A New Distribution Bound and Reduction Scheme for OFDM PAPR

X. Zhou & Dr. J. Caffery, Jr.
University of Cincinnati
Dept. of Electrical & Computer Engineering
& Computer Science
Cincinnati, OH 45221-0030

Introduction

- The demand of high data rate over the wireless channel.
  - ⋆ Wireless multimedia >2 Mbps
  - ⋆ 2-155 Mbps under investigation
- Some applications:
  - ⋆ DAB (useful bit rate 5.6 Mbps)
  - ⋆ DVB over the terrestrial networks, digital terrestrial television broadcasting (DTTB)
  - ⋆ HIPERLAN Phase II, supporting 20 Mbps in propagation environments with delay spreads up to 1 us
  - ⋆ IEEE 802.11A (48 subcarriers, 5,10,15,20,30 Mbps.)
  - ⋆ Asymmetric digital subscriber Line (ADSL)
  - ⋆ High speed cellular (Band Division Multiple Access, BDMA)
- How to transmit a large bit stream over a wireless channel to provide sufficient QoS? What modulation can be used?
- Wireless environment channel is harsh (path loss, shadowing, multipath, etc.).
- Possible solutions to high bit rate transmission over wireless channels?

Possible Solutions for High Data Rate Transmission

- Possible Solutions are
  - ⋆ **Equalization** — Practical difficulties in performing equalization in real-time at several Mbps with low complexity and compact hardware.
  - ⋆ **Spread spectrum** — Robust against fading, interference. However, if data rate is 20 Mbps, the spreading factor is 128, then 2.56Gcps; another difficulty is near-far problem.
  - ⋆ **Ultra wide band (UWB)** — High capacity, multipath-fading resistant, immunity to impulse noise, hard to design the hardware to accommodate the very wide frequency range, hard to design antenna, interference with existing GPS, RF devices? FCC approval.
  - ⋆ **Multicarrier/OFDM** — The transmission bandwidth B with data rate R is divided into many (N) narrow subchannels. Each subchannel has bandwidth B/N and date rate R/N. All subchannels contribute the high bit rate in parallel. For the end-users, a high bit rate is achieved!.
- As a special and important form of multicarrier, OFDM is implemented via IFFT/FFT in MODEM.

OFDM System Model

- Use IFFT/FFT to modulate/demodulate,reduces the complexity of MC.
- Carrier spacing is the reciprocal of the useful OFDM symbol period T.
- OFDM is a particular form of MC with overlapping orthogonal subcarriers.
- The continuous time OFDM signal, in one symbol period, is given by

$$s\left(t\right) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k e^{j2\pi kt/T} g(t)$$

  where S_k is the QAM value of the k^{th} subcarrier, $g(t)$ is the rectangular window of unit height over the OFDM symbol interval $[0, T]$ and N is the subcarrier number.

- The Nyquist-rate sampled OFDM signal is given by

$$s_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k e^{j2\pi kn/N}.$$

PAPR Problem Formulation

- An OFDM signal is the sum of independently modulated subcarrier signals.
- If those subcarriers are added coherently, the total instantaneous power will become much larger than the average power.
- Therefore, RF power amplifiers should operate in a very large linear region. Otherwise, if signal peaks move into the nonlinear region of the power amplifier, signal distortion results by introducing intermodulation among the subcarriers and out-of-band radiation. Thus, it is highly desirable to reduce the PAPR.
- The PAPR of the OFDM signal s_τ, where τ is used to represent both the continuous time index t and discrete time index n, is defined as

$$\mathrm{PAPR}\{s_\tau, \xi\} = \frac{\max\limits_{\tau \in \xi} |s_\tau|^2}{E\{|s_\tau|^2\}}$$

  where ξ denotes either T or N.
- PAPR distribution is better to describe the PAPR characteristics than the maximum PAPR in practical OFDM systems.

Assumptions and Observations

- Guard time is an important issue in OFDM systems, however it is just a partial replica of the signal during $[0, T]$, it changes neither the average nor peak power of the signal. Thus, it is removed in our analysis.

- Since the RF frequency is much higher than the subcarrier's frequency, the baseband OFDM signal has the same PAPR as its passband equivalent, so we use the baseband signal to analyze the PAPR.

- We assume an ideal bandlimited OFDM signal where the bandwidth $B = N/T$ and its power spectral density (PSD) is constant over $[-B/2, B/2]$.

- The baseband OFDM signal, $s(t)$, can be written in complex form as $s(t) = x(t) + jy(t)$. According to Central Limit Theorem, $x(t)$ and $y(t)$ can be approximated as two independent Gaussian random processes and the envelope of $s(t)$ can be approximated as Rayleigh process.

PAPR Distribution

- Define the normalized OFDM signal and PAPR cumulative CDF as followings:

$$r(t) = \frac{s(t)}{\sqrt{P_{AV}}} = \frac{x(t) + jy(t)}{\sqrt{P_{AV}}}$$

where $P_{AV} = E[|s(t)|^2]$ denotes the average power over the entire signal. Thus, $|r(t)|$ is also approximated by a Raleigh process.

- The PAPR cumulative CDF is defined as:

$$C_{\text{PAPR}}(\gamma) = \Pr\left\{ \max_{0<t<T} |r(t)|^2 \geq \gamma \right\}$$

where T denotes one OFDM symbol period.

- It is straightforward that

$$C_{\text{PAPR}}(\gamma) = \Pr\left\{ \max_{0<t<T} |r(t)| \geq \sqrt{\gamma} \right\}.$$

Previous Bounds or Approximations

- According to the Rayleigh distribution and Nyquist sampling rate, van Nee and de Wild (1998) gave a $lowerbound$ and an $experimental approximation$ as

$$C_{\mathrm{PAPR}}(\gamma) \geq 1 - (1 - e^{-\gamma})^N,$$

$$C_{\mathrm{PAPR}}(\gamma) = 1 - (1 - e^{-\gamma})^{2.8N}.$$

- An $upperbound$ based on oversampled signals is proposed by Sharif and Khalaj (2001) as

$$C_{\mathrm{PAPR}}(\mathrm{PAPR} > \gamma) \leq k_{opt} N e^{-\gamma(1-\pi/k_{opt})}.$$

where $k_{opt} > \pi$ and $\frac{\pi}{k_{opt}}(1 - \frac{\pi}{k_{opt}}) = \frac{1}{\gamma}$.

- Using the level crossing rate approach and numerical computations, an $approximation$ of the PAPR CCDF is given by Ochiai and Imai (2001) as:

$$C_{\mathrm{PAPR}}(\gamma) = \begin{cases} 1 - (1 - \frac{\sqrt{\gamma} e^{-\gamma}}{\sqrt{\overline{\gamma}} e^{-\overline{\gamma}}})^{\sqrt{\frac{\pi}{3}} N \sqrt{\overline{\gamma}} e^{-\overline{\gamma}}}, & \gamma > \overline{\gamma} \\ 1, & \gamma \leq \overline{\gamma} \end{cases}$$

where $\overline{\gamma}$ stands for the reference PAPR value whose probability is negligible (close to 0).

A New Simple Upper Bound

- One peak occurs if there is a positive level crossing at the same amplitude value. Thus, the probability that the PAPR is greater than γ^2 is equivalent to the probability that $|r(t)|$ will cross γ at least once during one OFDM symbol period T (The level crossing adopted is the one with positive slope.).
- $\Pr\left\{\max_{0<t<T}\left|r(t)^2\right| > \gamma^2\right\} = \Pr[C_r(\gamma,T) \geq 1]$ where $C_r(\gamma,T)$ denotes the number of times that $r(t)$ crosses level γ during time period T.
- Use the Markov inequality to convert the above into an inequality

$$\Pr\left\{\max_{0<t<T}|r(t)| > \gamma\right\} \leq E[C_r(\gamma,T)].$$

Thus, $E[C_r(\gamma,T)]$, the mean number of crossings at level γ, denotes an upper bound of the crest factor CCDF.

Derivation of the Upper Bound

- The level crossing rate of a Rayleigh or Ricean process $v_r(\gamma)$ is given by:

$$v_r(\gamma) = \sqrt{\frac{b_2}{b_0} - \frac{b_1^2}{b_0^2}} \frac{\gamma e^{-\gamma^2}}{\sqrt{\pi}}$$

where the n^{th} spectral moment b_n is defined as

$$b_n = \left. \frac{d^n \phi(\tau)}{j^n d\tau^n} \right|_{\tau=0}$$

and $\phi(\tau)$, the autocovariance of $r(t)$, is defined as

$$\phi(\tau) = \frac{E[r^*(t) r(t+\tau)] - |E[r(t)]|^2}{2}.$$

Final Result

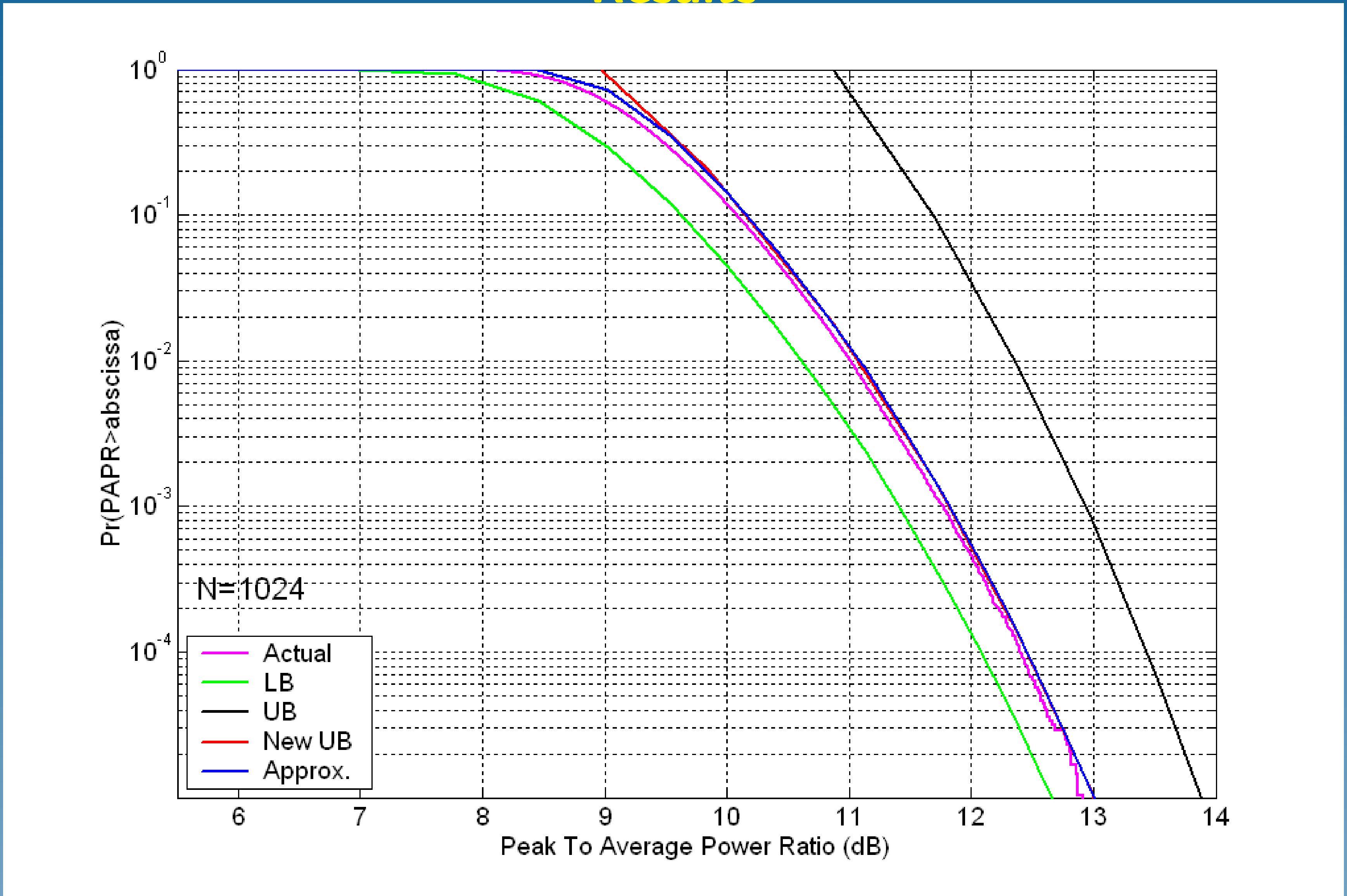
- $x(t)$ and $y(t)$ are uncorrelated and have the same autocorrelation function as $R_x(\tau) = R_y(\tau)$.
- For the Rayleigh process, we have $\phi(\tau) = R_x(\tau)$.
- If $S_x(f)$ denotes the PSD of $x(t)$, then $R_x(\tau) = \int\limits_{-\infty}^{\infty} S_x(f) e^{j2\pi f\tau} df$.
- Further, $b_n = (2\pi)^n \int\limits_{-\infty}^{\infty} S_x(f) f^n df$.
- For an ideal bandlimited OFDM signal, the PSD of $x(t)$ is expressed by

$$S_x(f) = \begin{cases} H & |f| \leq B/2 \\ 0 & |f| > B/2 \end{cases}$$

  where H is a constant and B is the bandwidth.
- The PAPR CCDF bound is obtained as

$$C_{\mathrm{PAPR}}(\gamma) \leq \sqrt{\frac{\pi}{3}} N \sqrt{\gamma} e^{-\gamma}.$$

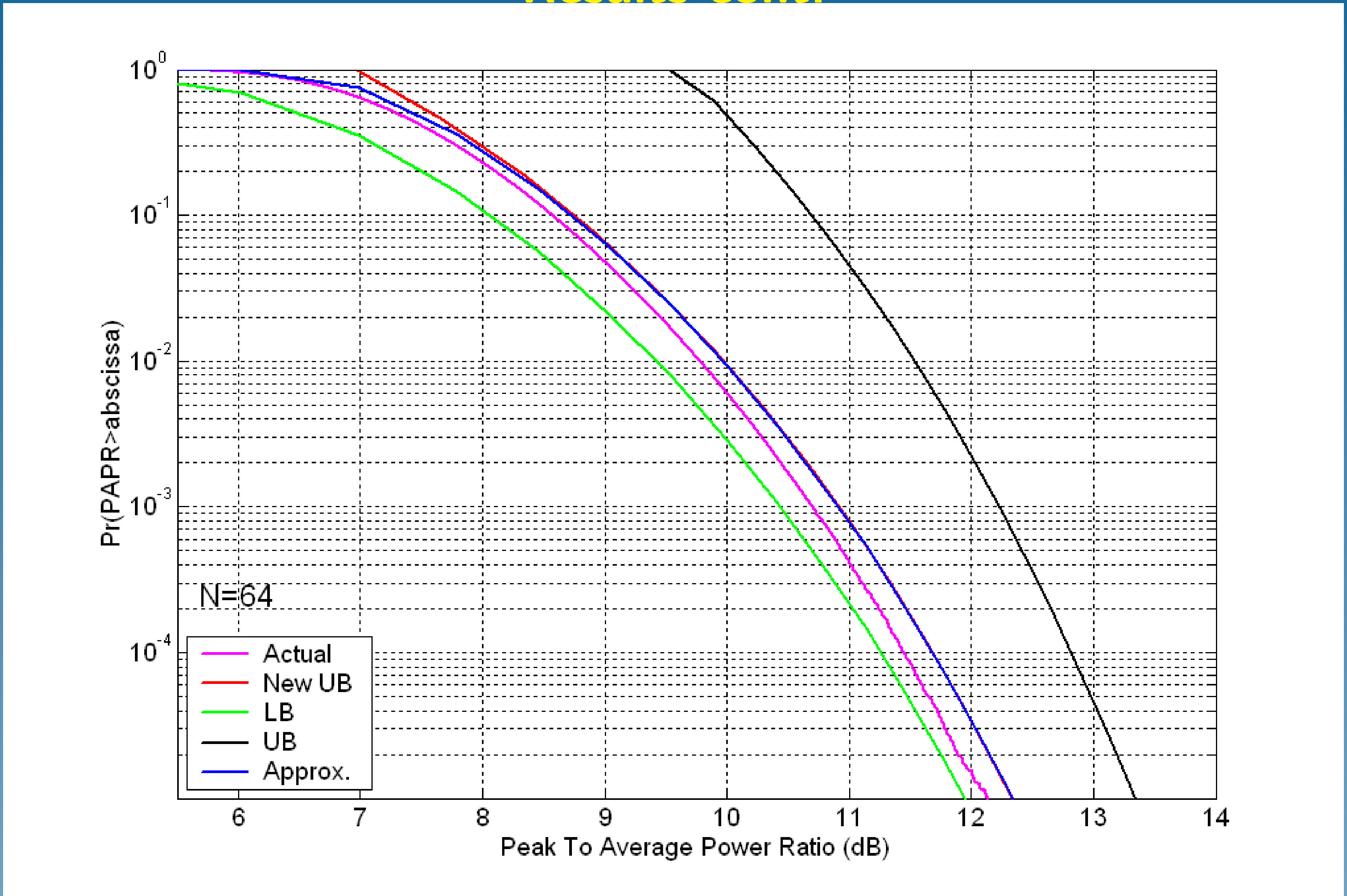
Results



Comparison of bounds and approximations (1024 subcarriers)

- Simulation condition: 4-ary QAM, 10^7 OFDM symbols and oversampling rate 4.

Results-cont.



Comparison of bounds and approximations (64 subcarriers)

- Simulation condition: 4-ary QAM, 10^7 OFDM symbols and oversampling rate 4.

Results Summary

- The simulation shows that the proposed bound matches the simulation results well when the subcarrier number $\geq$ 64. The higher the subcarrier number, the tighter the bound.

- When the subcarrier number is 32, none of these bounds or approximations are tight. The explanation is that $x(t)$ and $y(t)$ are not approximated well as Gaussian random processes for $N \leq 32$.

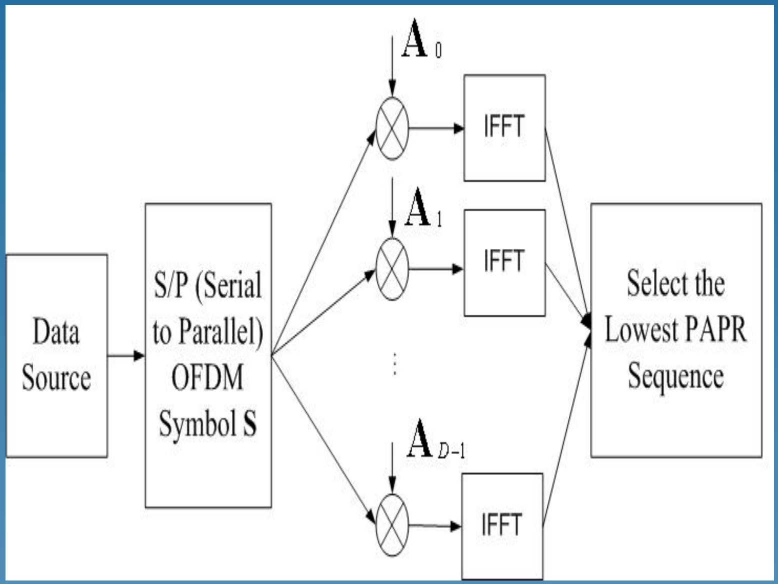
- The approximation in Ochiai and Imai (2001) approximates the PAPR well if the reference $\bar{\gamma}$ is carefully chosen.

- The maximum PAPR is directly proportional to the subcarrier number N, however the probability of the maximum PAPR is very small when $N \geq 8$. Instead of the actual PAPR being directly proportional to the subcarrier number N, we found the $probability$ that the PAPR is greater than a given value γ is directly proportional to the subcarrier number N!

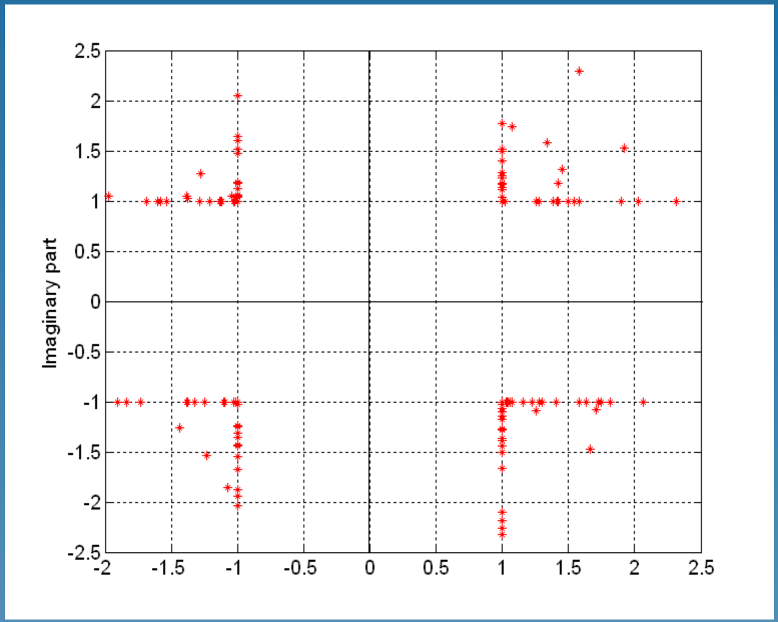
An Overview of PAPR Reduction Schemes

- Nonlinear processing
  - ⋆ Clipping (or with filtering) [Li and Cimini, 1997]
  - ⋆ Peak windowing [van Nee and de Wild,1998]
  - ⋆ Peak cancellation [May and Rohling,1998]
  - ⋆ Companding [Wang and Tjhung,1999]
- Linear processing
  - ⋆ Selective Mapping (SLM) [Bauml, Fischer and Huber,1996]
  - ⋆ Partial Transmit Sequence (PTS) [Muller and Huber 1997]
  - ⋆ Tone Reservation (TR) [Tellado and Cioffi 1998]
  - ⋆ Tone Injection (TI) [Tellado and Cioffi 1998]
- Coding (only for small subcarrier numbers) [Wulich,1996]
- Phase alignment–to choose the phase of subcarrier so that the sequence has lower PAPR (Newman Phase, P3 Phases and P4 Phases) [Stephen,1986][Levanon,2000].
- Pulse shaping function design [Slimane,2000] and dynamic constellation (DC)[Jones,1999].

Principles of DC and SLM



SLM principle.



DC principle.

SLM-DC for PAPR Reduction

- Selective mapping (SLM) and dynamic constellation (DC) are complementary and they benefit each other.

- SLM: Use D statistically independent sequences to represent the same information [Bauml, Fischer and Huber, 1996]. The probability that the PAPR of SLM exceeds γ is expressed by $\{\Pr(\mathrm{PAPR} > \gamma)\}^D$ where $\Pr(\mathrm{PAPR} > \gamma)$ is the PAPR probability before using SLM and D is the SLM order.

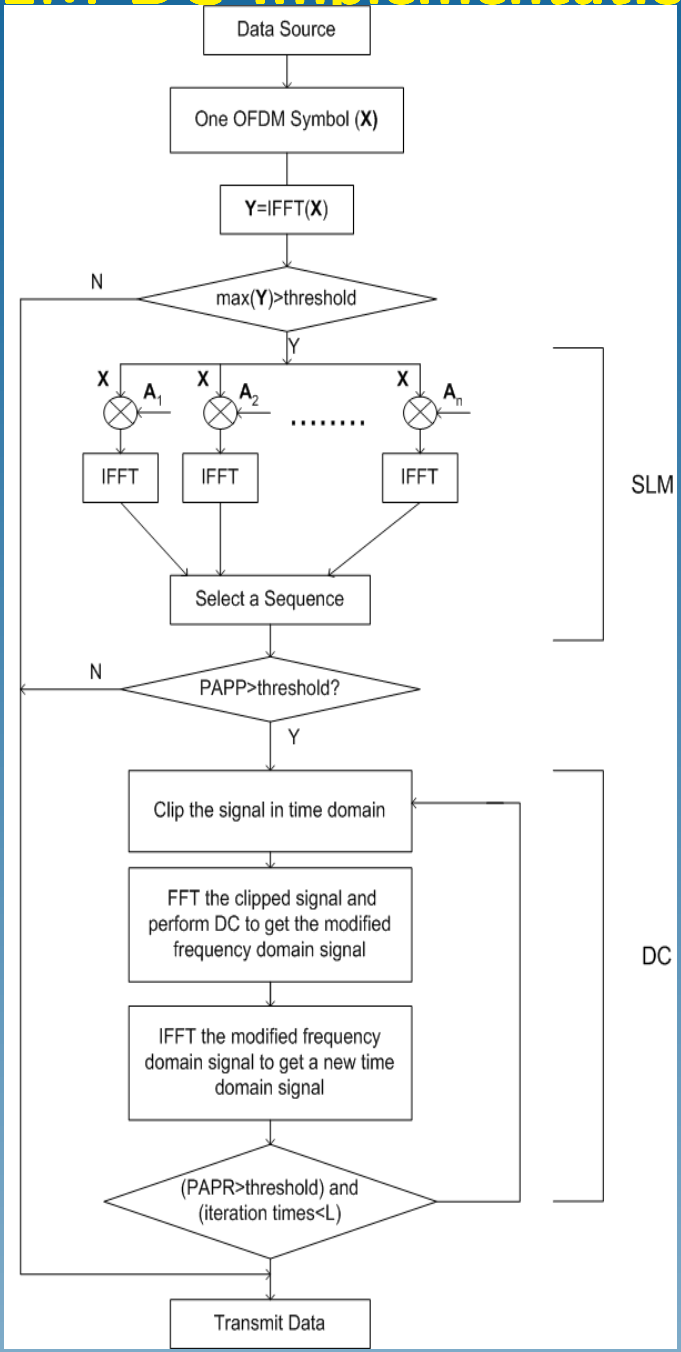
- The dynamic constellation (DC) method [Jones,1999] intentionally adds some noise to the input symbols so that the PAPR is reduced. The noise is added so that each symbol in 4-ary QAM will increase either its real or its imaginary value so that the minimum distance between two signal points is increased. For 4-ary QAM modulation, we can use the projection between the frequency domain and time domain to implement DC.

Limitation of DC and SLM

- Limitation of DC
  - ⋆ Very high computational complexity
  - ⋆ Has problem with unbalanced sequences (e.g., symbols are all $1+j$).
  - ⋆ Significant average power increase

- Limitation of SLM
  - ⋆ There are no ideal multiple representations for the same information in SLM, so we use randomly generated sequences to represent the same information. An order greater than $D=7$ does not provide much more improvement for PAPR reduction.

SLM-DC Implementation



Benefits achieved by combing SLM and DC

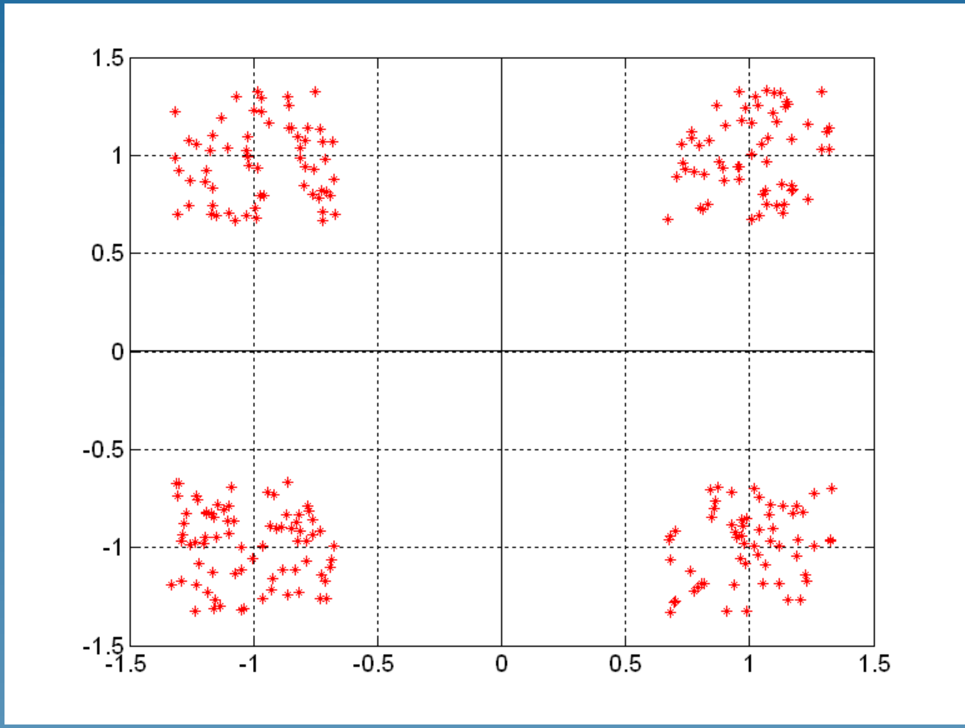
- A threshold value is set to invoke SLM or DC. If a sequences original PAPR is below this threshold, we do not use SLM-DC and we transmit the sequence directly. Otherwise, we pass this sequence to SLM. If after using SLM, the PAPR is below the threshold, we transmit the sequence directly without DC. Otherwise, we further employ DC to reduce the PAPR. This method has less overall computational complexity compared to DC only.

- Reduce average-power-increase; without SLM, DC only will increase the average power significantly!If we take the threshold to be 7.7 dB for 128 subcarriers, the average power increase will be about 0.25% for 7^{th} order SLM-DC. In the case without SLM, DC will have an average power increase of 40%.

- SLM makes the input sequence more randomly located in the signal space so that DC's ability to reduce PAPR is extended.

- More PAPR reduction.

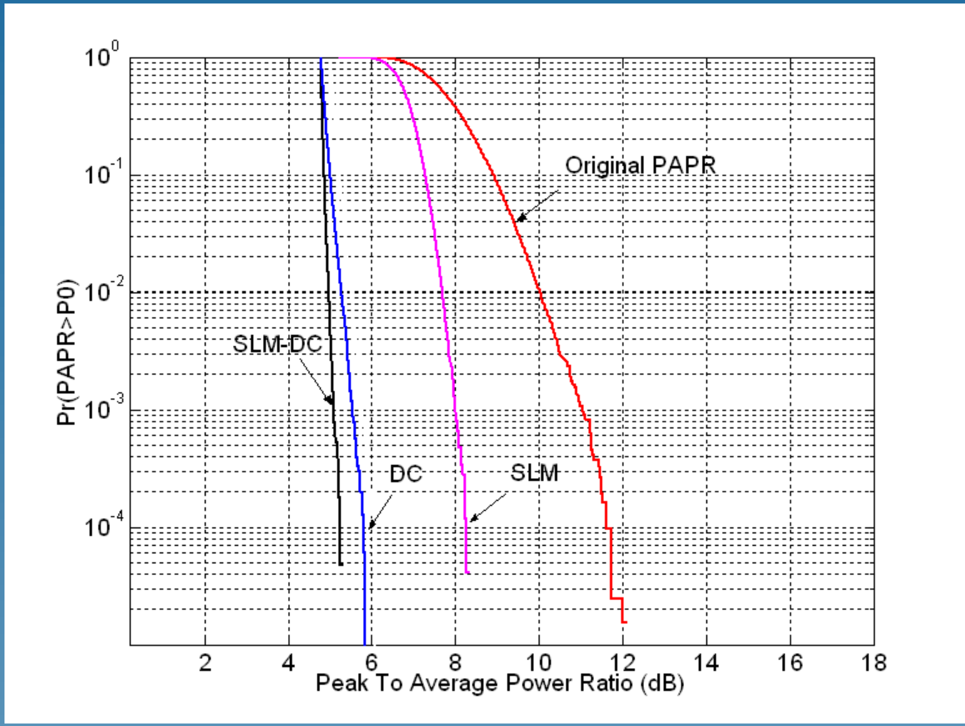
SLM extends DC's ability



SLM extends DC's ability by making symbols balanced.

- SLM-DC can work on all $1+j$ sequences. The sequence has balanced symbols (as shown in the plot) after SLM and the PAPR is reduced from 24 dB to 8 dB ($N = 256$), further the DC works on the balanced sequence and the final PAPR will be as low as 4.8 db (Threshold value is 4.77. dB and 7th order SLM-DC).

Effects of DC combined with SLM



Effect of DC combined with SLM.

- In this example of a 128-subcarrier OFDM system, it reduces the PAPR at a probability of 10^{-4} from 11.8 dB to 5.2 dB. The reduction is 2 dB more than SLM and 0.5 dB more than DC.
- SLM-DC slightly better than DC but with much less overall computational complexity, less average power increase and extended ability for unbalanced sequences.

Conclusions

- A new simpe bound on the PAPR distribution in OFDM is proposed in this paper and our simulations show that this bound is tight when the subcarrier number N is greater than 64.

- The $probability$ that the PAPR is greater than a given value is directly proportional to the subcarrier number N.

- We proposed SLM-DC for PAPR reduction and simulations show that this combination achieves good PAPR reduction.

- An estimate of the average power increase in SLM-DC via the proposed bound shows that SLM-DC can reduce the average power increase significantly, compared to DC only.