Multi-Window Semi-Blind Decorrelating Multiuser Detection for Asynchronous CDMA

Shu Wang, James Caffery, Jr. and Hanhong Shen
ECE&CS Department
University of Cincinnati
Cincinnati, OH 45211-0030

Abstract

Multiuser detection is one of the key techniques for combating multiple access interference (MAI) in CDMA systems. In this paper, we propose three effective one-shot semi-blind decorrelating multiuser detectors with multiple windows for asynchronous CDMA. They are a least-square (LS) semi-blind decorrelating multiuser detectors, a total least-square (TLS) semi-blind decorrelating multiuser detectors and a mixed LS and TLS (MLS) semi-blind decorrelating multiuser detector. All these three schemes are based on the classic single-truncated-window decorrelating detector and different assumptions regarding the noise in a semi-blind signature matrix. With the analysis of the single-truncated-window scheme, a multiple-window scheme is further proposed to detect several consecutive bits at the same time. By this way, the proposed detectors are expected to achieve the same or even better performance than that of the single-truncated-window schemes. The proposed algorithms are simple and direct. Only the desired user's timing, amplitude and signature are required. No knowledge of other users is involved. Also no search or convergence procedure is employed as in many other semi-blind or blind detectors. Theoretical analysis and computer simulations are also presented to support the performance of the proposed one-shot semi-blind decorrelating multiuser detectors.

1 Introduction

Direct-sequence code division multiple access (DS/CDMA) techniques have attracted increasing attention for efficient use of available bandwidth, resistance to interference and flexibility to variable traffic patterns. One of the problems of such a system is the so-called near-far problem resulting from excessive MAI energy from nearby users compared with the desired user's signal energy. Multiuser detection strategy is a method to minimize the effect of MAI and solve the near-far problem in CDMA systems without a significant reduction in the signal energies of the strong users in order for the weaker users to achieve reliable communication. It has been extensively investigated over the past several years [1], since MAI is the dominant impairment for CDMA systems and exists even in perfect power-controlled CDMA systems. Most early work on multiuser detection assumed that the receiver knew the spreading codes or had some knowledge of all users, then exploited this knowledge to combat MAI. For example, the classic decorrelating detector for the synchronous case or the single-truncated-window decorrelating detector for asynchronous system can achieve the optimum near-far resistance and completely eliminate MAI from other users with the expense of enhancement of background noise. However, in many practical cases, especially in a dynamic environment, e.g. in the downlink of a CDMA system, it is very difficult for a mobile user to obtain accurate information on other active users in the same channel. On the other hand, the frequent use of training sequence is certainly a waste of channel bandwidth. So blind multiuser detection has been proposed. Recent research has been devoted to the blind multiuser receivers and subspace-based signature waveform estimation schemes to achieve better performance and higher capacity [2, 3, 4, 5, 6]. The minimum output energy (MOE) method and subspace method were presented for multiuser blind detection with the knowledge of only the desired users' spreading code and possible timing.

An optimum near-far resistant multiuser detector which dose not need power control has been proposed by Verdú [7]. However, its complexity is exponential in terms of the number of users. That makes it unsuitable for practical situations. The large gaps in performance and complexity between the conventional single-user matched filter and optimum multiuser detector encourage the search for other multiuser detectors that exhibit good performance/complexity tradeoff. Suboptimum linear multiuser detectors, which are based on linear transformation of the sampled match filter outputs, were considered in [8] for synchronous case. The decorrelating detector [8] not only is a simple and nature strategy but also is optimal according to three different criteria: least squares, near-far resistance [9] and maximum-likelihood when the received amplitudes are unknown [8]. Though it does not require knowledge of the received amplitudes, it does require the information about other active users' signature waveforms. For the asynchronous cases, Verdú suggested to use a one-shot version of his decorrelating detector, the single-truncated-window decorrelating detector, where one full-code matched filter is used for the desired user and two partial-code matched filters are used for each of the other users, where one of the two partial-code mateched filter is matched to the *previous* part of its code and the other one is matched to the *current* part of the code. In this case, it not only requires the information about other active users' signatures but also the time delay of other users with respect to the desired user. Furthermore, since the signature information of the other users is divided in a signature matrix of the nearly doubled size, the performance of this one-short single-truncated-window decorrelating detector is worse than the complete asynchronous decorrelating detector.

It is shown in [2] that the multiuser detector with maximum output energy criteria is equivalent to that with the linear minimum mean square error (MMSE) criteria. Compared the optimal multiuser detection, they are near-far resistant and has much less complexity. The major limitation of MOE schemes to multiuser blind detection is that there is a saturation effect in the steady state, which causes a significant performance gap between the converged blind MOE and the true MMSE detector [2].

Multiuser blind detection using subspace techniques was first developed in depth by Wang and Poor [4, 10]. Such techniques were appropriate for the downlink environment where only the desired user's code is available. More recently, these subspace techniques were extended by Wang and Host-Madsen [11], named group multiuser blind detectors, to uplink environments where the base station knows the codes of in-cell users, but not those of users outside the cell. In the subspace-based blind detection approach [4], the linear detectors are constructed in the closed form once the signal subspace components are computed. That offers lower computational complexity and better performance than the blind MOE detector. For the subspace-based blind adaptive detector, the project approximation subspace tracking deflation (PASTd) algorithm [12] is used to estimate the signal subspace.

As we see, various multiuser detection schemes have been developed to combat the effects of MAI. These detection techniques either assume the knowledge of all the users in the system (Conventional) or assume the knowledge of the user of interest and without knowledge of the channel input (Blind). Due to the limitation of the blind algorithms in the presence of a large number of interferers, there is a significant performance gap between these two classes of detectors. In this work, we consider an asynchronous DS/CDMA system and develop some partial blind detectors, a new least squares semi-blind decorrelating detector (LS-DD), a new total least squares semi-blind decorrelating detector (TLS-DD) and a mixed LS/TLS semi-blind decorrelating detector (MLS-DD). They are expected to bridge the performance gap between the blind multiuser detectors and the conventional multiuser detectors. In the proposed semi-blind multiuser detectors, besides the desired user's timing and signature, the amplitude is also required. So, we call them semi-blind detectors. In the proposed algorithms, a new semi-blind signature matrix is constructed with the desired user's signature and amplitude and several previously received signal vectors. After this, the decorrelating operation based on this new semi-blind signature is proposed. With the analysis of the classic single-truncated-window scheme, a new multi-window scheme is proposed to detect several consecutive bits at the same time. That is expected to improve the performance of the single-truncate-window algorithm. At last, the LS, TLS and MLS version of the proposed semi-blind multi-window decorrelating detection scheme are respectively proposed with different assumptions regarding the noise in the proposed semi-blind signature matrix. All these three algorithms are simple and direct detection schemes. No knowledge of other users' information is required. And no search or converging procedure is employed as in many other semi-blind/blind detectors. Finally theoretical analysis and computer simulations are also presented to demonstrate the performance of the proposed semi-blind decorrelating detectors.

The rest of the paper is organized as follows. In Section II, we summarize the asynchronous signal model. In Section III, we review the classic single-truncated-window decorrelating detector for multiuser detection. In Section IV, the new data model with multiple consecutive windows is discussed. In Section V, the multi-window LS, TLS and MLS semi-blind decorrelating detection algorithms are introduced. Performance analysis and simulation results are provided in Section VI and VII. Section VIII concludes this papers.

2 Data Model and Problem Description

A single-cell symbol-asynchronous DS/CDMA system over the nondispersive additive white Gaussian noise (AWGN) channel is considered. Spreading sequences are preassigned to all the active users in the system and the signature waveform of the kth user can be expressed as

$$s_k(t) \quad = \quad \sum_{l=0}^{L-1} c_k^{(l)} \psi(t - lT_c) \tag{1}$$

where $c_k^{(l)} \in \{-1/\sqrt{L},\ 1/\sqrt{L}\}$ is the lth chip of the kth user's signature sequence, L is the spreading gain, T_c is the chip interval and $\psi(t)$ is the chip waveform. We assume that $\psi(t)$ satisfies the Nyquist criterion for zero inter-chip interference and $\int_{-\infty}^{+\infty} |\psi(t)|^2 dt = 1$.

We consider reverse link transmission. The baseband representation of the received signal due to the kth user is given by

$$r_k(t) \quad = \quad \sum_{i=-\infty}^{+\infty} A_k b_k^{(i)} s_k(t - iT_c - \tau_k) \tag{2}$$

where $b_k^{(i)}$ is the ith bit sent by the kth user. We assume that the $b_k^{(i)}$ is independent and identically distributed random variables with $E\{b_k^{(i)}\} = 0$ and $E\{|b_k^{(i)}|^2\} = 1$. Also, τ_k denotes the transmission delay from the kth user to the base station and A_k is the power of the received signal of the kth user. Then the baseband signal at the input of the receiver at the base station is

$$r(t) \quad = \quad \sum_{k=1}^{K} r_k(t) + n(t) \tag{3}$$

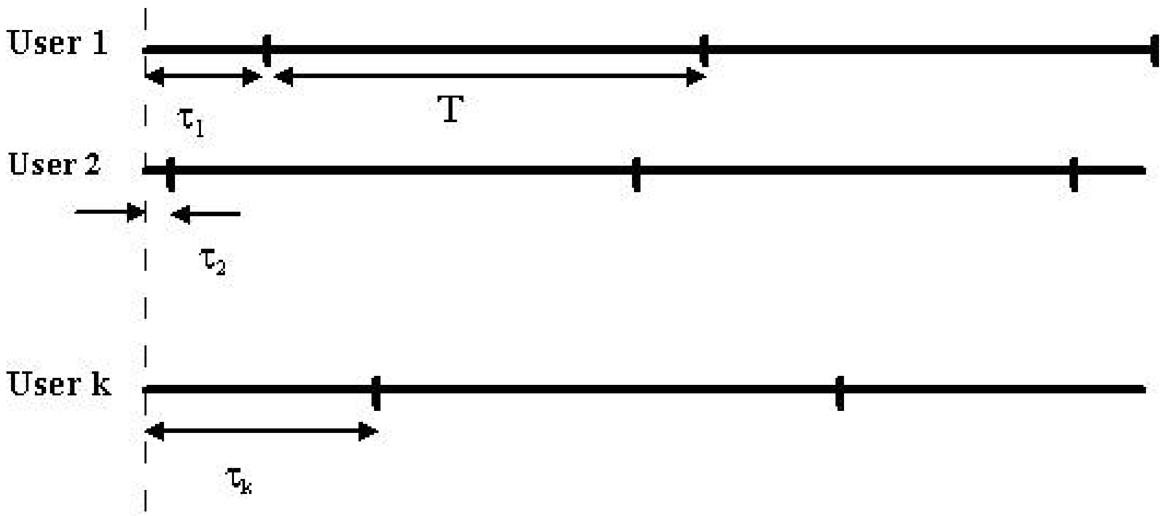

Figure 1: The demonstration of a basic asynchronous multiuser channel.

where $n(t)$ is additive white Gaussian noise with power spectral density σ_n^2. We assume that there are K simultaneous users in the system.

The received signal is synchronized for each user, passed through the corresponding chip matched filter (CMF), and sampled at the chip rate $1/T_c$. The vector of the output samples of the CMF for kth user in the nth symbol interval can be expressed as

$$\mathbf{r}_k^{(n)} = [\, r_k(nT + T_c + \tau_k) \quad r_k(nT + 2T_c + \tau_k) \quad \ldots \quad r_k(nT + LT_c + \tau_k)\,]^T \tag{4}$$

where

$$r_k(nT + lT_c + \tau_k) = \int\limits_{nT+lT_c+\tau_k}^{nT+(l+1)T_c+\tau_k} r_k(t)\psi(t)dt \tag{5}$$

for $1 \leq l \leq L$. To facilitate the analysis, we will assume the system to be chip-synchronous and restrict ourselves to the receiver that has an observation window of one symbol interval. Without loss of generality. we consider the detection of the first user. The observation window of the first user is marked with a thick line in figure 1. The signals of other users are treated as interference. A typical interferer has two different but consecutive symbols interfering the symbol of user 1, as shown in figure 1. This can be expressed as[1]

$$\mathbf{r}_1 = A_1 b_1^{(n)} \mathbf{s}_1 + \sum_{k=2}^{K} A_k b_k^{(n-1)} \mathbf{s}_{k-} + \sum_{k=2}^{K} A_k b_k^{(n)} \mathbf{s}_{k+} + \mathbf{n} \tag{6}$$

where $\mathbf{s}_{k_-}$ and $\mathbf{s}_{k_+}$ are named effective signature sequences or part signature sequences that are completely determined by the spreading sequences $\mathbf{s}_k$ and the delays relative to the first user $\tau_{k1} = \tau_k - \tau_1$, $\mathbf{n}$ is an L-dimension Gaussian vector with independent σ_n^2-variance components and $L \geq 2K - 1$.

Now, the received signal vector $\mathbf{r}_1$ is fed into a correlating receiver bank that is matched to each user's spreading waveform to yield the output signal vector

$$\mathbf{y}_1 = \mathbf{R}(1)\mathbf{A}\mathbf{b}^{(n-1)} + \mathbf{R}(0)\mathbf{A}\mathbf{b}^{(n)} + \mathbf{R}^T(1)\mathbf{A}\mathbf{b}^{(n+1)} + \bar{\mathbf{n}} \tag{7}$$

[1]Without loss of the generality, we will drop the superscript (n) from the vector $\mathbf{r}_1^{(n)}$ and $\mathbf{y}_1^{(n)}$.

where the zero-mean Gaussian process $\bar{\mathbf{n}}$ has autocovariance matrix

$$E\{\bar{\mathbf{n}}_i \bar{\mathbf{n}}_j^T\} = \begin{cases} \sigma_n^2 \mathbf{R}^T(1) & j = i+1 \\ \sigma_n^2 \mathbf{R}(0) & j = i \\ \sigma_n^2 \mathbf{R}(1) & j = i-1 \\ \mathbf{0} & |j-i| > 1 \end{cases} \tag{8}$$

and the matrices $\mathbf{R}(0)$ and $\mathbf{R}(1)$ are defined by

$$R_{ij}(0) = \begin{cases} 1 & j = i \\ \rho_{ij} & j < i \\ \rho_{ji} & j > i \end{cases} \tag{9}$$

$$R_{ij}(1) = \begin{cases} 0 & j \geq i \\ \rho_{ij} & j < i \end{cases} \tag{10}$$

where, if $i \geq j$, then we denote

$$\rho_{ij} = \int_{\tau}^{T} s_i(t) s_j(t-\tau) dt \ , \tag{11}$$

$$\rho_{ji} = \int_{0}^{\tau} s_i(t) s_j(t+T-\tau) dt \ . \tag{12}$$

Most of the linear multiuser detectors for demodulating the user's data bit $b_1^{(n)}$ in (6) is in the form of a correlator followed by a hard limiter, which can be expressed as

$$\hat{b}_1^{(n)} = \mathrm{sgn}\{\mathbf{w}_1^T \mathbf{r}_1\} \tag{13}$$

where $\mathbf{w}_1 \in \mathbb{R}^{L \times 1}$.

Linear multiuser detectors can be implemented in a decentralized fashion where only the user or users of interest need be demodulated.

3 Single-Truncated-Window Decorrelating Detector

The large gaps in performance and complexity between the conventional single-user matched filter and optimum multiuser detector encourage the search for other multiuser detectors that exhibit good performance/complexity tradeoffs. One of the earliest suggestions to eliminate multiuser interference with a linear receiver was proposed by Shnidman [13]. The derivation of the asymptotic efficiency of the decorrelating detector for synchronous channels and the proof of its optimum near-far resistant property are due to Verdú [9] in the case of nonsingular covariance matrices. The forerunner of the decorrelating detector in the single-user intersymbol interference (ISI) channel is the zero-forcing equalizer. And the counterpart of decovariance in antenna array subject to undesired sources is called null steering. Prior to developing the semi-blind decorrelating detector, we discuss the classic single-truncated-window decorrelating detector in the following, which is useful in understanding the decorrelating detector.

As in equation (6), the output vector of the receiver outputs can be written as

$$
\begin{aligned}
\mathbf{r}_1 &= A_1 b_1^{(n)} \mathbf{s}_1 + \sum_{k=2}^{K} A_k [\, \mathbf{s}_{k-} \quad \mathbf{s}_{k+} \,] \begin{bmatrix} b_k^{(n-1)} \\ b_k^{(n)} \end{bmatrix} + \mathbf{n} \\
&= [\, \mathbf{s}_1 \quad \mathbf{s}_{2-} \quad \mathbf{s}_{2+} \quad \ldots \quad \mathbf{s}_{K-} \quad \mathbf{s}_{K+} \,] \begin{bmatrix} A_1 & & & & & \\ & A_2 & & & & \\ & & A_2 & & & \\ & & & \ddots & & \\ & & & & A_K & \\ & & & & & A_K \end{bmatrix} \begin{bmatrix} b_1^{(n)} \\ b_2^{(n-1)} \\ b_2^{(n)} \\ \vdots \\ b_K^{(n-1)} \\ b_K^{(n)} \end{bmatrix} + \mathbf{n} \qquad (14) \\
&= \mathbf{S}_1 \mathbf{A}_1 \mathbf{b}_1 + \mathbf{n}
\end{aligned}
$$

where

$$\mathbf{S}_1 = [\, \mathbf{s}_1 \quad \mathbf{s}_{2-} \quad \mathbf{s}_{2+} \quad \ldots \quad \mathbf{s}_{K-} \quad \mathbf{s}_{K+} \,] \quad , \qquad (15)$$

$$\mathbf{A}_1 = \mathrm{diag}\,\{\, A_1 \quad A_2 \quad A_2 \quad \ldots \quad A_K \quad A_K \,\} \qquad (16)$$

and

$$\mathbf{b}_1 = [\, b_1^{(n)} \quad b_2^{(n-1)} \quad b_2^{(n)} \quad \ldots \quad b_K^{(n-1)} \quad b_K^{(n)} \,]^T \quad . \qquad (17)$$

The classic one-short decorrelating detector then performs the following operation.

$$\hat{\mathbf{b}}_1 = \mathrm{sgn}\{\mathbf{S}_1^{+} \mathbf{r}_1\} \qquad (18)$$

where $[\star]^+$ denotes the Moore-Penrose generalized inverse of $\star$.

In the classic single-truncated-window decorrelating detector, the received vector $\mathbf{r}_1$ is projected on the subspace which is orthogonal to the codes of the other users [1, 14]. Furthermore, in [15], it is shown that the above decorrelating detector is the *oblique projection* of the desired user's signature vector. That is to project $\mathbf{s}_1$ onto the orthogonal complement subspace of the space $\mathbb{S}_1$, which is spanned by the other users' partial signature vectors, along the null space of the space $\mathbb{S}_1$. The decorrelating detector is designed to completely eliminate MAI caused by other users, at the expense of enhancing the ambient noise. So, the decorrelating detector is a sensible choice when the received amplitudes are completely unknown. There are some desirable features of this multiuser detector. It does not require knowledge of the received amplitude, but it does require $\mathbf{S}_1^{+}$. It can readily be decentralized in the sense that the demodulation of each user can be implemented completely independently.

4 New Data Model With Semi-Blind Signature Matrix

Most of the multiuser detection schemes in the asynchronous case assume the knowledge of the timing, the spreading codes and/or channel parameters of all the users that contribute to the received signal. Then these receivers would exploit this knowledge to combat MAI. A second form of detectors, know as the blind detectors, which operate without the knowledge of the channel input and the information of other users. Usually, many practical systems lie in between these two extremes. The first form of detectors are too optimistic as we can not expect to know the signature waveforms of all the users, while the latter under-utilize the our knowledge of the system. In this section, we will develop a new data model, where a new semi-blind signature matrix with multiple consecutive windows is proposed. Based this data model, the semi-blind decorrelating detectors can be proposed in the next section and they are expected to bridge the performance gap between the conventional detectors and blind ones.

It is widely known that the performance of the single-truncated-window decorrelating detector is worse than that of the asynchronous decorrelator in terms of both bit-error rate (BER) and near-far resistance (NFR) [1]. This is because that, in the classic one-shot single-truncated-window decorrelating detector, the energy of each interfering signal is separated in a signature matrix of the larger columns, $2K-1$, so that the single-truncated-window crosscovariance matrix may be more easily singular, even when the corresponding $K \times K$ asynchronous crosscovariance matrix is not singular. Hence, we propose to use multiple consecutive windows instead of the single window in this section. We will see that, except the first and last window, there is no broken interferering signal vector or signature any more. This is expected to improve the performance of decorrelating detectors for asynchronous systems.

Without loss of the generality, we only detect the information signal bits sent by user 1 at P consecutive symbol intervals $t \in [(n-P+1)T_s,\ nT_s]$, where P is the length of the consecutive truncated windows. The P consecutive information bits for user 1 in the signal vector $\mathbf{b}_1$ can be detected simultaneously. To this end, the new $PL \times M$ semi-blind signature matrix $\boldsymbol{\mathcal{S}}$ for user 1 is defined as

$$\begin{aligned} \boldsymbol{\mathcal{S}} &= [\,\bar{\mathbf{s}}_1 \quad \bar{\mathbf{s}}_2 \quad \bar{\mathbf{s}}_3 \quad \dots \quad \bar{\mathbf{s}}_M\,] \\ &= [\,\mathbf{I}_P \otimes (A_1\mathbf{s}_1) \quad \bar{\mathbf{r}}_1 \quad \bar{\mathbf{r}}_2 \quad \dots \quad \bar{\mathbf{r}}_{M-P}\,] \\ &= [\,\mathbf{SAE} \quad \mathbf{SA}\bar{\mathbf{b}}_1 \quad \mathbf{SA}\bar{\mathbf{b}}_2 \quad \dots \quad \mathbf{SA}\bar{\mathbf{b}}_{M-P}\,] + \bar{\mathbf{N}} \\ &= \mathbf{SA}\,[\,\mathbf{E} \quad \bar{\mathbf{b}}_1 \quad \bar{\mathbf{b}}_2 \quad \dots \quad \bar{\mathbf{b}}_{M-P}\,] + \bar{\mathbf{N}} \\ &= \mathbf{SA}\,[\,\mathbf{E} \quad \mathbf{D}\,] + \bar{\mathbf{N}} \\ &= \mathbf{SAB} + \bar{\mathbf{N}} \end{aligned} \tag{19}$$

where the $PL \times (PK+K-1)$ matrix $\mathbf{S} = [\,\bar{\mathbf{S}}_1 \quad \bar{\mathbf{S}}_2 \quad \bar{\mathbf{S}}_3 \quad \dots \quad \bar{\mathbf{S}}_K\,]$ is the multi-window signature matrix, in which

$$\bar{\mathbf{S}}_1 \;=\; \mathbf{I}_P \otimes \mathbf{s}_1 \;=\; \mathrm{diag}\{\,\mathbf{s}_1 \quad \mathbf{s}_1 \quad \dots \quad \mathbf{s}_1\,\}_{PL\times P} \tag{20}$$

and

$$\bar{\mathbf{S}}_k \;=\; \mathrm{diag}\{\,\mathbf{s}_{k_-} \quad \mathbf{s}_k \quad \dots \quad \mathbf{s}_k \quad \mathbf{s}_{k_+}\,\}_{PL\times(P+1)}\ , \tag{21}$$

the $(PK+K-1)\times(PK+K-1)$ diagonal matrix $\mathbf{A} = \mathrm{diag}\{\,\bar{\mathbf{A}}_1 \quad \bar{\mathbf{A}}_2 \quad \dots \quad \bar{\mathbf{A}}_K\,\}$ denotes the multi-window signal amplitude matrix, in which

$$\bar{\mathbf{A}}_1 \;=\; \mathrm{diag}\{\,A_1 \quad A_1 \quad \dots \quad A_1\,\}_{P\times P} \tag{22}$$

and

$$\bar{\mathbf{A}}_k \;=\; \mathrm{diag}\{\,A_k \quad A_k \quad \dots \quad A_k\,\}_{(P+1)\times(P+1)}\ , \tag{23}$$

$\bar{\mathbf{r}}_i$ are the arbitrary received vectors, $\otimes$ denotes the Kronecker product, $\mathbf{E} = [\,\mathbf{I}_P \quad \mathbf{0}\,]^T$, the $(PK+K-1)\times 1$ vector $\bar{\mathbf{b}}_i$ is one of the multi-window bit vectors consisting of bits previously sent by the $K-1$ interfering users, the $PL \times (M-P)$ matrix $\mathbf{D} = [\bar{\mathbf{D}}^T\ \tilde{\mathbf{D}}^T]^T$ denotes the multi-window information bit matrix, in which the $(M-P)\times P$ vector $\bar{\mathbf{D}}$ is the information vector consisting of the known bits previously sent by the desired user and rank$\{\tilde{\mathbf{D}}\} = PK + K - P - 1$, the $PL \times M$ matrix $\bar{\mathbf{N}} = [\mathbf{0}\ \tilde{\mathbf{N}}]$ denotes the multi-window noise matrix,

$$\begin{aligned}
\mathbf{B} &= [\mathbf{E} \quad \mathbf{D}] \\
&= \begin{bmatrix} & \mathbf{C} \\ \mathbf{0} & \tilde{\mathbf{D}} \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{I}_P & \bar{\mathbf{D}} \\ \mathbf{0} & \tilde{\mathbf{D}} \end{bmatrix}
\end{aligned} \tag{24}$$

and rank$\{\mathbf{B}\} = PK + K - 1$, $i = 1,\ 2,\ \ldots,\ M - P$, $k = 2,\ 3,\ \ldots,\ K$. We maintain $PL \geq M \geq PK + K - 1$.

With equation (6) and (19), the relationship between the current received signal vector $\mathbf{r}$, which consists of P consecutively received signal vectors by the desired user, and the new semi-blind signature matrix $\boldsymbol{\mathcal{S}}$ can be expressed as

$$\begin{aligned}
\mathbf{r} &= [\mathbf{r}_1^{(n)T} \quad \mathbf{r}_1^{(n-1)T} \quad \ldots \quad \mathbf{r}_1^{(n-P+1)T}]^T \\
&= \mathbf{SAb} + \mathbf{n} \\
&= \mathbf{SABB}^+\mathbf{b} + \mathbf{n} \\
&= (\boldsymbol{\mathcal{S}} - \bar{\mathbf{N}})\mathbf{B}^+\mathbf{b} + \mathbf{n} \\
&= \boldsymbol{\mathcal{S}}\mathbf{B}^+\mathbf{b} - \bar{\mathbf{N}}\mathbf{B}^+\mathbf{b} + \mathbf{n} \\
&= \boldsymbol{\mathcal{S}}\mathbf{d} + \tilde{\mathbf{n}}
\end{aligned} \tag{25}$$

where

$$\begin{aligned}
\mathbf{b} &= [\bar{\mathbf{b}}^T \quad \tilde{\mathbf{b}}^T]^T \\
&= \begin{bmatrix} \begin{bmatrix} b_1^{(n)} \\ b_1^{(n-1)} \\ \vdots \\ b_1^{(n-P+1)} \end{bmatrix}^T & \begin{bmatrix} b_2^{(n)} \\ b_2^{(n-1)} \\ \vdots \\ b_2^{(n-P+1)} \\ b_2^{(n-P)} \end{bmatrix}^T & \cdots & \begin{bmatrix} b_K^{(n)} \\ b_K^{(n-1)} \\ \vdots \\ b_K^{(n-P+1)} \\ b_K^{(n-P)} \end{bmatrix}^T \end{bmatrix}^T,
\end{aligned} \tag{26}$$

in which the $P \times 1$ vector $\bar{\mathbf{b}} = \begin{bmatrix} b_1^{(n)} & b_1^{(n-1)} & \cdots & b_1^{(n-P+1)} \end{bmatrix}^T$ denotes the desired bit vector of user 1, the $M \times 1$ vector $\mathbf{d}$ denotes the new detection vector and is defined as

$$\begin{aligned}
\mathbf{d} &= \mathbf{B}^+\mathbf{b} \\
&= [\mathbf{E} \quad \mathbf{D}]^+\,\mathbf{b} \\
&= \begin{bmatrix} \mathbf{I}_P & \bar{\mathbf{D}} \\ \mathbf{0} & \tilde{\mathbf{D}} \end{bmatrix}^+ \begin{bmatrix} \bar{\mathbf{b}} \\ \tilde{\mathbf{b}} \end{bmatrix}
\end{aligned} \tag{27}$$

and $\tilde{\mathbf{n}}$ is the new noise vector and defined as

$$\tilde{\mathbf{n}} = \mathbf{n} - \bar{\mathbf{N}}\mathbf{B}^+\mathbf{b} \tag{28}$$

With the following lemma, it is easy to see that the new semi-blind noise item $\tilde{\mathbf{n}}$ is enhanced, compared with the former noise item $\mathbf{n}$. This enhancement is because there is the noise matrix $\bar{\mathbf{N}}$ existing in the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$.

Now, the following result can be easily proved.

Proposition 1. *The Moor-Penrose general inverse of* $\mathbf{B}$ *is*

$$\mathbf{B}^{+} = \begin{bmatrix} \mathbf{E} & \begin{matrix} \bar{\mathbf{D}}\tilde{\mathbf{D}}^{+} \\ \tilde{\mathbf{D}}^{+} \end{matrix} \end{bmatrix} \tag{29}$$

Proof. Based on the definition of $\mathbf{B}$,

$$\mathbf{B} = \begin{bmatrix} \mathbf{E} & \begin{matrix} \bar{\mathbf{D}} \\ \tilde{\mathbf{D}} \end{matrix} \end{bmatrix} \tag{30}$$

so that

$$\begin{bmatrix} \mathbf{E} & \begin{matrix} \bar{\mathbf{D}} \\ \tilde{\mathbf{D}} \end{matrix} \end{bmatrix} \begin{bmatrix} \mathbf{E} & \begin{matrix} \bar{\mathbf{D}}\tilde{\mathbf{D}}^{+} \\ \tilde{\mathbf{D}}^{+} \end{matrix} \end{bmatrix} = \begin{bmatrix} \mathbf{I}_P & \bar{\mathbf{D}} \\ \mathbf{0} & \tilde{\mathbf{D}} \end{bmatrix} \begin{bmatrix} \mathbf{I}_P & -\bar{\mathbf{D}}\tilde{\mathbf{D}}^{+} \\ \mathbf{0} & \tilde{\mathbf{D}}^{+} \end{bmatrix} = \mathbf{I}_{PK+K-1} \tag{31}$$

where $\mathbf{I}_{PK+K-1}$ is the unitary matrix of the size $(PK + K - 1) \times (PK + K - 1)$. □

With Proposition 1, the detection vector $\mathbf{d}$ can be re-written as

$$\begin{aligned} \mathbf{d} &= \begin{bmatrix} \bar{\mathbf{d}} \\ \tilde{\mathbf{d}} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{E} & \begin{matrix} \bar{\mathbf{D}} \\ \tilde{\mathbf{D}} \end{matrix} \end{bmatrix}^{+} \begin{bmatrix} \bar{\mathbf{b}} \\ \tilde{\mathbf{b}} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{I}_P & -\bar{\mathbf{D}}\tilde{\mathbf{D}}^{+} \\ \mathbf{0} & \tilde{\mathbf{D}}^{+} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{b}} \\ \tilde{\mathbf{b}} \end{bmatrix} \\ &= \begin{bmatrix} \bar{\mathbf{b}} - \bar{\mathbf{D}}\tilde{\mathbf{D}}^{+}\tilde{\mathbf{b}} \\ \tilde{\mathbf{D}}^{+}\tilde{\mathbf{b}} \end{bmatrix} \end{aligned} \tag{32}$$

where the $P \times 1$ matrix $\bar{\mathbf{d}}$ denotes the first consecutive P elements of the detection vector $\mathbf{d}$ as

$$\bar{\mathbf{d}} = \bar{\mathbf{b}} - \bar{\mathbf{D}}\tilde{\mathbf{D}}^{+}\tilde{\mathbf{b}} \tag{33}$$

and $\tilde{\mathbf{d}}$ is the complement vector as

$$\tilde{\mathbf{d}} = \tilde{\mathbf{D}}^{+}\tilde{\mathbf{b}} \tag{34}$$

Now, the following result can be very easily reached.

Lemma 1. *The bits vector* $\bar{\mathbf{b}}$ *which consists of the bits sent by user 1 at* P *consecutive time intervals* $t = n - P + 1,\ n - P + 2,\ \ldots,\ n$ *can be got with the following equation.*

$$\begin{aligned} \bar{\mathbf{b}} &= \mathbf{C}\mathbf{d} \\ &= [\mathbf{I}_P \quad \bar{\mathbf{D}}]\mathbf{d} \\ &= \bar{\mathbf{d}} + \bar{\mathbf{D}}\tilde{\mathbf{d}} \end{aligned} \tag{35}$$

We can see that, after the definition of the new semi-blind signature matrix $\boldsymbol{\mathcal{S}}$ in equation (19), the form of the classic multiuser detection model can still be kept as in equation (25). But the different is that, the information bit vector $\mathbf{b}$ is replaced by the detection vector $\mathbf{d}$ as in equation (27) and the original AWGN noise vector $\mathbf{n}$ is replace by the new noise vector $\tilde{\mathbf{n}}$ as in equation (28). Fortunately, with lemma 1, it still is possible for us to calculate the desired bit vector $\bar{\mathbf{b}}$ with the detection vector $\mathbf{d}$ and the previously detected bits matrix $\mathbf{C}$. The next question then is how to estimate the detection vector $\mathbf{d}$ as efficiently as possible. In the following section, there are three estimation schemes proposed for the estimation of $\mathbf{d}$. They are LS, TLS and MLS estimation schemes.

On the other hand, as we may see, the classic single-truncated-window scheme for the asynchronous case can be taken as a special case of the presented multi-window scheme with $P = 1$. In the single-truncated-window scheme, there is no complete signature of other user existing in the signature matrix. This makes its inverse unstable. But, in the presented multi-window scheme, there are $P - 1$ complete signature vectors of any other user in the new constructed signature matrix. Hence, the inverse of the multi-window signature matrix should be much more stable than that of the single-truncated-window signature matrix.

5 One-Shot Multi-Windows Semi-Blind Decorrelating Detection

In the previous section, the principles of the proposed semi-blind decorrelating detection scheme are described. With lemma 1, the detection of the desired information bit vector $\bar{\mathbf{b}}$ can be decided by the estimation of the detection vector $\mathbf{d}$. The left problem is how to estimate the vector $\mathbf{d}$. In this section, we are going to propose three algorithms to estimate $\mathbf{d}$ with the difference assumptions regarding the noise matrix $\bar{\mathbf{N}}$ in the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$. Also, following the different estimation schemes of the detection vector $\mathbf{d}$, three different multi-window semi-blind detectors are proposed.

5.1 Least-Square Semi-Blind Decorrelating Detector

At first, the measurements of $\boldsymbol{\mathcal{S}}$ is assumed to be free of error and all errors are confined to the received vector $\mathbf{r}$ as in equation (25). The following least-square estimation of the detection vector $\mathbf{d}$ is then proposed.

Lemma 2. *[16] Suppose* $\mathbf{U}^T\boldsymbol{\mathcal{S}}\mathbf{V} = \mathbf{\Sigma}$ *is the SVD of* $\boldsymbol{\mathcal{S}} \in \mathbb{R}^{PL\times M}$ *with* $r = rank(\boldsymbol{\mathcal{S}})$*. And if* $\mathbf{U} = [\,\mathbf{u}_1 \quad \mathbf{u}_2 \quad \dots \quad \mathbf{u}_{PL}\,]$, $\mathbf{V} = [\,\mathbf{v}_1 \quad \mathbf{v}_2 \quad \dots \quad \mathbf{v}_M\,]$, $\mathbf{\Sigma} = diag\{[\,\sigma_1 \quad \dots \sigma_r \quad 0 \quad \dots \quad 0\,]\}$ *and* $\mathbf{r} \in \mathbb{R}^{PL\times 1}$*, then*

$$\mathbf{d}_{LS} = \sum_{i=1}^{r} \frac{\mathbf{u}_i^T\mathbf{r}}{\sigma_i}\mathbf{v}_i = \boldsymbol{\mathcal{S}}^{+}\mathbf{r} \tag{36}$$

minimizes $\|\boldsymbol{\mathcal{S}}\mathbf{d} - \mathbf{r}\|_2$ *and has the smallest 2-norm of all minimizers. Moreover*

$$\varepsilon_{LS}^2 = \min_{\mathbf{x}\in\mathbb{R}} \|\boldsymbol{\mathcal{S}}\mathbf{x} - \mathbf{r}\|_2^2 \quad = \quad \sum_{i=r+1}^{L} (\mathbf{u}_i^T\mathbf{r})^2 \tag{37}$$

Proof. For any $\mathbf{x} \in \mathbb{R}^{K\times 1}$, we have

$$\begin{aligned} \|\boldsymbol{\mathcal{S}}\mathbf{x} - \mathbf{r}\|_2^2 &= \|(\mathbf{U}^T\boldsymbol{\mathcal{S}}\mathbf{V})(\mathbf{V}^T\mathbf{x}) - \mathbf{U}^T\mathbf{r}\|_2^2 \\ &= \|\mathbf{\Sigma}\alpha - \mathbf{U}^T\mathbf{r}\|_2^2 \\ &= \sum_{i=1}^{r}(\sigma_i\alpha_i - \mathbf{u}_i^T\mathbf{r}) + \sum_{i=r+1}^{m}(\mathbf{u}_i^T\mathbf{r})^2 \end{aligned} \tag{38}$$

where $\alpha = \mathbf{V}^T\mathbf{x}$. Clearly, if $\mathbf{x}$ solves the least-square problem, then $\alpha_i = \mathbf{u}_i^T\mathbf{r}/\sigma_i$ for $i = 1,\ 2,\ \dots,\ r$. If we set $\alpha_{r+1} = \alpha_{r+2} = \dots = \alpha_M$, then the resulting $\mathbf{x} = \mathbf{d}_{LS}$ clearly has minimal 2-norm. □

So, the least-square estimation of $\mathbf{d}$ is

$$\begin{aligned}\mathbf{d}_{LS} &= \boldsymbol{\mathcal{S}}^{+}\mathbf{r} \\ &= \mathbf{d} + \boldsymbol{\mathcal{S}}^{+}\tilde{\mathbf{n}}\end{aligned} \tag{39}$$

And, the linear filter bank representation $\mathbf{W}_{LS}$ in the proposed LS semi-blind decorrelating detector is

$$\mathbf{W}_{LS} = (\mathbf{C}\boldsymbol{\mathcal{S}}^{+})^{T} \tag{40}$$

And, the block of bits $\bar{\mathbf{b}}$ sent by the first user during the time interval $t \in [(n-P)T,\ nT]$ can be detected with the following equation.

$$\begin{aligned}\bar{\mathbf{b}}_{LS} &= \mathrm{sgn}\{\mathbf{W}_{LS}^{T}\mathbf{r}\} \\ &= \mathrm{sgn}\{\mathbf{C}\mathbf{d}_{LS}\} \\ &= \mathrm{sgn}\{[\,\mathbf{I}_P \quad \bar{\mathbf{D}}\,](\mathbf{d} + \boldsymbol{\mathcal{S}}^{+}\tilde{\mathbf{n}})\} \\ &= \mathrm{sgn}\{\bar{\mathbf{b}} + [\,\mathbf{I}_P \quad \bar{\mathbf{D}}\,]\boldsymbol{\mathcal{S}}^{+}\tilde{\mathbf{n}}\}\end{aligned} \tag{41}$$

5.2 Total Least-Square Semi-Blind Decorrelating Detector

It is easy to find that the above least-square estimation of detection matrix $\mathbf{d}$ is the minimum norm solution to the following equation,

$$\min_{\mathbf{x}} \|\mathbf{r} - \boldsymbol{\mathcal{S}}\mathbf{x}\|_2 \quad \text{subject to} \quad \mathbf{r} \subseteq \mathbb{R}(\boldsymbol{\mathcal{S}}) \ . \tag{42}$$

It is assumed that the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$ is error-free. However, this assumption is not true with its definition as in the equation (19). Obviously, there is the noise item $\bar{\mathbf{N}}$ exiting.

On the other hand, $\mathbf{r}$ can also be expressed as

$$\begin{aligned}\mathbf{r} &= \mathbf{SAb} + \mathbf{n} \\ &= \mathbf{SABB}^{+}\mathbf{b} + \mathbf{n} \\ &= (\boldsymbol{\mathcal{S}} - \bar{\mathbf{N}})\mathbf{B}^{+}\mathbf{b} + \mathbf{n} \\ &= \hat{\boldsymbol{\mathcal{S}}}\mathbf{d} + \mathbf{n}\end{aligned} \tag{43}$$

where $\hat{\boldsymbol{\mathcal{S}}} = \boldsymbol{\mathcal{S}} - \bar{\mathbf{N}} = \mathbf{SAB}$. Hence, the estimation of $\mathbf{d}$ can easily be transformed into the following total least-square problem

$$\min_{\bar{\boldsymbol{\mathcal{S}}},\ \mathbf{x}} \|[\,\boldsymbol{\mathcal{S}} \quad \mathbf{r}\,] - [\,\bar{\boldsymbol{\mathcal{S}}} \quad \bar{\boldsymbol{\mathcal{S}}}\mathbf{x}\,]\|_2 \quad \text{subject to} \quad \mathbf{r} \subseteq \mathbb{R}(\bar{\boldsymbol{\mathcal{S}}}) \quad . \tag{44}$$

Lemma 3. *[17] Let $\boldsymbol{\mathcal{S}} = \mathbf{U}'\mathbf{\Sigma}'\mathbf{V}'^{T}$ and $[\,\boldsymbol{\mathcal{S}} \quad \mathbf{r}\,] = \mathbf{U\Sigma V}^{T}$ be the SVD of $\boldsymbol{\mathcal{S}}$ and $[\,\boldsymbol{\mathcal{S}} \quad \mathbf{r}\,]$, respectively. If $\sigma'_M > \sigma_{M+1}$, then*

$$\mathbf{d}_{TLS} = (\boldsymbol{\mathcal{S}}^{T}\boldsymbol{\mathcal{S}} - \sigma_{M+1}^{2}\mathbf{I})^{-1}\boldsymbol{\mathcal{S}}^{T}\mathbf{r} \tag{45}$$

and

$$\varepsilon_{TLS}^{2} = \min_{\mathbf{x}\in\mathbb{R}^{M\times 1}} \|\boldsymbol{\mathcal{S}}\mathbf{x} - \mathbf{r}\|_2^2 = \sigma_{M+1}^{2}\left[1 + \sum_{i+1}^{M} \frac{(\mathbf{u}_i'^{T}\mathbf{r})^2}{\sigma_i'^{2} - \sigma_{M+1}^{2}}\right] \tag{46}$$

where
$\mathbf{U} = [\,\mathbf{u}_1 \quad \mathbf{u}_2 \quad \ldots \quad \mathbf{u}_{PL}\,]$, $\mathbf{V} = [\,\mathbf{v}_1 \quad \mathbf{v}_2 \quad \ldots \quad \mathbf{v}_{M+1}\,]$, $\mathbf{\Sigma} = diag\{[\,\sigma_1 \quad \ldots \quad \sigma_{M+\min\{PL-M,\ 1\}}\,]\}$ *and* $\mathbf{U}' = [\,\mathbf{u}_1' \quad \mathbf{u}_2' \quad \ldots \quad \mathbf{u}_{PL}'\,]$, $\mathbf{V}' = [\,\mathbf{v}_1' \quad \mathbf{v}_2' \quad \ldots \quad \mathbf{v}_M'\,]$, $\mathbf{\Sigma}' = diag\{[\,\sigma_1' \quad \sigma_2' \quad \ldots \sigma_M'\,]\}$;

So, the total least-square estimation of $\mathbf{d}$ is

$$\mathbf{d}_{TLS} \;=\; (\boldsymbol{\mathcal{S}}^T\boldsymbol{\mathcal{S}} - \sigma^2_{M+1}\mathbf{I})^{-1}\boldsymbol{\mathcal{S}}^T\mathbf{r} \tag{47}$$

The linear filter representation of the presented total least-squaer semi-blind decorrelating detector is

$$\mathbf{W}_{TLS} \;=\; \mathbf{C}^T(\boldsymbol{\mathcal{S}}^T\boldsymbol{\mathcal{S}} - \sigma^2_{M+1}\mathbf{I})^{-1}\boldsymbol{\mathcal{S}}^T \tag{48}$$

And, the block of bits $\bar{\mathbf{b}}$ sent by the first user during the time interval $t \in [(n-P)T,\ nT]$ can be detected with the following equation,

$$\begin{aligned}\bar{\mathbf{b}}_{TLS} &= \mathrm{sgn}\{\mathbf{W}^T_{TLS}\mathbf{r}\}\\ &= \mathrm{sgn}\{\mathbf{C}^T\mathbf{d}_{TLS}\}\\ &= \mathrm{sgn}\{[\,\mathbf{I}_P \quad \bar{\mathbf{D}}\,](\boldsymbol{\mathcal{S}}^T\boldsymbol{\mathcal{S}} - \sigma^2_{M+1}\mathbf{I})^{-1}\boldsymbol{\mathcal{S}}^T\mathbf{r}\}\end{aligned} \tag{49}$$

5.3 Mixed LS/TLS Semi-Blind Decorrelating Detector

In the least-square estimation of $\mathbf{d}$, it assumes that the signature matrix $\boldsymbol{\mathcal{S}}$ is noise-free. In total least-square estimation of $\mathbf{d}$, it assumes that there is noise in each element of $\boldsymbol{\mathcal{S}}$. However, we can see that, except the first P columns of $\boldsymbol{\mathcal{S}}$ are exactly known to be noise/error-free, there are noise existing in each element of its rest $M-P$ columns. To maximize the accuracy of the estimated detection vector $\mathbf{d}$, it is natural to seek the best fitting solution that is appropriate to the received signal vector $\mathbf{r}$ and the polluted $M-P$ columns in the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$, while keeping the exactly known P columns of $\boldsymbol{\mathcal{S}}$ unperturbed. Thus, the problem to estimate the detection vector $\mathbf{d}$ can easily be transformed into the following mixed LS/TLS estimation problem.

$$\min_{\hat{\bar{\boldsymbol{\mathcal{S}}}},\,\mathbf{x}} \big\|[\,\bar{\boldsymbol{\mathcal{S}}} \quad \mathbf{r}\,] - [\,\hat{\bar{\boldsymbol{\mathcal{S}}}} \quad [\,\bar{\mathbf{S}}_1\bar{\mathbf{A}}_1 \quad \hat{\bar{\boldsymbol{\mathcal{S}}}}\,]\mathbf{x}\,]\big\|_2 \quad \text{subject to} \quad \mathbf{r} \subseteq \mathbb{R}([\,\bar{\mathbf{S}}_1\bar{\mathbf{A}}_1 \quad \hat{\bar{\boldsymbol{\mathcal{S}}}}\,]) \tag{50}$$

This mixed LS/TLS problem can be solved with the following lemma.

Lemma 4. *[17] Consider the mixed least squares and total least squares problem in equation (50) and perform the householder transformations Q on the matrix $[\,\boldsymbol{\mathcal{S}} \quad \mathbf{r}\,]$ so that*

$$Q^H[\,\bar{\mathbf{S}}_1\bar{\mathbf{A}}_1 \quad \bar{\boldsymbol{\mathcal{S}}} \quad \mathbf{r}\,] \;=\; \begin{bmatrix}\mathbf{R}_{11} & \mathbf{R}_{12} & \mathbf{R}_{1r}\\ \mathbf{0} & \mathbf{R}_{22} & \mathbf{R}_{2r}\end{bmatrix} \tag{51}$$

where $\mathbf{R}_{11}$ is a $P\times P$ upper triangle matrix, $\mathbf{R}_{12}$ is a $P\times(M-P)$ matrix, $\mathbf{R}_{22}$ is a $(L-P)\times(M-P)$ matrix , $\mathbf{R}_{1r}$ is a $P\times 1$ vector and $\mathbf{R}_{2r}$ is a $(L-P)\times 1$ vector.

Denote σ' the smallest singular value of $\mathbf{R}_{22}$ and σ the smallest singular value of $[\,\mathbf{R}_{22} \quad \mathbf{R}_{2r}\,]$. If $\sigma' > \sigma$, then the MLS solution uniquely exists and is given by

$$\mathbf{d}_{MLS} \;=\; \left(\boldsymbol{\mathcal{S}}^T\boldsymbol{\mathcal{S}} - \sigma^2\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{M-P}\end{bmatrix}\right)^{-1}\boldsymbol{\mathcal{S}}^T\mathbf{r}\ . \tag{52}$$

So, the linear filter representation of the presented MLS semi-blind decorrelating detector is

$$\mathbf{W}_{MLS} \;=\; \boldsymbol{\mathcal{S}}\left(\boldsymbol{\mathcal{S}}^T\boldsymbol{\mathcal{S}} - \sigma^2\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{M-P}\end{bmatrix}\right)^{-1}\mathbf{C}^T \tag{53}$$

And, the block of bits $\bar{\mathbf{b}}$ sent by the first user during the time interval $t \in [(n-P)T \;\; nT]$ can be detected with the following equation.

$$
\begin{aligned}
\bar{\mathbf{b}}_{MLS} &= \operatorname{sgn}\{\mathbf{W}_{MLS}^T \mathbf{r}\} \\
&= \operatorname{sgn}\left\{ [\mathbf{I}_P \quad \bar{\mathbf{D}}] \left(\boldsymbol{\mathcal{S}}^T \boldsymbol{\mathcal{S}} - \sigma^2 \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{M-P} \end{bmatrix} \right)^{-1} \boldsymbol{\mathcal{S}}^T \mathbf{r} \right\} \\
&= \operatorname{sgn}\{\mathbf{C}\mathbf{d}_{MLS}\} \\
&= \operatorname{sgn}\{[\mathbf{I}_P \quad \bar{\mathbf{D}}]\mathbf{d}_{MLS}\} \quad .
\end{aligned} \tag{54}
$$

6 Performance Analysis

6.1 The Relationship Between the Semi-Blind Signature Matrix $\boldsymbol{\mathcal{S}}$ and the Signature Matrix S

As we may see, one of the most different between the classic single-truncated-window decorrelating detector and the proposed semi-blind decorrelating detector is the definition of the signature matrix. When $P = 1$, the proposed semi-blind signature matrix $\boldsymbol{\mathcal{S}}$ is composed with the desired user's signature and $M-1$ uncorrelated previously received vector as the rest columns while the original signature matrix $\mathbf{S}$ is defined with the desired user's complete signature vector and all the other users' partial signature vectors. When $\bar{\mathbf{N}} = \mathbf{0}$, it is very easy to see that these two signal subspaces, span$\{\mathbf{S}\}$ and span$\{\boldsymbol{\mathcal{S}}\}$, are the same one.

6.2 The Relationship Between Decorrelating Detector and LS Semi-Blind Detector

When there is no noise the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$, $\hat{\boldsymbol{\mathcal{S}}} = \boldsymbol{\mathcal{S}}$ and $\tilde{\mathbf{n}} = \mathbf{n}$. When $\bar{\mathbf{N}} = \mathbf{0}$ and $P = 1$, there is the following relationship between the first user's LS semi-blind decorrelating detector $\mathbf{w}_{LS}$ and its classic single-truncated-window decorrelating detector $\mathbf{w}_{DD}$.

$$
\mathbf{w}_{LS} = A_1^{-1} \mathbf{w}_{DD} \tag{55}
$$

Furthermore, the classic decorrelating detector can be taken as a special case of the semi-blind decorrelating detector with $\mathbf{B} = \mathbf{I}$. And the output of the limiter of the LS semi-blind decorrelating detector is

$$
b_{LS} = \operatorname{sgn}\{b_1 + A_1^{-1} \mathbf{S}^{+} \mathbf{n}\} \tag{56}
$$

At this time, the bit-error-rate of the user 1 with the presented LS semi-blind decorrelating detector is same to that with single-truncated-window decorrelating detector. It can be expressed as

$$
P_e = Q\left(\frac{A_1}{\sigma_n \sqrt{R_{11}^{+}}} \right) \tag{57}
$$

where R_{11}^{+} is a shorthand for $(\mathbf{R}^{-1})_{11}$, the 1th row and 1th column element in the matrix $\mathbf{R}^{-1} = (\mathbf{S}^T\mathbf{S})^{-1}$.

Thus, the presented LS semi-blind detector at this time achieves the maximum near-far resistance as the classic single-truncated-window decorrelating detector does.

6.3 The Relationship Between $\bar{\mathbf{b}}$ and $\mathbf{d}$

We can see that the detection vector $\mathbf{d}$ plays the key pole in the proposed semi-blind decorrelating detectors. When there is no estimation error in the detection vector $\mathbf{d}$, with Lemma 1, the desired bit vector $\bar{\mathbf{b}}$ could exactly be estimated from the detection vector $\mathbf{d}$ and $\mathbf{C}$. When the detection vector $\mathbf{d}$ cannot be exactly estimated, the relationship between the estimation errors of $\bar{\mathbf{b}}$ and $\mathbf{d}$ is

$$\Delta b_p \quad \leq \quad \|\Delta\tilde{\mathbf{d}}\|_1 + |\bar{d}_p| \tag{58}$$

where b_p denotes the pth element of vector $\mathbf{b}$, $\hat{b}_p$ is a estimate of b_p, $\Delta b_p = \hat{b}_p - b_p$, $\Delta\tilde{\mathbf{d}} = \hat{\tilde{\mathbf{d}}} - \tilde{\mathbf{d}}$, $\bar{d}_p$ denotes the pth element of $\bar{\mathbf{d}}$ and $\| \star \|_1$ denotes the 1-norm of the vector $\star$.

6.4 The Relationship between the Noise Items $\tilde{\mathbf{n}}$ and $\mathbf{n}$

With the following lemma, it is easy to see that the new semi-blind noise item $\tilde{\mathbf{n}}$ is enhanced compared to the former noise item $\mathbf{n}$. This enhancement is because there is the noise $\bar{\mathbf{N}}$ existing in the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$.

The mean of the semi-blind noise item $\tilde{\mathbf{n}}$ which is defined in equation (28) is

$$\tilde{m} \quad = \quad E\{\tilde{\mathbf{n}}\} \quad = \quad 0 \tag{59}$$

The variance of the semi-blind noise item $\tilde{\mathbf{n}}$ satisfies the following inequation

$$\max\{var\{\tilde{\mathbf{n}}\}\} \quad = \quad \max\{E\{(\tilde{\mathbf{n}} - \tilde{m})^2\}\} \quad \leq \quad \sigma_n^2 + (P+1)(K-1)\|\tilde{\mathbf{D}}^{+}\|_2^2\sigma_{\bar{n}}^2 \tag{60}$$

where $\max\{\star\}$ denotes the maximum item in the vector $\star$ and $\sigma_{\bar{n}}^2$ is the power of the noise item $\bar{\mathbf{N}}$ in the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$.

7 Computer Simulations

In this section, various computer simulations and analytical results are presented. In the computer simulations, two users are sending the signals in asynchronous CDMA system. The spreading gain $g = 24$. the covariance matrix between these two users are $\mathbf{R}$.

$$\begin{aligned}
\mathbf{R} \quad &= \quad \begin{bmatrix} \mathbf{s}_1^T \\ \mathbf{s}_{2_-}^T \\ \mathbf{s}_{2_+}^T \end{bmatrix} \begin{bmatrix} \mathbf{s}_1 & \mathbf{s}_{2_-} & \mathbf{s}_{2_+} \end{bmatrix} \\
&= \quad \begin{bmatrix} 1.0000 & -0.6250 & -0.0417 \\ -0.6250 & 0.8750 & 0 \\ -0.0417 & 0 & 0.1250 \end{bmatrix}
\end{aligned} \tag{61}$$

and the channel is AWGN channel. We do the simulations with $P = 1$, 2, 3, 4, 5, 7 and 9. Correspondingly, the number of columns in the semi-blind signature matrix is $M = (P+1) \times K - 1$. We will compare our algorithms with single-user matched filter (MF) detector and the single-truncated-window decorrelating detector (DD).

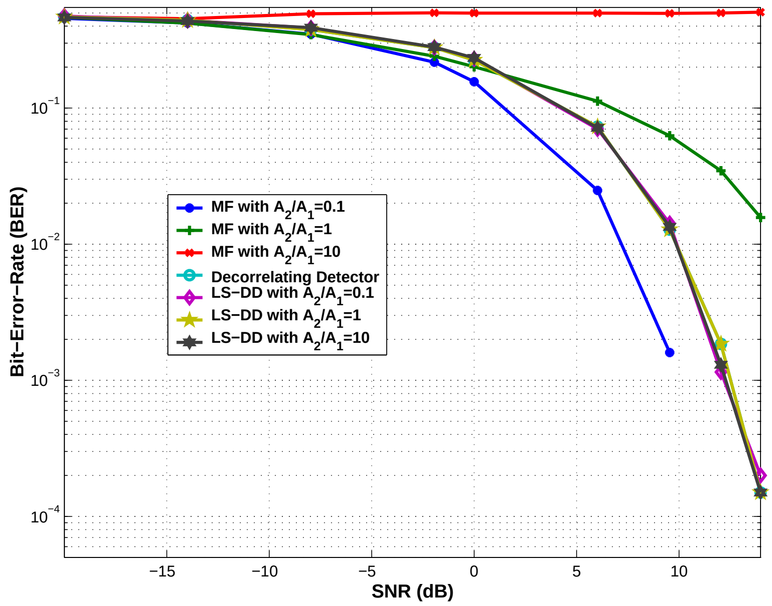

Figure 2: Bit-error-rate comparison of the signal-user matched filter, decorrelating detector, and the presented LS semi-blind detector with $P = 1$ for the first user within two users.

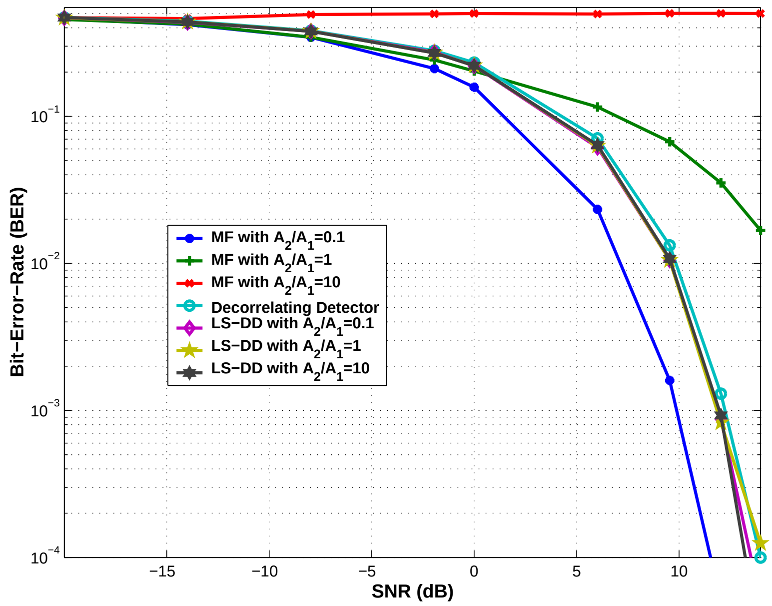

Figure 3: Bit-error-rate comparison of the signal-user matched filter, decorrelating detector, and the presented LS semi-blind detector with $P = 4$ for the first user within two users.

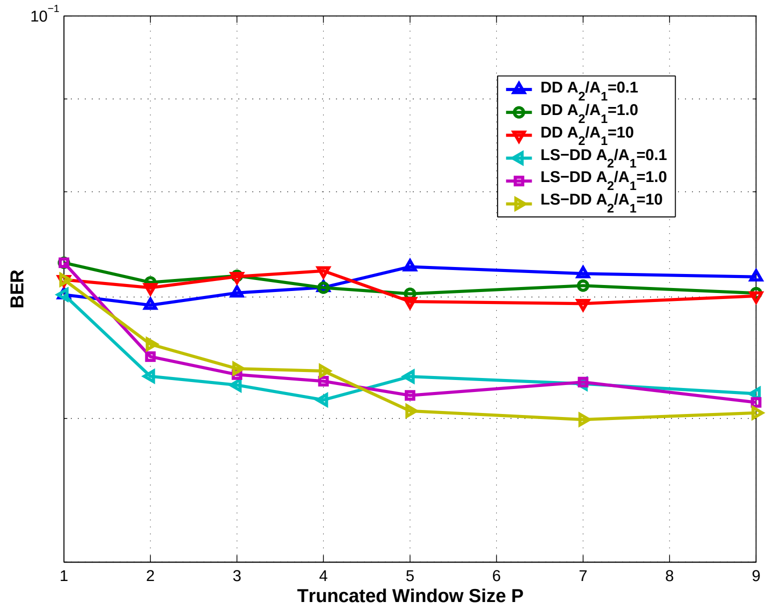

Figure 4: Bit-error-rate comparison of the classic signal-truncated-window decorrelating detector and the proposed LS semi-blind detector against the window size P.

case 1: $\bar{\mathbf{N}} = \mathbf{0}$

In this case, we suppose that there is no noise in the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$ or the noise is negligible, compared to the noise in the received signal vectors. With the theoretical analysis before, since there is no noise or the noise is negligible in the semi-blind signature matrix, only the LS semi-blind detector is examined and compared to the single-user matched filter detector and the single-truncated-window decorrelating detector in this case.

As we see in figure 2, the performance of the LS semi-blind detector with $P = 1$ is very close to that of the single-truncated window decorrelating detector. This also supports that the classic single-truncated-window decorrelating detector just is a special case of the proposed LS-DD detector with $\bar{\mathbf{N}} = \mathbf{0}$ and $\mathbf{B} = \mathbf{I}$. Comparing figure 3 to figure 2, the performance of the LS-DD with four consecutive windows ($P = 4$) is a little bit better than that of the LS-DD with single truncated window ($P = 1$) and the classic single-truncated-window decorrelating detector. As we analyzed before, it is because that more complete signatures of other users are employed in th semi-blind signature matrix $\boldsymbol{\mathcal{S}}$.

In figure 4, the performance of the proposed LS semi-blind detector is checked with the changing of the window size P. It is easy to see that, when the single window is employed ($P = 1$), the performance of the proposed LS semi-blind detector is same to that of the classic decorrelating detector. When multiple windows are employed and $P > 1$, the performance of the proposed LS semi-blind detector is better than that of the classic decorrelating detector. And the BER performance of the LS semi-blind detector and the classic decorrelating detector is kept unchanged against A_2/A_1.

Now, the NFR performance of the signal-user matched filter, the classic single-truncated-window decorrelating detector and the proposed LS semi-blind detector are checked in figure 5. As we see, the classic single-truncated-window decorrelating detector and the proposed LS semi-blind detector have the same optimum near-far resistance and their BERs are not change against A_2/A_1. But the BER performance of the MF detector is decreasing when A_2/A_1 becomes larger.

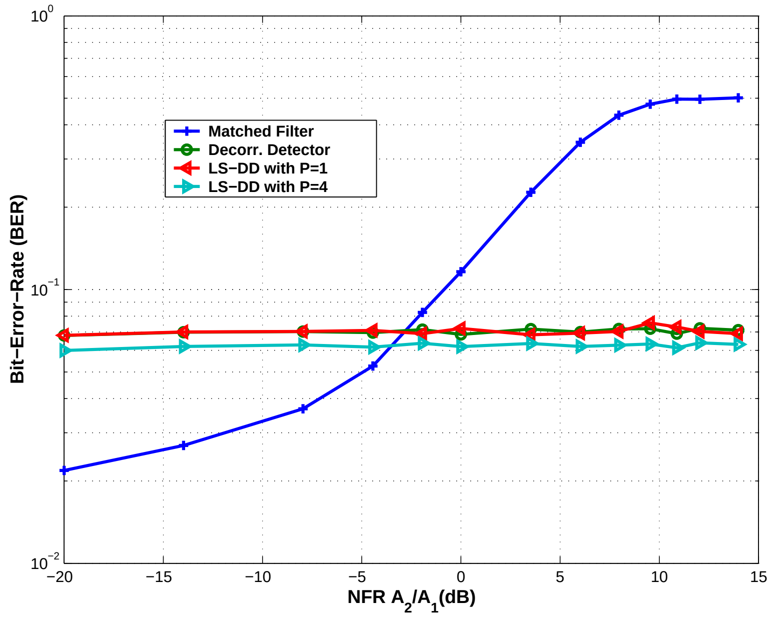

Figure 5: NFR performance of the signal-user matched filter, decorrelating detector, and the proposed LS semi-blind detector with $P = 1$ and $P = 4$ for the first user within two users.

Furthermore, the performance of the proposed LS semi-blind detector with $P = 4$ is better than that of the classic single-window decorrelating detector. And the proposed LS semi-blind detector with $P = 1$ has the similar performance to the classic single-window decorrelating detector.

case 2: $\bar{\mathbf{N}} \neq \mathbf{0}$

In the case, we are dealing some more practical situations. So, there would be the same level white Gaussian noise existing in the blind signature matrix $\boldsymbol{\mathcal{S}}$ as in the received signal vectors. Then, both LS and MLS semi-blind detectors are used to detect the expected signal.

In figure 6 and 7, the performance of the proposed LS semi-blind detector with $P = 1$ and $P = 4$ is checked against SNR and A_2/A_1, respectively. The most interesting is that, when $A_2/A_1 = 0.1$, the performance of the proposed LS semi-blind detector is very close to the MF detector and better than that of the classic decorrelating detector. When $A_2/A_1 = 1$ or 10, the performance of the LS semi-blind decorrelating detector is decreased and is worse than that of the classic decorrelating detector. But it is still better than that of the MF detector. In this case, the performance of LS semi-blind detector is always between those of the decorrelating detector and single-user matched-filter detector.

On the other hand, when $A_2/A_1 = 0.1$, the performance of the proposed LS semi-blind detector with $P = 4$ is a little better than that with $P = 1$. But, when $A_2/A_1 = 10$, the performance of the proposed LS semi-blind detector with $P = 1$ is better than that with $P = 4$.

The BER performance of the proposed MLS semi-blind detector is checked is checked in figure 8 and 9 against SNR and A_2/A_1. As we see, the performance of the proposed MLS detector is basically between that of the classic decorrelating detector and MF detector when $A_2/A_1 = 10$.

In figure 4, we checked the performance of the proposed LS and MLS multi-window semi-blind detector and the classic single-truncated-window decorrelating detector, with the changing of the window size P. As we see, except the performance of the LS semi-blind detector with

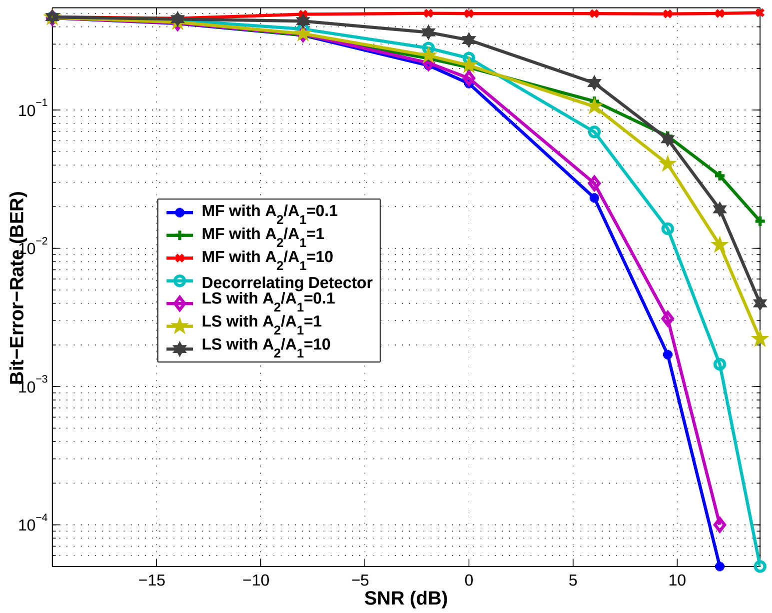

Figure 6: Bit-error-rate comparison of the signal-user matched filter, the single-truncated-window decorrelating detector, and the proposed LS semi-blind detector with $P = 1$ for the first user within users.

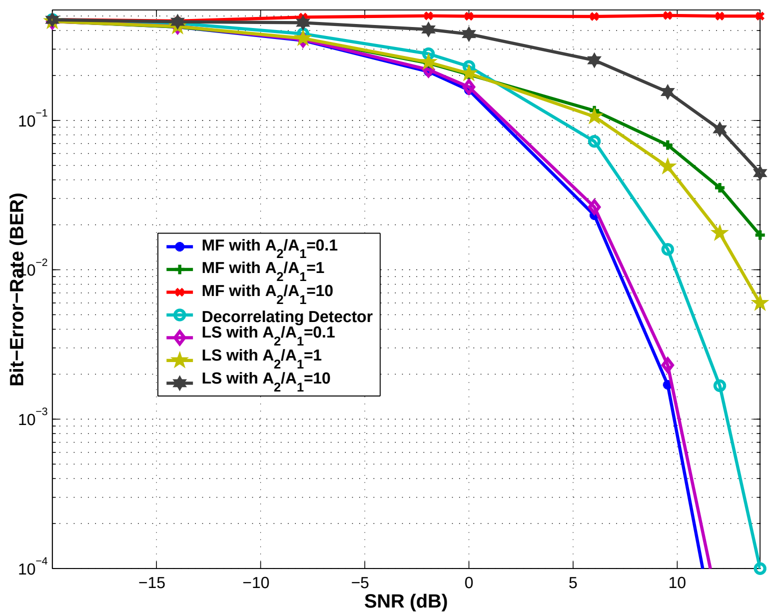

Figure 7: Bit-error-rate comparison of the signal-user matched filter, the single-truncated-window decorrelating detector, and the proposed LS semi-blind detector with $P = 4$ for the first user within users.

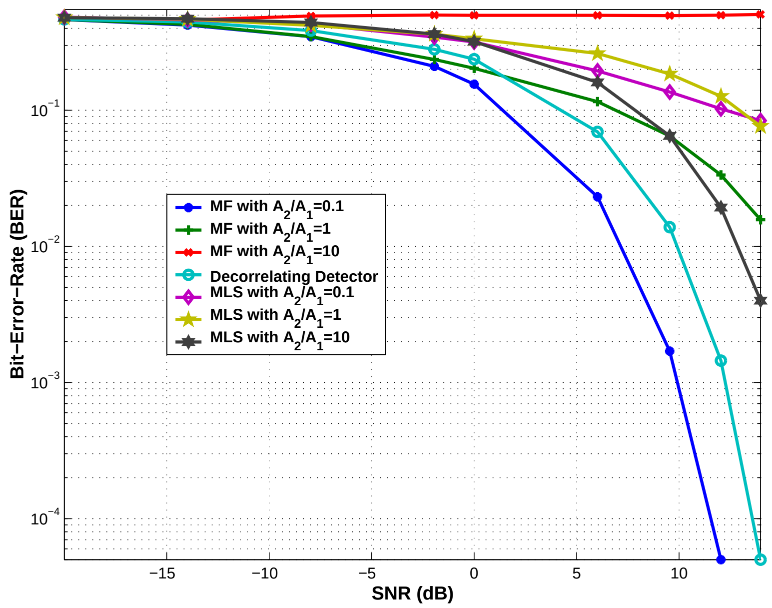

Figure 8: Bit-error-rate comparison of the signal-user matched filter, the single-truncated-window decorrelating detector, and the proposed MLS semi-blind detector with $P = 1$ for the first user within two users.

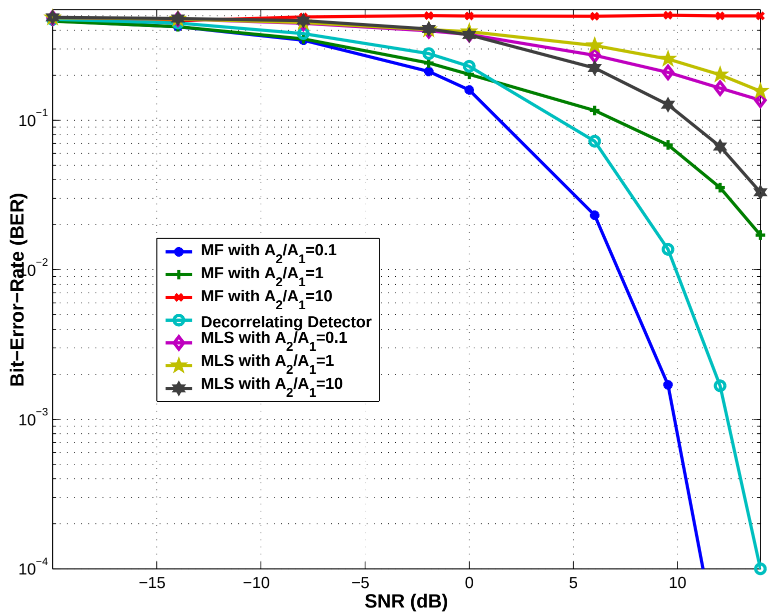

Figure 9: Bit-error-rate comparison of the signal-user matched filter, the single-truncated-window decorrelating detector, and the proposed MLS semi-blind detector with $P = 4$ for the first user within two users.

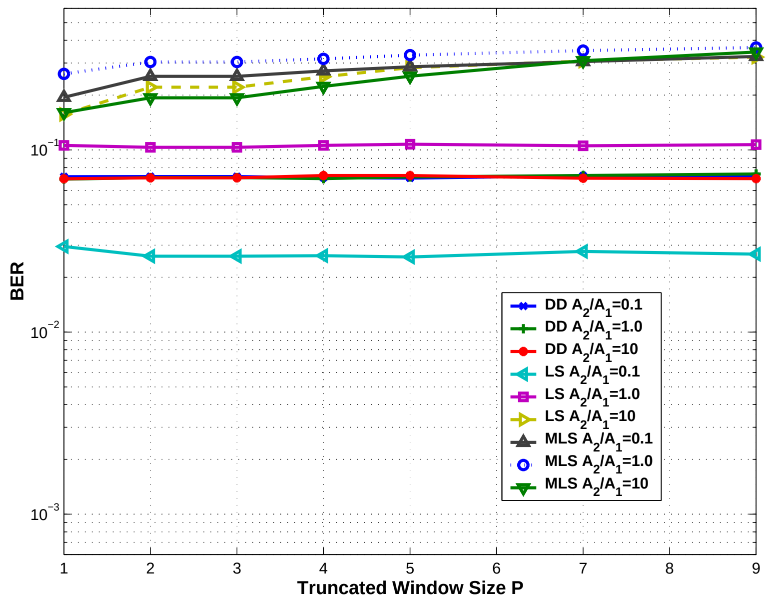

Figure 10: Bit-error-rate comparison of the classic signal-truncated-window decorrelating detector and the proposed LS and MLS semi-blind detectors with against the window size P

$A_2/A_1 = 0.1$ is better than that of the classic decorrelating detector, the performance of the proposed semi-blind detectors in the other case are not as good as that of the classic decorrelating detector.

In figure 11, we check the NFR performance of the signal-user matched filter, the classic single-truncated-window decorrelating detector, and the proposed LS and MLS semi-blind detector with $P = 1$ and $P = 4$. Firstly, the BER performance of the proposed MLS semi-blind detector with $P = 1$ or $P = 4$ is basically not changed against A_2/A_1. From figure 6 and 7, we can see that the performance of the LS semi-blind detector is between that of the MF detector and the classic decorrelating detector. When A_2/A_1 is large enough, the performance of the LS semi-blind detector with $P = 1$ and $P = 4$ is closed to that of the MLS semi-blind detector with $P = 1$ and $P = 4$, respectively.

8 Conclusions

In this paper, we presented the one-shot LS and MLS semi-blind decorrelating detectors with the multiple consecutive truncated windows. In these two semi-blind multi-windows decorrelating detector, besides the signature and timing of the desired user, the amplitude of this user is also required. This is why they are called semi-blind detectors. In the theoretical analysis and computer simulations, as we may see, when there is no noise in the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$ and $P = 1$, the classic single-truncated-window decorrelating detector is just one special case of the presented LS semi-blind detector with $\mathbf{B} = \mathbf{I}$. And when there is no noise in the semi-blind signature matrix, the performance of the LS semi-blind detector with multiple windows is better than that with $P = 1$ and the classic single-truncated-window decorrelating detector.

These two semi-blind decorrelating detectors are simple and direct. No searching or converging procedure is required as in other semi-blind/semi-blind detectors.

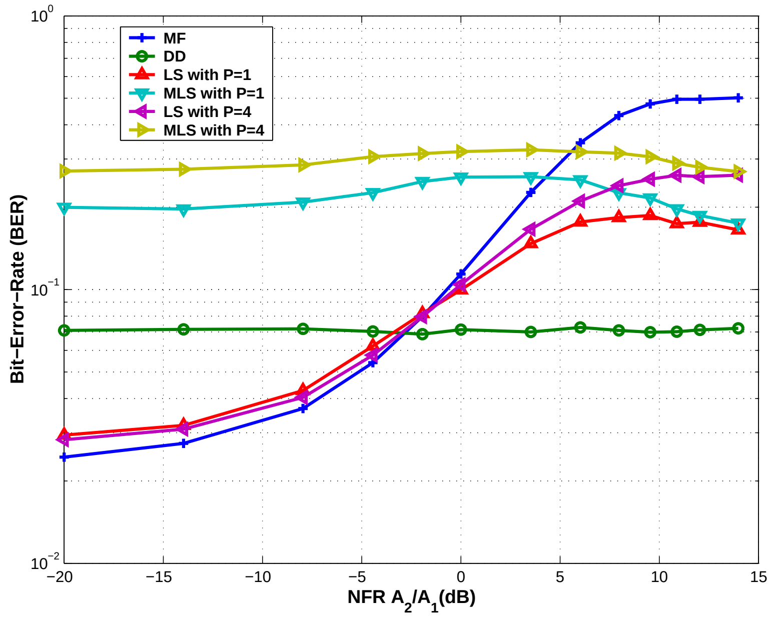

Figure 11: Near-far resistance comparison of the signal-user matched filter, single-truncated-window decorrelating detector, and the proposed LS and MLS semi-blind detectors for the first user within two users. $SNR = 6dB$

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