

# Blind Adaptive Multiuser Detection

Shu Wang, Sang G. Kim, Li-Hsiang Sun, Hobin Kim,  
Suk W. Lee, S. R. Subramanya, Ki Y. Kim and Byung K. Yi  
LGE Mobile Research (LGEMR), San Diego, CA 92131

**Abstract**—We propose a new blind multiuser signal model and detection framework for solving the near-far problem in synchronous CDMA in this paper. Compared with existing blind detectors, the proposed framework requires a minimum number of previously received signals, which is about the number of interfering users, and no subspace separation or sequence estimation. Hence its computation complexity and detection delay are much reduced. Following this framework, several blind multiuser detectors are developed using least squares (LS) estimation, best least unbiased (BLU) estimation and minimum mean-square error (MMSE) estimation criteria and a recursively adaptive procedure is developed for further decreasing the complexity. All these can be easily extended for asynchronous CDMA. The near-far performance of this framework and the trade-off between the complexity and performance are discussed. Computer simulations are provided to demonstrate the performance of the proposed schemes too.

## I. INTRODUCTION

Multiuser detection strategy is a method for mitigating multiple access interference (MAI) effects and solving the near-far problem with exploiting interference structure [1]. Recent research has been devoted to blind multiuser detection and subspace-based signature waveform estimation for reducing the computation complexity and prior knowledge [2, 3, 4, 5, 6, 7]. Blind multiuser detectors can achieve good performance with only the knowledge of desired users' timing and signature waveform. This assumption is also much closer to practical applications. There are two popular approaches for designing blind multiuser detectors. One is to use the conventional multiuser signal model [1], where received signals and multiuser receivers are taken as linear combinations of actual spreading sequences and noise, and statistical signal estimation techniques for blind multiuser detection, e.g., the blind multiuser receiver design using Wiener filters [2, 3] or Kalman filters [7] techniques. The other approach is based on parametric signal modelling and signal spectrum estimation, where received signals and multiuser receivers are taken as a linear combination of desired users' spreading sequences and the signal/noise subspace bases. Many subspace-based schemes are examples of this approach, which essentially is a method for blindly reconstructing existing conventional multiuser detectors using subspace concept [5, 6]. Though both the

conventional multiuser signal model and subspace-based signal model provide natural and straightforward representations of received signals and multiuser detectors, the blind detectors based on these two models are difficult to be implemented in practices since these two signal models are mostly unknown beforehand and it is nontrivial to accurately estimate them. Their computation complexity are known to be very high for many practical applications.

In order to solve the near-far problem with minimum prior knowledge and computation complexity, we propose a new blind multiuser framework based on a blind multiuser signal model in this paper. This blind signal model is different from the traditional linear prediction model [8]<sup>1</sup> and the widely-discussed conventional signal model and subspace-based parametric signal model. In the proposed model, each received signal can be taken as a linear combination of desired users' spreading sequences, several previously received signals and/or noise. Based on this new blind multiuser signal model and detection framework, several blind multiuser detectors are then developed using best linear unbiased and minimum mean squared error estimation criteria in addition to the least-squares-based schemes in [9, 10]. The proposed algorithms are simple and direct only using the signatures and timing of desired users. There is no statistical signal estimation or subspace separation procedure employed by many other blind detectors [2, 3, 5, 6]. Compared with existing blind detection schemes, they require a minimum number of previously received signals. Hence the computation complexity and detection delay can be much reduced. A recursively adaptive implementation is provided to further lessen the required computation complexity. The near-far performance and the trade-off between the complexity and performance are also discussed. Computer simulations are finally presented to demonstrate the performance of these blind detectors.

## II. SYSTEM MODEL AND PROBLEM DESCRIPTION

We consider forward-link transmissions in a single-cell DS/CDMA system. There are  $K$  active users over the multipath channel with  $P$  strong paths<sup>2</sup> and

<sup>1</sup>The discussion of the relationship between linear prediction and the proposed framework is omitted because of the paper length limitation.

<sup>2</sup>Strong paths are those to be explicitly combined by RAKE receiver.

the channel is an additive white Gaussian noise (AWGN) channel. The baseband representation of the received signal due to user  $k$  is given by

$$r_k(t) = \sum_{p=1}^P \alpha_{pk} A_k[n] b_k[n] c_k(t - nT - \tau_p) \quad (1)$$

where  $\alpha_{pk}$  is the  $p$ th path loss of user  $k$ 's signal,  $b_k[n]$  is the  $n$ th bit sent by user  $k$ . We assume that the  $\{b_k[n]\}$  are independent and identically distributed random variables with  $E\{b_k[i]\} = 0$  and  $E\{|b_k[i]|^2\} = 1$ . The parameters  $c_k(t)$  denote the normalized spreading signal waveform of user  $k$  during the interval  $[0, T]$ ,  $0 \leq \tau_1 \leq \tau_2 \leq \dots \leq \tau_P$ , denotes  $P$  different transmission delays from the base station to user  $k$  and  $A_k[n]$  is the received signal amplitude for user  $k$  at time  $t = n$ , which depends on the possible channel statistics. The total baseband signal received by user  $k$  is

$$\tilde{r}(t) = \sum_{k=1}^K r_k(t) \quad (2)$$

The received signal  $\tilde{r}(t)$  is passed through the corresponding chip matched filter (CMF),  $\phi(t)$ , and RAKE combiner. The combined output  $r(t)$  is<sup>3</sup>

$$r(t) = A_k b_k c_k(t - nT - \tau_1) \otimes \phi(t - \tau_1) + m_{\text{ISI}}(t) + m_{\text{MAI}}(t) + n(t) \quad (3)$$

where

$$m_{\text{ISI}}(t) = \sum_{p \neq q}^P \beta_{qk} \alpha_{pk} A_k b_k c_k(t - nT + \tau_{q1} - \tau_1) \otimes \phi(t - \tau_1) \quad (4)$$

is the intersymbol interference (ISI) to user  $k$ ,

$$m_{\text{MAI}}(t) = \sum_{i \neq k}^K A_i b_i c_i(t - nT - \tau_1) \otimes \phi(t - \tau_1) + \sum_{i \neq k}^K \sum_{p \neq q}^P \beta_{qk} \alpha_{pi} A_i b_i c_i(t - nT + \tau_{q1} - \tau_p) \otimes \phi(t - \tau_1) \quad (5)$$

is the MAI to user  $k$ ,  $\beta_{qk}$  is the weight of the  $q$ th RAKE finger with  $\sum_{q=1}^P \beta_{qk} \alpha_{qk} = 1$  and  $\tau_{q1} = \tau_q - \tau_1$  is the propagation delay difference between the 1st path and  $p$ th path.  $\otimes$  denotes the convolutional product.  $n(t)$  is AWGN with variance  $\sigma^2$ . The user  $k$ 's RAKE output can be sampled at  $f_s = 1/T_s$  and straightforwardly expressed by

$$\begin{aligned} \mathbf{r} &= [r(nT + T_s + \tau_1) \dots r(nT + LT_s + \tau_1)]^T \\ &= \sum_{k=1}^K A_k b_k \mathbf{s}_k + \mathbf{n} \\ &= \mathbf{S} \mathbf{A} \mathbf{b} + \mathbf{n} \end{aligned} \quad (6)$$

<sup>3</sup>Without loss of the generality, we drop the time index  $n$  in the following discussion.

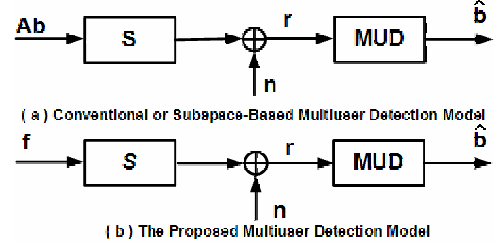


Fig. 1. Multiuser detection models.

where  $\mathbf{S} = [\mathbf{s}_1 \mathbf{s}_2 \dots \mathbf{s}_K]$  is the received spreading sequence matrix combined with both ISI and MAI information, and  $L = T/T_s$  is the number of samples per symbol, which should not be less than the spreading gain  $L_c$ .

Because of  $m_{\text{MAI}}(t)$  existing in the received signal  $r(t)$ , the performance of conventional matched filter receiver suffers from the so-called near-far problem [1]. Multiuser detection is the receiver technique for solving this problem and most multiuser detectors are developed using the conventional system model like (6). These are well documented in [1]. One of the difficulties in developing blind multiuser detectors using (6) is that the  $\mathbf{S}$  is hard to be known beforehand. And it normally takes much effort to determine it later. The similar situation arises in developing blind detectors using the parametric subspace signal model proposed in [5].

### III. BLIND MULTIUSER DETECTION FRAMEWORK

Instead of using the conventional signal model or the parametric subspace signal model, we develop a new blind multiuser signal model shown in Fig. 1, which uses a blind but "faked" spreading matrix  $\mathbf{S}$ . We call  $\mathbf{S}$  the blind but "faked" spreading matrix because 1) it is composed by only desired users' spreading sequences and previously received signals and 2) it isn't the original one but works like the original signal bases for representing received signals and even multiuser detectors. With the construction of  $\mathbf{S}$ , we can construct group-wise or single blind detectors in the following. Without loss of the generality, only the bits sent for first  $G$  users are considered here and the  $L \times M$  blind spreading sequence matrix  $\mathbf{S}$  is defined by

$$\mathbf{S} = [\mathbf{s}_1 \dots \mathbf{s}_G \mathbf{r}_1 \mathbf{r}_2 \dots \mathbf{r}_{M-G}] \quad (7)$$

where  $\mathbf{s}_g$ ,  $g = 1, 2, \dots, G$ , denote the group of  $G$  spreading waveforms which are already known to user 1.  $\mathbf{r}_m$ ,  $m = 1, 2, \dots, M - G$ , are  $M - G$  previously received independent signal vectors<sup>4</sup>.  $K \leq M \leq L$ , where

<sup>4</sup>The first  $M - G$  received signal  $\mathbf{r}_m$  may be obtained from some receiver initialization procedure. After that, there will be some possible adaptive procedure for updating  $\mathbf{S}$ , e.g. the procedure proposed in Section V. Compared with existing blind detectors, this number,  $M - G$ , of required previously received signals are very small.

$M = K$  is the minimum number for blind multiuser detector to unambiguously distinguish different interfering signals and  $M \leq L$  is the constraint for guaranteeing the uniqueness of designed blind multiuser receiver. The relationship between the proposed blind spreading matrix  $\mathcal{S}$  and the original spreading matrix  $\mathbf{S}$  can be given by

$$\mathcal{S} = \mathbf{S}\mathbf{B} + \mathbf{N} \quad (8)$$

where the first  $G$  columns of  $\mathcal{S}$  and  $\mathbf{S}$  are same,

$$\mathbf{B} = \begin{bmatrix} \mathbf{I} & \bar{\mathbf{D}} \\ \mathbf{0} & \tilde{\mathbf{D}} \end{bmatrix} = \begin{bmatrix} \mathbf{E} & \bar{\mathbf{D}} \\ \mathbf{0} & \tilde{\mathbf{D}} \end{bmatrix} = \begin{bmatrix} \mathbf{G} & \tilde{\mathbf{D}} \\ \mathbf{0} & \tilde{\mathbf{D}} \end{bmatrix} \quad (9)$$

is the  $K \times M$  data matrix associated with  $\mathcal{S}$ .  $\mathbf{E} = [\mathbf{I} \ \mathbf{0}]^T$  is a  $K \times G$  matrix,  $\mathbf{G} = [\mathbf{I} \ \bar{\mathbf{D}}]$  is the  $G \times M$  matrix, where  $\bar{\mathbf{D}}$  is previously detected and known matrix for desired users.  $\text{rank}\{\bar{\mathbf{D}}\} = K - G$ , where  $\tilde{\mathbf{D}}$  is unknown matrix for unknown  $K - G$  users, and  $\text{rank}\{\mathbf{B}\} \leq K$ . Combining (6) and (8), the received signal vector  $\mathbf{r}$  in (6) can be expressed as the linear combination of the columns in  $\mathcal{S}$  instead of  $\mathbf{S}$ , which is written by

$$\mathbf{r} = \mathcal{S}\mathbf{f} + \bar{\mathbf{n}} \quad (10)$$

where the  $M \times 1$  vector  $\mathbf{f}$  is termed the detection vector defined by

$$\mathbf{f} = \mathbf{B}^+ \bar{\mathbf{b}} \quad (11)$$

where  $[\cdot]^+$  denotes the general inverse operator and  $\bar{\mathbf{b}} = \mathbf{A}\mathbf{b}$ .  $\bar{\mathbf{n}}$  is the new  $L \times 1$  AWGN vector defined by

$$\bar{\mathbf{n}} = \mathbf{n} - \mathbf{N}\mathbf{B}^+ \bar{\mathbf{b}} \quad (12)$$

We can see that (10) may be taken as a modified linear prediction model and the multiuser detection problem may be taken a modified linear prediction problem if  $\bar{\mathbf{n}} = \mathbf{0}$ . On the other hand, if  $\mathbf{f}$  can be estimated, the amplitudes  $A_g$  and bits  $b_g$  for the first  $G$  users can be estimated and detected with (11) by

$$\hat{\mathbf{b}}_1 = [\hat{b}_1 \ \hat{b}_2 \ \dots \ \hat{b}_G]^T = \text{sgn}\{\mathbf{G}\mathbf{f}\} \quad (13)$$

$$\hat{\mathbf{a}}_1 = [\hat{A}_1 \ \hat{A}_2 \ \dots \ \hat{A}_G]^T = |\mathbf{G}\mathbf{f}| \quad (14)$$

In the following, we will present various schemes for estimating  $\mathbf{f}$  and detecting  $\mathbf{b}_1$ . An adaptive implementation will be presented in Section V.

#### IV. BLIND MULTIUSER DETECTORS

##### A. LEAST SQUARES DETECTION

At first, we assume that the measurements of  $\mathcal{S}$  are assumed to be free of error. All errors are confined to the received vector  $\mathbf{r}$ . Hence, the detection vector can be estimated with solving the following equation [11, 12]

$$\mathbf{f}_{\text{LS}} = \arg \min_{\mathbf{x}} \|\mathbf{r} - \mathcal{S}\mathbf{x}\|_2 = \mathcal{S}^+ \mathbf{r} \quad (15)$$

and the bit vector for the first  $G$  users can be detected by

$$\hat{\mathbf{b}}_1 = \text{sign}\{\mathbf{G}\mathcal{S}^+ \mathbf{r}\} \quad (16)$$

##### B. TOTAL LEAST SQUARES DETECTION

The previous LS estimation assumes  $\mathcal{S}$  to be error-free. This assumption is not entirely accurate with  $\mathcal{S}$  because of  $\mathbf{N}$ . Problem (15) can then be transformed into the TLS problem:

$$\begin{bmatrix} \mathcal{S}_{\text{TLS}} \\ \mathbf{f}_{\text{TLS}} \end{bmatrix} = \arg \min_{\mathcal{S}, \mathbf{x}} \left\| \begin{bmatrix} \mathcal{S} \\ \mathbf{r} \end{bmatrix} - \begin{bmatrix} \bar{\mathcal{S}} \\ \bar{\mathcal{S}}\mathbf{x} \end{bmatrix} \right\|_2 \quad (17)$$

Let  $\mathcal{S} = \mathbf{U}' \Sigma' \mathbf{V}'^T$  and  $[\mathcal{S} \ \mathbf{r}] = \mathbf{U} \Sigma \mathbf{V}^T$  be the SVD of  $\mathcal{S}$  and  $[\mathcal{S} \ \mathbf{r}]$ , respectively. If  $\sigma_K > \sigma_{K+1}$ , TLS estimation of  $\mathbf{f}$  then is [11, 12]

$$\mathbf{f}_{\text{TLS}} = \left( \mathcal{S}^T \mathcal{S} - \sigma_{K+1}^2 \mathbf{I} \right)^{-1} \mathcal{S}^T \mathbf{r} \quad (18)$$

and the bit vector for the first  $G$  users can be detected by

$$\hat{\mathbf{b}}_1 = \text{sign} \left\{ \mathbf{G} \left( \mathcal{S}^T \mathcal{S} - \sigma_{K+1}^2 \mathbf{I} \right)^{-1} \mathcal{S}^T \mathbf{r} \right\} \quad (19)$$

##### C. MIXED LS/TLS DETECTION

Though there exists a noise or error matrix  $\mathbf{N}$  in  $\mathcal{S}$ , its first  $G$  columns are exactly known to be noise-free or error-free. Hence, to maximize the estimation accuracy of the detection vector  $\mathbf{f}$ , it is natural to require the corresponding columns of  $\mathcal{S}$  to be unperturbed since they are known exactly. Problem (15) and (17) can then be transformed into the following MLS problem:

$$\begin{bmatrix} \mathcal{S}_{\text{MLS}} \\ \mathbf{f}_{\text{MLS}} \end{bmatrix} = \arg \min_{\mathcal{S}, \mathbf{x}} \left\| \begin{bmatrix} \bar{\mathcal{S}} \\ \mathbf{r} \end{bmatrix} - \begin{bmatrix} \bar{\mathcal{S}} \\ [\mathbf{s}_1 \ \bar{\mathcal{S}}] \mathbf{x} \end{bmatrix} \right\|_2 \quad (20)$$

Perform the Householder transformation  $\mathbf{Q}$  on the matrix  $[\mathcal{S} \ \mathbf{r}]$  so that

$$\mathbf{Q}^T [\mathbf{s}_1 \ \dots \ \mathbf{s}_G \ \bar{\mathcal{S}} \ \mathbf{r}] = \begin{bmatrix} \mathbf{R}_{11} & \mathbf{R}_{12} & \mathbf{r}_{1r} \\ \mathbf{0} & \mathbf{R}_{22} & \mathbf{r}_{2r} \end{bmatrix} \quad (21)$$

where  $\mathbf{R}_{11}$  is a  $G \times G$  upper triangle matrix,  $\mathbf{r}_{1r}$  is a  $G \times 1$  vector and  $\mathbf{r}_{2r}$  is a  $(L - G) \times 1$  vector. Denote  $\sigma'$  as the smallest singular value of  $\mathbf{R}_{22}$  and  $\sigma$  as the smallest singular value of  $[\mathbf{R}_{22} \ \mathbf{r}_{2r}]$ . If  $\sigma' > \sigma$ , then the MLS solution uniquely exists and is given by [11]

$$\mathbf{f}_{\text{MLS}} = \left( \mathcal{S}^T \mathcal{S} - \sigma^2 \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{M-G} \end{bmatrix} \right)^{-1} \mathcal{S}^T \mathbf{r} \quad (22)$$

and the bit vector for the first  $G$  users can be detected by

$$\hat{\mathbf{b}}_1 = \text{sign} \left\{ \mathbf{G} \left( \mathcal{S}^T \mathcal{S} - \sigma^2 \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{M-G} \end{bmatrix} \right)^{-1} \mathcal{S}^T \mathbf{r} \right\} \quad (23)$$

#### D. BEST LINEAR UNBIASED DETECTION

We assume the linear structure

$$\mathbf{f}_{\text{BLU}} = \mathbf{W}_{\text{BLU}}^T \mathbf{r} \quad (24)$$

for this so-called best linear unbiased estimator (BLUE), which is equal to the optimal minimum variance unbiased estimator (MVUE) in linear signal models if the data are truly Gaussian. Matrix  $\mathbf{W}_{\text{BLU}}$  is designed such that: 1)  $\mathbf{S}$  must be deterministic, 2)  $\mathbf{n}$  must be zero mean with positive definite known covariance matrix  $\mathbf{C}_{\mathbf{n}}$ , 3)  $\mathbf{f}_{\text{BLU}}$  is an unbiased estimator of  $\mathbf{f}$ , 4) and the error variance for each of the  $M$  parameters is minimized as

$$\mathbf{W}_{\text{BLU}} = \min_{\mathbf{W}_{\mathbf{f}}} \text{var} \{ \mathbf{W}_{\mathbf{f}}^T \mathbf{r} \}. \quad (25)$$

The resulting best linear unbiased estimator is (Gauss-Markov Theorem):

$$\mathbf{f}_{\text{BLU}} = (\mathbf{S}^T \mathbf{C}_{\mathbf{n}}^{-1} \mathbf{S})^{-1} \mathbf{S}^T \mathbf{C}_{\mathbf{n}}^{-1} \mathbf{r}. \quad (26)$$

The covariance matrix of  $\mathbf{f}_{\text{BLU}}$  given by

$$\mathbf{C}_{\mathbf{f}_{\text{BLU}}} = (\mathbf{S}^T \mathbf{C}_{\mathbf{n}}^{-1} \mathbf{S})^{-1}. \quad (27)$$

Though the PDF of  $\mathbf{B}$  may be determined, the PDF of  $\mathbf{B}^+$  is largely unknown. However, with Girko's Law, when  $\alpha = (K - G)/(M - G)$  is fixed,  $K, M \rightarrow \infty$ , the diagonal element of  $\frac{1}{M-G} \tilde{\mathbf{D}}^+ \tilde{\mathbf{b}} \tilde{\mathbf{b}}^T \tilde{\mathbf{D}}^{+T}$  may be approximated by [13]

$$\lim_{M \rightarrow \infty} \frac{1}{M-G} \left[ \tilde{\mathbf{D}}^+ \tilde{\mathbf{b}} \tilde{\mathbf{b}}^T \tilde{\mathbf{D}}^{+T} \right]_{ii}^{-1} = 1 - \alpha. \quad (28)$$

Hence, the covariance matrix of  $\mathbf{f}$ ,  $\mathbf{C}_{\mathbf{f}}$ , can be decided by

$$\mathbf{C}_{\mathbf{f}} = \begin{bmatrix} \frac{2M-K-G}{M-K} \mathbf{A}_1^2 & \mathbf{0}^T \\ \mathbf{0} & \frac{1}{M-K} \mathbf{I} \end{bmatrix}, \quad (29)$$

where  $\mathbf{A}_1 = \text{diag} \{ \hat{A}_1 \ \hat{A}_2 \ \dots \ \hat{A}_G \}$ ,

$$\mathbf{C}_{\mathbf{n}} = \sigma^2 \frac{2M-K-G}{M-K} \mathbf{I} \quad (30)$$

and the bit vector for the first  $G$  users can be detected by

$$\hat{\mathbf{b}}_1 = \text{sign} \left\{ \mathbf{G} (\mathbf{S}^T \mathbf{S})^{-1} \mathbf{S}^T \mathbf{r} \right\} \quad (31)$$

#### E. MINIMUM MEAN SQUARED ERROR DETECTION

Given measurements  $\mathbf{r}$ , the MMSE estimator of  $\mathbf{f}$ ,  $\mathbf{f}_{\text{MMSE}} = f(\mathbf{r})$ , minimizes the MSE  $J_{\text{MSE}} = E\{\|\mathbf{f} - \hat{\mathbf{f}}\|_2^2\}$ . When  $\mathbf{f}$  and  $\mathbf{r}$  are jointly Gaussian, the linear estimator  $\mathbf{W}_{\text{MMSE}}$  that minimizes the MSE  $J_{\text{MSE}}$  is (Bayesian Gauss-Markov Theorem) [8]

$$\mathbf{f}_{\text{MMSE}} = (\mathbf{C}_{\mathbf{f}}^{-1} + \mathbf{S}^T \mathbf{C}_{\mathbf{n}}^{-1} \mathbf{S})^{-1} \mathbf{S}^T \mathbf{C}_{\mathbf{n}}^{-1} \mathbf{r}, \quad (32)$$

which is also termed Wiener filter, and the bit vector for the first  $G$  users can be detected by

$$\hat{\mathbf{b}}_1 = \text{sign} \left\{ \mathbf{G} (\mathbf{C}_{\mathbf{f}}^{-1} + \mathbf{S}^T \mathbf{C}_{\mathbf{n}}^{-1} \mathbf{S})^{-1} \mathbf{S}^T \mathbf{C}_{\mathbf{n}}^{-1} \mathbf{r} \right\} \quad (33)$$

Since (33) requires channel noise variance estimation, this detector is not included in the followed discussion and simulations.

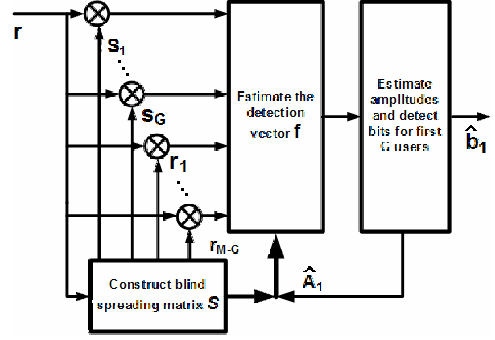


Fig. 2. The proposed adaptive blind multiuser detection structure.

#### V. ADAPTIVE IMPLEMENTATION

In Fig. 2, an adaptive structure of the presented blind multiuser detection scheme is presented for time-variant channels. Following the well-known Sherman-Morrison-Woodbury matrix inverse lemma [8, 12], a recursively adaptive implementation of the proposed LS and BLU blind detector can be expressed by

$$\hat{\mathbf{b}}_1(n) = \text{sign} \left\{ \mathbf{G}(n) \mathbf{C}_{\mathbf{S}}^+(n) \mathbf{S}^T(n) \mathbf{r}(n) \right\} \quad (34)$$

$$\mathbf{C}_{\mathbf{S}}^+(n) = \mathbf{C}_{\mathbf{S}}^+(n-1) - \frac{\mathbf{C}_{\mathbf{S}}^+(n-1) \mathbf{u}(n-1) \mathbf{u}^T(n-1) \mathbf{C}_{\mathbf{S}}^+(n-1)}{1 + \mathbf{u}^T(n-1) \mathbf{C}_{\mathbf{S}}^+(n-1) \mathbf{u}(n-1)} \quad (35)$$

where

$$\mathbf{C}_{\mathbf{S}}(n) = \mathbf{S}(n)^T \mathbf{S}(n) \quad (36)$$

and  $\mathbf{u}(n-1)$  is designed using SVD so that

$$\mathbf{u}(n-1) \mathbf{u}^T(n-1) = \mathbf{C}_{\mathbf{S}}(n) - \mathbf{C}_{\mathbf{S}}(n-1) \quad (37)$$

#### VI. PERFORMANCE ANALYSIS

##### A. ASYMPTOTIC MULTIUSER EFFICIENCY

A commonly used performance measure for a multiuser detector is asymptotic multiuser efficiency (AME) [1]. The AME of the proposed schemes is

$$\bar{\eta}_k = \sqrt{\frac{M-K}{2M-K-G}} [\mathbf{R}_{\mathbf{S}}^+]_{kk}^{-1}. \quad (38)$$

where  $\mathbf{R}_{\mathbf{S}} = \mathbf{S}^T \mathbf{S}$  and  $[\cdot]_{kk}$  denotes the  $k$ th diagonal element of the matrix.

##### B. CRLB FOR $\mathbf{f}$ ESTIMATION

The Cramér-Rao Lower Bound (CRLB) is given by the inverse of the Fisher information matrix (FIM). Providing the blind spreading matrix  $\mathbf{S}$  is known beforehand, we first define the parameter vector  $\phi = [\bar{\sigma}^2 \ \mathbf{f}^T]^T$ , where  $\bar{\sigma}^2 = (1 + \frac{M-G}{M-K}) \sigma^2$ , for computing the FIM

$$\mathbf{I}(\phi) = E \left\{ \left( \frac{\partial \ln \mathcal{L}}{\partial \phi} \right) \left( \frac{\partial \ln \mathcal{L}}{\partial \phi} \right)^H \right\} \quad (39)$$

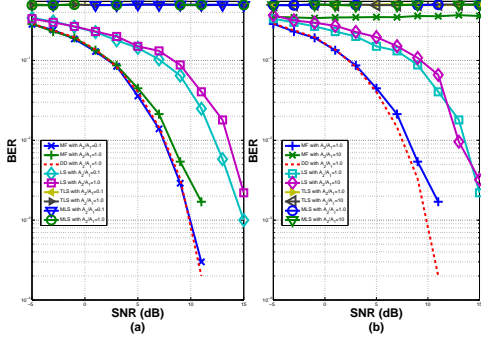


Fig. 3. The BER performance of the proposed schemes against SNR.  $K = 10$ ,  $M = 40$  and  $G = 3$

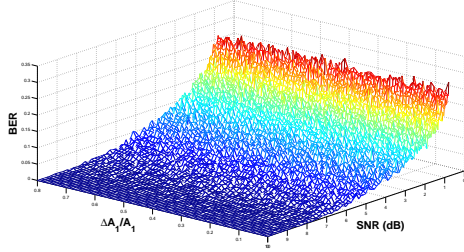


Fig. 4. The performance of the LS detector against amplitude estimation error  $\Delta A_1/A_1$  and SNR,  $M = 63$ .

where  $\ln \mathcal{L}$  is the log-likelihood function given by

$$\ln \mathcal{L} = C - L \ln \sigma^2 - \frac{1}{2\sigma^2} \|\mathbf{e}\|_2^2, \quad (40)$$

$C$  is a constant and  $\mathbf{e} = \mathbf{r} - \mathbf{S}\mathbf{f}$ . Providing  $\mathbf{S}$  is known, the closed-form CRLB expression of  $\mathbf{f}$  is then given by

$$\text{CRLB}(\mathbf{f} | \mathbf{S}) = (1 + \frac{M-G}{M-K})\sigma^2(\mathbf{S}^T \mathbf{S})^+. \quad (41)$$

It shows that the accuracy of estimating  $\mathbf{f}$  may increase with increasing  $M$ .

## VII. COMPUTER SIMULATIONS

There are  $K = 10$  users with the group size  $G = 3$  and the spreading sequences used in simulations

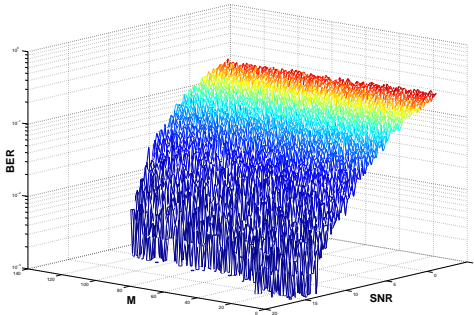


Fig. 5. The performance of the LS blind MUD against  $M$  and SNR.

are 64-chip ( $L = 64$ ) random sequences. In the computer simulations, the previous amplitude estimation from (14) is directly use for the next detection without any amplitude filtering. From Fig. 3, it is interesting to see that the performance of the simplest LS detector has very stable performance against the change of interfering signal power. We then check the performance of the proposed LS blind detector against the amplitude estimation errors. From Fig. 4, we can see that the BER of the LS detector basically is unchanged against amplitude estimation error when SNR is large enough. From Fig. 5, we can see that the performance of the LS detector can be better providing  $M$  is larger enough. This confirms (30), which shows that the variance of  $\hat{\mathbf{n}}$  decreases with increasing  $M$ .

## VIII. CONCLUSIONS

In this paper, we proposed a blind multiuser detection framework as well as several blind detectors. The proposed blind detectors are direct and simple without any sequence estimation or subspace separation. The performance of the proposed blind LS and BLU detectors are comparable with the conventional decorrelating detector.

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