Multiuser Receiver Design Maximizing Mutual Information Rate

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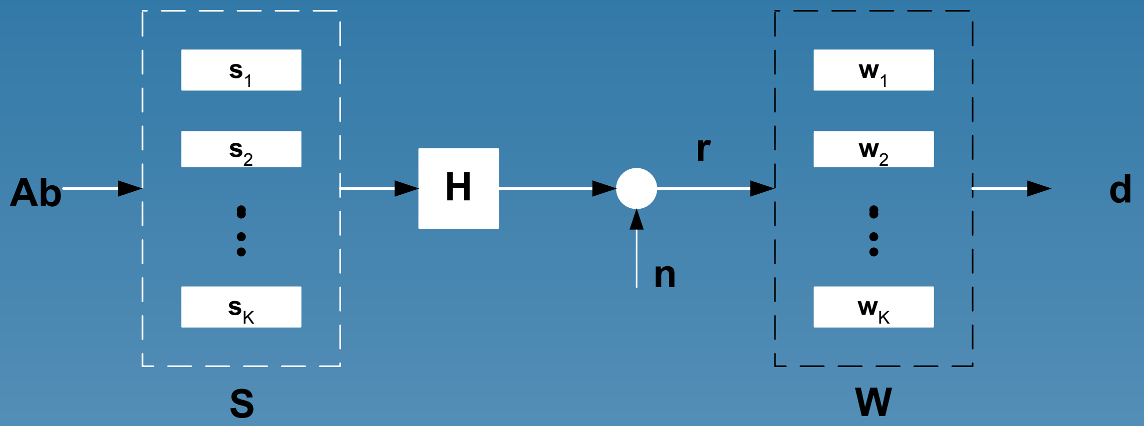
Multi-Access Interference (MAI)

- In TDMA systems (GSM and IS136), MAI comes from overlapping slots originating in other cells and from users in the same cell due to channel dispersion.
- In orthogonal CDMA systems (IS95 and CDMA2000), the orthogonality can not be guaranteed at each receiver once they go through the channel. Especially in asynchronous case. MAI is hard to be avoided even in the absence of channel distortion or other-cell interference.
- In FDMA systems (AMPS), extra guard bands have to be used to separate different users.
- In some other MIMO system, such as OFDM and DMT, MAI can appear as ISI or cross talk.

Multiuser Detection

- Multiuser detection refers to data detection in non-orthogonal multiplexes.
- The MAI has considerable structure and much less randomness than the white Gaussian background noise.
- By exploiting this structure, multiuser detection can increase spectral efficiency, receiver sensitivity and the number of substantiable users. It can also reduce the reliance on tight and accurate power control.
- Multiuser detection comes in various flavors.
  - ⋆ Optimal: jointly and individually optimal multiuser detection.
  - ⋆ Linear: decorrelating detection, MMSE detection and MAME detection.
  - ⋆ Nonlinear: hard interference cancellations.

Multiuser Channel Model



where $\mathbf{r} = \mathbf{HSAb} + \mathbf{n}$ and $\mathbf{d} = \mathbf{W}^T\mathbf{HSAb} + \mathbf{W}^T\mathbf{n}$. For the sake of simplification, $\mathbf{H}$ is set to be $\mathbf{I}$ in the following.

Multiuser Receiver Design Criteria I

- Minimum bit-error rate (BER). Individually and jointly optimum multiuser receivers.

$$\mathbf{w}_k^{IO} = \text{arg}\min_{\mathbf{w}} P_e^{IO} \text{ and } \mathbf{W}^{JO} = \text{arg}\min_{\mathbf{W}} P_e^{JO}$$

- Decorrelating detector (DD), which minimizes MAI.

$$\mathbf{W}^{DD} = \mathbf{S}^{+}$$

- Minimum mean-square errors (MMSE) receiver.

$$\mathbf{W}^{MMSE} = \mathbf{S}(\mathbf{S}^T\mathbf{S} + \sigma^2\mathbf{A}^{-2})^{-1}$$

Multiuser Receiver Design Criteria II

- Maximum asymptotical multiuser efficiency (MAME) receiver.

$$\mathbf{w}_k^{MAME} = \text{arg}\max_{\mathbf{w}} \eta_k$$

- Decision-driven multiuser detector. (hard interference cancellation)

$$\hat{b}_k = \text{sgn}\left\{\mathbf{s}_k^T(\mathbf{r} - \text{Est. MAI})\right\}$$

- Maximum mutual information rate (MMIR) receiver. (my topic)

$$\mathbf{W}^{MMIR} = \text{arg}\max_{\mathbf{W}} \mathbf{I}(\mathbf{b};\ \mathbf{d})$$

Mutual Information Rate I

The mutual information per input symbol between any input block $\mathbf{b}$ of K users/channels and the corresponding output block $\mathbf{d}$ of M receivers is maximized when the additive noise $\mathbf{m}$ is Gaussian and is given by (N. Al-Dhahir and J. M. Cioffi, 1996)

$$\mathbf{I}(\mathbf{b};\ \mathbf{d}) \quad = \quad \frac{1}{K}\log_2\left|(\mathbf{R}_b^+ + \mathbf{G}\mathbf{R}_m^{-1}\mathbf{G}^H)\mathbf{R}_b\right|$$

- $\mathbf{d} = \mathbf{G}^H\mathbf{b} + \mathbf{m}$ and $\mathbf{b}$ are the output/input of a MIMO system.
- $\mathbf{b}$ and $\mathbf{m}$ are zero-mean independent vectors with covariance matrices $\mathbf{R}_b$ and $\mathbf{R}_m$. And $\mathbf{m}$ is additive Gaussian noise.

Mutual Information Rate II

- It shows that $\mathbf{I}(\mathbf{b};\ \mathbf{d})$ is maximized when the eigenvectors of $\mathbf{R}_b$ are matched to those of $\mathbf{G}\mathbf{R}_m^{-1}\mathbf{G}^H$. (Hadamard inequality)

- The choice of $\mathbf{G}$ effects the mutual information rate of the blocks though MIMO channel.

- The choice of $\mathbf{G}$ is not unique. When $\mathbf{m} = \mathbf{G}^H\mathbf{n}$, $\mathbf{G}$ can be written as $\mathbf{UF}$, where $\mathbf{n}$ is AWGN vector, $\mathbf{U}$ is the eigenvector matrix of $\mathbf{R}_b$ and $\mathbf{F}$ can be any compatible and invertible matrix.

MMIR Multiuser Receiver

- $\mathbf{W}^{MMIR} = \mathbf{UF}$, where $\mathbf{U}$ is the eigenvector matrix of $\mathbf{SS}^T$ and $\mathbf{F}$ is any invertible $L \times K$ matrix.
- The final output is $\hat{\mathbf{b}} = \text{sgn}\{\mathbf{F}^T\mathbf{U}^T\mathbf{SAb} + \mathbf{F}^T\mathbf{U}^T\mathbf{n}\}$
- The maximization of multiuser channel capacity can't guarantee the minimization of final BERs.
- The choice of $\mathbf{F}$ can greatly effect the performance of MMIR multiuser receivers.

The Choice of $\mathbf{F}$

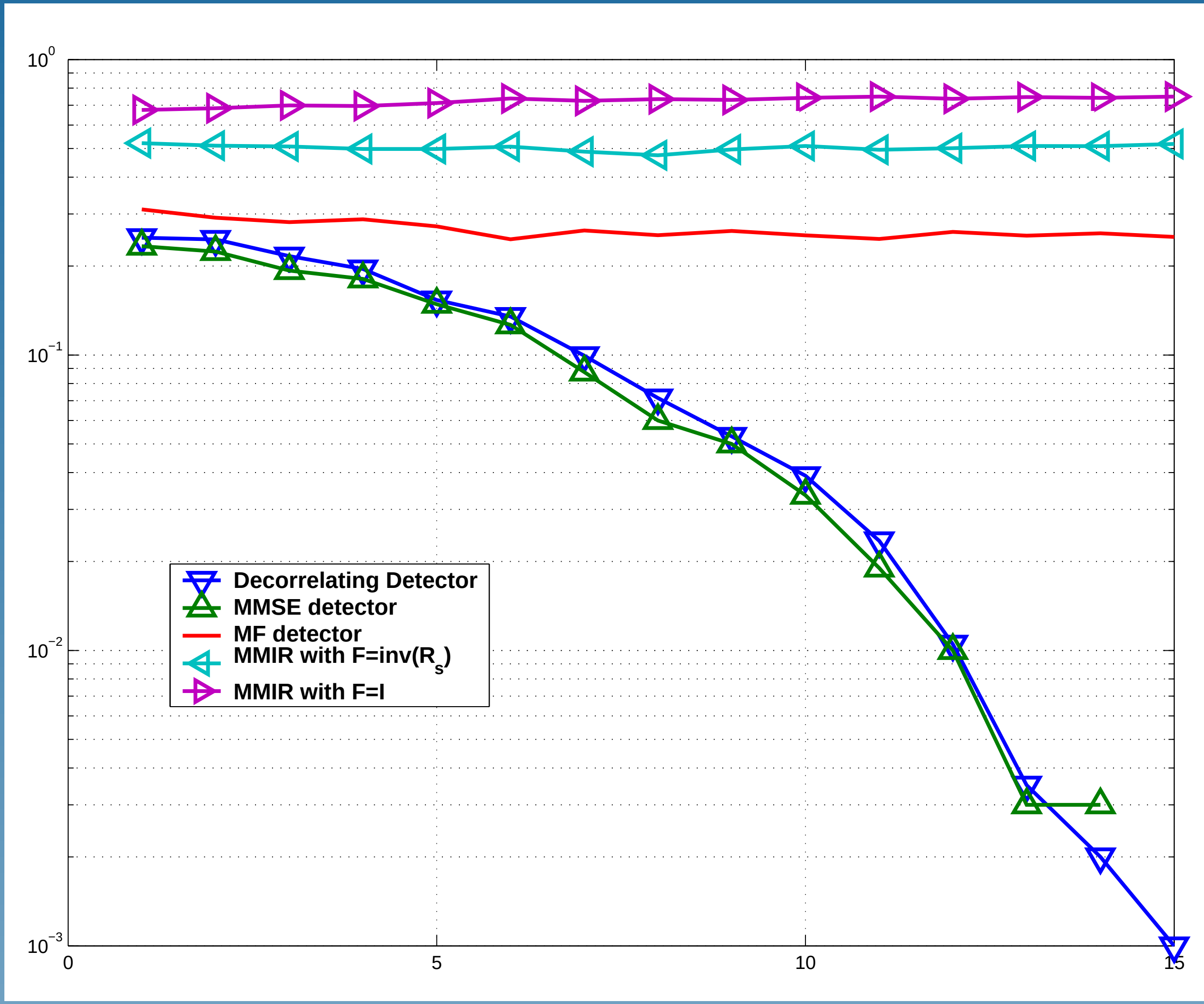
- When $\mathbf{F} = \mathbf{E}$, where $\mathbf{E} = \mathbf{U}^T\mathbf{S}$, $\mathbf{W}^{MMIR}$ is equal to $\mathbf{S}$ and single-user matched filter is one of the solutions to maximize $\mathbf{I}(\mathbf{b};\ \mathbf{d})$.

- When $\mathbf{F} = \mathbf{E}\mathbf{R}_s^{-1} = \mathbf{R}_s^{-1/2}$, where $\mathbf{R}_s = \mathbf{S}^T\mathbf{S}$, $\mathbf{W}^{MMIR}$ is equal to $\mathbf{W}^{DD}$ and decorrelating detector is one of the solutions to maximize $\mathbf{I}(\mathbf{b};\ \mathbf{d})$, too.

- When $\mathbf{F} = \mathbf{E}(\mathbf{R}_s + \sigma^2\mathbf{I})^{-1}$, $\mathbf{W}^{MMIR}$ becomes $\mathbf{W}^{MMSE}$ and MMSE detector also is one of the solutions to maximize $\mathbf{I}(\mathbf{b};\ \mathbf{d})$.

- What about other possible choices?

Computer Simulations



Blind MMIR Multiuser Receiver

- Autocorrelation matrix of the receiver signal $\mathbf{r}$ is given by (X. Wang and H. V. Poor, 1998)

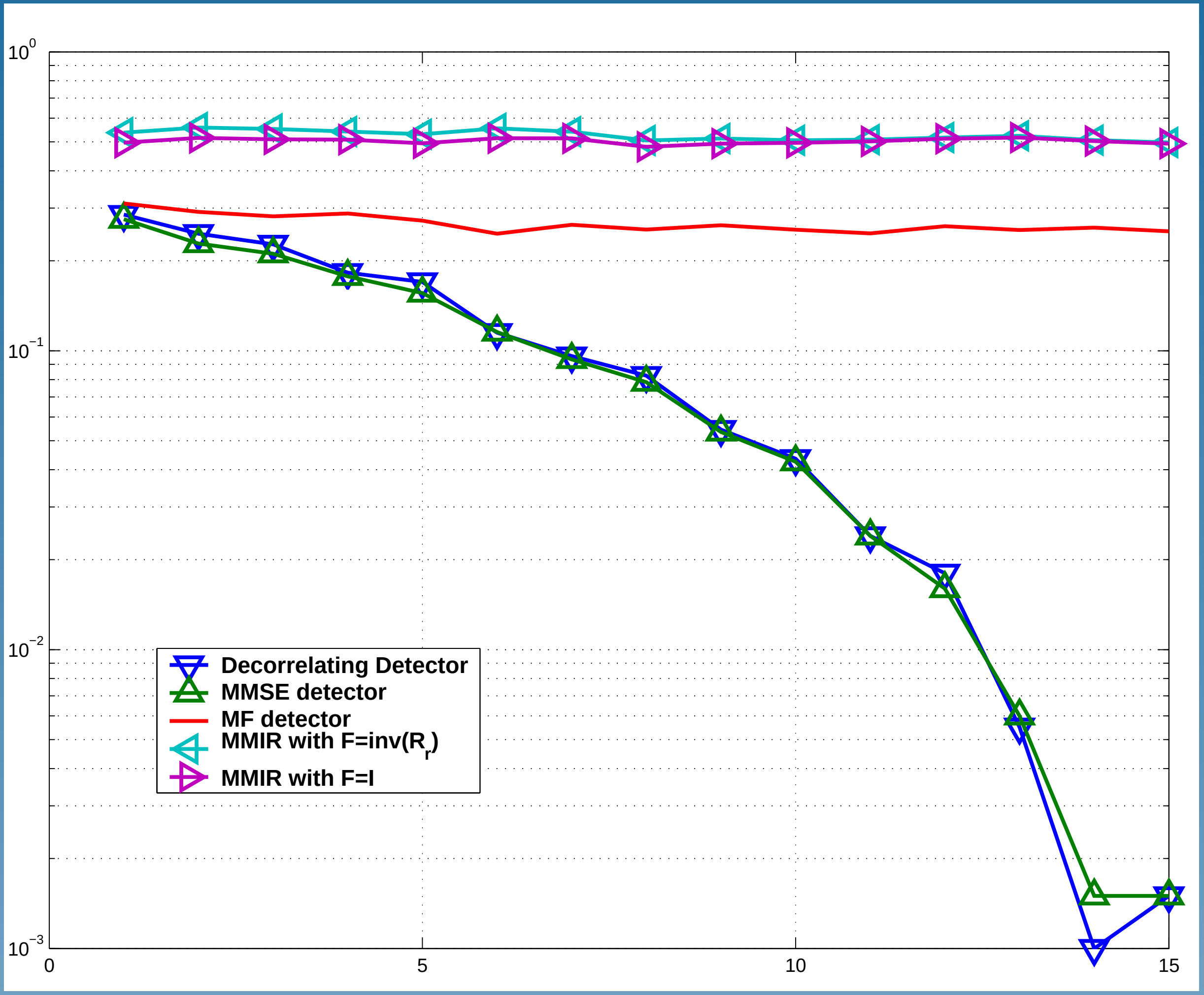
$$\mathbf{R}_r = \mathsf{E}\{\mathbf{r}\mathbf{r}^T\} = \mathbf{S}\mathbf{A}^2\mathbf{S}^T + \sigma^2\mathbf{I} = \mathbf{U}\bar{\mathbf{\Lambda}}\mathbf{U}^T$$

where $\mathbf{U} = [\,\mathbf{U}_s \quad \mathbf{U}_n\,]$ and $\bar{\mathbf{\Lambda}} = \begin{bmatrix} \bar{\mathbf{\Lambda}}_s & \mathbf{0} \\ \mathbf{0} & \bar{\mathbf{\Lambda}}_n \end{bmatrix} = \mathbf{A}^2 + \sigma^2\mathbf{I}$.

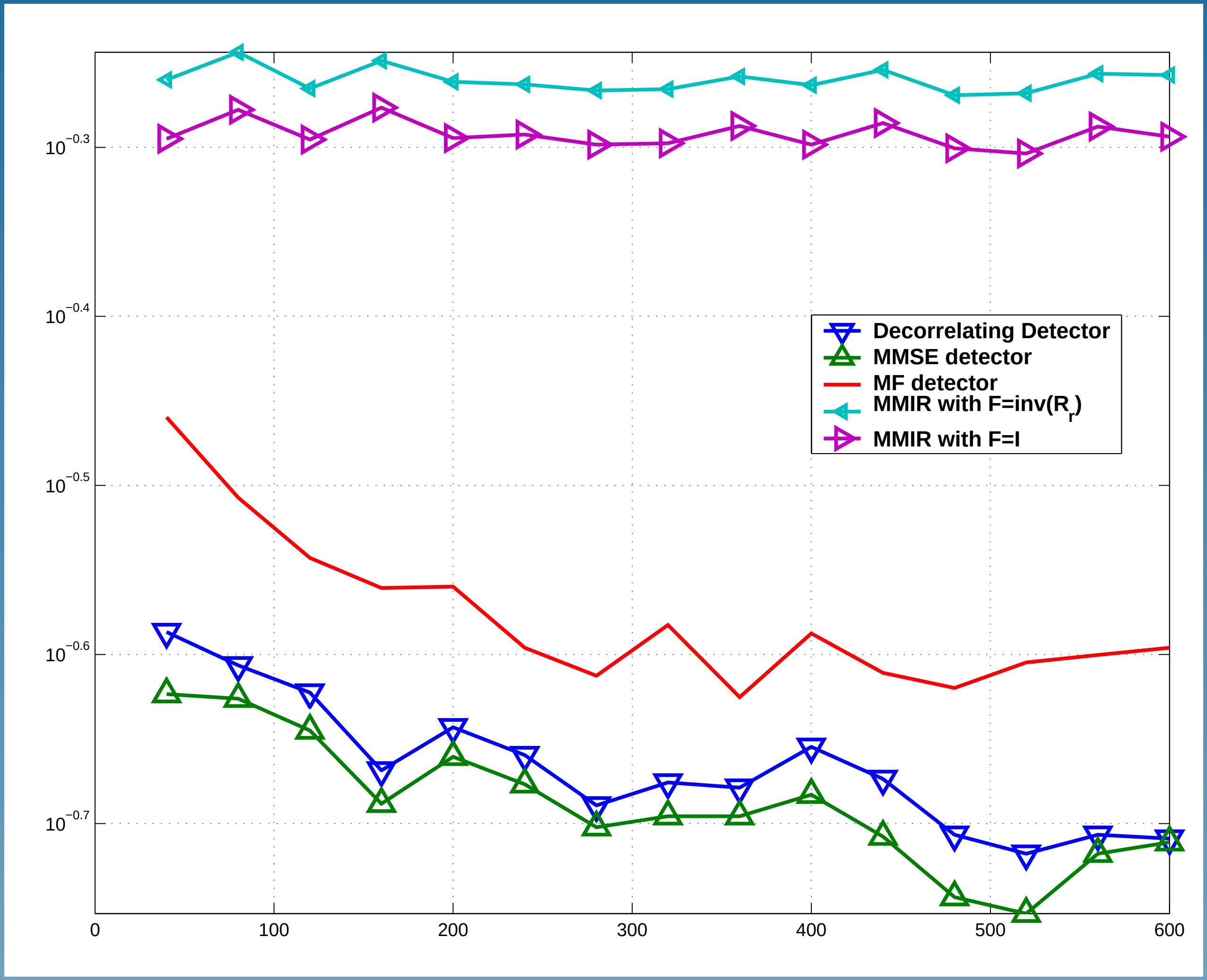
- Blind $\mathbf{W}^{DD}$ can be expressed as $\mathbf{U}_s(\bar{\mathbf{\Lambda}}_s - \sigma^2\mathbf{I})^{-1}\mathbf{U}_s^T\mathbf{S}$. Blind $\mathbf{W}^{MMSE}$ can be expressed as $\mathbf{U}_s\bar{\mathbf{\Lambda}}_s^{-1}\mathbf{U}_s^T\mathbf{S}$

- Blind $\mathbf{W}^{MMIR}$ can also be easily constructed if $\mathbf{F}$ is available.

Computer Simulations I



Computer Simulations II



Conclusions & Future Directions

- It is interesting to find that many existing multiuser detectors satisfy the proposed MMIR constrain.
- Multiuser receiver can not be completely decided only by MMIR constrain. Additional constrain is needed. In the future, it is very important to look for additional constrain to decide $\mathbf{F}$.
- It is also very interesting to look for adaptive realization of blind MMIR receiver.
- Since the model is the basic MIMO system model, many previous discussions are applicable in many other MIMO systems, such as OFDM, DMT and multi-antenna system.

Selected References

- Naofal Al-Dhahir and John M. Cioffi, Block Transmission over Dispersive Channels: Transmit Filter Optimization and Realization, and MMSE-DFE Receiver Performance, IEEE Transactions On Information Theory, Vol. 42, January 1996, pp. 137-160.

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