One-Shot Semi-Blind Decorrelating Detection Algorithms with Multiple Windows for Asynchroneous CDMA

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Abstract

Multiuser detection is one of the key techniques for combating multiple access interference (MAI) in CDMA systems. In this paper, we propose two effective one-shot semi-blind multiuser decorrelating detectors with multiple consecutive windows for asynchronous CDMA. Both of them are based on the classic single-truncated-window decorrelating detector and a new semi-blind signature matrix. Beside least squares (LS) estimation, a mixed least squares and total least squares (MLS) estimation scheme is also presented with different assumption of the noise in the proposed semi-blind signature matrix. Furthermore, with the analysis of the single-truncated-window scheme, the multiple-truncated-window scheme is proposed to detect several consecutive bits at the same time with LS or MLS estimation. These two detectors can achieve the same or even better performance of the classic single-truncated-window decorrelating detector in many situations. They are simple and direct. Only the desired user's timing, amplitude and signature are required. No knowledge of other users is involved. And no search or convergence procedure is employed as in other semi-blind or blind detectors. Theoretical analysis and computer simulations are also presented to support the performance of the proposed two one-shot semi-blind detectors.

1 Introduction

Direct-sequence code division multiple access (DS/CDMA) techniques have attracted increasing attention for efficient use of available bandwidth, resistance to interference and flexibility to variable traffic patterns. One of the problems of such a system is the so-called near-far problem resulting from excessive MAI energy from nearby users compared with the desired user's signal energy. Multiuser detection strategy is a method to minimize the effect of MAI and solve the near-far problem in CDMA systems without a significant reduction in the signal energies of the strong users in order for the weaker users to achieve reliable communication. It has been extensively investigated over the past several years [1], since MAI is the dominant impairment for CDMA systems and exists even in perfect power-controlled CDMA systems. Most early work on multiuser detection assumed that the receiver knew the spreading codes or had some knowledge of all users, then exploited this knowledge to combat MAI. For example, the classic decorrelating detector for the synchronous case or the single-truncated-window decorrelating detector can achieve the optimum near-far resistance and completely eliminate MAI from other users with the expense of enhancement of background noise. However, in many practical cases, especially in a dynamic environment, e.g. in the downlink of the CDMA system, it is very difficult for a mobile user to obtain accurate information on other active users in the same channel. On the other hand, the frequent use of training sequence is certainly a waste of channel bandwidth. So blind multiuser detection has been proposed. Recent research has been devoted to the multiuser blind receivers and subspace-based signature waveform estimation schemes to achieve better performance and higher capacity [2, 3, 4, 5, 6]. The Minimum output energy (MOE) method and subspace method were presented for multiuser blind detection with the knowledge of only the desired users' spreading code and possible timing.

An optimum near-far resistant multiuser detector which dose not need power control has been proposed by Verdú [7]. However, its complexity is exponential in terms of he number of users, which makes it unsuitable for practical situations. The large gaps in performance and complexity between the conventional single-user matched filter and optimum multiuser detector encourage the search for other multiuser detectors that exhibit good performance/complexity tradeoffs. Suboptimum linear detectors, which are based on linear transformation of the sampled match filter outputs, were considered in [8] for synchronous case. The decorrelating detector [8] not only is a simple and nature strategy but it is optimal according to three different criteria: least squares, near-far resistance [9] and maximum-likelihood when the received amplitudes are unknown [8]. Though it does not require knowledge of the received amplitudes, it does require the information about other active users' signature waveforms. For the asynchronous cases, Verdú suggested to use a one-shot version of his decorrelating detector, the single-truncated-window decorrelating detector, in which $1 + 2(K - 1) = 2K - 1$ filters, where K is the number of users, are matched to one full code of the desired user (without loss of generality, called the first user) and two filters matched to each of the other users. One filter is matched to the "previous" part of its code, corresponding to the time between zero and τ_k, where τ_k is the delay of user k with respect to the first user, and to the "current" part of the code, corresponding to the time between τ_k and T. In this case, it not only requires the information about other active users' signatures but also the time delay of other users with respect to the first user. However, since only the signature information of the other users is divided in a signature matrix of the nearly doubled size, the performance of this one-short single-truncated-window decorrelating detector is worse than the complete asynchronous decorrelating detector.

Multiuser blind detection using subspace techniques was first developed in depth by Wang and Poor [4, 10]. Such techniques were appropriate for the downlink environment where only the desired user's code is available. More recently, these subspace techniques were extended by Wang and Host-Madsen [11], named group multiuser blind detectors, to uplink environments where the base station knows the codes of in-cell users, but not those of users outside the cell. In the subspace-based blind detection approach [4], the linear detectors are constructed in the closed form once the signal subspace components are computed and offer lower computational complexity and better performance than the blind MOE detector. For the subspace-based blind adaptive detector, the project approximation subspace tracking deflation (PASTd) algorithm [12] is used to estimate the signal subspace.

It is shown in [2] that the MOE receiver is equivalent to the linear minimum mean square error (MMSE) detector, which is near-far resistant and has much less complexity compared the optimal multiuser detection. The major limitation of MOE schemes to multiuser blind detection is that there is a satuaration effect in the steady state, which causes a significant performance gap between the converged blind MOE and the true MMSE detector [2].

As we see, various multiuser detection schemes have been developed to combat the effects of MAI. These detection techniques either assume the knowledge of all the users in the system (Conventional) or assume the knowledge of the user of interest and without knowledge of the channel input (Blind). Due to the limitation of the blind algorithms in the presence of a large number of interferers, there is a significant performance gap between these two classes of detectors.

In this work, we consider an asynchronous DS/CDMA system and develop some partial blind detectors, a new least squares semi-blind decorrelating detector (LS-DD) and a mixed least squares and total least squares semi-blind decorrelating detector (MLS-DD), with multiple consecutive windows. They are expected to bridge the performance gap between the blind and the conventional multiuser detectors. In the proposed semi-blind multiuser detectors, besides the desired user's timing and signature, the amplitude is also required. So, we call them semi-blind detectors. In the proposed algorithms, firstly, a new semi-blind signature matrix is constructed with the desired user's signature and amplitude and several previously received signal vectors. After this, the decorrelating operation based on this new semi-blind signature is proposed. Furthermore, with the analysis of the single-truncated-window scheme, a new multi-window scheme is proposed to detect several consecutive bits at the same time. It is expected to improve the performance of the single-truncate-window algorithm. At last, both LS and MLS version of this semi-blind multi-window decorrelating detection schemes are proposed with different assumptions of the noise in the semi-blind signature matrix. These two algorithms are simple and direct detection algorithms. No knowledge of other users' information is required. And no search or converging procedure is employed as in other semi-blind/blind detectors. Theoretical analysis and computer simulations are also presented to demonstrate the performance of these two proposed semi-blind decorrelating detectors.

The rest of the paper is organized as follows. In Section II, we summarize the signal model. In Section III, we review the classic single-truncated-window decorrelating detector in multiuser detection. In Section IV, the new data model with multiple consecutive windows is discussed discussed. In Section V, the LS semi-blind multiuser decorrelating detection algorithm with multiple windows and the MLS semi-blind multiuser decorrelating detection algorithm with multiple windows are introduced. Performance analysis and simulation results are provided in Section VI and VII. Section VIII concludes this papers.

2 Data Model and Problem Description

A single-cell symbol-asynchronous DS-CDMA system over the nondispersive additive white Gaussian noise (AWGN) channel is considered. Spreading sequences are assigned to users in the system. The signature waveform of the kth user can be expressed as

$$s_k(t) \quad = \quad \sum_{l=0}^{L-1} c_k^{(l)} \psi(t - lT_c) \tag{1}$$

where $c_k^{(l)} \in \{-1/\sqrt{L},\ 1/\sqrt{L}\}$ is the lth chip of the kth user, L is the spreading gain, T_c is the chip interval and $\psi(t)$ is the chip waveform. We assume that $\psi(t)$ satisfies the Nyquist criterion for zero inter-chip interference and $\int_{-\infty}^{+\infty} |\psi(t)|^2 dt = 1$.

We consider reverse link transmission. The baseband representation of the received signal due to the kth user is given by

$$r_k(t) \quad = \quad \sum_{i=-\infty}^{+\infty} A_k b_k^{(i)} s_k(t - iT_c - \tau_k) \tag{2}$$

where $b_k^{(i)}$ is the ith bit of the kth user. We assume that the $b_k^{(i)}$ are independent and identically distributed random variables with $E\{b_k^{(i)}\} = 0$ and $E\{|b_k^{(i)}|^2\} = 1$. Also, τ_k denotes the

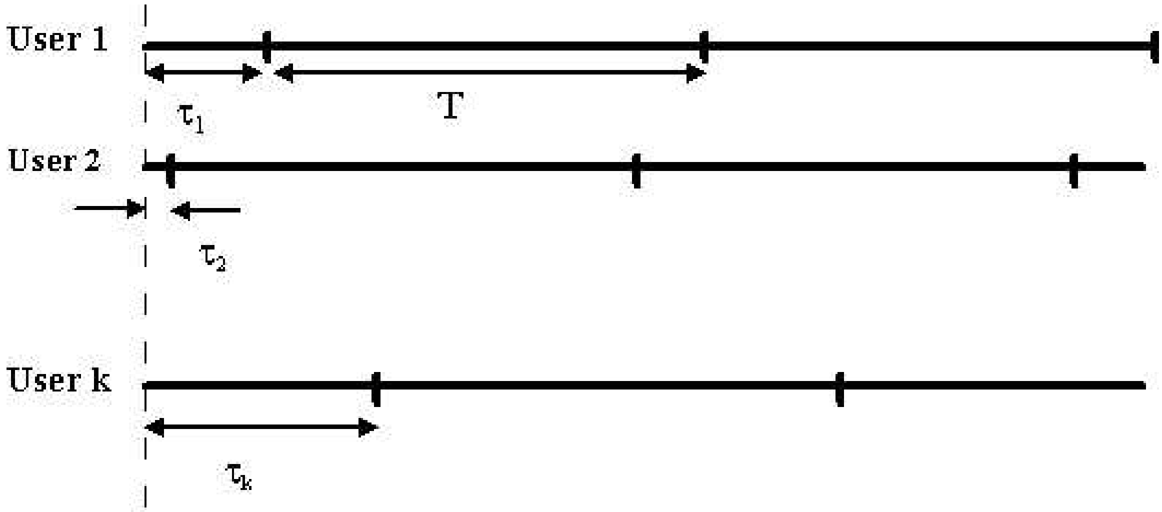

Figure 1: The demonstration of a basic asynchronous multiuser channel.

transmission delay from the kth user to the base station and A_k is the power of the received signal of the kth user. Then the baseband signal at the input to the receiver at the base station is

$$r(t) = \sum_{k=1}^{K} r_k(t) + n(t) \tag{3}$$

where $n(t)$ is additive white Gaussian noise with power spectral density σ_n^2. We assume that there are K simultaneous users in the system.

The received signal is synchronized for each user, passed through the corresponding chip matched filter (CMF), and sampled at the chip rate $1/T_c$. The vector of the output samples of the CMF for kth user in the nth symbol interval can be expressed as

$$\mathbf{r}_k^{(n)} = [\, r_k(nT + T_c + \tau_k) \quad r_k(nT + 2T_c + \tau_k) \quad \dots \quad r_k(nT + LT_c + \tau_k)\,]^T \tag{4}$$

where

$$r_k(nT + lT_c + \tau_k) = \int_{nT+lT_c+\tau_k}^{nT+(l+1)T_c+\tau_k} r_k(t)\psi(t)dt \tag{5}$$

for $1 \le l \le L$. To facilitate the analysis, we will assume the system to be chip-synchronous and restrict ourselves to the receiver that has an observation window of one symbol interval. Without loss of generality. we consider the detection of the first user. The observation window of the first user is marked with a thick line in figure 1. The signals of other users are treated as interference. A typical interferer has two different but consecutive symbols interfering the symbol of user 1, as shown in figure 1.

$$\begin{aligned} \mathbf{r}_1^{(n)} &= A_1 b_1^{(n)} \mathbf{s}_1 + \sum_{k=2}^{K} A_k b_k^{(n-1)} \mathbf{s}_{k_-} \\ &\quad \sum_{k=2}^{K} A_k b_k^{(n)} \mathbf{s}_{k_+} + \mathbf{n} \end{aligned} \tag{6}$$

where $\mathbf{s}_{k_-}$ and $\mathbf{s}_{k_+}$ are named effective signature sequences or part signature sequences that are completely determined by the spreading sequences $\mathbf{s}_k$ and the delays relative to the first user

$\tau_{k1} = \tau_k - \tau_1$, $\mathbf{n}$ is an L-dimension Gaussian vector with independent σ_n^2-variance components and $L \geq 2K - 1$.

Now, the received signal vector $\mathbf{r}_1^{(n)}$ is fed into a correlating receiver bank that is matched to each user's spreading waveform to yield the output signal vector

$$\mathbf{y}_1^{(n)} = \mathbf{R}(1)\mathbf{A}\mathbf{b}^{(n-1)} + \mathbf{R}(0)\mathbf{A}\mathbf{b}^{(n)} + \mathbf{R}^T(1)\mathbf{A}\mathbf{b}^{(n+1)} + \bar{\mathbf{n}}_n \tag{7}$$

where the zero-mean Gaussian process $\bar{\mathbf{n}}$ has autocovariance matrix

$$E\{\bar{\mathbf{n}}_i \bar{\mathbf{n}}_j^T\} = \begin{cases} \sigma^2 \mathbf{R}^T(1) & j = i+1 \\ \sigma^2 \mathbf{R}(0) & j = i \\ \sigma^2 \mathbf{R}(1) & j = i-1 \\ \mathbf{0} & |j-i| > 1 \end{cases} \tag{8}$$

and the matrices $\mathbf{R}(0)$ and $\mathbf{R}(1)$ are defined by

$$R_{ij}(0) = \begin{cases} 1 & j = i \\ \rho_{ij} & j < i \\ \rho_{ji} & j > i \end{cases} \tag{9}$$

$$R_{ij}(1) = \begin{cases} 0 & j \geq i \\ \rho_{ij} & j < i \end{cases} \tag{10}$$

where, if $i \geq j$, then we denote

$$\rho_{ij} = \int_{\tau}^{T} s_i(t) s_j(t-\tau) dt \ , \tag{11}$$

$$\rho_{ji} = \int_{0}^{\tau} s_i(t) s_j(t+T-\tau) dt \ . \tag{12}$$

Most of the linear multiuser detectors for demodulating the user's data bit $b_1^{(n)}$ in (6) is in the form of a correlator followed by a hard limiter, which can be expressed as

$$\hat{b}_1^{(n)} = \mathrm{sgn}\{\mathbf{w}_1^T \mathbf{r}_1^{(n)}\} \tag{13}$$

where $\mathbf{w}_1 \in \mathbb{R}^{L \times 1}$.

Linear multiuser detectors can be implemented in a decentralized fashion where only the user or users of interest need be demodulated.

3 Single-Truncated-Window Decorrelating Detector

The large gaps in performance and complexity between the conventional single-user matched filter and optimum multiuser detector encourage the search for other multiuser detectors that exhibit good performance/complexity tradeoffs. One of the earliest suggestions to eliminate multiuser interference with a linear receiver was proposed by Shnidman [13]. The derivation of the asymptotic efficiency of the decorrelating detector for synchronous channels and the proof of its optimum near-far resistant property are due to Verdú [9] in the case of nonsingular covariance matrices. The forerunner of the decorrelating detector in the single-user intersymbol interference (ISI) channel is the zero-forcing equalizer. And the counterpart of decovariance in antenna array subject to undesired sources is called null steering. Prior to developing the semi-blind decorrelating detector, we will discuss general decorrelating detectors in the following, which is useful in understanding the decorrelating detector.

As in equation (6), the output vector of the receiver outputs can be written as

$$\begin{aligned}
\mathbf{r}_1^{(n)} &= A_1 b_1^{(n)} \mathbf{s}_1 + \sum_{k=2}^{K} A_k [\, \mathbf{s}_{k_-} \quad \mathbf{s}_{k_+} \,] \begin{bmatrix} b_k^{(n-1)} \\ b_k^{(n)} \end{bmatrix} + \mathbf{n} \\
&= [\, \mathbf{s}_1 \quad \mathbf{s}_{2_-} \quad \mathbf{s}_{2_+} \quad \ldots \quad \mathbf{s}_{K_-} \quad \mathbf{s}_{K_+} \,] \begin{bmatrix} A_1 & & & & & \\ & A_2 & & & & \\ & & A_2 & & & \\ & & & \ddots & & \\ & & & & A_K & \\ & & & & & A_K \end{bmatrix} \begin{bmatrix} b_1^{(n)} \\ b_2^{(n-1)} \\ b_2^{(n)} \\ \vdots \\ b_K^{(n-1)} \\ b_K^{(n)} \end{bmatrix} + \mathbf{n} \\
&= \mathbf{S}_1 \mathbf{A}_1 \mathbf{b}^{(n)} + \mathbf{n}
\end{aligned} \tag{14}$$

where

$$\mathbf{S}_1 = [\, \mathbf{s}_1 \quad \mathbf{s}_{2_-} \quad \mathbf{s}_{2_+} \quad \ldots \quad \mathbf{s}_{K_-} \quad \mathbf{s}_{K_+} \,] \quad , \tag{15}$$

$$\mathbf{A}_1 = diag\{\, A_1 \quad A_2 \quad A_2 \quad \ldots \quad A_K \quad A_K \,\} \tag{16}$$

and

$$\mathbf{b}^{(n)} = [\, b_1^{(n)} \quad b_2^{(n-1)} \quad b_2^{(n)} \quad \ldots \quad b_K^{(n-1)} \quad b_K^{(n)} \,]^T \quad . \tag{17}$$

The classic one-short decorrelating detector then performs the following operation.

$$\hat{\mathbf{b}}^{(n)} = \mathrm{sgn}\{\mathbf{S}_1^{+} \mathbf{r}_1^{(n)}\} \tag{18}$$

where $[\star]^+$ denotes the Moore-Penrose generalized inverse of $\star$.

In the above decorrelating detector, the received vector $\mathbf{r}_n$ is projected on the subspace orthogonal to the codes of the other users [1, 14]. Furthermore, in [15], it is shown that the above decorrelating detector is the *oblique projection* of the desired user's signature vector. That is to project $\mathbf{s}_1$ onto the orthogonal complement subspace of the space $\mathbb{S}_1$, which is spanned by the other users' signature vectors, along the null space of the space $\mathbb{S}_1$.

The decorrelating detector is designed to completely eliminate MAI caused by other users, at the expense of enhancing the ambient noise. So, when the received amplitudes are completely unknown the decorrelating detector is a sensible choice. There are some desirable features of this multiuser detector. It does not require knowledge of the received amplitude, but it requires $\mathbf{S}^+$. It can readily be decentralized in the sense that the demodulation of each user can be implemented completely independently.

4 New Data Model With Semi-Blind Signature Matrix

Most of the multiuser detection schemes in the asynchronous case assume the knowledge of the timing, the spreading codes and/or channel parameters of all the users that contribute to the received signal. Then these receivers would exploit this knowledge to combat MAI. A second form of detectors, know as the blind detectors, which operate without the knowledge of the channel input and the information of other users. Usually, many practical systems lie in between these two extremes. The first form of detectors are too optimistic as we can not expect to know the signature waveforms of all the users, while the latter under-utilize the our knowledge of the system. In this section, we would develop semi-blind detectors with multiple consecutive windows that are expected to bridge the performance gap between the conventional detectors and blind ones.

It is widely known that the performance of the single-truncated-window decorrelating detector is worse than that of the asynchronous decorrelator in terms of both bit-error rate (BER) and near-far resistance (NFR) [1]. This is because that, in the classic one-shot single-truncated-window decorrelating detector, the energy of each interfering signal is separated in a signature matrix of the larger columns, $2K-1$, so that the single-truncated-window crosscovariance matrix may be more easily singular, even when the corresponding $K \times K$ asynchronous crosscovariance matrix is not singular. In the following, we are going to present the semi-blind detection schemes with multiple consecutive truncated-windows.

Without loss of the generality, we are going to detect the information signal bits sent by user 1 at P consecutive time intervals $t = n-P+1,\ n-P+2,\ \ldots,\ n$, where P is the length of the consecutive truncated windows. The P consecutive information bits in the signal vector $\mathbf{b}_1^{(n)}$ are detected simultaneously. To this end, the new $PL \times M$ semi-blind signature matrix $\boldsymbol{\mathcal{S}}$ is defined as

$$\begin{aligned} \boldsymbol{\mathcal{S}} &= [\,\bar{\mathbf{s}}_1 \quad \bar{\mathbf{s}}_2 \quad \bar{\mathbf{s}}_3 \quad \ldots \quad \bar{\mathbf{s}}_M\,] \\ &= [\,\mathbf{I}_P \otimes (A_1 \mathbf{s}_1) \quad \bar{\mathbf{r}}_1 \quad \bar{\mathbf{r}}_2 \quad \ldots \quad \bar{\mathbf{r}}_{M-P}\,] \\ &= [\,\mathbf{SAE} \quad \mathbf{SA}\bar{\mathbf{b}}_1 \quad \mathbf{SA}\bar{\mathbf{b}}_2 \quad \ldots \quad \mathbf{SA}\bar{\mathbf{b}}_{M-P}\,] + \bar{\mathbf{N}} \\ &= \mathbf{SA}\,[\,\mathbf{E} \quad \bar{\mathbf{b}}_1 \quad \bar{\mathbf{b}}_2 \quad \ldots \quad \bar{\mathbf{b}}_{M-P}\,] + \bar{\mathbf{N}} \\ &= \mathbf{SA}\,[\,\mathbf{E} \quad \mathbf{D}\,] + \bar{\mathbf{N}} \\ &= \mathbf{SAB} + \bar{\mathbf{N}} \end{aligned} \tag{19}$$

where $\mathbf{S} = [\,\bar{\mathbf{S}}_1 \quad \bar{\mathbf{S}}_2 \quad \bar{\mathbf{S}}_3 \quad \ldots \quad \bar{\mathbf{S}}_K\,]$ is a $PL \times (PK+K-1)$ matrix with

$$\bar{\mathbf{S}}_1 = \mathbf{I}_P \otimes \mathbf{s}_1 = diag\{\,\mathbf{s}_1 \quad \mathbf{s}_1 \quad \ldots \quad \mathbf{s}_1\,\}_{PL \times P} \tag{20}$$

$$\bar{\mathbf{S}}_k = diag\{\,\mathbf{s}_{k_-} \quad \mathbf{s}_k \quad \ldots \quad \mathbf{s}_k \quad \mathbf{s}_{k_+}\,\}_{PL \times (P+1)} \tag{21}$$

$$\mathbf{A} = diag\{\,\bar{\mathbf{A}}_1 \quad \bar{\mathbf{A}}_2 \quad \ldots \quad \bar{\mathbf{A}}_K\,\}_{(PK+K-1)\times(PK+K-1)} \tag{22}$$

$$\bar{\mathbf{A}}_1 = diag\{\,A_1 \quad A_1 \quad \ldots \quad A_1\,\}_{P \times P} \tag{23}$$

$$\bar{\mathbf{A}}_k = diag\{\,A_k \quad A_k \quad \ldots \quad A_k\,\}_{(P+1)\times(P+1)} \tag{24}$$

$k = 2,\ 3,\ \ldots,\ K$ and $PL \geq M \geq PK+K-1$, and $\bar{\mathbf{r}}_i$, $i = 1,\ 2,\ \ldots,\ M-P$, are the arbitrary received vectors, where $\otimes$ denotes the Kronecker product. $\mathbf{E} = [\,\mathbf{I}_P \quad \mathbf{0}\,]^T$, the $(PK+K-1)\times 1$ vector $\bar{\mathbf{b}}_i$ is the corresponding information vector which includes $(P+1)$ consecutive bits sent by the other $K-1$ users, $\mathbf{D} = [\bar{\mathbf{D}}^T\ \tilde{\mathbf{D}}^T]^T$, the $(M-P)\times P$ vector $\bar{\mathbf{D}}$ is the information vector consist of the known bits sent by the desired user previously, $\mathrm{rank}\{\tilde{\mathbf{D}}\} = PK+K-P-1$, $\bar{\mathbf{N}} = [\mathbf{0}\ \tilde{\mathbf{N}}]$ and

$$\begin{aligned} \mathbf{B} &= [\,\mathbf{E} \quad \mathbf{D}\,] \\ &= \begin{bmatrix} \mathbf{E} & \begin{matrix} \bar{\mathbf{D}} \\ \tilde{\mathbf{D}} \end{matrix} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{C} & \\ \mathbf{0} & \tilde{\mathbf{D}} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{I}_P & \bar{\mathbf{D}} \\ \mathbf{0} & \tilde{\mathbf{D}} \end{bmatrix} \end{aligned} \tag{25}$$

and $\text{rank}\{\mathbf{B}\} = PK + K - 1$.

As in equation (6) and (19), the relationship between the received signal vector $\mathbf{r}_n$, which consists of P consecutively received signal vectors of the desired user, and the new semi-blind signature matrix $\boldsymbol{\mathcal{S}}$ can be expressed as

$$\begin{aligned}
\mathbf{r}_n &= [\mathbf{r}_1^{(n)T} \quad \mathbf{r}_1^{(n-1)T} \quad \dots \quad \mathbf{r}_1^{(n-P+1)T}]^T \\
&= \mathbf{SAb}_n + \mathbf{n} \\
&= \mathbf{SABB}^+\mathbf{b}_n + \mathbf{n} \\
&= (\boldsymbol{\mathcal{S}} - \bar{\mathbf{N}})\mathbf{B}^+\mathbf{b}_n + \mathbf{n} \\
&= \boldsymbol{\mathcal{S}}\mathbf{B}^+\mathbf{b}_n - \bar{\mathbf{N}}\mathbf{B}^+\mathbf{b}_n + \mathbf{n} \\
&= \boldsymbol{\mathcal{S}}\mathbf{d}_n + \tilde{\mathbf{n}}
\end{aligned} \tag{26}$$

where

$$\mathbf{b}_n = \begin{bmatrix} \begin{bmatrix} b_1^{(n)} \\ b_1^{(n-1)} \\ \vdots \\ b_1^{(n-P+1)} \end{bmatrix}^T & \begin{bmatrix} b_2^{(n)} \\ b_2^{(n-1)} \\ \vdots \\ b_2^{(n-P+1)} \\ b_2^{(n-P)} \end{bmatrix}^T & \dots & \begin{bmatrix} b_K^{(n)} \\ b_K^{(n-1)} \\ \vdots \\ b_K^{(n-P+1)} \\ b_K^{(n-P)} \end{bmatrix}^T \end{bmatrix}^T . \tag{27}$$

and $\mathbf{d}_n$ denotes the new $M \times 1$ detection vector and is defined as

$$\begin{aligned}
\mathbf{d}_n &= \mathbf{B}^+\mathbf{b}_n \\
&= [\mathbf{E} \quad \mathbf{D}]^+ \mathbf{b}_1^{(n)} \\
&= \begin{bmatrix} \mathbf{I}_P & \bar{\mathbf{D}} \\ \mathbf{0} & \tilde{\mathbf{D}} \end{bmatrix}^+ \begin{bmatrix} \mathbf{b}_1^{(n)} \\ \tilde{\mathbf{b}}_n \end{bmatrix}
\end{aligned} \tag{28}$$

and $\tilde{\mathbf{n}}$ is the new noise vector and defined as

$$\tilde{\mathbf{n}} = \mathbf{n} - \bar{\mathbf{N}}\mathbf{B}^+\mathbf{b}_n \tag{29}$$

With the following lemma, it is easy to see that the new semi-blind noise item $\tilde{\mathbf{n}}$ is enhanced, compared with the former noise item $\mathbf{n}$. This enhancement is because there is the noise $\bar{\mathbf{N}}$ existing in the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$.

Now, the following result can be easily proved.

Proposition 1. *The Moor-Penrose general inverse of* $\mathbf{B}$ *is*

$$\mathbf{B}^+ = \begin{bmatrix} \mathbf{E} & \begin{matrix} \bar{\mathbf{D}}\tilde{\mathbf{D}}^+ \\ \tilde{\mathbf{D}}^+ \end{matrix} \end{bmatrix} \tag{30}$$

Proof. Based on the definition of $\mathbf{B}$,

$$\mathbf{B} = \begin{bmatrix} \mathbf{E} & \begin{matrix} \bar{\mathbf{D}} \\ \tilde{\mathbf{D}} \end{matrix} \end{bmatrix} \tag{31}$$

so that,

$$\begin{bmatrix} \mathbf{E} & \begin{matrix} \bar{\mathbf{D}} \\ \tilde{\mathbf{D}} \end{matrix} \end{bmatrix} \begin{bmatrix} \mathbf{E} & \begin{matrix} \bar{\mathbf{D}}\tilde{\mathbf{D}}^+ \\ \tilde{\mathbf{D}}^+ \end{matrix} \end{bmatrix} = \begin{bmatrix} \mathbf{I}_P & \bar{\mathbf{D}} \\ \mathbf{0} & \tilde{\mathbf{D}} \end{bmatrix} \begin{bmatrix} \mathbf{I}_P & -\bar{\mathbf{D}}\tilde{\mathbf{D}}^+ \\ \mathbf{0} & \tilde{\mathbf{D}}^+ \end{bmatrix} = \mathbf{I}_{PK+K-1} \tag{32}$$

where $\mathbf{I}_{PK+K-1}$ is the unitary matrix of the size $(PK + K - 1) \times (PK + K - 1)$. $\square$

So, with the above lemma, the detection vector $\mathbf{d}_n$ can be re-written as

$$\begin{aligned}\mathbf{d}_n &= \begin{bmatrix}\mathbf{d}_n^1\\ \tilde{\mathbf{d}}_n\end{bmatrix}\\ &= \begin{bmatrix}\mathbf{E} & \bar{\mathbf{D}}\\ & \tilde{\mathbf{D}}\end{bmatrix}^+ \begin{bmatrix}\mathbf{b}_1^{(n)}\\ \tilde{\mathbf{b}}_n\end{bmatrix}\\ &= \begin{bmatrix}\mathbf{I}_P & -\bar{\mathbf{D}}\tilde{\mathbf{D}}^+\\ \mathbf{0} & \tilde{\mathbf{D}}^+\end{bmatrix}\begin{bmatrix}\mathbf{b}_1^{(n)}\\ \tilde{\mathbf{b}}_n\end{bmatrix}\\ &= \begin{bmatrix}\mathbf{b}_1^{(n)} - \bar{\mathbf{D}}\tilde{\mathbf{D}}^+\tilde{\mathbf{b}}_n\\ \tilde{\mathbf{D}}^+\tilde{\mathbf{b}}_n\end{bmatrix}\end{aligned} \tag{33}$$

where $\mathbf{d}_n^1$ is the first consecutive P elements of the detection vector $\mathbf{d}_n$ as

$$\mathbf{d}_n^1 = \mathbf{b}_1^{(n)} - \bar{\mathbf{D}}\tilde{\mathbf{D}}^+\tilde{\mathbf{b}}_n \tag{34}$$

and $\tilde{\mathbf{d}}_n$ is the corresponding complement vector as

$$\tilde{\mathbf{d}}_n = \tilde{\mathbf{D}}^+\tilde{\mathbf{b}}_n \tag{35}$$

Now, the following result can be very easily reached.

Lemma 1. *The bits vector $\mathbf{b}_1^{(n)}$ which consists of the bits sent by user 1 at P consecutive time intervals $t = n - P + 1,\ n - P + 2,\ \ldots,\ n$ can be got with the following equation.*

$$\begin{aligned}\mathbf{b}_1^{(n)} &= \mathbf{C}\mathbf{d}_n\\ &= [\mathbf{I}_P \quad \bar{\mathbf{D}}]\mathbf{d}_n\\ &= \mathbf{d}_n^1 + \bar{\mathbf{D}}\tilde{\mathbf{d}}_n\end{aligned} \tag{36}$$

By now, as we may see, after the definition of the new semi-blind signature matrix $\boldsymbol{\mathcal{S}}$ in equation (19), the form of the classic multiuser detection model is still kept as in equation (26). But the different is that, the information bit vector $\mathbf{b}_n$ is replaced by the detection vector $\mathbf{d}_n$ as in equation (28) and the original AWGN noise vector $\mathbf{n}$ is replace by the new noise vector $\tilde{\mathbf{n}}$ as in equation (29). Fortunately, with lemma 1, it still is possible for us to calculate the bit vector $\mathbf{b}_1^{(n)}$ with the detection vector $\mathbf{d}_n$ and the previously detected bits matrix $\mathbf{C}$. So, the next question is how to estimate the detection vector $\mathbf{d}_n$ as efficiently as possible. In the following sections, there are two estimation schemes, LS and MLS estimation, proposed. With different estimation of $\mathbf{d}_n$, two different semi-blind decorrelating detectors are proposed.

On the other hand, as we may see, the classic single-truncated-window scheme for the asynchronous case is just the special case of the presented multi-window scheme with $P = 1$. In the single-truncated-window scheme, there is no other user's complete signature in the signature matrix. But, in the presented multi-window scheme, there are $P - 1$ complete signature vectors of any other user in the new constructed signature matrix. Hence, the inverse of the multi-window signature matrix is not so easily singular as that of the single-truncated-window signature matrix.

5 One-Shot LS/MLS Semi-Blind Decorrelating Detectors

In the previous section, the principles of the proposed semi-blind decorrelating detection scheme are described. With lemma 1, the detection of the desired information bit vector $\mathbf{b}_1^{(n)}$ is decided by the estimation of the detection vector $\mathbf{d}_n$. In this section, we are going to propose two algorithms to estimate $\mathbf{d}_n$, which are based on the difference assumptions of the noise matrix $\bar{\mathbf{N}}$ in the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$. following the different estimation schemes of the detection vector $\mathbf{d}_n$, two different semi-blind detectors are proposed.

5.1 Least Squares Semi-Blind Decorrelator Detector

At first, we assume the measurements of $\boldsymbol{\mathcal{S}}$ is assumed to be free of error and all errors are confined to the received vector $\mathbf{r}_n$ as in equation (26). Then the following LS estimation of the detection vector $\mathbf{d}_n$ is proposed.

Lemma 2. *[16] Suppose* $\mathbf{U}^T\boldsymbol{\mathcal{S}}\mathbf{V} = \boldsymbol{\Sigma}$ *is the SVD of* $\boldsymbol{\mathcal{S}} \in \mathbb{R}^{L\times K}$ *with* $r = rank(\boldsymbol{\mathcal{S}})$*. And if* $\mathbf{U} = [\,\mathbf{u}_1 \quad \mathbf{u}_2 \quad \ldots \quad \mathbf{u}_L\,]$, $\mathbf{V} = [\,\mathbf{v}_1 \quad \mathbf{v}_2 \quad \ldots \quad \mathbf{v}_K\,]$, $\boldsymbol{\Sigma} = diag\{[\,\sigma_1 \quad \ldots\sigma_r \quad 0 \quad \ldots \quad 0]\}$ *and* $\mathbf{r}_n \in \mathbb{R}^{L\times 1}$*, then*

$$\mathbf{d}_{LSn} \quad = \quad \sum_{i=1}^{r} \frac{\mathbf{u}_i^T\mathbf{r}_n}{\sigma_i}\mathbf{v}_i \quad = \quad \boldsymbol{\mathcal{S}}^{+}\mathbf{r}_n \tag{37}$$

minimizes $\|\boldsymbol{\mathcal{S}}\mathbf{d}_n - \mathbf{r}_n\|_2$ *and has the smallest 2-norm of all minimizers. Moreover*

$$\varepsilon_{LS}^2 \quad = \quad \min_{\mathbf{x}\in\mathbb{R}} \|\boldsymbol{\mathcal{S}}\mathbf{x} - \mathbf{r}_n\|_2^2 \quad = \quad \sum_{i=r+1}^{L} (\mathbf{u}_i^T\mathbf{r}_n)^2 \tag{38}$$

Proof. For any $\mathbf{x} \in \mathbb{R}^{K\times 1}$, we have

$$\begin{aligned} \|\boldsymbol{\mathcal{S}}\mathbf{x} - \mathbf{r}_n\|_2^2 &= \|(\mathbf{U}^T\boldsymbol{\mathcal{S}}\mathbf{V})(\mathbf{V}^T\mathbf{x}) - \mathbf{U}^T\mathbf{r}_n\|_2^2 \\ &= \|\boldsymbol{\Sigma}\alpha - \mathbf{U}^T\mathbf{r}_n\|_2^2 \\ &= \sum_{i=1}^{r}(\sigma_i\alpha_i - \mathbf{u}_i^T\mathbf{r}_n) + \sum_{i=r+1}^{m}(\mathbf{u}_i^T\mathbf{r}_n)^2 \end{aligned} \tag{39}$$

where $\alpha = \mathbf{V}^T\mathbf{x}$. Clearly, if $\mathbf{x}$ solves the LS problem, then $\alpha_i = \mathbf{u}_i^T\mathbf{r}_n/\sigma_i$ for $i = 1,\ 2,\ \ldots,\ r$. If we set $\alpha_{r+1} = \alpha_{r+2} = \ldots = \alpha_n$, then the resulting $\mathbf{x} = \mathbf{d}_{LSn}$ clearly has minimal 2-norm. □

So, the least squares estimation of $\mathbf{d}_n$ is

$$\begin{aligned} \mathbf{d}_{LSn} &= \boldsymbol{\mathcal{S}}^{+}\mathbf{r}_n \\ &= \mathbf{d}_n + \boldsymbol{\mathcal{S}}^{+}\tilde{\mathbf{n}} \end{aligned} \tag{40}$$

And, the linear filter bank representation $\mathbf{W}_{LS}$ in the proposed LS semi-blind decorrelating detector is

$$\mathbf{W}_{LS} \quad = \quad (\mathbf{C}\boldsymbol{\mathcal{S}}^{+})^T \tag{41}$$

And, the bit $\mathbf{b}_1^{(n)}$ sent by the first user during the time interval $t \in [(n-P)T,\ nT]$ can be detected with the following equation.

$$\begin{aligned} \hat{\mathbf{b}}_{LS1}^{(n)} &= \operatorname{sgn}\{\mathbf{W}_{LS}^T\mathbf{r}_n\} \\ &= \operatorname{sgn}\{\mathbf{C}\mathbf{d}_{LSn}\} \\ &= \operatorname{sgn}\{[\,\mathbf{I}_P \quad \bar{\mathbf{D}}\,]\hat{\mathbf{d}}_n\} \\ &= \operatorname{sgn}\{\mathbf{b}_1^{(n)} + [\,\mathbf{I}_P \quad \bar{\mathbf{D}}\,]\boldsymbol{\mathcal{S}}^{+}\tilde{\mathbf{n}}\} \end{aligned} \tag{42}$$

5.2 Mixed Least Squares and Total Least Squares Semi-Blind Decorrelating Detector

It is easy to find that the above least squares estimation of detection matrix $\mathbf{d}_n$ defined in (26), (28) and (40) is the solution to the following equation.

$$\min_{\mathbf{d}_n} \|\mathbf{r}_n - \boldsymbol{\mathcal{S}}\mathbf{d}_n\|_2 \quad \text{subject to} \quad \mathbf{r}_n \subseteq \mathbb{R}(\boldsymbol{\mathcal{S}}) \tag{43}$$

and it assumed the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$ is error-free. Obviously, this assumption is not true with its definition in equation (19). There is the noise or error matrix $\bar{\mathbf{N}}$ exiting. Furthermore, $\mathbf{r}_n$ could also be expressed as

$$\begin{aligned} \mathbf{r}_n &= \mathbf{SAb}_n + \mathbf{n} \\ &= \mathbf{SABB}^{-1}\mathbf{b}_n + \mathbf{n} \\ &= (\boldsymbol{\mathcal{S}} - \bar{\mathbf{N}})\mathbf{B}^{-1}\mathbf{b}_n + \mathbf{n} \\ &= \hat{\boldsymbol{\mathcal{S}}}\mathbf{d}_n + \mathbf{n} \end{aligned} \tag{44}$$

where $\hat{\boldsymbol{\mathcal{S}}} = \boldsymbol{\mathcal{S}} - \bar{\mathbf{N}} = \mathbf{SAB}$.

Now, it is easy to see that, except the first P columns of $\boldsymbol{\mathcal{S}}$ are exactly known to be noise/error-free, there are noise existing in each element of its rest $M - P$ columns. To maximize the accuracy of the estimated detection vector $\mathbf{d}_n$, it is natural to seek the best fitting solution that is appropriate to the received signal vector $\mathbf{r}_n$ and the polluted $M - P$ columns in the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$, while keeping the exactly known P columns of $\boldsymbol{\mathcal{S}}$ unperturbed. Thus, the problem to estimate the detection vector $\mathbf{d}_n$ can easily be transformed into the following MLS problem from equation (43).

$$\min_{\hat{\bar{\boldsymbol{\mathcal{S}}}},\, \mathbf{d}_n} \left\| [\,\bar{\boldsymbol{\mathcal{S}}} \quad \mathbf{r}_n\,] - [\,\hat{\bar{\boldsymbol{\mathcal{S}}}} \quad [\,\bar{\mathbf{S}}_1\bar{\mathbf{A}}_1 \quad \hat{\bar{\boldsymbol{\mathcal{S}}}}\,]\mathbf{d}_n\,] \right\|_2 \quad \text{subject to} \quad \mathbf{r}_n \subseteq \mathbb{R}([\,\bar{\mathbf{S}}_1\bar{\mathbf{A}}_1 \quad \hat{\bar{\boldsymbol{\mathcal{S}}}}\,]) \tag{45}$$

This MLS problem can be solved with the following lemma.

Lemma 3. *[17] Consider the mixed least squares and total least squares problem in equation (45) and perform the householder transformations Q on the matrix $[\,\boldsymbol{\mathcal{S}} \quad \mathbf{r}\,]$ so that*

$$Q^H[\,\bar{\mathbf{S}}_1\bar{\mathbf{A}}_1 \quad \bar{\boldsymbol{\mathcal{S}}} \quad \mathbf{r}_n\,] = \begin{bmatrix} \mathbf{R}_{11} & \mathbf{R}_{12} & \mathbf{R}_{1r} \\ \mathbf{0} & \mathbf{R}_{22} & \mathbf{R}_{2r} \end{bmatrix} \tag{46}$$

where $\mathbf{R}_{11}$ is a $P\times P$ upper triangle matrix, $\mathbf{R}_{12}$ is a $P\times(M-P)$ matrix, $\mathbf{R}_{22}$ is a $(L-P)\times(M-P)$ matrix , $\mathbf{R}_{1r}$ is a $P\times 1$ vector and $\mathbf{R}_{2r}$ is a $(L-P)\times 1$ vector.

Denote σ' the smallest singular value of $\mathbf{R}_{22}$ and σ the smallest singular value of $[\,\mathbf{R}_{22} \quad \mathbf{R}_{2r}\,]$. If $\sigma' > \sigma$, then the MLS solution uniquely exists and is given by

$$\mathbf{d}_{MLSn} = \left(\boldsymbol{\mathcal{S}}^T\boldsymbol{\mathcal{S}} - \sigma^2\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{M-P}\end{bmatrix}\right)^{-1}\boldsymbol{\mathcal{S}}^T\mathbf{r}_n\ . \tag{47}$$

So, the linear filter representation of the presented MLS semi-blind decorrelating detector is

$$\mathbf{W}_{MLS} = \boldsymbol{\mathcal{S}}\left(\boldsymbol{\mathcal{S}}^T\boldsymbol{\mathcal{S}} - \sigma^2\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{M-P}\end{bmatrix}\right)^{-1}\mathbf{C}^T \tag{48}$$

And, the bit $\mathbf{b}_1^{(n)}$ sent by the first user during the time interval $t \in [(n-P)T \;\; nT]$ can be detected with the following equation.

$$\begin{aligned}\hat{\mathbf{b}}_{MLS1}^{(n)} &= \text{sgn}\{\mathbf{W}_{MLS}^T \mathbf{r}_n\} \\ &= \text{sgn}\left\{[\,\mathbf{I}_P \quad \bar{\mathbf{D}}\,]\left(\boldsymbol{\mathcal{S}}^T\boldsymbol{\mathcal{S}} - \sigma^2\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{M-P}\end{bmatrix}\right)^{-1}\boldsymbol{\mathcal{S}}^T\mathbf{r}_n\right\} \\ &= \text{sgn}\{\mathbf{C}\mathbf{d}_{MLSn}\} \\ &= \text{sgn}\{[\,\mathbf{I}_P \quad \bar{\mathbf{D}}\,]\mathbf{d}_{MLSn}\}\ .\end{aligned} \tag{49}$$

6 Performance Analysis

6.1 The Relationship Between the Semi-Blind Signature Matrix $\boldsymbol{\mathcal{S}}$ and the Signature Matrix S

As we may see, one of the most different between the classic single-truncated-window decorrelating detector and the proposed semi-blind decorrelating detector is the definition of the signature matrix. The original signature matrix $\mathbf{S}$ is defined with the desired user's signature vector as the first column and the other users' signatures as the rest columns. When $P = 1$, the proposed semi-blind signature matrix $\boldsymbol{\mathcal{S}}$ is composed with the desired user's signature as the first column and $M-1$ uncorrelated previously received vector $\bar{\mathbf{r}}_k$ as the rest columns. When $\bar{\mathbf{N}} = \mathbf{0}$, it is very easy to see that these two signal subspaces, span$\{\mathbf{S}\}$ and span$\{\boldsymbol{\mathcal{S}}\}$, are the same one.

6.2 The Relationship Between Decorrelating Detector $\mathbf{w}_{DD}$ and LS Semi-Blind Detector $\mathbf{w}_{LS}$

When there is no noise the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$, $\hat{\boldsymbol{\mathcal{S}}} = \boldsymbol{\mathcal{S}}$ and $\tilde{\mathbf{n}} = \mathbf{n}$. With the following lemma, we can see that, when $P = 1$, the presented LS semi-blind decorrelating detector actually is same to the classic single-truncated-window decorrelating detector at this time.

Lemma 4. *When $\bar{\mathbf{N}} = \mathbf{0}$ and $P = 1$, there is the following relationship between the desired user's LS semi-blind decorrelating detector $\mathbf{w}_{LS}$ and its classic single-truncated-window decorrelating detector $\mathbf{w}_{DD}$.*

$$\mathbf{w}_{LS} = A_k^{-1}\mathbf{w}_{DD} \tag{50}$$

Furthermore, the classic decorrelating detector is a special case of the semi-blind decorrelating detector with $\mathbf{B} = \mathbf{I}$. And the output of the limiter of the LS semi-blind decorrelating detector is

$$\hat{b}_{LSk}^{(n)} = sgn\{b_k^{(n)} + A_k^{-1}[\mathbf{S}^+\mathbf{n}]_k\} \tag{51}$$

At this time, the bit-error-rate $P_{ek}^{LS}(\sigma_n)$ of the kth user in the presented LS semi-blind decorrelating detector is

$$\begin{aligned}P_{ek}^{LS}(A_k,\ \sigma_n) &= P_{ek}^{DD}(A_k,\ \sigma_n) \\ &= Q\left(\frac{A_k}{\sigma_n\sqrt{R_{kk}^+}}\right) \\ &= Q\left(\frac{A_k}{\sigma_n}\sqrt{1 - \mathbf{a}_k^T\mathbf{R}_k^{-1}\mathbf{a}_k}\right)\end{aligned} \tag{52}$$

where R^+_{kk} *is a shorthand for* $(\mathbf{R}^{-1})_{kk}$, *the* k*th row and* k*th column element in the matrix* $\mathbf{R}^{-1}$, $\mathbf{a}_k$ *is the* k*th column of* $\mathbf{R} = \mathbf{S}^T\mathbf{S}$ *without the diagonal element and* $\mathbf{R}_k$ *is the* $(K-1)\times(K-1)$ *matrix that results by striking out the* k*th row and column from* $\mathbf{R}$.

Thus, we can see that, without noise in the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$ and $P = 1$, the presented LS semi-blind decorrelating detector has the same performance to the classic single-truncated-window decorrelating detector. The multiuser efficiency of the presented LS semi-blind detector in this case is

$$\eta_k^{LS} \quad = \quad \frac{1}{R^+_{kk}} \quad . \tag{53}$$

which does not depend on either the noise or the interfering powers. So, its asymptotic multiuser efficiency and near-far resistance is

$$\bar{\eta}_k^{LS} \quad = \quad \frac{1}{R^+_{kk}} \quad . \tag{54}$$

Thus, the presented LS semi-blind detector at this time achieves the maximum near-far resistance as the classic single-truncated-window decorrelating detector does.

6.3 The Relationship Between $b_1^{(n)}$ and $\mathbf{d}_n$

With lemma 1, it is easy to see that $\mathbf{d}_n$ plays the key pole in the proposed semi-blind detectors. When there is no estimation error in the detection vector $\mathbf{d}_n$, with the lemma 1, the desired bit $b_1^{(n)}$ could directly be estimated from the detection vector $\mathbf{d}_n$ and $\mathbf{c}$. When the detection vector $\mathbf{d}_n$ cannot be exactly estimated, the following lemma describes the relationship between the estimation errors of $b_1^{(n)}$ and $\mathbf{d}_n$.

Lemma 5. *The following relationship between the estimation errors of* $b_1^{(p)}$, $p = n,\ n-1,\ \ldots,$ $n-P+1$, *and* $\mathbf{d}_n$ *is*

$$\Delta b_1^{(p)} \quad \leq \quad \|\Delta\tilde{\mathbf{d}}_n\|_1 + |\mathbf{d}_n^{(n-p+1)}| \tag{55}$$

where $\Delta b_1^{(p)} = \hat{b}_1^{(p)} - b_1^{(p)}$, $\Delta\tilde{\mathbf{d}}_n = \hat{\tilde{\mathbf{d}}}_n - \tilde{\mathbf{d}}_n$, $\mathbf{d}_n^{(n-p+1)}$ *denotes the* $(n-p+1)$*th element of* $\mathbf{d}_n$ *and* $\|\star\|_1$ *denotes the 1-norm of the vector* $\star$.

6.4 The Relationship between the Noise Items $\tilde{\mathbf{n}}$ and $\mathbf{n}$

With the following lemma, it is easy to see that the new semi-blind noise item $\tilde{\mathbf{n}}$ is enhanced compared to the former noise item $\mathbf{n}$. This enhancement is because there is the noise $\bar{\mathbf{N}}$ existing in the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$.

Lemma 6. *The mean of the semi-blind noise item* $\tilde{\mathbf{n}}$ *which is defined in equation (29) is*

$$\tilde{m} \quad = \quad E\{\tilde{\mathbf{n}}\} \quad = \quad 0 \tag{56}$$

The variance of the semi-blind noise item $\tilde{\mathbf{n}}$ *satisfies the following inequation*

$$\max\{var\{\tilde{\mathbf{n}}\}\} \quad = \quad \max\{E\{(\tilde{\mathbf{n}}-\tilde{m})^2\}\} \quad \leq \quad \sigma_n^2 + (P+1)(K-1)\|\tilde{\mathbf{D}}^+\|_2^2\sigma_{\bar{n}}^2 \tag{57}$$

where $\max\{\star\}$ *denotes the maximum item in the vector* $\star$ *and* $\sigma_{\bar{n}}^2$ *is the power of the noise item* $\bar{\mathbf{N}}$ *in the semi-blind signature matrix* $\boldsymbol{\mathcal{S}}$.

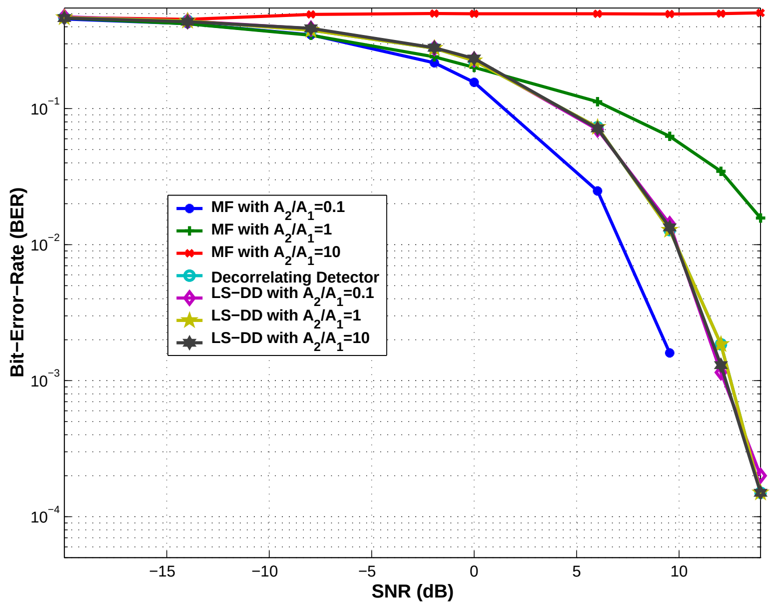

Figure 2: Bit-error-rate comparison of the signal-user matched filter, decorrelating detector, and the presented LS semi-blind detector with $P = 1$ for the first user within two users.

7 Computer Simulations

In this section, various computer simulations and analytical results are presented. In the computer simulations, two users are sending the signals in asynchronous CDMA system. The spreading gain $g = 24$. the covariance matrix between these two users are $\mathbf{R}$.

$$
\begin{aligned}
\mathbf{R} &= \begin{bmatrix} \mathbf{s}_1^T \\ \mathbf{s}_{2-}^T \\ \mathbf{s}_{2+}^T \end{bmatrix} \begin{bmatrix} \mathbf{s}_1 & \mathbf{s}_{2-} & \mathbf{s}_{2+} \end{bmatrix} \\
&= \begin{bmatrix} 1.0000 & -0.6250 & -0.0417 \\ -0.6250 & 0.8750 & 0 \\ -0.0417 & 0 & 0.1250 \end{bmatrix}
\end{aligned} \tag{58}
$$

and the channel is AWGN channel. We do the simulations with $P = 1, 2, 3, 4, 5, 7$ and 9. Correspondingly, the number of columns in the semi-blind signature matrix is $M = (P+1) \times K - 1$. We will compare our algorithms with single-user matched filter (MF) detector and the single-truncated-window decorrelating detector (DD).

case 1: $\bar{\mathrm{N}} = 0$

In this case, we suppose that there is no noise in the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$ or the noise is negligible, compared to the noise in the received signal vectors. With the theoretical analysis before, since there is no noise or the noise is negligible in the semi-blind signature matrix, only the LS semi-blind detector is examined and compared to the single-user matched filter detector and the single-truncated-window decorrelating detector in this case.

As we see in figure 2, the performance of the LS semi-blind detector with $P = 1$ is very close to that of the single-truncated window decorrelating detector. This also supports that the classic single-truncated-window decorrelating detector just is a special case of the proposed LS-DD

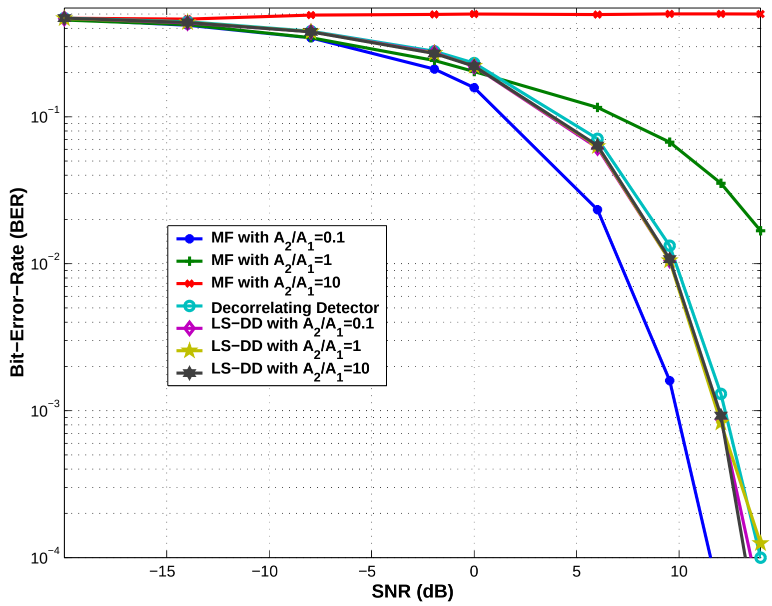

Figure 3: Bit-error-rate comparison of the signal-user matched filter, decorrelating detector, and the presented LS semi-blind detector with $P = 4$ for the first user within two users.

detector with $\bar{\mathbf{N}} = \mathbf{0}$ and $\mathbf{B} = \mathbf{I}$. Comparing figure 3 to figure 2, the performance of the LS-DD with four consecutive windows ($P = 4$) is a little bit better than that of the LS-DD with single truncated window ($P = 1$) and the classic single-truncated-window decorrelating detector. As we analyzed before, it is because that more complete signatures of other users are employed in th semi-blind signature matrix $\boldsymbol{\mathcal{S}}$.

In figure 4, the performance of the proposed LS semi-blind detector is checked with the changing of the window size P. It is easy to see, that, when the single window is employed ($P = 1$), the performance of the proposed LS semi-blind detector is same to that of the classic decorrelating detector. When multiple windows are employed and $P > 1$, the performance of the proposed LS semi-blind detector is better than that of the classic decorrelating detector. And the BER performance of the LS semi-blind detector and the classic decorrelating detector is kept unchanged against A_2/A_1.

Now, the NFR performance of the signal-user matched filter, the classic single-truncated-window decorrelating detector and the proposed LS semi-blind detector are checked in figure 5. As we see, the classic single-truncated-window decorrelating detector and the proposed LS semi-blind detector have the same optimum near-far resistance and their BERs are not change against A_2/A_1. But the BER performance of the MF detector is decreasing when A_2/A_1 becomes larger. Furthermore, the performance of the proposed LS semi-blind detector with $P = 4$ is better than that of the classic single-window decorrelating detector. And the proposed LS semi-blind detector with $P = 1$ has the similar performance to the classic single-window decorrelating detector.

case 2: $\bar{\mathbf{N}} \neq \mathbf{0}$

In the case, we are dealing some more practical situations. So, there would be the same level white Gaussian noise existing in the blind signature matrix $\boldsymbol{\mathcal{S}}$ as in the received signal vectors. Then, both LS and MLS semi-blind detectors are used to detect the expected signal.

In figure 6 and 7, the performance of the proposed LS semi-blind detector with $P = 1$

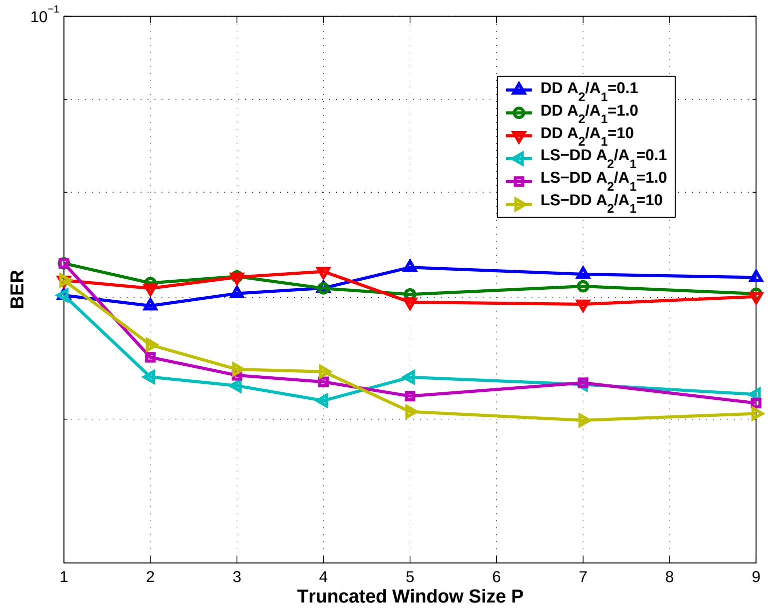

Figure 4: Bit-error-rate comparison of the classic signal-truncated-window decorrelating detector and the proposed LS semi-blind detector against the window size P.

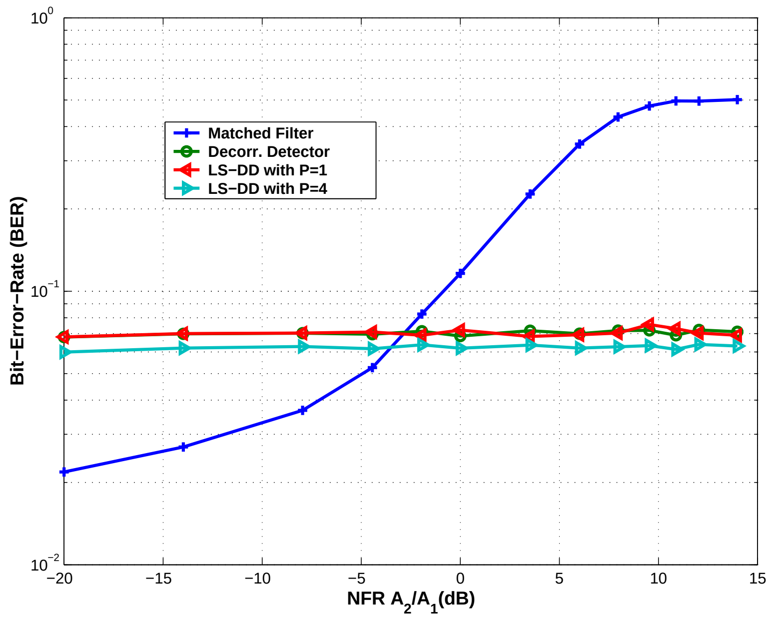

Figure 5: NFR performance of the signal-user matched filter, decorrelating detector, and the proposed LS semi-blind detector with $P = 1$ and $P = 4$ for the first user within two users.

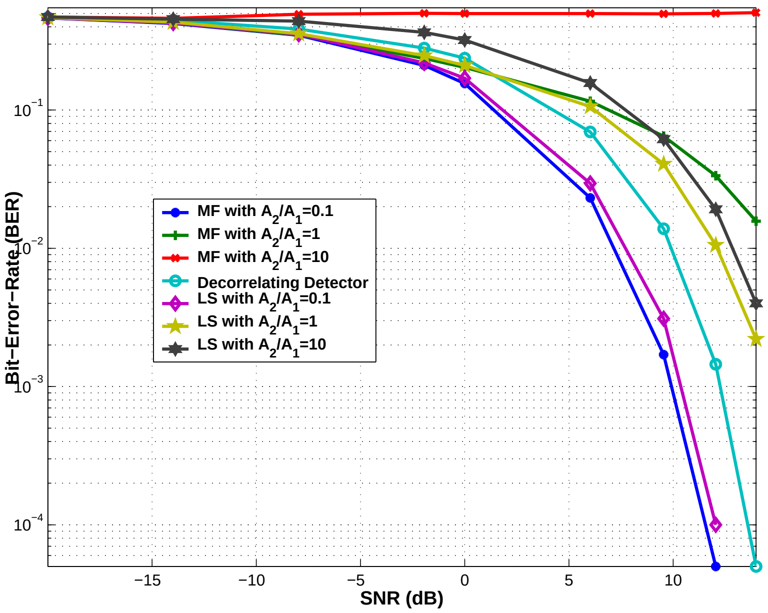

Figure 6: Bit-error-rate comparison of the signal-user matched filter, the single-truncated-window decorrelating detector, and the proposed LS semi-blind detector with $P = 1$ for the first user within users.

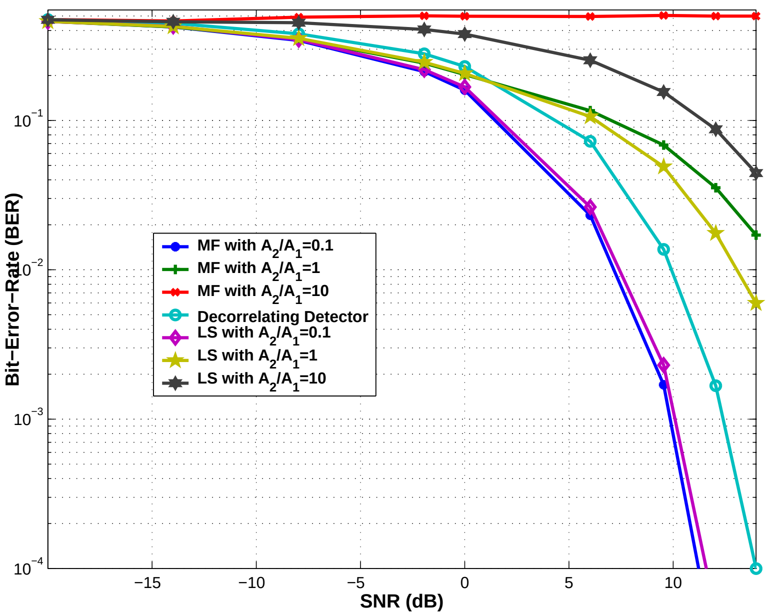

Figure 7: Bit-error-rate comparison of the signal-user matched filter, the single-truncated-window decorrelating detector, and the proposed LS semi-blind detector with $P = 4$ for the first user within users.

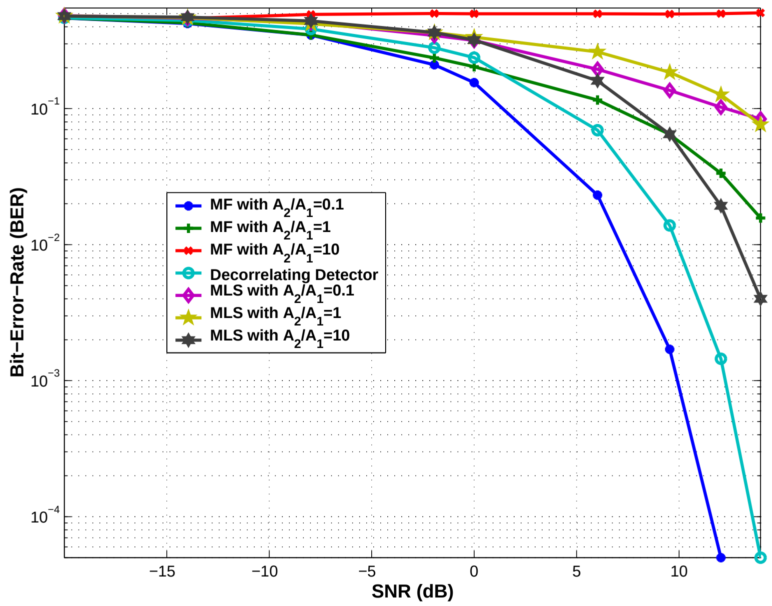

Figure 8: Bit-error-rate comparison of the signal-user matched filter, the single-truncated-window decorrelating detector, and the proposed MLS semi-blind detector with $P = 1$ for the first user within two users.

and $P = 4$ is checked against SNR and A_2/A_1, respectively. The most interesting is that, when $A_2/A_1 = 0.1$, the performance of the proposed LS semi-blind detector is very close to the MF detector and better than that of the classic decorrelating detector. When $A_2/A_1 = 1$ or 10, the performance of the LS semi-blind decorrelating detector is decreased and is worse than that of the classic decorrelating detector. But it is still better than that of the MF detector. In this case, the performance of LS semi-blind detector is always between those of the decorrelating detector and single-user matched-filter detector.

On the other hand, when $A_2/A_1 = 0.1$, the performance of the proposed LS semi-blind detector with $P = 4$ is a little better than that with $P = 1$. But, when $A_2/A_1 = 10$, the performance of the proposed LS semi-blind detector with $P = 1$ is better than that with $P = 4$.

The BER performance of the proposed MLS semi-blind detector is checked is checked in figure 8 and 9 against SNR and A_2/A_1. As we see, the performance of the proposed MLS detector is basically between that of the classic decorrelating detector and MF detector when $A_2/A_1 = 10$.

In figure 4, we checked the performance of the proposed LS and MLS multi-window semi-blind detector and the classic single-truncated-window decorrelating detector, with the changing of the window size P. As we see, except the performance of the LS semi-blind detector with $A_2/A_1 = 0.1$ is better than that of the classic decorrelating detector, the performance of the proposed semi-blind detectors in the other case are not as good as that of the classic decorrelating detector.

In figure 11, we check the NFR performance of the signal-user matched filter, the classic single-truncated-window decorrelating detector, and the proposed LS and MLS semi-blind detector with $P = 1$ and $P = 4$. Firstly, the BER performance of the proposed MLS semi-blind detector with $P = 1$ or $P = 4$ is basically not changed against A_2/A_1. From figure 6 and 7, we can see that the performance of the LS semi-blind detector is between that of the MF detector and the classic decorrelating detector. When A_2/A_1 is large enough, the performance of the LS semi-blind detector with $P = 1$ and $P = 4$ is closed to that of the MLS semi-blind detector with $P = 1$ and

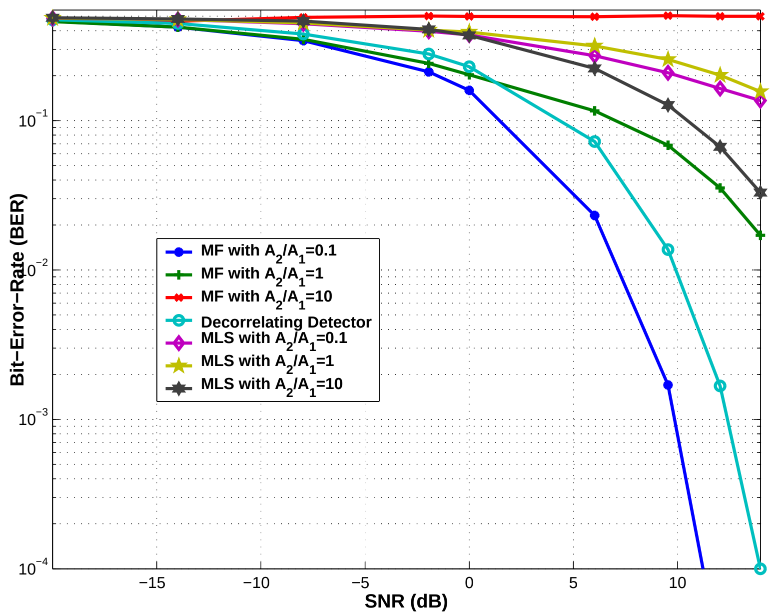

Figure 9: Bit-error-rate comparison of the signal-user matched filter, the single-truncated-window decorrelating detector, and the proposed MLS semi-blind detector with $P = 4$ for the first user within two users.

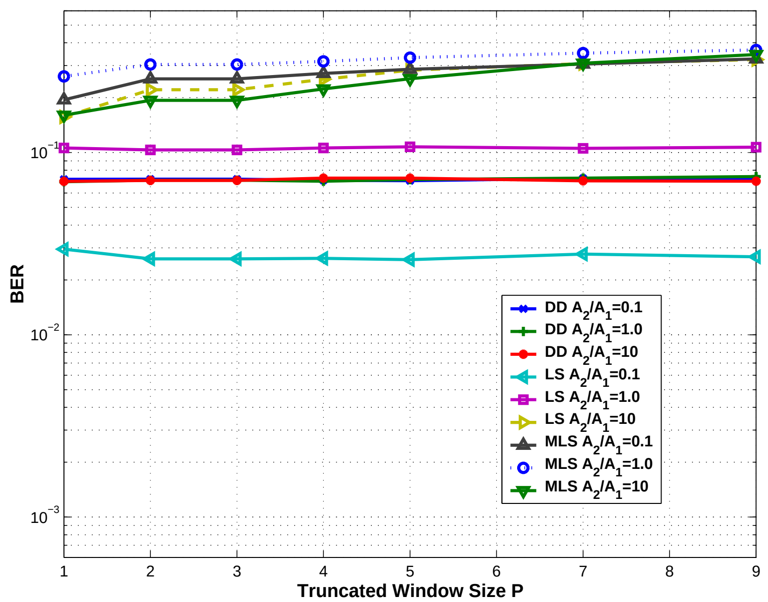

Figure 10: Bit-error-rate comparison of the classic signal-truncated-window decorrelating detector and the proposed LS and MLS semi-blind detectors with against the window size P

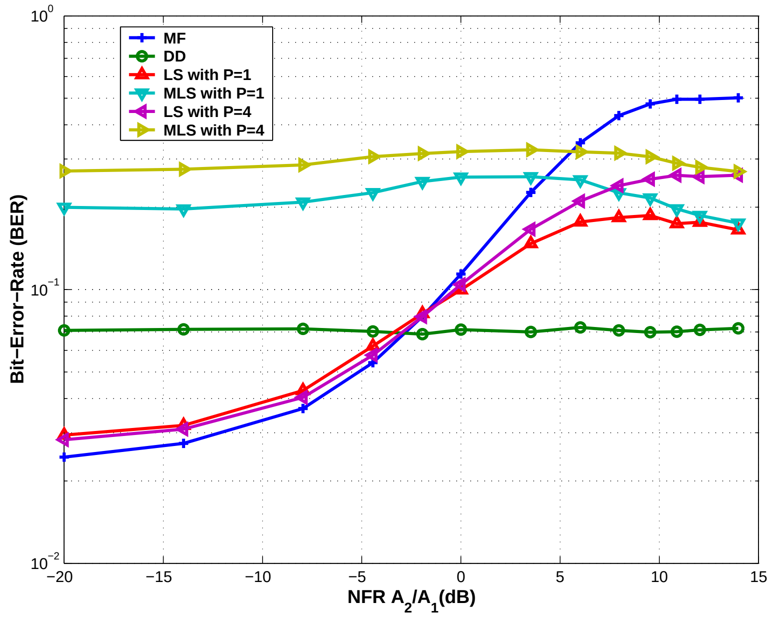

Figure 11: Near-far resistance comparison of the signal-user matched filter, single-truncated-window decorrelating detector, and the proposed LS and MLS semi-blind detectors for the first user within two users. $SNR = 6dB$

$P = 4$, respectively.

8 Conclusions

In this paper, we presented the one-shot LS and MLS semi-blind decorrelating detectors with the multiple consecutive truncated windows. In these two semi-blind multi-windows decorrelating detector, besides the signature and timing of the desired user, the amplitude of this user is also required. This is why they are called semi-blind detectors. In the theoretical analysis and computer simulations, as we may see, when there is no noise in the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$ and $P = 1$, the classic single-truncated-window decorrelating detector is just one special case of the presented LS semi-blind detector with $\mathbf{B} = \mathbf{I}$. And when there is no noise in the semi-blind signature matrix, the performance of the LS semi-blind detector with multiple windows is better than that with $P = 1$ and the classic single-truncated-window decorrelating detector.

These two semi-blind decorrelating detectors are simple and direct. No searching or converging procedure is required as in other semi-blind/semi-blind detectors.

References

[1] Sergio Verdu. *Multiuser Detection*. Cambridge University Press, 1998.

[2] U. Madhow M. Honig and S. Verdu. Blind adaptive multiuser detection. *IEEE Trans. On Information Theory*, 41:944–960, July 1995.

[3] H. V. Poor and X. Wang. Code-aided interference suppression for ds/cdma communications–part ii: Parallel blind adaptive implementations. *IEEE Trans. On Communications*, 45:1112–1122, September 1997.

[4] U. Madhow M. Honig and S. Verdu. Blind adaptive multiuser detection: A subsequence approach. *IEEE Trans. On Information Theory*, 44:677–691, March 1998.

[5] M. Torlak and G. Xu. Blind multiuser channel estimation in asynchronous cdma systems. *IEEE Trans. On Signal Processing*, 45:137–147, January 1997.

[6] H. Liu and G. Xu. A subspace method for signature waveform estimation in sychronouse cdma systems. *IEEE Trans. On Communications*, 44:1346–1354, October 1996.

[7] Sergio Verdu. *Recent progress in multuser detection, in Advances in Communication and Signal Processing.* New York: Springer-Verlag, 1989.

[8] R. Lupas and S. Verdu. Linear multiuser detectors for synchronous code-division multiple-access channels. *IEEE Trans. On Information Theory*, 35:123–136, 1989.

[9] S. Verdu. Near-far resistant receivers for ds/cdma communications. *U.S. Army Research Proposal*, 1986.

[10] H. V. Poor and S. Verdu. Blind multiuser detection: A subspace approach. *IEEE Trans. On Information Theory*, 44:677–690, March 1998.

[11] X Wang and A. Host-Madsen. Group-blind multiuser detection for uplink cdma. *IEEE Trans. On Select Area Communications*, 17:1971–1984, November 1999.

[12] B. Yang. Projection approximation subspace tracking. *IEEE Trans. On Signal Processing*, 43:95–107, January 1995.

[13] D. Shnidman. A generalized nyquist criterion and an optimum linear receiver for a pulse modulation system. *Bell System Technical Journal*, 46:2163–2177, November 1967.

[14] D. N. C. Tse and S. V. Hanly. Linear multiuser receivers: Effective interference, effective bandwidth and user capacity. *IEEE Trans. On Information Theory*, 45:641–657, March 1999.

[15] Yonina C. Eldar. On geometric properties of the decorrelator. *IEEE Communications Letters*, 6:16–18, January 2002.

[16] Charles F. Van Loan Gene H. Golub. *Matrix Computations.* The Johns Hopkins University Press, 1996.

[17] Sabine Van Huffel and Joos Vandewalle. *The total least squares problem: computational aspects and analysis.* Society for Industrial and Applied Mathematics, 1991.

[18] S. Van Huffel. On the accuracy of total least squares and least squares techniques in the presence of errors on all data. *Automatica*, 25:765–769, 1989.