Semi-Blind Decorrelating Multiuser Detection for Synchronous CDMA

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Introduction

- CDMA techniques have attracted attention for their spectral efficiency, interference resistance and flexible traffic control.

- MAI is the dominant impairment for CDMA systems and exits even with perfect power control.

- Multiuser detection promising approach to mitigate MAI: classic/non-blind multiuser detection and blind multiuser detection

- With exploring a new data model and the trade-off between classic multiuser detection and blind multiuser detection, we propose semi-blind decorrelating multiuser detection.

  ⋆ Least squares semi-blind decorrelating multiuser detection (LS-DD)
  ⋆ Total least squares semi-blind decorrelating multiuser detection (TLS-DD)
  ⋆ Mixed LS/TLS semi-blind decorrelating multiuser detection (MLS-DD)

Data Model

- Consider a synchronous CDMA system with L chips per bit ($T_b = LT_c$).
- Received signal, after chip-matched filter followed by chip-rate sampling:

$$\mathbf{r} = \sum_{k=1}^{K} A_k b_k \mathbf{s}_k + \mathbf{n} \;=\; \mathbf{SAb} + \mathbf{n}$$

  where $\mathbf{A} = \text{diag}\{A_1\ A_2\ \ldots\ A_K\}$, $\mathbf{S} = [\mathbf{s}_1\ \mathbf{s}_2\ \ldots\ \mathbf{s}_K]$ and $\mathbf{b} = [b_1\ b_2\ \ldots\ b_K]^T$ with $b_i \in \{+1, -1\}$.
- Linear multiuser detectors can be written in the form

$$\mathbf{b} = \text{sgn}\{\mathbf{Wr}\}$$

  or for the kth user $b_k = \text{sgn}\left\{\mathbf{w}_k^T \mathbf{r}\right\}$ where $\mathbf{w}_k^T$ is the kth row of $\mathbf{W}$.
- We consider detection of user 1's data only.
- Maintain the restriction that $L > K$.

New Data Model w.r.t. User 1 – I

- In terms of user 1, a new semi-blind signature matrix $\boldsymbol{\mathcal{S}}$ is constructed by

$$\begin{aligned}\boldsymbol{\mathcal{S}} &= [\,A_1\mathbf{s}_1 \quad \mathbf{r}_1 \quad \mathbf{r}_2 \quad \ldots \quad \mathbf{r}_{M-1}\,] \\ &= \mathbf{SA}\,[\,\mathbf{e}_1 \quad \mathbf{D}\,] + \mathbf{N} \\ &= \mathbf{SAB} + \mathbf{N}\end{aligned}$$

where $\mathbf{B} = \begin{bmatrix} 1 & \bar{\mathbf{d}}^T \\ \mathbf{0} & \tilde{\mathbf{D}} \end{bmatrix} = \begin{bmatrix} & \mathbf{c}^T \\ \mathbf{0} & \tilde{\mathbf{D}} \end{bmatrix}$.

- The relationship between $\mathbf{r}$ and $\boldsymbol{\mathcal{S}}$ is

$$\mathbf{r} = \boldsymbol{\mathcal{S}}\mathbf{d} + \mathbf{z}$$

where $\mathbf{d}$ is named the detection vector.

$$\mathbf{d} = \mathbf{B}^{+}\mathbf{b} = \begin{bmatrix} 1 & \bar{\mathbf{d}}^T \\ \mathbf{0} & \tilde{\mathbf{D}} \end{bmatrix}^{+} \begin{bmatrix} b_1 \\ \tilde{\mathbf{b}} \end{bmatrix}$$

New Data Model w.r.t. User 1 – II

- $\mathbf{N}$ is a noise matrix of the form, $\mathbf{N} = [\mathbf{0}\ \tilde{\mathbf{N}}]$. $\mathbf{z}$ is the new noise vector defined as $\mathbf{z} = \mathbf{n} - \mathbf{N}\mathbf{B}^{+}\mathbf{b}$.

- The bit sent by the first user, b_1, during the time interval $t \in [(n-1)T,\ nT]$ can be estimated as

$$b_1 = \mathbf{c}^T \mathbf{d}$$

where $\mathbf{c} = [\,1 \quad \bar{\mathbf{d}}^T\,]^T$ and $\mathbf{d} = [\,d_1 \quad \tilde{\mathbf{d}}\,]^T$.

- Now, the semi-blind multiuser detection problem becomes how to estimate $\mathbf{d}$.

Least-Square Semi-blind Multiuser Detection (LS-DD)

- The LS estimation problem is

$$\mathbf{d}_{\mathrm{LS}} = \min_{\mathbf{x}} \|\boldsymbol{\mathcal{S}}\mathbf{x} - \mathbf{r}\|_2$$

- The LS solution is

$$\mathbf{d}_{\mathrm{LS}} = \boldsymbol{\mathcal{S}}^{+}\mathbf{r}$$

  and the bit sent by the first user, b_1, in the nth signaling interval is detected with

$$\begin{aligned} \hat{b}_1^{\mathrm{LS}} &= \mathrm{sgn}\{\mathbf{w}_{\mathrm{LS}}^T \mathbf{r}\} \\ &= \mathrm{sgn}\left\{b_1 + \mathbf{c}^T \boldsymbol{\mathcal{S}}^{+} \tilde{\mathbf{n}}\right\} \quad . \end{aligned}$$

- In LS estimation, it assumes that $\boldsymbol{\mathcal{S}}$ is error-free. However, this assumption is not entirely accurate according since there is a noise term, $\mathbf{N}$.

Total Least-Square Semi-blind Multiuser Detection (TLS-DD)

- $\mathbf{r}$ can also be expressed as $\mathbf{r} = \hat{\boldsymbol{\mathcal{S}}}\mathbf{d} + \mathbf{n}$, where $\hat{\boldsymbol{\mathcal{S}}} = \boldsymbol{\mathcal{S}} - \mathbf{N} = \mathbf{SAB}$.
- The LS problem can then be transformed into the following TLS problem:

$$\mathbf{d}_{\mathrm{TLS}} = \min_{\bar{\boldsymbol{\mathcal{S}}},\ \mathbf{x}} \left\| [\,\boldsymbol{\mathcal{S}} \quad \mathbf{r}\,] - [\,\bar{\boldsymbol{\mathcal{S}}} \quad \bar{\boldsymbol{\mathcal{S}}}\mathbf{x}\,] \right\|_2 .$$

- The TLS solution is

$$\mathbf{d}_{\mathrm{TLS}} = \left(\boldsymbol{\mathcal{S}}^T\boldsymbol{\mathcal{S}} - \sigma_{K+1}^2\mathbf{I}\right)^{-1}\boldsymbol{\mathcal{S}}^T\mathbf{r}$$

and the nth bit sent by the first user, b_1, can be detected with

$$\begin{aligned}\hat{b}_1^{\mathrm{TLS}} &= \mathrm{sgn}\{\mathbf{w}_{\mathrm{TLS}}^T\mathbf{r}\} \\ &= \mathrm{sgn}\left\{\mathbf{c}^T\left(\boldsymbol{\mathcal{S}}^T\boldsymbol{\mathcal{S}} - \sigma_{K+1}^2\mathbf{I}\right)^{-1}\boldsymbol{\mathcal{S}}^T\mathbf{r}\right\} .\end{aligned}$$

Mixed LS/TLS Semi-blind Multiuser Detection (MLS-DD) I

- In the LS problem, it assumed the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$ is error-free. Again, this assumption is not completely accurate.

- In the TLS problem, it assumed that in each column of the semi-blind signature matrix, $\boldsymbol{\mathcal{S}}$, some noise or error exists. This assumption also is not complete.

- Hence, to maximize the estimation accuracy of the detection vector $\mathbf{d}$, it is natural to require that the corresponding columns of $\boldsymbol{\mathcal{S}}$ be unperturbed since they are known exactly. The estimation problem becomes the following MLS problem.

$$\mathbf{d}_{\mathrm{MLS}} = \min_{\bar{\boldsymbol{\mathcal{S}}},\ \mathbf{x}} \| [\, \tilde{\boldsymbol{\mathcal{S}}} \quad \mathbf{r}\,] - [\, \bar{\boldsymbol{\mathcal{S}}} \quad [A_1 \mathbf{s}_1 \ \bar{\boldsymbol{\mathcal{S}}}]\mathbf{x}\,] \|_2$$

Mixed LS/TLS Semi-blind Multiuser Detection (MLS-DD) II

- The MLS solution is given by

$$\mathbf{d}_{\mathrm{MLS}} = \left(\boldsymbol{\mathcal{S}}^T\boldsymbol{\mathcal{S}} - \sigma^2 \begin{bmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{M-1} \end{bmatrix}\right)^{-1} \boldsymbol{\mathcal{S}}^T\mathbf{r} \ .$$

and the bit sent by the first user, b_1, can be detected with

$$\begin{aligned} \hat{b}_1^{\mathrm{MLS}} &= \mathsf{sgn}\{\mathbf{w}_{\mathrm{MLS}}^T\mathbf{r}\} \\ &= \mathsf{sgn}\left\{\mathbf{c}^T\left(\boldsymbol{\mathcal{S}}^T\boldsymbol{\mathcal{S}} - \sigma^2 \begin{bmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{M-1} \end{bmatrix}\right)^{-1} \boldsymbol{\mathcal{S}}^T\mathbf{r}\right\}. \end{aligned}$$

Performance Analysis I

- When $\mathbf{N} = \mathbf{0}$, the proposed LS-DD and the classic decorrelating detector share the same signature space and

$$\mathbf{w}_{\mathrm{LS}} = A_k^{-1}\mathbf{w}_{\mathrm{DD}} \quad .$$

  Hence, they share the same performance for user 1, too.

- It is straightforward to develop the following relationship between the estimation errors of b_1 and $\mathbf{d}$:

$$\Delta b_1 \leq \|\Delta \mathbf{d}\|_1$$

  where $\Delta b_1 = \hat{b}_1 - b_1$, $\Delta \mathbf{d} = \hat{\mathbf{d}} - \mathbf{d}$ and $\|\mathbf{m}\|_1$ denotes the 1-norm of the vector $\mathbf{m}$.

Performance Analysis II

- The mean of the semi-blind noise item $\mathbf{z}$ is

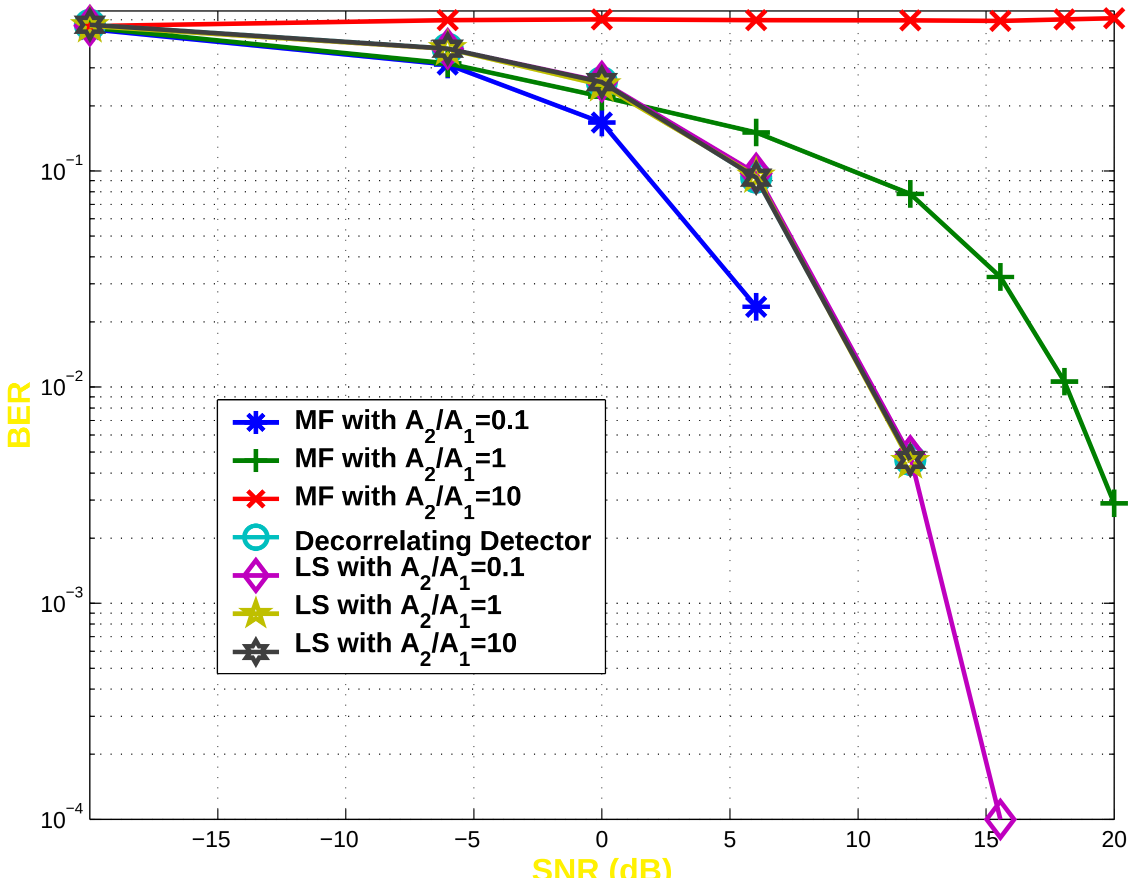
$$\mu = E\{\mathbf{z}\} = 0 \; .$$

- The variance of the semi-blind noise item $\mathbf{z}$ satisfies the following inequality

$$\begin{aligned} \max\{var\{\mathbf{z}\}\} &= \max\{E\{(\mathbf{z}-\mu)^2\}\} \\ &\leq \sigma^2 + (K-1)\|\tilde{\mathbf{D}}^{+}\|_2^2 \sigma_N^2 \end{aligned}$$

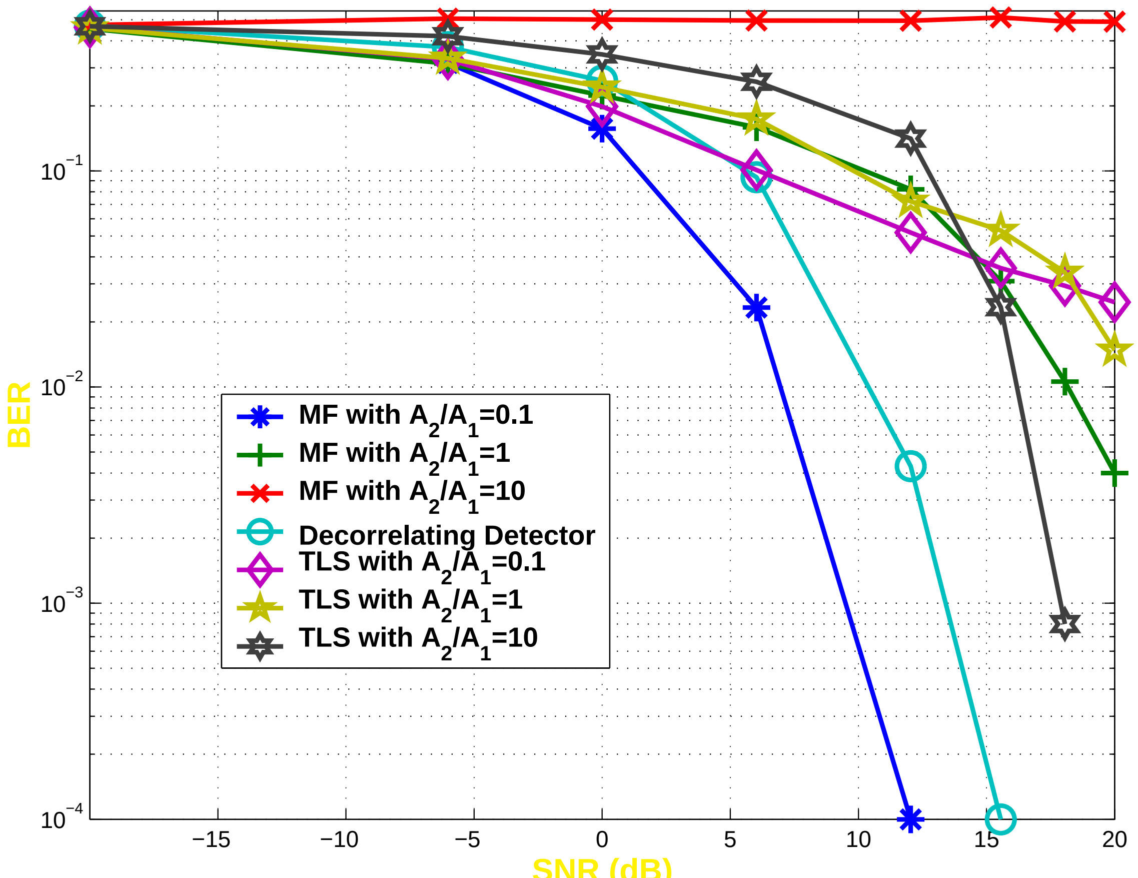
where $\max\{\mathbf{m}\}$ denotes the maximum item in the vector $\mathbf{m}$ and σ_N^2 is the power of the noise item $\mathbf{N}$ in the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$.

Computer Simulations I



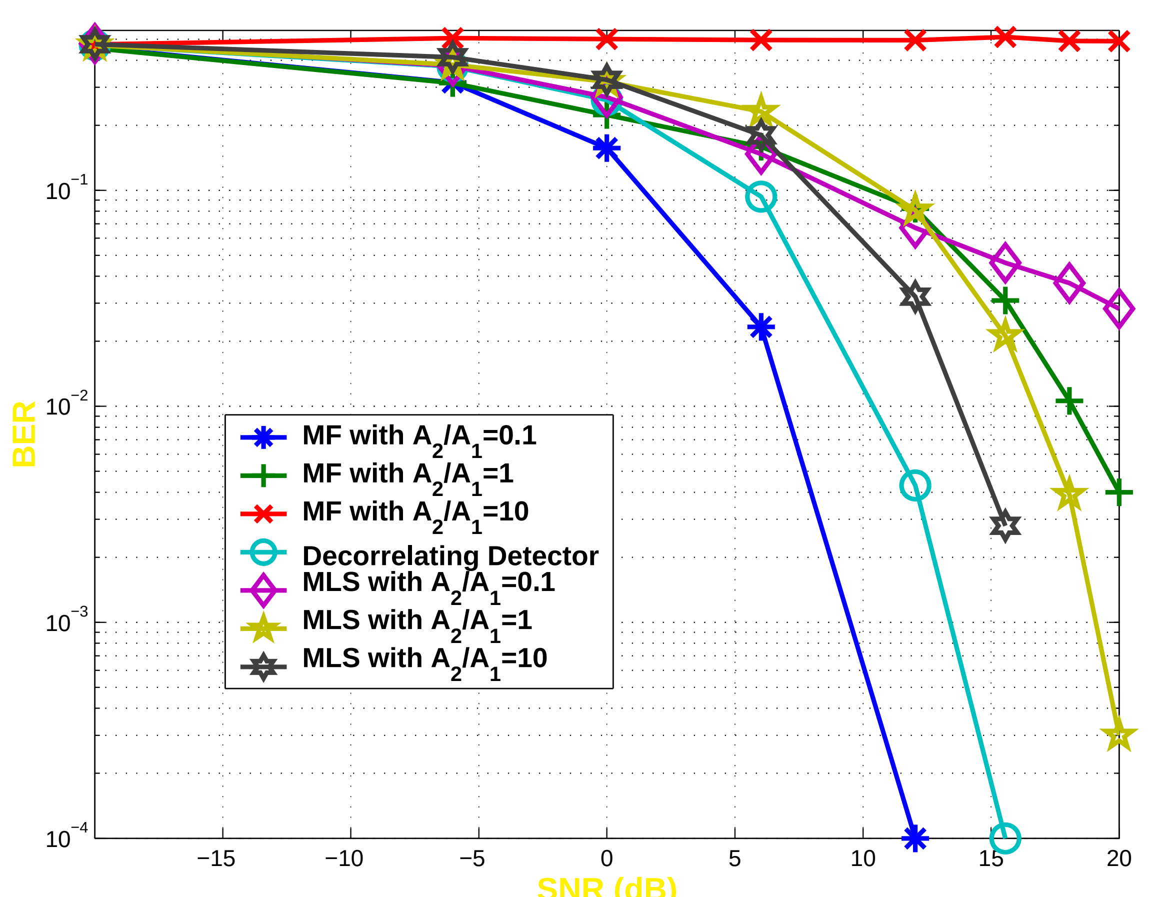
BER comparison of the single-user matched filter, decorrelating detector, and the LS semi-blind detector for the first user in a two user system with $\rho = 0.75$.

Computer Simulations II



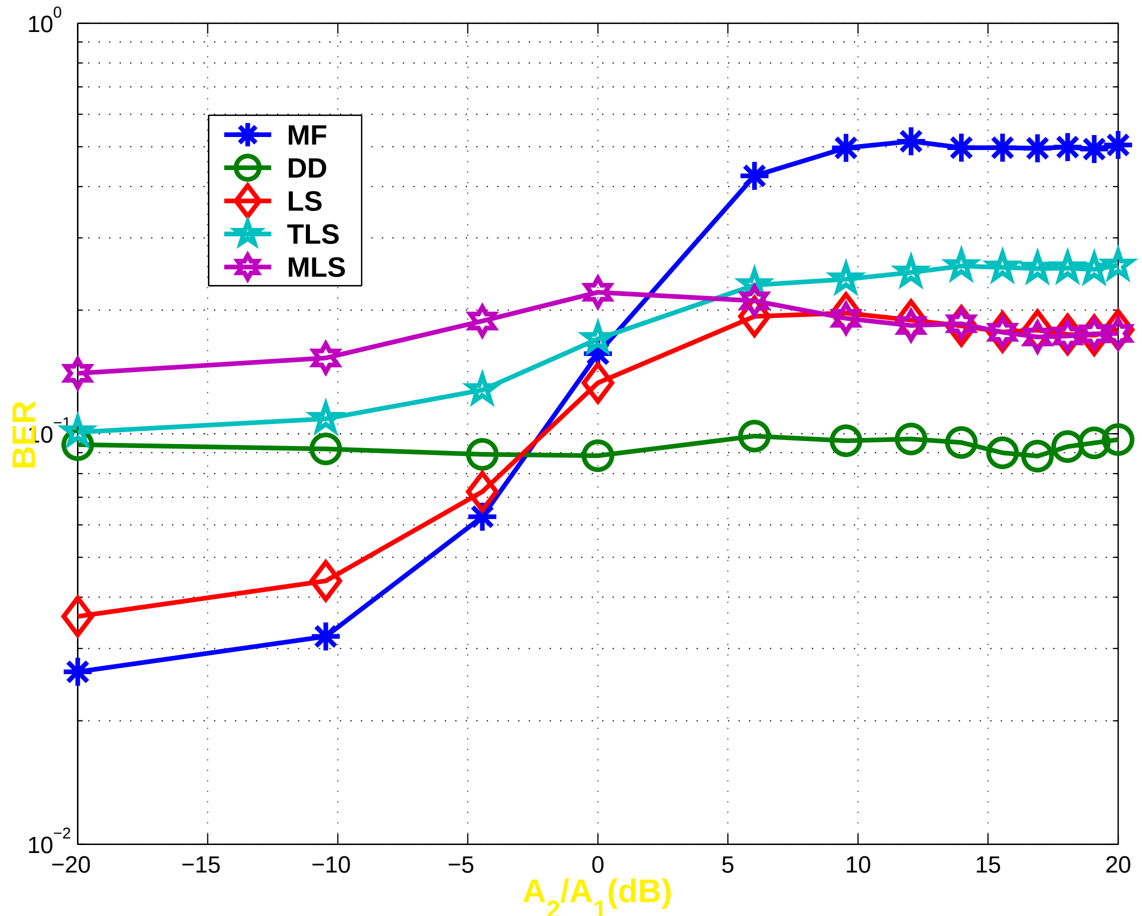
BER comparison of the single-user matched filter, decorrelating detector, and the TLS semi-blind detector for the first user in a two user system with $\rho = 0.75$.

Computer Simulations III



BER comparison of the single-user matched filter, decorrelating detector, and the MLS semi-blind detector for the first user in a two user system with $\rho = 0.75$.

Computer Simulations IV



Near-far resistance comparison of the single-user matched filter, decorrelating detector, and the LS, TLS and MLS semi-blind detectors for the first user in a two user system with $\rho = 0.75$ and $SNR = 6dB$.

Conclusion & Future Directions

- Multiuser precoding provides another alternative to solve the near-far problem in multiuser systems.
- With linear system theory, it can be shown that many linear multiuser precoders and detectors can achieve the same performance.
- With considering the input information bits, nonlinear multiuser precoding can be developed to further enhance system performance.
- Additional performance enhancement may be achievable by jointly optimizing transmitter multiuser precoding and adaptive receivers/transmitters.
- For unknown or time-variable channels, adaptive multiuser precoding with feedback from receivers amd multiuser precoding with channel coding can be interesting topics.