On Soft Interference Cancellation for Synchronous CDMA

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Abstract

Compensating for near-far effects is critical for satisfactory performance of Code Division Multiple Access (CDMA) systems. Interference cancellation is one of the multiuser detection (MUD) methods for suppressing the effects from the multiple access interference (MAI) and consequently improving the system performance. In this paper, the principles of interference cancellation (IC) for synchronous CDMA are reviewed. We introduce several soft interference cancellation schemes, including individually and jointly optimum interference cancellation, direct interference cancellation, maximum asymptotic multiuser efficiency (MAME) interference cancellation and minimum mean-square error (MMSE) interference cancellation algorithms. In addition to showing that optimum multiuser detection and MAME multiuser detection can be realized as interference cancellers, the direct interference cancellation detector can be taken as the special realization of the classic decorrelating detection while the linear MMSE interference cancellation has the same performance as the linear MMSE multiuser detector.

1 Introduction

Direct-sequence code division multiple access (DS/CDMA) techniques have attracted increasing attention for efficient use of available bandwidth, resistance to interference and flexibility to variable traffic patterns. Multiuser detection strategy is a method to minimize the effects of MAI and mitigate the near-far problem in CDMA systems. It has been extensively investigated over the past several years [1], since MAI is the dominant impairment for CDMA systems and exists even with perfect power control. Since 1980s, there have been lots of work conducted on the derivation and analysis of the optimum multiuser receivers. The optimum multiuser receiver was developed by Verdú [2, 3, 4], but has overwhelming complexity. The large gaps in performance and complexity between the conventional single-user matched filter and optimum multiuser detector encouraged the search for other multiuser detectors that exhibit good performance/complexity tradeoffs. There has been considerable interest in linear multiuser detection based on decorrelating, MAME or MMSE criterion [5, 6, 7].

The classic decorrelating detector [5] not only is a simple and natural strategy but also is optimal according to three different criteria: least squares, near-far resistance [8] and maximum-likelihood when the received amplitudes are unknown [5]. Linear multiuser detectors can be implemented in a decentralized fashion where only the user(s) of interest need be demodulated. In the MAME linear multiuser detector [6], the computational complexity is prohibitive for large number of user. For MMSE linear multiuser detector [7], although it doesn't achieve minimum bit-error rate (BER), it has been proved to achieve optimal near-far resistance. It is shown that there is no explicit knowledge of interference parameters required in MMSE detector. More recently, it suggested that the output of MMSE detector can be accurately approximated by Gaussian noise in many cases [9]. Compared to decorrelating detector and MAME detector, MMSE detector can be blindly realized without any knowledge of interference parameters.

Interference cancellation [10, 11, 12, 13, 14, 15] provides a promising alternative to the conventional or optimum detectors in multiuser detection. Interference cancellation methods typically require less implementation complexity while practically offering similar performance. The idea behind interference cancellation is to estimate the multiple access and/or multipath induced interference and then to subtract the interference estimate from the received signal. Hence, compared to other multiuser detection schemes, interference cancellation pays more attention on the estimation of the MAI. Different schemes for the MAI estimation lead to different interference cancellation schemes. Actually, interference cancellation detector will cancel the interfering signal exactly provided that the decision was correct and channel information is known. Otherwise it will double the contribution of the interferers. The main alternatives for implementation of interference cancellation are parallel hard interference cancellation (PIC) and serial/successive hard interference cancellation (SIC) [11, 12], while many other variants on these basic principles have also been developed. With conventional PIC [13, 14], all user are simultaneously demodulated and detected in a parallel behave. With conventional SIC, a decision for the symbol of the stronger user is made first, the interference from this user is subsequently removed in the the next stronger user's receiver before the next user's receiver make its decision and so on.

In this work, we consider a synchronous DS/CDMA system and review the principles of interference cancellation. several soft interference cancellation schemes, including individually and jointly optimum interference cancellation, direct interference cancellation detector, MAME interference cancellation detector and MMSE interference detector, are introduced with different MAI estimation schemes. With the discussion of the relationship between soft interference cancellation and multiuser detection, we show that, besides optimum interference cancellation and MAME interefence cancellation, the proposed direct interference cancellation detector has the same performance of the classic decorrelating detector while the MMSE multiuser detection and the MMSE interference cancellation actually are the same detectors.

The rest of the paper is organized as follows. In Section II, we summarize the signal model. In Section III, we review the interference cancellation in multiuser detection and proposed the direct interference cancellation, maximum asymptotic multiuser efficiency interference cancellation and minimum mean-square error interference cancellation detectors, besides those optimum interference cancellations. In Section IV, some theoretical analysis results are present. In section V, the conclusion are presented, respectively.

2 Data Model and Problem Description

The basic CDMA K-user channel model, consisting of the sum of antipodally modulated synchronous signature waveforms embedded in additive white Gaussian noise (AWGN), is considered here. The received base-band signal during one symbol interval in such a channel can be modeled as:

$$r(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + n(t) \tag{1}$$

where $n(t)$ represents the Gaussian channel noise, K is the number of users, A_k and b_k denote the received amplitude and nth data bit of the kth user, respectively, $t \in [(n-1)T,\ nT]$, T is the symbol interval. It is assumed that $b_k \in \{-1,\ +1\}$ is a collection of independent equiprobable ± 1 random variables transmitted by the kth user during $[(n-1)T,\ nT]$ and $s_k(t)$ denotes the normalized signal waveform of the kth user on the interval $[(n-1)T,\ nT]$, i.e., $\|s_k(t)\| = 1$.

The received signal $r(t)$ is passed through a chip-matched filter followed by a chip-rate sampler. As a result, $r(t)$, $t \in [(n-1)T,\ nT]$, is converted into a $L \times 1$ column vector $\mathbf{r}$ of the samples of the chip-matched filter outputs within a symbol interval $[(n-1)T,\ nT]$ as [1]

$$\begin{aligned} \mathbf{r} &= \sum_{k=1}^{K} A_k b_k \mathbf{s}_k + \mathbf{n} \\ &= \mathbf{SAb} + \mathbf{n} \end{aligned} \tag{2}$$

where $\mathbf{A} = \mathrm{diag}\{\, A_1 \quad A_2 \quad \ldots \quad A_K \,\}$ is the received amplitude diagonal matrix, $\mathbf{S} = [\mathbf{s}_1\, \mathbf{s}_2\, \ldots\, \mathbf{s}_K]$ is the $L \times K$ signature matrix with the kth column $\mathbf{s}_k$ being the signature vector of the kth user, $\mathbf{b} = [b_1\ b_2\ \ldots\ b_K]^T = [b_1\ \tilde{\mathbf{b}}^T]^T$ is the information vector sent by all the K users at time $t = n$ and b_1 is the bit sent by the first user at time $t = n$, and $\mathbf{n}$ is an L-dimensional Gaussian vector with independent σ^2-variance components. We maintain the restriction that $L \geq K$.

Most of the linear multiuser detectors for demodulating the kth user's data bit in (2) is in the form of a correlator followed by a hard limiter, which can be expressed as

$$\hat{b}_k = \mathrm{sgn}\{\mathbf{w}_k^T \mathbf{r}\} \tag{3}$$

where $\mathbf{w}_k \in \mathbb{R}^{L \times 1}$ is the linear representation of multiuser detector.

Linear multiuser detectors can be implemented in a decentralized fashion where only the user(s) of interest need be demodulated.

3 Interference Cancellation

Interference cancellation is one of several multiuser detection methods to suppress the effects from the MAI and consequently in return increase the capacity of CDMA systems. As in

[1] Without loss of generality, the subscript index n is dropped. Hence, it is assumed $\mathbf{r} = \mathbf{r}_n$, $\mathbf{b} = \mathbf{b}_n$ and $b_k = b_k^{(n)}$, $k = 1, 2, \ldots, K$.

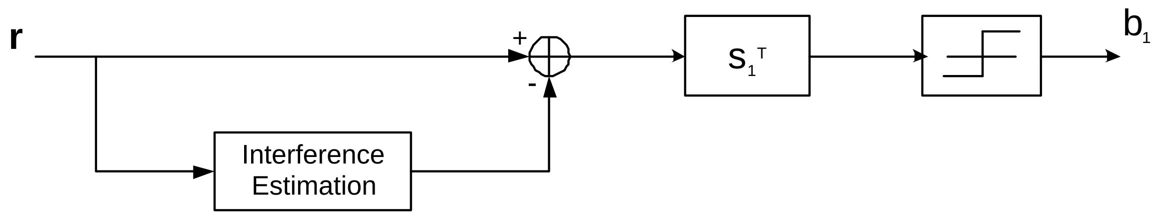

Figure 1: The block structure of a basic interference cancellation detector for user 1.

figure 1, there are usually two basic stages in interference cancellation realization. At the first stage, the MAI from other users are reconstructed. At the second stage, the MAI is removed from the received signal and the final decision is made from the rest signal through a matched filter. Thus, the key part in interference cancellation is how to estimate MAI as efficiently as possible.

Without loss of the generality, assume only the bits b_1 sent by the first user is considered in the following description. With equation (2), the received signature vector $\mathbf{r}$ can be expressed as the combination of the desired signal vector, MAI and AWGN vector as the following equation.

$$\begin{aligned} \mathbf{r} &= b_1 A_1 \mathbf{s}_1 + \sum_{k=2}^{K} b_k A_k \mathbf{s}_k + \mathbf{n} \\ &= b_1 A_1 \mathbf{s}_1 + \tilde{\mathbf{S}}\tilde{\mathbf{A}}\tilde{\mathbf{b}} + \mathbf{n} \\ &= b_1 A_1 \mathbf{s}_1 + \mathbf{m} + \mathbf{n} \end{aligned} \tag{4}$$

where $\tilde{\mathbf{S}} = [\, \mathbf{s}_2 \quad \mathbf{s}_3 \quad \ldots \quad \mathbf{s}_K \,]$, $\tilde{\mathbf{A}} = diag\{\, A_2 \quad A_3 \quad \ldots \quad A_K \,\}$, $\tilde{\mathbf{b}} = [\, b_2 \quad b_3 \quad \ldots \quad b_K \,]^T$ and

$$\mathbf{m} \;=\; \sum_{k=2}^{K} b_k A_k \mathbf{s}_k \;=\; \tilde{\mathbf{S}}\tilde{\mathbf{A}}\tilde{\mathbf{b}} \tag{5}$$

denotes the MAI [2].

Before further discussion about interference cancellation, it will be helpful to clarify the geometric meaning of single-user matched filter. For this, the following proposition is presented.

Proposition 1. *If $\hat{\mathbf{r}}_1$ is an observation of $b_1 A_1 \mathbf{s}_1$, the output of this observation through the single-user matched filter followed by a limiter*

$$\hat{b}_1 \;=\; sgn\left\{\mathbf{s}_1^T \hat{\mathbf{r}}_1\right\} \tag{6}$$

actually is the solution to the following equation

$$\min_b \|\hat{\mathbf{r}}_1 - bA_1\mathbf{s}_1\|_2 \quad subject\ to \quad b \in \{\pm 1\} \tag{7}$$

where A_1 denotes the unknown amplitude.

[2]Without loss of generality, $\mathbf{m}$ specially denotes the MAI to user 1 in this paper.

When the interference is estimated as $\hat{\mathbf{m}}$, the output of interference cancellation is the solution to the following equation

$$\hat{b}_1 = \arg\min_b \|\mathbf{r} - \hat{\mathbf{m}} - bA_1\mathbf{s}_1\|_2 \quad \text{subject to} \quad b \in \{\pm 1\} \tag{8}$$

and the result is

$$\hat{b}_1 \quad = \quad \mathrm{sgn}\left\{\mathbf{s}_1^T(\mathbf{r} - \hat{\mathbf{m}})\right\}. \tag{9}$$

In the following, how to the estimation of the MAI will be presented in details with different criteria.

3.1 Perfect Interference Cancellation

In perfect interference cancellation (PEIC), the MAI is supposed to be accurately estimated. Hence, with the equation (4), the estimated interference $\mathbf{m}^{PEIC}$ for user 1 is

$$\mathbf{m}^{PEIC} \quad = \quad \mathbf{m} \ . \tag{10}$$

In order to estimation b_1, the best strategy clearly is to subtract $\mathbf{m}$ from the received signal vector $\mathbf{r}$. The output of perfect interference cancellation detector is

$$b_1^{PEIC} \quad = \quad \mathrm{sgn}\{\mathbf{s}_1^T(\mathbf{r} - \mathbf{m}^{PEIC})\} \quad = \quad \mathrm{sgn}\{b_1 + \mathbf{s}_1^T\mathbf{n}\} \quad . \tag{11}$$

Obviously, this result is the same to the single-user case with AWGN channel.

3.2 Individually Optimum Interference Cancellation

In individually optimum interference cancellation (IO-IC), the interference $\mathbf{m}$ is estimated and removed from the observation so that the optimum output b_1^{IO} is expected to minimizes the following error probability function

$$\begin{aligned} b_1^{IO} \quad &= \quad \arg\min_b \ P[b \neq b_1] \\ &= \quad \arg\min_b \left\{\sum_{\mathbf{d}} P[b \neq b_1 \mid \mathbf{d}]P[\tilde{\mathbf{b}} = \mathbf{d}]\right\} \quad . \end{aligned} \tag{12}$$

Due to the absolute value functions and alphabet combinations in equation (12), it is very hard to give a close-form solution to this kind of optimization problem. However, with the following theorem, we can show that there does exist an possible estimate of MAI $\hat{\mathbf{m}}$ so that the output of interference canceller can be is equal to b_1^{IO}.

Lemma 1. *For any possible observation* $\mathbf{r}$ *as in equation (2), there exists a vector* $\tilde{\mathbf{d}} \in \{\pm 1\}^{(K-1)\times 1}$ *so that the output of individually optimum multiuser detection* b_1^{IO} *is also the solution to the following equation*

$$b_1^{IO} = arg\min_b \left\|\mathbf{r} - \tilde{\mathbf{S}}\tilde{\mathbf{A}}\tilde{\mathbf{d}} - bA_1\mathbf{s}_1\right\|_2 \quad \textit{subject to} \quad b \in \{\pm 1\} \ . \tag{13}$$

Proof. Equation (12) can be expressed as

$$\begin{aligned}\arg\min_b P[b \neq b_1] &= \arg\min_b \left\{ \sum_{\mathbf{d}} P[b \neq b_1 \mid \mathbf{d}] P[\tilde{\mathbf{b}} = \mathbf{d}] \right\} \\ &= \arg\max_b \left\{ \sum_{\mathbf{d}} \exp\left\{ -\tfrac{2}{\sigma^2} \|\mathbf{r} - \tilde{\mathbf{S}}\tilde{\mathbf{A}}\mathbf{d} - bA_1\mathbf{s}_1\|_2^2 \right\} P[\tilde{\mathbf{b}} = \mathbf{d}] \right\} \quad .\end{aligned} \tag{14}$$

Now there is the claim that, for any observation $\mathbf{r}$, if there exists $b_1' \in \{\pm 1\}$, which satisfies the following function,

$$\left\| \mathbf{r} - \tilde{\mathbf{S}}\tilde{\mathbf{A}}\mathbf{d} - b_1' A_1 \mathbf{s}_1 \right\|_2 \quad > \quad \left\| \mathbf{r} - \tilde{\mathbf{S}}\tilde{\mathbf{A}}\mathbf{d} + b_1' A_1 \mathbf{s}_1 \right\|_2, \quad \forall\ \mathbf{d} \in \{\pm 1\}^{(K-1)\times 1} \quad , \tag{15}$$

it will never be the optimum solution to equation (12).

Hence, for any possible observation $\mathbf{r}$, there exist $\hat{b}_1 \in \{\pm 1\}$ and $\tilde{\mathbf{d}} \in \{\pm 1\}^{(K-1)\times 1}$, which satisfy equation (12) and (13). □

With lemma 1, we show that the individually optimum multiuser detection can be presented as an interference canceller. At this time, the desired MAI estimation can be set to $\mathbf{m}^{IO} = \tilde{\mathbf{S}}\tilde{\mathbf{A}}\tilde{\mathbf{d}}$ and the output of the individually optimum interference cancellation is

$$\begin{aligned} b_1^{IO} &= \mathrm{sgn}\{\mathbf{s}_1^T(\mathbf{r} - \mathbf{m}^{IO})\} \\ &= \mathrm{sgn}\{b_1 + \mathbf{s}_1^T(\mathbf{m} - \mathbf{m}^{IO}) + \mathbf{s}_1^T\mathbf{n}\} \quad . \end{aligned} \tag{16}$$

3.3 Jointly Optimum Interference Cancellation

Similar to the jointly optimum multiuser detector, it is expected that the output of the jointly optimum interference cancellation detector is the solution to the minimization of the following joint error probability function

$$\mathbf{b}^{JO} = \arg\min_{\bar{\mathbf{b}}} P[\bar{\mathbf{b}} \neq \mathbf{b}] \quad \text{subject to} \quad \bar{\mathbf{b}} \in \{\pm 1\}^{K\times 1} \quad . \tag{17}$$

It is not easy to give a close-form solution to equation (17) for interference cancellation, too. But, we can show that the desired MAI estimate $\hat{\mathbf{m}}$ for the jointly optimum interference cancellation exists and can be estimated with the following theorem.

Lemma 2. *Suppose $\mathbf{b}^{JO} = [b_1^{JO}\ (\tilde{\mathbf{b}}^{JO})^T]^T \in \{\pm 1\}^{K\times 1}$ is the jointly optimum solution which minimizes the joint error probability function as in equation (17). If $\hat{\mathbf{m}}$ is set to be $\mathbf{m}^{JO} = \tilde{\mathbf{S}}\tilde{\mathbf{A}}\tilde{\mathbf{b}}^{JO}$, the output $\hat{b}_1$ with equation (8) is equal to b_1^{JO}.*

Proof. As we know, the vector $\mathbf{b}^{JO}$ defined as in equation (17) actually also is the optimum solution to the following equation.

$$\mathbf{b}^{JO} = \arg\min_{\bar{\mathbf{b}}} \left\| \mathbf{r} - \mathbf{S}\mathbf{A}\bar{\mathbf{b}} \right\| \quad \text{subject to} \quad \bar{\mathbf{b}} \in \{\pm 1\}^{K\times 1} \quad . \tag{18}$$

At this time, with equation (8) and the MAI estimate $\hat{\mathbf{m}} = \tilde{\mathbf{S}}\tilde{\mathbf{A}}\tilde{\mathbf{b}}^{JO}$, the output $\hat{b}_1$ is the optimum solution to

$$\hat{b}_1 = \arg\min_b \left\| \mathbf{r} - \tilde{\mathbf{S}}\tilde{\mathbf{A}}\tilde{\mathbf{b}}^{JO} - bA_1\mathbf{s}_1 \right\|_2 \quad \text{subject to} \quad b \in \{+1,\ -1\} \tag{19}$$

□

With Lemma 2, it reveals that jointly optimum multiuser detection can be realized as interference canceller. In the jointly optimum interference cancellation, the key then is how to optimally estimate the MAI $\mathbf{m}^{JO}$. After the interference $\mathbf{m}^{JO}$ is estimated, the output of the jointly optimum interference cancellation can be expressed as

$$\begin{aligned} b_1^{JO} &= \operatorname{sgn}\{\mathbf{s}_1^T(\mathbf{r}-\mathbf{m}^{JO})\} \\ &= \operatorname{sgn}\{b_1+\mathbf{s}_1^T(\mathbf{m}-\mathbf{m}^{JO})+\mathbf{s}_1^T\mathbf{n}\} \ . \end{aligned} \tag{20}$$

3.4 Direct Interference Cancellation

As we see, there are many optimum schemes proposed to estimate the MAI $\mathbf{m}$ with the known signature vectors $\mathbf{s}_k$, $k=1,\,2,\,\ldots,\,K$, and different criterion. Though these optimum algorithms have very good performance, their computation usually is so complex and depends not only on the decision rule but also on the information vector alphabet. Hence, it is reasonable to look for some suboptimum algorithms with low computation complexity and good performance. In this section, another scheme is proposed to estimate MAI $\mathbf{m}$ directly with $\mathbf{S}$. It is expected that the estimated MAI $\hat{\mathbf{m}}$ minimize the following equation

$$\mathbf{m}^{D-IC} = \arg\min_{\hat{\mathbf{m}}} \|\hat{\mathbf{m}}-(\mathbf{r}-b_1A_1\mathbf{s}_1)\|_2 \ . \tag{21}$$

Before estimating the MAI $\mathbf{m}$, a Household transformation $\mathbf{Q}$ is defined to be performed on both sides of the equation (2) so that

$$\mathbf{Q}^T\mathbf{r} = \bar{\mathbf{r}} = \begin{bmatrix} r_1 \\ \bar{\mathbf{r}}_2 \end{bmatrix} \tag{22}$$

and

$$\mathbf{Q}^T\mathbf{S} = \mathbf{Q}^T[\,\mathbf{s}_1 \quad \tilde{\mathbf{S}}\,] = \begin{bmatrix} c & \bar{\mathbf{s}}_{21}^T \\ \mathbf{0} & \tilde{\mathbf{S}}_{22} \end{bmatrix} \tag{23}$$

where r_1 is the first element in the vector $\bar{\mathbf{r}}$, $\bar{\mathbf{r}}_2$ is the $(L-1)\times 1$ complement subvector of r_1, $\bar{\mathbf{s}}_{21}$ is a $(K-1)\times 1$ vector and $\tilde{\mathbf{S}}_{22}$ is a $(L-1)\times(K-1)$ matrix and $c=\pm\|\mathbf{s}_1\|_2=\pm 1$. This Household transformation can then be written as

$$\mathbf{Q} = \mathbf{I}-2\mathbf{q}\mathbf{q}^T = \mathbf{I}-\frac{1}{c(c-s_{11})}\begin{bmatrix} (s_{11}-c) \\ s_{12} \\ \vdots \\ s_{1L} \end{bmatrix}[\,(s_{11}-c) \quad s_{12} \quad \cdots \quad s_{1L}\,] \tag{24}$$

where $\mathbf{q}=\frac{1}{\sqrt{2c(c-s_{11})}}[\,(s_{11}-c) \quad s_{12} \quad \cdots \quad s_{1L}\,]^T$ and s_{11} is the first element of $\mathbf{s}_1$.

Now, the direct estimation of the MAI $\mathbf{m}$ can be realization with the following lemma.

Lemma 3. *The minimum norm (or least squares)solutions to the following equation*

$$\mathbf{x} = arg\min_{\mathbf{x}} \left\|\tilde{\mathbf{S}}\mathbf{x}-(\mathbf{r}-b_1A_1\mathbf{s}_1)\right\|_2 \tag{25}$$

is given by

$$\mathbf{x} = \tilde{\mathbf{S}}_{22}^{+}\bar{\mathbf{r}}_2 \tag{26}$$

Before proving the above theorem, we introduce the following conclusion.

Proposition 2. $$\begin{bmatrix} c & \bar{\mathbf{s}}_{21}^T \\ \mathbf{0} & \tilde{\mathbf{S}}_{22} \end{bmatrix}^+ = \begin{bmatrix} \frac{1}{c} & -\frac{1}{c}\bar{\mathbf{s}}_{21}^T\tilde{\mathbf{S}}_{22}^+ \\ \mathbf{0} & \tilde{\mathbf{S}}_{22}^+ \end{bmatrix}$$

Proof. We know that

$$\begin{aligned} \tilde{\mathbf{S}}\mathbf{x} - (\mathbf{r} - b_1A_1\mathbf{s}_1) &= \mathbf{S}\begin{bmatrix} b_1A_1 \\ \mathbf{x} \end{bmatrix} - \mathbf{r} \\ &= \begin{bmatrix} c & \bar{\mathbf{s}}_{21}^T \\ \mathbf{0} & \tilde{\mathbf{S}}_{22} \end{bmatrix}\begin{bmatrix} b_1A_1 \\ \mathbf{x} \end{bmatrix} - \begin{bmatrix} r_1 \\ \bar{\mathbf{r}}_2 \end{bmatrix} \end{aligned} \tag{28}$$

Then,

$$\begin{aligned} \arg\min_{\mathbf{x}} \left\|\tilde{\mathbf{S}}\mathbf{x} - (\mathbf{r} - b_1A_1\mathbf{s}_1)\right\| &= \arg\min_{\mathbf{x}} \left\|\begin{bmatrix} c & \bar{\mathbf{s}}_{21}^T \\ \mathbf{0} & \tilde{\mathbf{S}}_{22} \end{bmatrix}\begin{bmatrix} b_1A_1 \\ \mathbf{x} \end{bmatrix} - \begin{bmatrix} r_1 \\ \bar{\mathbf{r}}_2 \end{bmatrix}\right\| \\ &= \arg\min_{\mathbf{x}} \left\|\begin{bmatrix} b_1A_1 \\ \mathbf{x} \end{bmatrix} - \begin{bmatrix} \frac{1}{c} & -\frac{1}{c}\bar{\mathbf{s}}_{21}^T\tilde{\mathbf{S}}_{22}^+ \\ \mathbf{0} & \tilde{\mathbf{S}}_{22}^+ \end{bmatrix}\begin{bmatrix} r_1 \\ \bar{\mathbf{r}}_2 \end{bmatrix}\right\| \\ &= \arg\min_{\mathbf{x}} \left\|\begin{bmatrix} b_1A_1 \\ \mathbf{x} \end{bmatrix} - \begin{bmatrix} \frac{r_1}{c} - \frac{1}{c}\bar{\mathbf{s}}_{21}^T\tilde{\mathbf{S}}_{22}^+\bar{\mathbf{r}}_2 \\ \tilde{\mathbf{S}}_{22}^+\bar{\mathbf{r}}_2 \end{bmatrix}\right\| \end{aligned} \tag{29}$$

Thus,

$$\arg\min_{\mathbf{x}} \left\|\tilde{\mathbf{S}}\mathbf{x} - (\mathbf{r} - b_1A_1\mathbf{s}_1)\right\| = \tilde{\mathbf{S}}_{22}^+\bar{\mathbf{r}}_2 \tag{30}$$

□

Hence, the estimation of $\mathbf{m}$ with the minimum norm estimation of the vector $\tilde{\mathbf{A}}\tilde{\mathbf{b}}$ is

$$\mathbf{m}^{D-IC} = \tilde{\mathbf{S}}\tilde{\mathbf{S}}_{22}^+\bar{\mathbf{r}}_2 \tag{31}$$

After the interference is estimated, it can be removed out from the received signal. Thus, the output of the proposed direct interference cancellation detector is

$$b_1^{D-IC} = \operatorname{sgn}\left\{\mathbf{s}_1^T\left(\mathbf{I}_L - \tilde{\mathbf{S}}\tilde{\mathbf{S}}_{22}^+\,[\,\mathbf{0} \quad \mathbf{I}_{(L-1)}\,]\,\mathbf{Q}^T\right)\mathbf{r}\right\} \tag{32}$$

where $\mathbf{I}_L$ is a $L \times L$ identity matrix and $\mathbf{I}_{L-1}$ is a $(L-1) \times (L-1)$ identity matrix.

Thus, the linear filter representation of the direct interference cancellation detector is

$$\mathbf{w}_1^{D-IC} = \left(\mathbf{I}_L - \mathbf{Q}\begin{bmatrix} \mathbf{0}^T \\ \mathbf{I}_{(L-1)} \end{bmatrix}\tilde{\mathbf{S}}_{22}^{+T}\tilde{\mathbf{S}}^T\right)\mathbf{s}_1 \quad . \tag{33}$$

3.5 Maximum Asymptotic Multiuser Efficiency Interference Cancellation

As we know, the effective energy of user 1 $e_1(\sigma)$ is always upper bounded by the actual energy A_1^2. Multiuser efficiency or ratio between the effective energy and actual energy, $e_1(\sigma)/A_1^2$, which depends on the signature waveforms, received signal-to-noise ratio (SNR) and the employed detector, is always not larger than 1. The asymptotic multiuser efficiency (AME) for user 1 is defined as

$$\eta_1 = \lim_{\sigma\to 0} \frac{e_1(\sigma)}{A_1^2} \quad , \tag{34}$$

which measure the slope when user 1's error probability

$$P_{e1} = Q\left(\frac{e_1(\sigma)}{\sigma}\right) \tag{35}$$

in logarithmic scale goes to 0 in the high SNR region.

Now, we discuss the linear transformation which can maximize the possible asymptotic multiuser efficiency. We denote the linear transformation for MAI estimation in MAME interference cancellation by $\mathbf{W}_1^m$ so that

$$\mathbf{W}_1^{mT}\mathbf{r} = \mathbf{m}^{MAME-IC} \tag{36}$$

The the error probability achieve by $\mathbf{W}_1^m$ and $\mathbf{s}_1$ can be expressed as

$$\begin{aligned} P_{e1} &= \mathrm{E}\left[Q\left(\frac{\mathbf{s}_1^T\mathbf{r}-\mathbf{s}_1^T\mathbf{W}_1^{mT}\mathbf{r}}{\sigma\|\mathbf{s}_1-\mathbf{W}_1^m\mathbf{s}_1\|_2}\right)\right] \\ &= \mathrm{E}\left[Q\left(\frac{A_1b_1+\sum_{k=2}^{K}A_kb_k\rho_{1k}-\sum_{k=1}^{K}A_kb_k\mathbf{w}_1^{mT}\mathbf{s}_k}{\sigma\|\mathbf{s}_1-\mathbf{w}_1^m\|_2}\right)\right] \end{aligned} \tag{37}$$

Where $\mathbf{w}_1^m = \mathbf{W}_1^m\mathbf{s}_1$ and the expectation is with respect to b_j, $j \neq 1$. The asymptotic multiuser efficiency of user 1 is given by the square of the smallest argument in equation (37) normalized by A_1^2/σ^2

$$\eta_1(\mathbf{w}_1^m) = \max{}^2\left\{0, \frac{1}{\|\mathbf{s}_1-\mathbf{w}_1^m\|_2}\left(1-\mathbf{s}_1^T\mathbf{w}_1^m-\sum_{k=2}^{K}\frac{A_k}{A_1}\left|\rho_{1k}-\mathbf{s}_k^T\mathbf{w}_1^m\right|\right)\right\}. \tag{38}$$

As the MAME linear multiuser detector [5], due to the presence of the absolute value function in equation (38), solve this optimization problem for K-user case doesn't admit a closed-form solution.

After the estimation of the interference $\mathbf{m}^{MAME-IC}$, it is removed from the observation $\mathbf{r}$ and the output of the MAME interference cancellation is

$$\begin{aligned} b_1^{MAME-IC} &= \mathrm{sgn}\{\mathbf{s}_1^T(\mathbf{r}-\mathbf{m}^{MAME-IC})\} \\ &= \mathrm{sgn}\{b_1+\mathbf{s}_1^T(\mathbf{m}-\mathbf{m}^{MAME-IC})+\mathbf{s}_1^T\mathbf{n}\} \quad . \end{aligned} \tag{39}$$

3.5.1 Two-user Case

When there are only two active users, equation (38) can be simplified as

$$\eta_1 = \max^2\left\{0, \frac{1}{\|\mathbf{s}_1-\mathbf{w}_1^m\|_2}\left(1-\mathbf{s}_1^T\mathbf{w}_1^m-\frac{A_2}{A_1}\left|\rho-\mathbf{s}_2^T\mathbf{w}_1^m\right|\right)\right\}, \tag{40}$$

where $\mathbf{w}_1^m$ is a linear combination of $\mathbf{s}_1$ and $\mathbf{s}_2$.

Solving equation (40) can lead to the following solution.

$$\mathbf{w}_1^m = \begin{cases} \frac{A_1}{A_2}\mathrm{sgn}(\rho)\mathbf{s}_2, & \text{if } \frac{A_2}{A_1} < |\rho|, \\ \rho\mathbf{s}_2, & \text{otherwise.} \end{cases} \tag{41}$$

Compared to the MAME detector for user 1,

$$\mathbf{w}_1 = \begin{cases} \mathbf{s}_1 - \frac{A_1}{A_2}\mathrm{sgn}(\rho)\mathbf{s}_2, & \text{if } \frac{A_2}{A_1} < |\rho|, \\ \mathbf{s}_1 - \rho\mathbf{s}_2, & \text{otherwise,} \end{cases} \tag{42}$$

we can see that they are actually identical one.

3.6 Minimum Mean-Square Error Interference Cancellation

In this section, we are going to propose a minimum mean-square error interference cancellation (MMSE-IC) scheme, in which the MAI is estimated with minimizing the output variance as in the following function

$$\mathbf{m}^{MMSE-IC} = \arg\min_{\hat{\mathbf{m}}_n} E\left\{[\mathbf{s}_1^T(\mathbf{r}-b_1A_1\mathbf{s}_1)-\mathbf{s}_1^T\hat{\mathbf{m}}]^2\right\} \quad . \tag{43}$$

Hence, the output of the MMSE interference cancellation can be written as

$$\begin{aligned} b_1^{MMSE-IC} &= \mathrm{sgn}\{\mathbf{s}_1^T(\mathbf{r}-\mathbf{m}^{MMSE})\} \\ &= \mathrm{sgn}\{b_1+\mathbf{s}_1^T(\mathbf{m}-\mathbf{m}^{MMSE})+\mathbf{s}_1^T\mathbf{n}\} \quad . \end{aligned} \tag{44}$$

One possible linear solutions to equation (43) is to estimate

$$\mathbf{m}^{MMSE-IC} = \mathbf{s}_1\bar{\mathbf{w}}^T\mathbf{r} \quad \text{with} \quad \bar{\mathbf{w}} = \arg\min_{\hat{\mathbf{w}}} E\left\{[\mathbf{s}_1^T(\mathbf{r}-b_1A_1\mathbf{s}_1)-\hat{\mathbf{w}}^T\mathbf{r}]^2\right\} \tag{45}$$

The close-form solution to the equation (45) can be presented in the following theorem.

Lemma 4. *The solution to*

$$\bar{\mathbf{w}} = arg\min_{\hat{\mathbf{w}}} E\left\{[\mathbf{s}_1^T(\mathbf{r}-b_1A_1\mathbf{s}_1)-\hat{\mathbf{w}}^T\mathbf{r}]^2\right\} \tag{46}$$

is

$$\bar{\mathbf{w}} = (\mathbf{S}\mathbf{A}^2\mathbf{S}^T+\sigma^2\mathbf{I}_L)^{-1}(\tilde{\mathbf{S}}\tilde{\mathbf{A}}^2\tilde{\mathbf{S}}^T+\sigma^2\mathbf{I}_L)\mathbf{s}_1 \tag{47}$$

Proof. It is easy to verify the following conclusion.

$$(\mathbf{S}\mathbf{A}^2\mathbf{S}^T + \sigma^2\mathbf{I}_L)^{-1}(\tilde{\mathbf{S}}\tilde{\mathbf{A}}^2\tilde{\mathbf{S}}^T + \sigma^2\mathbf{I}_L) \;=\; \arg\min_{\hat{\mathbf{W}}} \; E\left\|(\mathbf{r} - b_1A_1\mathbf{s}_1) - \hat{\mathbf{W}}^T\mathbf{r}\right\|_2 \tag{48}$$

Then,

$$\bar{\mathbf{w}} \;=\; (\mathbf{S}\mathbf{A}^2\mathbf{S}^T + \sigma^2\mathbf{I}_L)^{-1}(\tilde{\mathbf{S}}\tilde{\mathbf{A}}^2\tilde{\mathbf{S}}^T + \sigma^2\mathbf{I}_L)\mathbf{s}_1 \tag{49}$$

□

Thus, the output of the proposed linear MMSE interference cancellation detector is

$$b_1^{MMSE-IC} \;=\; \mathrm{sgn}\left\{\mathbf{s}_1^T\left[\mathbf{I}_L - (\tilde{\mathbf{S}}\tilde{\mathbf{A}}^2\tilde{\mathbf{S}}^T + \sigma^2\mathbf{I}_L)(\mathbf{S}\mathbf{A}^2\mathbf{S}^T + \sigma^2\mathbf{I}_L)^{-1}\right]\mathbf{r}\right\} \tag{50}$$

and the linear filter representation of the linear MMSE interference cancellation detector is

$$\begin{aligned}\mathbf{w}_1^{MMSE-IC} &= \left[\mathbf{I}_L - (\mathbf{S}\mathbf{A}^2\mathbf{S}^T + \sigma^2\mathbf{I}_L)^{-1}(\tilde{\mathbf{S}}\tilde{\mathbf{A}}^2\tilde{\mathbf{S}}^T + \sigma^2\mathbf{I}_L)\right]\mathbf{s}_1 \\ &= A_1^2(\mathbf{S}\mathbf{A}^2\mathbf{S}^T + \sigma^2\mathbf{I}_L)^{-1}\mathbf{s}_1 \;\;.\end{aligned} \tag{51}$$

4 Performance Analysis

As we see, in interference cancellation, After the MAI is estimated and removed from the received signal, the decision is simply made by projecting the rest signal vector onto the signature vector of the desired user. Hence, the key is the interference estimation in the first stage and the second stage is fixed. Different MAI estimation schemes lead to different interference cancellation algorithms. In the following, the proposed interference cancellation detection schemes would be compared with other multiuser detection algorithms.

In the perfect interference cancellation, the MAI is completely removed with some help so that the result is same to that of the optimally demodulating the signal, which is sent through a single-user channel. Obviously, the performance of EIC is same to that of the optimal receiver for the single-user channel and its BER is

$$P_{e1}^{PEIC} \;=\; Q\left(\frac{A_1}{\sigma}\right) \;\;. \tag{52}$$

Obviously, the BER performance of the perfect interference cancellation is the best we can obtain through the multiuser channel with the assumption that the MAI is known. And its asymptotic efficiency is always kept as 1, which are are better than any other multiuser scheme, since the MAI can be perfectly removed from the received signal vector. Thus, the perfect interference cancellation can be taken as the lowest bound for all the possible multiuser detection schemes.

In the individually or jointly optimum interference cancellation, the interference is estimated and cancelled so that the output of the matched filter is a optimum solution to minimize the individual or joint error probability function as in equations (12) and (17), respectively. These two definitions sound basically the same to that of the individually and jointly optimum multiuser detection, respectively. Furthermore with Lemma 1 and 2, the optimum interference cancellations can be taken the special realization of the optimum

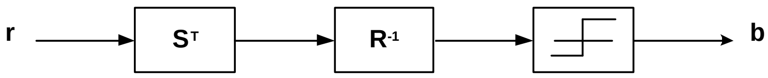

Figure 2: The structure of decorrelating detector.

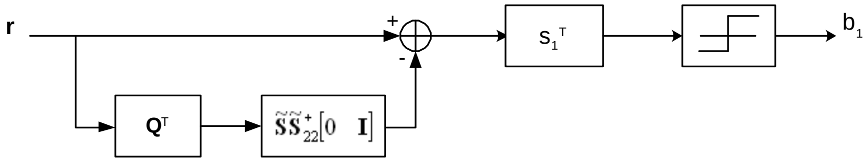

Figure 3: The structure of direct interference cancellation for user 1

multiuser detections, respectively. Hence, the performance of the individually or jointly optimum interference cancellation can achieve the same performance of the individually or jointly optimum multiuser detection scheme, respectively.

In decorrelating detection, the receiver signal passes the single-user matched filter first. The decorrelating operation then is performed on the output. The final results are obtained with limiters. The basic structure for decorrelating detection can be shown in figure 2. Compared with figure 3, we can see that this is different to direct inference cancellation. However, though decorrelating detection and direct interference cancellation have different operation structure, with the following lemma, we can see that the decorrelating detector and the proposed direct interference cancellation have the same performance. The proposed interference cancellation then can be taken as a special realization of the decorrelating detection.

Lemma 5. $\mathbf{w}_1^{D-IC} = \alpha \mathbf{w}_1^{DD}$*, where* α *is a positive factor, so that the proposed direct interference cancellation detector has the same performance as that of the classic decorrelating detector.*

Proof. With the unitary transformation $\mathbf{Q}$ on both sides of the equation (2), we got

$$\mathbf{Q}^T\mathbf{r} \;=\; \begin{bmatrix} c & \bar{\mathbf{s}}_{21}^T \\ \mathbf{0} & \tilde{\mathbf{S}}_{22} \end{bmatrix} \mathbf{Ab} + \mathbf{Q}^T\mathbf{n} \quad . \tag{53}$$

So that, as the same to the decorrelating detection, the least-square estimation of $\mathbf{Ab}$ is

$$\mathbf{Ab} \;=\; \begin{bmatrix} c & \bar{\mathbf{s}}_{21}^T \\ \mathbf{0} & \tilde{\mathbf{S}}_{22} \end{bmatrix}^+ \mathbf{Q}^T\mathbf{r} \quad . \tag{54}$$

Hence, the decorrelation detector can be written in the interference cancellation form as

$$b_1^{DD} \;=\; \operatorname{sgn}\left\{ [\, c^{-1} \quad -c^{-1}\bar{\mathbf{s}}_{21}^T \tilde{\mathbf{S}}_{22}^+ \,]\, \mathbf{Q}^T\mathbf{r} \right\} \quad . \tag{55}$$

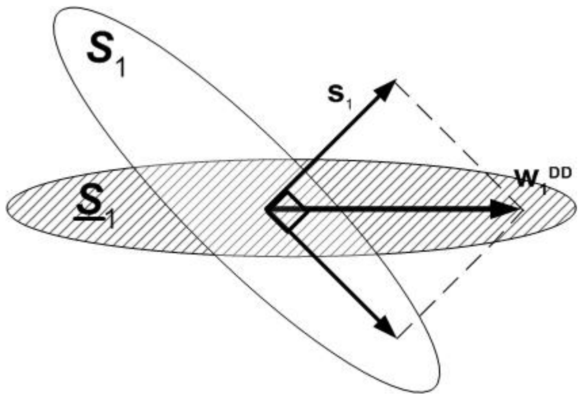

Figure 4: The geometric demonstration of the decorrelating detector for user 1.

On the other hand, with equation (32), the output of the interference detection detector is

$$\begin{aligned} b_1^{D-IC} &= \operatorname{sgn}\left\{\mathbf{s}_1^T\left(\mathbf{I}_L - \tilde{\mathbf{S}}\tilde{\mathbf{S}}_{22}^{+}\left[\,\mathbf{0} \quad \mathbf{I}_{(L-1)}\,\right]\mathbf{Q}^T\right)\mathbf{r}\right\} \\ &= \operatorname{sgn}\left\{\left(\left[\,c \quad 0 \quad \ldots \quad 0\,\right] - \left[\,c \quad 0 \quad \ldots \quad 0\,\right]\begin{bmatrix}\bar{\mathbf{s}}_{21} \\ \tilde{\mathbf{S}}_{22}\end{bmatrix}\left[\,\mathbf{0} \quad \tilde{\mathbf{S}}_{22}^{+}\,\right]\right)\mathbf{Q}^T\mathbf{r}\right\} \\ &= \operatorname{sgn}\left\{\left[\,c \quad -c\bar{\mathbf{s}}_{21}^T\tilde{\mathbf{S}}_{22}^{+}\,\right]\mathbf{Q}^T\mathbf{r}\right\} \quad . \end{aligned} \tag{56}$$

Thus, $\mathbf{w}_1^{D-IC} = c^2\mathbf{w}_1^{DD}$ and the proposed interference cancellation detector has the same performance as that of the classic decorrelating detector. □

As we know, with decorrelating detector, the received signal vector $\mathbf{r}$ is projected on the subspace orthogonal to the codes of the other users [1, 16]. It is shown in [17] that the decorrelating detector is the *oblique projection* of the desired user's signature vector onto the space $\underline{\mathcal{S}}_1$, which is the orthogonal complement of subspace spanned by other users' signature vectors, along $\mathcal{S}_1$ the orthogonal complement space of this space. For example, the decorrelating detector $\mathbf{w}_1^{DD}$ for the first user is

$$\begin{aligned} \mathbf{w}_1^{DD} &= \mathbf{P}_1^O\mathbf{s}_1 \\ &= \frac{1}{\mathbf{s}_1^H\mathbf{P}_1^{\perp}\mathbf{s}_1}\mathbf{P}_1^{\perp}\mathbf{s}_1 \end{aligned} \tag{57}$$

where

$$\mathbf{P}_1^O\mathbf{s}_1 = \frac{1}{\mathbf{s}_1^H\mathbf{P}_1^{\perp}\mathbf{s}_1}\mathbf{P}_1^{\perp}\mathbf{s}_1\mathbf{s}_1^H \tag{58}$$

denotes the oblique projection of $\mathbf{s}_1$, and $\mathbf{P}_1^{\perp}$ is the orthogonal projection for user 1 onto the orthogonal complement of the space spanned by the other users' signature vectors. This can be demonstrated on figure 4.

Different to the decorrelating multiuser detection, the output of the direct interference cancellation is the result of the different between the observation $\mathbf{r}$ and the MAI $\mathbf{m}^{D-IC}$. Hence, the linear filter representation of direct interference cancellation can actually be take as the difference between $\mathbf{s}_1$ and $\mathbf{w}_{m1}^{D-IC}$ as in equation (33), where $\mathbf{w}_{m1}^{D-IC}$ is the linear

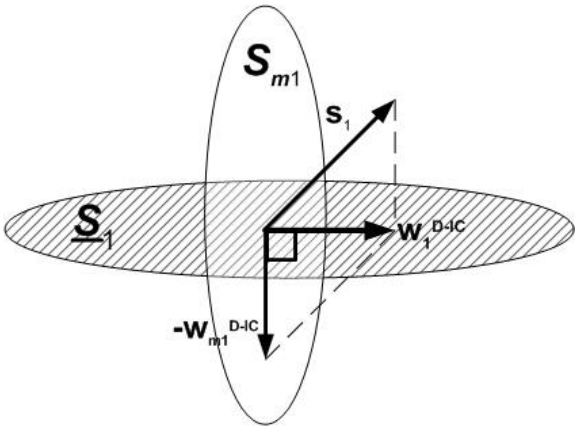

Figure 5: The geometric demonstration of the direct interference cancellation for user 1.

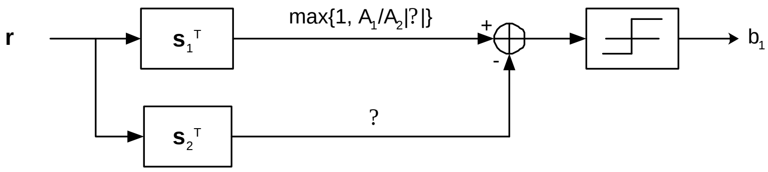

Figure 6: The structure of two-user MAME detector for user 1.

filter representation of the MAI estimation in the direct interference cancellation and can be expressed as

$$\mathbf{w}_{m1}^{D-IC} = \mathbf{Q}\begin{bmatrix}\mathbf{0}^T \\ \mathbf{I}_{(L-1)}\end{bmatrix}\tilde{\mathbf{S}}_{22}^{+T}\tilde{\mathbf{S}}^T\mathbf{s}_1 \quad . \tag{59}$$

Furthermore, with the definition of $\mathbf{m}$ as in equation (5), $\mathbf{w}_{m1}^{D-IC}$ belongs to the linear subspace $\mathcal{S}_{m1}$ spanned by other signature vectors. Thus, the linear filter representation of the direct interference cancellation can be taken the oblique projection of $\mathbf{s}_1$ on to $\underline{\mathcal{S}}_1$ along $-\mathbf{w}_{m1}^{D-IC}$. This can be demonstrated in figure 5

Based on the definition of the $\mathbf{m}^{MAME-IC}$, it can be take as the combination of the single-user matched filter and the direction interference cancellation. The basic structure for tow-user MAME multiuser detection and MAME interference cancellation are shown in figure 6 and 7, respectively. With Lemma 5, the performance of the proposed MAME interference cancellation detector obviously is same to that of the MAME multiuser detector.

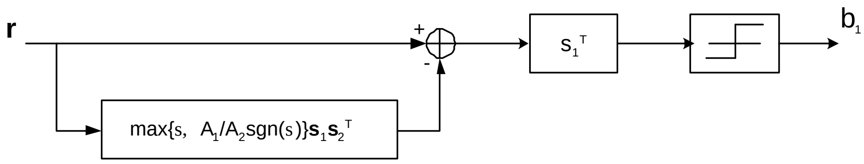

Figure 7: The structure of two-user MAME interference cancellation for user 1

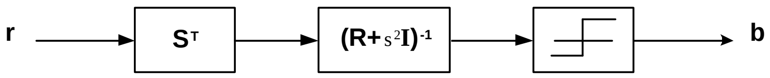

Figure 8: The structure of MMSE detector.

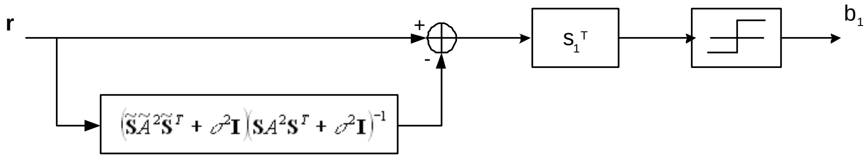

Figure 9: The structure of MMSE interference cancellation for user 1

The classic linear MMSE detector can be seen as a compromise solution that takes into account the relative importance of each interfering user and the background noise. The basic structure for MMSE detector is shown in figure 8. Obviously, it is different to the structure of MMSE interference cancellation, which is shown in figure 9. With the following theorem, we can see that the proposed linear MMSE interference cancellation detector and the classic linear MMSE detector are also of the same performance.

Proposition 3. *There is the following relationship.*

$$(\mathbf{S}\mathbf{A}^2\mathbf{S}^T + \sigma^2\mathbf{I}_L)^{-1}\mathbf{S}\mathbf{A}^2 \quad = \quad \mathbf{S}(\mathbf{S}^T\mathbf{S} + \sigma^2\mathbf{A}^{-2})^{-1} \tag{60}$$

Proof. It easy to see the following relationship.

$$\begin{aligned} \mathbf{S}\mathbf{A}^2\mathbf{S}^T\mathbf{S} + \sigma^2\mathbf{S} &= \mathbf{S}\mathbf{A}^2(\mathbf{S}^T\mathbf{S} + \sigma^2\mathbf{A}^{-2}) \\ &= (\mathbf{S}\mathbf{A}^2\mathbf{S}^T + \sigma^2\mathbf{I}_L)\mathbf{S} \quad . \end{aligned} \tag{61}$$

Thus,

$$(\mathbf{S}\mathbf{A}^2\mathbf{S}^T + \sigma^2\mathbf{I}_L)^{-1}\mathbf{S}\mathbf{A}^2 \quad = \quad \mathbf{S}(\mathbf{S}^T\mathbf{S} + \sigma^2\mathbf{A}^{-2})^{-1} \tag{62}$$

□

Lemma 6. $\mathbf{w}_1^{MMSE-IC} = \beta\mathbf{w}_1^{MMSE}$, *where β is a positive factor, so that the proposed MMSE interference cancellation detector has the same performance as the classic MMSE multiuser detector.*

Proof. As we know, the linear filter representation of the MMSE multiuser detector for user 1 is

$$\mathbf{w}_1^{MMSE} \quad = \quad \mathbf{S}(\mathbf{S}^T\mathbf{S} + \sigma^2\mathbf{A}^{-2})^{-1}\mathbf{e}_1 \quad , \tag{63}$$

where $\mathbf{e}_1 = [\,1 \quad 0 \quad \ldots \quad 0\,]^T$ is a $K \times 1$ vector.

With Proposition 3,

$$\mathbf{w}_1^{MMSE-IC} \quad = \quad A_1^2\mathbf{w}_1^{MMSE} \quad . \tag{64}$$

□

5 Conclusions

In this paper, the principles of the interference detection is reviewed. Besides individually and jointly optimum interference cancellation, direct interference cancellation, MAME interference cancellation and MMSE interference cancellation are proposed. With theoretical analysis, the optimum multiuser detectors can be realized as interference cancellation. the performance of direct interference cancellation, MAME interference cancellation and MMSE interference cancellation are same to that of the decorrelating detection, MAME multiuser detector and linear MMSE detector, respectively.

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