0149044 & -0.736075 & -0.065410 \\
-0.66056 & 0.199758 & 0.475706 & -0.0149044 & 1.39879 & -1.0114 & 0.025997 \\
0.421898 & 9.86533 & -9.55115 & -0.736075 & -1.0114 & 20.143 & 1.67411 \\
-0.0171366 & 0.876674 & -0.794127 & -0.0654107 & 0.0259972 & 1.67411 & 0.148123 \\
-0.0231626 & 0.822866 & -0.738307 & -0.0613959 & 0.0394147 & 1.55635 & 0.138948 \\
-0.612187 & 0.0280208 & 0.586257 & -0.00209069 & 1.29821 & -1.24387 & -0.002488 \\
-0.397151 & -9.04068 & 8.76329 & 0.674546 & 0.949175 & -18.4815 & -1.5343 \\
-0.00993557 & 0.960139 & -0.878565 & -0.0716382 & 0.00973712 & 1.85223 & 0.16233 \\
0.0311137 & -0.771033 & 0.682391 & 0.0575286 & -0.0568923 & -1.43835 & -0.130085 \\
-0.371846 & -0.287751 & 0.638127 & 0.0214698 & 0.792139 & -1.35017 & -0.053075 \\
0.25316 & 5.29309 & -5.15132 & -0.39493 & -0.599493 & 10.8642 & 0.898536 \\
0.00549974 & 0.869643 & -0.810257 & -0.0648861 & -0.0219349 & 1.70841 & 0.147201 \\
-0.0376132 & 0.484676 & -0.4109 & -0.0361628 & 0.07406 & 0.865859 & 0.081558 \\
0.0441495 & -1.86192 & 1.67885 & 0.138922 & -0.0716615 & -3.5391 & -0.314499
\end{array}\right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\; 8.49006 \quad 113.789 \quad 8.49006 \quad 0.633463 \quad -19.3524 \quad -19.3524 \quad -1.44393 \quad -1.44393 \quad -21.5145$$

Gauss point = {s → 0.861136, t → −0.339981}　　Weight = 0.226852

$$\boldsymbol{N}^{T} =$$

{0.0465187, 0.623472, 0.307096, 0.0229132, −0.106035, −0.503986, −0.0522287, −0.0376036, −0.117882, 0.221207, −0.0580638, 0.0165047, −0.109646, 0.0812318, −0.0540071, 0.00606089, 0.0857143}

$$\partial \boldsymbol{N}^{T}/\partial s = (\ -0.334995 \quad 0.334995 \quad 0.165005 \quad -0.165005 \quad 0.70662 \quad -0.270795 \quad 0.348052 \quad 0.270795 \quad 0$$

$$\partial \boldsymbol{N}^{T}/\partial t = (\ -0.0347159 \quad -0.465284 \quad 0.465284 \quad 0.0347159 \quad 0.0791321 \quad -0.387479 \quad -0.0791321 \quad -0.0$$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 39.9204 & 0. \\ -5.54349 & 133.333 \end{pmatrix} \qquad \det J = 0.000187874$$

$$\boldsymbol{B}^{T} = \begin{pmatrix} -13.3731 & 13.3731 & 6.58705 & -6.58705 & 28.2086 & -10.8102 & 13.8944 & 10.8102 & 25.89 \\ -2.77175 & -63.8949 & 61.1232 & 5.54349 & 6.6338 & -50.1628 & -12.4804 & -5.35591 & 8.13 \end{pmatrix}$$

$$k_x = 45. \qquad k_y = 45. \qquad p = 0. \qquad q = 5. \times 10^{6}$$

$$\boldsymbol{k}_{k} = \begin{pmatrix} 0.35773 & -0.00333785 & -0.493868 & 0.139477 & -0.75876 & 0.543921 & -0.29002 \\ -0.00333785 & 8.17286 & -7.32126 & -0.84826 & -0.0894287 & 5.86982 & 1.88574 \\ -0.493868 & -7.32126 & 7.2485 & 0.566631 & 1.13402 & -6.01699 & -1.2875 \\ 0.139477 & -0.84826 & 0.566631 & 0.142152 & -0.285835 & -0.39675 & -0.308218 \\ -0.75876 & -0.0894287 & 1.13402 & -0.285835 & 1.6105 & -1.22305 & 0.592908 \\ 0.543921 & 5.86982 & -6.01699 & -0.39675 & -1.22305 & 5.05009 & 0.912619 \\ -0.29002 & 1.88574 & -1.2875 & -0.308218 & 0.592908 & 0.912619 & 0.668981 \\ -0.248791 & 0.93359 & -0.491289 & -0.19351 & 0.516699 & 0.291146 & 0.416267 \\ -0.707404 & -0.332572 & 1.28064 & -0.240664 & 1.50444 & -1.3194 & 0.495362 \\ 0.222431 & 8.05455 & -7.52879 & -0.748192 & -0.566922 & 6.12957 & 1.67593 \\ -0.255371 & 1.98158 & -1.42154 & -0.304674 & 0.518282 & 1.03443 & 0.663047 \\ 0.143606 & 0.38343 & -0.543153 & 0.0161171 & -0.309138 & 0.494936 & -0.027773 \\ -0.437844 & -0.686378 & 1.22338 & -0.0991596 & 0.93684 & -1.16206 & 0.195886 \\ -0.551452 & -11.6373 & 11.1973 & 0.991515 & 1.30713 & -9.20747 & -2.23538 \\ -0.1291 & 1.65742 & -1.30635 & -0.221971 & 0.254268 & 0.994249 & 0.486259 \\ 0.00549571 & -0.948174 & 0.84228 & 0.100398 & -0.000460782 & -0.673186 & -0.222905 \\ 0.5213 & -2.0496 & 1.11315 & 0.41515 & -1.08155 & -0.677199 & -0.893742 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (9.91303 \quad 132.86 \quad 65.4415 \quad 4.88274 \quad -22.5959 \quad -107.398 \quad -11.1298 \quad -8.01324 \quad -25.1204 \quad 4$

Gauss point = $\{s \to 0.861136, t \to 0.339981\}$ Weight = 0.226852

$\boldsymbol{N}^{\mathrm{T}} =$

$\{0.0229132, 0.307096, 0.623472, 0.0465187, -0.0522287, -0.503986, -0.106035, -0.0376036, -0.0580638, -0.221207, -0.117882, -0.0165047, -0.0540071, 0.0812318, -0.109646, 0.00606089, 0.0857143\}$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial s = (-0.165005 \quad 0.165005 \quad 0.334995 \quad -0.334995 \quad 0.348052 \quad -0.270795 \quad 0.70662 \quad 0.270795 \quad 0$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial t = (-0.0347159 \quad -0.465284 \quad 0.465284 \quad 0.0347159 \quad 0.0791321 \quad 0.387479 \quad -0.0791321 \quad 0.0289$

$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 50.125 & 0. \\ -6.96054 & 133.333 \end{pmatrix}$ detJ = 0.000149626

$\boldsymbol{B}^{\mathrm{T}} = \Big(\begin{matrix} -8.27086 & 8.27086 & 16.7916 & -16.7916 & 17.4461 & -13.5736 & 35.4193 & 13.5736 & 16.01 \\ -3.48027 & -63.1864 & 59.7061 & 6.96054 & 8.12831 & 53.5488 & -15.4694 & 1.96989 & 9.5 \end{matrix}$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^6$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{matrix}
0.122988 & 0.231404 & -0.529522 & 0.17513 & -0.263609 & -0.113181 & -0.365225 & -0 \\
0.231404 & 6.20279 & -5.55028 & -0.883914 & -0.564088 & -5.33963 & 1.94046 & -0 \\
-0.529522 & -5.55028 & 5.8757 & 0.204108 & 1.18874 & 4.53536 & -0.502327 & 0 \\
0.17513 & -0.883914 & 0.204108 & 0.504675 & -0.36104 & 0.917454 & -1.0729 & -0 \\
-0.263609 & -0.564088 & 1.18874 & -0.36104 & 0.565815 & 0.303127 & 0.751784 & 0 \\
-0.113181 & -5.33963 & 4.53536 & 0.917454 & 0.303127 & 4.66129 & -1.99962 & -0 \\
-0.365225 & 1.94046 & -0.502327 & -1.0729 & 0.751784 & -1.99962 & 2.28173 & 0 \\
-0.181949 & -0.0186419 & 0.527784 & -0.327193 & 0.386162 & -0.120297 & 0.687794 & 0 \\
-0.252857 & -0.715103 & 1.27766 & -0.309702 & 0.544793 & 0.445453 & 0.641838 & 0 \\
0.411489 & 6.02911 & -5.92095 & -0.519648 & -0.944025 & -5.04977 & 1.17217 & -0 \\
-0.324409 & 1.9786 & -0.647541 & -1.00665 & 0.664758 & -2.00282 & 2.14291 & 0 \\
-0.0454514 & 0.616527 & -0.358649 & -0.212427 & 0.0891291 & -0.582225 & 0.454825 & 0 \\
-0.170178 & -0.807439 & 1.11743 & -0.139814 & 0.370508 & 0.589724 & 0.283371 & 0 \\
0.482026 & 9.28091 & -8.6875 & -1.07544 & -1.13204 & -7.88726 & 2.38376 & -0 \\
-0.169754 & 1.55147 & -0.746375 & -0.635341 & 0.341753 & -1.5068 & 1.35647 & 0 \\
0.0639303 & 0.631261 & -0.67872 & -0.0164715 & -0.14306 & -0.513041 & 0.0429531 & -0 \\
0.387279 & 0.102595 & -1.17307 & 0.683193 & -0.822689 & 0.200125 & -1.4353 & -0
\end{matrix}\right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = \left(\begin{matrix} 3.8887 & 52.1188 & 105.812 & 7.89491 & -8.86397 & -85.5339 & -17.9958 & -6.38189 & -9.85428 & -3 \end{matrix}\right.$$

Gauss point = {s → 0.861136, t → 0.861136}   Weight = 0.121003

$$\boldsymbol{N}^{T} = \{0.00482078,\ 0.0646111,\ 0.865957,\ 0.0646111,$$
$$-0.0109886,\ -0.147276,\ -0.147276,\ -0.0109886,\ -0.0122162,\ -0.16373,$$
$$-0.16373,\ -0.0122162,\ -0.0113627,\ -0.15229,\ -0.15229,\ -0.0113627,\ 0.0250475\}$$

$$\partial \boldsymbol{N}^{T}/\partial s = (\ -0.0347159\ \ 0.0347159\ \ 0.465284\ \ -0.465284\ \ 0.0732278\ \ -0.0791321\ \ 0.981444\ \ 0.079$$

$$\partial \boldsymbol{N}^{T}/\partial t = (\ -0.0347159\ \ -0.465284\ \ 0.465284\ \ 0.0347159\ \ 0.0791321\ \ 0.981444\ \ -0.0791321\ \ 0.0732$$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 62.3384 & 0. \\ -8.65654 & 133.333 \end{pmatrix} \qquad \det J = 0.000120311$$

$$\boldsymbol{B}^{T} = \begin{pmatrix} -2.16413 & 2.16413 & 29.0051 & -29.0051 & 4.56491 & -4.93297 & 61.1817 & 4.93297 & 4.190 \\ -4.32827 & -62.3384 & 58.0101 & 8.65654 & 9.91704 & 131.544 & -19.0469 & 9.0787 & 11.147 \end{pmatrix}$$

$$k_x = 45. \qquad k_y = 45. \qquad p = 0. \qquad q = 5. \times 10^{6}$$

$$\boldsymbol{k}_{k} = \begin{pmatrix}
0.015341 & 0.173692 & -0.205609 & 0.0165763 & -0.0345916 & -0.365999 & -0.0327329 & - \\
0.173692 & 2.54888 & -2.32793 & -0.394642 & -0.398525 & -5.37907 & 0.864585 & - \\
-0.205609 & -2.32793 & 2.7557 & -0.222165 & 0.463618 & 4.90535 & 0.438706 & \\
0.0165763 & -0.394642 & -0.222165 & 0.600231 & -0.0305007 & 0.83972 & -1.27056 & - \\
-0.0345916 & -0.398525 & 0.463618 & -0.0305007 & 0.07808 & 0.839859 & 0.059222 & \\
-0.365999 & -5.37907 & 4.90535 & 0.83972 & 0.839859 & 11.3519 & -1.8391 & \\
-0.0327329 & 0.864585 & 0.438706 & -1.27056 & 0.059222 & -1.8391 & 2.68987 & \\
-0.0327363 & -0.363767 & 0.438752 & -0.0422486 & 0.0737342 & 0.766424 & 0.0844348 & \\
-0.0375507 & -0.449318 & 0.503277 & -0.016408 & 0.0849565 & 0.94713 & 0.028861 & \\
-0.335007 & -4.94473 & 4.48997 & 0.78977 & 0.768991 & 10.4355 & -1.72824 & \\
-0.0242529 & 0.877159 & 0.325052 & -1.17796 & 0.0410862 & -1.86443 & 2.49479 & \\
-0.0310304 & -0.327162 & 0.415888 & -0.0576953 & 0.0696834 & 0.68905 & 0.11747 & \\
-0.0334627 & -0.428064 & 0.448488 & 0.0130388 & 0.0760343 & 0.902693 & -0.0329532 & \\
-0.195357 & -2.92504 & 2.6183 & 0.502104 & 0.448923 & 6.17355 & -1.09596 & \\
-0.00293122 & 0.680004 & 0.039286 & -0.716359 & -0.00181224 & -1.44286 & 1.51904 & \\
-0.0201891 & -0.179387 & 0.270587 & -0.0710108 & 0.0449425 & 0.377313 & 0.14743 & \\
0.0737606 & 0.835124 & -0.988584 & 0.0797 & -0.166319 & -1.75975 & -0.157382 & -
\end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \left(\begin{array}{ccccccccccccccccc}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{array}\right)$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\; 0.350905 \quad 4.70304 \quad 63.033 \quad 4.70304 \quad -0.799858 \quad -10.7202 \quad -10.7202 \quad -0.799858 \quad -0.88922$$

Adding contributions from all Gauss points

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{array}{cccccc}
25.1402 & 19.8598 & -5.96963 & -39.0304 & -29.6457 & \\
19.8598 & 70.1402 & -39.0304 & -50.9696 & -43.839 & \\
-5.96963 & -39.0304 & 44.3107 & 0.689252 & 14.1933 & - \\
-39.0304 & -50.9696 & 0.689252 & 89.3107 & 59.2914 & \\
-29.6457 & -43.839 & 14.1933 & 59.2914 & 58.5981 & \\
-5.43695 & 5.43695 & -20.0595 & 20.0595 & 4.76636 & \\
4.17787 & 32.5645 & -28.3866 & -8.35574 & -13.6963 & \\
5.43695 & 31.3054 & -16.6829 & -20.0595 & -34.7664 & \\
7.65449 & -7.65449 & 15.309 & -15.309 & -8.97663 & \\
-1.29299 & 1.29299 & -2.58597 & 2.58597 & -0.904903 & - \\
-1.07872 & 1.07872 & -2.15744 & 2.15744 & 6.33432 & \\
1.29299 & -1.29299 & 2.58597 & -2.58597 & 0.904903 & - \\
-2.22045\times 10^{-16} & 1.11022\times 10^{-15} & -2.22045\times 10^{-16} & 8.88178\times 10^{-16} & -1.65193 & \\
-0.152988 & 0.152988 & -0.305977 & 0.305977 & -0.10707 & \\
1.33227\times 10^{-15} & 9.99201\times 10^{-16} & 1.33227\times 10^{-15} & -2.66454\times 10^{-15} & -0.624062 & \\
0.152988 & -0.152988 & 0.305977 & -0.305977 & 0.10707 & \\
10.2336 & -10.2336 & 20.4673 & -20.4673 & -7.82921 & -
\end{array}\right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = \left(\begin{array}{cccccccccc} 937.5 & 937.5 & 750. & 750. & -765.466 & -688.919 & -612.372 & -688.919 & 4.77396\times 10^{-15} & 59.292 \end{array}\right.$$

Natural boundary conditions

Specified NBC values for side 3: $\alpha = -55$ $\beta = 1100$

Interpolation functions for mapping: $\left\{0,\ 0,\ \frac{1-a}{2},\ \frac{a+1}{2}\right\}$

Interpolation functions for solution:

$$\left\{0,\ 0,\ \frac{1-a}{2},\ \frac{a+1}{2},\ 0,\ 0,\ \frac{\frac{3a^2}{2}-\frac{3}{2}}{\sqrt{6}},\ 0,\ 0,\ 0,\ \frac{\frac{5a}{2}-\frac{5a^3}{2}}{\sqrt{10}},\ 0,\ 0,\ 0,\ \frac{\frac{35a^4}{8}-\frac{21a^2}{4}+\frac{7}{8}}{\sqrt{14}},\ 0,\ 0\right\}$$

$x(a) = 0.045 - 0.015\,a$ $y(a) = 0.015$

$J_c = 0.015$

Value in mapped coordinate: $\alpha(a) = -55$ $\beta(a) = 1100$

Gauss point = −0.861136 Weight = 0.347855

$\boldsymbol{N}_c^T = \{0.,\ 0.,\ 0.930568,\ 0.0694318,\ 0.,\ 0.,\ -0.158264,\ 0.,\ 0.,\ 0.,\ -0.175946,\ 0.,\ 0.,\ 0.,\ -0.163653,\ 0.,\ 0.\}$

$J_c = 0.015$ $\alpha = -55.$ $\beta = 1100.$

$$\boldsymbol{k}_\alpha = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & -0.248513 & -0.0185421 & 0. & 0. & 0.0422652 & 0. & 0. & 0. & 0.0469871 & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & -0.0185421 & -0.00138347 & 0. & 0. & 0.0031535 & 0. & 0. & 0. & 0.00350582 & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0.0422652 & 0.0031535 & 0. & 0. & -0.00718815 & 0. & 0. & 0. & -0.00799122 & 0. & 0. & 0. & 0. & - \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0.0469871 & 0.00350582 & 0. & 0. & -0.00799122 & 0. & 0. & 0. & -0.00888402 & 0. & 0. & 0. & 0. & - \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0.0437043 & 0.00326088 & 0. & 0. & -0.00743291 & 0. & 0. & 0. & -0.00826333 & 0. & 0. & 0. & 0. & - \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

Gauss point = −0.339981 Weight = 0.652145

$\boldsymbol{N}_c^T = \{0.,\ 0.,\ 0.669991,\ 0.330009,\ 0.,\ 0.,\ -0.54159,\ 0.,\ 0.,\ 0.,\ -0.237711,\ 0.,\ 0.,\ 0.,\ 0.0872927,\ 0.,\ 0.\}$

$J_c = 0.015$ $\alpha = -55.$ $\beta = 1100.$

$$\boldsymbol{k}_{\alpha} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & -0.24151 & -0.118958 & 0. & 0. & 0.195226 & 0. & 0. & 0. & 0.0856873 & 0. & 0. & 0. & -0.(\\
0. & 0. & -0.118958 & -0.0585937 & 0. & 0. & 0.0961602 & 0. & 0. & 0. & 0.042206 & 0. & 0. & 0. & -0.(\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0.195226 & 0.0961602 & 0. & 0. & -0.157812 & 0. & 0. & 0. & -0.0692658 & 0. & 0. & 0. & 0.(\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0.0856873 & 0.042206 & 0. & 0. & -0.0692658 & 0. & 0. & 0. & -0.0304017 & 0. & 0. & 0. & 0.(\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & -0.0314662 & -0.015499 & 0. & 0. & 0.0254359 & 0. & 0. & 0. & 0.0111642 & 0. & 0. & 0. & -0.(\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

Gauss point = 0.339981 Weight = 0.652145

$\boldsymbol{N}_{c}^{T}$ = {0., 0., 0.330009, 0.669991, 0., 0., −0.54159, 0., 0., 0., 0.237711, 0., 0., 0., 0.0872927, 0., 0.}

J_c = 0.015 α = −55. β = 1100.

$$\boldsymbol{k}_{\alpha} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & -0.0585937 & -0.118958 & 0. & 0. & 0.0961602 & 0. & 0. & 0. & -0.042206 & 0. & 0. & 0. & -0.(\\
0. & 0. & -0.118958 & -0.24151 & 0. & 0. & 0.195226 & 0. & 0. & 0. & -0.0856873 & 0. & 0. & 0. & -0.(\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0.0961602 & 0.195226 & 0. & 0. & -0.157812 & 0. & 0. & 0. & 0.0692658 & 0. & 0. & 0. & 0.(\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & -0.042206 & -0.0856873 & 0. & 0. & 0.0692658 & 0. & 0. & 0. & -0.0304017 & 0. & 0. & 0. & -0.(\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & -0.015499 & -0.0314662 & 0. & 0. & 0.0254359 & 0. & 0. & 0. & -0.0111642 & 0. & 0. & 0. & -0.(\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

Gauss point = 0.861136 Weight = 0.347855

$\boldsymbol{N}_c^T$ = {0., 0., 0.0694318, 0.930568, 0., 0., −0.158264, 0., 0., 0., 0.175946, 0., 0., 0., −0.163653, 0., 0.}

J_c = 0.015 α = −55. β = 1100.

$$\boldsymbol{k}_\alpha = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & -0.00138347 & -0.0185421 & 0. & 0. & 0.0031535 & 0. & 0. & 0. & -0.00350582 & 0. & 0. & 0. \\
0. & 0. & -0.0185421 & -0.248513 & 0. & 0. & 0.0422652 & 0. & 0. & 0. & -0.0469871 & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0.0031535 & 0.0422652 & 0. & 0. & -0.00718815 & 0. & 0. & 0. & 0.00799122 & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & -0.00350582 & -0.0469871 & 0. & 0. & 0.00799122 & 0. & 0. & 0. & -0.00888402 & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0.00326088 & 0.0437043 & 0. & 0. & -0.00743291 & 0. & 0. & 0. & 0.00826333 & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

Adding contributions from all Gauss points

$$\boldsymbol{k}_{\alpha} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & -0.55 & -0.275 & 0. & 0. & 0.336805 & 0. & 0. & 0. & 0.0869626 \\
0. & 0. & -0.275 & -0.55 & 0. & 0. & 0.336805 & 0. & 0. & 0. & -0.0869626 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0.336805 & 0.336805 & 0. & 0. & -0.33 & 0. & 0. & 0. & 3.46945\times 10 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0.0869626 & -0.0869626 & 0. & 0. & 3.46945\times 10^{-18} & 0. & 0. & 0. & -0.0785714 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 4.77049\times 10^{-18} & 6.93889\times 10^{-18} & 0. & 0. & 0.036006 & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$$\begin{pmatrix}
25.1402 & 19.8598 & -5.96963 & -39.0304 & -29.6457 & -5.43695 & 4.17787 & 5.43695 \\
19.8598 & 70.1402 & -39.0304 & -50.9696 & -43.839 & 5.43695 & 32.5645 & 31.3054 \\
-5.96963 & -39.0304 & 43.7607 & 0.414252 & 14.1933 & -20.0595 & -28.0498 & -16.6829 \\
-39.0304 & -50.9696 & 0.414252 & 88.7607 & 59.2914 & 20.0595 & -8.01893 & -20.0595 \\
-29.6457 & -43.839 & 14.1933 & 59.2914 & 58.5981 & 4.76636 & -13.6963 & -34.7664 \\
-5.43695 & 5.43695 & -20.0595 & 20.0595 & 4.76636 & 97.1145 & 5.46729 & 37.8855 \\
4.17787 & 32.5645 & -28.0498 & -8.01893 & -13.6963 & 5.46729 & 49.5625 & 24.5327 \\
5.43695 & 31.3054 & -16.6829 & -20.0595 & -34.7664 & 37.8855 & 24.5327 & 97.1145 \\
7.65449 & -7.65449 & 15.309 & -15.309 & -8.97663 & -8.97663 & -16.9036 & 8.97663 \\
-1.29299 & 1.29299 & -2.58597 & 2.58597 & -0.904903 & -15.0485 & 1.80981 & 3.42958 \\
-1.07872 & 1.07872 & -2.07048 & 2.07048 & 6.33432 & 6.33432 & -12.6686 & -6.33432 \\
1.29299 & -1.29299 & 2.58597 & -2.58597 & 0.904903 & -19.8083 & -1.80981 & -15.0485 \\
0 & 0 & 0 & 0 & -1.65193 & 0 & 9.19574 & 0 \\
-0.152988 & 0.152988 & -0.305977 & 0.305977 & -0.10707 & -0.749487 & 0.214139 & 0.749487 \\
0 & 0 & 0 & 0 & -0.624062 & 0 & 1.28413 & 0 \\
0.152988 & -0.152988 & 0.305977 & -0.305977 & 0.10707 & 0.749487 & -0.214139 & -0.749487 \\
10.2336 & -10.2336 & 20.4673 & -20.4673 & -7.82921 & -60.0926 & -24.7582 & -50.1344
\end{pmatrix}$$

Equations for element 2

Element coordinates: ({0, 0} {0.03, 0.015} {0.03, 0.03} {0, 0.03})

$x(s,t) = 0.015\,s + 0.015$

$y(s,t) = -0.00375\,t\,s + 0.00375\,s + 0.01125\,t + 0.01875$

$$\boldsymbol{J} = \begin{pmatrix} 0.015 & 0 \\ 0.00375 - 0.00375\,t & 0.01125 - 0.00375\,s \end{pmatrix}$$

$\det J = 0.00016875 - 0.00005625\,s$

Given element data

$k_x = 45$ $k_y = 45$ $p = 0$ $q = 5000000$

Element data in mapped coordinates

$k_x = 45$ $k_y = 45$ $p = 0$ $q = 5000000$

Gauss point = {s → −0.861136, t → −0.861136} Weight = 0.121003

$\boldsymbol{N}^T$ = {0.865957, 0.0646111, 0.00482078, 0.0646111, −0.147276, −0.0109886, −0.0109886, −0.147276, 0.16373, 0.0122162, 0.0122162, 0.16373, −0.15229, −0.0113627, −0.0113627, −0.15229, 0.0250475}

$\partial\boldsymbol{N}^T/\partial s$ = (−0.465284 0.465284 0.0347159 −0.0347159 −0.981444 −0.0791321 −0.0732278 0.0

$\partial\boldsymbol{N}^T/\partial t$ = (−0.465284 −0.0347159 0.0347159 0.465284 0.0791321 −0.0732278 −0.0791321 −0.

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 66.6667 & -32.1345 \\ 0. & 69.0643 \end{pmatrix} \qquad \det J = 0.000217189$$

$$\boldsymbol{B}^T = \begin{pmatrix} -16.0673 & 32.1345 & 1.19882 & -17.2661 & -67.9725 & -2.92233 & -2.33899 & 36.8137 & 62.89 \\ -32.1345 & -2.39763 & 2.39763 & 32.1345 & 5.4652 & -5.05743 & -5.4652 & -67.7828 & -6.07 \end{pmatrix}$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5.\times 10^6$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{matrix}
1.52651 & -0.519487 & -0.113897 & -0.893127 & 1.08389 & 0.247726 & 0.252138 \\
-0.519487 & 1.22801 & 0.0387601 & -0.74728 & -2.59866 & -0.096717 & -0.073392 \\
-0.113897 & 0.0387601 & 0.00849808 & 0.0666383 & -0.0808712 & -0.0184834 & -0.0188126 \\
-0.893127 & -0.74728 & 0.0666383 & 1.57377 & 1.59564 & -0.132526 & -0.159934 \\
1.08389 & -2.59866 & -0.0808712 & 1.59564 & 5.49935 & 0.202226 & 0.152698 \\
0.247726 & -0.096717 & -0.0184834 & -0.132526 & 0.202226 & 0.0403483 & 0.0407711 \\
0.252138 & -0.073392 & -0.0188126 & -0.159934 & 0.152698 & 0.0407711 & 0.041793 \\
1.87643 & 1.59123 & -0.140005 & -3.32766 & -3.3974 & 0.278182 & 0.336267 \\
-0.964127 & 2.40728 & 0.0719358 & -1.51509 & -5.09482 & -0.181013 & -0.134696 \\
-0.246831 & 0.127625 & 0.0184166 & 0.10079 & -0.267797 & -0.0405716 & -0.0402547 \\
-0.262337 & 0.0456503 & 0.0195736 & 0.197113 & -0.0937334 & -0.0420576 & -0.0438462 \\
-1.70314 & -1.49958 & 0.127075 & 3.07564 & 3.20095 & -0.251838 & -0.305864 \\
0.507274 & -1.4601 & -0.0378489 & 0.990679 & 3.09111 & 0.0975248 & 0.0685848 \\
0.183381 & -0.151217 & -0.0136825 & -0.0184824 & 0.318588 & 0.0308083 & 0.029241 \\
0.214948 & 0.0156611 & -0.0160377 & -0.214571 & -0.0357579 & 0.0338336 & 0.0365523 \\
0.964886 & 0.959113 & -0.0719924 & -1.85201 & -2.04582 & 0.141381 & 0.174576 \\
-0.547623 & 0.186362 & 0.0408594 & 0.320402 & -0.388835 & -0.0888697 & -0.0904525
\end{matrix}\right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = \left(\begin{matrix} 113.789 & 8.49006 & 0.633463 & 8.49006 & -19.3524 & -1.44393 & -1.44393 & -19.3524 & 21.5145 \end{matrix}\right.$$

Gauss point = $\{s \to -0.861136,\ t \to -0.339981\}$ Weight = 0.226852

$\boldsymbol{N}^{T}$ = {0.623472, 0.0465187, 0.0229132, 0.307096, −0.106035, −0.0376036, −0.0522287, −0.503986, 0.117882, 0.0165047, 0.0580638, 0.221207, −0.109646, 0.00606089, −0.0540071, 0.0812318, 0.0857143}

$\partial \boldsymbol{N}^{T}/\partial s$ = (−0.334995 0.334995 0.165005 −0.165005 −0.70662 −0.270795 −0.348052 0.27079

$\partial \boldsymbol{N}^{T}/\partial t$ = (−0.465284 −0.0347159 0.0347159 0.465284 0.0791321 −0.0289107 −0.0791321 −0.

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 66.6667 & -23.1362 \\ 0. & 69.0643 \end{pmatrix} \qquad \det J = 0.000217189$$

$$\boldsymbol{B}^{T} = \begin{pmatrix} -11.5681 & 23.1362 & 10.1971 & -21.7652 & -48.9388 & -17.3841 & -21.3726 & 27.0178 & 45.2 \\ -32.1345 & -2.39763 & 2.39763 & 32.1345 & 5.4652 & -1.9967 & -5.4652 & -26.761 & -6.0 \end{pmatrix}$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^{6}$

$$\boldsymbol{k}_{k} = \begin{pmatrix} 2.58618 & -0.422576 & -0.43236 & -1.73124 & 0.86581 & 0.588127 & 0.937544 & 1. \\ -0.422576 & 1.19954 & 0.510328 & -1.2873 & -2.53943 & -0.881124 & -1.06728 & 1. \\ -0.43236 & 0.510328 & 0.243286 & -0.321254 & -1.07738 & -0.403642 & -0.512254 & 0. \\ -1.73124 & -1.2873 & -0.321254 & 3.33979 & 2.75099 & 0.696638 & 0.641991 & -3. \\ 0.86581 & -2.53943 & -1.07738 & 2.75099 & 5.37629 & 1.86205 & 2.2528 & -3. \\ 0.588127 & -0.881124 & -0.403642 & 0.696638 & 1.86205 & 0.678875 & 0.847959 & -0. \\ 0.937544 & -1.06728 & -0.512254 & 0.641991 & 2.2528 & 0.847959 & 1.07899 & -0. \\ 1.21368 & 1.52817 & 0.468572 & -3.21041 & -3.25581 & -0.922877 & -0.956002 & 3. \\ -0.728474 & 2.355 & 0.991418 & -2.61795 & -4.98672 & -1.71834 & -2.07204 & 3. \\ -0.0480687 & 0.462175 & 0.184734 & -0.59884 & -0.979776 & -0.326415 & -0.384777 & 0. \\ -0.926997 & 0.955938 & 0.467856 & -0.496797 & -2.01674 & -0.769439 & -0.986528 & 0. \\ 2.28277 & 0.340325 & -0.104204 & -2.51889 & -0.748837 & 0.02379 & 0.250776 & 2. \\ 0.299059 & -1.43343 & -0.588492 & 1.72286 & 3.037 & 1.02946 & 1.22794 & -1. \\ -0.306054 & 0.0661526 & 0.0574453 & 0.182457 & -0.136706 & -0.0808109 & -0.124005 & -0. \\ 0.675789 & -0.516265 & -0.270822 & 0.111299 & 1.0871 & 0.435503 & 0.573141 & -0. \\ -3.02707 & -1.26311 & -0.177558 & 4.46773 & 2.71499 & 0.532186 & 0.323871 & -4. \\ -1.26184 & 1.85095 & 0.850652 & -1.43977 & -3.91126 & -1.4291 & -1.78736 & 1. \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (153.591 \quad 11.4598 \quad 5.64463 \quad 75.6527 \quad -26.1217 \quad -9.26359 \quad -12.8665 \quad -124.156 \quad 29.0401 \quad 4.0$$

Gauss point = {s → −0.861136, t → 0.339981} Weight = 0.226852

$\boldsymbol{N}^{\mathrm{T}}$ =

{0.307096, 0.0229132, 0.0465187, 0.623472, −0.0522287, −0.0376036, −0.106035, −0.503986, 0.0580638, −0.0165047, 0.117882, −0.221207, −0.0540071, 0.00606089, −0.109646, 0.0812318, 0.0857143}

$$\partial\boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} = (-0.165005 \quad 0.165005 \quad 0.334995 \quad -0.334995 \quad -0.348052 \quad -0.270795 \quad -0.70662 \quad 0.27079$$

$$\partial\boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} = (-0.465284 \quad -0.0347159 \quad 0.0347159 \quad 0.465284 \quad 0.0791321 \quad 0.0289107 \quad -0.0791321 \quad 0.387$$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 66.6667 & -11.3959 \\ 0. & 69.0643 \end{pmatrix} \qquad \det J = 0.000217189$$

$$\boldsymbol{B}^{\mathrm{T}} = \Bigg(\begin{matrix} -5.69797 & 11.3959 & 21.9374 & -27.6354 & -24.1052 & -18.3825 & -46.2062 & 13.6373 & 22.3 \\ -32.1345 & -2.39763 & 2.39763 & 32.1345 & 5.4652 & 1.9967 & -5.4652 & 26.761 & -6.0 \end{matrix}$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^6$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{matrix}
2.36146 & 0.0268562 & -0.447962 & -1.94035 & -0.0848514 & 0.0899709 & 0.973109 \\
0.0268562 & 0.300679 & 0.541532 & -0.869068 & -0.638104 & -0.475072 & -1.13841 \\
-0.447962 & 0.541532 & 1.07974 & -1.17331 & -1.14338 & -0.883476 & -2.27644 \\
-1.94035 & -0.869068 & -1.17331 & 3.98273 & 1.86634 & 1.26858 & 2.44174 \\
-0.0848514 & -0.638104 & -1.14338 & 1.86634 & 1.35452 & 1.00664 & 2.40325 \\
0.0899709 & -0.475072 & -0.883476 & 1.26858 & 1.00664 & 0.758043 & 1.85901 \\
0.973109 & -1.13841 & -2.27644 & 2.44174 & 2.40325 & 1.85901 & 4.79985 \\
-2.07891 & 0.202307 & 0.805553 & 1.07105 & -0.404575 & -0.437339 & -1.72135 \\
0.151119 & 0.595819 & 1.05249 & -1.79943 & -1.26561 & -0.935897 & -2.21124 \\
0.271376 & -0.176715 & -0.378685 & 0.284023 & 0.371634 & 0.295325 & 0.799894 \\
-0.966535 & 1.03501 & 2.08689 & -2.15537 & -2.18401 & -1.69475 & -4.40117 \\
2.19542 & 0.515013 & 0.47533 & -3.18577 & -1.11835 & -0.693081 & -0.970629 \\
-0.232395 & -0.370523 & -0.625392 & 1.22831 & 0.788681 & 0.57424 & 1.31205 \\
0.218702 & 0.108553 & 0.151534 & -0.478789 & -0.23284 & -0.159783 & -0.315773 \\
0.712564 & -0.589816 & -1.22328 & 1.10053 & 1.24268 & 0.974857 & 2.58177 \\
3.46061 & 0.39603 & -0.00738451 & -3.84925 & -0.880905 & -0.433483 & 0.0611174 \\
-0.166293 & 1.00531 & 1.86448 & -2.70349 & -2.13046 & -1.60276 & -3.92291
\end{matrix}\right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\; 75.6527 \quad 5.64463 \quad 11.4598 \quad 153.591 \quad -12.8665 \quad -9.26359 \quad -26.1217 \quad -124.156 \quad 14.3039 \quad -$$

Gauss point = $\{s \to -0.861136,\ t \to 0.861136\}$ Weight = 0.121003

$\boldsymbol{N}^{\mathrm{T}}$ = {0.0646111, 0.00482078, 0.0646111, 0.865957, −0.0109886, −0.0109886, −0.147276, −0.147276, 0.0122162, −0.0122162, 0.16373, −0.16373, −0.0113627, −0.0113627, −0.15229, −0.15229, 0.0250475}

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s}$ = (−0.0347159 0.0347159 0.465284 −0.465284 −0.0732278 −0.0791321 −0.981444 0.0

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t}$ = (−0.465284 −0.0347159 0.0347159 0.465284 0.0791321 0.0732278 −0.0791321 0.981

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 66.6667 & -2.39763 \\ 0. & 69.0643 \end{pmatrix} \qquad \mathrm{detJ} = 0.000217189$$

$$\boldsymbol{B}^{\mathrm{T}} = \begin{pmatrix} -1.19882 & 2.39763 & 30.9357 & -32.1345 & -5.07159 & -5.45104 & -65.2399 & 2.92233 & 4.6 \\ -32.1345 & -2.39763 & 2.39763 & 32.1345 & 5.4652 & 5.05743 & -5.4652 & 67.7828 & -6.0 \end{pmatrix}$$

$\mathrm{k_x} = 45.$ $\mathrm{k_y} = 45.$ $\mathrm{p} = 0.$ $\mathrm{q} = 5. \times 10^{6}$

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 1.22291 & 0.087718 & -0.134976 & -1.17565 & -0.200504 & -0.184469 & 0.300188 & -2 \\ 0.087718 & 0.0135969 & 0.0809195 & -0.182234 & -0.029877 & -0.0297967 & -0.169491 & -0 \\ -0.134976 & 0.0809195 & 1.13859 & -1.08453 & -0.170049 & -0.185088 & -2.40232 & 0 \\ -1.17565 & -0.182234 & -1.08453 & 2.44242 & 0.40043 & 0.399354 & 2.27162 & 2 \\ -0.200504 & -0.029877 & -0.170049 & 0.40043 & 0.0657413 & 0.0653817 & 0.355971 & 0 \\ -0.184469 & -0.0297967 & -0.185088 & 0.399354 & 0.0653817 & 0.065389 & 0.387883 & 0 \\ 0.300188 & -0.169491 & -2.40232 & 2.27162 & 0.355971 & 0.387883 & 5.06886 & -0 \\ -2.58009 & -0.183911 & 0.299112 & 2.46489 & 0.420571 & 0.386572 & -0.663568 & 5 \\ 0.224245 & 0.0305333 & 0.154447 & -0.409225 & -0.0674137 & -0.0665895 & -0.322773 & -0 \\ -0.167893 & -0.0302511 & -0.2073 & 0.405444 & 0.0661498 & 0.0666151 & 0.434927 & 0 \\ -0.315755 & 0.152485 & 2.20697 & -2.0437 & -0.319716 & -0.349506 & -4.65719 & 0 \\ -2.36996 & -0.165932 & 0.311973 & 2.22392 & 0.379953 & 0.348283 & -0.688004 & 5 \\ -0.210747 & -0.0240634 & -0.0877023 & 0.322512 & 0.0535307 & 0.0520777 & 0.182222 & 0 \\ -0.0960287 & -0.023489 & -0.195297 & 0.314814 & 0.0509578 & 0.05213 & 0.410537 & 0 \\ 0.264633 & -0.0837098 & -1.30285 & 1.12193 & 0.174436 & 0.192946 & 2.7503 & -0 \\ -1.39842 & -0.0920362 & 0.256935 & 1.23353 & 0.211736 & 0.192189 & -0.559565 & 2 \\ 0.421755 & 0.0653751 & 0.389067 & -0.876197 & -0.143651 & -0.143265 & -0.814924 & -0 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (8.49006 \quad 0.633463 \quad 8.49006 \quad 113.789 \quad -1.44393 \quad -1.44393 \quad -19.3524 \quad -19.3524 \quad 1.60524$

Gauss point = $\{s \to -0.339981,\ t \to -0.861136\}$ Weight = 0.226852

$\boldsymbol{N}^{\mathrm{T}} = \{0.623472, 0.307096, 0.0229132, 0.0465187, -0.503986, -0.0522287, -0.0376036, -0.106035, 0.221207, 0.0580638, 0.0165047, 0.117882, 0.0812318, -0.0540071, 0.00606089, -0.109646, 0.0857143\}$

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial s = (-0.465284 \quad 0.465284 \quad 0.0347159 \quad -0.0347159 \quad -0.387479 \quad -0.0791321 \quad -0.0289107$

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial t = (-0.334995 \quad -0.165005 \quad 0.165005 \quad 0.334995 \quad 0.270795 \quad -0.348052 \quad -0.270795 \quad -0.70662$

$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 66.6667 & -37.1486 \\ 0. & 79.8408 \end{pmatrix}$ detJ = 0.000187874

$\boldsymbol{B}^{\mathrm{T}} = \begin{pmatrix} -18.5743 & 37.1486 & -3.81531 & -14.759 & -35.8916 & 7.65418 & 8.13229 & 31.5255 \\ -26.7463 & -13.1741 & 13.1741 & 26.7463 & 21.6205 & -27.7887 & -21.6205 & -56.4171 \end{pmatrix}$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5.\times 10^{6}$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{array}{cccccccc}
2.03366 & -0.647577 & -0.539867 & -0.846216 & 0.169531 & 1.15279 & 0.819349 & 1. \\
-0.647577 & 2.97958 & -0.604689 & -1.72731 & -3.10343 & 1.24745 & 1.12567 & 3. \\
-0.539867 & -0.604689 & 0.360779 & 0.783777 & 0.808901 & -0.758128 & -0.605777 & -1. \\
-0.846216 & -1.72731 & 0.783777 & 1.78975 & 2.125 & -1.64211 & -1.33924 & -3. \\
0.169531 & -3.10343 & 0.808901 & 2.125 & 3.36713 & -1.67916 & -1.4563 & -4. \\
1.15279 & 1.24745 & -0.758128 & -1.64211 & -1.67916 & 1.59337 & 1.27165 & 3. \\
0.819349 & 1.12567 & -0.605777 & -1.33924 & -1.4563 & 1.27165 & 1.02334 & 2. \\
1.77094 & 3.67154 & -1.65614 & -3.78634 & -4.50944 & 3.46956 & 2.83106 & 8. \\
1.4708 & -1.72827 & -0.0376404 & 0.295119 & 1.50796 & 0.100249 & -0.0373399 & -0. \\
-1.09466 & -1.07234 & 0.68848 & 1.47852 & 1.47111 & -1.44771 & -1.15143 & -3. \\
-0.244332 & -0.724654 & 0.289566 & 0.67942 & 0.861969 & -0.605656 & -0.499636 & -1. \\
-1.58931 & -3.44327 & 1.5278 & 3.50477 & 4.21 & -3.20005 & -2.61484 & -7. \\
-1.5813 & 3.60816 & -0.449576 & -1.57728 & -3.5455 & 0.911014 & 0.915093 & 3. \\
0.715907 & 0.488865 & -0.390776 & -0.813996 & -0.728508 & 0.823122 & 0.646821 & 1. \\
-0.248007 & 0.0504583 & 0.0738221 & 0.123727 & 0.0106785 & -0.157184 & -0.114177 & -0. \\
0.864587 & 2.17022 & -0.914315 & -2.12049 & -2.61689 & 1.91377 & 1.571 & 4. \\
-1.73997 & -2.35107 & 1.27539 & 2.81564 & 3.04927 & -2.67754 & -2.15347 & -5.
\end{array}\right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = \left(\begin{array}{cccccccccc} 132.86 & 65.4415 & 4.88274 & 9.91303 & -107.398 & -11.1298 & -8.01324 & -22.5959 & 47.1386 & 12. \end{array}\right.$$

Gauss point = $\{s \to -0.339981,\ t \to -0.339981\}$ Weight = 0.425293

$\boldsymbol{N}^{T}$ = {0.448887, 0.221103, 0.108906, 0.221103, −0.36286, −0.17873, −0.17873, −0.36286, 0.159264, 0.078447, 0.078447, 0.159264, 0.0584853, 0.0288074, 0.0288074, 0.0584853, 0.29332}

$\partial \boldsymbol{N}^{T}/\partial s = (\ -0.334995 \quad 0.334995 \quad 0.165005 \quad -0.165005 \quad -0.278977 \quad -0.270795 \quad -0.137413 \quad 0.27079$

$\partial \boldsymbol{N}^{T}/\partial t = (\ -0.334995 \quad -0.165005 \quad 0.165005 \quad 0.334995 \quad 0.270795 \quad -0.137413 \quad -0.270795 \quad -0.27897$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 66.6667 & -26.7463 \\ 0. & 79.8408 \end{pmatrix} \qquad \det J = 0.000187874$$

$$\boldsymbol{B}^{T} = \begin{pmatrix} -13.3731 & 26.7463 & 6.58705 & -19.9602 & -25.8413 & -14.3777 & -1.91808 & 25.5146 & -19 \\ -26.7463 & -13.1741 & 13.1741 & 26.7463 & 21.6205 & -10.9711 & -21.6205 & -22.2738 & -9 \end{pmatrix}$$

$k_x = 45.$ $\quad k_y = 45.$ $\quad p = 0.$ $\quad q = 5. \times 10^{6}$

$$\boldsymbol{k}_k = \begin{pmatrix} 3.21517 & -0.0191408 & -1.58366 & -1.61237 & -0.836649 & 1.74641 & 2.17143 \\ -0.0191408 & 3.19617 & 0.00942794 & -3.18646 & -3.50923 & -0.862993 & 0.839669 \\ -1.58366 & 0.00942794 & 0.780045 & 0.794187 & 0.412098 & -0.86021 & -1.06956 \\ -1.61237 & -3.18646 & 0.794187 & 4.00465 & 3.93378 & -0.0232088 & -1.94154 \\ -0.836649 & -3.50923 & 0.412098 & 3.93378 & 4.08175 & 0.483019 & -1.50252 \\ 1.74641 & -0.862993 & -0.86021 & -0.0232088 & 0.483019 & 1.17606 & 0.952031 \\ 2.17143 & 0.839669 & -1.06956 & -1.94154 & -1.50252 & 0.952031 & 1.69396 \\ 0.91518 & 3.50876 & -0.45078 & -3.97316 & -4.10218 & -0.440362 & 1.55555 \\ 1.86888 & -1.46308 & -0.920534 & 0.514734 & 1.11017 & 1.40247 & 0.874856 \\ 0.708376 & 1.84491 & -0.348917 & -2.20437 & -2.21753 & -0.108508 & 0.971698 \\ -0.213412 & -1.84786 & 0.105118 & 1.95615 & 2.08873 & 0.377362 & -0.637414 \\ 2.59268 & 1.43652 & -1.27705 & -2.75216 & -2.27116 & 1.02096 & 2.13835 \\ -1.37534 & 3.586 & 0.677436 & -2.88809 & -3.57607 & -1.70149 & 0.0255692 \\ -1.76476 & -1.33536 & 0.869247 & 2.23087 & 1.93906 & -0.599548 & -1.55089 \\ -1.09385 & 1.35238 & 0.538783 & -0.797316 & -1.1952 & -0.953183 & -0.37972 \\ -3.15887 & -3.559 & 1.55593 & 5.16195 & 4.75596 & -0.761398 & -3.08784 \\ -2.1644 & 0.0128852 & 1.06609 & 1.08542 & 0.563217 & -1.17565 & -1.46177 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (179.334 \quad 88.3324 \quad 43.5089 \quad 88.3324 \quad -144.965 \quad -71.4039 \quad -71.4039 \quad -144.965 \quad 63.6273 \quad 31$

Gauss point = $\{s \to -0.339981,\ t \to 0.339981\}$ Weight = 0.425293

$\boldsymbol{N}^{\mathrm{T}} = \{0.221103, 0.108906, 0.221103, 0.448887, -0.17873, -0.17873, -0.36286, -0.36286, 0.078447, -0.078447, 0.159264, -0.159264, 0.0288074, 0.0288074, 0.0584853, 0.0584853, 0.29332\}$

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial s = (-0.165005 \quad 0.165005 \quad 0.334995 \quad -0.334995 \quad -0.137413 \quad -0.270795 \quad -0.278977 \quad 0.27079$

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial t = (-0.334995 \quad -0.165005 \quad 0.165005 \quad 0.334995 \quad 0.270795 \quad 0.137413 \quad -0.270795 \quad 0.278977$

$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 66.6667 & -13.1741 \\ 0. & 79.8408 \end{pmatrix}$ detJ = 0.000187874

$\boldsymbol{B}^{\mathrm{T}} = \Bigg(\begin{matrix} -6.58705 & 13.1741 & 20.1592 & -26.7463 & -12.7283 & -19.8633 & -15.031 & 14.3777 & -9.795 \\ -26.7463 & -13.1741 & 13.1741 & 26.7463 & 21.6205 & 10.9711 & -21.6205 & 22.2738 & -9.489 \end{matrix}$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5.\times 10^6$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{array}{cccccccc}
2.72815 & 0.95491 & -1.74438 & -1.93867 & -1.77774 & -0.584627 & 2.4352 & -2.4 \\
0.95491 & 1.24807 & 0.330874 & -2.53386 & -1.62705 & -1.46058 & 0.312133 & -0.3 \\
-1.74438 & 0.330874 & 2.08526 & -0.671746 & 0.101529 & -0.920084 & -2.11363 & 2.0 \\
-1.93867 & -2.53386 & -0.671746 & 5.14428 & 3.30326 & 2.96529 & -0.633697 & 0.7 \\
-1.77774 & -1.62705 & 0.101529 & 3.30326 & 2.26325 & 1.76193 & -0.992829 & 1.0 \\
-0.584627 & -1.46058 & -0.920084 & 2.96529 & 1.76193 & 1.85142 & 0.220639 & -0.1 \\
2.4352 & 0.312133 & -2.11363 & -0.633697 & -0.992829 & 0.220639 & 2.49308 & -2.5 \\
-2.48255 & -0.374023 & 2.09723 & 0.759348 & 1.07351 & -0.148211 & -2.50856 & 2.5 \\
1.1446 & -0.0145163 & -1.15955 & 0.0294712 & -0.289378 & 0.32529 & 1.26712 & -1.2 \\
1.44305 & 0.375562 & -1.05614 & -0.762472 & -0.797903 & -0.131206 & 1.36468 & -1.3 \\
-0.329183 & -1.61632 & -1.33597 & 3.28147 & 1.86503 & 2.1336 & 0.59358 & -0.5 \\
2.36104 & 1.89981 & -0.403818 & -3.85703 & -2.71877 & -1.98121 & 1.47294 & -1.5 \\
-0.0798599 & 0.995032 & 1.10496 & -2.02013 & -1.07278 & -1.38885 & -0.676051 & 0.6 \\
1.80353 & 1.25781 & -0.50771 & -2.55364 & -1.86414 & -1.24758 & 1.23947 & -1.3 \\
-1.13636 & 1.43741 & 2.6172 & -2.91825 & -1.27735 & -2.27867 & -2.09924 & 2.0 \\
3.87039 & 2.13598 & -1.66987 & -4.3365 & -3.38109 & -1.90314 & 2.99292 & -3.0 \\
1.30508 & 1.70575 & 0.452207 & -3.46303 & -2.22369 & -1.99618 & 0.426593 & -0.5
\end{array}\right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = \left(\begin{array}{ccccccccc} 88.3324 & 43.5089 & 88.3324 & 179.334 & -71.4039 & -71.4039 & -144.965 & -144.965 & 31.3402 \end{array}\right.$$

Gauss point = {s → −0.339981, t → 0.861136} Weight = 0.226852

$$\boldsymbol{N}^{\mathrm{T}} =$$

{0.0465187, 0.0229132, 0.307096, 0.623472, −0.0376036, −0.0522287, −0.503986, −0.106035, 0.0165047, −0.0580638, 0.221207, −0.117882, 0.00606089, −0.0540071, 0.0812318, −0.109646, 0.0857143}

$$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} = (\ -0.0347159 \quad 0.0347159 \quad 0.465284 \quad -0.465284 \quad -0.0289107 \quad -0.0791321 \quad -0.387479 \quad 0.0$$

$$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} = (\ -0.334995 \quad -0.165005 \quad 0.165005 \quad 0.334995 \quad 0.270795 \quad 0.348052 \quad -0.270795 \quad 0.70662 \quad -$$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 66.6667 & -2.77175 \\ 0. & 79.8408 \end{pmatrix} \qquad \det J = 0.000187874$$

$$\boldsymbol{B}^{\mathrm{T}} = \begin{pmatrix} -1.38587 & 2.77175 & 30.5616 & -31.9475 & -2.67796 & -6.24018 & -25.0814 & 3.3169 & -2.0 \\ -26.7463 & -13.1741 & 13.1741 & 26.7463 & 21.6205 & 27.7887 & -21.6205 & 56.4171 & -9.4 \end{pmatrix}$$

$$k_x = 45. \qquad k_y = 45. \qquad p = 0. \qquad q = 5. \times 10^{6}$$

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 1.37566 & 0.668414 & -0.757012 & -1.28707 & -1.10193 & -1.40887 & 1.17571 & -2.90 \\ 0.668414 & 0.347596 & -0.1704 & -0.84561 & -0.560506 & -0.735292 & 0.412941 & -1.40 \\ -0.757012 & -0.1704 & 2.12418 & -1.19677 & 0.389306 & 0.336361 & -2.01638 & 1.61 \\ -1.28707 & -0.84561 & -1.19677 & 3.32945 & 1.27313 & 1.8078 & 0.427723 & 2.69 \\ -1.10193 & -0.560506 & 0.389306 & 1.27313 & 0.910258 & 1.18432 & -0.767686 & 2.32 \\ -1.40887 & -0.735292 & 0.336361 & 1.8078 & 1.18432 & 1.55569 & -0.852101 & 2.96 \\ 1.17571 & 0.412941 & -2.01638 & 0.427723 & -0.767686 & -0.852101 & 2.103 & -2.49 \\ -2.9028 & -1.40782 & 1.61987 & 2.69075 & 2.32233 & 2.96708 & -2.49892 & 6.12 \\ 0.492254 & 0.22881 & -0.360569 & -0.360495 & -0.382902 & -0.481082 & 0.492629 & -1.03 \\ -1.29062 & -0.680427 & 0.248875 & 1.72217 & 1.09245 & 1.44035 & -0.733064 & 2.71 \\ -0.400745 & -0.411828 & -1.65742 & 2.46999 & 0.559728 & 0.893122 & 1.1635 & 0.82 \\ -2.66747 & -1.28694 & 1.54694 & 2.40748 & 2.12664 & 2.71153 & -2.34316 & 5.62 \\ 0.168956 & 0.107644 & 0.128026 & -0.404626 & -0.163431 & -0.22984 & -0.0328303 & -0.35 \\ -0.75468 & -0.411319 & 0.0292392 & 1.13676 & 0.653586 & 0.872126 & -0.335397 & 1.58 \\ -0.305446 & 0.165335 & 2.88186 & -2.74175 & -0.100311 & -0.38473 & -2.43733 & 0.68 \\ -1.5761 & -0.747195 & 1.02825 & 1.29504 & 1.24202 & 1.57277 & -1.47608 & 3.32 \\ 2.32348 & 1.18404 & -0.801968 & -2.70555 & -1.92173 & -2.50207 & 1.60355 & -4.89 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (9.91303 \quad 4.88274 \quad 65.4415 \quad 132.86 \quad -8.01324 \quad -11.1298 \quad -107.398 \quad -22.5959 \quad 3.51712 \quad -1$

Gauss point = $\{s \to 0.339981,\ t \to -0.861136\}$ Weight = 0.226852

$\boldsymbol{N}^{\mathrm{T}} =$

$\{0.307096, 0.623472, 0.0465187, 0.0229132, -0.503986, -0.106035, -0.0376036, -0.0522287, -0.221207, 0.117882, -0.0165047, 0.0580638, 0.0812318, -0.109646, 0.00606089, -0.0540071, 0.0857143\}$

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial s = (-0.465284 \quad 0.465284 \quad 0.0347159 \quad -0.0347159 \quad 0.387479 \quad -0.0791321 \quad 0.0289107 \quad 0.079$

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial t = (-0.165005 \quad -0.334995 \quad 0.334995 \quad 0.165005 \quad 0.270795 \quad -0.70662 \quad -0.270795 \quad -0.34805$

$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 66.6667 & -46.6447 \\ 0. & 100.25 \end{pmatrix}$ detJ = 0.000149626

$\boldsymbol{B}^{\mathrm{T}} = \Big(\begin{matrix} -23.3223 & 46.6447 & -13.3114 & -10.011 & 13.2008 & 27.6846 & 14.5585 & 21.5102 & -37.5 \\ -16.5417 & -33.5832 & 33.5832 & 16.5417 & 27.1472 & -70.8386 & -27.1472 & -34.8922 & 11.9 \end{matrix}$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^6$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{array}{llllllll}
1.24877 & -0.813112 & -0.374332 & -0.0613241 & -1.15617 & 0.803616 & 0.167287 & 0.1 \\
-0.813112 & 5.04597 & -2.67108 & -1.56178 & -0.452033 & 5.60618 & 2.42979 & 3.3 \\
-0.374332 & -2.67108 & 1.99334 & 1.05207 & 1.12414 & -4.19664 & -1.68855 & -2.2 \\
-0.0613241 & -1.56178 & 1.05207 & 0.571028 & 0.484055 & -2.21316 & -0.908526 & -1.2 \\
-1.15617 & -0.452033 & 1.12414 & 0.484055 & 1.39184 & -2.37914 & -0.832122 & -1.0 \\
0.803616 & 5.60618 & -4.19664 & -2.21316 & -2.37914 & 8.83551 & 3.55299 & 4.6 \\
0.167287 & 2.42979 & -1.68855 & -0.908526 & -0.832122 & 3.55299 & 1.44941 & 1.9 \\
0.115332 & 3.32237 & -2.22718 & -1.21051 & -1.0131 & 4.68497 & 1.92515 & 2.5 \\
1.03775 & -3.28881 & 1.37534 & 0.87573 & -0.263711 & -2.87846 & -1.3298 & -1.8 \\
-0.774123 & -5.07364 & 3.83172 & 2.01605 & 2.20465 & -8.06774 & -3.2389 & -4.2 \\
0.188716 & 0.835886 & -0.675326 & -0.349276 & -0.430487 & 1.4226 & 0.564198 & 0.7 \\
-0.069467 & -3.12273 & 2.06532 & 1.12687 & 0.909417 & -4.344 & -1.79 & -2.3 \\
1.74033 & -3.0351 & 0.705754 & 0.589018 & -1.10391 & -1.46117 & -0.835921 & -1.2 \\
0.527153 & 2.84509 & -2.21567 & -1.15657 & -1.3386 & 4.66617 & 1.86277 & 2.4 \\
0.0889829 & -0.623522 & 0.338335 & 0.196204 & 0.0684931 & -0.710298 & -0.305991 & -0.4 \\
-0.0303932 & 1.98147 & -1.25689 & -0.694181 & -0.495166 & 2.64266 & 1.09858 & 1.4 \\
-0.341189 & -5.14862 & 3.5684 & 1.92141 & 1.74862 & -7.50833 & -3.0646 & -4.0
\end{array}\right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\, 52.1188 \quad 105.812 \quad 7.89491 \quad 3.8887 \quad -85.5339 \quad -17.9958 \quad -6.38189 \quad -8.86397 \quad -37.542 \quad 20.$$

Gauss point = {s → 0.339981, t → −0.339981}   Weight = 0.425293

$\boldsymbol{N}^{T}$ = {0.221103, 0.448887, 0.221103, 0.108906, −0.36286, −0.36286, −0.17873, −0.17873, −0.159264, 0.159264, −0.078447, 0.078447, 0.0584853, 0.0584853, 0.0288074, 0.0288074, 0.29332}

$\partial \boldsymbol{N}^{T}/\partial s$ = (−0.334995 0.334995 0.165005 −0.165005 0.278977 −0.270795 0.137413 0.270795

$\partial \boldsymbol{N}^{T}/\partial t$ = (−0.165005 −0.334995 0.334995 0.165005 0.270795 −0.278977 −0.270795 −0.13741

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 66.6667 & -33.5832 \\ 0. & 100.25 \end{pmatrix} \qquad \det J = 0.000149626$$

$$\boldsymbol{B}^{T} = \begin{pmatrix} -16.7916 & 33.5832 & -0.249912 & -16.5417 & 9.50432 & -8.68404 & 18.255 & 22.6678 & -2 \\ -16.5417 & -33.5832 & 33.5832 & 16.5417 & 27.1472 & -27.9675 & -27.1472 & -13.7756 & 1 \end{pmatrix}$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^{6}$

$$\boldsymbol{k}_k = \begin{pmatrix} 1.59096 & -0.0240336 & -1.57877 & 0.0118379 & -1.74292 & 1.74234 & 0.408143 & -0.4 \\ -0.0240336 & 6.45927 & -3.25367 & -3.18157 & -1.69668 & 1.85445 & 4.36624 & 3.5 \\ -1.57877 & -3.25367 & 3.22982 & 1.60262 & 2.60389 & -2.68336 & -2.62376 & -1.3 \\ 0.0118379 & -3.18157 & 1.60262 & 1.56711 & 0.835714 & -0.913425 & -2.15063 & -1.7 \\ -1.74292 & -1.69668 & 2.60389 & 0.835714 & 2.36904 & -2.41048 & -1.61353 & -0.4 \\ 1.74234 & 1.85445 & -2.68336 & -0.913425 & -2.41048 & 2.45578 & 1.72018 & 0.5 \\ 0.408143 & 4.36624 & -2.62376 & -2.15063 & -1.61353 & 1.72018 & 3.06464 & 2.2 \\ -0.437429 & 3.50469 & -1.341 & -1.72626 & -0.453954 & 0.539557 & 2.25584 & 2.0 \\ 0.736673 & -3.74803 & 1.16523 & 1.84612 & 0.189837 & -0.281383 & -2.34074 & -2.2 \\ 0.703323 & 5.21524 & -3.34975 & -2.56881 & -2.16458 & 2.29196 & 3.71812 & 2.6 \\ 0.918798 & 0.437087 & -1.14059 & -0.215291 & -1.12686 & 1.13753 & 0.540992 & -0.0 \\ 0.915098 & 1.43147 & -1.64148 & -0.705082 & -1.38806 & 1.42302 & 1.21316 & 0.5 \\ 1.79111 & -2.74692 & -0.397215 & 1.35302 & -1.23663 & 1.16954 & -1.38174 & -1.9 \\ -1.75722 & -6.36231 & 4.98572 & 3.13381 & 3.62938 & -3.78478 & -4.77578 & -2.9 \\ 0.678074 & -2.19146 & 0.433964 & 1.07942 & -0.160968 & 0.107441 & -1.30264 & -1.3 \\ -0.656706 & -3.55147 & 2.45887 & 1.7493 & 1.66948 & -1.75623 & -2.57938 & -1.7 \\ 0.016179 & -4.34827 & 2.19031 & 2.14177 & 1.14217 & -1.24838 & -2.93928 & -2.3 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\ 70.3495 \quad 142.825 \quad 70.3495 \quad 34.6512 \quad -115.453 \quad -115.453 \quad -56.8673 \quad -56.8673 \quad -50.6739$$

Gauss point = $\{s \to 0.339981,\ t \to 0.339981\}$ Weight = 0.425293

$\boldsymbol{N}^{\mathrm{T}} = \{0.108906, 0.221103, 0.448887, 0.221103, -0.17873, -0.36286, -0.36286, -0.17873, -0.078447, -0.159264, -0.159264, -0.078447, 0.0288074, 0.0584853, 0.0584853, 0.0288074, 0.29332\}$

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial s = (\ -0.165005 \quad 0.165005 \quad 0.334995 \quad -0.334995 \quad 0.137413 \quad -0.270795 \quad 0.278977 \quad 0.270795$

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial t = (\ -0.165005 \quad -0.334995 \quad 0.334995 \quad 0.165005 \quad 0.270795 \quad 0.278977 \quad -0.270795 \quad 0.137413$

$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 66.6667 & -16.5417 \\ 0. & 100.25 \end{pmatrix}$ detJ = 0.000149626

$$\boldsymbol{B}^{\mathrm{T}} = \left(\begin{matrix} -8.27086 & 16.5417 & 16.7916 & -25.0625 & 4.68143 & -22.6678 & 23.0779 & 15.78 & -13.32 \\ -16.5417 & -33.5832 & 33.5832 & 16.5417 & 27.1472 & 27.9675 & -27.1472 & 13.7756 & 11.91 \end{matrix} \right.$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^6$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{array}{cccccccc}
0.979443 & 1.19901 & -1.98848 & -0.18997 & -1.39679 & -0.787906 & 0.739337 & -1.02 \\
1.19901 & 4.01319 & -2.43424 & -2.77795 & -2.38894 & -3.76331 & 3.70386 & -0.57 \\
-1.98848 & -2.43424 & 4.03705 & 0.385679 & 2.83579 & 1.59962 & -1.50101 & 2.08 \\
-0.18997 & -2.77795 & 0.385679 & 2.58224 & 0.94994 & 2.9516 & -2.94218 & -0.47 \\
-1.39679 & -2.38894 & 2.83579 & 0.94994 & 2.17312 & 1.87026 & -1.80099 & 1.28 \\
-0.787906 & -3.76331 & 1.59962 & 2.9516 & 1.87026 & 3.71121 & -3.67214 & 0.07 \\
0.739337 & 3.70386 & -1.50101 & -2.94218 & -1.80099 & -3.67214 & 3.63548 & -0.02 \\
-1.02626 & -0.577302 & 2.08354 & -0.479972 & 1.28243 & 0.0789551 & -0.0280642 & 1.25 \\
-0.248747 & -1.77719 & 0.50501 & 1.52093 & 0.7476 & 1.81938 & -1.80704 & -0.13 \\
1.69517 & 3.23154 & -3.44156 & -1.48515 & -2.72598 & -2.63514 & 2.55108 & -1.46 \\
1.06416 & 0.146356 & -2.16048 & 0.949962 & -1.20914 & 0.415415 & -0.468186 & -1.42 \\
0.554867 & 2.15193 & -1.1265 & -1.5803 & -1.18416 & -2.06564 & 2.03812 & -0.18 \\
0.591527 & -0.347741 & -1.20093 & 0.957141 & -0.557645 & 0.702722 & -0.732055 & -0.90 \\
2.17957 & 5.5176 & -4.42501 & -3.27217 & -3.86844 & -4.88642 & 4.77834 & -1.52 \\
0.624693 & -2.0847 & -1.26826 & 2.72827 & -0.130754 & 2.63055 & -2.66152 & -1.41 \\
1.17643 & 2.51202 & -2.3884 & -1.30005 & -1.96365 & -2.12494 & 2.0666 & -0.94 \\
1.33861 & 1.63869 & -2.71767 & -0.259632 & -1.90901 & -1.07683 & 1.01045 & -1.40
\end{array}\right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \left(\begin{array}{ccccccccccccccccc}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{array}\right)$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\,34.6512 \quad 70.3495 \quad 142.825 \quad 70.3495 \quad -56.8673 \quad -115.453 \quad -115.453 \quad -56.8673 \quad -24.9599$$

Gauss point = {s → 0.339981, t → 0.861136}   Weight = 0.226852

$$\boldsymbol{N}^{T} =$$

{0.0229132, 0.0465187, 0.623472, 0.307096, −0.0376036, −0.106035, −0.503986, −0.0522287, −0.0165047, −0.117882, −0.221207, −0.0580638, 0.00606089, −0.109646, 0.0812318, −0.0540071, 0.0857143}

$$\partial \boldsymbol{N}^{T}/\partial s = (\ -0.0347159\ \ 0.0347159\ \ 0.465284\ \ -0.465284\ \ 0.0289107\ \ -0.0791321\ \ 0.387479\ \ 0.079$$

$$\partial \boldsymbol{N}^{T}/\partial t = (\ -0.165005\ \ -0.334995\ \ 0.334995\ \ 0.165005\ \ 0.270795\ \ 0.70662\ \ -0.270795\ \ 0.348052\ \ 0$$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 66.6667 & -3.48027 \\ 0. & 100.25 \end{pmatrix} \qquad \det J = 0.000149626$$

$$\boldsymbol{B}^{T} = \begin{pmatrix} -1.74013 & 3.48027 & 29.8531 & -31.5932 & 0.984943 & -7.7347 & 26.7744 & 4.06416 & -2.8 \\ -16.5417 & -33.5832 & 33.5832 & 16.5417 & 27.1472 & 70.8386 & -27.1472 & 34.8922 & 11.9 \end{pmatrix}$$

$$k_x = 45. \qquad k_y = 45. \qquad p = 0. \qquad q = 5. \times 10^{6}$$

$$\boldsymbol{k}_{k} = \begin{pmatrix} 0.422574 & 0.839276 & -0.927874 & -0.333976 & -0.688528 & -1.76928 & 0.614745 & -0.89 \\ 0.839276 & 1.74119 & -1.564 & -1.01647 & -1.38731 & -3.67487 & 1.53488 & -1.76 \\ -0.927874 & -1.564 & 3.08395 & -0.592078 & 1.43746 & 3.28106 & -0.171672 & 1.97 \\ -0.333976 & -1.01647 & -0.592078 & 1.94253 & 0.63838 & 2.16308 & -1.97795 & 0.68 \\ -0.688528 & -1.38731 & 1.43746 & 0.63838 & 1.12715 & 2.92572 & -1.08539 & 1.45 \\ -1.76928 & -3.67487 & 3.28106 & 2.16308 & 2.92572 & 7.75621 & -3.25368 & 3.72 \\ 0.614745 & 1.53488 & -0.171672 & -1.97795 & -1.08539 & -3.25368 & 2.22064 & -1.28 \\ -0.8924 & -1.76823 & 1.97515 & 0.685476 & 1.45294 & 3.72736 & -1.28061 & 1.88 \\ -0.293602 & -0.626114 & 0.483345 & 0.436371 & 0.489854 & 1.32238 & -0.608749 & 0.61 \\ -1.62147 & -3.37894 & 2.9654 & 2.03502 & 2.68426 & 7.13224 & -3.02866 & 3.41 \\ 0.385112 & 0.443095 & -2.05324 & 1.22504 & -0.54165 & -0.915626 & -0.79925 & -0.83 \\ -0.821936 & -1.61779 & 1.85978 & 0.579946 & 1.33533 & 3.40963 & -1.1338 & 1.73 \\ 0.119627 & 0.206303 & -0.3801 & 0.0541709 & -0.186569 & -0.433111 & 0.041834 & -0.25 \\ -0.949507 & -2.00038 & 1.65495 & 1.29494 & 1.57766 & 4.22357 & -1.86532 & 1.99 \\ 0.0168622 & -0.479281 & -1.96145 & 2.42387 & 0.109315 & 1.03979 & -2.14191 & -0.06 \\ -0.489329 & -0.942022 & 1.18643 & 0.244924 & 0.789337 & 1.98416 & -0.585802 & 1.03 \\ 1.45321 & 2.92457 & -3.04704 & -1.33075 & -2.37805 & -6.16748 & 2.27606 & -3.06 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_q^T = (3.8887 \quad 7.89491 \quad 105.812 \quad 52.1188 \quad -6.38189 \quad -17.9958 \quad -85.5339 \quad -8.86397 \quad -2.8011 \quad -2$$

Gauss point = $\{s \to 0.861136,\ t \to -0.861136\}$ Weight = 0.121003

$$\boldsymbol{N}^T = \{0.0646111, 0.865957, 0.0646111, 0.00482078, -0.147276, -0.147276, -0.0109886, -0.0109886, -0.16373, 0.16373, -0.0122162, 0.0122162, -0.15229, -0.15229, -0.0113627, -0.0113627, 0.0250475\}$$

$$\partial\boldsymbol{N}^T/\partial s = (-0.465284 \quad 0.465284 \quad 0.0347159 \quad -0.0347159 \quad 0.981444 \quad -0.0791321 \quad 0.0732278 \quad 0.079$$

$$\partial\boldsymbol{N}^T/\partial t = (-0.0347159 \quad -0.465284 \quad 0.465284 \quad 0.0347159 \quad 0.0791321 \quad -0.981444 \quad -0.0791321 \quad -0.0$$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 66.6667 & -58.0101 \\ 0. & 124.677 \end{pmatrix} \qquad \det J = 0.000120311$$

$$\boldsymbol{B}^T = \left(\begin{matrix} -29.0051 & 58.0101 & -24.6768 & -4.32827 & 60.8392 & 51.6582 & 9.47232 & 9.52343 & 54. \\ -4.32827 & -58.0101 & 58.0101 & 4.32827 & 9.86593 & -122.363 & -9.86593 & -9.12981 & 10. \end{matrix} \right.$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^6$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{array}{llllllll}
0.563413 & -0.937793 & 0.304409 & 0.0699709 & -1.18401 & -0.634624 & -0.152014 & -0. \\
-0.937793 & 4.40912 & -3.14235 & -0.328975 & 1.93714 & 6.61335 & 0.734912 & 0. \\
0.304409 & -3.14235 & 2.60349 & 0.234458 & -0.608592 & -5.48528 & -0.528065 & -0. \\
0.0699709 & -0.328975 & 0.234458 & 0.0245456 & -0.144534 & -0.493437 & -0.0548335 & -0. \\
-1.18401 & 1.93714 & -0.608592 & -0.144534 & 2.48859 & 1.26804 & 0.313766 & 0. \\
-0.634624 & 6.61335 & -5.48528 & -0.493437 & 1.26804 & 11.557 & 1.11143 & 1. \\
-0.152014 & 0.734912 & -0.528065 & -0.0548335 & 0.313766 & 1.11143 & 0.122546 & 0. \\
-0.155072 & 0.708879 & -0.500916 & -0.0528911 & 0.320561 & 1.05415 & 0.118105 & 0. \\
-1.07544 & 1.67185 & -0.471673 & -0.124741 & 2.26143 & 0.980749 & 0.270164 & 0. \\
0.563163 & -6.03218 & 5.01894 & 0.450075 & -1.12332 & -10.5747 & -1.01394 & -0. \\
-0.151026 & 0.781077 & -0.571772 & -0.0582779 & 0.311126 & 1.20359 & 0.130368 & 0. \\
0.161775 & -0.689587 & 0.476361 & 0.0514517 & -0.335006 & -1.00229 & -0.114762 & -0. \\
-0.610808 & 0.776059 & -0.107348 & -0.0579036 & 1.28645 & 0.218537 & 0.12409 & 0. \\
-0.293595 & 3.47608 & -2.92312 & -0.259358 & 0.581713 & 6.15934 & 0.584656 & 0. \\
-0.111414 & 0.668384 & -0.5071 & -0.0498696 & 0.228433 & 1.06774 & 0.111769 & 0. \\
-0.133295 & 0.482136 & -0.312868 & -0.0359733 & 0.277046 & 0.657948 & 0.0799994 & 0. \\
0.336425 & -1.58174 & 1.12729 & 0.118017 & -0.694932 & -2.37248 & -0.263644 & -0.
\end{array}\right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = \left(\begin{array}{lllllllll} 4.70304 & 63.033 & 4.70304 & 0.350905 & -10.7202 & -10.7202 & -0.799858 & -0.799858 & -11.9179 \end{array}\right.$$

Gauss point = $\{s \to 0.861136,\ t \to -0.339981\}$ Weight = 0.226852

$\boldsymbol{N}^{\mathrm{T}} =$

{0.0465187, 0.623472, 0.307096, 0.0229132, −0.106035, −0.503986, −0.0522287, −0.0376036, −0.117882,
0.221207, −0.0580638, 0.0165047, −0.109646, 0.0812318, −0.0540071, 0.00606089, 0.0857143}

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} = (\ -0.334995\ \ 0.334995\ \ 0.165005\ \ -0.165005\ \ 0.70662\ \ -0.270795\ \ 0.348052\ \ 0.270795\ \ 0$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} = (\ -0.0347159\ \ -0.465284\ \ 0.465284\ \ 0.0347159\ \ 0.0791321\ \ -0.387479\ \ -0.0791321\ \ -0.0$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 66.6667 & -41.7661 \\ 0. & 124.677 \end{pmatrix} \qquad \mathrm{detJ} = 0.000120311$$

$$\boldsymbol{B}^{\mathrm{T}} = \begin{pmatrix} -20.8831 & 41.7661 & -8.4328 & -12.4503 & 43.803 & -1.86949 & 26.5085 & 19.2605 & 39.5 \\ -4.32827 & -58.0101 & 58.0101 & 4.32827 & 9.86593 & -48.3097 & -9.86593 & -3.6045 & 10.9 \end{pmatrix}$$

$\mathrm{k_x} = 45. \qquad \mathrm{k_y} = 45. \qquad \mathrm{p} = 0. \qquad \mathrm{q} = 5. \times 10^{6}$

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix}
0.558619 & -0.762846 & -0.0920895 & 0.296317 & -1.17591 & 0.304757 & -0.627446 & -0. \\
-0.762846 & 6.27547 & -4.5656 & -0.947025 & 1.54401 & 3.346 & 2.0627 & 1. \\
-0.0920895 & -4.5656 & 4.22036 & 0.437322 & 0.249249 & -3.42254 & -0.977462 & -0. \\
0.296317 & -0.947025 & 0.437322 & 0.213387 & -0.61735 & -0.228222 & -0.457791 & -0. \\
-1.17591 & 1.54401 & 0.249249 & -0.61735 & 2.47605 & -0.685947 & 1.30655 & 0. \\
0.304757 & 3.346 & -3.42254 & -0.228222 & -0.685947 & 2.87064 & 0.524508 & 0. \\
-0.627446 & 2.0627 & -0.977462 & -0.457791 & 1.30655 & 0.524508 & 0.982586 & 0. \\
-0.474833 & 1.2448 & -0.456288 & -0.313675 & 0.992495 & 0.169642 & 0.670742 & 0. \\
-1.07322 & 1.24838 & 0.371614 & -0.546774 & 2.26172 & -0.741629 & 1.1554 & 0. \\
-0.39952 & 5.7049 & -4.55879 & -0.746589 & 0.780078 & 3.49073 & 1.63746 & 0. \\
-0.582254 & 2.06256 & -1.04011 & -0.4402 & 1.21069 & 0.593428 & 0.946014 & 0. \\
0.188582 & -0.0111267 & -0.251953 & 0.0744978 & -0.39988 & 0.280002 & -0.155046 & -0. \\
-0.619717 & 0.404121 & 0.4985 & -0.282904 & 1.30974 & -0.655928 & 0.594197 & 0. \\
0.218147 & -7.75354 & 6.64704 & 0.888351 & -0.371166 & -5.24035 & -1.96282 & -1. \\
-0.355132 & 1.54558 & -0.892154 & -0.29829 & 0.735037 & 0.568658 & 0.643254 & 0. \\
0.0964744 & -0.738905 & 0.528336 & 0.114095 & -0.195912 & -0.383815 & -0.248252 & -0. \\
1.0036 & -2.67992 & 1.00827 & 0.668049 & -2.09713 & -0.393733 & -1.42894 & -1.
\end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = ($ 6.34812   85.0814   41.9076   3.12682   −14.47   −68.776   −7.12733   −5.13154   −16.0866   30.1

Gauss point = {s → 0.861136, t → 0.339981}   Weight = 0.226852

$\boldsymbol{N}^{\mathrm{T}} =$

{0.0229132, 0.307096, 0.623472, 0.0465187, −0.0522287, −0.503986, −0.106035, −0.0376036, −0.0580638, −0.221207, −0.117882, −0.0165047, −0.0540071, 0.0812318, −0.109646, 0.00606089, 0.0857143}

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial\mathrm{s} = ($ −0.165005   0.165005   0.334995   −0.334995   0.348052   −0.270795   0.70662   0.270795   0

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial\mathrm{t} = ($ −0.0347159   −0.465284   0.465284   0.0347159   0.0791321   0.387479   −0.0791321   0.0289

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 66.6667 & -20.5723 \\ 0. & 124.677 \end{pmatrix} \qquad \mathrm{detJ} = 0.000120311$$

$$\boldsymbol{B}^{\mathrm{T}} = \Bigg(\begin{matrix} -10.2861 & 20.5723 & 12.7611 & -23.0472 & 21.5755 & -26.0243 & 48.736 & 17.4582 & 19.49 \\ -4.32827 & -58.0101 & 58.0101 & 4.32827 & 9.86593 & 48.3097 & -9.86593 & 3.6045 & 10.96 \end{matrix}$$

$k_x = 45.$ $k_y = 45.$ $p = 0.$ $q = 5. \times 10^6$

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{array}{cccccccc}
0.152955 & 0.0484816 & -0.469587 & 0.268151 & -0.325014 & 0.0719617 & -0.563244 & - \\
0.0484816 & 4.65281 & -3.8106 & -0.890693 & -0.157779 & -4.09944 & 1.93429 & \\
-0.469587 & -3.8106 & 4.33303 & -0.0528404 & 1.04106 & 3.03403 & 0.0609163 & \\
0.268151 & -0.890693 & -0.0528404 & 0.675383 & -0.558271 & 0.993453 & -1.43197 & - \\
-0.325014 & -0.157779 & 1.04106 & -0.558271 & 0.691267 & -0.104234 & 1.17189 & \\
0.0719617 & -4.09944 & 3.03403 & 0.993453 & -0.104234 & 3.69815 & -2.14309 & - \\
-0.563244 & 1.93429 & 0.0609163 & -1.43197 & 1.17189 & -2.14309 & 3.0367 & \\
-0.239714 & 0.184298 & 0.530428 & -0.475012 & 0.506294 & -0.344143 & 1.00131 & \\
-0.304537 & -0.288982 & 1.08692 & -0.493403 & 0.649382 & 0.0277964 & 1.03375 & \\
0.293712 & 4.31843 & -4.23808 & -0.374067 & -0.674002 & -3.61774 & 0.843498 & - \\
-0.510879 & 1.91981 & -0.0753096 & -1.33363 & 1.06098 & -2.09083 & 2.82971 & \\
-0.0856559 & 0.537349 & -0.18276 & -0.268933 & 0.175344 & -0.542086 & 0.572607 & \\
-0.191426 & -0.45246 & 0.897053 & -0.253167 & 0.411387 & 0.258198 & 0.526414 & \\
0.252017 & 6.81321 & -6.0904 & -0.974829 & -0.615019 & -5.86673 & 2.13968 & - \\
-0.288743 & 1.4128 & -0.30135 & -0.822705 & 0.595786 & -1.47305 & 1.74864 & \\
0.0583057 & 0.429345 & -0.503425 & 0.0157747 & -0.128745 & -0.337788 & -0.0275158 & - \\
0.507621 & -0.342516 & -1.16095 & 0.995842 & -1.0727 & 0.686311 & -2.09863 & -
\end{array}\right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\; 3.12682 \quad 41.9076 \quad 85.0814 \quad 6.34812 \quad -7.12733 \quad -68.776 \quad -14.47 \quad -5.13154 \quad -7.92361 \quad -30.$$

Gauss point = {s → 0.861136, t → 0.861136}   Weight = 0.121003

$$\boldsymbol{N}^{T} = \{0.00482078,\ 0.0646111,\ 0.865957,\ 0.0646111,$$
$$-0.0109886,\ -0.147276,\ -0.147276,\ -0.0109886,\ -0.0122162,\ -0.16373,$$
$$-0.16373,\ -0.0122162,\ -0.0113627,\ -0.15229,\ -0.15229,\ -0.0113627,\ 0.0250475\}$$

$$\partial \boldsymbol{N}^{T}/\partial s = (\ -0.0347159\ \ 0.0347159\ \ 0.465284\ \ -0.465284\ \ 0.0732278\ \ -0.0791321\ \ 0.981444\ \ 0.079$$

$$\partial \boldsymbol{N}^{T}/\partial t = (\ -0.0347159\ \ -0.465284\ \ 0.465284\ \ 0.0347159\ \ 0.0791321\ \ 0.981444\ \ -0.0791321\ \ 0.0732$$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 66.6667 & -4.32827 \\ 0. & 124.677 \end{pmatrix} \qquad \det J = 0.000120311$$

$$\boldsymbol{B}^{T} = \begin{pmatrix} -2.16413 & 4.32827 & 29.0051 & -31.1692 & 4.53935 & -9.52343 & 65.7721 & 4.95852 & 4.100 \\ -4.32827 & -58.0101 & 58.0101 & 4.32827 & 9.86593 & 122.363 & -9.86593 & 9.12981 & 10.968 \end{pmatrix}$$

$$k_x = 45. \qquad k_y = 45. \qquad p = 0. \qquad q = 5. \times 10^{6}$$

$$\boldsymbol{k}_k = \begin{pmatrix}
0.015341 & 0.158351 & -0.205609 & 0.0319172 & -0.0344104 & -0.333459 & -0.0652734 & -0. \\
0.158351 & 2.21683 & -2.12232 & -0.252867 & -0.362064 & -4.67718 & 0.561432 & -0. \\
-0.205609 & -2.12232 & 2.7557 & -0.427774 & 0.46119 & 4.46922 & 0.874835 & 0. \\
0.0319172 & -0.252867 & -0.427774 & 0.648725 & -0.0647154 & 0.541422 & -1.37099 & -0. \\
-0.0344104 & -0.362064 & 0.46119 & -0.0647154 & 0.0772652 & 0.762547 & 0.131825 & 0. \\
-0.333459 & -4.67718 & 4.46922 & 0.541422 & 0.762547 & 9.86824 & -1.20121 & 0. \\
-0.0652734 & 0.561432 & 0.874835 & -1.37099 & 0.131825 & -1.20121 & 2.89776 & 0. \\
-0.0329174 & -0.332901 & 0.44118 & -0.0753618 & 0.073754 & 0.700923 & 0.154644 & 0. \\
-0.036914 & -0.405196 & 0.494744 & -0.0526341 & 0.083085 & 0.853641 & 0.105803 & 0. \\
-0.304665 & -4.29653 & 4.0833 & 0.517891 & 0.696976 & 9.06538 & -1.14674 & 0. \\
-0.0545953 & 0.588215 & 0.73172 & -1.26534 & 0.108859 & -1.25633 & 2.67534 & 0. \\
-0.031667 & -0.302703 & 0.424421 & -0.0900503 & 0.070745 & 0.637068 & 0.185999 & 0. \\
-0.0321667 & -0.381223 & 0.431117 & -0.0177274 & 0.0727321 & 0.803542 & 0.0325118 & 0. \\
-0.176569 & -2.53575 & 2.36648 & 0.345837 & 0.404472 & 5.35076 & -0.761517 & 0. \\
-0.0217199 & 0.488996 & 0.291104 & -0.75838 & 0.040297 & -1.04067 & 1.6052 & 0. \\
-0.0214852 & -0.172576 & 0.287958 & -0.0938964 & 0.0476112 & 0.36266 & 0.195768 & 0. \\
0.0737606 & 0.761363 & -0.988584 & 0.153461 & -0.165448 & -1.60329 & -0.31384 & -0.
\end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}}=\begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}}=(\,0.350905 \quad 4.70304 \quad 63.033 \quad 4.70304 \quad -0.799858 \quad -10.7202 \quad -10.7202 \quad -0.799858 \quad -0.88922$$

Adding contributions from all Gauss points

$$\boldsymbol{k}_{\mathrm{k}} = \left(\begin{array}{lllll}
22.5818 & -0.163551 & -11.0864 & -11.3318 & -9.5862 \\
-0.163551 & 45.3271 & -22.8271 & -22.3364 & -17.5699 \\
-11.0864 & -22.8271 & 34.0771 & -0.163551 & 8.38435 \\
-11.3318 & -22.3364 & -0.163551 & 33.8318 & 18.7718 \\
-9.5862 & -17.5699 & 8.38435 & 18.7718 & 35.7126 \\
1.04447 & -2.08893 & -7.09665 & 8.14112 & 6.13318 \\
9.5862 & 17.5699 & -17.5699 & -9.5862 & -1.96262 \\
-7.41142 & 14.8228 & 3.54833 & -10.9597 & -8.69159 \\
2.40126 & -4.80252 & 4.80252 & -2.40126 & -3.04952 \\
-0.26968 & 0.53936 & -0.53936 & 0.26968 & -1.58358 \\
-2.40126 & 4.80252 & -4.80252 & 2.40126 & 0.144784 \\
1.91362 & -3.82724 & 3.82724 & -1.91362 & 2.24416 \\
0.284121 & -0.568242 & 0.568242 & -0.284121 & -1.3919 \\
4.44089\times 10^{-16} & 0. & -3.33067\times 10^{-16} & 8.88178\times 10^{-16} & -2.22045\times 10 \\
-0.284121 & 0.568242 & -0.568242 & 0.284121 & 1.3919 \\
-8.88178\times 10^{-16} & 1.33227\times 10^{-15} & -6.66134\times 10^{-16} & 0. & 0. \\
2.55841 & -5.11682 & 5.11682 & -2.55841 & -12.5336
\end{array} \right.$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\; 937.5 \quad 750. \quad 750. \quad 937.5 \quad -688.919 \quad -612.372 \quad -688.919 \quad -765.466 \quad 59.2927 \quad 3.55271\times 10^{-1}$$

Natural boundary conditions

Specified NBC values for side 2: $\alpha = -55$ $\beta = 1100$

Interpolation functions for mapping: $\left\{0, \frac{1-a}{2}, \frac{a+1}{2}, 0\right\}$

Interpolation functions for solution:

$$\left\{0, \frac{1-a}{2}, \frac{a+1}{2}, 0, 0, \frac{\frac{3a^2}{2}-\frac{3}{2}}{\sqrt{6}}, 0, 0, 0, \frac{\frac{5a^3}{2}-\frac{5a}{2}}{\sqrt{10}}, 0, 0, 0, \frac{\frac{35a^4}{8}-\frac{21a^2}{4}+\frac{7}{8}}{\sqrt{14}}, 0, 0, 0\right\}$$

$x(a) = 0.03$ $y(a) = 0.0075\,a + 0.0225$

$J_c = 0.0075$

Value in mapped coordinate: $\alpha(a) = -55$ $\beta(a) = 1100$

Gauss point = −0.861136 Weight = 0.347855

$\boldsymbol{N}_c^T$ = {0., 0.930568, 0.0694318, 0., 0., −0.158264, 0., 0., 0., 0.175946, 0., 0., 0., −0.163653, 0., 0., 0.}

$J_c = 0.0075$ $\alpha = -55.$ $\beta = 1100.$

$$\boldsymbol{k}_\alpha = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & (\\
0. & -0.124256 & -0.00927105 & 0. & 0. & 0.0211326 & 0. & 0. & 0. & -0.0234936 & 0. & 0. & 0. & (\\
0. & -0.00927105 & -0.000691734 & 0. & 0. & 0.00157675 & 0. & 0. & 0. & -0.00175291 & 0. & 0. & 0. & (\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & (\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & (\\
0. & 0.0211326 & 0.00157675 & 0. & 0. & -0.00359407 & 0. & 0. & 0. & 0.00399561 & 0. & 0. & 0. & -(\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & (\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & (\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & (\\
0. & -0.0234936 & -0.00175291 & 0. & 0. & 0.00399561 & 0. & 0. & 0. & -0.00444201 & 0. & 0. & 0. & (\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & (\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & (\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & (\\
0. & 0.0218522 & 0.00163044 & 0. & 0. & -0.00371645 & 0. & 0. & 0. & 0.00413166 & 0. & 0. & 0. & -(\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & (\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & (\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & (
\end{pmatrix}$$

Gauss point = −0.339981 Weight = 0.652145

$\boldsymbol{N}_c^T$ = {0., 0.669991, 0.330009, 0., 0., −0.54159, 0., 0., 0., 0.237711, 0., 0., 0., 0.0872927, 0., 0., 0.}

$J_c = 0.0075$ $\alpha = -55.$ $\beta = 1100.$

$$\boldsymbol{k}_{\alpha} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & -0.120755 & -0.059479 & 0. & 0. & 0.097613 & 0. & 0. & 0. & -0.0428437 & 0. & 0. & 0. & -0.01 \\
0. & -0.059479 & -0.0292969 & 0. & 0. & 0.0480801 & 0. & 0. & 0. & -0.021103 & 0. & 0. & 0. & -0.00 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0.097613 & 0.0480801 & 0. & 0. & -0.0789059 & 0. & 0. & 0. & 0.0346329 & 0. & 0. & 0. & 0.01 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & -0.0428437 & -0.021103 & 0. & 0. & 0.0346329 & 0. & 0. & 0. & -0.0152008 & 0. & 0. & 0. & -0.00 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & -0.0157331 & -0.00774948 & 0. & 0. & 0.0127179 & 0. & 0. & 0. & -0.00558208 & 0. & 0. & 0. & -0.00 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

Gauss point = 0.339981 Weight = 0.652145

$\boldsymbol{N}_{\mathrm{c}}^{\mathrm{T}} = \{0., 0.330009, 0.669991, 0., 0., -0.54159, 0., 0., 0., -0.237711, 0., 0., 0., 0.0872927, 0., 0., 0.\}$

$\mathrm{J_c} = 0.0075 \qquad \alpha = -55. \qquad \beta = 1100.$

$$\boldsymbol{k}_{\alpha} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & -0.0292969 & -0.059479 & 0. & 0. & 0.0480801 & 0. & 0. & 0. & 0.021103 & 0. & 0. & 0. & -0.00 \\
0. & -0.059479 & -0.120755 & 0. & 0. & 0.097613 & 0. & 0. & 0. & 0.0428437 & 0. & 0. & 0. & -0.01 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0.0480801 & 0.097613 & 0. & 0. & -0.0789059 & 0. & 0. & 0. & -0.0346329 & 0. & 0. & 0. & 0.01 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0.021103 & 0.0428437 & 0. & 0. & -0.0346329 & 0. & 0. & 0. & -0.0152008 & 0. & 0. & 0. & 0.00 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & -0.00774948 & -0.0157331 & 0. & 0. & 0.0127179 & 0. & 0. & 0. & 0.00558208 & 0. & 0. & 0. & -0.00 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

Gauss point = 0.861136 Weight = 0.347855

$\boldsymbol{N}_c^T = \{0., 0.0694318, 0.930568, 0., 0., -0.158264, 0., 0., 0., -0.175946, 0., 0., 0., -0.163653, 0., 0., 0.\}$

$J_c = 0.0075 \qquad \alpha = -55. \qquad \beta = 1100.$

$$\boldsymbol{k}_\alpha = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & -0.000691734 & -0.00927105 & 0. & 0. & 0.00157675 & 0. & 0. & 0. & 0.00175291 & 0. & 0. & 0. & 0. \\
0. & -0.00927105 & -0.124256 & 0. & 0. & 0.0211326 & 0. & 0. & 0. & 0.0234936 & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0.00157675 & 0.0211326 & 0. & 0. & -0.00359407 & 0. & 0. & 0. & -0.00399561 & 0. & 0. & 0. & -0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0.00175291 & 0.0234936 & 0. & 0. & -0.00399561 & 0. & 0. & 0. & -0.00444201 & 0. & 0. & 0. & -0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0.00163044 & 0.0218522 & 0. & 0. & -0.00371645 & 0. & 0. & 0. & -0.00413166 & 0. & 0. & 0. & -0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

Adding contributions from all Gauss points

$$\boldsymbol{k}_{\alpha} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & -0.275 & -0.1375 & 0. & 0. & 0.168402 & 0. & 0. & 0. & -0.0434813 \\
0. & -0.1375 & -0.275 & 0. & 0. & 0.168402 & 0. & 0. & 0. & 0.0434813 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0.168402 & 0.168402 & 0. & 0. & -0.165 & 0. & 0. & 0. & -1.73472 \times 10^{-18} \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & -0.0434813 & 0.0434813 & 0. & 0. & -1.73472 \times 10^{-18} & 0. & 0. & 0. & -0.0392857 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 2.38524 \times 10^{-18} & 3.46945 \times 10^{-18} & 0. & 0. & 0.018003 & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

Specified NBC values for side 3: $\alpha = -55$ $\beta = 1100$

Interpolation functions for mapping: $\left\{0, 0, \frac{1-a}{2}, \frac{a+1}{2}\right\}$

Interpolation functions for solution:

$$\left\{0, 0, \frac{1-a}{2}, \frac{a+1}{2}, 0, 0, \frac{\frac{3a^2}{2} - \frac{3}{2}}{\sqrt{6}}, 0, 0, 0, \frac{\frac{5a}{2} - \frac{5a^3}{2}}{\sqrt{10}}, 0, 0, 0, \frac{\frac{35a^4}{8} - \frac{21a^2}{4} + \frac{7}{8}}{\sqrt{14}}, 0, 0\right\}$$

$x(a) = 0.015 - 0.015\,a$ $y(a) = 0.03$

$J_c = 0.015$

Value in mapped coordinate: $\alpha(a) = -55$ $\beta(a) = 1100$

Gauss point = −0.861136 Weight = 0.347855

$\boldsymbol{N}_c^T = \{0., 0., 0.930568, 0.0694318, 0., 0., -0.158264, 0., 0., 0., -0.175946, 0., 0., 0., -0.163653, 0., 0.\}$

$J_c = 0.015$ $\alpha = -55.$ $\beta = 1100.$

$$\boldsymbol{k}_{\alpha} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & -0.248513 & -0.0185421 & 0. & 0. & 0.0422652 & 0. & 0. & 0. & 0.0469871 & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & -0.0185421 & -0.00138347 & 0. & 0. & 0.0031535 & 0. & 0. & 0. & 0.00350582 & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0.0422652 & 0.0031535 & 0. & 0. & -0.00718815 & 0. & 0. & 0. & -0.00799122 & 0. & 0. & 0. & 0. & - \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0.0469871 & 0.00350582 & 0. & 0. & -0.00799122 & 0. & 0. & 0. & -0.00888402 & 0. & 0. & 0. & 0. & - \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0.0437043 & 0.00326088 & 0. & 0. & -0.00743291 & 0. & 0. & 0. & -0.00826333 & 0. & 0. & 0. & 0. & - \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

Gauss point = -0.339981 Weight = 0.652145

$\boldsymbol{N}_{c}^{T}$ = {0., 0., 0.669991, 0.330009, 0., 0., −0.54159, 0., 0., 0., −0.237711, 0., 0., 0., 0.0872927, 0., 0.}

J_c = 0.015 α = −55. β = 1100.

$$\boldsymbol{k}_{\alpha} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & -0.24151 & -0.118958 & 0. & 0. & 0.195226 & 0. & 0. & 0. & 0.0856873 & 0. & 0. & 0. & -0.(\\
0. & 0. & -0.118958 & -0.0585937 & 0. & 0. & 0.0961602 & 0. & 0. & 0. & 0.042206 & 0. & 0. & 0. & -0.(\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0.195226 & 0.0961602 & 0. & 0. & -0.157812 & 0. & 0. & 0. & -0.0692658 & 0. & 0. & 0. & 0.(\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0.0856873 & 0.042206 & 0. & 0. & -0.0692658 & 0. & 0. & 0. & -0.0304017 & 0. & 0. & 0. & 0.(\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & -0.0314662 & -0.015499 & 0. & 0. & 0.0254359 & 0. & 0. & 0. & 0.0111642 & 0. & 0. & 0. & -0.(\\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

Gauss point = 0.339981 Weight = 0.652145

$\boldsymbol{N}_c^T$ = {0., 0., 0.330009, 0.669991, 0., 0., −0.54159, 0., 0., 0., 0.237711, 0., 0., 0., 0.0872927, 0., 0.}

J_c = 0.015 α = −55. β = 1100.

$$\boldsymbol{k}_\alpha = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & -0.0585937 & -0.118958 & 0. & 0. & 0.0961602 & 0. & 0. & 0. & -0.042206 & 0. & 0. & 0. & -0.0 \\
0. & 0. & -0.118958 & -0.24151 & 0. & 0. & 0.195226 & 0. & 0. & 0. & -0.0856873 & 0. & 0. & 0. & -0.0 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0.0961602 & 0.195226 & 0. & 0. & -0.157812 & 0. & 0. & 0. & 0.0692658 & 0. & 0. & 0. & 0.0 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & -0.042206 & -0.0856873 & 0. & 0. & 0.0692658 & 0. & 0. & 0. & -0.0304017 & 0. & 0. & 0. & -0.0 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & -0.015499 & -0.0314662 & 0. & 0. & 0.0254359 & 0. & 0. & 0. & -0.0111642 & 0. & 0. & 0. & -0.0 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

Gauss point = 0.861136 Weight = 0.347855

$\boldsymbol{N}_c^T$ = {0., 0., 0.0694318, 0.930568, 0., 0., −0.158264, 0., 0., 0., 0.175946, 0., 0., 0., −0.163653, 0., 0.}

J_c = 0.015 α = −55. β = 1100.

$$\boldsymbol{k}_{\alpha} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & -0.00138347 & -0.0185421 & 0. & 0. & 0.0031535 & 0. & 0. & 0. & -0.00350582 & 0. & 0. & 0. \\
0. & 0. & -0.0185421 & -0.248513 & 0. & 0. & 0.0422652 & 0. & 0. & 0. & -0.0469871 & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0.0031535 & 0.0422652 & 0. & 0. & -0.00718815 & 0. & 0. & 0. & 0.00799122 & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & -0.00350582 & -0.0469871 & 0. & 0. & 0.00799122 & 0. & 0. & 0. & -0.00888402 & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0.00326088 & 0.0437043 & 0. & 0. & -0.00743291 & 0. & 0. & 0. & 0.00826333 & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

Adding contributions from all Gauss points

$$\boldsymbol{k}_{\alpha} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & -0.55 & -0.275 & 0. & 0. & 0.336805 & 0. & 0. & 0.0869626 \\
0. & 0. & -0.275 & -0.55 & 0. & 0. & 0.336805 & 0. & 0. & -0.0869626 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0.336805 & 0.336805 & 0. & 0. & -0.33 & 0. & 0. & 3.46945\times 10 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0.0869626 & -0.0869626 & 0. & 0. & 3.46945\times 10^{-18} & 0. & 0. & -0.0785714 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 4.77049\times 10^{-18} & 6.93889\times 10^{-18} & 0. & 0. & 0.036006 & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}$$

Specified NBC values for side 4: $\alpha = 0$ $\beta = 8000$

Interpolation functions for mapping: $\left\{\frac{a+1}{2}, 0, 0, \frac{1-a}{2}\right\}$

Interpolation functions for solution:

$$\left\{\frac{a+1}{2}, 0, 0, \frac{1-a}{2}, 0, 0, 0, \frac{\frac{3a^2}{2}-\frac{3}{2}}{\sqrt{6}}, 0, 0, 0, \frac{\frac{5a}{2}-\frac{5a^3}{2}}{\sqrt{10}}, 0, 0, 0, \frac{\frac{35a^4}{8}-\frac{21a^2}{4}+\frac{7}{8}}{\sqrt{14}}, 0\right\}$$

$x(\mathrm{a}) = 0 \qquad y(\mathrm{a}) = 0.015 - 0.015\,\mathrm{a}$

$\mathrm{J_c} = 0.015$

Value in mapped coordinate: $\alpha(\mathrm{a}) = 0 \qquad \beta(\mathrm{a}) = 8000$

Gauss point = −0.861136 Weight = 0.347855

$\boldsymbol{N}_c^T = \{0.0694318, 0., 0., 0.930568, 0., 0., 0., -0.158264, 0., 0., 0., -0.175946, 0., 0., 0., -0.163653, 0.\}$

$\mathrm{J_c} = 0.015 \qquad \alpha = 0. \qquad \beta = 8000.$

$$\boldsymbol{k}_\alpha = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix} \qquad \boldsymbol{r}_\beta = \begin{pmatrix}
-2.89826 \\ 0. \\ 0. \\ -38.8443 \\ 0. \\ 0. \\ 0. \\ 6.60635 \\ 0. \\ 0. \\ 0. \\ 7.34443 \\ 0. \\ 0. \\ 0. \\ 6.8313 \\ 0.
\end{pmatrix}$$

Gauss point = −0.339981 Weight = 0.652145

$\boldsymbol{N}_c^T = \{0.330009, 0., 0., 0.669991, 0., 0., 0., -0.54159, 0., 0., 0., -0.237711, 0., 0., 0., 0.0872927, 0.\}$

$\mathrm{J_c} = 0.015 \qquad \alpha = 0. \qquad \beta = 8000.$

$$\boldsymbol{k}_{\alpha} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix} \qquad \boldsymbol{r}_{\beta} = \begin{pmatrix}
-25.8257 \\ 0. \\ 0. \\ -52.4317 \\ 0. \\ 0. \\ 0. \\ 42.3834 \\ 0. \\ 0. \\ 0. \\ 18.6027 \\ 0. \\ 0. \\ 0. \\ -6.8313 \\ 0.
\end{pmatrix}$$

Gauss point = 0.339981 Weight = 0.652145

$\boldsymbol{N}_{\mathrm{c}}^{\mathrm{T}} = \{0.669991, 0., 0., 0.330009, 0., 0., 0., -0.54159, 0., 0., 0., 0.237711, 0., 0., 0., 0.0872927, 0.\}$

$\mathrm{J}_{\mathrm{c}} = 0.015$ $\alpha = 0.$ $\beta = 8000.$

$$\boldsymbol{k}_{\alpha} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix} \qquad \boldsymbol{r}_{\beta} = \begin{pmatrix}
-52.4317 \\ 0. \\ 0. \\ -25.8257 \\ 0. \\ 0. \\ 0. \\ 42.3834 \\ 0. \\ 0. \\ 0. \\ -18.6027 \\ 0. \\ 0. \\ 0. \\ -6.8313 \\ 0.
\end{pmatrix}$$

Gauss point = 0.861136 Weight = 0.347855

$\boldsymbol{N}_{\mathrm{c}}^{\mathrm{T}} = \{0.930568, 0., 0., 0.0694318, 0., 0., 0., -0.158264, 0., 0., 0., 0.175946, 0., 0., 0., -0.163653, 0.\}$

$\mathrm{J}_{\mathrm{c}} = 0.015$ $\alpha = 0.$ $\beta = 8000.$

$$\boldsymbol{k}_{\alpha} = \begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix} \qquad \boldsymbol{r}_{\beta} = \begin{pmatrix}
-38.8443 \\ 0. \\ 0. \\ -2.89826 \\ 0. \\ 0. \\ 0. \\ 6.60635 \\ 0. \\ 0. \\ 0. \\ -7.34443 \\ 0. \\ 0. \\ 0. \\ 6.8313 \\ 0.
\end{pmatrix}$$

Adding contributions from all Gauss points

$$\boldsymbol{k}_{\alpha}=\begin{pmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{pmatrix}
\qquad
\boldsymbol{r}_{\beta}=\begin{pmatrix}
-120. \\ 0. \\ 0. \\ -120. \\ 0. \\ 0. \\ 0. \\ 97.9796 \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \\ 7.10543\times10^{-15} \\ 0.
\end{pmatrix}$$

$$\left(\begin{array}{cccccccc}
22.5818 & -0.163551 & -11.0864 & -11.3318 & -9.5862 & 1.04447 & 9.5862 & -7.41142 \\
-0.163551 & 45.0521 & -22.9646 & -22.3364 & -17.5699 & -1.92053 & 17.5699 & 14.8228 \\
-11.0864 & -22.9646 & 33.2521 & -0.438551 & 8.38435 & -6.92825 & -17.2331 & 3.54833 \\
-11.3318 & -22.3364 & -0.438551 & 33.2818 & 18.7718 & 8.14112 & -9.2494 & -10.9597 \\
-9.5862 & -17.5699 & 8.38435 & 18.7718 & 35.7126 & 6.13318 & -1.96262 & -8.69159 \\
1.04447 & -1.92053 & -6.92825 & 8.14112 & 6.13318 & 58.307 & 1.36682 & 15.764 \\
9.5862 & 17.5699 & -17.2331 & -9.2494 & -1.96262 & 1.36682 & 35.3826 & 1.19159 \\
-7.41142 & 14.8228 & 3.54833 & -10.9597 & -8.69159 & 15.764 & 1.19159 & 47.243 \\
2.40126 & -4.80252 & 4.80252 & -2.40126 & -3.04952 & -0.452451 & -5.66469 & 0.226226 \\
-0.26968 & 0.495879 & -0.495879 & 0.26968 & -1.58358 & -3.16716 & 1.58358 & 1.58358 \\
-2.40126 & 4.80252 & -4.71556 & 2.3143 & 0.144784 & 0.452451 & -3.04952 & -0.226226 \\
1.91362 & -3.82724 & 3.82724 & -1.91362 & 2.24416 & -4.2259 & -2.24416 & -2.24416 \\
0.284121 & -0.568242 & 0.568242 & -0.284121 & -1.3919 & -0.0535348 & 1.3919 & 0.0267674 \\
0 & 0 & 0 & 0 & 0 & 0.330034 & 0 & -0.156016 \\
-0.284121 & 0.568242 & -0.568242 & 0.284121 & 1.3919 & 0.0535348 & -1.3559 & -0.0267674 \\
0 & 0 & 0 & 0 & 0 & 2.29894 & 0 & -0.412982 \\
2.55841 & -5.11682 & 5.11682 & -2.55841 & -12.5336 & -31.2997 & -15.0232 & -23.8482
\end{array}\right.$$

Global equations after assembling all elements

$$\begin{pmatrix}
47.722 & 19.8598 & -5.96963 & -39.1939 & -11.0864 & -11.3318 & -29.6457 & -4.14925 \\
19.8598 & 70.1402 & -39.0304 & -50.9696 & 0 & 0 & -43.839 & 31.3054 \\
-5.96963 & -39.0304 & 43.7607 & 0.414252 & 0 & 0 & 14.1933 & -16.6829 \\
-39.1939 & -50.9696 & 0.414252 & 133.813 & -22.9646 & -22.3364 & 59.2914 & -37.6294 \\
-11.0864 & 0 & 0 & -22.9646 & 33.2521 & -0.438551 & 0 & 8.38435 \\
-11.3318 & 0 & 0 & -22.3364 & -0.438551 & 33.2818 & 0 & 18.7718 \\
-29.6457 & -43.839 & 14.1933 & 59.2914 & 0 & 0 & 58.5981 & -34.7664 \\
-4.14925 & 31.3054 & -16.6829 & -37.6294 & 8.38435 & 18.7718 & -34.7664 & 132.827 \\
-7.41142 & 0 & 0 & 14.8228 & 3.54833 & -10.9597 & 0 & -8.69159 \\
-5.43695 & 5.43695 & -20.0595 & 20.0595 & 0 & 0 & 4.76636 & 37.8855 \\
4.17787 & 32.5645 & -28.0498 & -8.01893 & 0 & 0 & -13.6963 & 24.5327 \\
1.04447 & 0 & 0 & -1.92053 & -6.92825 & 8.14112 & 0 & 6.13318 \\
9.5862 & 0 & 0 & 17.5699 & -17.2331 & -9.2494 & 0 & -1.96262 \\
7.65449 & -7.65449 & 15.309 & -15.309 & 0 & 0 & -8.97663 & 8.97663 \\
3.69425 & -1.29299 & 2.58597 & -7.3885 & 4.80252 & -2.40126 & 0.904903 & -18.0981 \\
1.91362 & 0 & 0 & -3.82724 & 3.82724 & -1.91362 & 0 & 2.24416 \\
-1.29299 & 1.29299 & -2.58597 & 2.58597 & 0 & 0 & -0.904903 & 3.42958 \\
-1.07872 & 1.07872 & -2.07048 & 2.07048 & 0 & 0 & 6.33432 & -6.33432 \\
-0.26968 & 0 & 0 & 0.495879 & -0.495879 & 0.26968 & 0 & -1.58358 \\
-2.40126 & 0 & 0 & 4.80252 & -4.71556 & 2.3143 & 0 & 0.144784 \\
0 & 0 & 0 & 0 & 0 & 0 & -1.65193 & 0 \\
0.43711 & -0.152988 & 0.305977 & -0.874219 & 0.568242 & -0.284121 & 0.10707 & -2.14139 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.152988 & 0.152988 & -0.305977 & 0.305977 & 0 & 0 & -0.10707 & 0.749487 \\
0 & 0 & 0 & 0 & 0 & 0 & -0.624062 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.284121 & 0 & 0 & 0.568242 & -0.568242 & 0.284121 & 0 & 1.3919 \\
10.2336 & -10.2336 & 20.4673 & -20.4673 & 0 & 0 & -7.82921 & -50.1344 \\
2.55841 & 0 & 0 & -5.11682 & 5.11682 & -2.55841 & 0 & -12.5336
\end{pmatrix}$$

Essential boundary conditions

On element 1, side 1, specified value = 110

$$\left\{T_1,\ T_2,\ \delta_1^{\{1,2\}},\ \delta_2^{\{1,2\}},\ \delta_3^{\{1,2\}}\right\} = \{110,\ 110,\ 0,\ 0,\ 0\}$$

Known values from EBC

$$\left\{T_1 = 110,\ T_2 = 110,\ \delta_1^{\{1,2\}} = 0,\ \delta_2^{\{1,2\}} = 0,\ \delta_3^{\{1,2\}} = 0\right\}$$

Global equations after EBC

$$\begin{pmatrix}
43.7607 & 0.414252 & 0 & 0 & -16.6829 & 0 & -20.0595 & -28.0498 \\
0.414252 & 133.813 & -22.9646 & -22.3364 & -37.6294 & 14.8228 & 20.0595 & -8.01893 \\
0 & -22.9646 & 33.2521 & -0.438551 & 8.38435 & 3.54833 & 0 & 0 \\
0 & -22.3364 & -0.438551 & 33.2818 & 18.7718 & -10.9597 & 0 & 0 \\
-16.6829 & -37.6294 & 8.38435 & 18.7718 & 132.827 & -8.69159 & 37.8855 & 24.5327 \\
0 & 14.8228 & 3.54833 & -10.9597 & -8.69159 & 47.243 & 0 & 0 \\
-20.0595 & 20.0595 & 0 & 0 & 37.8855 & 0 & 97.1145 & 5.46729 \\
-28.0498 & -8.01893 & 0 & 0 & 24.5327 & 0 & 5.46729 & 49.5625 \\
0 & -1.92053 & -6.92825 & 8.14112 & 6.13318 & 15.764 & 0 & 0 \\
0 & 17.5699 & -17.2331 & -9.2494 & -1.96262 & 1.19159 & 0 & 0 \\
2.58597 & -7.3885 & 4.80252 & -2.40126 & -18.0981 & 0.226226 & -19.8083 & -1.80981 \\
0 & -3.82724 & 3.82724 & -1.91362 & 2.24416 & -2.24416 & 0 & 0 \\
-2.58597 & 2.58597 & 0 & 0 & 3.42958 & 0 & -15.0485 & 1.80981 \\
-2.07048 & 2.07048 & 0 & 0 & -6.33432 & 0 & 6.33432 & -12.6686 \\
0 & 0.495879 & -0.495879 & 0.26968 & -1.58358 & 1.58358 & 0 & 0 \\
0 & 4.80252 & -4.71556 & 2.3143 & 0.144784 & -0.226226 & 0 & 0 \\
0.305977 & -0.874219 & 0.568242 & -0.284121 & -2.14139 & 0.0267674 & 0.749487 & -0.214139 \\
0 & 0 & 0 & 0 & 0 & -0.412982 & 0 & 0 \\
-0.305977 & 0.305977 & 0 & 0 & 0.749487 & 0 & -0.749487 & 0.214139 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.28413 \\
0 & 0 & 0 & 0 & 0 & -0.156016 & 0 & 0 \\
0 & 0.568242 & -0.568242 & 0.284121 & 1.3919 & -0.0267674 & 0 & 0 \\
20.4673 & -20.4673 & 0 & 0 & -50.1344 & 0 & -60.0926 & -24.7582 \\
0 & -5.11682 & 5.11682 & -2.55841 & -12.5336 & -23.8482 & 0 & 0
\end{pmatrix}$$

Solving the final system of global equations we get

$\{T_3 = 126.672,\ T_4 = 139.179,\ T_5 = 161.022,\ T_6 = 157.838,\ \delta_1^{\{1,4\}} = -10.2723,\ \delta_1^{\{1,6\}} = -18.1555,\ \delta_1^{\{2,3\}} = -3.1219,$
$\delta_1^{\{3,4\}} = 9.11823,\ \delta_1^{\{4,5\}} = -7.80844,\ \delta_1^{\{5,6\}} = -0.417322,\ \delta_2^{\{1,4\}} = -1.97672,\ \delta_2^{\{1,6\}} = -0.489402,\ \delta_2^{\{2,3\}} = 0.628592,$
$\delta_2^{\{3,4\}} = 1.57953,\ \delta_2^{\{4,5\}} = -0.116878,\ \delta_2^{\{5,6\}} = 2.87472,\ \delta_3^{\{1,4\}} = -0.453533,\ \delta_3^{\{1,6\}} = -0.0551927,$
$\delta_3^{\{2,3\}} = 0.290689,\ \delta_3^{\{3,4\}} = -0.356589,\ \delta_3^{\{4,5\}} = 0.0993939,\ \delta_3^{\{5,6\}} = 1.207,\ \delta_1^1 = 5.59898,\ \delta_1^2 = -5.68136\}$

Solution for element 1

DOF values for the element

$\{T_1 = 110,\ T_2 = 110,\ T_3 = 126.672,\ T_4 = 139.179,\ \delta_1^{\{1,2\}} = 0,\ \delta_1^{\{2,3\}} = -3.1219,$
$\delta_1^{\{3,4\}} = 9.11823,\ \delta_1^{\{1,4\}} = -10.2723,\ \delta_2^{\{1,2\}} = 0,\ \delta_2^{\{2,3\}} = 0.628592,\ \delta_2^{\{3,4\}} = 1.57953,\ \delta_2^{\{1,4\}} = -1.97672,$
$\delta_3^{\{1,2\}} = 0,\ \delta_3^{\{2,3\}} = 0.290689,\ \delta_3^{\{3,4\}} = -0.356589,\ \delta_3^{\{1,4\}} = -0.453533,\ \delta_1^1 = 5.59898\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 110\ \ 110\ \ 126.672\ \ 139.179\ \ 0\ \ -3.1219\ \ 9.11823\ \ -10.2723\ \ 0\ \ 0.628592\ \ 1.57953\ \ -1.97672\ \ 0\ \ 0.$

Mapping

$x(s,t) = -0.0075\,t\,s + 0.0225\,s + 0.0075\,t + 0.0375$

$y(s,t) = 0.0075\,t + 0.0075$

$\boldsymbol{J} = \begin{pmatrix} 0.0225 - 0.0075\,t & 0.0075 - 0.0075\,s \\ 0 & 0.0075 \end{pmatrix}$

Element solution at $\{s \to 0,\ t \to 0\}$

Location: {0.0375, 0.0075}

$\boldsymbol{N}^{\mathrm{T}} = (\ 0.25\ \ 0.25\ \ 0.25\ \ 0.25\ \ -0.306186\ \ -0.306186\ \ -0.306186\ \ -0.306186\ \ 0.\ \ 0.\ \ 0.\ \ 0.\ \ 0.1169$

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial s = (\ -0.25\ \ 0.25\ \ 0.25\ \ -0.25\ \ 0.\ \ -0.306186\ \ 0.\ \ 0.306186\ \ -0.395285\ \ 0.\ \ -0.395285\ \ 0.\ \ ($

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial t = (\ -0.25\ \ -0.25\ \ 0.25\ \ 0.25\ \ 0.306186\ \ 0.\ \ -0.306186\ \ 0.\ \ 0.\ \ -0.395285\ \ 0.\ \ -0.395285\ \ -$

$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 44.4444 & 0. \\ -44.4444 & 133.333 \end{pmatrix}$

$\boldsymbol{B}_x^{\mathrm{T}} = \partial\boldsymbol{N}^{\mathrm{T}}/\partial x = (\ -11.1111\ \ 11.1111\ \ 11.1111\ \ -11.1111\ \ 0.\ \ -13.6083\ \ 0.\ \ 13.6083\ \ -17.5682\ \ 0.$

$\boldsymbol{B}_y^{\mathrm{T}} = \partial\boldsymbol{N}^{\mathrm{T}}/\partial y = (\ -22.2222\ \ -44.4444\ \ 22.2222\ \ 44.4444\ \ 40.8248\ \ 13.6083\ \ -40.8248\ \ -13.6083\ \ 1$

$T = \boldsymbol{N}^{\mathrm{T}}\boldsymbol{d} = 124.811$

$\partial T/\partial x = \boldsymbol{B}_x^{\mathrm{T}}\boldsymbol{d} = -260.161$

$\partial T/\partial y = \boldsymbol{B}_y^{\mathrm{T}}\boldsymbol{d} = 1481.78$

	x	y	T	$\partial T/\partial x$	$\partial T/\partial y$
1	0.0375	0.0075	124.811	−260.161	1481.78

Solution for element 2

DOF values for the element

$\{T_1 = 110,\ T_4 = 139.179,\ T_5 = 161.022,\ T_6 = 157.838,\ \delta_1^{\{1,4\}} = -10.2723,\ \delta_1^{\{4,5\}} = -7.80844,\ \delta_1^{\{5,6\}} = -0.417322,$
$\delta_1^{\{1,6\}} = -18.1555,\ \delta_2^{\{1,4\}} = -1.97672,\ \delta_2^{\{4,5\}} = -0.116878,\ \delta_2^{\{5,6\}} = 2.87472,\ \delta_2^{\{1,6\}} = -0.489402,$
$\delta_3^{\{1,4\}} = -0.453533,\ \delta_3^{\{4,5\}} = 0.0993939,\ \delta_3^{\{5,6\}} = 1.207,\ \delta_3^{\{1,6\}} = -0.0551927,\ \delta_1^2 = -5.68136\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 110\ \ 139.179\ \ 161.022\ \ 157.838\ \ -10.2723\ \ -7.80844\ \ -0.417322\ \ -18.1555\ \ -1.97672\ \ -0.116878$

Mapping

$x(s,t) = 0.015\,s + 0.015$

$y(s,t) = -0.00375\,t\,s + 0.00375\,s + 0.01125\,t + 0.01875$

$\boldsymbol{J} = \begin{pmatrix} 0.015 & 0 \\ 0.00375 - 0.00375\,t & 0.01125 - 0.00375\,s \end{pmatrix}$

Element solution at $\{s \to 0,\ t \to 0\}$

Location: {0.015, 0.01875}

$\boldsymbol{N}^{\mathrm{T}} = (\ 0.25\ \ 0.25\ \ 0.25\ \ 0.25\ \ -0.306186\ \ -0.306186\ \ -0.306186\ \ -0.306186\ \ 0.\ \ 0.\ \ 0.\ \ 0.\ \ 0.1169$

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial s = (\ -0.25\ \ 0.25\ \ 0.25\ \ -0.25\ \ 0.\ \ -0.306186\ \ 0.\ \ 0.306186\ \ -0.395285\ \ 0.\ \ -0.395285\ \ 0.\ \ ($

$\partial\boldsymbol{N}^{\mathrm{T}}/\partial t = (\ -0.25\ \ -0.25\ \ 0.25\ \ 0.25\ \ 0.306186\ \ 0.\ \ -0.306186\ \ 0.\ \ 0.\ \ -0.395285\ \ 0.\ \ -0.395285\ \ -$

$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 66.6667 & -22.2222 \\ 0. & 88.8889 \end{pmatrix}$

$\boldsymbol{B}_x^{\mathrm{T}} = \partial\boldsymbol{N}^{\mathrm{T}}/\partial x = (\ -11.1111\ \ 22.2222\ \ 11.1111\ \ -22.2222\ \ -6.80414\ \ -20.4124\ \ 6.80414\ \ 20.4124\ \ -$

$\boldsymbol{B}_y^{\mathrm{T}} = \partial\boldsymbol{N}^{\mathrm{T}}/\partial y = (\ -22.2222\ \ -22.2222\ \ 22.2222\ \ 22.2222\ \ 27.2166\ \ 0.\ \ -27.2166\ \ 0.\ \ 0.\ \ -35.1364\ \ ($

$\mathrm{T} = \boldsymbol{N}^{\mathrm{T}}\boldsymbol{d} = 151.195$

$\partial\mathrm{T}/\partial x = \boldsymbol{B}_x^{\mathrm{T}}\boldsymbol{d} = -23.981$

$\partial\mathrm{T}/\partial y = \boldsymbol{B}_y^{\mathrm{T}}\boldsymbol{d} = 1318.8$

	x	y	T	$\partial\mathrm{T}/\partial x$	$\partial\mathrm{T}/\partial y$
1	0.015	0.01875	151.195	−23.981	1318.8

Nodal solution summary

dof	x	y	Value
T_1	0	0	110
T_2	0.06	0	110
T_3	0.06	0.015	126.672
T_4	0.03	0.015	139.179
T_5	0.03	0.03	161.022
T_6	0	0.03	157.838

Element solution summary

	x	y	T	$\partial T/\partial x$	$\partial T/\partial y$
1	0.0375	0.0075	124.811	−260.161	1481.78
2	0.015	0.01875	151.195	−23.981	1318.8

Example 9.8: Seepage through soil (p. 627)

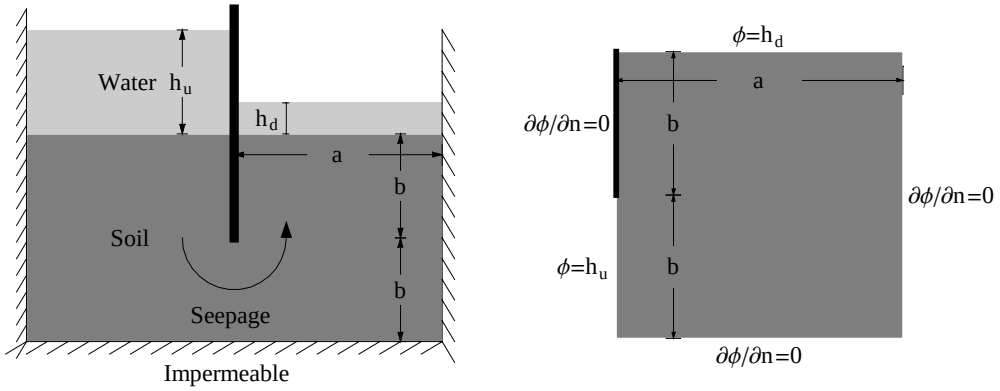
The problem of determining the amount of water that seeps through dams or from underneath sheet piles can be formulated in terms of the following equation

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial \phi}{\partial y}\right) = 0$$

where $\phi(x, y)$ is the hydraulic head (or hydraulic potential) and k_x and k_y are coefficients of permeability in the x and y directions. Typical units for ϕ are meters and those for k_x and k_y are m / day. The fluid velocity components in the x and y directions are related to the hydraulic head as follows.

$$v_x = -k_x \frac{\partial \phi}{\partial x} \text{ and } v_y = -k_y \frac{\partial \phi}{\partial y}$$

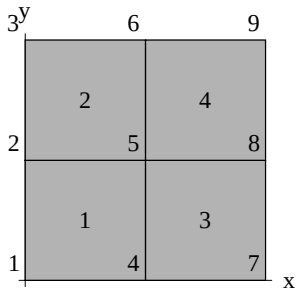
A typical situation is illustrated in Figure. The analytical model consists of the soil on the downstream side. On the impermeable sides the no flow condition is expressed in terms of normal derivative of ϕ being zero. On the top $\phi = h_d$ the hydraulic head on the downstream side. On the left side the boundary condition on the soil below the pile is $\phi = h_u$, the hydraulic head on the upstream side.



As a numerical example, consider the following numerical values.

$$a = b = 10\,m; \quad h_u = 10\,m; \quad h_d = 1\,m; \quad k_x = k_y = 1\,m/s$$

The solution domain is divided into four elements. Solution using second-order ($n = 2$) p formulation is as follows.



Interpolation functions for mapping: $\left\{\frac{1}{4}(1-s)(1-t),\ \frac{1}{4}(s+1)(1-t),\ \frac{1}{4}(s+1)(t+1),\ \frac{1}{4}(1-s)(t+1)\right\}$

Interpolation functions for assumed solution: $\boldsymbol{N}^{\mathrm{T}} = \left\{\frac{1}{4}(1-s)(1-t),\ \frac{1}{4}(s+1)(1-t),\ \frac{1}{4}(s+1)(t+1),\right.$

$$\left.\frac{1}{4}(1-s)(t+1),\ \frac{\left(\frac{3s^2}{2}-\frac{3}{2}\right)(1-t)}{2\sqrt{6}},\ \frac{(s+1)\left(\frac{3t^2}{2}-\frac{3}{2}\right)}{2\sqrt{6}},\ \frac{\left(\frac{3s^2}{2}-\frac{3}{2}\right)(t+1)}{2\sqrt{6}},\ \frac{(1-s)\left(\frac{3t^2}{2}-\frac{3}{2}\right)}{2\sqrt{6}}\right\}$$

$$\partial\boldsymbol{N}^{\mathrm{T}}/\partial s = \left\{\frac{t-1}{4},\ \frac{1-t}{4},\ \frac{t+1}{4},\ \frac{1}{4}(-t-1),\ \frac{1}{2}\sqrt{\frac{3}{2}}\,s\,(1-t),\ \frac{\frac{3t^2}{2}-\frac{3}{2}}{2\sqrt{6}},\ \frac{1}{2}\sqrt{\frac{3}{2}}\,s\,(t+1),\ -\frac{\frac{3t^2}{2}-\frac{3}{2}}{2\sqrt{6}}\right\}$$

$$\partial\boldsymbol{N}^{\mathrm{T}}/\partial t = \left\{\frac{s-1}{4},\ \frac{1}{4}(-s-1),\ \frac{s+1}{4},\ \frac{1-s}{4},\ -\frac{\frac{3s^2}{2}-\frac{3}{2}}{2\sqrt{6}},\ \frac{1}{2}\sqrt{\frac{3}{2}}\,(s+1)\,t,\ \frac{\frac{3s^2}{2}-\frac{3}{2}}{2\sqrt{6}},\ \frac{1}{2}\sqrt{\frac{3}{2}}\,(1-s)\,t\right\}$$

Use 2×2 Gauss quadrature for integration.

Global equations at start of the element assembly process

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \\ \phi_9 \\
\delta_1^{\{1,2\}} \\ \delta_1^{\{1,4\}} \\ \delta_1^{\{2,3\}} \\ \delta_1^{\{2,5\}} \\ \delta_1^{\{3,6\}} \\ \delta_1^{\{4,5\}} \\ \delta_1^{\{4,7\}} \\ \delta_1^{\{5,6\}} \\ \delta_1^{\{5,8\}} \\ \delta_1^{\{6,9\}} \\ \delta_1^{\{7,8\}} \\ \delta_1^{\{8,9\}}
\end{pmatrix}
=
\begin{pmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{pmatrix}$$

Equations for element 1

Element coordinates: $\begin{pmatrix} \{0, 0\} & \{5, 0\} & \{5, 5\} & \{0, 5\} \end{pmatrix}$

$$x(\mathrm{s,t}) = \frac{5\,\mathrm{s}}{2} + \frac{5}{2}$$

$$y(\mathrm{s,t}) = \frac{5\,\mathrm{t}}{2} + \frac{5}{2}$$

$$\boldsymbol{J} = \begin{pmatrix} \frac{5}{2} & 0 \\ 0 & \frac{5}{2} \end{pmatrix}$$

$$\det J = \frac{25}{4}$$

Given element data

$k_x = 1$ $k_y = 1$ $p = 0$ $q = 0$

Element data in mapped coordinates

$k_x = 1$ $k_y = 1$ $p = 0$ $q = 0$

Gauss point = {s → −0.57735, t → −0.57735} Weight = 1.

$$\boldsymbol{N}^T = \{0.622008, 0.166667, 0.0446582, 0.166667, -0.321975, -0.086273, -0.086273, -0.321975\}$$

$$\partial \boldsymbol{N}^T/\partial s =$$
$$(\ -0.394338 \quad 0.394338 \quad 0.105662 \quad -0.105662 \quad -0.557678 \quad -0.204124 \quad -0.149429 \quad 0.204124\ )$$

$$\partial \boldsymbol{N}^T/\partial t =$$
$$(\ -0.394338 \quad -0.105662 \quad 0.105662 \quad 0.394338 \quad 0.204124 \quad -0.149429 \quad -0.204124 \quad -0.557678\ )$$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix} \qquad \det J = 6.25$$

$$\boldsymbol{B}^T =$$
$$\begin{pmatrix} -0.157735 & 0.157735 & 0.042265 & -0.042265 & -0.223071 & -0.0816497 & -0.0597717 & 0.0816497 \\ -0.157735 & -0.042265 & 0.042265 & 0.157735 & 0.0816497 & -0.0597717 & -0.0816497 & -0.223071 \end{pmatrix}$$

$k_x = 1.$ $k_y = 1.$ $p = 0.$ $q = 0.$

$$\boldsymbol{k}_k = \begin{pmatrix} 0.311004 & -0.113835 & -0.0833333 & -0.113835 & 0.139419 & 0.139419 & 0.139419 \\ -0.113835 & 0.166667 & 0.0305021 & -0.0833333 & -0.241481 & -0.0647048 & -0.0373573 \\ -0.0833333 & 0.0305021 & 0.0223291 & 0.0305021 & -0.0373573 & -0.0373573 & -0.0373573 \\ -0.113835 & -0.0833333 & 0.0305021 & 0.166667 & 0.139419 & -0.0373573 & -0.0647048 \\ 0.139419 & -0.241481 & -0.0373573 & 0.139419 & 0.352671 & 0.0833333 & 0.0416667 \\ 0.139419 & -0.0647048 & -0.0373573 & -0.0373573 & 0.0833333 & 0.0639958 & 0.0610042 \\ 0.139419 & -0.0373573 & -0.0373573 & -0.0647048 & 0.0416667 & 0.0610042 & 0.0639958 \\ 0.139419 & 0.139419 & -0.0373573 & -0.241481 & -0.227671 & 0.0416667 & 0.0833333 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_q^T = (\ 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0.\ )$$

Gauss point = {s → −0.57735, t → 0.57735} Weight = 1.

$\boldsymbol{N}^{\mathrm{T}} = \{0.166667,\ 0.0446582,\ 0.166667,\ 0.622008,\ -0.086273,\ -0.086273,\ -0.321975,\ -0.321975\}$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} =$

$(\ -0.105662\quad 0.105662\quad 0.394338\quad -0.394338\quad -0.149429\quad -0.204124\quad -0.557678\quad 0.204124\ )$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} =$

$(\ -0.394338\quad -0.105662\quad 0.105662\quad 0.394338\quad 0.204124\quad 0.149429\quad -0.204124\quad 0.557678\ )$

$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix}$ $\qquad \det \mathrm{J} = 6.25$

$\boldsymbol{B}^{\mathrm{T}} =$

$$\begin{pmatrix} -0.042265 & 0.042265 & 0.157735 & -0.157735 & -0.0597717 & -0.0816497 & -0.223071 & 0.0816497 \\ -0.157735 & -0.042265 & 0.042265 & 0.157735 & 0.0816497 & 0.0597717 & -0.0816497 & 0.223071 \end{pmatrix}$$

$\mathrm{k_x} = 1.$ $\qquad \mathrm{k_y} = 1.$ $\qquad \mathrm{p} = 0.$ $\qquad \mathrm{q} = 0.$

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 0.166667 & 0.0305021 & -0.0833333 & -0.113835 & -0.0647048 & -0.0373573 & 0.139419 \\ 0.0305021 & 0.0223291 & 0.0305021 & -0.0833333 & -0.0373573 & -0.0373573 & -0.0373573 \\ -0.0833333 & 0.0305021 & 0.166667 & -0.113835 & -0.0373573 & -0.0647048 & -0.241481 \\ -0.113835 & -0.0833333 & -0.113835 & 0.311004 & 0.139419 & 0.139419 & 0.139419 \\ -0.0647048 & -0.0373573 & -0.0373573 & 0.139419 & 0.0639958 & 0.0610042 & 0.0416667 \\ -0.0373573 & -0.0373573 & -0.0647048 & 0.139419 & 0.0610042 & 0.0639958 & 0.0833333 \\ 0.139419 & -0.0373573 & -0.241481 & 0.139419 & 0.0416667 & 0.0833333 & 0.352671 \\ -0.241481 & -0.0373573 & 0.139419 & 0.139419 & 0.0833333 & 0.0416667 & -0.227671 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\ 0.\quad 0.\quad 0.\quad 0.\quad 0.\quad 0.\quad 0.\quad 0.\ )$

Gauss point = $\{\mathrm{s} \to 0.57735,\ \mathrm{t} \to -0.57735\}$ $\qquad$ Weight = 1.

$\boldsymbol{N}^{\mathrm{T}} = \{0.166667,\ 0.622008,\ 0.166667,\ 0.0446582,\ -0.321975,\ -0.321975,\ -0.086273,\ -0.086273\}$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} =$

$(\ -0.394338\quad 0.394338\quad 0.105662\quad -0.105662\quad 0.557678\quad -0.204124\quad 0.149429\quad 0.204124\ )$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} =$

$(\ -0.105662\quad -0.394338\quad 0.394338\quad 0.105662\quad 0.204124\quad -0.557678\quad -0.204124\quad -0.149429\ )$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix} \qquad \det J = 6.25$$

$$\boldsymbol{B}^T =$$

$$\begin{pmatrix} -0.157735 & 0.157735 & 0.042265 & -0.042265 & 0.223071 & -0.0816497 & 0.0597717 & 0.0816497 \\ -0.042265 & -0.157735 & 0.157735 & 0.042265 & 0.0816497 & -0.223071 & -0.0816497 & -0.0597717 \end{pmatrix}$$

$k_x = 1.$ $\quad k_y = 1.$ $\quad p = 0.$ $\quad q = 0.$

$$\boldsymbol{k}_k = \begin{pmatrix} 0.166667 & -0.113835 & -0.0833333 & 0.0305021 & -0.241481 & 0.139419 & -0.0373573 \\ -0.113835 & 0.311004 & -0.113835 & -0.0833333 & 0.139419 & 0.139419 & 0.139419 \\ -0.0833333 & -0.113835 & 0.166667 & 0.0305021 & 0.139419 & -0.241481 & -0.0647048 \\ 0.0305021 & -0.0833333 & 0.0305021 & 0.0223291 & -0.0373573 & -0.0373573 & -0.0373573 \\ -0.241481 & 0.139419 & 0.139419 & -0.0373573 & 0.352671 & -0.227671 & 0.0416667 \\ 0.139419 & 0.139419 & -0.241481 & -0.0373573 & -0.227671 & 0.352671 & 0.0833333 \\ -0.0373573 & 0.139419 & -0.0647048 & -0.0373573 & 0.0416667 & 0.0833333 & 0.0639958 \\ -0.0647048 & 0.139419 & -0.0373573 & -0.0373573 & 0.0833333 & 0.0416667 & 0.0610042 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_q^T = (\,0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0.\,)$$

Gauss point = {s → 0.57735, t → 0.57735} $\qquad$ Weight = 1.

$$\boldsymbol{N}^T = \{0.0446582,\ 0.166667,\ 0.622008,\ 0.166667,\ -0.086273,\ -0.321975,\ -0.321975,\ -0.086273\}$$

$$\partial \boldsymbol{N}^T/\partial s =$$

$$(\,-0.105662 \quad 0.105662 \quad 0.394338 \quad -0.394338 \quad 0.149429 \quad -0.204124 \quad 0.557678 \quad 0.204124\,)$$

$$\partial \boldsymbol{N}^T/\partial t =$$

$$(\,-0.105662 \quad -0.394338 \quad 0.394338 \quad 0.105662 \quad 0.204124 \quad 0.557678 \quad -0.204124 \quad 0.149429\,)$$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix} \qquad \det J = 6.25$$

$$\boldsymbol{B}^T =$$

$$\begin{pmatrix} -0.042265 & 0.042265 & 0.157735 & -0.157735 & 0.0597717 & -0.0816497 & 0.223071 & 0.0816497 \\ -0.042265 & -0.157735 & 0.157735 & 0.042265 & 0.0816497 & 0.223071 & -0.0816497 & 0.0597717 \end{pmatrix}$$

$k_x = 1.$ $k_y = 1.$ $p = 0.$ $q = 0.$

$$\boldsymbol{k}_k = \begin{pmatrix} 0.0223291 & 0.0305021 & -0.0833333 & 0.0305021 & -0.0373573 & -0.0373573 & -0.0373573 \\ 0.0305021 & 0.166667 & -0.113835 & -0.0833333 & -0.0647048 & -0.241481 & 0.139419 \\ -0.0833333 & -0.113835 & 0.311004 & -0.113835 & 0.139419 & 0.139419 & 0.139419 \\ 0.0305021 & -0.0833333 & -0.113835 & 0.166667 & -0.0373573 & 0.139419 & -0.241481 \\ -0.0373573 & -0.0647048 & 0.139419 & -0.0373573 & 0.0639958 & 0.0833333 & 0.0416667 \\ -0.0373573 & -0.241481 & 0.139419 & 0.139419 & 0.0833333 & 0.352671 & -0.227671 \\ -0.0373573 & 0.139419 & 0.139419 & -0.241481 & 0.0416667 & -0.227671 & 0.352671 \\ -0.0373573 & -0.0373573 & 0.139419 & -0.0647048 & 0.0610042 & 0.0416667 & 0.0833333 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_q^T = (\,0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0.\,)$$

Adding contributions from all Gauss points

$$\boldsymbol{k}_k = \begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.2041 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.2041 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.2041 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.2041 \\ -0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.833333 & 1.38778\times10^{-17} & 0.1666 \\ 0.204124 & -0.204124 & -0.204124 & 0.204124 & 1.38778\times10^{-17} & 0.833333 & 6.9388 \\ 0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.166667 & 6.93889\times10^{-18} & 0.8333 \\ -0.204124 & 0.204124 & 0.204124 & -0.204124 & -2.77556\times10^{-17} & 0.166667 & -1.3877 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_q^T = (\,0.\quad 0.\quad 0.\quad 0.\quad 0.\quad 0.\quad 0.\quad 0.\,)$$

$$\begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\ -0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.833333 & 0 & 0.166667 & 0 \\ 0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.833333 & 0 & 0.166667 \\ 0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.166667 & 0 & 0.833333 & 0 \\ -0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.166667 & 0 & 0.833333 \end{pmatrix}$$

$$\begin{pmatrix} \phi_1 \\ \phi_4 \\ \phi_5 \\ \phi_2 \\ \delta_1^{\{1,4\}} \\ \delta_1^{\{4,5\}} \\ \delta_1^{\{2,5\}} \\ \delta_1^{\{1,2\}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 2

Element coordinates: ({0, 5} {5, 5} {5, 10} {0, 10})

$$x(s,t) = \frac{5\,s}{2} + \frac{5}{2}$$

$$y(s,t) = \frac{5\,t}{2} + \frac{15}{2}$$

$$\boldsymbol{J} = \begin{pmatrix} \frac{5}{2} & 0 \\ 0 & \frac{5}{2} \end{pmatrix}$$

$$\det J = \frac{25}{4}$$

Given element data

$k_x = 1$ $k_y = 1$ $p = 0$ $q = 0$

Element data in mapped coordinates

$k_x = 1$ $k_y = 1$ $p = 0$ $q = 0$

Gauss point = {s → −0.57735, t → −0.57735} Weight = 1.

$\boldsymbol{N}^{T}$ = {0.622008, 0.166667, 0.0446582, 0.166667, −0.321975, −0.086273, −0.086273, −0.321975}

$\partial \boldsymbol{N}^{T}/\partial s$ =

(−0.394338 0.394338 0.105662 −0.105662 −0.557678 −0.204124 −0.149429 0.204124)

$\partial \boldsymbol{N}^{T}/\partial t$ =

(−0.394338 −0.105662 0.105662 0.394338 0.204124 −0.149429 −0.204124 −0.557678)

$\boldsymbol{J}^{-T} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix}$ detJ = 6.25

$\boldsymbol{B}^{T}$ =

$$\begin{pmatrix} -0.157735 & 0.157735 & 0.042265 & -0.042265 & -0.223071 & -0.0816497 & -0.0597717 & 0.0816497 \\ -0.157735 & -0.042265 & 0.042265 & 0.157735 & 0.0816497 & -0.0597717 & -0.0816497 & -0.223071 \end{pmatrix}$$

k_x = 1. k_y = 1. p = 0. q = 0.

$$\boldsymbol{k}_k = \begin{pmatrix} 0.311004 & -0.113835 & -0.0833333 & -0.113835 & 0.139419 & 0.139419 & 0.139419 \\ -0.113835 & 0.166667 & 0.0305021 & -0.0833333 & -0.241481 & -0.0647048 & -0.0373573 \\ -0.0833333 & 0.0305021 & 0.0223291 & 0.0305021 & -0.0373573 & -0.0373573 & -0.0373573 \\ -0.113835 & -0.0833333 & 0.0305021 & 0.166667 & 0.139419 & -0.0373573 & -0.0647048 \\ 0.139419 & -0.241481 & -0.0373573 & 0.139419 & 0.352671 & 0.0833333 & 0.0416667 \\ 0.139419 & -0.0647048 & -0.0373573 & -0.0373573 & 0.0833333 & 0.0639958 & 0.0610042 \\ 0.139419 & -0.0373573 & -0.0373573 & -0.0647048 & 0.0416667 & 0.0610042 & 0.0639958 \\ 0.139419 & 0.139419 & -0.0373573 & -0.241481 & -0.227671 & 0.0416667 & 0.0833333 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$\boldsymbol{r}_q^{T}$ = (0. 0. 0. 0. 0. 0. 0. 0.)

Gauss point = {s → −0.57735, t → 0.57735} Weight = 1.

$\boldsymbol{N}^{T}$ = {0.166667, 0.0446582, 0.166667, 0.622008, −0.086273, −0.086273, −0.321975, −0.321975}

$\partial \boldsymbol{N}^{T}/\partial s$ =

(−0.105662 0.105662 0.394338 −0.394338 −0.149429 −0.204124 −0.557678 0.204124)

$\partial \boldsymbol{N}^{T}/\partial t =$

$$\begin{pmatrix} -0.394338 & -0.105662 & 0.105662 & 0.394338 & 0.204124 & 0.149429 & -0.204124 & 0.557678 \end{pmatrix}$$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix} \qquad \det J = 6.25$$

$\boldsymbol{B}^{T} =$

$$\begin{pmatrix} -0.042265 & 0.042265 & 0.157735 & -0.157735 & -0.0597717 & -0.0816497 & -0.223071 & 0.0816497 \\ -0.157735 & -0.042265 & 0.042265 & 0.157735 & 0.0816497 & 0.0597717 & -0.0816497 & 0.223071 \end{pmatrix}$$

$k_x = 1. \qquad k_y = 1. \qquad p = 0. \qquad q = 0.$

$$\boldsymbol{k}_k = \begin{pmatrix} 0.166667 & 0.0305021 & -0.0833333 & -0.113835 & -0.0647048 & -0.0373573 & 0.139419 \\ 0.0305021 & 0.0223291 & 0.0305021 & -0.0833333 & -0.0373573 & -0.0373573 & -0.0373573 \\ -0.0833333 & 0.0305021 & 0.166667 & -0.113835 & -0.0373573 & -0.0647048 & -0.241481 \\ -0.113835 & -0.0833333 & -0.113835 & 0.311004 & 0.139419 & 0.139419 & 0.139419 \\ -0.0647048 & -0.0373573 & -0.0373573 & 0.139419 & 0.0639958 & 0.0610042 & 0.0416667 \\ -0.0373573 & -0.0373573 & -0.0647048 & 0.139419 & 0.0610042 & 0.0639958 & 0.0833333 \\ 0.139419 & -0.0373573 & -0.241481 & 0.139419 & 0.0416667 & 0.0833333 & 0.352671 \\ -0.241481 & -0.0373573 & 0.139419 & 0.139419 & 0.0833333 & 0.0416667 & -0.227671 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_q^{T} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

Gauss point = $\{s \to 0.57735, t \to -0.57735\}$ Weight = 1.

$\boldsymbol{N}^{T} = \{0.166667, 0.622008, 0.166667, 0.0446582, -0.321975, -0.321975, -0.086273, -0.086273\}$

$\partial \boldsymbol{N}^{T}/\partial s =$

$$\begin{pmatrix} -0.394338 & 0.394338 & 0.105662 & -0.105662 & 0.557678 & -0.204124 & 0.149429 & 0.204124 \end{pmatrix}$$

$\partial \boldsymbol{N}^{T}/\partial t =$

$$\begin{pmatrix} -0.105662 & -0.394338 & 0.394338 & 0.105662 & 0.204124 & -0.557678 & -0.204124 & -0.149429 \end{pmatrix}$$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix} \qquad \det J = 6.25$$

$\boldsymbol{B}^{\mathrm{T}} =$

$$\begin{pmatrix} -0.157735 & 0.157735 & 0.042265 & -0.042265 & 0.223071 & -0.0816497 & 0.0597717 & 0.0816497 \\ -0.042265 & -0.157735 & 0.157735 & 0.042265 & 0.0816497 & -0.223071 & -0.0816497 & -0.0597717 \end{pmatrix}$$

$k_x = 1.$ $k_y = 1.$ $p = 0.$ $q = 0.$

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 0.166667 & -0.113835 & -0.0833333 & 0.0305021 & -0.241481 & 0.139419 & -0.0373573 \\ -0.113835 & 0.311004 & -0.113835 & -0.0833333 & 0.139419 & 0.139419 & 0.139419 \\ -0.0833333 & -0.113835 & 0.166667 & 0.0305021 & 0.139419 & -0.241481 & -0.0647048 \\ 0.0305021 & -0.0833333 & 0.0305021 & 0.0223291 & -0.0373573 & -0.0373573 & -0.0373573 \\ -0.241481 & 0.139419 & 0.139419 & -0.0373573 & 0.352671 & -0.227671 & 0.0416667 \\ 0.139419 & 0.139419 & -0.241481 & -0.0373573 & -0.227671 & 0.352671 & 0.0833333 \\ -0.0373573 & 0.139419 & -0.0647048 & -0.0373573 & 0.0416667 & 0.0833333 & 0.0639958 \\ -0.0647048 & 0.139419 & -0.0373573 & -0.0373573 & 0.0833333 & 0.0416667 & 0.0610042 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\ 0.\ \ 0.\ \ 0.\ \ 0.\ \ 0.\ \ 0.\ \ 0.\ \ 0.\ )$

Gauss point = {s → 0.57735, t → 0.57735} Weight = 1.

$\boldsymbol{N}^{\mathrm{T}} = \{0.0446582, 0.166667, 0.622008, 0.166667, -0.086273, -0.321975, -0.321975, -0.086273\}$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} =$

$$(\ -0.105662\ \ 0.105662\ \ 0.394338\ \ -0.394338\ \ 0.149429\ \ -0.204124\ \ 0.557678\ \ 0.204124\ )$$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} =$

$$(\ -0.105662\ \ -0.394338\ \ 0.394338\ \ 0.105662\ \ 0.204124\ \ 0.557678\ \ -0.204124\ \ 0.149429\ )$$

$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix}$ $\mathrm{detJ} = 6.25$

$\boldsymbol{B}^{\mathrm{T}} =$

$$\begin{pmatrix} -0.042265 & 0.042265 & 0.157735 & -0.157735 & 0.0597717 & -0.0816497 & 0.223071 & 0.0816497 \\ -0.042265 & -0.157735 & 0.157735 & 0.042265 & 0.0816497 & 0.223071 & -0.0816497 & 0.0597717 \end{pmatrix}$$

$k_x = 1.$ $k_y = 1.$ $p = 0.$ $q = 0.$

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 0.0223291 & 0.0305021 & -0.0833333 & 0.0305021 & -0.0373573 & -0.0373573 & -0.0373573 \\ 0.0305021 & 0.166667 & -0.113835 & -0.0833333 & -0.0647048 & -0.241481 & 0.139419 \\ -0.0833333 & -0.113835 & 0.311004 & -0.113835 & 0.139419 & 0.139419 & 0.139419 \\ 0.0305021 & -0.0833333 & -0.113835 & 0.166667 & -0.0373573 & 0.139419 & -0.241481 \\ -0.0373573 & -0.0647048 & 0.139419 & -0.0373573 & 0.0639958 & 0.0833333 & 0.0416667 \\ -0.0373573 & -0.241481 & 0.139419 & 0.139419 & 0.0833333 & 0.352671 & -0.227671 \\ -0.0373573 & 0.139419 & 0.139419 & -0.241481 & 0.0416667 & -0.227671 & 0.352671 \\ -0.0373573 & -0.0373573 & 0.139419 & -0.0647048 & 0.0610042 & 0.0416667 & 0.0833333 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

Adding contributions from all Gauss points

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.2041 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.2041 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.2041 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.2041 \\ -0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.833333 & 1.38778\times10^{-17} & 0.1666 \\ 0.204124 & -0.204124 & -0.204124 & 0.204124 & 1.38778\times10^{-17} & 0.833333 & 6.9388 \\ 0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.166667 & 6.93889\times10^{-18} & 0.8333 \\ -0.204124 & 0.204124 & 0.204124 & -0.204124 & -2.77556\times10^{-17} & 0.166667 & -1.3877 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\ -0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.833333 & 0 & 0.166667 & 0 \\ 0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.833333 & 0 & 0.166667 \\ 0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.166667 & 0 & 0.833333 & 0 \\ -0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.166667 & 0 & 0.833333 \end{pmatrix}$$

$$\begin{pmatrix} \phi_2 \\ \phi_5 \\ \phi_6 \\ \phi_3 \\ \delta_1^{\{2,5\}} \\ \delta_1^{\{5,6\}} \\ \delta_1^{\{3,6\}} \\ \delta_1^{\{2,3\}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 3

Element coordinates: ({5, 0} {10, 0} {10, 5} {5, 5})

$$x(s,t) = \frac{5\,s}{2} + \frac{15}{2}$$

$$y(s,t) = \frac{5\,t}{2} + \frac{5}{2}$$

$$\boldsymbol{J} = \begin{pmatrix} \frac{5}{2} & 0 \\ 0 & \frac{5}{2} \end{pmatrix}$$

$$\text{detJ} = \frac{25}{4}$$

Given element data

$k_x = 1$ $k_y = 1$ $p = 0$ $q = 0$

Element data in mapped coordinates

$k_x = 1$ $k_y = 1$ $p = 0$ $q = 0$

Gauss point = {s → −0.57735, t → −0.57735} Weight = 1.

$\boldsymbol{N}^{T}$ = {0.622008, 0.166667, 0.0446582, 0.166667, −0.321975, −0.086273, −0.086273, −0.321975}

$\partial\boldsymbol{N}^{T}/\partial s$ =

(−0.394338 0.394338 0.105662 −0.105662 −0.557678 −0.204124 −0.149429 0.204124)

$\partial\boldsymbol{N}^{T}/\partial t$ =

(−0.394338 −0.105662 0.105662 0.394338 0.204124 −0.149429 −0.204124 −0.557678)

$\boldsymbol{J}^{-T} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix}$ detJ = 6.25

$\boldsymbol{B}^{T}$ =

$$\begin{pmatrix} -0.157735 & 0.157735 & 0.042265 & -0.042265 & -0.223071 & -0.0816497 & -0.0597717 & 0.0816497 \\ -0.157735 & -0.042265 & 0.042265 & 0.157735 & 0.0816497 & -0.0597717 & -0.0816497 & -0.223071 \end{pmatrix}$$

k_x = 1. k_y = 1. p = 0. q = 0.

$$\boldsymbol{k}_k = \begin{pmatrix} 0.311004 & -0.113835 & -0.0833333 & -0.113835 & 0.139419 & 0.139419 & 0.139419 \\ -0.113835 & 0.166667 & 0.0305021 & -0.0833333 & -0.241481 & -0.0647048 & -0.0373573 \\ -0.0833333 & 0.0305021 & 0.0223291 & 0.0305021 & -0.0373573 & -0.0373573 & -0.0373573 \\ -0.113835 & -0.0833333 & 0.0305021 & 0.166667 & 0.139419 & -0.0373573 & -0.0647048 \\ 0.139419 & -0.241481 & -0.0373573 & 0.139419 & 0.352671 & 0.0833333 & 0.0416667 \\ 0.139419 & -0.0647048 & -0.0373573 & -0.0373573 & 0.0833333 & 0.0639958 & 0.0610042 \\ 0.139419 & -0.0373573 & -0.0373573 & -0.0647048 & 0.0416667 & 0.0610042 & 0.0639958 \\ 0.139419 & 0.139419 & -0.0373573 & -0.241481 & -0.227671 & 0.0416667 & 0.0833333 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$\boldsymbol{r}_q^{T}$ = (0. 0. 0. 0. 0. 0. 0. 0.)

Gauss point = {s → −0.57735, t → 0.57735} Weight = 1.

$\boldsymbol{N}^{T}$ = {0.166667, 0.0446582, 0.166667, 0.622008, −0.086273, −0.086273, −0.321975, −0.321975}

$\partial\boldsymbol{N}^{T}/\partial s$ =

(−0.105662 0.105662 0.394338 −0.394338 −0.149429 −0.204124 −0.557678 0.204124)

$\partial\boldsymbol{N}^{T}/\partial t$ =

(−0.394338 −0.105662 0.105662 0.394338 0.204124 0.149429 −0.204124 0.557678)

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix} \qquad \text{detJ} = 6.25$$

$$\boldsymbol{B}^{T} =$$

$$\begin{pmatrix} -0.042265 & 0.042265 & 0.157735 & -0.157735 & -0.0597717 & -0.0816497 & -0.223071 & 0.0816497 \\ -0.157735 & -0.042265 & 0.042265 & 0.157735 & 0.0816497 & 0.0597717 & -0.0816497 & 0.223071 \end{pmatrix}$$

$k_x = 1.$ $\quad k_y = 1.$ $\quad p = 0.$ $\quad q = 0.$

$$\boldsymbol{k}_k = \begin{pmatrix} 0.166667 & 0.0305021 & -0.0833333 & -0.113835 & -0.0647048 & -0.0373573 & 0.139419 \\ 0.0305021 & 0.0223291 & 0.0305021 & -0.0833333 & -0.0373573 & -0.0373573 & -0.0373573 \\ -0.0833333 & 0.0305021 & 0.166667 & -0.113835 & -0.0373573 & -0.0647048 & -0.241481 \\ -0.113835 & -0.0833333 & -0.113835 & 0.311004 & 0.139419 & 0.139419 & 0.139419 \\ -0.0647048 & -0.0373573 & -0.0373573 & 0.139419 & 0.0639958 & 0.0610042 & 0.0416667 \\ -0.0373573 & -0.0373573 & -0.0647048 & 0.139419 & 0.0610042 & 0.0639958 & 0.0833333 \\ 0.139419 & -0.0373573 & -0.241481 & 0.139419 & 0.0416667 & 0.0833333 & 0.352671 \\ -0.241481 & -0.0373573 & 0.139419 & 0.139419 & 0.0833333 & 0.0416667 & -0.227671 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_q^{T} = (\,0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0.\,)$$

Gauss point = {s → 0.57735, t → −0.57735} $\qquad$ Weight = 1.

$$\boldsymbol{N}^{T} = \{0.166667,\ 0.622008,\ 0.166667,\ 0.0446582,\ -0.321975,\ -0.321975,\ -0.086273,\ -0.086273\}$$

$$\partial \boldsymbol{N}^{T}/\partial s =$$

$$(\,-0.394338 \quad 0.394338 \quad 0.105662 \quad -0.105662 \quad 0.557678 \quad -0.204124 \quad 0.149429 \quad 0.204124\,)$$

$$\partial \boldsymbol{N}^{T}/\partial t =$$

$$(\,-0.105662 \quad -0.394338 \quad 0.394338 \quad 0.105662 \quad 0.204124 \quad -0.557678 \quad -0.204124 \quad -0.149429\,)$$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix} \qquad \text{detJ} = 6.25$$

$$\boldsymbol{B}^{T} =$$

$$\begin{pmatrix} -0.157735 & 0.157735 & 0.042265 & -0.042265 & 0.223071 & -0.0816497 & 0.0597717 & 0.0816497 \\ -0.042265 & -0.157735 & 0.157735 & 0.042265 & 0.0816497 & -0.223071 & -0.0816497 & -0.0597717 \end{pmatrix}$$

$k_x = 1.$ $k_y = 1.$ $p = 0.$ $q = 0.$

$$\boldsymbol{k}_k = \begin{pmatrix} 0.166667 & -0.113835 & -0.0833333 & 0.0305021 & -0.241481 & 0.139419 & -0.0373573 \\ -0.113835 & 0.311004 & -0.113835 & -0.0833333 & 0.139419 & 0.139419 & 0.139419 \\ -0.0833333 & -0.113835 & 0.166667 & 0.0305021 & 0.139419 & -0.241481 & -0.0647048 \\ 0.0305021 & -0.0833333 & 0.0305021 & 0.0223291 & -0.0373573 & -0.0373573 & -0.0373573 \\ -0.241481 & 0.139419 & 0.139419 & -0.0373573 & 0.352671 & -0.227671 & 0.0416667 \\ 0.139419 & 0.139419 & -0.241481 & -0.0373573 & -0.227671 & 0.352671 & 0.0833333 \\ -0.0373573 & 0.139419 & -0.0647048 & -0.0373573 & 0.0416667 & 0.0833333 & 0.0639958 \\ -0.0647048 & 0.139419 & -0.0373573 & -0.0373573 & 0.0833333 & 0.0416667 & 0.0610042 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_q^T = (\,0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0.\,)$$

Gauss point = $\{s \to 0.57735,\ t \to 0.57735\}$ Weight = 1.

$$\boldsymbol{N}^T = \{0.0446582,\ 0.166667,\ 0.622008,\ 0.166667,\ -0.086273,\ -0.321975,\ -0.321975,\ -0.086273\}$$

$\partial \boldsymbol{N}^T / \partial s =$

$$(\,-0.105662 \quad 0.105662 \quad 0.394338 \quad -0.394338 \quad 0.149429 \quad -0.204124 \quad 0.557678 \quad 0.204124\,)$$

$\partial \boldsymbol{N}^T / \partial t =$

$$(\,-0.105662 \quad -0.394338 \quad 0.394338 \quad 0.105662 \quad 0.204124 \quad 0.557678 \quad -0.204124 \quad 0.149429\,)$$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix} \qquad \det J = 6.25$$

$\boldsymbol{B}^T =$

$$\begin{pmatrix} -0.042265 & 0.042265 & 0.157735 & -0.157735 & 0.0597717 & -0.0816497 & 0.223071 & 0.0816497 \\ -0.042265 & -0.157735 & 0.157735 & 0.042265 & 0.0816497 & 0.223071 & -0.0816497 & 0.0597717 \end{pmatrix}$$

$k_x = 1.$ $k_y = 1.$ $p = 0.$ $q = 0.$

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 0.0223291 & 0.0305021 & -0.0833333 & 0.0305021 & -0.0373573 & -0.0373573 & -0.0373573 \\ 0.0305021 & 0.166667 & -0.113835 & -0.0833333 & -0.0647048 & -0.241481 & 0.139419 \\ -0.0833333 & -0.113835 & 0.311004 & -0.113835 & 0.139419 & 0.139419 & 0.139419 \\ 0.0305021 & -0.0833333 & -0.113835 & 0.166667 & -0.0373573 & 0.139419 & -0.241481 \\ -0.0373573 & -0.0647048 & 0.139419 & -0.0373573 & 0.0639958 & 0.0833333 & 0.0416667 \\ -0.0373573 & -0.241481 & 0.139419 & 0.139419 & 0.0833333 & 0.352671 & -0.227671 \\ -0.0373573 & 0.139419 & 0.139419 & -0.241481 & 0.0416667 & -0.227671 & 0.352671 \\ -0.0373573 & -0.0373573 & 0.139419 & -0.0647048 & 0.0610042 & 0.0416667 & 0.0833333 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\,0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0.\,)$$

Adding contributions from all Gauss points

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.2041 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.2041 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.2041 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.2041 \\ -0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.833333 & 1.38778\times10^{-17} & 0.1666 \\ 0.204124 & -0.204124 & -0.204124 & 0.204124 & 1.38778\times10^{-17} & 0.833333 & 6.9388 \\ 0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.166667 & 6.93889\times10^{-18} & 0.8333 \\ -0.204124 & 0.204124 & 0.204124 & -0.204124 & -2.77556\times10^{-17} & 0.166667 & -1.3877 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\,0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0.\,)$$

$$\begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\ -0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.833333 & 0 & 0.166667 & 0 \\ 0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.833333 & 0 & 0.166667 \\ 0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.166667 & 0 & 0.833333 & 0 \\ -0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.166667 & 0 & 0.833333 \end{pmatrix}$$

$$\begin{pmatrix} \phi_4 \\ \phi_7 \\ \phi_8 \\ \phi_5 \\ \delta_1^{\{4,7\}} \\ \delta_1^{\{7,8\}} \\ \delta_1^{\{5,8\}} \\ \delta_1^{\{4,5\}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 4

Element coordinates: ({5, 5} {10, 5} {10, 10} {5, 10})

$$x(s,t) = \frac{5\,s}{2} + \frac{15}{2}$$

$$y(s,t) = \frac{5\,t}{2} + \frac{15}{2}$$

$$\boldsymbol{J} = \begin{pmatrix} \frac{5}{2} & 0 \\ 0 & \frac{5}{2} \end{pmatrix}$$

$$\det J = \frac{25}{4}$$

Given element data

$k_x = 1$ $k_y = 1$ $p = 0$ $q = 0$

Element data in mapped coordinates

$k_x = 1$ $k_y = 1$ $p = 0$ $q = 0$

Gauss point = {s → −0.57735, t → −0.57735} Weight = 1.

$\boldsymbol{N}^{T}$ = {0.622008, 0.166667, 0.0446582, 0.166667, −0.321975, −0.086273, −0.086273, −0.321975}

$\partial\boldsymbol{N}^{T}/\partial s$ =

(−0.394338 0.394338 0.105662 −0.105662 −0.557678 −0.204124 −0.149429 0.204124)

$\partial\boldsymbol{N}^{T}/\partial t$ =

(−0.394338 −0.105662 0.105662 0.394338 0.204124 −0.149429 −0.204124 −0.557678)

$\boldsymbol{J}^{-T} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix}$ detJ = 6.25

$\boldsymbol{B}^{T}$ =

$$\begin{pmatrix} -0.157735 & 0.157735 & 0.042265 & -0.042265 & -0.223071 & -0.0816497 & -0.0597717 & 0.0816497 \\ -0.157735 & -0.042265 & 0.042265 & 0.157735 & 0.0816497 & -0.0597717 & -0.0816497 & -0.223071 \end{pmatrix}$$

k_x = 1. k_y = 1. p = 0. q = 0.

$$\boldsymbol{k}_k = \begin{pmatrix} 0.311004 & -0.113835 & -0.0833333 & -0.113835 & 0.139419 & 0.139419 & 0.139419 \\ -0.113835 & 0.166667 & 0.0305021 & -0.0833333 & -0.241481 & -0.0647048 & -0.0373573 \\ -0.0833333 & 0.0305021 & 0.0223291 & 0.0305021 & -0.0373573 & -0.0373573 & -0.0373573 \\ -0.113835 & -0.0833333 & 0.0305021 & 0.166667 & 0.139419 & -0.0373573 & -0.0647048 \\ 0.139419 & -0.241481 & -0.0373573 & 0.139419 & 0.352671 & 0.0833333 & 0.0416667 \\ 0.139419 & -0.0647048 & -0.0373573 & -0.0373573 & 0.0833333 & 0.0639958 & 0.0610042 \\ 0.139419 & -0.0373573 & -0.0373573 & -0.0647048 & 0.0416667 & 0.0610042 & 0.0639958 \\ 0.139419 & 0.139419 & -0.0373573 & -0.241481 & -0.227671 & 0.0416667 & 0.0833333 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$\boldsymbol{r}_q^{T}$ = (0. 0. 0. 0. 0. 0. 0. 0.)

Gauss point = {s → −0.57735, t → 0.57735} Weight = 1.

$\boldsymbol{N}^{T}$ = {0.166667, 0.0446582, 0.166667, 0.622008, −0.086273, −0.086273, −0.321975, −0.321975}

$\partial\boldsymbol{N}^{T}/\partial s$ =

(−0.105662 0.105662 0.394338 −0.394338 −0.149429 −0.204124 −0.557678 0.204124)

$\partial\boldsymbol{N}^{T}/\partial t$ =

(−0.394338 −0.105662 0.105662 0.394338 0.204124 0.149429 −0.204124 0.557678)

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix} \qquad \text{detJ} = 6.25$$

$$\boldsymbol{B}^{T} =$$

$$\begin{pmatrix} -0.042265 & 0.042265 & 0.157735 & -0.157735 & -0.0597717 & -0.0816497 & -0.223071 & 0.0816497 \\ -0.157735 & -0.042265 & 0.042265 & 0.157735 & 0.0816497 & 0.0597717 & -0.0816497 & 0.223071 \end{pmatrix}$$

$k_x = 1.$ $\quad k_y = 1.$ $\quad p = 0.$ $\quad q = 0.$

$$\boldsymbol{k}_k = \begin{pmatrix} 0.166667 & 0.0305021 & -0.0833333 & -0.113835 & -0.0647048 & -0.0373573 & 0.139419 \\ 0.0305021 & 0.0223291 & 0.0305021 & -0.0833333 & -0.0373573 & -0.0373573 & -0.0373573 \\ -0.0833333 & 0.0305021 & 0.166667 & -0.113835 & -0.0373573 & -0.0647048 & -0.241481 \\ -0.113835 & -0.0833333 & -0.113835 & 0.311004 & 0.139419 & 0.139419 & 0.139419 \\ -0.0647048 & -0.0373573 & -0.0373573 & 0.139419 & 0.0639958 & 0.0610042 & 0.0416667 \\ -0.0373573 & -0.0373573 & -0.0647048 & 0.139419 & 0.0610042 & 0.0639958 & 0.0833333 \\ 0.139419 & -0.0373573 & -0.241481 & 0.139419 & 0.0416667 & 0.0833333 & 0.352671 \\ -0.241481 & -0.0373573 & 0.139419 & 0.139419 & 0.0833333 & 0.0416667 & -0.227671 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_q^{T} = (\,0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0.\,)$$

Gauss point = $\{s \to 0.57735,\ t \to -0.57735\}$ $\qquad$ Weight = 1.

$$\boldsymbol{N}^{T} = \{0.166667,\ 0.622008,\ 0.166667,\ 0.0446582,\ -0.321975,\ -0.321975,\ -0.086273,\ -0.086273\}$$

$$\partial \boldsymbol{N}^{T}/\partial s =$$

$$(\,-0.394338 \quad 0.394338 \quad 0.105662 \quad -0.105662 \quad 0.557678 \quad -0.204124 \quad 0.149429 \quad 0.204124\,)$$

$$\partial \boldsymbol{N}^{T}/\partial t =$$

$$(\,-0.105662 \quad -0.394338 \quad 0.394338 \quad 0.105662 \quad 0.204124 \quad -0.557678 \quad -0.204124 \quad -0.149429\,)$$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix} \qquad \text{detJ} = 6.25$$

$$\boldsymbol{B}^{T} =$$

$$\begin{pmatrix} -0.157735 & 0.157735 & 0.042265 & -0.042265 & 0.223071 & -0.0816497 & 0.0597717 & 0.0816497 \\ -0.042265 & -0.157735 & 0.157735 & 0.042265 & 0.0816497 & -0.223071 & -0.0816497 & -0.0597717 \end{pmatrix}$$

$k_x = 1.$ $k_y = 1.$ $p = 0.$ $q = 0.$

$$\boldsymbol{k}_k = \begin{pmatrix} 0.166667 & -0.113835 & -0.0833333 & 0.0305021 & -0.241481 & 0.139419 & -0.0373573 \\ -0.113835 & 0.311004 & -0.113835 & -0.0833333 & 0.139419 & 0.139419 & 0.139419 \\ -0.0833333 & -0.113835 & 0.166667 & 0.0305021 & 0.139419 & -0.241481 & -0.0647048 \\ 0.0305021 & -0.0833333 & 0.0305021 & 0.0223291 & -0.0373573 & -0.0373573 & -0.0373573 \\ -0.241481 & 0.139419 & 0.139419 & -0.0373573 & 0.352671 & -0.227671 & 0.0416667 \\ 0.139419 & 0.139419 & -0.241481 & -0.0373573 & -0.227671 & 0.352671 & 0.0833333 \\ -0.0373573 & 0.139419 & -0.0647048 & -0.0373573 & 0.0416667 & 0.0833333 & 0.0639958 \\ -0.0647048 & 0.139419 & -0.0373573 & -0.0373573 & 0.0833333 & 0.0416667 & 0.0610042 \end{pmatrix}$$

$$\boldsymbol{k}_p = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_q^T = (\,0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0.\,)$$

Gauss point = {s → 0.57735, t → 0.57735} Weight = 1.

$$\boldsymbol{N}^T = \{0.0446582,\ 0.166667,\ 0.622008,\ 0.166667,\ -0.086273,\ -0.321975,\ -0.321975,\ -0.086273\}$$

$\partial \boldsymbol{N}^T / \partial s =$

$$(\,-0.105662 \quad 0.105662 \quad 0.394338 \quad -0.394338 \quad 0.149429 \quad -0.204124 \quad 0.557678 \quad 0.204124\,)$$

$\partial \boldsymbol{N}^T / \partial t =$

$$(\,-0.105662 \quad -0.394338 \quad 0.394338 \quad 0.105662 \quad 0.204124 \quad 0.557678 \quad -0.204124 \quad 0.149429\,)$$

$$\boldsymbol{J}^{-T} = \begin{pmatrix} 0.4 & 0. \\ 0. & 0.4 \end{pmatrix} \qquad \det J = 6.25$$

$\boldsymbol{B}^T =$

$$\begin{pmatrix} -0.042265 & 0.042265 & 0.157735 & -0.157735 & 0.0597717 & -0.0816497 & 0.223071 & 0.0816497 \\ -0.042265 & -0.157735 & 0.157735 & 0.042265 & 0.0816497 & 0.223071 & -0.0816497 & 0.0597717 \end{pmatrix}$$

$k_x = 1.$ $k_y = 1.$ $p = 0.$ $q = 0.$

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 0.0223291 & 0.0305021 & -0.0833333 & 0.0305021 & -0.0373573 & -0.0373573 & -0.0373573 \\ 0.0305021 & 0.166667 & -0.113835 & -0.0833333 & -0.0647048 & -0.241481 & 0.139419 \\ -0.0833333 & -0.113835 & 0.311004 & -0.113835 & 0.139419 & 0.139419 & 0.139419 \\ 0.0305021 & -0.0833333 & -0.113835 & 0.166667 & -0.0373573 & 0.139419 & -0.241481 \\ -0.0373573 & -0.0647048 & 0.139419 & -0.0373573 & 0.0639958 & 0.0833333 & 0.0416667 \\ -0.0373573 & -0.241481 & 0.139419 & 0.139419 & 0.0833333 & 0.352671 & -0.227671 \\ -0.0373573 & 0.139419 & 0.139419 & -0.241481 & 0.0416667 & -0.227671 & 0.352671 \\ -0.0373573 & -0.0373573 & 0.139419 & -0.0647048 & 0.0610042 & 0.0416667 & 0.0833333 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\,0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0.\,)$$

Adding contributions from all Gauss points

$$\boldsymbol{k}_{\mathrm{k}} = \begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.2041 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.2041 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.2041 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.2041 \\ -0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.833333 & 1.38778\times10^{-17} & 0.1666 \\ 0.204124 & -0.204124 & -0.204124 & 0.204124 & 1.38778\times10^{-17} & 0.833333 & 6.9388 \\ 0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.166667 & 6.93889\times10^{-18} & 0.8333 \\ -0.204124 & 0.204124 & 0.204124 & -0.204124 & -2.77556\times10^{-17} & 0.166667 & -1.3877 \end{pmatrix}$$

$$\boldsymbol{k}_{\mathrm{p}} = \begin{pmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{q}}^{\mathrm{T}} = (\,0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0.\,)$$

$$\begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\ -0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.833333 & 0 & 0.166667 & 0 \\ 0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.833333 & 0 & 0.166667 \\ 0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.166667 & 0 & 0.833333 & 0 \\ -0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.166667 & 0 & 0.833333 \end{pmatrix}$$

$$\begin{pmatrix} \phi_5 \\ \phi_8 \\ \phi_9 \\ \phi_6 \\ \delta_1^{\{5,8\}} \\ \delta_1^{\{8,9\}} \\ \delta_1^{\{6,9\}} \\ \delta_1^{\{5,6\}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Global equations after assembling all elements

$$\begin{pmatrix}
0.666667 & -0.166667 & 0 & -0.166667 & -0.333333 & 0 & 0 & 0 & 0 \\
-0.166667 & 1.33333 & -0.166667 & -0.333333 & -0.333333 & -0.333333 & 0 & 0 & 0 \\
0 & -0.166667 & 0.666667 & 0 & -0.333333 & -0.166667 & 0 & 0 & 0 \\
-0.166667 & -0.333333 & 0 & 1.33333 & -0.333333 & 0 & -0.166667 & -0.333333 & 0 \\
-0.333333 & -0.333333 & -0.333333 & -0.333333 & 2.66667 & -0.333333 & -0.333333 & -0.333333 & -0.333333 \\
0 & -0.333333 & -0.166667 & 0 & -0.333333 & 1.33333 & 0 & -0.333333 & -0.166667 \\
0 & 0 & 0 & -0.166667 & -0.333333 & 0 & 0.666667 & -0.166667 & 0 \\
0 & 0 & 0 & -0.333333 & -0.333333 & -0.333333 & -0.166667 & 1.33333 & -0.166667 \\
0 & 0 & 0 & 0 & -0.333333 & -0.166667 & 0 & -0.166667 & 0.666667 \\
-0.204124 & -0.204124 & 0 & 0.204124 & 0.204124 & 0 & 0 & 0 & 0 \\
-0.204124 & 0.204124 & 0 & -0.204124 & 0.204124 & 0 & 0 & 0 & 0 \\
0 & -0.204124 & -0.204124 & 0 & 0.204124 & 0.204124 & 0 & 0 & 0 \\
0.204124 & -0.408248 & 0.204124 & 0.204124 & -0.408248 & 0.204124 & 0 & 0 & 0 \\
0 & 0.204124 & -0.204124 & 0 & 0.204124 & -0.204124 & 0 & 0 & 0 \\
0.204124 & 0.204124 & 0 & -0.408248 & -0.408248 & 0 & 0.204124 & 0.204124 & 0 \\
0 & 0 & 0 & -0.204124 & 0.204124 & 0 & -0.204124 & 0.204124 & 0 \\
0 & 0.204124 & 0.204124 & 0 & -0.408248 & -0.408248 & 0 & 0.204124 & 0.204124 \\
0 & 0 & 0 & 0.204124 & -0.408248 & 0.204124 & 0.204124 & -0.408248 & 0.204124 \\
0 & 0 & 0 & 0 & 0.204124 & -0.204124 & 0 & 0.204124 & -0.204124 \\
0 & 0 & 0 & 0.204124 & 0.204124 & 0 & -0.204124 & -0.204124 & 0 \\
0 & 0 & 0 & 0 & 0.204124 & 0.204124 & 0 & -0.204124 & -0.204124
\end{pmatrix}$$

Essential boundary conditions

On element 1, side 4, specified value = 10

$$\left\{\phi_2,\ \phi_1,\ \delta_1^{\{1,2\}}\right\} = \{10,\ 10,\ 0\}$$

On element 2, side 3, specified value = 1

$$\left\{\phi_6,\ \phi_3,\ \delta_1^{\{3,6\}}\right\} = \{1,\ 1,\ 0\}$$

On element 4, side 3, specified value = 1

$$\left\{\phi_9,\ \phi_6,\ \delta_1^{\{6,9\}}\right\} = \{1,\ 1,\ 0\}$$

Known values from EBC

$$\left\{\phi_1 = 10,\ \phi_2 = 10,\ \phi_3 = 1,\ \phi_6 = 1,\ \phi_9 = 1,\ \delta_1^{\{1,2\}} = 0,\ \delta_1^{\{3,6\}} = 0,\ \delta_1^{\{6,9\}} = 0\right\}$$

Global equations after EBC

$$\begin{pmatrix}
1.33333 & -0.333333 & -0.166667 & -0.333333 & -0.204124 & 0 & 0.204124 & -0.408248 & -0.204124 \\
-0.333333 & 2.66667 & -0.333333 & -0.333333 & 0.204124 & 0.204124 & -0.408248 & -0.408248 & 0.204124 \\
-0.166667 & -0.333333 & 0.666667 & -0.166667 & 0 & 0 & 0 & 0.204124 & -0.204124 \\
-0.333333 & -0.333333 & -0.166667 & 1.33333 & 0 & 0 & 0 & 0.204124 & 0.204124 \\
-0.204124 & 0.204124 & 0 & 0 & 0.833333 & 0 & 0.166667 & 0 & 0 \\
0 & 0.204124 & 0 & 0 & 0 & 0.833333 & 0 & 0 & 0 \\
0.204124 & -0.408248 & 0 & 0 & 0.166667 & 0 & 1.66667 & 0 & 0 \\
-0.408248 & -0.408248 & 0.204124 & 0.204124 & 0 & 0 & 0 & 1.66667 & 0 \\
-0.204124 & 0.204124 & -0.204124 & 0.204124 & 0 & 0 & 0 & 0 & 0.833333 \\
0 & -0.408248 & 0 & 0.204124 & 0 & 0.166667 & 0 & 0 & 0 \\
0.204124 & -0.408248 & 0.204124 & -0.408248 & 0 & 0 & 0 & 0 & 0.166667 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0 & 0 & 0 & 0.166667 & 0 \\
0 & 0.204124 & 0 & -0.204124 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Solving the final system of global equations we get

$$\{\phi_4 = 6.12921,\ \phi_5 = 4.66596,\ \phi_7 = 5.08248,\ \phi_8 = 3.96195,$$
$$\delta_1^{\{1,4\}} = 0.0857292,\ \delta_1^{\{2,3\}} = 1.44833,\ \delta_1^{\{2,5\}} = 1.36347,\ \delta_1^{\{4,5\}} = -0.887807,\ \delta_1^{\{4,7\}} = 0.54473,$$
$$\delta_1^{\{5,6\}} = -0.708827,\ \delta_1^{\{5,8\}} = 0.440829,\ \delta_1^{\{7,8\}} = -0.251281,\ \delta_1^{\{8,9\}} = -0.0306799\}$$

Solution for element 1

DOF values for the element

$$\{\phi_1 = 10,\ \phi_4 = 6.12921,\ \phi_5 = 4.66596,\ \phi_2 = 10,\ \delta_1^{\{1,4\}} = 0.0857292,\ \delta_1^{\{4,5\}} = -0.887807,\ \delta_1^{\{2,5\}} = 1.36347,\ \delta_1^{\{1,2\}} = 0\}$$

$$\boldsymbol{d}^{\mathrm{T}} = (\ 10 \quad 6.12921 \quad 4.66596 \quad 10 \quad 0.0857292 \quad -0.887807 \quad 1.36347 \quad 0\ )$$

Mapping

$$x(s,t) = \frac{5\,s}{2} + \frac{5}{2}$$

$$y(s,t) = \frac{5\,t}{2} + \frac{5}{2}$$

$$\boldsymbol{J} = \begin{pmatrix} \frac{5}{2} & 0 \\ 0 & \frac{5}{2} \end{pmatrix}$$

Element solution at $\{s \to 0,\ t \to 0\}$

Location: {2.5, 2.5}

$\boldsymbol{N}^{\mathrm{T}} = (\,0.25 \quad 0.25 \quad 0.25 \quad 0.25 \quad -0.306186 \quad -0.306186 \quad -0.306186 \quad -0.306186\,)$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} = (\,-0.25 \quad 0.25 \quad 0.25 \quad -0.25 \quad 0. \quad -0.306186 \quad 0. \quad 0.306186\,)$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} = (\,-0.25 \quad -0.25 \quad 0.25 \quad 0.25 \quad 0.306186 \quad 0. \quad -0.306186 \quad 0.\,)$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} \frac{2}{5} & 0 \\ 0 & \frac{2}{5} \end{pmatrix}$$

$\boldsymbol{B}_{\mathrm{x}}^{\mathrm{T}} = \partial \boldsymbol{N}^{\mathrm{T}}/\partial x = (\,-0.1 \quad 0.1 \quad 0.1 \quad -0.1 \quad 0. \quad -0.122474 \quad 0. \quad 0.122474\,)$

$\boldsymbol{B}_{\mathrm{y}}^{\mathrm{T}} = \partial \boldsymbol{N}^{\mathrm{T}}/\partial y = (\,-0.1 \quad -0.1 \quad 0.1 \quad 0.1 \quad 0.122474 \quad 0. \quad -0.122474 \quad 0.\,)$

$\phi = \boldsymbol{N}^{\mathrm{T}}\boldsymbol{d} = 7.5269$

$\partial\phi/\partial x = \boldsymbol{B}_{\mathrm{x}}^{\mathrm{T}}\boldsymbol{d} = -0.811749$

$\partial\phi/\partial y = \boldsymbol{B}_{\mathrm{y}}^{\mathrm{T}}\boldsymbol{d} = -0.302817$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.5269	−0.811749	−0.302817

Solution for element 2

DOF values for the element

$\left\{\phi_2 = 10,\ \phi_5 = 4.66596,\ \phi_6 = 1,\ \phi_3 = 1,\ \delta_1^{\{2,5\}} = 1.36347,\ \delta_1^{\{5,6\}} = -0.708827,\ \delta_1^{\{3,6\}} = 0,\ \delta_1^{\{2,3\}} = 1.44833\right\}$

$\boldsymbol{d}^{\mathrm{T}} = (\,10 \quad 4.66596 \quad 1 \quad 1 \quad 1.36347 \quad -0.708827 \quad 0 \quad 1.44833\,)$

Mapping

$$x(\mathrm{s},\mathrm{t}) = \frac{5\,\mathrm{s}}{2} + \frac{5}{2}$$

$$y(\mathrm{s},\mathrm{t}) = \frac{5\,\mathrm{t}}{2} + \frac{15}{2}$$

$$\boldsymbol{J} = \begin{pmatrix} \frac{5}{2} & 0 \\ 0 & \frac{5}{2} \end{pmatrix}$$

Element solution at $\{\mathrm{s} \to 0,\ \mathrm{t} \to 0\}$

Location: {2.5, 7.5}

$\boldsymbol{N}^{\mathrm{T}} = (\,0.25 \quad 0.25 \quad 0.25 \quad 0.25 \quad -0.306186 \quad -0.306186 \quad -0.306186 \quad -0.306186\,)$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} = (\,-0.25 \quad 0.25 \quad 0.25 \quad -0.25 \quad 0. \quad -0.306186 \quad 0. \quad 0.306186\,)$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} = (\,-0.25 \quad -0.25 \quad 0.25 \quad 0.25 \quad 0.306186 \quad 0. \quad -0.306186 \quad 0.\,)$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} \frac{2}{5} & 0 \\ 0 & \frac{2}{5} \end{pmatrix}$$

$\boldsymbol{B}_{\mathrm{x}}^{\mathrm{T}} = \partial \boldsymbol{N}^{\mathrm{T}}/\partial x = (\,-0.1 \quad 0.1 \quad 0.1 \quad -0.1 \quad 0. \quad -0.122474 \quad 0. \quad 0.122474\,)$

$\boldsymbol{B}_{\mathrm{y}}^{\mathrm{T}} = \partial \boldsymbol{N}^{\mathrm{T}}/\partial y = (\,-0.1 \quad -0.1 \quad 0.1 \quad 0.1 \quad 0.122474 \quad 0. \quad -0.122474 \quad 0.\,)$

$\phi = \boldsymbol{N}^{\mathrm{T}}\boldsymbol{d} = 3.52259$

$\partial\phi/\partial x = \boldsymbol{B}_{\mathrm{x}}^{\mathrm{T}}\boldsymbol{d} = -0.269207$

$\partial\phi/\partial y = \boldsymbol{B}_{\mathrm{y}}^{\mathrm{T}}\boldsymbol{d} = -1.09961$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	7.5	3.52259	−0.269207	−1.09961

Solution for element 3

DOF values for the element

$\{\phi_4 = 6.12921,\ \phi_7 = 5.08248,\ \phi_8 = 3.96195,\ \phi_5 = 4.66596,$
$\delta_1^{\{4,7\}} = 0.54473,\ \delta_1^{\{7,8\}} = -0.251281,\ \delta_1^{\{5,8\}} = 0.440829,\ \delta_1^{\{4,5\}} = -0.887807\}$

$\boldsymbol{d}^{\mathrm{T}} = (\,6.12921 \quad 5.08248 \quad 3.96195 \quad 4.66596 \quad 0.54473 \quad -0.251281 \quad 0.440829 \quad -0.887807\,)$

Mapping

$$x(\mathrm{s},\mathrm{t}) = \frac{5\,\mathrm{s}}{2} + \frac{15}{2}$$

$$y(\mathrm{s},\mathrm{t}) = \frac{5\,\mathrm{t}}{2} + \frac{5}{2}$$

$$\boldsymbol{J} = \begin{pmatrix} \frac{5}{2} & 0 \\ 0 & \frac{5}{2} \end{pmatrix}$$

Element solution at $\{\mathrm{s} \to 0,\ \mathrm{t} \to 0\}$

Location: {7.5, 2.5}

$\boldsymbol{N}^{\mathrm{T}} = (\,0.25 \quad 0.25 \quad 0.25 \quad 0.25 \quad -0.306186 \quad -0.306186 \quad -0.306186 \quad -0.306186\,)$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{s} = (\,-0.25 \quad 0.25 \quad 0.25 \quad -0.25 \quad 0. \quad -0.306186 \quad 0. \quad 0.306186\,)$

$\partial \boldsymbol{N}^{\mathrm{T}}/\partial \mathrm{t} = (\,-0.25 \quad -0.25 \quad 0.25 \quad 0.25 \quad 0.306186 \quad 0. \quad -0.306186 \quad 0.\,)$

$$\boldsymbol{J}^{-\mathrm{T}} = \begin{pmatrix} \frac{2}{5} & 0 \\ 0 & \frac{2}{5} \end{pmatrix}$$

$\boldsymbol{B}_x^T = \partial \boldsymbol{N}^T/\partial x = (\,-0.1 \quad 0.1 \quad 0.1 \quad -0.1 \quad 0. \quad -0.122474 \quad 0. \quad 0.122474\,)$

$\boldsymbol{B}_y^T = \partial \boldsymbol{N}^T/\partial y = (\,-0.1 \quad -0.1 \quad 0.1 \quad 0.1 \quad 0.122474 \quad 0. \quad -0.122474 \quad 0.\,)$

$\phi = \boldsymbol{N}^T \boldsymbol{d} = 5.00691$

$\partial\phi/\partial x = \boldsymbol{B}_x^T \boldsymbol{d} = -0.253032$

$\partial\phi/\partial y = \boldsymbol{B}_y^T \boldsymbol{d} = -0.245653$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	2.5	5.00691	−0.253032	−0.245653

Solution for element 4

DOF values for the element

$\left\{\phi_5 = 4.66596,\ \phi_8 = 3.96195,\ \phi_9 = 1,\ \phi_6 = 1,\ \delta_1^{\{5,8\}} = 0.440829,\ \delta_1^{\{8,9\}} = -0.0306799,\ \delta_1^{\{6,9\}} = 0,\ \delta_1^{\{5,6\}} = -0.708827\right\}$

$\boldsymbol{d}^T = (\,4.66596 \quad 3.96195 \quad 1 \quad 1 \quad 0.440829 \quad -0.0306799 \quad 0 \quad -0.708827\,)$

Mapping

$x(s,t) = \dfrac{5\,s}{2} + \dfrac{15}{2}$

$y(s,t) = \dfrac{5\,t}{2} + \dfrac{15}{2}$

$\boldsymbol{J} = \begin{pmatrix} \frac{5}{2} & 0 \\ 0 & \frac{5}{2} \end{pmatrix}$

Element solution at $\{s \to 0,\ t \to 0\}$

Location: {7.5, 7.5}

$\boldsymbol{N}^T = (\,0.25 \quad 0.25 \quad 0.25 \quad 0.25 \quad -0.306186 \quad -0.306186 \quad -0.306186 \quad -0.306186\,)$

$\partial \boldsymbol{N}^T/\partial s = (\,-0.25 \quad 0.25 \quad 0.25 \quad -0.25 \quad 0. \quad -0.306186 \quad 0. \quad 0.306186\,)$

$\partial \boldsymbol{N}^T/\partial t = (\,-0.25 \quad -0.25 \quad 0.25 \quad 0.25 \quad 0.306186 \quad 0. \quad -0.306186 \quad 0.\,)$

$\boldsymbol{J}^{-T} = \begin{pmatrix} \frac{2}{5} & 0 \\ 0 & \frac{2}{5} \end{pmatrix}$

$\boldsymbol{B}_x^T = \partial \boldsymbol{N}^T/\partial x = (\,-0.1 \quad 0.1 \quad 0.1 \quad -0.1 \quad 0. \quad -0.122474 \quad 0. \quad 0.122474\,)$

$\boldsymbol{B}_y^T = \partial \boldsymbol{N}^T/\partial y = (\,-0.1 \quad -0.1 \quad 0.1 \quad 0.1 \quad 0.122474 \quad 0. \quad -0.122474 \quad 0.\,)$

$\phi = \boldsymbol{N}^T \boldsymbol{d} = 2.74843$

$$\partial\phi/\partial x = \boldsymbol{B}_x^{\mathrm{T}}\boldsymbol{d} = -0.153456$$

$$\partial\phi/\partial y = \boldsymbol{B}_y^{\mathrm{T}}\boldsymbol{d} = -0.608801$$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	7.5	2.74843	−0.153456	−0.608801

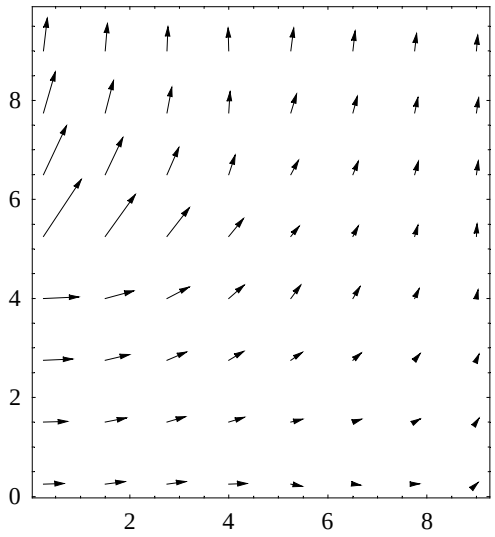
Nodal solution summary

dof	x	y	Value
ϕ_1	0	0	10
ϕ_2	0	5	10
ϕ_3	0	10	1
ϕ_4	5	0	6.12921
ϕ_5	5	5	4.66596
ϕ_6	5	10	1
ϕ_7	10	0	5.08248
ϕ_8	10	5	3.96195
ϕ_9	10	10	1

Element solution summary

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.5269	−0.811749	−0.302817
2	2.5	7.5	3.52259	−0.269207	−1.09961
3	7.5	2.5	5.00691	−0.253032	−0.245653
4	7.5	7.5	2.74843	−0.153456	−0.608801

Computing $\partial\phi/\partial x$ and $\partial\phi/\partial y$ at several points within each element, the velocity field as shown in Figure is obtained.



The $n = 1$ solution corresponds to the conventional rectangular element with bi-linear solution and no p-modes.

Equations for element 1

$$\begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_4 \\ \phi_5 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 2

$$\begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 \end{pmatrix} \begin{pmatrix} \phi_2 \\ \phi_5 \\ \phi_6 \\ \phi_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 3

$$\begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 \end{pmatrix} \begin{pmatrix} \phi_4 \\ \phi_7 \\ \phi_8 \\ \phi_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 4

$$\begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 \end{pmatrix} \begin{pmatrix} \phi_5 \\ \phi_8 \\ \phi_9 \\ \phi_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Global equations after assembling all elements

$$\begin{pmatrix} 0.666667 & -0.166667 & 0 & -0.166667 & -0.333333 & 0 & 0 & 0 & 0 \\ -0.166667 & 1.33333 & -0.166667 & -0.333333 & -0.333333 & -0.333333 & 0 & 0 & 0 \\ 0 & -0.166667 & 0.666667 & 0 & -0.333333 & -0.166667 & 0 & 0 & 0 \\ -0.166667 & -0.333333 & 0 & 1.33333 & -0.333333 & 0 & -0.166667 & -0.333333 & 0 \\ -0.333333 & -0.333333 & -0.333333 & -0.333333 & 2.66667 & -0.333333 & -0.333333 & -0.333333 & -0.333333 \\ 0 & -0.333333 & -0.166667 & 0 & -0.333333 & 1.33333 & 0 & -0.333333 & -0.166667 \\ 0 & 0 & 0 & -0.166667 & -0.333333 & 0 & 0.666667 & -0.166667 & 0 \\ 0 & 0 & 0 & -0.333333 & -0.333333 & -0.333333 & -0.166667 & 1.33333 & -0.166667 \\ 0 & 0 & 0 & 0 & -0.333333 & -0.166667 & 0 & -0.166667 & 0.666667 \end{pmatrix}$$

Essential boundary conditions

On element 1, side 4, specified value = 10

$\{\phi_2, \phi_1\} = \{10, 10\}$

On element 2, side 3, specified value = 1

$\{\phi_6, \phi_3\} = \{1, 1\}$

On element 4, side 3, specified value = 1

$\{\phi_9, \phi_6\} = \{1, 1\}$

Known values from EBC

$\{\phi_1 = 10, \phi_2 = 10, \phi_3 = 1, \phi_6 = 1, \phi_9 = 1\}$

Global equations after EBC

$$\begin{pmatrix} 1.33333 & -0.333333 & -0.166667 & -0.333333 \\ -0.333333 & 2.66667 & -0.333333 & -0.333333 \\ -0.166667 & -0.333333 & 0.666667 & -0.166667 \\ -0.333333 & -0.333333 & -0.166667 & 1.33333 \end{pmatrix} \begin{pmatrix} \phi_4 \\ \phi_5 \\ \phi_7 \\ \phi_8 \end{pmatrix} = \begin{pmatrix} 5. \\ 7.66667 \\ 0 \\ 0.5 \end{pmatrix}$$

Solving the final system of global equations we get

$$\{\phi_4 = 6.52857,\ \phi_5 = 4.79286,\ \phi_7 = 4.98571,\ \phi_8 = 3.82857\}$$

Solution for element 1

DOF values for the element

$$\{\phi_1 = 10,\ \phi_4 = 6.52857,\ \phi_5 = 4.79286,\ \phi_2 = 10\}$$

$$\boldsymbol{d}^{\mathrm{T}} = (\ 10 \quad 6.52857 \quad 4.79286 \quad 10\ )$$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.83036	−0.867857	−0.173571

Solution for element 2

DOF values for the element

$$\{\phi_2 = 10,\ \phi_5 = 4.79286,\ \phi_6 = 1,\ \phi_3 = 1\}$$

$$\boldsymbol{d}^{\mathrm{T}} = (\ 10 \quad 4.79286 \quad 1 \quad 1\ )$$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	7.5	4.19821	−0.520714	−1.27929

Solution for element 3

DOF values for the element

$$\{\phi_4 = 6.52857,\ \phi_7 = 4.98571,\ \phi_8 = 3.82857,\ \phi_5 = 4.79286\}$$

$$\boldsymbol{d}^{\mathrm{T}} = (\ 6.52857 \quad 4.98571 \quad 3.82857 \quad 4.79286\ )$$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	2.5	5.03393	−0.250714	−0.289286

Solution for element 4

DOF values for the element

$$\{\phi_5 = 4.79286,\ \phi_8 = 3.82857,\ \phi_9 = 1,\ \phi_6 = 1\}$$

$$\boldsymbol{d}^{\mathrm{T}} = (\ 4.79286 \quad 3.82857 \quad 1 \quad 1\ )$$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	7.5	2.65536	−0.0964286	−0.662143

Nodal solution summary

dof	x	y	Value
ϕ_1	0	0	10
ϕ_2	0	5	10
ϕ_3	0	10	1
ϕ_4	5	0	6.52857
ϕ_5	5	5	4.79286
ϕ_6	5	10	1
ϕ_7	10	0	4.98571
ϕ_8	10	5	3.82857
ϕ_9	10	10	1

Element solution summary

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.83036	−0.867857	−0.173571
2	2.5	7.5	4.19821	−0.520714	−1.27929
3	7.5	2.5	5.03393	−0.250714	−0.289286
4	7.5	7.5	2.65536	−0.0964286	−0.662143

Solution with $n = 2$

Equations for element 1

$$\begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\ -0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.833333 & 0 & 0.166667 & 0 \\ 0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.833333 & 0 & 0.166667 \\ 0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.166667 & 0 & 0.833333 & 0 \\ -0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.166667 & 0 & 0.833333 \end{pmatrix}$$

$$\begin{pmatrix} \phi_1 \\ \phi_4 \\ \phi_5 \\ \phi_2 \\ \delta_1^{\{1,4\}} \\ \delta_1^{\{4,5\}} \\ \delta_1^{\{2,5\}} \\ \delta_1^{\{1,2\}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 2

$$\begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\ -0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.833333 & 0 & 0.166667 & 0 \\ 0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.833333 & 0 & 0.166667 \\ 0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.166667 & 0 & 0.833333 & 0 \\ -0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.166667 & 0 & 0.833333 \end{pmatrix}$$

$$\begin{pmatrix} \phi_2 \\ \phi_5 \\ \phi_6 \\ \phi_3 \\ \delta_1^{\{2,5\}} \\ \delta_1^{\{5,6\}} \\ \delta_1^{\{3,6\}} \\ \delta_1^{\{2,3\}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 3

$$\begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\ -0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.833333 & 0 & 0.166667 & 0 \\ 0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.833333 & 0 & 0.166667 \\ 0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.166667 & 0 & 0.833333 & 0 \\ -0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.166667 & 0 & 0.833333 \end{pmatrix}$$

$$\begin{pmatrix} \phi_4 \\ \phi_7 \\ \phi_8 \\ \phi_5 \\ \delta_1^{\{4,7\}} \\ \delta_1^{\{7,8\}} \\ \delta_1^{\{5,8\}} \\ \delta_1^{\{4,5\}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 4

$$\begin{pmatrix} 0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\ -0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\ -0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\ -0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\ -0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.833333 & 0 & 0.166667 & 0 \\ 0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.833333 & 0 & 0.166667 \\ 0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.166667 & 0 & 0.833333 & 0 \\ -0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.166667 & 0 & 0.833333 \end{pmatrix}$$

$$\begin{pmatrix} \phi_5 \\ \phi_8 \\ \phi_9 \\ \phi_6 \\ \delta_1^{\{5,8\}} \\ \delta_1^{\{8,9\}} \\ \delta_1^{\{6,9\}} \\ \delta_1^{\{5,6\}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Global equations after assembling all elements

$$\begin{pmatrix}
0.666667 & -0.166667 & 0 & -0.166667 & -0.333333 & 0 & 0 & 0 & 0 \\
-0.166667 & 1.33333 & -0.166667 & -0.333333 & -0.333333 & -0.333333 & 0 & 0 & 0 \\
0 & -0.166667 & 0.666667 & 0 & -0.333333 & -0.166667 & 0 & 0 & 0 \\
-0.166667 & -0.333333 & 0 & 1.33333 & -0.333333 & 0 & -0.166667 & -0.333333 & 0 \\
-0.333333 & -0.333333 & -0.333333 & -0.333333 & 2.66667 & -0.333333 & -0.333333 & -0.333333 & -0.333333 \\
0 & -0.333333 & -0.166667 & 0 & -0.333333 & 1.33333 & 0 & -0.333333 & -0.166667 \\
0 & 0 & 0 & -0.166667 & -0.333333 & 0 & 0.666667 & -0.166667 & 0 \\
0 & 0 & 0 & -0.333333 & -0.333333 & -0.333333 & -0.166667 & 1.33333 & -0.166667 \\
0 & 0 & 0 & 0 & -0.333333 & -0.166667 & 0 & -0.166667 & 0.666667 \\
-0.204124 & -0.204124 & 0 & 0.204124 & 0.204124 & 0 & 0 & 0 & 0 \\
-0.204124 & 0.204124 & 0 & -0.204124 & 0.204124 & 0 & 0 & 0 & 0 \\
0 & -0.204124 & -0.204124 & 0 & 0.204124 & 0.204124 & 0 & 0 & 0 \\
0.204124 & -0.408248 & 0.204124 & 0.204124 & -0.408248 & 0.204124 & 0 & 0 & 0 \\
0 & 0.204124 & -0.204124 & 0 & 0.204124 & -0.204124 & 0 & 0 & 0 \\
0.204124 & 0.204124 & 0 & -0.408248 & -0.408248 & 0 & 0.204124 & 0.204124 & 0 \\
0 & 0 & 0 & -0.204124 & 0.204124 & 0 & -0.204124 & 0.204124 & 0 \\
0 & 0.204124 & 0.204124 & 0 & -0.408248 & -0.408248 & 0 & 0.204124 & 0.204124 \\
0 & 0 & 0 & 0.204124 & -0.408248 & 0.204124 & 0.204124 & -0.408248 & 0.204124 \\
0 & 0 & 0 & 0 & 0.204124 & -0.204124 & 0 & 0.204124 & -0.204124 \\
0 & 0 & 0 & 0.204124 & 0.204124 & 0 & -0.204124 & -0.204124 & 0 \\
0 & 0 & 0 & 0 & 0.204124 & 0.204124 & 0 & -0.204124 & -0.204124
\end{pmatrix}$$

Essential boundary conditions

On element 1, side 4, specified value = 10

$$\left\{\phi_2,\ \phi_1,\ \delta_1^{\{1,2\}}\right\} = \{10,\ 10,\ 0\}$$

On element 2, side 3, specified value = 1

$$\left\{\phi_6,\ \phi_3,\ \delta_1^{\{3,6\}}\right\} = \{1,\ 1,\ 0\}$$

On element 4, side 3, specified value = 1

$$\left\{\phi_9,\ \phi_6,\ \delta_1^{\{6,9\}}\right\} = \{1,\ 1,\ 0\}$$

Known values from EBC

$$\left\{\phi_1 = 10,\ \phi_2 = 10,\ \phi_3 = 1,\ \phi_6 = 1,\ \phi_9 = 1,\ \delta_1^{\{1,2\}} = 0,\ \delta_1^{\{3,6\}} = 0,\ \delta_1^{\{6,9\}} = 0\right\}$$

Global equations after EBC

$$\begin{pmatrix}
1.33333 & -0.333333 & -0.166667 & -0.333333 & -0.204124 & 0 & 0.204124 & -0.408248 & -0.204124 \\
-0.333333 & 2.66667 & -0.333333 & -0.333333 & 0.204124 & 0.204124 & -0.408248 & -0.408248 & 0.204124 \\
-0.166667 & -0.333333 & 0.666667 & -0.166667 & 0 & 0 & 0 & 0.204124 & -0.204124 \\
-0.333333 & -0.333333 & -0.166667 & 1.33333 & 0 & 0 & 0 & 0.204124 & 0.204124 \\
-0.204124 & 0.204124 & 0 & 0 & 0.833333 & 0 & 0.166667 & 0 & 0 \\
0 & 0.204124 & 0 & 0 & 0 & 0.833333 & 0 & 0 & 0 \\
0.204124 & -0.408248 & 0 & 0 & 0.166667 & 0 & 1.66667 & 0 & 0 \\
-0.408248 & -0.408248 & 0.204124 & 0.204124 & 0 & 0 & 0 & 1.66667 & 0 \\
-0.204124 & 0.204124 & -0.204124 & 0.204124 & 0 & 0 & 0 & 0 & 0.833333 \\
0 & -0.408248 & 0 & 0.204124 & 0 & 0.166667 & 0 & 0 & 0 \\
0.204124 & -0.408248 & 0.204124 & -0.408248 & 0 & 0 & 0 & 0 & 0.166667 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0 & 0 & 0 & 0.166667 & 0 \\
0 & 0.204124 & 0 & -0.204124 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Solving the final system of global equations we get

$$\{\phi_4 = 6.12921,\ \phi_5 = 4.66596,\ \phi_7 = 5.08248,\ \phi_8 = 3.96195,$$
$$\delta_1^{\{1,4\}} = 0.0857292,\ \delta_1^{\{2,3\}} = 1.44833,\ \delta_1^{\{2,5\}} = 1.36347,\ \delta_1^{\{4,5\}} = -0.887807,\ \delta_1^{\{4,7\}} = 0.54473,$$
$$\delta_1^{\{5,6\}} = -0.708827,\ \delta_1^{\{5,8\}} = 0.440829,\ \delta_1^{\{7,8\}} = -0.251281,\ \delta_1^{\{8,9\}} = -0.0306799\}$$

Solution for element 1

DOF values for the element

$$\{\phi_1 = 10,\ \phi_4 = 6.12921,\ \phi_5 = 4.66596,\ \phi_2 = 10,\ \delta_1^{\{1,4\}} = 0.0857292,\ \delta_1^{\{4,5\}} = -0.887807,\ \delta_1^{\{2,5\}} = 1.36347,\ \delta_1^{\{1,2\}} = 0\}$$

$$\boldsymbol{d}^{\mathrm{T}} = (\ 10 \quad 6.12921 \quad 4.66596 \quad 10 \quad 0.0857292 \quad -0.887807 \quad 1.36347 \quad 0\ )$$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.5269	−0.811749	−0.302817

Solution for element 2

DOF values for the element

$$\{\phi_2 = 10,\ \phi_5 = 4.66596,\ \phi_6 = 1,\ \phi_3 = 1,\ \delta_1^{\{2,5\}} = 1.36347,\ \delta_1^{\{5,6\}} = -0.708827,\ \delta_1^{\{3,6\}} = 0,\ \delta_1^{\{2,3\}} = 1.44833\}$$

$$\boldsymbol{d}^{\mathrm{T}} = (\ 10 \quad 4.66596 \quad 1 \quad 1 \quad 1.36347 \quad -0.708827 \quad 0 \quad 1.44833\ )$$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	7.5	3.52259	−0.269207	−1.09961

Solution for element 3

DOF values for the element

$\{\phi_4 = 6.12921,\ \phi_7 = 5.08248,\ \phi_8 = 3.96195,\ \phi_5 = 4.66596,$
$\delta_1^{\{4,7\}} = 0.54473,\ \delta_1^{\{7,8\}} = -0.251281,\ \delta_1^{\{5,8\}} = 0.440829,\ \delta_1^{\{4,5\}} = -0.887807\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 6.12921 \quad 5.08248 \quad 3.96195 \quad 4.66596 \quad 0.54473 \quad -0.251281 \quad 0.440829 \quad -0.887807\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	2.5	5.00691	−0.253032	−0.245653

Solution for element 4

DOF values for the element

$\{\phi_5 = 4.66596,\ \phi_8 = 3.96195,\ \phi_9 = 1,\ \phi_6 = 1,\ \delta_1^{\{5,8\}} = 0.440829,\ \delta_1^{\{8,9\}} = -0.0306799,\ \delta_1^{\{6,9\}} = 0,\ \delta_1^{\{5,6\}} = -0.708827\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 4.66596 \quad 3.96195 \quad 1 \quad 1 \quad 0.440829 \quad -0.0306799 \quad 0 \quad -0.708827\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	7.5	2.74843	−0.153456	−0.608801

Nodal solution summary

dof	x	y	Value
ϕ_1	0	0	10
ϕ_2	0	5	10
ϕ_3	0	10	1
ϕ_4	5	0	6.12921
ϕ_5	5	5	4.66596
ϕ_6	5	10	1
ϕ_7	10	0	5.08248
ϕ_8	10	5	3.96195
ϕ_9	10	10	1

Element solution summary

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.5269	−0.811749	−0.302817
2	2.5	7.5	3.52259	−0.269207	−1.09961
3	7.5	2.5	5.00691	−0.253032	−0.245653
4	7.5	7.5	2.74843	−0.153456	−0.608801

Solution with $n = 3$

Equations for element 1

$$\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 & 0.0 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 & -0.0 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 & 0.0 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 & -0.0 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 & 0.7 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0.3 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Equations for element 2

$$\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 & 0.0 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 & -0.0 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 & 0.0 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 & -0.0 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 & 0.7 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0.3 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Equations for element 3

$$\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 & 0.0\\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 & -0.0\\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 & 0.0\\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 & -0.0\\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 & 0\\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 & 0\\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 & 0\\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 & 0\\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 & 0.7\\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0\\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0.3\\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Equations for element 4

$$\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 & 0.0\\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 & -0.0\\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 & 0.0\\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 & -0.0\\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 & 0\\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 & 0\\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 & 0\\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 & 0\\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 & 0.7\\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0\\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0.3\\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Global equations after assembling all elements

$$\begin{pmatrix}
0.666667 & -0.166667 & 0 & -0.166667 & -0.333333 & 0 & 0 & 0\\
-0.166667 & 1.33333 & -0.166667 & -0.333333 & -0.333333 & -0.333333 & 0 & 0\\
0 & -0.166667 & 0.666667 & 0 & -0.333333 & -0.166667 & 0 & 0\\
-0.166667 & -0.333333 & 0 & 1.33333 & -0.333333 & 0 & -0.166667 & -0.333333
\end{pmatrix}$$

$$\begin{pmatrix}
-0.333333 & -0.333333 & -0.333333 & -0.333333 & 2.66667 & -0.333333 & -0.333333 & -0.333333 \\
0 & -0.333333 & -0.166667 & 0 & -0.333333 & 1.33333 & 0 & -0.333333 \\
0 & 0 & 0 & -0.166667 & -0.333333 & 0 & 0.666667 & -0.166667 \\
0 & 0 & 0 & -0.333333 & -0.333333 & -0.333333 & -0.166667 & 1.33333 \\
0 & 0 & 0 & 0 & -0.333333 & -0.166667 & 0 & -0.166667 \\
-0.204124 & -0.204124 & 0 & 0.204124 & 0.204124 & 0 & 0 & 0 \\
-0.204124 & 0.204124 & 0 & -0.204124 & 0.204124 & 0 & 0 & 0 \\
0 & -0.204124 & -0.204124 & 0 & 0.204124 & 0.204124 & 0 & 0 \\
0.204124 & -0.408248 & 0.204124 & 0.204124 & -0.408248 & 0.204124 & 0 & 0 \\
0 & 0.204124 & -0.204124 & 0 & 0.204124 & -0.204124 & 0 & 0 \\
0.204124 & 0.204124 & 0 & -0.408248 & -0.408248 & 0 & 0.204124 & 0.204124 \\
0 & 0 & 0 & -0.204124 & 0.204124 & 0 & -0.204124 & 0.204124 \\
0 & 0.204124 & 0.204124 & 0 & -0.408248 & -0.408248 & 0 & 0.204124 \\
0 & 0 & 0 & 0.204124 & -0.408248 & 0.204124 & 0.204124 & -0.408248 \\
0 & 0 & 0 & 0 & 0.204124 & -0.204124 & 0 & 0.204124 \\
0 & 0 & 0 & 0.204124 & 0.204124 & 0 & -0.204124 & -0.204124 \\
0 & 0 & 0 & 0 & 0.204124 & 0.204124 & 0 & -0.204124 \\
0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 & 0 & 0 & 0 \\
0 & 0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 & 0 & 0 \\
-0.0527046 & 0.105409 & -0.0527046 & 0.0527046 & -0.105409 & 0.0527046 & 0 & 0 \\
0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 & -0.0527046 & 0 & 0 \\
-0.0527046 & 0.0527046 & 0 & 0.105409 & -0.105409 & 0 & -0.0527046 & 0.0527046 \\
0 & 0 & 0 & 0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 \\
0 & -0.0527046 & 0.0527046 & 0 & 0.105409 & -0.105409 & 0 & -0.0527046 \\
0 & 0 & 0 & -0.0527046 & 0.105409 & -0.0527046 & 0.0527046 & -0.105409 \\
0 & 0 & 0 & 0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 \\
0 & 0 & 0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 & -0.0527046 \\
0 & 0 & 0 & 0 & -0.0527046 & 0.0527046 & 0 & 0.0527046
\end{pmatrix}$$

Essential boundary conditions

On element 1, side 4, specified value = 10

$$\left\{\phi_2,\ \phi_1,\ \delta_1^{\{1,2\}},\ \delta_2^{\{1,2\}}\right\} = \{10,\ 10,\ 0,\ 0\}$$

On element 2, side 3, specified value = 1

$$\left\{\phi_6,\ \phi_3,\ \delta_1^{\{3,6\}},\ \delta_2^{\{3,6\}}\right\} = \{1,\ 1,\ 0,\ 0\}$$

On element 4, side 3, specified value = 1

$$\left\{\phi_9,\ \phi_6,\ \delta_1^{\{6,9\}},\ \delta_2^{\{6,9\}}\right\} = \{1,\ 1,\ 0,\ 0\}$$

Known values from EBC

$$\left\{\phi_1 = 10,\ \phi_2 = 10,\ \phi_3 = 1,\ \phi_6 = 1,\ \phi_9 = 1,\ \delta_1^{\{1,2\}} = 0,\ \delta_1^{\{3,6\}} = 0,\ \delta_1^{\{6,9\}} = 0,\ \delta_2^{\{1,2\}} = 0,\ \delta_2^{\{3,6\}} = 0,\ \delta_2^{\{6,9\}} = 0\right\}$$

Global equations after EBC

$$\begin{pmatrix}
1.33333 & -0.333333 & -0.166667 & -0.333333 & -0.204124 & 0 & 0.204124 & -0.408248 & -0.204 \\
-0.333333 & 2.66667 & -0.333333 & -0.333333 & 0.204124 & 0.204124 & -0.408248 & -0.408248 & 0.204 \\
-0.166667 & -0.333333 & 0.666667 & -0.166667 & 0 & 0 & 0 & 0.204124 & -0.204 \\
-0.333333 & -0.333333 & -0.166667 & 1.33333 & 0 & 0 & 0 & 0.204124 & 0.204 \\
-0.204124 & 0.204124 & 0 & 0 & 0.866667 & 0 & 0.133333 & 0 & 0 \\
0 & 0.204124 & 0 & 0 & 0 & 0.866667 & 0 & 0 & 0 \\
0.204124 & -0.408248 & 0 & 0 & 0.133333 & 0 & 1.73333 & 0 & 0 \\
-0.408248 & -0.408248 & 0.204124 & 0.204124 & 0 & 0 & 0 & 1.73333 & 0 \\
-0.204124 & 0.204124 & -0.204124 & 0.204124 & 0 & 0 & 0 & 0 & 0.866 \\
0 & -0.408248 & 0 & 0.204124 & 0 & 0.133333 & 0 & 0 & 0 \\
0.204124 & -0.408248 & 0.204124 & -0.408248 & 0 & 0 & 0 & 0 & 0.133 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0 & 0 & 0 & 0.133333 & 0 \\
0 & 0.204124 & 0 & -0.204124 & 0 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.0527046 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.105409 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.105409 & -0.105409 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.105409 & 0 & -0.0527046 & 0 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.105409 & 0.0527046 & -0.105409 & 0 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.0527046 & 0 & 0.0527046 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Solving the final system of global equations we get

$\{\phi_4 = 6.1987,\ \phi_5 = 4.62455,\ \phi_7 = 5.04395,\ \phi_8 = 3.95026,\ \delta_1^{\{1,4\}} = 0.172592,\ \delta_1^{\{2,3\}} = 1.36673,\ \delta_1^{\{2,5\}} = 1.28806,$
$\delta_1^{\{4,5\}} = -0.842123,\ \delta_1^{\{4,7\}} = 0.564241,\ \delta_1^{\{5,6\}} = -0.654287,\ \delta_1^{\{5,8\}} = 0.416699,\ \delta_1^{\{7,8\}} = -0.301232,$
$\delta_1^{\{8,9\}} = -0.0581536,\ \delta_2^{\{1,4\}} = 0.25397,\ \delta_2^{\{2,3\}} = -0.537466,\ \delta_2^{\{2,5\}} = -0.316048,\ \delta_2^{\{4,5\}} = -0.093706,$
$\delta_2^{\{4,7\}} = -0.0363905,\ \delta_2^{\{5,6\}} = 0.309716,\ \delta_2^{\{5,8\}} = 0.000501449,\ \delta_2^{\{7,8\}} = 0.0763353,\ \delta_2^{\{8,9\}} = -0.081967\}$

Solution for element 1

DOF values for the element

$\{\phi_1 = 10,\ \phi_4 = 6.1987,\ \phi_5 = 4.62455,\ \phi_2 = 10,\ \delta_1^{\{1,4\}} = 0.172592,\ \delta_1^{\{4,5\}} = -0.842123,$
$\delta_1^{\{2,5\}} = 1.28806,\ \delta_1^{\{1,2\}} = 0,\ \delta_2^{\{1,4\}} = 0.25397,\ \delta_2^{\{4,5\}} = -0.093706,\ \delta_2^{\{2,5\}} = -0.316048,\ \delta_2^{\{1,2\}} = 0\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 10\quad 6.1987\quad 4.62455\quad 10\quad 0.172592\quad -0.842123\quad 1.28806\quad 0\quad 0.25397\quad -0.093706\quad -0.316048\quad 0\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.51643	−0.804721	−0.279215

Solution for element 2

DOF values for the element

$\{\phi_2 = 10,\ \phi_5 = 4.62455,\ \phi_6 = 1,\ \phi_3 = 1,\ \delta_1^{\{2,5\}} = 1.28806,\ \delta_1^{\{5,6\}} = -0.654287,\ \delta_1^{\{3,6\}} = 0,$
$\delta_1^{\{2,3\}} = 1.36673,\ \delta_2^{\{2,5\}} = -0.316048,\ \delta_2^{\{5,6\}} = 0.309716,\ \delta_2^{\{3,6\}} = 0,\ \delta_2^{\{2,3\}} = -0.537466\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 10\quad 4.62455\quad 1\quad 1\quad 1.28806\quad -0.654287\quad 0\quad 1.36673\quad -0.316048\quad 0.309716\quad 0\quad -0.537466\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	7.5	3.54361	−0.240051	−1.06869

Solution for element 3

DOF values for the element

$\{\phi_4 = 6.1987,\ \phi_7 = 5.04395,\ \phi_8 = 3.95026,\ \phi_5 = 4.62455,\ \delta_1^{\{4,7\}} = 0.564241,\ \delta_1^{\{7,8\}} = -0.301232,\ \delta_1^{\{5,8\}} = 0.416699,$
$\delta_1^{\{4,5\}} = -0.842123,\ \delta_2^{\{4,7\}} = -0.0363905,\ \delta_2^{\{7,8\}} = 0.0763353,\ \delta_2^{\{5,8\}} = 0.000501449,\ \delta_2^{\{4,5\}} = -0.093706\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 6.1987\quad 5.04395\quad 3.95026\quad 4.62455\quad 0.564241\quad -0.301232\quad 0.416699\quad -0.842123\quad -0.0363905\quad 0.0763$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	2.5	5.00409	−0.243475	−0.245966

Solution for element 4

DOF values for the element

$\{\phi_5 = 4.62455,\ \phi_8 = 3.95026,\ \phi_9 = 1,\ \phi_6 = 1,\ \delta_1^{\{5,8\}} = 0.416699,\ \delta_1^{\{8,9\}} = -0.0581536,\ \delta_1^{\{6,9\}} = 0,$
$\delta_1^{\{5,6\}} = -0.654287,\ \delta_2^{\{5,8\}} = 0.000501449,\ \delta_2^{\{8,9\}} = -0.081967,\ \delta_2^{\{6,9\}} = 0,\ \delta_2^{\{5,6\}} = 0.309716\}$

$\boldsymbol{d}^{\mathrm{T}}$ = (4.62455 3.95026 1 1 0.416699 −0.0581536 0 −0.654287 0.000501449 −0.081967 0 0.309716

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	7.5	2.73426	−0.140519	−0.642457

Nodal solution summary

dof	x	y	Value
ϕ_1	0	0	10
ϕ_2	0	5	10
ϕ_3	0	10	1
ϕ_4	5	0	6.1987
ϕ_5	5	5	4.62455
ϕ_6	5	10	1
ϕ_7	10	0	5.04395
ϕ_8	10	5	3.95026
ϕ_9	10	10	1

Element solution summary

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.51643	−0.804721	−0.279215
2	2.5	7.5	3.54361	−0.240051	−1.06869
3	7.5	2.5	5.00409	−0.243475	−0.245966
4	7.5	7.5	2.73426	−0.140519	−0.642457

Solution summary with $n = 4$

Equations for element 1

$$\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 \\
0 & 0 & 0 & 0 & -0.408248 & -0.408248 & -0.408248 & -0.408248
\end{pmatrix}$$

Equations for element 2

$$\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 \\
0 & 0 & 0 & 0 & -0.408248 & -0.408248 & -0.408248 & -0.408248
\end{pmatrix}$$

Equations for element 3

$$\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 \\
0 & 0 & 0 & 0 & -0.408248 & -0.408248 & -0.408248 & -0.408248
\end{pmatrix}$$

Equations for element 4

$$\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 \\
0 & 0 & 0 & 0 & -0.408248 & -0.408248 & -0.408248 & -0.408248
\end{pmatrix}$$

Global equations after assembling all elements

$$\begin{pmatrix}
0.666667 & -0.166667 & 0 & -0.166667 & -0.333333 & 0 & 0 & 0 \\
-0.166667 & 1.33333 & -0.166667 & -0.333333 & -0.333333 & -0.333333 & 0 & 0 \\
0 & -0.166667 & 0.666667 & 0 & -0.333333 & -0.166667 & 0 & 0 \\
-0.166667 & -0.333333 & 0 & 1.33333 & -0.333333 & 0 & -0.166667 & -0.333333 \\
-0.333333 & -0.333333 & -0.333333 & -0.333333 & 2.66667 & -0.333333 & -0.333333 & -0.333333 \\
0 & -0.333333 & -0.166667 & 0 & -0.333333 & 1.33333 & 0 & -0.333333 \\
0 & 0 & 0 & -0.166667 & -0.333333 & 0 & 0.666667 & -0.166667 \\
0 & 0 & 0 & -0.333333 & -0.333333 & -0.333333 & -0.166667 & 1.33333 \\
0 & 0 & 0 & 0 & -0.333333 & -0.166667 & 0 & -0.166667 \\
-0.204124 & -0.204124 & 0 & 0.204124 & 0.204124 & 0 & 0 & 0 \\
-0.204124 & 0.204124 & 0 & -0.204124 & 0.204124 & 0 & 0 & 0 \\
0 & -0.204124 & -0.204124 & 0 & 0.204124 & 0.204124 & 0 & 0
\end{pmatrix}$$

$$\left(\begin{matrix}
0.204124 & -0.408248 & 0.204124 & 0.204124 & -0.408248 & 0.204124 & 0 & 0 \\
0 & 0.204124 & -0.204124 & 0 & 0.204124 & -0.204124 & 0 & 0 \\
0.204124 & 0.204124 & 0 & -0.408248 & -0.408248 & 0 & 0.204124 & 0.204124 \\
0 & 0 & 0 & -0.204124 & 0.204124 & 0 & -0.204124 & 0.204124 \\
0 & 0.204124 & 0.204124 & 0 & -0.408248 & -0.408248 & 0 & 0.204124 \\
0 & 0 & 0 & 0.204124 & -0.408248 & 0.204124 & 0.204124 & -0.408248 \\
0 & 0 & 0 & 0 & 0.204124 & -0.204124 & 0 & 0.204124 \\
0 & 0 & 0 & 0.204124 & 0.204124 & 0 & -0.204124 & -0.204124 \\
0 & 0 & 0 & 0 & 0.204124 & 0.204124 & 0 & -0.204124 \\
0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 & 0 & 0 & 0 \\
0 & 0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 & 0 & 0 \\
-0.0527046 & 0.105409 & -0.0527046 & 0.0527046 & -0.105409 & 0.0527046 & 0 & 0 \\
0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 & -0.0527046 & 0 & 0 \\
-0.0527046 & 0.0527046 & 0 & 0.105409 & -0.105409 & 0 & -0.0527046 & 0.0527046 \\
0 & 0 & 0 & 0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 \\
0 & -0.0527046 & 0.0527046 & 0 & 0.105409 & -0.105409 & 0 & -0.0527046 \\
0 & 0 & 0 & -0.0527046 & 0.105409 & -0.0527046 & 0.0527046 & -0.105409 \\
0 & 0 & 0 & 0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 \\
0 & 0 & 0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 & -0.0527046 \\
0 & 0 & 0 & 0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{matrix}\right.$$

Essential boundary conditions

On element 1, side 4, specified value = 10

$$\left\{\phi_2,\ \phi_1,\ \delta_1^{\{1,2\}},\ \delta_2^{\{1,2\}},\ \delta_3^{\{1,2\}}\right\} = \{10,\ 10,\ 0,\ 0,\ 0\}$$

On element 2, side 3, specified value = 1

$$\left\{\phi_6,\ \phi_3,\ \delta_1^{\{3,6\}},\ \delta_2^{\{3,6\}},\ \delta_3^{\{3,6\}}\right\} = \{1,\ 1,\ 0,\ 0,\ 0\}$$

On element 4, side 3, specified value = 1

$$\left\{\phi_9,\ \phi_6,\ \delta_1^{\{6,9\}},\ \delta_2^{\{6,9\}},\ \delta_3^{\{6,9\}}\right\} = \{1,\ 1,\ 0,\ 0,\ 0\}$$

Known values from EBC

$$\left\{\phi_1 = 10,\ \phi_2 = 10,\ \phi_3 = 1,\ \phi_6 = 1,\ \phi_9 = 1,\ \delta_1^{\{1,2\}} = 0,\ \delta_1^{\{3,6\}} = 0,\right.$$
$$\left.\delta_1^{\{6,9\}} = 0,\ \delta_2^{\{1,2\}} = 0,\ \delta_2^{\{3,6\}} = 0,\ \delta_2^{\{6,9\}} = 0,\ \delta_3^{\{1,2\}} = 0,\ \delta_3^{\{3,6\}} = 0,\ \delta_3^{\{6,9\}} = 0\right\}$$

Global equations after EBC

$$\begin{pmatrix}
1.33333 & -0.333333 & -0.166667 & -0.333333 & -0.204124 & 0 & 0.204124 & -0.408248 \\
-0.333333 & 2.66667 & -0.333333 & -0.333333 & 0.204124 & 0.204124 & -0.408248 & -0.408248 \\
-0.166667 & -0.333333 & 0.666667 & -0.166667 & 0 & 0 & 0 & 0.204124 \\
-0.333333 & -0.333333 & -0.166667 & 1.33333 & 0 & 0 & 0 & 0.204124 \\
-0.204124 & 0.204124 & 0 & 0 & 0.866667 & 0 & 0.133333 & 0 \\
0 & 0.204124 & 0 & 0 & 0 & 0.866667 & 0 & 0 \\
0.204124 & -0.408248 & 0 & 0 & 0.133333 & 0 & 1.73333 & 0 \\
-0.408248 & -0.408248 & 0.204124 & 0.204124 & 0 & 0 & 0 & 1.73333 \\
-0.204124 & 0.204124 & -0.204124 & 0.204124 & 0 & 0 & 0 & 0 \\
0 & -0.408248 & 0 & 0.204124 & 0 & 0.133333 & 0 & 0 \\
0.204124 & -0.408248 & 0.204124 & -0.408248 & 0 & 0 & 0 & 0 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0 & 0 & 0 & 0.133333 \\
0 & 0.204124 & 0 & -0.204124 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.0527046 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.105409 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.105409 & -0.105409 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0.0527046 & -0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0.105409 & 0 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.105409 & 0.0527046 & -0.105409 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & -0.0527046 & 0 & 0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0436436 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0436436 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.408248 & 0 & -0.408248 & -0.408248 \\
0 & 0 & 0 & 0 & 0 & -0.408248 & -0.408248 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.408248 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Solving the final system of global equations we get

$$\{\phi_4 = 6.24941,\ \phi_5 = 4.78965,\ \phi_7 = 4.95572,\ \phi_8 = 3.83463,\ \delta_1^{\{1,4\}} = 0.522616,\ \delta_1^{\{2,3\}} = 2.25151,$$
$$\delta_1^{\{2,5\}} = 2.04415,\ \delta_1^{\{4,5\}} = -0.49774,\ \delta_1^{\{4,7\}} = 0.559187,\ \delta_1^{\{5,6\}} = -0.14886,\ \delta_1^{\{5,8\}} = 0.462738,$$
$$\delta_1^{\{7,8\}} = -0.429955,\ \delta_1^{\{8,9\}} = -0.163152,\ \delta_2^{\{1,4\}} = 0.224069,\ \delta_2^{\{2,3\}} = -0.489115,\ \delta_2^{\{2,5\}} = -0.290896,$$
$$\delta_2^{\{4,5\}} = -0.0775514,\ \delta_2^{\{4,7\}} = -0.0172751,\ \delta_2^{\{5,6\}} = 0.261655,\ \delta_2^{\{5,8\}} = -0.0192845,$$
$$\delta_2^{\{7,8\}} = 0.0560094,\ \delta_2^{\{8,9\}} = -0.0341941,\ \delta_3^{\{1,4\}} = -0.0848399,\ \delta_3^{\{2,3\}} = 0.110627,\ \delta_3^{\{2,5\}} = 0.0770081,$$
$$\delta_3^{\{4,5\}} = -0.0107503,\ \delta_3^{\{4,7\}} = 0.000383013,\ \delta_3^{\{5,6\}} = -0.0719367,\ \delta_3^{\{5,8\}} = 0.00577932,$$
$$\delta_3^{\{7,8\}} = 0.00720908,\ \delta_3^{\{8,9\}} = 0.0332466,\ \delta_1^1 = 1.05585,\ \delta_1^2 = 2.11616,\ \delta_1^3 = 0.0480867,\ \delta_1^4 = 0.076917\}$$

Solution for element 1

DOF values for the element

$$\{\phi_1 = 10,\ \phi_4 = 6.24941,\ \phi_5 = 4.78965,\ \phi_2 = 10,\ \delta_1^{\{1,4\}} = 0.522616,\ \delta_1^{\{4,5\}} = -0.49774,$$
$$\delta_1^{\{2,5\}} = 2.04415,\ \delta_1^{\{1,2\}} = 0,\ \delta_2^{\{1,4\}} = 0.224069,\ \delta_2^{\{4,5\}} = -0.0775514,\ \delta_2^{\{2,5\}} = -0.290896,\ \delta_2^{\{1,2\}} = 0,$$
$$\delta_3^{\{1,4\}} = -0.0848399,\ \delta_3^{\{4,5\}} = -0.0107503,\ \delta_3^{\{2,5\}} = 0.0770081,\ \delta_3^{\{1,2\}} = 0,\ \delta_1^1 = 1.05585\}$$

$\boldsymbol{d}^{\mathrm{T}} = (\ 10\quad 6.24941\quad 4.78965\quad 10\quad 0.522616\quad -0.49774\quad 2.04415\quad 0\quad 0.224069\quad -0.0775514\quad -0.290896\quad 0\quad -$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.52003	−0.825071	−0.312494

Solution for element 2

DOF values for the element

$\{\phi_2 = 10,\ \phi_5 = 4.78965,\ \phi_6 = 1,\ \phi_3 = 1,\ \delta_1^{\{2,5\}} = 2.04415,\ \delta_1^{\{5,6\}} = -0.14886,\ \delta_1^{\{3,6\}} = 0,$
$\delta_1^{\{2,3\}} = 2.25151,\ \delta_2^{\{2,5\}} = -0.290896,\ \delta_2^{\{5,6\}} = 0.261655,\ \delta_2^{\{3,6\}} = 0,\ \delta_2^{\{2,3\}} = -0.489115,$
$\delta_3^{\{2,5\}} = 0.0770081,\ \delta_3^{\{5,6\}} = -0.0719367,\ \delta_3^{\{3,6\}} = 0,\ \delta_3^{\{2,3\}} = 0.110627,\ \delta_1^{2} = 2.11616\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 10\quad 4.78965\quad 1\quad 1\quad 2.04415\quad -0.14886\quad 0\quad 2.25151\quad -0.290896\quad 0.261655\quad 0\quad -0.489115\quad 0.0770081$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	7.5	3.73481	−0.189595	−0.996245

Solution for element 3

DOF values for the element

$\{\phi_4 = 6.24941,\ \phi_7 = 4.95572,\ \phi_8 = 3.83463,\ \phi_5 = 4.78965,\ \delta_1^{\{4,7\}} = 0.559187,\ \delta_1^{\{7,8\}} = -0.429955,\ \delta_1^{\{5,8\}} = 0.462738,$
$\delta_1^{\{4,5\}} = -0.49774,\ \delta_2^{\{4,7\}} = -0.0172751,\ \delta_2^{\{7,8\}} = 0.0560094,\ \delta_2^{\{5,8\}} = -0.0192845,\ \delta_2^{\{4,5\}} = -0.0775514,$
$\delta_3^{\{4,7\}} = 0.000383013,\ \delta_3^{\{7,8\}} = 0.00720908,\ \delta_3^{\{5,8\}} = 0.00577932,\ \delta_3^{\{4,5\}} = -0.0107503,\ \delta_1^{3} = 0.0480867\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 6.24941\quad 4.95572\quad 3.83463\quad 4.78965\quad 0.559187\quad -0.429955\quad 0.462738\quad -0.49774\quad -0.0172751\quad 0.0560($

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	2.5	4.94684	−0.226552	−0.242614

Solution for element 4

DOF values for the element

$\{\phi_5 = 4.78965,\ \phi_8 = 3.83463,\ \phi_9 = 1,\ \phi_6 = 1,\ \delta_1^{\{5,8\}} = 0.462738,\ \delta_1^{\{8,9\}} = -0.163152,\ \delta_1^{\{6,9\}} = 0,$
$\delta_1^{\{5,6\}} = -0.14886,\ \delta_2^{\{5,8\}} = -0.0192845,\ \delta_2^{\{8,9\}} = -0.0341941,\ \delta_2^{\{6,9\}} = 0,\ \delta_2^{\{5,6\}} = 0.261655,$
$\delta_3^{\{5,8\}} = 0.00577932,\ \delta_3^{\{8,9\}} = 0.0332466,\ \delta_3^{\{6,9\}} = 0,\ \delta_3^{\{5,6\}} = -0.0719367,\ \delta_1^{4} = 0.076917\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 4.78965\quad 3.83463\quad 1\quad 1\quad 0.462738\quad -0.163152\quad 0\quad -0.14886\quad -0.0192845\quad -0.0341941\quad 0\quad 0.261655$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	7.5	2.63491	−0.0857831	−0.641989

Nodal solution summary

dof	x	y	Value
ϕ_1	0	0	10
ϕ_2	0	5	10
ϕ_3	0	10	1
ϕ_4	5	0	6.24941
ϕ_5	5	5	4.78965
ϕ_6	5	10	1
ϕ_7	10	0	4.95572
ϕ_8	10	5	3.83463
ϕ_9	10	10	1

Element solution summary

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.52003	−0.825071	−0.312494
2	2.5	7.5	3.73481	−0.189595	−0.996245
3	7.5	2.5	4.94684	−0.226552	−0.242614
4	7.5	7.5	2.63491	−0.0857831	−0.641989

Solution summary with $n = 5$

Equations for element 1

$$\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.408248 & -0.408248 & -0.408248 & -0.408248 \\
0 & 0 & 0 & 0 & 0.105409 & 0 & -0.105409 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.105409 & 0 & 0.105409
\end{pmatrix}$$

Equations for element 2

$$
\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.408248 & -0.408248 & -0.408248 & -0.408248 \\
0 & 0 & 0 & 0 & 0.105409 & 0 & -0.105409 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.105409 & 0 & 0.105409
\end{pmatrix}
$$

Equations for element 3

$$\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.408248 & -0.408248 & -0.408248 & -0.408248 \\
0 & 0 & 0 & 0 & 0.105409 & 0 & -0.105409 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.105409 & 0 & 0.105409
\end{pmatrix}$$

Equations for element 4

$$\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.408248 & -0.408248 & -0.408248 & -0.408248 \\
0 & 0 & 0 & 0 & 0.105409 & 0 & -0.105409 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.105409 & 0 & 0.105409
\end{pmatrix}$$

Global equations after assembling all elements

$$\begin{pmatrix}
0.666667 & -0.166667 & 0 & -0.166667 & -0.333333 & 0 & 0 & 0 \\
-0.166667 & 1.33333 & -0.166667 & -0.333333 & -0.333333 & -0.333333 & 0 & 0 \\
0 & -0.166667 & 0.666667 & 0 & -0.333333 & -0.166667 & 0 & 0
\end{pmatrix}$$

$$
\begin{vmatrix}
-0.166667 & -0.333333 & 0 & 1.33333 & -0.333333 & 0 & -0.166667 & -0.333333 \\
-0.333333 & -0.333333 & -0.333333 & -0.333333 & 2.66667 & -0.333333 & -0.333333 & -0.333333 \\
0 & -0.333333 & -0.166667 & 0 & -0.333333 & 1.33333 & 0 & -0.333333 \\
0 & 0 & 0 & -0.166667 & -0.333333 & 0 & 0.666667 & -0.166667 \\
0 & 0 & 0 & -0.333333 & -0.333333 & -0.333333 & -0.166667 & 1.33333 \\
0 & 0 & 0 & 0 & -0.333333 & -0.166667 & 0 & -0.166667 \\
-0.204124 & -0.204124 & 0 & 0.204124 & 0.204124 & 0 & 0 & 0 \\
-0.204124 & 0.204124 & 0 & -0.204124 & 0.204124 & 0 & 0 & 0 \\
0 & -0.204124 & -0.204124 & 0 & 0.204124 & 0.204124 & 0 & 0 \\
0.204124 & -0.408248 & 0.204124 & 0.204124 & -0.408248 & 0.204124 & 0 & 0 \\
0 & 0.204124 & -0.204124 & 0 & 0.204124 & -0.204124 & 0 & 0 \\
0.204124 & 0.204124 & 0 & -0.408248 & -0.408248 & 0 & 0.204124 & 0.204124 \\
0 & 0 & 0 & -0.204124 & 0.204124 & 0 & -0.204124 & 0.204124 \\
0 & 0.204124 & 0.204124 & 0 & -0.408248 & -0.408248 & 0 & 0.204124 \\
0 & 0 & 0 & 0.204124 & -0.408248 & 0.204124 & 0.204124 & -0.408248 \\
0 & 0 & 0 & 0 & 0.204124 & -0.204124 & 0 & 0.204124 \\
0 & 0 & 0 & 0.204124 & 0.204124 & 0 & -0.204124 & -0.204124 \\
0 & 0 & 0 & 0 & 0.204124 & 0.204124 & 0 & -0.204124 \\
0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 & 0 & 0 & 0 \\
0 & 0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 & 0 & 0 \\
-0.0527046 & 0.105409 & -0.0527046 & 0.0527046 & -0.105409 & 0.0527046 & 0 & 0 \\
0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 & -0.0527046 & 0 & 0 \\
-0.0527046 & 0.0527046 & 0 & 0.105409 & -0.105409 & 0 & -0.0527046 & 0.0527046 \\
0 & 0 & 0 & 0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 \\
0 & -0.0527046 & 0.0527046 & 0 & 0.105409 & -0.105409 & 0 & -0.0527046 \\
0 & 0 & 0 & -0.0527046 & 0.105409 & -0.0527046 & 0.0527046 & -0.105409 \\
0 & 0 & 0 & 0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 \\
0 & 0 & 0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 & -0.0527046 \\
0 & 0 & 0 & 0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{vmatrix}
$$

$$\left(\begin{matrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{matrix}\right.$$

Essential boundary conditions

On element 1, side 4, specified value = 10

$$\left\{\phi_2,\ \phi_1,\ \delta_1^{\{1,2\}},\ \delta_2^{\{1,2\}},\ \delta_3^{\{1,2\}},\ \delta_4^{\{1,2\}}\right\} = \{10,\ 10,\ 0,\ 0,\ 0,\ 0\}$$

On element 2, side 3, specified value = 1

$$\left\{\phi_6,\ \phi_3,\ \delta_1^{\{3,6\}},\ \delta_2^{\{3,6\}},\ \delta_3^{\{3,6\}},\ \delta_4^{\{3,6\}}\right\} = \{1,\ 1,\ 0,\ 0,\ 0,\ 0\}$$

On element 4, side 3, specified value = 1

$$\left\{\phi_9,\ \phi_6,\ \delta_1^{\{6,9\}},\ \delta_2^{\{6,9\}},\ \delta_3^{\{6,9\}},\ \delta_4^{\{6,9\}}\right\} = \{1,\ 1,\ 0,\ 0,\ 0,\ 0\}$$

Known values from EBC

$$\left\{\phi_1 = 10,\ \phi_2 = 10,\ \phi_3 = 1,\ \phi_6 = 1,\ \phi_9 = 1,\ \delta_1^{\{1,2\}} = 0,\ \delta_1^{\{3,6\}} = 0,\ \delta_1^{\{6,9\}} = 0,\ \delta_2^{\{1,2\}} = 0,\right.$$
$$\left.\delta_2^{\{3,6\}} = 0,\ \delta_2^{\{6,9\}} = 0,\ \delta_3^{\{1,2\}} = 0,\ \delta_3^{\{3,6\}} = 0,\ \delta_3^{\{6,9\}} = 0,\ \delta_4^{\{1,2\}} = 0,\ \delta_4^{\{3,6\}} = 0,\ \delta_4^{\{6,9\}} = 0\right\}$$

Global equations after EBC

$$\begin{pmatrix}
1.33333 & -0.333333 & -0.166667 & -0.333333 & -0.204124 & 0 & 0.204124 & -0.408248 \\
-0.333333 & 2.66667 & -0.333333 & -0.333333 & 0.204124 & 0.204124 & -0.408248 & -0.408248 \\
-0.166667 & -0.333333 & 0.666667 & -0.166667 & 0 & 0 & 0 & 0.204124 \\
-0.333333 & -0.333333 & -0.166667 & 1.33333 & 0 & 0 & 0 & 0.204124 \\
-0.204124 & 0.204124 & 0 & 0 & 0.866667 & 0 & 0.133333 & 0 \\
0 & 0.204124 & 0 & 0 & 0 & 0.866667 & 0 & 0 \\
0.204124 & -0.408248 & 0 & 0 & 0.133333 & 0 & 1.73333 & 0 \\
-0.408248 & -0.408248 & 0.204124 & 0.204124 & 0 & 0 & 0 & 1.73333 \\
-0.204124 & 0.204124 & -0.204124 & 0.204124 & 0 & 0 & 0 & 0 \\
0 & -0.408248 & 0 & 0.204124 & 0 & 0.133333 & 0 & 0 \\
0.204124 & -0.408248 & 0.204124 & -0.408248 & 0 & 0 & 0 & 0 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0 & 0 & 0 & 0.133333 \\
0 & 0.204124 & 0 & -0.204124 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.0527046 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.105409 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.105409 & -0.105409 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0.105409 & 0 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.105409 & 0.0527046 & -0.105409 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & -0.0527046 & 0 & 0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0
\end{pmatrix}$$

$$
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0436436 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0436436 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.408248 & 0 & -0.408248 & -0.408248 \\
0 & 0 & 0 & 0 & 0.105409 & 0 & -0.105409 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.105409 \\
0 & 0 & 0 & 0 & 0 & -0.408248 & -0.408248 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.105409 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.105409 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.408248 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.105409 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

Solving the final system of global equations we get

$$\{\phi_4 = 6.13561,\ \phi_5 = 4.75006,\ \phi_7 = 4.92791,\ \phi_8 = 3.79922,\ \delta_1^{\{1,4\}} = 0.407813,\ \delta_1^{\{2,3\}} = 2.51783,$$
$$\delta_1^{\{2,5\}} = 2.2194,\ \delta_1^{\{4,5\}} = -0.615162,\ \delta_1^{\{4,7\}} = 0.478187,\ \delta_1^{\{5,6\}} = -0.219334,\ \delta_1^{\{5,8\}} = 0.443003,$$
$$\delta_1^{\{7,8\}} = -0.444232,\ \delta_1^{\{8,9\}} = -0.177441,\ \delta_2^{\{1,4\}} = -0.0619089,\ \delta_2^{\{2,3\}} = -1.25615,\ \delta_2^{\{2,5\}} = -0.853448,$$
$$\delta_2^{\{4,5\}} = 0.0924994,\ \delta_2^{\{4,7\}} = 0.000292546,\ \delta_2^{\{5,6\}} = -0.0886744,\ \delta_2^{\{5,8\}} = -0.0239389,\ \delta_2^{\{7,8\}} = 0.0401044,$$
$$\delta_2^{\{8,9\}} = 0.0182006,\ \delta_3^{\{1,4\}} = -0.096009,\ \delta_3^{\{2,3\}} = 0.122953,\ \delta_3^{\{2,5\}} = 0.0855238,\ \delta_3^{\{4,5\}} = -0.0152258,$$
$$\delta_3^{\{4,7\}} = -0.00200726,\ \delta_3^{\{5,6\}} = -0.080264,\ \delta_3^{\{5,8\}} = 0.00691246,\ \delta_3^{\{7,8\}} = 0.0122907,\ \delta_3^{\{8,9\}} = 0.0375753,$$
$$\delta_4^{\{1,4\}} = 0.0212248,\ \delta_4^{\{2,3\}} = -0.026227,\ \delta_4^{\{2,5\}} = -0.0181012,\ \delta_4^{\{4,5\}} = 0.00151976,\ \delta_4^{\{4,7\}} = 0.000640333,$$
$$\delta_4^{\{5,6\}} = 0.0161698,\ \delta_4^{\{5,8\}} = -0.000534594,\ \delta_4^{\{7,8\}} = -0.00156006,\ \delta_4^{\{8,9\}} = -0.00611264,$$
$$\delta_1^1 = 1.02677,\ \delta_2^1 = 0.46184,\ \delta_3^1 = -0.885507,\ \delta_1^2 = 2.30553,\ \delta_2^2 = -1.58099,\ \delta_3^2 = -1.28613,$$
$$\delta_1^3 = -0.070527,\ \delta_2^3 = 0.101823,\ \delta_3^3 = 0.016889,\ \delta_1^4 = 0.0235907,\ \delta_2^4 = -0.152386,\ \delta_3^4 = -0.0108173\}$$

Solution for element 1

DOF values for the element

$$\{\phi_1 = 10,\ \phi_4 = 6.13561,\ \phi_5 = 4.75006,\ \phi_2 = 10,\ \delta_1^{\{1,4\}} = 0.407813,\ \delta_1^{\{4,5\}} = -0.615162,$$
$$\delta_1^{\{2,5\}} = 2.2194,\ \delta_1^{\{1,2\}} = 0,\ \delta_2^{\{1,4\}} = -0.0619089,\ \delta_2^{\{4,5\}} = 0.0924994,\ \delta_2^{\{2,5\}} = -0.853448,\ \delta_2^{\{1,2\}} = 0,$$
$$\delta_3^{\{1,4\}} = -0.096009,\ \delta_3^{\{4,5\}} = -0.0152258,\ \delta_3^{\{2,5\}} = 0.0855238,\ \delta_3^{\{1,2\}} = 0,\ \delta_4^{\{1,4\}} = 0.0212248,$$
$$\delta_4^{\{4,5\}} = 0.00151976,\ \delta_4^{\{2,5\}} = -0.0181012,\ \delta_4^{\{1,2\}} = 0,\ \delta_1^1 = 1.02677,\ \delta_2^1 = 0.46184,\ \delta_3^1 = -0.885507\}$$

$$\boldsymbol{d}^{\mathrm{T}} = (\ 10\quad 6.13561\quad 4.75006\quad 10\quad 0.407813\quad -0.615162\quad 2.2194\quad 0\quad -0.0619089\quad 0.0924994\quad -0.853448\quad 0\ \cdots$$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.48739	−0.863053	−0.276887

Solution for element 2

DOF values for the element

$$\{\phi_2 = 10,\ \phi_5 = 4.75006,\ \phi_6 = 1,\ \phi_3 = 1,\ \delta_1^{\{2,5\}} = 2.2194,\ \delta_1^{\{5,6\}} = -0.219334,\ \delta_1^{\{3,6\}} = 0,$$
$$\delta_1^{\{2,3\}} = 2.51783,\ \delta_2^{\{2,5\}} = -0.853448,\ \delta_2^{\{5,6\}} = -0.0886744,\ \delta_2^{\{3,6\}} = 0,\ \delta_2^{\{2,3\}} = -1.25615,$$
$$\delta_3^{\{2,5\}} = 0.0855238,\ \delta_3^{\{5,6\}} = -0.080264,\ \delta_3^{\{3,6\}} = 0,\ \delta_3^{\{2,3\}} = 0.122953,\ \delta_4^{\{2,5\}} = -0.0181012,$$
$$\delta_4^{\{5,6\}} = 0.0161698,\ \delta_4^{\{3,6\}} = 0,\ \delta_4^{\{2,3\}} = -0.026227,\ \delta_1^2 = 2.30553,\ \delta_2^2 = -1.58099,\ \delta_3^2 = -1.28613\}$$

$$\boldsymbol{d}^{\mathrm{T}} = (\ 10\quad 4.75006\quad 1\quad 1\quad 2.2194\quad -0.219334\quad 0\quad 2.51783\quad -0.853448\quad -0.0886744\quad 0\quad -1.25615\quad 0.0855238$$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	7.5	3.68376	−0.316262	−1.10231

Solution for element 3

DOF values for the element

$\{\phi_4 = 6.13561,\ \phi_7 = 4.92791,\ \phi_8 = 3.79922,\ \phi_5 = 4.75006,\ \delta_1^{\{4,7\}} = 0.478187,$
$\delta_1^{\{7,8\}} = -0.444232,\ \delta_1^{\{5,8\}} = 0.443003,\ \delta_1^{\{4,5\}} = -0.615162,\ \delta_2^{\{4,7\}} = 0.000292546,\ \delta_2^{\{7,8\}} = 0.0401044,$
$\delta_2^{\{5,8\}} = -0.0239389,\ \delta_2^{\{4,5\}} = 0.0924994,\ \delta_3^{\{4,7\}} = -0.00200726,\ \delta_3^{\{7,8\}} = 0.0122907,$
$\delta_3^{\{5,8\}} = 0.00691246,\ \delta_3^{\{4,5\}} = -0.0152258,\ \delta_4^{\{4,7\}} = 0.000640333,\ \delta_4^{\{7,8\}} = -0.00156006,$
$\delta_4^{\{5,8\}} = -0.000534594,\ \delta_4^{\{4,5\}} = 0.00151976,\ \delta_1^3 = -0.070527,\ \delta_2^3 = 0.101823,\ \delta_3^3 = 0.016889\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 6.13561\ \ 4.92791\ \ 3.79922\ \ 4.75006\ \ 0.478187\ \ -0.444232\ \ 0.443003\ \ -0.615162\ \ 0.000292546\ \ 0.040$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	2.5	4.9193	−0.228476	−0.247952

Solution for element 4

DOF values for the element

$\{\phi_5 = 4.75006,\ \phi_8 = 3.79922,\ \phi_9 = 1,\ \phi_6 = 1,\ \delta_1^{\{5,8\}} = 0.443003,\ \delta_1^{\{8,9\}} = -0.177441,\ \delta_1^{\{6,9\}} = 0,$
$\delta_1^{\{5,6\}} = -0.219334,\ \delta_2^{\{5,8\}} = -0.0239389,\ \delta_2^{\{8,9\}} = 0.0182006,\ \delta_2^{\{6,9\}} = 0,\ \delta_2^{\{5,6\}} = -0.0886744,$
$\delta_3^{\{5,8\}} = 0.00691246,\ \delta_3^{\{8,9\}} = 0.0375753,\ \delta_3^{\{6,9\}} = 0,\ \delta_3^{\{5,6\}} = -0.080264,\ \delta_4^{\{5,8\}} = -0.000534594,$
$\delta_4^{\{8,9\}} = -0.00611264,\ \delta_4^{\{6,9\}} = 0,\ \delta_4^{\{5,6\}} = 0.0161698,\ \delta_1^4 = 0.0235907,\ \delta_2^4 = -0.152386,\ \delta_3^4 = -0.0108173\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 4.75006\ \ 3.79922\ \ 1\ \ 1\ \ 0.443003\ \ -0.177441\ \ 0\ \ -0.219334\ \ -0.0239389\ \ 0.0182006\ \ 0\ \ -0.088674$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	7.5	2.62783	−0.0930979	−0.617761

Nodal solution summary

dof	x	y	Value
ϕ_1	0	0	10
ϕ_2	0	5	10
ϕ_3	0	10	1
ϕ_4	5	0	6.13561
ϕ_5	5	5	4.75006
ϕ_6	5	10	1
ϕ_7	10	0	4.92791
ϕ_8	10	5	3.79922
ϕ_9	10	10	1

Element solution summary

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.48739	−0.863053	−0.276887
2	2.5	7.5	3.68376	−0.316262	−1.10231
3	7.5	2.5	4.9193	−0.228476	−0.247952
4	7.5	7.5	2.62783	−0.0930979	−0.617761

Solution summary with $n = 6$

Equations for element 1

$$
\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.408248 & -0.408248 & -0.408248 & -0.408248 \\
0 & 0 & 0 & 0 & 0.105409 & 0 & -0.105409 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.105409 & 0 & 0.105409 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

Equations for element 2

$$
\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.408248 & -0.408248 & -0.408248 & -0.408248 \\
0 & 0 & 0 & 0 & 0.105409 & 0 & -0.105409 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.105409 & 0 & 0.105409 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

Equations for element 3

$$
\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.408248 & -0.408248 & -0.408248 & -0.408248 \\
0 & 0 & 0 & 0 & 0.105409 & 0 & -0.105409 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.105409 & 0 & 0.105409 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

Equations for element 4

$$
\begin{pmatrix}
0.666667 & -0.166667 & -0.333333 & -0.166667 & -0.204124 & 0.204124 & 0.204124 & -0.204124 \\
-0.166667 & 0.666667 & -0.166667 & -0.333333 & -0.204124 & -0.204124 & 0.204124 & 0.204124 \\
-0.333333 & -0.166667 & 0.666667 & -0.166667 & 0.204124 & -0.204124 & -0.204124 & 0.204124 \\
-0.166667 & -0.333333 & -0.166667 & 0.666667 & 0.204124 & 0.204124 & -0.204124 & -0.204124 \\
-0.204124 & -0.204124 & 0.204124 & 0.204124 & 0.866667 & 0 & 0.133333 & 0 \\
0.204124 & -0.204124 & -0.204124 & 0.204124 & 0 & 0.866667 & 0 & 0.133333 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0.133333 & 0 & 0.866667 & 0 \\
-0.204124 & 0.204124 & 0.204124 & -0.204124 & 0 & 0.133333 & 0 & 0.866667 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0218218 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.408248 & -0.408248 & -0.408248 & -0.408248 \\
0 & 0 & 0 & 0 & 0.105409 & 0 & -0.105409 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.105409 & 0 & 0.105409 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

Global equations after assembling all elements

$$
\begin{pmatrix}
0.666667 & -0.166667 & 0 & -0.166667 & -0.333333 & 0 & 0 & 0 \\
-0.166667 & 1.33333 & -0.166667 & -0.333333 & -0.333333 & -0.333333 & 0 & 0 \\
0 & -0.166667 & 0.666667 & 0 & -0.333333 & -0.166667 & 0 & 0 \\
-0.166667 & -0.333333 & 0 & 1.33333 & -0.333333 & 0 & -0.166667 & -0.333333 \\
-0.333333 & -0.333333 & -0.333333 & -0.333333 & 2.66667 & -0.333333 & -0.333333 & -0.333333 \\
0 & -0.333333 & -0.166667 & 0 & -0.333333 & 1.33333 & 0 & -0.333333 \\
0 & 0 & 0 & -0.166667 & -0.333333 & 0 & 0.666667 & -0.166667 \\
0 & 0 & 0 & -0.333333 & -0.333333 & -0.333333 & -0.166667 & 1.33333 \\
0 & 0 & 0 & 0 & -0.333333 & -0.166667 & 0 & -0.166667 \\
-0.204124 & -0.204124 & 0 & 0.204124 & 0.204124 & 0 & 0 & 0 \\
-0.204124 & 0.204124 & 0 & -0.204124 & 0.204124 & 0 & 0 & 0 \\
0 & -0.204124 & -0.204124 & 0 & 0.204124 & 0.204124 & 0 & 0 \\
0.204124 & -0.408248 & 0.204124 & 0.204124 & -0.408248 & 0.204124 & 0 & 0 \\
0 & 0.204124 & -0.204124 & 0 & 0.204124 & -0.204124 & 0 & 0 \\
0.204124 & 0.204124 & 0 & -0.408248 & -0.408248 & 0 & 0.204124 & 0.204124 \\
0 & 0 & 0 & -0.204124 & 0.204124 & 0 & -0.204124 & 0.204124 \\
0 & 0.204124 & 0.204124 & 0 & -0.408248 & -0.408248 & 0 & 0.204124 \\
0 & 0 & 0 & 0.204124 & -0.408248 & 0.204124 & 0.204124 & -0.408248 \\
0 & 0 & 0 & 0 & 0.204124 & -0.204124 & 0 & 0.204124 \\
0 & 0 & 0 & 0.204124 & 0.204124 & 0 & -0.204124 & -0.204124 \\
0 & 0 & 0 & 0 & 0.204124 & 0.204124 & 0 & -0.204124 \\
0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 & 0 & 0 & 0 \\
0 & 0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 & 0 & 0 \\
-0.0527046 & 0.105409 & -0.0527046 & 0.0527046 & -0.105409 & 0.0527046 & 0 & 0 \\
0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 & -0.0527046 & 0 & 0 \\
-0.0527046 & 0.0527046 & 0 & 0.105409 & -0.105409 & 0 & -0.0527046 & 0.0527046 \\
0 & 0 & 0 & 0.0527046 & -0.0527046 & 0 & -0.0527046 & 0.0527046 \\
0 & -0.0527046 & 0.0527046 & 0 & 0.105409 & -0.105409 & 0 & -0.0527046 \\
0 & 0 & 0 & -0.0527046 & 0.105409 & -0.0527046 & 0.0527046 & -0.105409 \\
0 & 0 & 0 & 0 & -0.0527046 & 0.0527046 & 0 & 0.0527046 \\
0 & 0 & 0 & 0.0527046 & 0.0527046 & 0 & 0.0527046 & 0.0527046
\end{pmatrix}
$$

0	0	0	−0.0527046	0.0527046	0	0.0527046	−0.0527046
0	0	0	0	−0.0527046	0.0527046	0	0.0527046
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

$$\left(\begin{array}{cccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right.$$

Essential boundary conditions

On element 1, side 4, specified value = 10

$$\left\{\phi_2,\ \phi_1,\ \delta_1^{\{1,2\}},\ \delta_2^{\{1,2\}},\ \delta_3^{\{1,2\}},\ \delta_4^{\{1,2\}},\ \delta_5^{\{1,2\}}\right\} = \{10,\ 10,\ 0,\ 0,\ 0,\ 0,\ 0\}$$

On element 2, side 3, specified value = 1

$$\left\{\phi_6,\ \phi_3,\ \delta_1^{\{3,6\}},\ \delta_2^{\{3,6\}},\ \delta_3^{\{3,6\}},\ \delta_4^{\{3,6\}},\ \delta_5^{\{3,6\}}\right\} = \{1,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0\}$$

On element 4, side 3, specified value = 1

$$\left\{\phi_9,\ \phi_6,\ \delta_1^{\{6,9\}},\ \delta_2^{\{6,9\}},\ \delta_3^{\{6,9\}},\ \delta_4^{\{6,9\}},\ \delta_5^{\{6,9\}}\right\} = \{1,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0\}$$

Known values from EBC

$$\left\{\phi_1 = 10,\ \phi_2 = 10,\ \phi_3 = 1,\ \phi_6 = 1,\ \phi_9 = 1,\ \delta_1^{\{1,2\}} = 0,\ \delta_1^{\{3,6\}} = 0,\ \delta_1^{\{6,9\}} = 0,\ \delta_2^{\{1,2\}} = 0,\ \delta_2^{\{3,6\}} = 0,\ \delta_2^{\{6,9\}} = 0,\right.$$
$$\left.\delta_3^{\{1,2\}} = 0,\ \delta_3^{\{3,6\}} = 0,\ \delta_3^{\{6,9\}} = 0,\ \delta_4^{\{1,2\}} = 0,\ \delta_4^{\{3,6\}} = 0,\ \delta_4^{\{6,9\}} = 0,\ \delta_5^{\{1,2\}} = 0,\ \delta_5^{\{3,6\}} = 0,\ \delta_5^{\{6,9\}} = 0\right\}$$

Global equations after EBC

$$\begin{pmatrix}
1.33333 & -0.333333 & -0.166667 & -0.333333 & -0.204124 & 0 & 0.204124 & -0.408248 \\
-0.333333 & 2.66667 & -0.333333 & -0.333333 & 0.204124 & 0.204124 & -0.408248 & -0.408248 \\
-0.166667 & -0.333333 & 0.666667 & -0.166667 & 0 & 0 & 0 & 0.204124 \\
-0.333333 & -0.333333 & -0.166667 & 1.33333 & 0 & 0 & 0 & 0.204124 \\
-0.204124 & 0.204124 & 0 & 0 & 0.866667 & 0 & 0.133333 & 0 \\
0 & 0.204124 & 0 & 0 & 0 & 0.866667 & 0 & 0 \\
0.204124 & -0.408248 & 0 & 0 & 0.133333 & 0 & 1.73333 & 0 \\
-0.408248 & -0.408248 & 0.204124 & 0.204124 & 0 & 0 & 0 & 1.73333 \\
-0.204124 & 0.204124 & -0.204124 & 0.204124 & 0 & 0 & 0 & 0 \\
0 & -0.408248 & 0 & 0.204124 & 0 & 0.133333 & 0 & 0 \\
0.204124 & -0.408248 & 0.204124 & -0.408248 & 0 & 0 & 0 & 0 \\
0.204124 & 0.204124 & -0.204124 & -0.204124 & 0 & 0 & 0 & 0.133333 \\
0 & 0.204124 & 0 & -0.204124 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.0527046 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.105409 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.105409 & -0.105409 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0.0527046 & -0.0527046 & -0.0527046 & 0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0.105409 & 0 & -0.0527046 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.105409 & 0.0527046 & -0.105409 & 0 & 0 & 0 & 0 \\
-0.0527046 & 0.0527046 & 0.0527046 & -0.0527046 & 0 & 0 & 0 & 0 \\
0 & -0.0527046 & 0 & 0.0527046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & 0.0218218 & 0
\end{pmatrix}$$

$$\left|\begin{array}{cccccccc}
0 & 0 & 0 & 0 & -0.0218218 & 0 & 0.0218218 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0218218 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.0218218 & 0 & -0.0436436 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0436436 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0218218 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0218218 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.408248 & 0 & -0.408248 & -0.408248 \\
0 & 0 & 0 & 0 & 0.105409 & 0 & -0.105409 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.105409 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.408248 & -0.408248 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.105409 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.105409 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right.$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.408248 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.105409 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Solving the final system of global equations we get

$$\{\phi_4 = 6.18542,\ \phi_5 = 4.69347,\ \phi_7 = 4.91823,\ \phi_8 = 3.78303,\ \delta_1^{\{1,4\}} = 0.500969,\ \delta_1^{\{2,3\}} = 2.6612,\ \delta_1^{\{2,5\}} = 2.27686,$$
$$\delta_1^{\{4,5\}} = -0.578562,\ \delta_1^{\{4,7\}} = 0.548653,\ \delta_1^{\{5,6\}} = -0.225498,\ \delta_1^{\{5,8\}} = 0.396936,\ \delta_1^{\{7,8\}} = -0.441505,$$
$$\delta_1^{\{8,9\}} = -0.179644,\ \delta_2^{\{1,4\}} = 0.129744,\ \delta_2^{\{2,3\}} = -1.72136,\ \delta_2^{\{2,5\}} = -1.20961,\ \delta_2^{\{4,5\}} = -0.0572092,$$
$$\delta_2^{\{4,7\}} = -0.0365082,\ \delta_2^{\{5,6\}} = 0.130747,\ \delta_2^{\{5,8\}} = -0.0044276,\ \delta_2^{\{7,8\}} = 0.0249143,\ \delta_2^{\{8,9\}} = 0.0385166,$$
$$\delta_3^{\{1,4\}} = 0.102927,\ \delta_3^{\{2,3\}} = 0.541385,\ \delta_3^{\{2,5\}} = 0.384263,\ \delta_3^{\{4,5\}} = 0.0486485,\ \delta_3^{\{4,7\}} = 0.00763707,$$
$$\delta_3^{\{5,6\}} = 0.114595,\ \delta_3^{\{5,8\}} = 0.00731432,\ \delta_3^{\{7,8\}} = 0.0150098,\ \delta_3^{\{8,9\}} = 0.0270124,\ \delta_4^{\{1,4\}} = 0.0341926,$$
$$\delta_4^{\{2,3\}} = -0.0425956,\ \delta_4^{\{2,5\}} = -0.0280251,\ \delta_4^{\{4,5\}} = -0.00156893,\ \delta_4^{\{4,7\}} = -0.00068026,\ \delta_4^{\{5,6\}} = 0.0288099,$$
$$\delta_4^{\{5,8\}} = 0.000376932,\ \delta_4^{\{7,8\}} = 0.0020261,\ \delta_4^{\{8,9\}} = -0.0150241,\ \delta_5^{\{1,4\}} = -0.00475221,\ \delta_5^{\{2,3\}} = 0.00555068,$$
$$\delta_5^{\{2,5\}} = 0.00429799,\ \delta_5^{\{4,5\}} = 0.000528368,\ \delta_5^{\{4,7\}} = -0.0000148321,\ \delta_5^{\{5,6\}} = -0.00312662,$$
$$\delta_5^{\{5,8\}} = 0.0000365732,\ \delta_5^{\{7,8\}} = -0.000575281,\ \delta_5^{\{8,9\}} = 0.000702557,\ \delta_1^1 = 1.1616,\ \delta_2^1 = 0.330829,$$
$$\delta_3^1 = 0.158754,\ \delta_4^1 = -1.01333,\ \delta_5^1 = -0.772856,\ \delta_6^1 = 0.56158,\ \delta_1^2 = 2.4836,\ \delta_2^2 = -1.79584,$$
$$\delta_3^2 = 0.846441,\ \delta_4^2 = -1.61156,\ \delta_5^2 = 1.69435,\ \delta_6^2 = 0.596853,\ \delta_1^3 = -0.0344364,\ \delta_2^3 = -0.0589146,$$
$$\delta_3^3 = 0.0550923,\ \delta_4^3 = -0.00457352,\ \delta_5^3 = 0.0632004,\ \delta_6^3 = 0.0103521,\ \delta_1^4 = 0.00331013,$$
$$\delta_2^4 = 0.0550457,\ \delta_3^4 = 0.130399,\ \delta_4^4 = 0.00610997,\ \delta_5^4 = -0.0485898,\ \delta_6^4 = 0.00704368\}$$

Solution for element 1

DOF values for the element

$\{\phi_1 = 10,\ \phi_4 = 6.18542,\ \phi_5 = 4.69347,\ \phi_2 = 10,\ \delta_1^{\{1,4\}} = 0.500969,\ \delta_1^{\{4,5\}} = -0.578562,\ \delta_1^{\{2,5\}} = 2.27686,$
$\delta_1^{\{1,2\}} = 0,\ \delta_2^{\{1,4\}} = 0.129744,\ \delta_2^{\{4,5\}} = -0.0572092,\ \delta_2^{\{2,5\}} = -1.20961,\ \delta_2^{\{1,2\}} = 0,\ \delta_3^{\{1,4\}} = 0.102927,$
$\delta_3^{\{4,5\}} = 0.0486485,\ \delta_3^{\{2,5\}} = 0.384263,\ \delta_3^{\{1,2\}} = 0,\ \delta_4^{\{1,4\}} = 0.0341926,\ \delta_4^{\{4,5\}} = -0.00156893,$
$\delta_4^{\{2,5\}} = -0.0280251,\ \delta_4^{\{1,2\}} = 0,\ \delta_5^{\{1,4\}} = -0.00475221,\ \delta_5^{\{4,5\}} = 0.000528368,\ \delta_5^{\{2,5\}} = 0.00429799,$
$\delta_5^{\{1,2\}} = 0,\ \delta_1^1 = 1.1616,\ \delta_2^1 = 0.330829,\ \delta_3^1 = 0.158754,\ \delta_4^1 = -1.01333,\ \delta_5^1 = -0.772856,\ \delta_6^1 = 0.56158\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 10\quad 6.18542\quad 4.69347\quad 10\quad 0.500969\quad -0.578562\quad 2.27686\quad 0\quad 0.129744\quad -0.0572092\quad -1.20961\quad 0\quad 0$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.44143	−0.8635	−0.280942

Solution for element 2

DOF values for the element

$\{\phi_2 = 10,\ \phi_5 = 4.69347,\ \phi_6 = 1,\ \phi_3 = 1,\ \delta_1^{\{2,5\}} = 2.27686,\ \delta_1^{\{5,6\}} = -0.225498,\ \delta_1^{\{3,6\}} = 0,$
$\delta_1^{\{2,3\}} = 2.6612,\ \delta_2^{\{2,5\}} = -1.20961,\ \delta_2^{\{5,6\}} = 0.130747,\ \delta_2^{\{3,6\}} = 0,\ \delta_2^{\{2,3\}} = -1.72136,\ \delta_3^{\{2,5\}} = 0.384263,$
$\delta_3^{\{5,6\}} = 0.114595,\ \delta_3^{\{3,6\}} = 0,\ \delta_3^{\{2,3\}} = 0.541385,\ \delta_4^{\{2,5\}} = -0.0280251,\ \delta_4^{\{5,6\}} = 0.0288099,\ \delta_4^{\{3,6\}} = 0,$
$\delta_4^{\{2,3\}} = -0.0425956,\ \delta_5^{\{2,5\}} = 0.00429799,\ \delta_5^{\{5,6\}} = -0.00312662,\ \delta_5^{\{3,6\}} = 0,\ \delta_5^{\{2,3\}} = 0.00555068,$
$\delta_1^2 = 2.4836,\ \delta_2^2 = -1.79584,\ \delta_3^2 = 0.846441,\ \delta_4^2 = -1.61156,\ \delta_5^2 = 1.69435,\ \delta_6^2 = 0.596853\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 10\quad 4.69347\quad 1\quad 1\quad 2.27686\quad -0.225498\quad 0\quad 2.6612\quad -1.20961\quad 0.130747\quad 0\quad -1.72136\quad 0.384263\quad 0.1$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	7.5	3.57625	−0.322093	−1.10679

Solution for element 3

DOF values for the element

$\{\phi_4 = 6.18542,\ \phi_7 = 4.91823,\ \phi_8 = 3.78303,\ \phi_5 = 4.69347,\ \delta_1^{\{4,7\}} = 0.548653,\ \delta_1^{\{7,8\}} = -0.441505,$
$\delta_1^{\{5,8\}} = 0.396936,\ \delta_1^{\{4,5\}} = -0.578562,\ \delta_2^{\{4,7\}} = -0.0365082,\ \delta_2^{\{7,8\}} = 0.0249143,\ \delta_2^{\{5,8\}} = -0.0044276,$
$\delta_2^{\{4,5\}} = -0.0572092,\ \delta_3^{\{4,7\}} = 0.00763707,\ \delta_3^{\{7,8\}} = 0.0150098,\ \delta_3^{\{5,8\}} = 0.00731432,\ \delta_3^{\{4,5\}} = 0.0486485,$
$\delta_4^{\{4,7\}} = -0.00068026,\ \delta_4^{\{7,8\}} = 0.0020261,\ \delta_4^{\{5,8\}} = 0.000376932,\ \delta_4^{\{4,5\}} = -0.00156893,$
$\delta_5^{\{4,7\}} = -0.0000148321,\ \delta_5^{\{7,8\}} = -0.000575281,\ \delta_5^{\{5,8\}} = 0.0000365732,\ \delta_5^{\{4,5\}} = 0.000528368,$
$\delta_1^3 = -0.0344364,\ \delta_2^3 = -0.0589146,\ \delta_3^3 = 0.0550923,\ \delta_4^3 = -0.00457352,\ \delta_5^3 = 0.0632004,\ \delta_6^3 = 0.0103521\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 6.18542\quad 4.91823\quad 3.78303\quad 4.69347\quad 0.548653\quad -0.441505\quad 0.396936\quad -0.578562\quad -0.0365082\quad 0.024$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	2.5	4.90475	−0.23055	−0.25038

Solution for element 4

DOF values for the element

$\{\phi_5 = 4.69347,\ \phi_8 = 3.78303,\ \phi_9 = 1,\ \phi_6 = 1,\ \delta_1^{\{5,8\}} = 0.396936,\ \delta_1^{\{8,9\}} = -0.179644,\ \delta_1^{\{6,9\}} = 0,\ \delta_1^{\{5,6\}} = -0.225498,$
$\delta_2^{\{5,8\}} = -0.0044276,\ \delta_2^{\{8,9\}} = 0.0385166,\ \delta_2^{\{6,9\}} = 0,\ \delta_2^{\{5,6\}} = 0.130747,\ \delta_3^{\{5,8\}} = 0.00731432,$
$\delta_3^{\{8,9\}} = 0.0270124,\ \delta_3^{\{6,9\}} = 0,\ \delta_3^{\{5,6\}} = 0.114595,\ \delta_4^{\{5,8\}} = 0.000376932,\ \delta_4^{\{8,9\}} = -0.0150241,\ \delta_4^{\{6,9\}} = 0,$
$\delta_4^{\{5,6\}} = 0.0288099,\ \delta_5^{\{5,8\}} = 0.0000365732,\ \delta_5^{\{8,9\}} = 0.000702557,\ \delta_5^{\{6,9\}} = 0,\ \delta_5^{\{5,6\}} = -0.00312662,$
$\delta_1^4 = 0.00331013,\ \delta_2^4 = 0.0550457,\ \delta_3^4 = 0.130399,\ \delta_4^4 = 0.00610997,\ \delta_5^4 = -0.0485898,\ \delta_6^4 = 0.00704368\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 4.69347\ \ 3.78303\ \ 1\ \ 1\ \ 0.396936\ \ -0.179644\ \ 0\ \ -0.225498\ \ -0.0044276\ \ 0.0385166\ \ 0\ \ 0.130747\ \ ($

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	7.5	7.5	2.62078	−0.0989251	−0.613286

Nodal solution summary

dof	x	y	Value
ϕ_1	0	0	10
ϕ_2	0	5	10
ϕ_3	0	10	1
ϕ_4	5	0	6.18542
ϕ_5	5	5	4.69347
ϕ_6	5	10	1
ϕ_7	10	0	4.91823
ϕ_8	10	5	3.78303
ϕ_9	10	10	1

Element solution summary

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	2.5	2.5	7.44143	−0.8635	−0.280942
2	2.5	7.5	3.57625	−0.322093	−1.10679
3	7.5	2.5	4.90475	−0.23055	−0.25038
4	7.5	7.5	2.62078	−0.0989251	−0.613286

Example 9.9: Groundwater flow — Free surface problem (p. 633)

The flow of water through porous media gives rise to the so-called free surface problem. A typical situation, illustrated in Figure, considers flow of groundwater towards a well. At a sufficient distance away from the well the groundwater level is equal to the established water table in the area and is unaffected by the well. Closer to the well the groundwater level is lower because of the water flowing into the well. The exact shape and depth below ground of the top of the groundwater surface depends on the coefficient of permeability of the soil and is not known a priori. An iterative procedure is used in the analysis to establish this free surface.

The governing differential equation for the problem is as follows.

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial \phi}{\partial y}\right) = 0$$

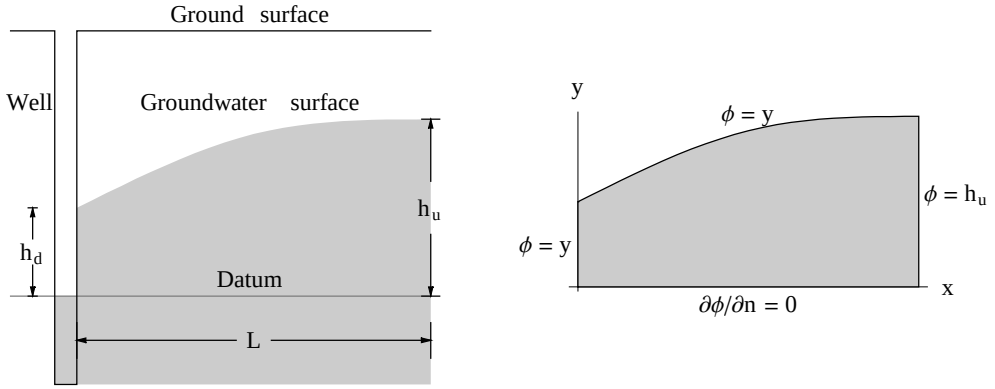
where $\phi(x, y)$ is the hydraulic head (or hydraulic potential) and k_x and k_y are coefficients of permeability in the x and y directions. Typical units for ϕ are meters and those for k_x and k_y are m / day. The fluid velocity components in the x and y directions are related to the hydraulic head as follows.

$$v_x = -k_x \frac{\partial \phi}{\partial x} \text{ and } v_y = -k_y \frac{\partial \phi}{\partial y}$$

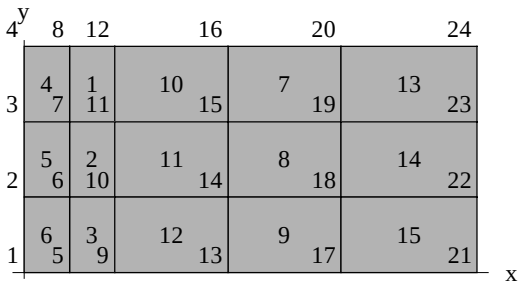
The computational domain consists of the region bounded by the *unknown* top of the groundwater suface, side of the well, line extending from the water line in the well and a vertical line at a sufficient distance away from the well. The boundary conditions on this computational doamin are as illustrated in Figure. Considering the water line in the well as datum for the hydraulic head, ϕ on the side away from the well is equal to the water depth h_u and that on the well side is equal to the elevation of the groundwater surface. On the bottom there is no flow and therefore the boundary condition is $\partial\phi/\partial n = 0$. On the top surface $\phi = y$ the height of the surface above the datum and also $\partial\phi/\partial n = 0$. It is unusual to have two specified boundary conditions for one boundary. However since the boundary itself is not known, there is no inconsistency. One of the conditions is used to establish the boundary while the other is used as a usual boundary condition. In finite elements a zero natural boundary condition does not require any work. Thus computationally we simply have to establish the free surface.

In an iterative procdure we arbitrarily choose the height of the free surface, say equal to the water level h_u. The finite model is now established with zero natural boundary conditions at the top and bottom, essential boundary condition on the far side and essential and natural boundary conditions on the well side. The problem is solved in the usual manner. In general the computed ϕ values on the top suface will not be equal to the assumed height. The top surface is now re-established by setting y equal to the computed ϕ values and the process is repeated until the difference between the computed ϕ values for the nodes on the top surface are equal to the y coordinate for these nodes. The procedure is illustrated using the following numerical data.

$$k_x = k_y = 1\,m/s; \quad h_u = 1\,m; \quad L = 2\,m$$



Initially the groundwater surface is assumed to be horizontal. Thus the computational domain is an $L \times h_u$ rectangle as shown in Figure. It is discretized into 15 elements. Essential boundary condition of $\phi = h_u$ is specified for element 13 side 23-24, element 14 side 22-23 and element 15 side 21-22. For element 5 side 2-3 and element 6 side 1-2 the specified boundary condition is $\phi = y$. It is important not to specify ϕ value at node 4 otherwise its elevation will not change and we will not be able to adjust the groundwater surface.



A linear (n = 1) element is used to solve the problem. The computed nodal ϕ values are as follows.

Equations for element 1

$$\begin{pmatrix} 0.755556 & -0.455556 & -0.377778 & 0.0777778 \\ -0.455556 & 0.755556 & 0.0777778 & -0.377778 \\ -0.377778 & 0.0777778 & 0.755556 & -0.455556 \\ 0.0777778 & -0.377778 & -0.455556 & 0.755556 \end{pmatrix} \begin{pmatrix} \phi_{12} \\ \phi_8 \\ \phi_7 \\ \phi_{11} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 2

$$\begin{pmatrix} 0.755556 & -0.455556 & -0.377778 & 0.0777778 \\ -0.455556 & 0.755556 & 0.0777778 & -0.377778 \\ -0.377778 & 0.0777778 & 0.755556 & -0.455556 \\ 0.0777778 & -0.377778 & -0.455556 & 0.755556 \end{pmatrix} \begin{pmatrix} \phi_{11} \\ \phi_7 \\ \phi_6 \\ \phi_{10} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 3

$$\begin{pmatrix} 0.755556 & -0.455556 & -0.377778 & 0.0777778 \\ -0.455556 & 0.755556 & 0.0777778 & -0.377778 \\ -0.377778 & 0.0777778 & 0.755556 & -0.455556 \\ 0.0777778 & -0.377778 & -0.455556 & 0.755556 \end{pmatrix} \begin{pmatrix} \phi_{10} \\ \phi_6 \\ \phi_5 \\ \phi_9 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 4

$$\begin{pmatrix} 0.755556 & -0.455556 & -0.377778 & 0.0777778 \\ -0.455556 & 0.755556 & 0.0777778 & -0.377778 \\ -0.377778 & 0.0777778 & 0.755556 & -0.455556 \\ 0.0777778 & -0.377778 & -0.455556 & 0.755556 \end{pmatrix} \begin{pmatrix} \phi_8 \\ \phi_4 \\ \phi_3 \\ \phi_7 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 5

$$\begin{pmatrix} 0.755556 & -0.455556 & -0.377778 & 0.0777778 \\ -0.455556 & 0.755556 & 0.0777778 & -0.377778 \\ -0.377778 & 0.0777778 & 0.755556 & -0.455556 \\ 0.0777778 & -0.377778 & -0.455556 & 0.755556 \end{pmatrix} \begin{pmatrix} \phi_7 \\ \phi_3 \\ \phi_2 \\ \phi_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 6

$$\begin{pmatrix} 0.755556 & -0.455556 & -0.377778 & 0.0777778 \\ -0.455556 & 0.755556 & 0.0777778 & -0.377778 \\ -0.377778 & 0.0777778 & 0.755556 & -0.455556 \\ 0.0777778 & -0.377778 & -0.455556 & 0.755556 \end{pmatrix} \begin{pmatrix} \phi_6 \\ \phi_2 \\ \phi_1 \\ \phi_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 7

$$\begin{pmatrix} 0.722222 & 0.0277778 & -0.361111 & -0.388889 \\ 0.0277778 & 0.722222 & -0.388889 & -0.361111 \\ -0.361111 & -0.388889 & 0.722222 & 0.0277778 \\ -0.388889 & -0.361111 & 0.0277778 & 0.722222 \end{pmatrix} \begin{pmatrix} \phi_{20} \\ \phi_{16} \\ \phi_{15} \\ \phi_{19} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 8

$$\begin{pmatrix} 0.722222 & 0.0277778 & -0.361111 & -0.388889 \\ 0.0277778 & 0.722222 & -0.388889 & -0.361111 \\ -0.361111 & -0.388889 & 0.722222 & 0.0277778 \\ -0.388889 & -0.361111 & 0.0277778 & 0.722222 \end{pmatrix} \begin{pmatrix} \phi_{19} \\ \phi_{15} \\ \phi_{14} \\ \phi_{18} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 9

$$\begin{pmatrix} 0.722222 & 0.0277778 & -0.361111 & -0.388889 \\ 0.0277778 & 0.722222 & -0.388889 & -0.361111 \\ -0.361111 & -0.388889 & 0.722222 & 0.0277778 \\ -0.388889 & -0.361111 & 0.0277778 & 0.722222 \end{pmatrix} \begin{pmatrix} \phi_{18} \\ \phi_{14} \\ \phi_{13} \\ \phi_{17} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 10

$$\begin{pmatrix} 0.722222 & 0.0277778 & -0.361111 & -0.388889 \\ 0.0277778 & 0.722222 & -0.388889 & -0.361111 \\ -0.361111 & -0.388889 & 0.722222 & 0.0277778 \\ -0.388889 & -0.361111 & 0.0277778 & 0.722222 \end{pmatrix} \begin{pmatrix} \phi_{16} \\ \phi_{12} \\ \phi_{11} \\ \phi_{15} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 11

$$\begin{pmatrix} 0.722222 & 0.0277778 & -0.361111 & -0.388889 \\ 0.0277778 & 0.722222 & -0.388889 & -0.361111 \\ -0.361111 & -0.388889 & 0.722222 & 0.0277778 \\ -0.388889 & -0.361111 & 0.0277778 & 0.722222 \end{pmatrix} \begin{pmatrix} \phi_{15} \\ \phi_{11} \\ \phi_{10} \\ \phi_{14} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 12

$$\begin{pmatrix} 0.722222 & 0.0277778 & -0.361111 & -0.388889 \\ 0.0277778 & 0.722222 & -0.388889 & -0.361111 \\ -0.361111 & -0.388889 & 0.722222 & 0.0277778 \\ -0.388889 & -0.361111 & 0.0277778 & 0.722222 \end{pmatrix} \begin{pmatrix} \phi_{14} \\ \phi_{10} \\ \phi_{9} \\ \phi_{13} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 13

$$\begin{pmatrix} 0.785185 & 0.114815 & -0.392593 & -0.507407 \\ 0.114815 & 0.785185 & -0.507407 & -0.392593 \\ -0.392593 & -0.507407 & 0.785185 & 0.114815 \\ -0.507407 & -0.392593 & 0.114815 & 0.785185 \end{pmatrix} \begin{pmatrix} \phi_{24} \\ \phi_{20} \\ \phi_{19} \\ \phi_{23} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 14

$$\begin{pmatrix} 0.785185 & 0.114815 & -0.392593 & -0.507407 \\ 0.114815 & 0.785185 & -0.507407 & -0.392593 \\ -0.392593 & -0.507407 & 0.785185 & 0.114815 \\ -0.507407 & -0.392593 & 0.114815 & 0.785185 \end{pmatrix} \begin{pmatrix} \phi_{23} \\ \phi_{19} \\ \phi_{18} \\ \phi_{22} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 15

$$\begin{pmatrix} 0.785185 & 0.114815 & -0.392593 & -0.507407 \\ 0.114815 & 0.785185 & -0.507407 & -0.392593 \\ -0.392593 & -0.507407 & 0.785185 & 0.114815 \\ -0.507407 & -0.392593 & 0.114815 & 0.785185 \end{pmatrix} \begin{pmatrix} \phi_{22} \\ \phi_{18} \\ \phi_{17} \\ \phi_{21} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Global equations after assembling all elements

$$\begin{pmatrix}
0.755556 & 0.0777778 & 0 & 0 & -0.455556 & -0.377778 & 0 & 0 & 0 \\
0.0777778 & 1.51111 & 0.0777778 & 0 & -0.377778 & -0.911111 & -0.377778 & 0 & 0 \\
0 & 0.0777778 & 1.51111 & 0.0777778 & 0 & -0.377778 & -0.911111 & -0.377778 & 0 \\
0 & 0 & 0.0777778 & 0.755556 & 0 & 0 & -0.377778 & -0.455556 & 0 \\
-0.455556 & -0.377778 & 0 & 0 & 1.51111 & 0.155556 & 0 & 0 & -0.4 \\
-0.377778 & -0.911111 & -0.377778 & 0 & 0.155556 & 3.02222 & 0.155556 & 0 & -0.3 \\
0 & -0.377778 & -0.911111 & -0.377778 & 0 & 0.155556 & 3.02222 & 0.155556 & 0 \\
0 & 0 & -0.377778 & -0.455556 & 0 & 0 & 0.155556 & 1.51111 & 0 \\
0 & 0 & 0 & 0 & -0.455556 & -0.377778 & 0 & 0 & 1.4 \\
0 & 0 & 0 & 0 & -0.377778 & -0.911111 & -0.377778 & 0 & -0.3 \\
0 & 0 & 0 & 0 & 0 & -0.377778 & -0.911111 & -0.377778 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -0.377778 & -0.455556 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Essential boundary conditions

On element 13, side 4, specified value = 1

$$\{\phi_{23}, \phi_{24}\} = \{1, 1\}$$

On element 14, side 4, specified value = 1

$$\{\phi_{22}, \phi_{23}\} = \{1, 1\}$$

On element 15, side 4, specified value = 1

$$\{\phi_{21}, \phi_{22}\} = \{1, 1\}$$

On element 5, side 2, specified value = y

$$\{\phi_3, \phi_2\} = \left\{\frac{2}{3}, \frac{1}{3}\right\}$$

On element 6, side 2, specified value = y

$$\{\phi_2, \phi_1\} = \left\{\frac{1}{3}, 0\right\}$$

Known values from EBC

$$\left\{\phi_1 = 0, \phi_2 = \frac{1}{3}, \phi_3 = \frac{2}{3}, \phi_{21} = 1, \phi_{22} = 1, \phi_{23} = 1, \phi_{24} = 1\right\}$$

Global equations after EBC

$$\begin{pmatrix}
0.755556 & 0 & 0 & -0.377778 & -0.455556 & 0 & 0 & 0 & 0 \\
0 & 1.51111 & 0.155556 & 0 & 0 & -0.455556 & -0.377778 & 0 & 0 \\
0 & 0.155556 & 3.02222 & 0.155556 & 0 & -0.377778 & -0.911111 & -0.377778 & 0 \\
-0.377778 & 0 & 0.155556 & 3.02222 & 0.155556 & 0 & -0.377778 & -0.911111 & -0.377 \\
-0.455556 & 0 & 0 & 0.155556 & 1.51111 & 0 & 0 & -0.377778 & -0.455 \\
0 & -0.455556 & -0.377778 & 0 & 0 & 1.47778 & -0.311111 & 0 & 0 \\
0 & -0.377778 & -0.911111 & -0.377778 & 0 & -0.311111 & 2.95556 & -0.311111 & 0 \\
0 & 0 & -0.377778 & -0.911111 & -0.377778 & 0 & -0.311111 & 2.95556 & -0.31 \\
0 & 0 & 0 & -0.377778 & -0.455556 & 0 & 0 & -0.311111 & 1.47 \\
0 & 0 & 0 & 0 & 0 & 0.0277778 & -0.361111 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.361111 & 0.0555556 & -0.361111 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -0.361111 & 0.0555556 & -0.36 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.361111 & 0.02 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Solving the final system of global equations we get

$$\{\phi_4 = 0.596157, \phi_5 = 0.306957, \phi_6 = 0.423017, \phi_7 = 0.581671, \phi_8 = 0.620208, \phi_9 = 0.463903,$$
$$\phi_{10} = 0.509264, \phi_{11} = 0.587326, \phi_{12} = 0.619842, \phi_{13} = 0.676938, \phi_{14} = 0.681975, \phi_{15} = 0.693494,$$
$$\phi_{16} = 0.700009, \phi_{17} = 0.828093, \phi_{18} = 0.829002, \phi_{19} = 0.830666, \phi_{20} = 0.831418\}$$

Solution for element 1

DOF values for the element

$$\{\phi_{12} = 0.619842, \phi_8 = 0.620208, \phi_7 = 0.581671, \phi_{11} = 0.587326\}$$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.619842 \quad 0.620208 \quad 0.581671 \quad 0.587326\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.3	0.833333	0.602262	0.0132221	0.10658

Solution for element 2

DOF values for the element

$\{\phi_{11} = 0.587326,\ \phi_7 = 0.581671,\ \phi_6 = 0.423017,\ \phi_{10} = 0.509264\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.587326 \quad 0.581671 \quad 0.423017 \quad 0.509264\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.3	0.5	0.52532	0.229755	0.355073

Solution for element 3

DOF values for the element

$\{\phi_{10} = 0.509264,\ \phi_6 = 0.423017,\ \phi_5 = 0.306957,\ \phi_9 = 0.463903\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.509264 \quad 0.423017 \quad 0.306957 \quad 0.463903\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.3	0.166667	0.425785	0.607983	0.242133

Solution for element 4

DOF values for the element

$\left\{\phi_8 = 0.620208,\ \phi_4 = 0.596157,\ \phi_3 = \frac{2}{3},\ \phi_7 = 0.581671\right\}$

$\boldsymbol{d}^{\mathrm{T}} = \left(\ 0.620208 \quad 0.596157 \quad \frac{2}{3} \quad 0.581671\ \right)$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.1	0.833333	0.616176	−0.152361	−0.0479582

Solution for element 5

DOF values for the element

$\left\{\phi_7 = 0.581671,\ \phi_3 = \frac{2}{3},\ \phi_2 = \frac{1}{3},\ \phi_6 = 0.423017\right\}$

$\boldsymbol{d}^{\mathrm{T}} = \left(\ 0.581671 \quad \frac{2}{3} \quad \frac{1}{3} \quad 0.423017\ \right)$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.1	0.5	0.501172	0.0117208	0.737981

Solution for element 6

DOF values for the element

$\left\{\phi_6 = 0.423017,\ \phi_2 = \frac{1}{3},\ \phi_1 = 0,\ \phi_5 = 0.306957\right\}$

$\boldsymbol{d}^{\mathrm{T}} = \left(\begin{array}{cccc} 0.423017 & \frac{1}{3} & 0 & 0.306957 \end{array}\right)$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.1	0.166667	0.265827	0.991601	0.674091

Solution for element 7

DOF values for the element

$\{\phi_{20} = 0.831418,\ \phi_{16} = 0.700009,\ \phi_{15} = 0.693494,\ \phi_{19} = 0.830666\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.831418 \quad 0.700009 \quad 0.693494 \quad 0.830666\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	1.15	0.833333	0.763897	0.26858	0.0108989

Solution for element 8

DOF values for the element

$\{\phi_{19} = 0.830666,\ \phi_{15} = 0.693494,\ \phi_{14} = 0.681975,\ \phi_{18} = 0.829002\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.830666 \quad 0.693494 \quad 0.681975 \quad 0.829002\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	1.15	0.5	0.758784	0.284199	0.019776

Solution for element 9

DOF values for the element

$\{\phi_{18} = 0.829002,\ \phi_{14} = 0.681975,\ \phi_{13} = 0.676938,\ \phi_{17} = 0.828093\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.829002 \quad 0.681975 \quad 0.676938 \quad 0.828093\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	1.15	0.166667	0.754002	0.298182	0.00891866

Solution for element 10

DOF values for the element

$\{\phi_{16} = 0.700009,\ \phi_{12} = 0.619842,\ \phi_{11} = 0.587326,\ \phi_{15} = 0.693494\}$

$\boldsymbol{d}^{\mathrm{T}}$ = (0.700009 0.619842 0.587326 0.693494)

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.65	0.833333	0.650168	0.186335	0.058547

Solution for element 11

DOF values for the element

$\{\phi_{15} = 0.693494, \phi_{11} = 0.587326, \phi_{10} = 0.509264, \phi_{14} = 0.681975\}$

$\boldsymbol{d}^{\mathrm{T}}$ = (0.693494 0.587326 0.509264 0.681975)

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.65	0.5	0.618015	0.278879	0.134371

Solution for element 12

DOF values for the element

$\{\phi_{14} = 0.681975, \phi_{10} = 0.509264, \phi_{9} = 0.463903, \phi_{13} = 0.676938\}$

$\boldsymbol{d}^{\mathrm{T}}$ = (0.681975 0.509264 0.463903 0.676938)

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.65	0.166667	0.58302	0.385746	0.0755986

Solution for element 13

DOF values for the element

$\{\phi_{24} = 1, \phi_{20} = 0.831418, \phi_{19} = 0.830666, \phi_{23} = 1\}$

$\boldsymbol{d}^{\mathrm{T}}$ = (1 0.831418 0.830666 1)

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	1.7	0.833333	0.915521	0.281597	0.00112653

Solution for element 14

DOF values for the element

$\{\phi_{23} = 1, \phi_{19} = 0.830666, \phi_{18} = 0.829002, \phi_{22} = 1\}$

$\boldsymbol{d}^{\mathrm{T}}$ = (1 0.830666 0.829002 1)

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	1.7	0.5	0.914917	0.28361	0.0024972

Solution for element 15

DOF values for the element

$\{\phi_{22} = 1, \phi_{18} = 0.829002, \phi_{17} = 0.828093, \phi_{21} = 1\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 1 \quad 0.829002 \quad 0.828093 \quad 1\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	1.7	0.166667	0.914274	0.285754	0.00136265

Nodal solution summary

dof	x	y	Value
ϕ_1	0	0	0
ϕ_2	0	$\frac{1}{3}$	$\frac{1}{3}$
ϕ_3	0	$\frac{2}{3}$	$\frac{2}{3}$
ϕ_4	0	1	0.596157
ϕ_5	0.2	0	0.306957
ϕ_6	0.2	$\frac{1}{3}$	0.423017
ϕ_7	0.2	$\frac{2}{3}$	0.581671
ϕ_8	0.2	1	0.620208
ϕ_9	0.4	0	0.463903
ϕ_{10}	0.4	$\frac{1}{3}$	0.509264
ϕ_{11}	0.4	$\frac{2}{3}$	0.587326
ϕ_{12}	0.4	1	0.619842
ϕ_{13}	0.9	0	0.676938
ϕ_{14}	0.9	$\frac{1}{3}$	0.681975
ϕ_{15}	0.9	$\frac{2}{3}$	0.693494
ϕ_{16}	0.9	1	0.700009
ϕ_{17}	1.4	0	0.828093
ϕ_{18}	1.4	$\frac{1}{3}$	0.829002
ϕ_{19}	1.4	$\frac{2}{3}$	0.830666
ϕ_{20}	1.4	1	0.831418
ϕ_{21}	2	0	1
ϕ_{22}	2	$\frac{1}{3}$	1
ϕ_{23}	2	$\frac{2}{3}$	1
ϕ_{24}	2	1	1

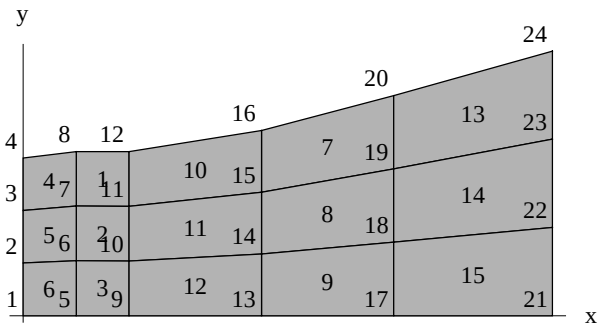
Element solution summary

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.3	0.833333	0.602262	0.0132221	0.10658
2	0.3	0.5	0.52532	0.229755	0.355073
3	0.3	0.166667	0.425785	0.607983	0.242133
4	0.1	0.833333	0.616176	−0.152361	−0.0479582
5	0.1	0.5	0.501172	0.0117208	0.737981
6	0.1	0.166667	0.265827	0.991601	0.674091
7	1.15	0.833333	0.763897	0.26858	0.0108989
8	1.15	0.5	0.758784	0.284199	0.019776
9	1.15	0.166667	0.754002	0.298182	0.00891866
10	0.65	0.833333	0.650168	0.186335	0.058547
11	0.65	0.5	0.618015	0.278879	0.134371
12	0.65	0.166667	0.58302	0.385746	0.0755986
13	1.7	0.833333	0.915521	0.281597	0.00112653
14	1.7	0.5	0.914917	0.28361	0.0024972
15	1.7	0.166667	0.914274	0.285754	0.00136265

For the next iteration, the y coordinates of the nodes on the top surface are set equal to the computed ϕ values at nodes 4, 8, 12, 16, 20, and 24.

New y coordinates of top surface = {0.596157, 0.620208, 0.619842, 0.700009, 0.831418, 1}

To avoid badly shaped elements a completely new finite element mesh, shown in Figure, is created.



Using this new finite element model we get the following nodal solution.

Equations for element 1

$$\begin{pmatrix} 0.667888 & -0.183174 & -0.334276 & -0.150437 \\ -0.183174 & 0.666166 & -0.15024 & -0.332751 \\ -0.334276 & -0.15024 & 0.667691 & -0.183174 \\ -0.150437 & -0.332751 & -0.183174 & 0.666363 \end{pmatrix} \begin{pmatrix} \phi_{12} \\ \phi_8 \\ \phi_7 \\ \phi_{11} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 2

$$\begin{pmatrix} 0.667582 & -0.183175 & -0.333971 & -0.150437 \\ -0.183175 & 0.66647 & -0.15024 & -0.333056 \\ -0.333971 & -0.15024 & 0.667385 & -0.183175 \\ -0.150437 & -0.333056 & -0.183175 & 0.666667 \end{pmatrix} \begin{pmatrix} \phi_{11} \\ \phi_7 \\ \phi_6 \\ \phi_{10} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 3

$$\begin{pmatrix} 0.667277 & -0.183175 & -0.333666 & -0.150437 \\ -0.183175 & 0.666775 & -0.15024 & -0.333361 \\ -0.333666 & -0.15024 & 0.66708 & -0.183175 \\ -0.150437 & -0.333361 & -0.183175 & 0.666972 \end{pmatrix} \begin{pmatrix} \phi_{10} \\ \phi_6 \\ \phi_5 \\ \phi_9 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 4

$$\begin{pmatrix} 0.613386 & -0.17176 & -0.284952 & -0.156674 \\ -0.17176 & 0.726849 & -0.169924 & -0.385165 \\ -0.284952 & -0.169924 & 0.626636 & -0.17176 \\ -0.156674 & -0.385165 & -0.17176 & 0.713599 \end{pmatrix} \begin{pmatrix} \phi_8 \\ \phi_4 \\ \phi_3 \\ \phi_7 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 5

$$\begin{pmatrix} 0.631336 & -0.172817 & -0.303938 & -0.154581 \\ -0.172817 & 0.704672 & -0.167789 & -0.364066 \\ -0.303938 & -0.167789 & 0.644544 & -0.172817 \\ -0.154581 & -0.364066 & -0.172817 & 0.691463 \end{pmatrix} \begin{pmatrix} \phi_7 \\ \phi_3 \\ \phi_2 \\ \phi_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 6

$$\begin{pmatrix} 0.650332 & -0.173346 & -0.323452 & -0.153534 \\ -0.173346 & 0.683562 & -0.166722 & -0.343495 \\ -0.323452 & -0.166722 & 0.663519 & -0.173346 \\ -0.153534 & -0.343495 & -0.173346 & 0.670374 \end{pmatrix} \begin{pmatrix} \phi_6 \\ \phi_2 \\ \phi_1 \\ \phi_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 7

$$\begin{pmatrix} 0.710312 & 0.173046 & -0.318777 & -0.56458 \\ 0.173046 & 1.00283 & -0.638081 & -0.537791 \\ -0.318777 & -0.638081 & 0.783812 & 0.173046 \\ -0.56458 & -0.537791 & 0.173046 & 0.929325 \end{pmatrix} \begin{pmatrix} \phi_{20} \\ \phi_{16} \\ \phi_{15} \\ \phi_{19} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 8

$$\begin{pmatrix} 0.734881 & 0.162999 & -0.352532 & -0.545347 \\ 0.162999 & 0.938065 & -0.617123 & -0.483941 \\ -0.352532 & -0.617123 & 0.806657 & 0.162999 \\ -0.545347 & -0.483941 & 0.162999 & 0.866289 \end{pmatrix} \begin{pmatrix} \phi_{19} \\ \phi_{15} \\ \phi_{14} \\ \phi_{18} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 9

$$\begin{pmatrix} 0.769067 & 0.157975 & -0.391311 & -0.535731 \\ 0.157975 & 0.883784 & -0.606645 & -0.435114 \\ -0.391311 & -0.606645 & 0.839981 & 0.157975 \\ -0.535731 & -0.435114 & 0.157975 & 0.81287 \end{pmatrix} \begin{pmatrix} \phi_{18} \\ \phi_{14} \\ \phi_{13} \\ \phi_{17} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 10

$$\begin{pmatrix} 0.824252 & 0.239508 & -0.392678 & -0.671083 \\ 0.239508 & 1.01368 & -0.7269 & -0.526289 \\ -0.392678 & -0.7269 & 0.88007 & 0.239508 \\ -0.671083 & -0.526289 & 0.239508 & 0.957864 \end{pmatrix} \begin{pmatrix} \phi_{16} \\ \phi_{12} \\ \phi_{11} \\ \phi_{15} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 11

$$\begin{pmatrix} 0.842571 & 0.235175 & -0.415066 & -0.662679 \\ 0.235175 & 0.978029 & -0.71797 & -0.495233 \\ -0.415066 & -0.71797 & 0.897862 & 0.235175 \\ -0.662679 & -0.495233 & 0.235175 & 0.922738 \end{pmatrix} \begin{pmatrix} \phi_{15} \\ \phi_{11} \\ \phi_{10} \\ \phi_{14} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 12

$$\begin{pmatrix} 0.865091 & 0.233008 & -0.439622 & -0.658477 \\ 0.233008 & 0.946841 & -0.713505 & -0.466344 \\ -0.439622 & -0.713505 & 0.920119 & 0.233008 \\ -0.658477 & -0.466344 & 0.233008 & 0.891813 \end{pmatrix} \begin{pmatrix} \phi_{14} \\ \phi_{10} \\ \phi_{9} \\ \phi_{13} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 13

$$\begin{pmatrix} 0.706367 & 0.177219 & -0.314511 & -0.569074 \\ 0.177219 & 1.01996 & -0.648529 & -0.548654 \\ -0.314511 & -0.648529 & 0.785821 & 0.177219 \\ -0.569074 & -0.548654 & 0.177219 & 0.940509 \end{pmatrix} \begin{pmatrix} \phi_{24} \\ \phi_{20} \\ \phi_{19} \\ \phi_{23} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 14

$$\begin{pmatrix} 0.731202 & 0.165692 & -0.349813 & -0.547081 \\ 0.165692 & 0.949019 & -0.624413 & -0.490298 \\ -0.349813 & -0.624413 & 0.808534 & 0.165692 \\ -0.547081 & -0.490298 & 0.165692 & 0.871687 \end{pmatrix} \begin{pmatrix} \phi_{23} \\ \phi_{19} \\ \phi_{18} \\ \phi_{22} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 15

$$\begin{pmatrix} 0.767033 & 0.159928 & -0.390877 & -0.536084 \\ 0.159928 & 0.890133 & -0.612355 & -0.437706 \\ -0.390877 & -0.612355 & 0.843304 & 0.159928 \\ -0.536084 & -0.437706 & 0.159928 & 0.813862 \end{pmatrix} \begin{pmatrix} \phi_{22} \\ \phi_{18} \\ \phi_{17} \\ \phi_{21} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Global equations after assembling all elements

$$\left(\begin{array}{lllllllll}
0.663519 & -0.166722 & 0 & 0 & -0.173346 & -0.323452 & 0 & 0 & 0 \\
-0.166722 & 1.32811 & -0.167789 & 0 & -0.343495 & -0.346163 & -0.303938 & 0 & 0 \\
0 & -0.167789 & 1.33131 & -0.169924 & 0 & -0.364066 & -0.344577 & -0.284952 & 0 \\
0 & 0 & -0.169924 & 0.726849 & 0 & 0 & -0.385165 & -0.17176 & 0 \\
-0.173346 & -0.343495 & 0 & 0 & 1.33745 & -0.303774 & 0 & 0 & -0.183175 \\
-0.323452 & -0.346163 & -0.364066 & 0 & -0.303774 & 2.67596 & -0.304821 & 0 & -0.333361 \\
0 & -0.303938 & -0.344577 & -0.385165 & 0 & -0.304821 & 2.6791 & -0.306914 & 0 \\
0 & 0 & -0.284952 & -0.17176 & 0 & 0 & -0.306914 & 1.27955 & 0 \\
0 & 0 & 0 & 0 & -0.183175 & -0.333361 & 0 & 0 & 1.58709 \\
0 & 0 & 0 & 0 & -0.333666 & -0.36635 & -0.333056 & 0 & -0.863941 \\
0 & 0 & 0 & 0 & 0 & -0.333971 & -0.366349 & -0.332751 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -0.334276 & -0.183174 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.233008 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.439622 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right.$$

Essential boundary conditions

On element 13, side 4, specified value = 1

$\{\phi_{23}, \phi_{24}\} = \{1, 1\}$

On element 14, side 4, specified value = 1

$\{\phi_{22}, \phi_{23}\} = \{1, 1\}$

On element 15, side 4, specified value = 1

$\{\phi_{21}, \phi_{22}\} = \{1, 1\}$

On element 5, side 2, specified value = y

$\{\phi_3, \phi_2\} = \{0.397438, 0.198719\}$

On element 6, side 2, specified value = y

$\{\phi_2, \phi_1\} = \{0.198719, 0.\}$

Known values from EBC

$\{\phi_1 = 0., \phi_2 = 0.198719, \phi_3 = 0.397438, \phi_{21} = 1, \phi_{22} = 1, \phi_{23} = 1, \phi_{24} = 1\}$

Global equations after EBC

$$\begin{pmatrix}
0.726849 & 0 & 0 & -0.385165 & -0.17176 & 0 & 0 & 0 & 0 \\
0 & 1.33745 & -0.303774 & 0 & 0 & -0.183175 & -0.333666 & 0 & 0 \\
0 & -0.303774 & 2.67596 & -0.304821 & 0 & -0.333361 & -0.36635 & -0.333971 & 0 \\
-0.385165 & 0 & -0.304821 & 2.6791 & -0.306914 & 0 & -0.333056 & -0.366349 & -0.334276 \\
-0.17176 & 0 & 0 & -0.306914 & 1.27955 & 0 & 0 & -0.332751 & -0.183174 \\
0 & -0.183175 & -0.333361 & 0 & 0 & 1.58709 & -0.863941 & 0 & 0 \\
0 & -0.333666 & -0.36635 & -0.333056 & 0 & -0.863941 & 3.17865 & -0.868407 & 0 \\
0 & 0 & -0.333971 & -0.366349 & -0.332751 & 0 & -0.868407 & 3.19204 & -0.877338 \\
0 & 0 & 0 & -0.334276 & -0.183174 & 0 & 0 & -0.877338 & 1.68157 \\
0 & 0 & 0 & 0 & 0 & 0.233008 & -0.466344 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.439622 & 0.468182 & -0.495233 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -0.415066 & 0.474683 & -0.526289 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.392678 & 0.239508 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Solving the final system of global equations we get

$\{\phi_4 = 0.398856, \phi_5 = 0.289648, \phi_6 = 0.32478, \phi_7 = 0.389915, \phi_8 = 0.420311, \phi_9 = 0.419679,$
$\phi_{10} = 0.430365, \phi_{11} = 0.452748, \phi_{12} = 0.46802, \phi_{13} = 0.637184, \phi_{14} = 0.640101, \phi_{15} = 0.648756,$
$\phi_{16} = 0.663723, \phi_{17} = 0.812016, \phi_{18} = 0.815298, \phi_{19} = 0.825998, \phi_{20} = 0.846926\}$

Solution for element 1

DOF values for the element

$\{\phi_{12} = 0.46802, \phi_8 = 0.420311, \phi_7 = 0.389915, \phi_{11} = 0.452748\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.46802 \quad 0.420311 \quad 0.389915 \quad 0.452748\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.3	0.516688	0.432748	0.276523	0.110486

Solution for element 2

DOF values for the element

$\{\phi_{11} = 0.452748,\ \phi_7 = 0.389915,\ \phi_6 = 0.32478,\ \phi_{10} = 0.430365\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.452748 \quad 0.389915 \quad 0.32478 \quad 0.430365\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.3	0.310013	0.399452	0.421239	0.211725

Solution for element 3

DOF values for the element

$\{\phi_{10} = 0.430365,\ \phi_6 = 0.32478,\ \phi_5 = 0.289648,\ \phi_9 = 0.419679\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.430365 \quad 0.32478 \quad 0.289648 \quad 0.419679\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.3	0.103338	0.366118	0.589074	0.110846

Solution for element 4

DOF values for the element

$\{\phi_8 = 0.420311,\ \phi_4 = 0.398856,\ \phi_3 = 0.397438,\ \phi_7 = 0.389915\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.420311 \quad 0.398856 \quad 0.397438 \quad 0.389915\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.1	0.506819	0.40163	0.0269648	0.0784678

Solution for element 5

DOF values for the element

$\{\phi_7 = 0.389915,\ \phi_3 = 0.397438,\ \phi_2 = 0.198719,\ \phi_6 = 0.32478\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.389915 \quad 0.397438 \quad 0.198719 \quad 0.32478\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.1	0.304091	0.327713	0.257215	0.650759

Solution for element 6

DOF values for the element

$\{\phi_6 = 0.32478,\ \phi_2 = 0.198719,\ \phi_1 = 0.,\ \phi_5 = 0.289648\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.32478 \quad 0.198719 \quad 0. \quad 0.289648\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.1	0.101364	0.203287	1.02771	0.576762

Solution for element 7

DOF values for the element

$\{\phi_{20} = 0.846926, \phi_{16} = 0.663723, \phi_{15} = 0.648756, \phi_{19} = 0.825998\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.846926 \quad 0.663723 \quad 0.648756 \quad 0.825998\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	1.15	0.638095	0.746351	0.345044	0.0703166

Solution for element 8

DOF values for the element

$\{\phi_{19} = 0.825998, \phi_{15} = 0.648756, \phi_{14} = 0.640101, \phi_{18} = 0.815298\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.825998 \quad 0.648756 \quad 0.640101 \quad 0.815298\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	1.15	0.382857	0.732538	0.347457	0.0379155

Solution for element 9

DOF values for the element

$\{\phi_{18} = 0.815298, \phi_{14} = 0.640101, \phi_{13} = 0.637184, \phi_{17} = 0.812016\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.815298 \quad 0.640101 \quad 0.637184 \quad 0.812016\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	1.15	0.127619	0.72615	0.349497	0.0121435

Solution for element 10

DOF values for the element

$\{\phi_{16} = 0.663723, \phi_{12} = 0.46802, \phi_{11} = 0.452748, \phi_{15} = 0.648756\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.663723 \quad 0.46802 \quad 0.452748 \quad 0.648756\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.65	0.549938	0.558312	0.382528	0.068734

Solution for element 11

DOF values for the element

$\{\phi_{15} = 0.648756,\ \phi_{11} = 0.452748,\ \phi_{10} = 0.430365,\ \phi_{14} = 0.640101\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.648756 \quad 0.452748 \quad 0.430365 \quad 0.640101\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.65	0.329963	0.542992	0.400088	0.0705481

Solution for element 12

DOF values for the element

$\{\phi_{14} = 0.640101,\ \phi_{10} = 0.430365,\ \phi_{9} = 0.419679,\ \phi_{13} = 0.637184\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.640101 \quad 0.430365 \quad 0.419679 \quad 0.637184\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.65	0.109988	0.531832	0.426415	0.0309182

Solution for element 13

DOF values for the element

$\{\phi_{24} = 1,\ \phi_{20} = 0.846926,\ \phi_{19} = 0.825998,\ \phi_{23} = 1\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 1 \quad 0.846926 \quad 0.825998 \quad 1\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	1.7	0.763091	0.918231	0.264537	0.0342814

Solution for element 14

DOF values for the element

$\{\phi_{23} = 1,\ \phi_{19} = 0.825998,\ \phi_{18} = 0.815298,\ \phi_{22} = 1\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 1 \quad 0.825998 \quad 0.815298 \quad 1\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	1.7	0.457854	0.910324	0.296458	0.0175266

Solution for element 15

DOF values for the element

$\{\phi_{22} = 1,\ \phi_{18} = 0.815298,\ \phi_{17} = 0.812016,\ \phi_{21} = 1\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 1 \quad 0.815298 \quad 0.812016 \quad 1\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	1.7	0.152618	0.906828	0.31032	0.00537721

Nodal solution summary

dof	x	y	Value
ϕ_1	0	0	0.
ϕ_2	0	0.198719	0.198719
ϕ_3	0	0.397438	0.397438
ϕ_4	0	0.596157	0.398856
ϕ_5	0.2	0	0.289648
ϕ_6	0.2	0.206736	0.32478
ϕ_7	0.2	0.413472	0.389915
ϕ_8	0.2	0.620208	0.420311
ϕ_9	0.4	0	0.419679
ϕ_{10}	0.4	0.206614	0.430365
ϕ_{11}	0.4	0.413228	0.452748
ϕ_{12}	0.4	0.619842	0.46802
ϕ_{13}	0.9	0	0.637184
ϕ_{14}	0.9	0.233336	0.640101
ϕ_{15}	0.9	0.466673	0.648756
ϕ_{16}	0.9	0.700009	0.663723
ϕ_{17}	1.4	0	0.812016
ϕ_{18}	1.4	0.277139	0.815298
ϕ_{19}	1.4	0.554278	0.825998
ϕ_{20}	1.4	0.831418	0.846926
ϕ_{21}	2	0	1
ϕ_{22}	2	$\frac{1}{3}$	1
ϕ_{23}	2	$\frac{2}{3}$	1
ϕ_{24}	2	1	1

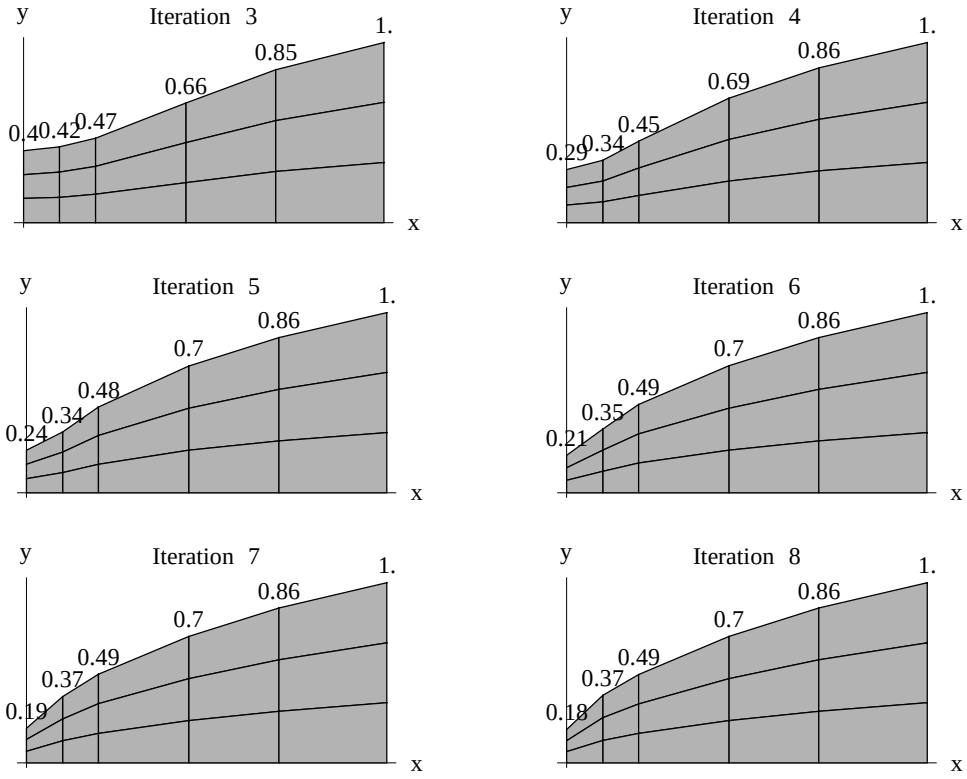
Element solution summary

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.3	0.516688	0.432748	0.276523	0.110486
2	0.3	0.310013	0.399452	0.421239	0.211725
3	0.3	0.103338	0.366118	0.589074	0.110846
4	0.1	0.506819	0.40163	0.0269648	0.0784678
5	0.1	0.304091	0.327713	0.257215	0.650759
6	0.1	0.101364	0.203287	1.02771	0.576762
7	1.15	0.638095	0.746351	0.345044	0.0703166
8	1.15	0.382857	0.732538	0.347457	0.0379155
9	1.15	0.127619	0.72615	0.349497	0.0121435
10	0.65	0.549938	0.558312	0.382528	0.068734
11	0.65	0.329963	0.542992	0.400088	0.0705481
12	0.65	0.109988	0.531832	0.426415	0.0309182
13	1.7	0.763091	0.918231	0.264537	0.0342814
14	1.7	0.457854	0.910324	0.296458	0.0175266
15	1.7	0.152618	0.906828	0.31032	0.00537721

A new finite element mesh is created using the new coordinate values from this solution.

New y coordinates of top surface = {0.398856, 0.420311, 0.46802, 0.663723, 0.846926, 1}

The process is repeated several more times. The finite element meshes and the coordinates of the groundwater surface are shown in the following figures.



The coordinates of the top surface have converged now. We can use the solution from this mesh to compute the flow rate and fluid velocity.

Equations for element 1

$$\begin{pmatrix} 0.520071 & 0.0476235 & -0.167023 & -0.400671 \\ 0.0476235 & 1.10997 & -0.509595 & -0.647996 \\ -0.167023 & -0.509595 & 0.628995 & 0.0476235 \\ -0.400671 & -0.647996 & 0.0476235 & 1.00104 \end{pmatrix} \begin{pmatrix} \phi_{12} \\ \phi_8 \\ \phi_7 \\ \phi_{11} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 2

$$\begin{pmatrix} 0.551899 & 0.0131356 & -0.22873 & -0.336304 \\ 0.0131356 & 0.940188 & -0.43601 & -0.517313 \\ -0.22873 & -0.43601 & 0.651604 & 0.0131356 \\ -0.336304 & -0.517313 & 0.0131356 & 0.840482 \end{pmatrix} \begin{pmatrix} \phi_{11} \\ \phi_7 \\ \phi_6 \\ \phi_{10} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 3

$$\begin{pmatrix} 0.61591 & -0.00410842 & -0.30768 & -0.304121 \\ -0.00410842 & 0.807201 & -0.399217 & -0.403875 \\ -0.30768 & -0.399217 & 0.711006 & -0.00410842 \\ -0.304121 & -0.403875 & -0.00410842 & 0.712104 \end{pmatrix} \begin{pmatrix} \phi_{10} \\ \phi_6 \\ \phi_5 \\ \phi_9 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 4

$$\begin{pmatrix} 0.743189 & 0.452772 & -0.290403 & -0.905557 \\ 0.452772 & 1.99895 & -1.37105 & -1.08067 \\ -0.290403 & -1.37105 & 1.20869 & 0.452772 \\ -0.905557 & -1.08067 & 0.452772 & 1.53345 \end{pmatrix} \begin{pmatrix} \phi_8 \\ \phi_4 \\ \phi_3 \\ \phi_7 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 5

$$\begin{pmatrix} 0.654138 & 0.303958 & -0.299642 & -0.658454 \\ 0.303958 & 1.49274 & -1.0229 & -0.773799 \\ -0.299642 & -1.0229 & 1.01859 & 0.303958 \\ -0.658454 & -0.773799 & 0.303958 & 1.12829 \end{pmatrix} \begin{pmatrix} \phi_7 \\ \phi_3 \\ \phi_2 \\ \phi_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 6

$$\begin{pmatrix} 0.688638 & 0.229551 & -0.383287 & -0.534902 \\ 0.229551 & 1.16061 & -0.848826 & -0.541339 \\ -0.383287 & -0.848826 & 1.00256 & 0.229551 \\ -0.534902 & -0.541339 & 0.229551 & 0.846691 \end{pmatrix} \begin{pmatrix} \phi_6 \\ \phi_2 \\ \phi_1 \\ \phi_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 7

$$\begin{pmatrix} 0.686181 & 0.171168 & -0.298912 & -0.558437 \\ 0.171168 & 1.03831 & -0.646143 & -0.563331 \\ -0.298912 & -0.646143 & 0.773887 & 0.171168 \\ -0.558437 & -0.563331 & 0.171168 & 0.9506 \end{pmatrix} \begin{pmatrix} \phi_{20} \\ \phi_{16} \\ \phi_{15} \\ \phi_{19} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 8

$$\begin{pmatrix} 0.711741 & 0.156774 & -0.337402 & -0.531113 \\ 0.156774 & 0.955169 & -0.61589 & -0.496053 \\ -0.337402 & -0.61589 & 0.796518 & 0.156774 \\ -0.531113 & -0.496053 & 0.156774 & 0.870392 \end{pmatrix} \begin{pmatrix} \phi_{19} \\ \phi_{15} \\ \phi_{14} \\ \phi_{18} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 9

$$\begin{pmatrix} 0.750962 & 0.149577 & -0.383089 & -0.517451 \\ 0.149577 & 0.887159 & -0.600764 & -0.435972 \\ -0.383089 & -0.600764 & 0.834275 & 0.149577 \\ -0.517451 & -0.435972 & 0.149577 & 0.803846 \end{pmatrix} \begin{pmatrix} \phi_{18} \\ \phi_{14} \\ \phi_{13} \\ \phi_{17} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 10

$$\begin{pmatrix} 0.815551 & 0.345523 & -0.36808 & -0.792994 \\ 0.345523 & 1.35995 & -0.985804 & -0.719671 \\ -0.36808 & -0.985804 & 1.00836 & 0.345523 \\ -0.792994 & -0.719671 & 0.345523 & 1.16714 \end{pmatrix} \begin{pmatrix} \phi_{16} \\ \phi_{12} \\ \phi_{11} \\ \phi_{15} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 11

$$\begin{pmatrix} 0.824643 & 0.311933 & -0.404809 & -0.731768 \\ 0.311933 & 1.2165 & -0.91267 & -0.615763 \\ -0.404809 & -0.91267 & 1.00555 & 0.311933 \\ -0.731768 & -0.615763 & 0.311933 & 1.0356 \end{pmatrix} \begin{pmatrix} \phi_{15} \\ \phi_{11} \\ \phi_{10} \\ \phi_{14} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 12

$$\begin{pmatrix} 0.864349 & 0.295139 & -0.458332 & -0.701155 \\ 0.295139 & 1.10961 & -0.876103 & -0.52865 \\ -0.458332 & -0.876103 & 1.0393 & 0.295139 \\ -0.701155 & -0.52865 & 0.295139 & 0.934667 \end{pmatrix} \begin{pmatrix} \phi_{14} \\ \phi_{10} \\ \phi_{9} \\ \phi_{13} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 13

$$\begin{pmatrix} 0.714147 & 0.163726 & -0.324158 & -0.553714 \\ 0.163726 & 0.973638 & -0.61763 & -0.519734 \\ -0.324158 & -0.61763 & 0.778063 & 0.163726 \\ -0.553714 & -0.519734 & 0.163726 & 0.909722 \end{pmatrix} \begin{pmatrix} \phi_{24} \\ \phi_{20} \\ \phi_{19} \\ \phi_{23} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 14

$$\begin{pmatrix} 0.738031 & 0.155811 & -0.355358 & -0.538483 \\ 0.155811 & 0.918093 & -0.6012 & -0.472704 \\ -0.355358 & -0.6012 & 0.800748 & 0.155811 \\ -0.538483 & -0.472704 & 0.155811 & 0.855376 \end{pmatrix} \begin{pmatrix} \phi_{23} \\ \phi_{19} \\ \phi_{18} \\ \phi_{22} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Equations for element 15

$$\begin{pmatrix} 0.76953 & 0.151853 & -0.390516 & -0.530867 \\ 0.151853 & 0.870763 & -0.592985 & -0.429631 \\ -0.390516 & -0.592985 & 0.831648 & 0.151853 \\ -0.530867 & -0.429631 & 0.151853 & 0.808645 \end{pmatrix} \begin{pmatrix} \phi_{22} \\ \phi_{18} \\ \phi_{17} \\ \phi_{21} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Global equations after assembling all elements

$$\left(\begin{array}{ccccccccc}
1.00256 & -0.848826 & 0 & 0 & 0.229551 & -0.383287 & 0 & 0 & 0 \\
-0.848826 & 2.1792 & -1.0229 & 0 & -0.541339 & 0.533508 & -0.299642 & 0 & 0 \\
0 & -1.0229 & 2.70143 & -1.37105 & 0 & -0.773799 & 0.756729 & -0.290403 & 0 \\
0 & 0 & -1.37105 & 1.99895 & 0 & 0 & -1.08067 & 0.452772 & 0 \\
0.229551 & -0.541339 & 0 & 0 & 1.5577 & -0.934119 & 0 & 0 & -0. \\
-0.383287 & 0.533508 & -0.773799 & 0 & -0.934119 & 3.27574 & -1.09446 & 0 & -0. \\
0 & -0.299642 & 0.756729 & -1.08067 & 0 & -1.09446 & 3.75677 & -1.41515 & 0 \\
0 & 0 & -0.290403 & 0.452772 & 0 & 0 & -1.41515 & 1.85316 & 0 \\
0 & 0 & 0 & 0 & -0.00410842 & -0.403875 & 0 & 0 & 1. \\
0 & 0 & 0 & 0 & -0.30768 & 0.00902714 & -0.517313 & 0 & -1. \\
0 & 0 & 0 & 0 & 0 & -0.22873 & 0.0607591 & -0.647996 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -0.167023 & 0.0476235 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0. \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0. \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right.$$

Essential boundary conditions

On element 13, side 4, specified value = 1

$\{\phi_{23}, \phi_{24}\} = \{1, 1\}$

On element 14, side 4, specified value = 1

$\{\phi_{22}, \phi_{23}\} = \{1, 1\}$

On element 15, side 4, specified value = 1

$\{\phi_{21}, \phi_{22}\} = \{1, 1\}$

On element 5, side 2, specified value = y

$\{\phi_3, \phi_2\} = \{0.122989, 0.0614946\}$

On element 6, side 2, specified value = y

$\{\phi_2, \phi_1\} = \{0.0614946, 0.\}$

Known values from EBC

$\{\phi_1 = 0., \phi_2 = 0.0614946, \phi_3 = 0.122989, \phi_{21} = 1, \phi_{22} = 1, \phi_{23} = 1, \phi_{24} = 1\}$

Global equations after EBC

$$\begin{pmatrix}
1.99895 & 0 & 0 & -1.08067 & 0.452772 & 0 & 0 & 0 \\
0 & 1.5577 & -0.934119 & 0 & 0 & -0.00410842 & -0.30768 & 0 \\
0 & -0.934119 & 3.27574 & -1.09446 & 0 & -0.403875 & 0.00902714 & -0.22873 \\
-1.08067 & 0 & -1.09446 & 3.75677 & -1.41515 & 0 & -0.517313 & 0.060759 \\
0.452772 & 0 & 0 & -1.41515 & 1.85316 & 0 & 0 & -0.647996 \\
0 & -0.00410842 & -0.403875 & 0 & 0 & 1.7514 & -1.18022 & 0 \\
0 & -0.30768 & 0.00902714 & -0.517313 & 0 & -1.18022 & 3.57155 & -1.24897 \\
0 & 0 & -0.22873 & 0.0607591 & -0.647996 & 0 & -1.24897 & 3.7778 \\
0 & 0 & 0 & -0.167023 & 0.0476235 & 0 & 0 & -1.38648 \\
0 & 0 & 0 & 0 & 0 & 0.295139 & -0.52865 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.458332 & 0.607072 & -0.615763 \\
0 & 0 & 0 & 0 & 0 & 0 & -0.404809 & 0.657457 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.36808 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Solving the final system of global equations we get

$$\{\phi_4 = 0.179251,\ \phi_5 = 0.284834,\ \phi_6 = 0.294747,\ \phi_7 = 0.333891,\ \phi_8 = 0.377971,\ \phi_9 = 0.423078,$$
$$\phi_{10} = 0.433334,\ \phi_{11} = 0.457718,\ \phi_{12} = 0.487647,\ \phi_{13} = 0.654265,\ \phi_{14} = 0.659864,\ \phi_{15} = 0.676055,$$
$$\phi_{16} = 0.700769,\ \phi_{17} = 0.824487,\ \phi_{18} = 0.828468,\ \phi_{19} = 0.840155,\ \phi_{20} = 0.859261\}$$

Solution for element 1

DOF values for the element

$\{\phi_{12} = 0.487647,\ \phi_8 = 0.377971,\ \phi_7 = 0.333891,\ \phi_{11} = 0.457718\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.487647 \quad 0.377971 \quad 0.333891 \quad 0.457718\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.357735	0.432418	0.457513	0.449434	0.212296
2	0.357735	0.342892	0.438507	0.513867	0.212296
3	0.242265	0.370467	0.392465	0.397391	0.309299
4	0.242265	0.293768	0.368742	0.472599	0.309299

Solution for element 2

DOF values for the element

$\{\phi_{11} = 0.457718,\ \phi_7 = 0.333891,\ \phi_6 = 0.294747,\ \phi_{10} = 0.433334\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.457718 \quad 0.333891 \quad 0.294747 \quad 0.433334\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.357735	0.277356	0.425738	0.573696	0.177371
2	0.357735	0.18783	0.409859	0.636004	0.177371
3	0.242265	0.23762	0.352446	0.541416	0.271175
4	0.242265	0.160921	0.331647	0.614143	0.271175

Solution for element 3

DOF values for the element

$\{\phi_{10} = 0.433334,\ \phi_6 = 0.294747,\ \phi_5 = 0.284834,\ \phi_9 = 0.423078\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.433334 \quad 0.294747 \quad 0.284834 \quad 0.423078\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.357735	0.122294	0.401895	0.682608	0.0656707
2	0.357735	0.0327685	0.396016	0.688915	0.0656707
3	0.242265	0.104773	0.321924	0.681168	0.0751652
4	0.242265	0.0280738	0.316158	0.688529	0.0751652

Solution for element 4

DOF values for the element

$\{\phi_8 = 0.377971,\ \phi_4 = 0.179251,\ \phi_3 = 0.122989,\ \phi_7 = 0.333891\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.377971 \quad 0.179251 \quad 0.122989 \quad 0.333891\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.157735	0.310534	0.326117	0.637141	0.418969
2	0.157735	0.246243	0.299181	0.748772	0.418969
3	0.042265	0.208745	0.2099	0.374221	0.717228
4	0.042265	0.165528	0.178903	0.540285	0.717228

Solution for element 5

DOF values for the element

$\{\phi_7 = 0.333891,\ \phi_3 = 0.122989,\ \phi_2 = 0.0614946,\ \phi_6 = 0.294747\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.333891 \quad 0.122989 \quad 0.0614946 \quad 0.294747\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.157735	0.199179	0.280052	0.855389	0.393937
2	0.157735	0.134888	0.254725	0.991805	0.393937
3	0.042265	0.133891	0.155561	0.649307	0.758419
4	0.042265	0.0906734	0.122784	0.852242	0.758419

Solution for element 6

DOF values for the element

$\{\phi_6 = 0.294747,\ \phi_2 = 0.0614946,\ \phi_1 = 0.,\ \phi_5 = 0.284834\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.294747 \quad 0.0614946 \quad 0. \quad 0.284834\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.157735	0.0878232	0.241056	1.17417	0.186911
2	0.157735	0.0235322	0.22904	1.35718	0.186911
3	0.042265	0.0590361	0.100095	1.05226	0.675898
4	0.042265	0.0158187	0.0708843	1.32452	0.675898

Solution for element 7

DOF values for the element

$\{\phi_{20} = 0.859261,\ \phi_{16} = 0.700769,\ \phi_{15} = 0.676055,\ \phi_{19} = 0.840155\}$

$\boldsymbol{d}^{\mathrm{T}}$ = (0.859261 0.700769 0.676055 0.840155)

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	1.29434	0.767498	0.82148	0.297608	0.0737282
2	1.29434	0.6086	0.809765	0.308586	0.0737282
3	1.00566	0.682353	0.72929	0.290991	0.0961616
4	1.00566	0.541083	0.715706	0.303339	0.0961616

Solution for element 8

DOF values for the element

$\{\phi_{19} = 0.840155, \phi_{15} = 0.676055, \phi_{14} = 0.659864, \phi_{18} = 0.828468\}$

$\boldsymbol{d}^{\mathrm{T}}$ = (0.840155 0.676055 0.659864 0.828468)

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	1.29434	0.492278	0.802806	0.321416	0.0459218
2	1.29434	0.33338	0.795509	0.329422	0.0459218
3	1.00566	0.437666	0.707513	0.318322	0.0622798
4	1.00566	0.296396	0.698714	0.327326	0.0622798

Solution for element 9

DOF values for the element

$\{\phi_{18} = 0.828468, \phi_{14} = 0.659864, \phi_{13} = 0.654265, \phi_{17} = 0.824487\}$

$\boldsymbol{d}^{\mathrm{T}}$ = (0.828468 0.659864 0.654265 0.824487)

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	1.29434	0.217059	0.791924	0.336582	0.0157086
2	1.29434	0.0581607	0.789428	0.339409	0.0157086
3	1.00566	0.192979	0.694383	0.3361	0.0214848
4	1.00566	0.0517085	0.691348	0.33928	0.0214848

Solution for element 10

DOF values for the element

$\{\phi_{16} = 0.700769, \phi_{12} = 0.487647, \phi_{11} = 0.457718, \phi_{15} = 0.676055\}$

$\boldsymbol{d}^{\mathrm{T}}$ = (0.700769 0.487647 0.457718 0.676055)

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.794338	0.609748	0.650276	0.382142	0.118072
2	0.794338	0.48351	0.63537	0.39775	0.118072
3	0.505662	0.496533	0.526593	0.364954	0.161899
4	0.505662	0.393734	0.50995	0.38412	0.161899

Solution for element 11

DOF values for the element

$\{\phi_{15} = 0.676055, \phi_{11} = 0.457718, \phi_{10} = 0.433334, \phi_{14} = 0.659864\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.676055 \quad 0.457718 \quad 0.433334 \quad 0.659864\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.794338	0.391097	0.626127	0.419517	0.081968
2	0.794338	0.264858	0.61578	0.435633	0.081968
3	0.505662	0.31848	0.499071	0.408132	0.127226
4	0.505662	0.21568	0.485992	0.427923	0.127226

Solution for element 12

DOF values for the element

$\{\phi_{14} = 0.659864, \phi_{10} = 0.433334, \phi_{9} = 0.423078, \phi_{13} = 0.654265\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 0.659864 \quad 0.433334 \quad 0.423078 \quad 0.654265\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.794338	0.172445	0.610601	0.451689	0.0301067
2	0.794338	0.0462065	0.606801	0.45951	0.0301067
3	0.505662	0.140426	0.479246	0.449253	0.0520705
4	0.505662	0.0376271	0.473893	0.458857	0.0520705

Solution for element 13

DOF values for the element

$\{\phi_{24} = 1, \phi_{20} = 0.859261, \phi_{19} = 0.840155, \phi_{23} = 1\}$

$\boldsymbol{d}^{\mathrm{T}} = (\ 1 \quad 0.859261 \quad 0.840155 \quad 1\ )$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	1.87321	0.901897	0.969405	0.238571	0.0124844
2	1.87321	0.715174	0.967074	0.257519	0.0124844
3	1.52679	0.826325	0.885818	0.2302	0.0508535
4	1.52679	0.655247	0.877119	0.250882	0.0508535

Solution for element 14

DOF values for the element

$\{\phi_{23} = 1, \phi_{19} = 0.840155, \phi_{18} = 0.828468, \phi_{22} = 1\}$

$\boldsymbol{d}^{\mathrm{T}} = (\,1 \quad 0.840155 \quad 0.828468 \quad 1\,)$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	1.87321	0.578483	0.965699	0.269456	0.00763633
2	1.87321	0.39176	0.964273	0.281047	0.00763633
3	1.52679	0.53001	0.871986	0.266172	0.0311056
4	1.52679	0.358933	0.866665	0.278823	0.0311056

Solution for element 15

DOF values for the element

$\{\phi_{22} = 1, \phi_{18} = 0.828468, \phi_{17} = 0.824487, \phi_{21} = 1\}$

$\boldsymbol{d}^{\mathrm{T}} = (\,1 \quad 0.828468 \quad 0.824487 \quad 1\,)$

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	1.87321	0.255069	0.963573	0.287128	0.00260162
2	1.87321	0.0683454	0.963087	0.291077	0.00260162
3	1.52679	0.233696	0.864054	0.286635	0.0105973
4	1.52679	0.0626186	0.862241	0.290945	0.0105973

Nodal solution summary

dof	x	y	Value
ϕ_1	0	0	0.
ϕ_2	0	0.0614946	0.0614946
ϕ_3	0	0.122989	0.122989
ϕ_4	0	0.184484	0.179251
ϕ_5	0.2	0	0.284834
ϕ_6	0.2	0.124716	0.294747
ϕ_7	0.2	0.249431	0.333891
ϕ_8	0.2	0.374147	0.377971
ϕ_9	0.4	0	0.423078
ϕ_{10}	0.4	0.163193	0.433334
ϕ_{11}	0.4	0.326387	0.457718
ϕ_{12}	0.4	0.48958	0.487647
ϕ_{13}	0.9	0	0.654265
ϕ_{14}	0.9	0.233512	0.659864
ϕ_{15}	0.9	0.467023	0.676055
ϕ_{16}	0.9	0.700535	0.700769
ϕ_{17}	1.4	0	0.824487
ϕ_{18}	1.4	0.286395	0.828468
ϕ_{19}	1.4	0.57279	0.840155
ϕ_{20}	1.4	0.859186	0.859261
ϕ_{21}	2	0	1
ϕ_{22}	2	$\frac{1}{3}$	1
ϕ_{23}	2	$\frac{2}{3}$	1
ϕ_{24}	2	1	1

Element solution summary

	x	y	ϕ	$\partial\phi/\partial x$	$\partial\phi/\partial y$
1	0.357735	0.432418	0.457513	0.449434	0.212296
2	0.357735	0.277356	0.425738	0.573696	0.177371
3	0.357735	0.122294	0.401895	0.682608	0.0656707
4	0.157735	0.310534	0.326117	0.637141	0.418969
5	0.157735	0.199179	0.280052	0.855389	0.393937
6	0.157735	0.0878232	0.241056	1.17417	0.186911
7	1.29434	0.767498	0.82148	0.297608	0.0737282
8	1.29434	0.492278	0.802806	0.321416	0.0459218
9	1.29434	0.217059	0.791924	0.336582	0.0157086
10	0.794338	0.609748	0.650276	0.382142	0.118072
11	0.794338	0.391097	0.626127	0.419517	0.081968
12	0.794338	0.172445	0.610601	0.451689	0.0301067
13	1.87321	0.901897	0.969405	0.238571	0.0124844
14	1.87321	0.578483	0.965699	0.269456	0.00763633
15	1.87321	0.255069	0.963573	0.287128	0.00260162