On Interference Cancellation for Synchronous CDMA

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Introduction

- CDMA techniques have attracted attention for their spectral efficiency, interference resistance and flexible traffic control.

- MAI is the dominant impairment for CDMA systems and exits even with perfect power control.

- Multiuser detection promising approach to mitigate MAI: multiuser detectors (linear/nonlinear) and interference cancellation

- We explore interference cancellation in a framework that leads to an interference cancellation representation of linear multiuser detectors (DD, MMSE, MAME)

  ⋆ Direct interference cancellation (D-IC)
  ⋆ MMSE interference cancellation (MMSE-IC)
  ⋆ MAME interference cancellation (MAME-IC)

Data Model

- Consider a synchronous CDMA system with L chips per bit $(T_b = LT_c)$.
- Received signal, after chip-matched filter followed by chip-rate sampling:

$$\mathbf{r} = \sum_{k=1}^{K} A_k b_k \mathbf{s}_k + \mathbf{n} \; = \; \mathbf{SAb} + \mathbf{n}$$

  where $\mathbf{A} = \text{diag}\{A_1 \; A_2 \; \ldots \; A_K\}$, $\mathbf{S} = [\mathbf{s}_1 \; \mathbf{s}_2 \; \ldots \; \mathbf{s}_K]$ and $\mathbf{b} = [b_1 \; b_2 \; \ldots \; b_K]^T$ with $b_i \in \{+1, -1\}$.
- Linear multiuser detectors can be written in the form

$$\mathbf{b} = \text{sgn}\{\mathbf{Wr}\}$$

  or for the kth user $b_k = \text{sgn}\left\{\mathbf{w}_k^T \mathbf{r}\right\}$ where $\mathbf{w}_k^T$ is the kth row of $\mathbf{W}$.
- We consider detection of user 1's data only.
- Maintain the restriction that $L > K$.

Data Model w.r.t. User 1

- In terms of user 1:

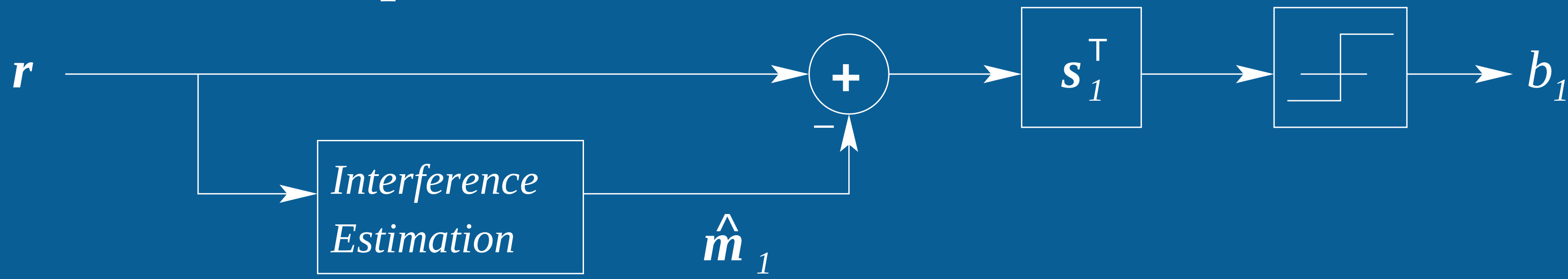
$$
\begin{aligned}
\mathbf{r} &= A_1 b_1 \mathbf{s}_1 + \sum_{k=2}^{K} A_k b_k \mathbf{s}_k + \mathbf{n} \\
&= \left[\mathbf{s}_1 \quad \tilde{\mathbf{S}}\right] \begin{bmatrix} A_1 & \mathbf{0}^T \\ \mathbf{0} & \tilde{\mathbf{A}} \end{bmatrix} \begin{bmatrix} b_1 \\ \tilde{\mathbf{b}} \end{bmatrix} + \mathbf{n} \\
&= A_1 b_1 \mathbf{s}_1 + \tilde{\mathbf{S}}\tilde{\mathbf{A}}\tilde{\mathbf{b}} + \mathbf{n} \\
&= A_1 b_1 \mathbf{s}_1 + \mathbf{m}_1 + \mathbf{n}
\end{aligned}
$$

where $\mathbf{m}_1 = \tilde{\mathbf{S}}\tilde{\mathbf{A}}\tilde{\mathbf{b}}$ is the MAI seen by user 1, and

$$
\begin{aligned}
\tilde{\mathbf{S}} &= \left[\mathbf{s}_2\ \mathbf{s}_3\ \ldots\ \mathbf{s}_K\right] \\
\tilde{\mathbf{A}} &= \mathrm{diag}\left\{A_2\ A_3\ \ldots\ A_K\right\} \\
\tilde{\mathbf{b}} &= \left[b_2\ b_3\ \ldots\ b_K\right]^T
\end{aligned}
$$

Interference Cancellation (IC)

- Goal: estimate $\mathbf{m}_1$ and remove from $\mathbf{r}$.



- Given an estimate of MAI, $\hat{\mathbf{m}}_1$, the estimate of user 1's data bit is

$$\begin{aligned}\hat{b}_1 &= \text{arg}\ \min_b \|\mathbf{r} - \hat{\mathbf{m}}_1 - A_1 b \mathbf{s}_1\| \\ &= \text{sgn}\left\{\mathbf{s}_1^T(\mathbf{r} - \hat{\mathbf{m}}_1)\right\} \\ &= \text{sgn}\left\{A_1 b_1 + \mathbf{s}_1^T(\mathbf{m}_1 - \hat{\mathbf{m}}_1) + \mathbf{s}_1^T \mathbf{n}\right\}\end{aligned}$$

- If we assume $\hat{\mathbf{m}}_1 = \mathbf{W}_1 \mathbf{r}$, then

$$\hat{b}_1 = \text{sgn}\left\{\mathbf{s}_1^T(\mathbf{I}_L - \mathbf{W}_1)\mathbf{r}\right\} = \text{sgn}\left\{\mathbf{s}_1^T \mathbf{W}_1^{IC} \mathbf{r}\right\} = \text{sgn}\left\{\left[\mathbf{w}_1^{IC}\right]^T \mathbf{r}\right\}$$

- Therefore, can represent IC as a linear filter.

Types of IC

- Parallel interference cancellation (PIC): All users are simultaneously demodulated and detected in a parallel behavior (Jacobi Iteration).
- Successive/Serial interference cancellation (SIC): A decision for the strongest user is made first and the interference from this user is subsequently removed for the detection of the next stronger user's data, etc. (Gauss-Siedel Iteration)
- Hybrid parallel/serial cancellation
- Groupwise interference cancellation
- Categories:
  - ⋆ Hard — make decision on other user's bits, re-form interfering signal and remove.
  - ⋆ Soft — estimate each user's interference, $A_k b_k \mathbf{s}_k$, and remove.

Direct Interference Cancellation (D-IC)

- Estimate $\hat{\mathbf{m}}_1$ according to the least squares (LS) criterion.
- Estimate MAI $\hat{\mathbf{m}}$ by solving

$$\hat{\mathbf{m}}_1^{D-IC} = \arg\min_{\mathbf{m}} \|\mathbf{r} - \mathbf{m} - A_1 b_1 \mathbf{s}_1\|_2$$

  subject to $\mathbf{m} \in \text{span}\{\tilde{\mathbf{S}}\}$.
- Since ideally $\hat{\mathbf{m}}_1 = \tilde{\mathbf{S}}\tilde{\mathbf{A}}\tilde{\mathbf{b}} = \tilde{\mathbf{S}}\hat{\mathbf{x}}_1$, we can formulate $\hat{\mathbf{m}}_1^{D-IC}$ by finding

$$\hat{\mathbf{x}}_1 = \arg\min_{\mathbf{x}} \left\| \mathbf{r} - A_1 b_1 \mathbf{s}_1 - \tilde{\mathbf{S}}\mathbf{x} \right\|_2$$

- To separate the MAI from the observation vector, we use a unitary matrix $\mathbf{Q}$ to perform the Householder transformation on $\mathbf{s}_1$ such that

$$\mathbf{Q}^T \mathbf{s}_1 = [\, c \quad \mathbf{0}^T \,]^T$$

  where $c = \pm\|\mathbf{s}_1\| = \pm 1$ and

$$\mathbf{Q} = \mathbf{I}_L - 2\mathbf{q}\mathbf{q}^T \quad \text{with} \quad \mathbf{q} = \kappa \begin{bmatrix} s_{11} - c \\ s_{12} \\ \vdots \\ s_{1L} \end{bmatrix}$$

Final Form of $\hat{\mathbf{m}}_1^{D-IC}$

- Applying $\mathbf{Q}$ to $\mathbf{r}$ results in

$$\mathbf{Q}^T\mathbf{r} = \begin{bmatrix} r_1 \\ \mathbf{r}_M \end{bmatrix} \quad \& \quad \mathbf{Q}^T\mathbf{S} = \mathbf{Q}^T \left[\, \mathbf{s}_1 \quad \tilde{\mathbf{S}} \,\right] = \begin{bmatrix} c & \mathbf{s}_{M1}^T \\ \mathbf{0} & \tilde{\mathbf{S}}_M \end{bmatrix}$$

- Using the Householder transformation, we can write

$$\begin{aligned} \arg\min_{\mathbf{x}} \left\| \tilde{\mathbf{S}}\mathbf{x} - (\mathbf{r} - A_1 b_1 \mathbf{s}_1) \right\| &= \arg\min_{\mathbf{x}} \left\| \mathbf{S} \begin{bmatrix} A_1 b_1 \\ \mathbf{x} \end{bmatrix} - \mathbf{r} \right\| \\ &= \arg\min_{\mathbf{x}} \left\| \begin{bmatrix} c & \mathbf{s}_{M1}^T \\ \mathbf{0} & \tilde{\mathbf{S}}_M \end{bmatrix} \begin{bmatrix} A_1 b_1 \\ \mathbf{x} \end{bmatrix} - \begin{bmatrix} r_1 \\ \mathbf{r}_M \end{bmatrix} \right\| \\ &= \arg\min_{\mathbf{x}} \left\| \begin{bmatrix} A_1 b_1 \\ \mathbf{x} \end{bmatrix} - \begin{bmatrix} \frac{r_1}{c} - \frac{1}{c}\mathbf{s}_{M1}^T \tilde{\mathbf{S}}_M \mathbf{r}_M \\ \tilde{\mathbf{S}}_M^+ \mathbf{r}_M \end{bmatrix} \right\| \end{aligned}$$

so that $\hat{\mathbf{x}}_1 = \tilde{\mathbf{S}}_M^+ \mathbf{r}_M$ which is an estimate of $\tilde{\mathbf{A}}\tilde{\mathbf{b}}$.

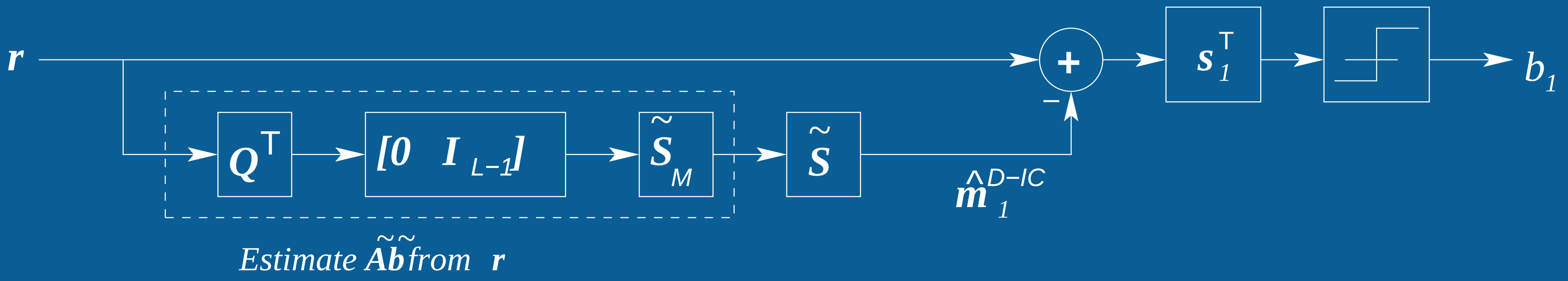
Detection of b_1

- Thus, $\hat{\mathbf{m}}_1^{D-IC} = \tilde{\mathbf{S}}\tilde{\mathbf{S}}_M^+ \mathbf{r}_M$.
- The decision for the data bit of user 1 is given by

$$
\begin{aligned}
\hat{b}_1^{D-IC} &= \operatorname{sgn}\left\{\mathbf{s}_1^T\left(\mathbf{r} - \hat{\mathbf{m}}_1^{D-IC}\right)\right\} \\
&= \operatorname{sgn}\left\{\mathbf{s}_1^T\left(\mathbf{I}_L - \tilde{\mathbf{S}}\tilde{\mathbf{S}}_M^+\left[\mathbf{0} \quad \mathbf{I}_{L-1}\right]\mathbf{Q}^T\right)\mathbf{r}\right\}
\end{aligned}
$$

- Can represent the decision for b_1 as the output of a linear filter: $\hat{b}_1 = \operatorname{sgn}\left\{\mathbf{w}_1^T\mathbf{r}\right\}$ where

$$
\mathbf{w}_1 = \left(\mathbf{I}_L - \mathbf{Q}\begin{bmatrix}\mathbf{0}^T \\ \mathbf{I}_{L-1}\end{bmatrix}\tilde{\mathbf{S}}_M^{+T}\tilde{\mathbf{S}}^T\right)\mathbf{s}_1
$$



MMSE Interference Cancellation (MMSE-IC)

- Minimize the MSE at the output of the MF for user 1 (i.e., input to the slicer)
- The MMSE estimate of the MAI, $\hat{\mathbf{m}}_1^{MMSE-IC}$ is

$$\hat{\mathbf{m}}_1^{MMSE-IC} = \arg\min_{\mathbf{m}} \mathsf{E}\left\{\left[\mathbf{s}_1^T\left(\mathbf{r} - A_1 b_1 \mathbf{s}_1 - \mathbf{m}\right)\right]^2\right\}$$

- Find MMSE result with (change of variables)

$$\hat{\mathbf{m}}_1^{MMSE-IC} = \mathbf{s}_1 \mathbf{w}_1^T \mathbf{r}$$

where
$$\mathbf{w}_1 = \arg\min_{\mathbf{w}} \mathsf{E}\left\{\left[\mathbf{s}_1^T\left(\mathbf{r} - A_1 b_1 \mathbf{s}_1\right) - \mathbf{w}^T \mathbf{r}\right]^2\right\}$$

- The optimal $\mathbf{w}_1$ is

$$\mathbf{w}_1 = \left(\mathbf{S}\mathbf{A}^2\mathbf{S}^T + \sigma^2 \mathbf{I}_L\right)^{-1}\left(\tilde{\mathbf{S}}\tilde{\mathbf{A}}^2\tilde{\mathbf{S}}^T + \sigma^2 \mathbf{I}_L\right)\mathbf{s}_1$$

so that

$$\hat{\mathbf{m}}_1^{MMSE-IC} = \left(\tilde{\mathbf{S}}\tilde{\mathbf{A}}^2\tilde{\mathbf{S}}^T + \sigma^2 \mathbf{I}_L\right)\left(\mathbf{S}\mathbf{A}^2\mathbf{S}^T + \sigma^2 \mathbf{I}_L\right)^{-1}\mathbf{r}$$

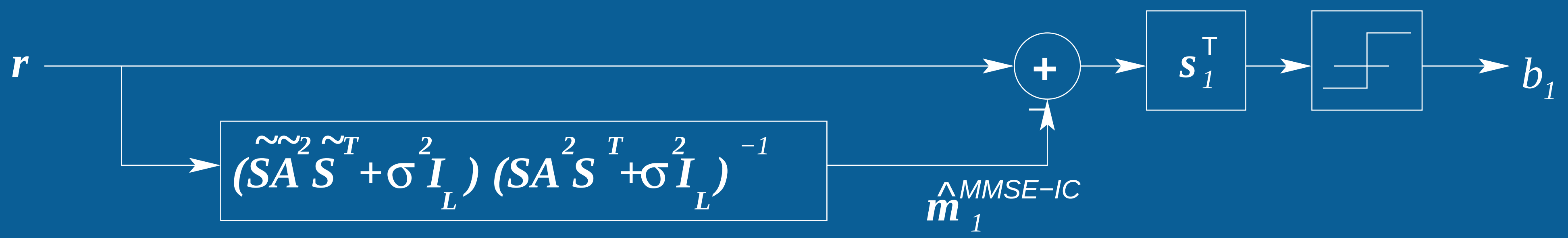
MMSE-IC (cont)

- The decision for user 1's data bit is

$$\begin{aligned}\hat{b}_1 &= \text{sgn}\left\{\mathbf{s}_1^T\left(\mathbf{r} - \hat{\mathbf{m}}_1^{MMSE-IC}\right)\right\} \\ &= \text{sgn}\left\{\mathbf{s}_1^T\left[\mathbf{I}_L - \left(\tilde{\mathbf{S}}\tilde{\mathbf{A}}^2\tilde{\mathbf{S}}^T + \sigma^2\mathbf{I}_L\right)\left(\mathbf{S}\mathbf{A}^2\mathbf{S}^T + \sigma^2\mathbf{I}_L\right)^{-1}\right]\mathbf{r}\right\}\end{aligned}$$

and the linear filter representation is

$$\mathbf{w}_1^{MMSE-IC} = \left[\mathbf{I}_L - \left(\mathbf{S}\mathbf{A}^2\mathbf{S}^T + \sigma^2\mathbf{I}_L\right)^{-1}\left(\tilde{\mathbf{S}}\tilde{\mathbf{A}}^2\tilde{\mathbf{S}}^T + \sigma^2\mathbf{I}_L\right)\right]\mathbf{s}_1$$



Maximum Asymptotic Multiuser Efficiency IC (MAME-IC)

- Let $\hat{\mathbf{m}}_1^{MAME-IC} = \mathbf{s}_1 \mathbf{w}_1^T \mathbf{r}$.
- The error probability for user 1 is

$$P_{e1} = \mathsf{E}\left[Q\left(\frac{A_1 b_1 + \sum_{k=2}^{K} A_k b_k \rho_{1k} - \sum_{k=1}^{K} A_k b_k \mathbf{w}_1^T \mathbf{s}_k}{\sigma \|\mathbf{s}_1 - \mathbf{w}_1\|_2} \right) \right]$$

- The asymptotic multiuser efficiency (AME) of user 1 is

$$\eta_1(\mathbf{w}_1) = \max{}^2 \left\{ 0 \, , \; \frac{1 - \mathbf{s}_1^T \mathbf{w}_1 - \sum_{k=2}^{K} \frac{A_k}{A_1} \left| \rho_{1k} - \mathbf{s}_k^T \mathbf{w}_1 \right|}{\|\mathbf{s}_1 - \mathbf{w}_1\|_2} \right\}$$

- Choose $\mathbf{w}_1$ to maximize the AME.

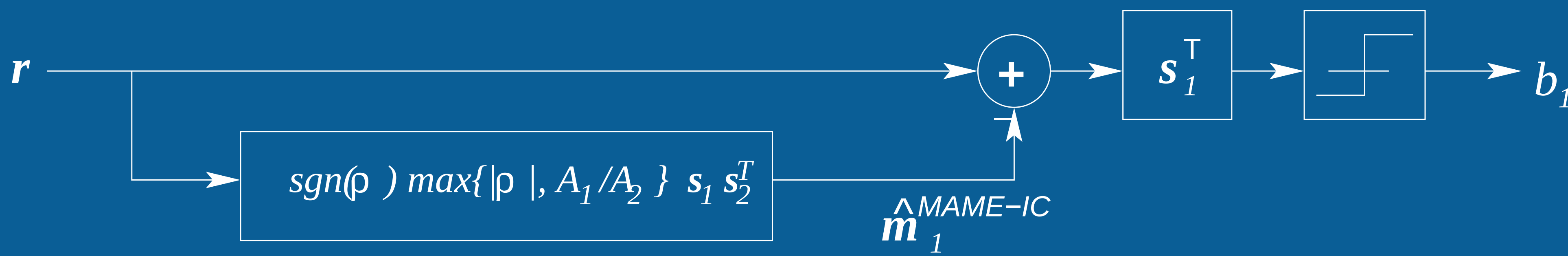
MAME-IC Solution

- Solving for $\mathbf{w}_1$ does not permit a close-form solution for $K > 2$.
- For $K = 2$, the $\mathbf{w}_1$ that maximizes AME is

$$\begin{aligned} \mathbf{w}_1 &= \begin{cases} \frac{A_1}{A_2}\text{sgn}(\rho)\mathbf{s}_2, & \text{if } \frac{A_2}{A_1} < |\rho|, \\ \rho\mathbf{s}_2, & \text{otherwise.} \end{cases} \\ &= \text{sgn}(\rho)\max\left\{|\rho|, \frac{A_1}{A_2}\right\}\mathbf{s}_2 \end{aligned}$$

- Thus, the MAME estimate of the MAI is

$$\hat{\mathbf{m}}_1^{MAME-IC} = \text{sgn}(\rho)\max\left\{|\rho|, \frac{A_1}{A_2}\right\}\mathbf{s}_1\mathbf{s}_2^T\mathbf{r}$$



Equivalence: D-IC and DD

- Consider the transformed received signal vector

$$\mathbf{Q}^T\mathbf{r} = \begin{bmatrix} s_{11} & \mathbf{s}_{M1}^T \\ \mathbf{0} & \tilde{\mathbf{S}}_M \end{bmatrix} \mathbf{A}\mathbf{b} + \mathbf{Q}^T\mathbf{n}$$

- The decorrelating detector (DD) provides the minimum norm estimate of $\mathbf{A}\mathbf{b}$ as

$$\mathbf{A}\mathbf{b} = \begin{bmatrix} s_{11} & \mathbf{s}_{M1}^T \\ \mathbf{0} & \tilde{\mathbf{S}}_M \end{bmatrix}^+ \mathbf{Q}^T\mathbf{r} = \mathbf{S}^+\mathbf{Q}^{-T}\mathbf{Q}^T\mathbf{r} = \mathbf{S}^+\mathbf{r}$$

- The DD decision rule is

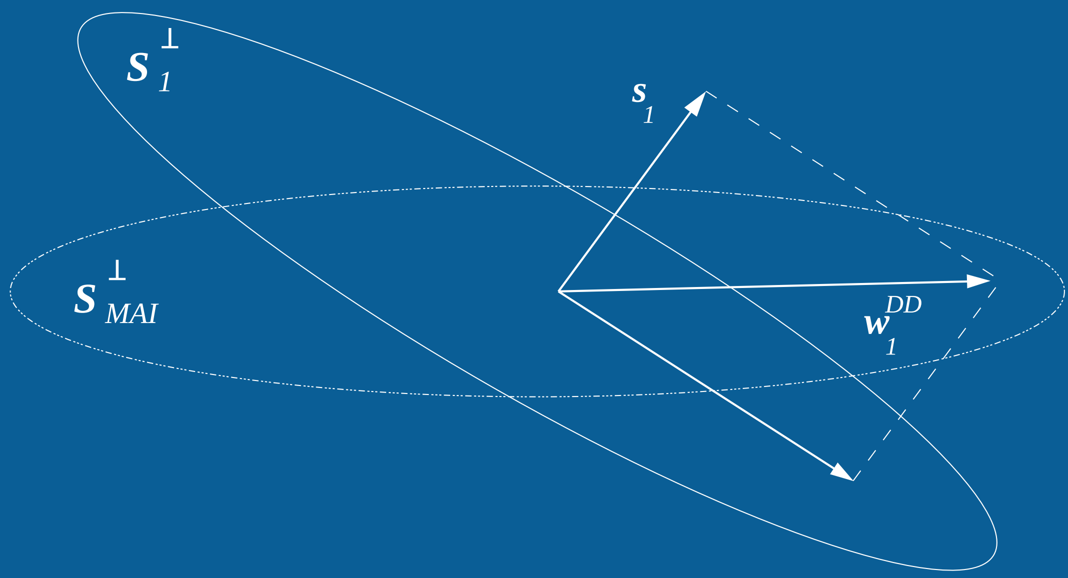
$$\hat{b}_1^{DD} = \text{sgn}\left\{\left[\, s_{11}^{-1} \quad -s_{11}^{-1}\mathbf{s}_{M1}^T\tilde{\mathbf{S}}_M^+ \,\right]\mathbf{Q}^T\mathbf{r}\right\}$$

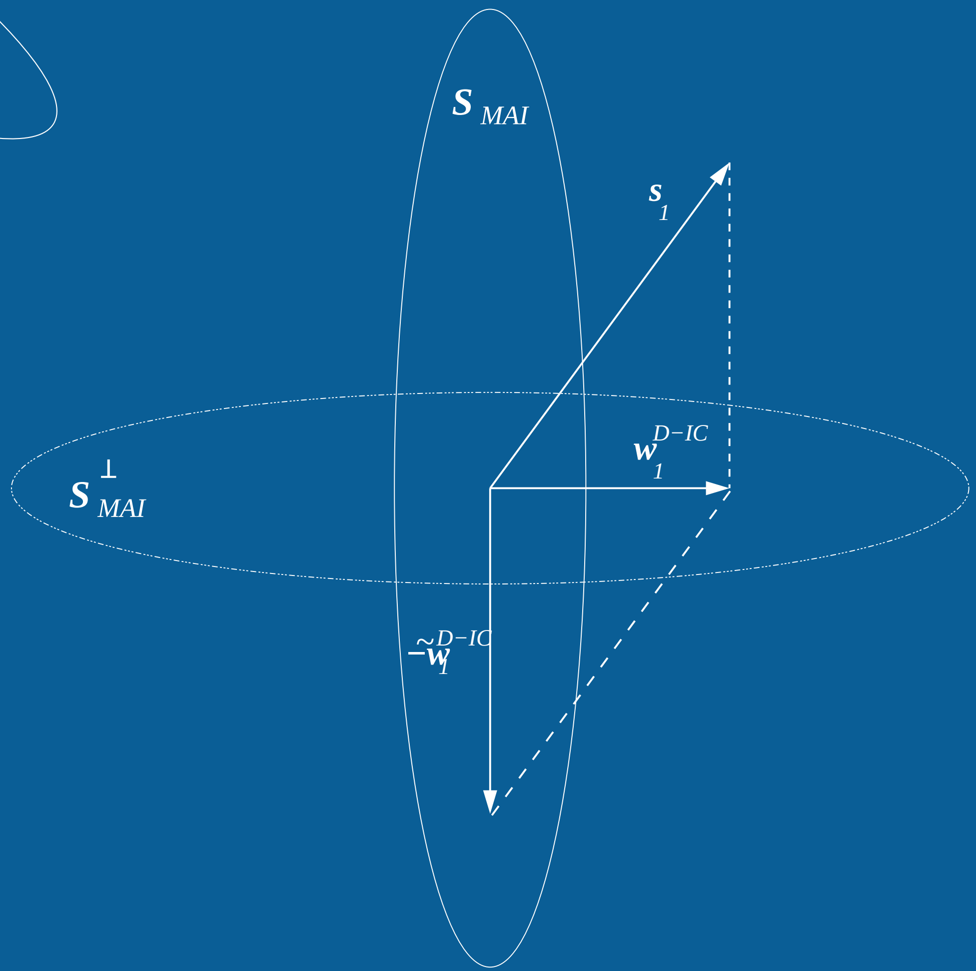
- The D-IC decision rule can be written as

$$\begin{aligned} \hat{b}_1^{D-IC} &= \operatorname{sgn}\left\{\mathbf{s}_1^T\left(\mathbf{I}_L - \tilde{\mathbf{S}}\tilde{\mathbf{S}}_M^+\left[\,\mathbf{0} \quad \mathbf{I}_{L-1}\,\right]\mathbf{Q}^T\right)\mathbf{r}\right\} \\ &= \text{sgn}\left\{\left[\, s_{11} \quad -s_{11}\mathbf{s}_{M1}^T\tilde{\mathbf{S}}_M^+ \,\right]\mathbf{Q}^T\mathbf{r}\right\} \end{aligned}$$

- Hence, $\mathbf{w}_1^{D-IC} = s_{11}^2\mathbf{w}_1^{DD}$.

Geometric Interpretation





Equivalences

- MMSE-IC and Linear MMSE
  - ⋆ Linear MMSE can be written as

$$\mathbf{w}_1^{MMSE} = \left(\mathbf{S}\mathbf{A}^2\mathbf{S}^T + \sigma^2\mathbf{I}_L\right)^{-1}\mathbf{s}_1$$

  - ⋆ MMSE-IC can be simplified to

$$\mathbf{w}_1^{MMSE-IC} = A_1^2\left(\mathbf{S}\mathbf{A}^2\mathbf{S}^T + \sigma^2\mathbf{I}_L\right)^{-1}\mathbf{s}_1$$

- MAME-IC and Linear MAME
  - ⋆ The decision variable for the MAME-IC can be written as

$$\hat{b}_1^{MAME-IC} = \text{sgn}\left\{\mathbf{s}_1^T\mathbf{r} - \mathbf{w}_1^T\mathbf{r}\right\}$$

which results in the over linear filter operation

$$\mathbf{w}_1^{MAME-IC} = \begin{cases} \mathbf{s}_1 - \frac{A_1}{A_2}\text{sgn}\{\rho\}\mathbf{s}_2 & \text{if } \frac{A_2}{A_1} < |\rho| \\ \mathbf{s}_1 - \rho\mathbf{s}_2, & \text{otherwise} \end{cases}$$

which is the same as the linear MAME.

Conclusions

- Considered a framework for interference cancellation in which MAI is removed before MF detection of user 1.
- Can be extended to other users.
- Interference cancelers could be written as a linear filtering operation.
- Proposed IC's (D-IC, MMSE-IC and MAME-IC) were shown to be equivalent to their linear multiuser detector counterparts (DD, MMSE, MAME).
- IC's essentially a form of re-formulating the linear multiuser detectors.