Semi-Blind Decorrelating Multiuser Detection for Synchronous CDMA

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***Abstract*— Multiuser detection is a key technology for combating multiple access interference (MAI) in CDMA systems. In this paper, we propose three effective one-shot multiuser semi-blind decorrelating detectors based on least squares (LS), total least squares (TLS) and mixed LS/TLS (MLS) formulations. All of the proposed algorithms are based on the classic decorrelating detector and a new semi-blind signature matrix. Different assumptions regarding the noise in the proposed semi-blind signature matrix lead to different detection algorithms. We also find that the classic decorrelating detector can be regarded as a special case of the proposed LS decorrelating detector. The proposed detection algorithms are simple and direct, where only the timing, signature and amplitude of the desired user are required. In addition, no search or convergence procedure is required. Finally, theoretical analysis and computer simulations are presented to illustrate the performance of the proposed algorithms.**

I. Introduction

Multiuser detection is a strategy for minimizing the effect of MAI and mitigating the near-far problem, especially in CDMA systems. MAI is the dominant impairment for CDMA systems and exists even in perfect power-controlled CDMA systems. Most early work on multiuser detection assumed that the receiver had some knowledge of the users' timing, received amplitude and/or signature waveform which was exploited to combat MAI. For example, the classic decorrelating detector can achieve the optimum near-far resistance and completely eliminate MAI at the expense of background noise enhancement. Though the decorrelating detector does not require knowledge of all users' received amplitudes, it does require information about their signature waveforms. However, in many practical cases, especially in a dynamic environment, it is very difficult for a mobile user to obtain accurate information of other active users. Most of the multiuser detection schemes assume knowledge of the spreading codes and/or channel parameters of all the users that contribute to the received signal. Blind detectors, on the other hand, operate without knowledge of the channel or information of other users. Usually, many practical systems lie in between these two extremes. The first form of detectors are too optimistic as we cannot expect to know the signature waveforms or received amplitudes of all the users, especially in the downlink of a CDMA system, while the latter under-utilizes the knowledge of the system.

Recent research has been devoted to blind multiuser receivers and subspace-based signature waveform estimation schemes to achieve better performance and higher capacity [1, 2, 3, 4, 5]. The minimum output energy (MOE) method and subspace methods were presented for multiuser blind detection with the knowledge of only the desired users' spreading code and possible timing. In the subspace-based blind detection approach [3], the linear detectors offer lower computational complexity and better performance than the blind MOE detector. However, the signal subspace components need to be computed before detection.

In this work, we consider a synchronous DS-CDMA system and develop a least squares multiuser semi-blind decorrelating detector (LS-DD), a total least squares multiuser semi-blind decorrelating detector (TLS-DD), and a mixed LS/TLS multiuser semi-blind decorrelating detector (MLS-DD). The detectors are semi-blind since they require the signature waveform and received amplitude of the desired user (timing is inherently assumed). A new semi-blind signature matrix is proposed on which the decorrelating operation is based. All algorithms are one-shot algorithms. No knowledge of the other users' information is required and no search or convergence procedure is employed. Theoretical analysis and computer simulations are also presented to demonstrate the performance of the proposed multiuser detectors. It is expected that these proposed algorithms can bridge the performance gap between the blind and the conventional multiuser detectors.

II. Data Model and Problem Description

The basic CDMA K-user channel model, consisting of the sum of antipodally modulated synchronous signature waveforms embedded in additive white Gaussian noise (AWGN), is considered here. The received base-band signal during one symbol interval in such a channel can be modeled as

$$r(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + n(t) \tag{1}$$

where K is the number of users, A_k and b_k denote the received amplitude and nth data bit for the kth user, respectively, $n(t)$ represents the Gaussian channel noise, and T is the symbol interval. It is assumed that $b_k \in \{-1, +1\}$ is a collection of independent, equiprobable random variables and $s_k(t)$ denotes the normalized signal waveform of the kth user during the interval $t \in [(n-1)T,\ nT]$, i.e., $\|s_k(t)\| = 1$.

The received signal, $r(t)$, is passed through a chip-matched filter followed by a chip-rate sampler. As a result, $r(t),\ t \in [(n-1)T,\ nT]$, is converted into a $L \times 1$ column vector, $\mathbf{r}$, containing the samples of the chip-matched filter outputs within the symbol interval $[(n-1)T,\ nT]$ as

$$\mathbf{r} = \mathbf{SAb} + \mathbf{n} \tag{2}$$

where $\mathbf{A} = \text{diag}\{A_1\ A_2\ \ldots\ A_K\}$ is the diagonal received amplitude matrix, $\mathbf{S} = [\mathbf{s}_1\ \mathbf{s}_2\ \ldots\ \mathbf{s}_K]$ is the $L \times K$ signature matrix with the kth column, $\mathbf{s}_k$, being the signature vector of the kth user, $\mathbf{b} = [b_1\ b_2\ \ldots\ b_K]^T = [b_1\ \tilde{\mathbf{b}}^T]^T$ is the vector containing data sent by all the K users at time $t = n$, and $\mathbf{n}$ is an L-dimensional Gaussian vector with independent σ_n^2-variance components. We maintain the restriction that $L \geq K$.

Most linear multiuser detectors for demodulating the kth user's data bit in (2) is in the form of a correlator followed by a hard limiter, which can be expressed as

$$\hat{b}_k = \text{sgn}\{\mathbf{w}_k^T \mathbf{r}\}, \tag{3}$$

where $\mathbf{w}_k \in \mathbb{R}^{L \times 1}$ is the linear representation of multiuser detector for user k.

III. Decorrelating Detector

The decorrelating detector, one of the earliest suggestions to eliminate multiuser interference with a linear receiver, was proposed by Shnidman [6]. The forerunner of the decorrelating detector in the single-user intersymbol interference (ISI) channel is the *zero-forcing equalizer*. Further, the counterpart of decovariance in an antenna array subject to undesired sources is called *null steering*. Prior to developing the semi-blind decorrelating detector, we first discuss decorrelating detection.

According to (2), the classic decorrelating detector performs the following operation:

$$\begin{aligned} \hat{\mathbf{b}} &= \text{sgn}\{\mathbf{W}_{\text{DD}}^T \mathbf{r}\} \\ &= \text{sgn}\{\mathbf{S}^+ \mathbf{r}\} \end{aligned} \tag{4}$$

where $\mathbf{W}_{\text{DD}}$ is the linear filter representation of the decorrelating detector bank and $\mathbf{M}^+$ denotes the Moore-Penrose generalized inverse of the matrix $\mathbf{M}$.

The decorrelating detector is one of the simplest multiuser detectors and is designed to completely eliminate MAI caused by the other users, but suffers from noise enhancement. When the received amplitudes are completely unknown, the decorrelating detector is a sensible choice. Desirable properties of the detector include not requiring knowledge of the received amplitudes, but it requires $\mathbf{S}^+$. It can readily be decentralized in the sense that the demodulation of each user can be implemented separately.

IV. One-Shot Semi-blind Decorrelating Detection Schemes

In this section, we develop three one-shot multiuser semi-blind decorrelating detectors that are expected to bridge the performance gap between the conventional and blind detectors.

A. *New Data Model with Semi-Blind Signature Matrix*

Without loss of the generality, we consider only the bit, b_1, sent by the first user at time $t = n$. We first define a new $L \times M$ semi-blind signature matrix $\boldsymbol{\mathcal{S}}$ for user 1 which is constructed as

$$\begin{aligned} \boldsymbol{\mathcal{S}} &= [\, A_1 \mathbf{s}_1 \quad \mathbf{r}_1 \quad \mathbf{r}_2 \quad \ldots \quad \mathbf{r}_{M-1} \,] \\ &= \mathbf{SA}\,[\, \mathbf{e}_1 \quad \mathbf{b}_1 \quad \mathbf{b}_2 \quad \ldots \quad \mathbf{b}_{M-1} \,] + \mathbf{N} \\ &= \mathbf{SA}\,[\, \mathbf{e}_1 \quad \mathbf{D} \,] + \mathbf{N} \\ &= \mathbf{SAB} + \mathbf{N} \end{aligned} \tag{5}$$

where $\mathbf{r}_m,\ m = 1, \ldots, M-1$, are $M-1$ previously received and linearly independent blind vectors that are used to compose $\boldsymbol{\mathcal{S}}$, $\mathbf{e}_1$ is a $K \times 1$ identity vector with a 1 on the first row and 0's elsewhere, and the $K \times 1$ vectors $\mathbf{b}_m$ are the corresponding data vectors which denote the unknown information sent by all K users in the previously received vector $\mathbf{r}_m$. The matrix $\mathbf{S}$ is the original signature matrix with the desired user's signature vector in the first column, $\mathbf{D} = [\bar{\mathbf{d}}\ \tilde{\mathbf{D}}^T]^T$ where the $(K-1) \times 1$ vector $\bar{\mathbf{d}}$ is the data vector consisting of the known bits sent by the desired user in previous signaling intervals. Further, note that $\text{rank}\{\tilde{\mathbf{D}}\} = K - 1$, $\mathbf{N} = [\mathbf{0}\ \tilde{\mathbf{N}}]$,

$$\mathbf{B} = \begin{bmatrix} 1 & \bar{\mathbf{d}}^T \\ \mathbf{0} & \tilde{\mathbf{D}} \end{bmatrix} \tag{6}$$

and $\text{rank}\{\mathbf{B}\} = K$.

From (2) and (5), the relationship between the received signal vector, $\mathbf{r}$, and the new semi-blind signature matrix, $\boldsymbol{\mathcal{S}}$, is

$$\mathbf{r} = \boldsymbol{\mathcal{S}}\mathbf{d} + \mathbf{z} \tag{7}$$

where $\mathbf{d}$ denotes a new $K \times 1$ detection vector defined as

$$\mathbf{d} = \mathbf{B}^+ \mathbf{b} = \begin{bmatrix} 1 & \bar{\mathbf{d}}^T \\ \mathbf{0} & \tilde{\mathbf{D}} \end{bmatrix}^+ \begin{bmatrix} b_1 \\ \tilde{\mathbf{b}} \end{bmatrix} \tag{8}$$

and $\mathbf{z}$ is the new noise vector defined as

$$\mathbf{z} = \mathbf{n} - \mathbf{N}\mathbf{B}^{+}\mathbf{b} \ . \tag{9}$$

Lemma 1. *The bit sent by the first user, b_1, during the time interval $t \in [(n-1)T,\ nT]$ can be estimated as*

$$b_1 = \mathbf{c}^T\mathbf{d} \tag{10}$$

where $\mathbf{c} = [\,1 \quad \bar{\mathbf{d}}^T\,]^T$ and $\mathbf{d} = [\,d_1 \quad \tilde{\mathbf{d}}\,]^T$.

With the definition of the new semi-blind signature matrix, $\boldsymbol{\mathcal{S}}$, in (5), the new multiuser detection model in (7) remains in the same form as (2). The difference is that the original signature matrix, $\mathbf{S}$, is replaced by the semi-blind signature matrix, $\boldsymbol{\mathcal{S}}$, the data vector $\mathbf{b}$ is replaced by the detection vector $\mathbf{d}$ in (8), and the original AWGN vector $\mathbf{n}$ is replace by the new noise vector $\mathbf{z}$ in (9). Fortunately, with Lemma 1, it is possible to calculate the desired bit b_1 with the detection vector $\mathbf{d}$ and the previously detected bit vector $\mathbf{c}$. The main question is how to estimate the detection vector $\mathbf{d}$ as efficiently as possible. The following sections propose three approaches that depend on assumptions regarding $\mathbf{N}$ in the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$.

B. *LS Semi-Blind Decorrelating Detector*

We first assume that the received measurements in $\boldsymbol{\mathcal{S}}$ are assumed to be free of error. Hence, all errors are confined to the received vector $\mathbf{r}$ due to $\mathbf{z}$ in (7) in the nth bit interval. The following LS estimator is proposed along the lines of the classic decorrelating detector.

Lemma 2. *[7] Suppose $\mathbf{U}^T\boldsymbol{\mathcal{S}}\mathbf{V} = \boldsymbol{\Sigma}$ is the SVD of $\boldsymbol{\mathcal{S}} \in \mathbb{R}^{L\times K}$ with $r = rank(\boldsymbol{\mathcal{S}})$. If $\mathbf{U} = [\mathbf{u}_1\ \mathbf{u}_2\ \ldots\ \mathbf{u}_L]$, $\mathbf{V} = [\mathbf{v}_1\ \mathbf{v}_2\ \ldots\ \mathbf{v}_K]$, $\boldsymbol{\Sigma} = \mathrm{diag}\{[\sigma_1\ \ldots\ \sigma_r\ 0\ \ldots\ 0]\}$ and $\mathbf{r} \in \mathbb{R}^{L\times 1}$, then*

$$\mathbf{d}_{\mathrm{LS}} = \textstyle\sum_{i=1}^{r} \frac{\mathbf{u}_i^T\mathbf{r}}{\sigma_i}\mathbf{v}_i = \boldsymbol{\mathcal{S}}^{+}\mathbf{r} \tag{11}$$

minimizes $\|\boldsymbol{\mathcal{S}}\mathbf{d} - \mathbf{r}\|_2$ and has the smallest 2-norm of all minimizers. Moreover, the minimum squared error achieved is

$$\varepsilon_{\mathrm{LS}}^2 = \min_{\mathbf{x}\in\mathbb{R}} \|\boldsymbol{\mathcal{S}}\mathbf{x} - \mathbf{r}\|_2^2 = \sum_{i=r+1}^{L} \left(\mathbf{u}_i^T\mathbf{r}\right)^2 \ . \tag{12}$$

The linear filter $\mathbf{w}_{\mathrm{LS}}$ in the LS semi-blind decorrelating detector for user 1 is then

$$\mathbf{w}_{\mathrm{LS}} = \left(\mathbf{c}^T\boldsymbol{\mathcal{S}}^{+}\right)^T \tag{13}$$

so that the bit sent by the first user, b_1, in the nth signaling interval is detected with

$$\begin{aligned} \hat{b}_1^{\mathrm{LS}} &= \mathrm{sgn}\{\mathbf{w}_{\mathrm{LS}}^T\mathbf{r}\} \\ &= \mathrm{sgn}\left\{b_1 + \mathbf{c}^T\boldsymbol{\mathcal{S}}^{+}\tilde{\mathbf{n}}\right\} \ . \end{aligned} \tag{14}$$

C. *TLS Semi-Blind Decorrelating Detector*

The previous LS estimate of the detection vector $\mathbf{d}$ from (7), (8) and (11) is the solution to

$$\hat{\mathbf{d}} = \min_{\mathbf{x}} \|\mathbf{r} - \boldsymbol{\mathcal{S}}\mathbf{x}\|_2 \quad \text{subject to} \quad \mathbf{r} \subseteq \mathbb{R}(\boldsymbol{\mathcal{S}}) \tag{15}$$

where the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$ is assumed to be error-free. However, this assumption is not entirely accurate according to the definition of $\boldsymbol{\mathcal{S}}$ in (5) since there is a noise term, $\mathbf{N}$.

Furthermore, $\mathbf{r}$ can also be expressed as

$$\begin{aligned} \mathbf{r} &= (\boldsymbol{\mathcal{S}} - \mathbf{N})\mathbf{B}^{+}\mathbf{b} + \mathbf{n} \\ &= \hat{\boldsymbol{\mathcal{S}}}\mathbf{d} + \mathbf{n} \end{aligned} \tag{16}$$

where $\hat{\boldsymbol{\mathcal{S}}} = \boldsymbol{\mathcal{S}} - \mathbf{N} = \mathbf{SAB}$. The minimization problem of (15) can then be transformed into the following TLS problem:

$$\left[\hat{\boldsymbol{\mathcal{S}}},\ \mathbf{x}\right] = \min_{\bar{\boldsymbol{\mathcal{S}}},\ \mathbf{x}} \|[\,\boldsymbol{\mathcal{S}} \quad \mathbf{r}\,] - [\,\bar{\boldsymbol{\mathcal{S}}} \quad \bar{\boldsymbol{\mathcal{S}}}\mathbf{x}\,]\|_2 \ , \tag{17}$$

subject to $\mathbf{r} \subseteq \mathbb{R}(\bar{\boldsymbol{\mathcal{S}}})$.

Lemma 3. *[8] Let $\boldsymbol{\mathcal{S}} = \mathbf{U}'\boldsymbol{\Sigma}'\mathbf{V}'^T$ and $[\boldsymbol{\mathcal{S}}\ \mathbf{r}] = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T$ be the SVD of $\boldsymbol{\mathcal{S}}$ and $[\boldsymbol{\mathcal{S}}\ \mathbf{r}]$, respectively. If $\sigma_K' > \sigma_{K+1}$, then*

$$\mathbf{d}_{\mathrm{TLS}} = \left(\boldsymbol{\mathcal{S}}^T\boldsymbol{\mathcal{S}} - \sigma_{K+1}^2\mathbf{I}\right)^{-1}\boldsymbol{\mathcal{S}}^T\mathbf{r} \tag{18}$$

and

$$\begin{aligned} \varepsilon_{\mathrm{TLS}}^2 &= \min_{\mathbf{x}\in\mathbb{R}^{K\times 1}} \|\boldsymbol{\mathcal{S}}\mathbf{x} - \mathbf{r}\|_2^2 \\ &= \sigma_{K+1}^2\left[1 + \textstyle\sum_{i+1}^{K} \frac{\left(\mathbf{u}_i'^T\mathbf{r}\right)^2}{\sigma_i'^2 - \sigma_{K+1}^2}\right] \end{aligned} \tag{19}$$

where $\mathbf{U} = [\mathbf{u}_1\ \mathbf{u}_2\ \ldots\ \mathbf{u}_L]$, $\mathbf{V} = [\mathbf{v}_1\ \mathbf{v}_2\ \ldots\ \mathbf{v}_{K+1}]$, $\boldsymbol{\Sigma} = \mathrm{diag}\{[\sigma_1\ \sigma_2 \ldots\ \sigma_{K+\min\{L-K,\,1\}}]\}$ and $\mathbf{U}' = [\mathbf{u}_1'\ \mathbf{u}_2'\ \ldots\ \mathbf{u}_L']$, $\mathbf{V}' = [\mathbf{v}_1'\ \mathbf{v}_2'\ \ldots\ \mathbf{v}_K']$, $\boldsymbol{\Sigma}' = \mathrm{diag}\{[\sigma_1'\ \sigma_2'\ \ldots\ \sigma_K']\}$.

The linear filter representation of the TLS semi-blind decorrelating detector is

$$\mathbf{w}_{\mathrm{TLS}} = \boldsymbol{\mathcal{S}}(\boldsymbol{\mathcal{S}}^T\boldsymbol{\mathcal{S}} - \sigma_{K+1}^2\mathbf{I})^{-1}\mathbf{c} \tag{20}$$

and the bit sent by the first user, b_1, in the nth signaling interval can be detected with

$$\begin{aligned} \hat{b}_1^{\mathrm{TLS}} &= \mathrm{sgn}\{\mathbf{w}_{\mathrm{TLS}}^T\mathbf{r}\} \\ &= \mathrm{sgn}\left\{\mathbf{c}^T\left(\boldsymbol{\mathcal{S}}^T\boldsymbol{\mathcal{S}} - \sigma_{K+1}^2\mathbf{I}\right)^{-1}\boldsymbol{\mathcal{S}}^T\mathbf{r}\right\} \ . \end{aligned} \tag{21}$$

D. *Mixed LS/TLS Semi-Blind Decorrelating Detector*

In the LS problem of (15), it assumed the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$ is error-free. Again, this assumption is not completely accurate. In the TLS problem of (17), it assumed that in each column of the semi-blind signature matrix, $\boldsymbol{\mathcal{S}}$, some noise or error exists. This assumption also is not complete. Though there exists a noise or error matrix $\mathbf{N}$ in $\boldsymbol{\mathcal{S}}$ from (5), its first column is exactly known to be noise-free or error-free. Hence, to maximize the estimation accuracy of the detection vector $\mathbf{d}$, it is natural to require that the corresponding columns of $\boldsymbol{\mathcal{S}}$ be unperturbed since they are known exactly. The problem of estimating the detection vector $\mathbf{d}$ can then be transformed into the following MLS problem by considering (15) and (17):

$$\min_{\bar{\boldsymbol{\mathcal{S}}},\,\mathbf{x}} \|[\,\tilde{\boldsymbol{\mathcal{S}}} \quad \mathbf{r}\,] - [\,\bar{\boldsymbol{\mathcal{S}}} \quad [A_1\mathbf{s}_1\ \bar{\boldsymbol{\mathcal{S}}}]\mathbf{x}\,]\|_2 \tag{22}$$

subject to $\mathbf{r} \subseteq \mathbb{R}([A_1\mathbf{s}_1\ \bar{\boldsymbol{\mathcal{S}}}])$. The following lemma outlines the MLS solution.

Lemma 4. *[8] Consider the MLS problem in (22) and perform the Householder transformation Q on the matrix $[\,\boldsymbol{\mathcal{S}} \quad \mathbf{r}\,]$ so that*

$$Q^H[\,A_1\mathbf{s}_1 \quad \bar{\boldsymbol{\mathcal{S}}} \quad \mathbf{r}\,] \quad = \quad \begin{bmatrix} R_{11} & \mathbf{R}_{12} & R_{1r} \\ \mathbf{0} & \mathbf{R}_{22} & \mathbf{R}_{2r} \end{bmatrix} \tag{23}$$

where $R_{11} \neq 0$, $\mathbf{R}_{12}$ is a $1 \times (M-1)$ vector, $\mathbf{R}_{22}$ is a $(L-1) \times (M-1)$ matrix and $\mathbf{R}_{2r}$ is a $(L-1) \times 1$ vector.

Denote σ' as the smallest singular value of $\mathbf{R}_{22}$ and σ as the smallest singular value of $[\,\mathbf{R}_{22} \quad \mathbf{R}_{2r}\,]$. If $\sigma' > \sigma$, then the MLS solution uniquely exists and is given by

$$\mathbf{d}_{\mathrm{MLS}} \quad = \quad \left(\boldsymbol{\mathcal{S}}^T\boldsymbol{\mathcal{S}} - \sigma^2 \begin{bmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{M-1} \end{bmatrix}\right)^{-1} \boldsymbol{\mathcal{S}}^T\mathbf{r} \ . \tag{24}$$

Therefore, the linear filter representation of the MLS semi-blind decorrelating detector is

$$\mathbf{w}_{\mathrm{MLS}} \quad = \quad \boldsymbol{\mathcal{S}}\left(\boldsymbol{\mathcal{S}}^T\boldsymbol{\mathcal{S}} - \sigma^2 \begin{bmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{M-1} \end{bmatrix}\right)^{-1} \mathbf{c} \tag{25}$$

and the bit sent by the first user, b_1, during the nth signaling interval can be detected with

$$\begin{aligned} &\hat{b}^{\mathrm{MLS}} = \mathrm{sgn}\{\mathbf{w}_{\mathrm{MLS}}^T\mathbf{r}\} \\ &= \mathrm{sgn}\left\{\mathbf{c}^T\left(\boldsymbol{\mathcal{S}}^T\boldsymbol{\mathcal{S}} - \sigma^2 \begin{bmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{M-1} \end{bmatrix}\right)^{-1} \boldsymbol{\mathcal{S}}^T\mathbf{r}\right\}. \end{aligned} \tag{26}$$

V. Performance Analysis

A. *Relationship between the proposed algorithms and decorrelating detection*

It is easy to see that when $\mathbf{N} = \mathbf{0}$, the signal subspaces span$\{\mathbf{S}\}$ and span$\{\boldsymbol{\mathcal{S}}\}$ are the same. Consequently, the following relationship between the desired user's LS semi-blind decorrelating detector, $\mathbf{w}_{\mathrm{LS}}$, and its classic decorrelating detector $\mathbf{w}_{\mathrm{DD}}$ holds:

$$\mathbf{w}_{\mathrm{LS}} = A_k^{-1}\mathbf{w}_{\mathrm{DD}} \ . \tag{27}$$

Thus, the classic decorrelating detector is a special case of the proposed LS semi-blind decorrelating detector and the output of the limiter of the LS semi-blind decorrelating detector is

$$\hat{b}_k^{\mathrm{LS}} \quad = \quad \mathrm{sgn}\left\{b_k + A_k^{-1}\left[\mathbf{S}^+\mathbf{n}\right]_k\right\}. \tag{28}$$

Furthermore, we can see that when there is no noise in the semi-blind signature matrix, the bit-error-rate $P_k^{\mathrm{LS}}(\sigma)$ of the kth user in the LS semi-blind decorrelating detector is

$$P_k^{\mathrm{LS}}(A_k,\,\sigma) \quad = \quad Q\left(\frac{A_k}{\sigma\sqrt{R_{kk}^+}}\right) \tag{29}$$

where R_{kk}^+ is shorthand for $\left[\mathbf{R}^{-1}\right]_{kk}$, the element in the kth row and kth column of matrix $\mathbf{R}^{-1}$.

B. *Relationship Between b_1 and $\mathbf{d}$*

It is straightforward to develop the following relationship between the estimation errors of b_1 and $\mathbf{d}$:

$$\Delta b_1 \leq \|\Delta\mathbf{d}\|_1 \tag{30}$$

where $\Delta b_1 = \hat{b}_1 - b_1$, $\Delta\mathbf{d} = \hat{\mathbf{d}} - \mathbf{d}$ and $\|\mathbf{m}\|_1$ denotes the 1-norm of the vector $\mathbf{m}$.

C. *Relationship between $\mathbf{z}$ and $\mathbf{n}$*

The mean of the semi-blind noise item $\mathbf{z}$ which is defined in (9) is

$$\mu \quad = \quad E\{\mathbf{z}\} \quad = \quad 0 \ . \tag{31}$$

The variance of the semi-blind noise item $\mathbf{z}$ satisfies the following inequality

$$\begin{aligned} \max\{var\{\mathbf{z}\}\} \quad &= \quad \max\{E\{(\mathbf{z}-\mu)^2\}\} \\ &\leq \quad \sigma^2 + (K-1)\|\tilde{\mathbf{D}}^+\|_2^2\sigma_N^2 \end{aligned} \tag{32}$$

where $\max\{\mathbf{m}\}$ denotes the maximum item in the vector $\mathbf{m}$ and σ_N^2 is the power of the noise item $\mathbf{N}$ in the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$.

VI. Simulation Results

In this section, various computer simulations and analytical results are presented. In the computer simulations, the system consists of two users in an AWGN channel. The spreading sequences for these two users are defined as

$$\mathrm{sgn}\{\mathbf{s}_1\} \quad = \quad [+ - + - + - + -]^T \tag{33}$$

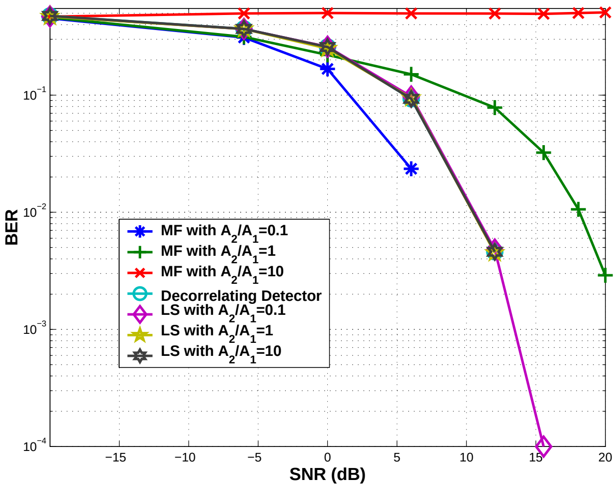

Fig. 1. BER comparison of the single-user matched filter, decorrelating detector, and the LS semi-blind detector for the first user in a two user system with $\rho = 0.75$.

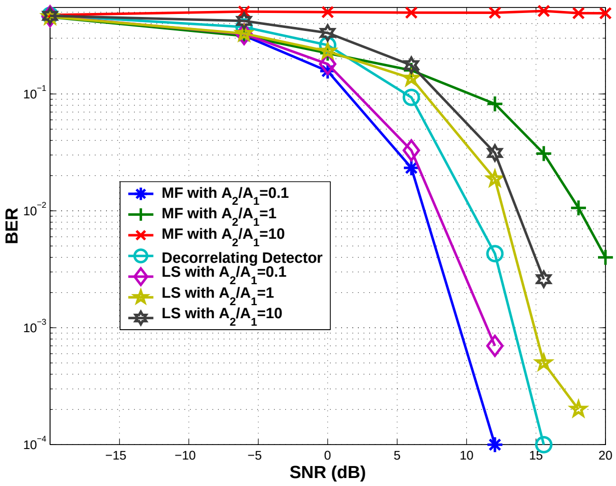

Fig. 2. BER comparison of the single-user matched filter, decorrelating detector, and the LS semi-blind detector for the first user in a two user system with $\rho = 0.75$.

and

$$\mathrm{sgn}\{\mathbf{s}_2\} = [+ + + - + - + -]^T . \quad (34)$$

The correlation between the spreading sequences of these two users is $\rho = 0.75$. We compared the proposed algorithms with single-user matched filter detector and decorrelating detector against SNR varying from -20 dB to $+20$ dB and the amplitude of the interfering signal.

CASE 1: $\bar{\mathbf{N}} = \mathbf{0}$ OR $SNR \gg 1$

In this case, there is no noise in the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$. Hence, only the LS semi-blind detector is applicable here. We examine the bit-error-rate (BER) performance of the proposed LS semi-blind detector versus SNR and A_2/A_1. As seen in Fig. 1, the performance of LS semi-blind detector is basically the same as the classic decorrelating detector. For increasing SNR, the BER of both decreases. The LS semi-blind detector has the optimum near-far resistance so that its BER remains constant with the changing amplitude of the interfering signal.

CASE 2: $\bar{\mathbf{N}} \neq \mathbf{0}$

It is assumed that the same level of AWGN exists in the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$ as in the received signal $\mathbf{r}$. The LS, TLS and MLS semi-blind detectors are used to detect the information bit of the desired user, b_1. In Fig. 2, the performance of the LS semi-blind detector is basically between that of decorrelating detector and single-user matched-filter detector. The BER performance of all the detectors becomes better when the SNR is increased. The difference from case 1 is that when the MAI becomes strong, the performance of the LS detector decreases.

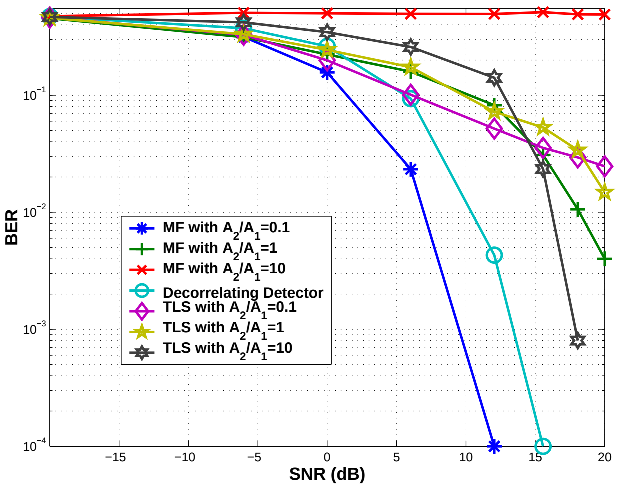

Fig. 3. BER comparison of the single-user matched filter, decorrelating detector, and the TLS semi-blind detector for the first user in a two user system with $\rho = 0.75$.

In Fig. 3, the performance of the proposed TLS semi-blind detector is evaluated. The BER performance of the TLS detector becomes better when SNR is increased. When the SNR is less than 7 dB, the BER performance of the TLS detector decreases with increasing MAI. But, when SNR is greater than about 12 dB, the BER performance of the TLS detector improves with increasing MAI. This is different from the LS semi-blind detector.

The BER performance of the proposed MLS semi-blind detector is shown in Fig. 4. The BER performance of the MLS detector improves with increasing SNR. Similar to the TLS semi-blind detector, when the SNR is less than 0 dB, the BER performance of the MLS detector decreases with increasing MAI. When the SNR is greater

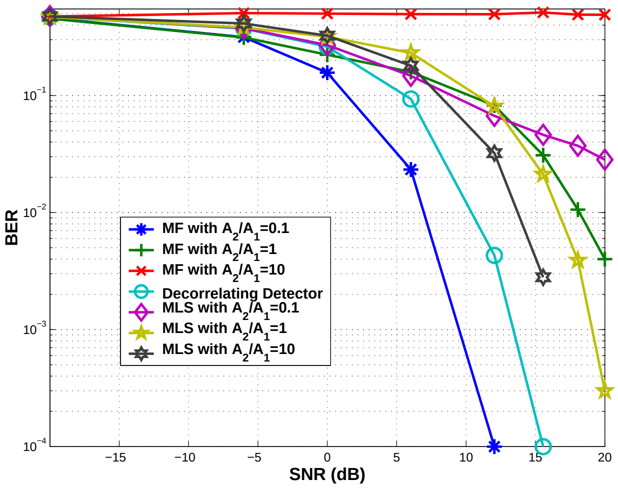

Fig. 4. BER comparison of the single-user matched filter, decorrelating detector, and the MLS semi-blind detector for the first user in a two user system with $\rho = 0.75$.

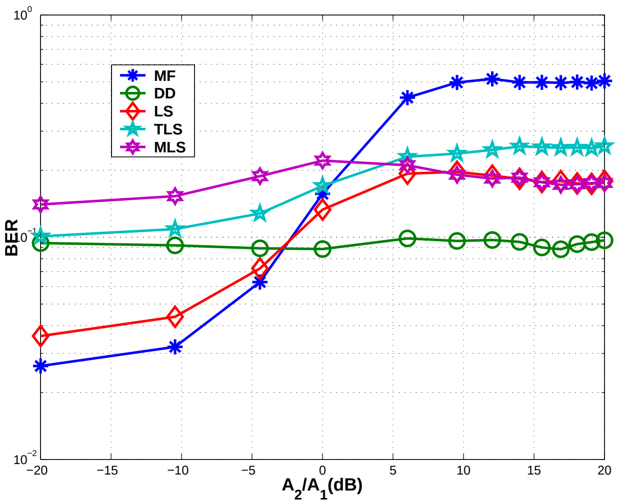

Fig. 5. Near-far resistance comparison of the single-user matched filter, decorrelating detector, and the LS, TLS and MLS semi-blind detectors for the first user in a two user system with $\rho = 0.75$ and $SNR = 6dB$.

than 13 dB, the BER performance of the MLS detector improves with increasing MAI.

In Fig. 5, the BER curves of all the proposed detectors are shown versus A_2/A_1 and are compared to the error probability of the single-user matched filter detector and the decorrelating detector. When $A_2/A_1 < -5$ dB, both the LS and the MF detector work better than the decorrelating detector. But, the performance of the TLS and MLS detectors are worse than that of the decorrelating detector in the same region. When $A_2/A_1 > 0$ dB, the performance of all the proposed semi-blind detectors is close to that of the decorrelating detector and much better than that of the single-user matched filter detector. Further, the performance of the LS detector is better than that of the other two proposed detectors. We note that the MLS detector performs closely to the LS detector.

VII. Conclusions

In this paper, we presented the LS, TLS and MLS one-shot semi-blind decorrelating detectors. In these semi-blind decorrelating detectors, only the signature waveform and received amplitude of the desired user are needed. In the performance analysis and computer simulations, the classic decorrelating detector is simply a special case of the presented LS semi-blind decorrelating detector when there is no noise in the semi-blind signature matrix. Likewise, the single-user matched filter is a special case when the SNR in the semi-blind signature matrix is close to 0. The presented semi-blind decorrelating detectors are one-shot algorithms which are simple and direct. No searching or adaptive procedure is required as in other semi-blind/blind detectors.

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