Blind Multiuser Detection Algorithms

Abstract

Multiuser detection is one of the key techniques for combating multiple access interference (MAI) in CDMA systems. In this paper, we propose a new blind multiuser model using a blind spreading sequence matrix and several blind multiuser detection algorithms using least-squares (LS) estimations, best least unbiased (BLU) estimation, minimum mean-squared error (MMSE) estimation and Kalman estimation criteria. Theoretical analysis and computer simulations are provided to demonstrate the proposed schemes too.

1 Introduction

Direct-sequence code division multiple access (DS/CDMA) techniques have attracted increasing attention for efficient use of available bandwidth, resistance to interference and flexibility to variable traffic patterns. Multiuser detection strategy is a method to minimize the effect of MAI and solve the near-far problem in CMDA systems. It has been extensively investigated over the past several years [1], since MAI is the dominant impairment for CDMA systems and exists even in perfect power-controlled CDMA systems. Most early work on multiuser detection assumed that the receiver knew the spreading codes or had some knowledge of all users, then exploited this knowledge to combat MAI. For example, the classic decorrelating detector could achieve the optimum near-far resistance and completely eliminate MAI from other users with the expense of enhancement of background noise. However, in many practical cases, especially in a dynamic environment, e.g. in the downlink of the CDMA system, it is very difficult for a mobile user to obtain accurate information on other active users in the same channel. On the other hand, the frequent use of training sequence is certainly a waste of channel bandwidth. So multiuser blind detection has been proposed. Recent research has been devoted to the blind multiuser receivers and subspace-based signature waveform estimation schemes to achieve better performance and higher capacity [2, 3, 4, 5, 6]. The Minimum output energy (MOE) method and subspace method were presented for multiuser blind detection with the knowledge of only the desired users' spreading code and possible timing.

The large gaps in performance and complexity between the conventional single-user matched filter and optimum multiuser detector encourage the search for other multiuser detectors that exhibit good performance/complexity tradeoffs. The classic decorrelating detector [7] not only is a simple and natural strategy but it is optimal according to three different criteria: least squares, near-far resistance [8] and maximum-likelihood when the received amplitudes are unknown [7]. Though it does not require knowledge of the received amplitudes, it does require some information about other active users' signature waveforms. This makes it difficult be put in practical applications.

Multiuser blind detection using subspace techniques was first developed in depth by Wang and Poor [4, 9]. Such techniques were appropriate for the downlink environment where only the desired users' code is available. More recently, these subspace techniques were extended by Wang and Host-Madsen [10], named group multiuser blind detectors, to uplink environments where the base station knows the codes of in-cell users, but not those of users outside the cell. In the subspace-based blind detection approach [4], the linear detectors are constructed in the closed form once the signal subspace components are computed and offer lower computational complexity and better performance than the blind MOE detector. For the subspace-based blind adaptive detector, the project approximation subspace tracking deflation (PASTd) algorithm [11] is used to estimate the signal subspace.

It is shown in [2] that the MOE receiver is equivalent to the linear MMSE detector, which is near-far resistant and has much less complexity compared the optimal multiuser detection. The major limitation of MOE schemes to multiuser blind detection is that there is a satuaration effect in the steady state, which causes a significant performance gap between the converged blind MOE and the true MMSE detector [2].

As we see, various multiuser detection schemes have been developed to mitigate the effects of MAI. Blind detectors are obviously more close to the practical application. However, since only the signature information of the desired user could be available, nearly all the proposed blind detectors need some adaptive or search procedure to collection information of other users and/or channel before detection. In this work, we consider another different idea to develop blind multiuser detectors. Instead of searching for the original spreading sequences, a blind spreading matrix consisting of the desired user's original spreading sequence vector and some previous received signal vectors is constructed. We show that the bits sent for desired user can be clearly detected if there is no noise or distortion in the proposed blind spreading sequence matrix. In practical situations, when the proposed spreading matrix is corrupted by channels, we proposed many different estimation schemes for blind multiuser detection, which include LS-liked estimations, BLU estimation, linear MMSE estimation and Kalman filter estimation. In the present multiuser blind detectors, only the signature and timing of the desired user are utilized. There are no any adaptive or search procedures employed as in other multiuser blind detection algorithms. Theoretical analysis and computer simulations are also presented to demonstrate the performance of these blind detectors.

The rest of the paper is organized as follows. In Section II, we summarize the signal model. In Section III, we propose a new blind multiuser model and present the blind multiuser detection framework. In Section IV, various estimation schemes are discussed for blind detection. Performance analysis and simulation results are provided in Section V and VI. Section VII concludes this papers.

2 Classic Channel Model And Problem Description

The basic CDMA K-user channel model, consisting of the sum of antipodally modulated synchronous signature waveforms embedded in additive white Gaussian noise (AWGN), is considered here. The received base-band signal during one symbol interval in such a channel can be modelled as:

$$r(t) = \sum_{k=1}^{K} A_k b_k[n] s_k(t) + n(t) \tag{1}$$

where $t \in [nT,\ (n+1)T]$, T is the symbol interval. $n(t)$ represents the Gaussian channel noise, K is the number of users and A_k, $b_k[n]$ denote the received amplitude and data bit of the kth user, respectively. It is assumed that $b_k[n] \in \{-1,\ +1\}$ is a collection of independent equiprobable ± 1 random variables transmitted by the kth user during $[nT,\ (n-1)T]$ and $s_k(t)$ denotes the normalized signal waveform of the kth user on the interval $[(n-1)T,\ nT]$, i.e., $\|s_k(t)\| = 1$. The received signal $r(t)$ is passed through a chip-matched filter followed by a chip-rate sampler. As a result, $r(t)$, $t \in [(n-1)T,\ nT]$, is converted into a $L \times 1$ column vector $\mathbf{r}$ [1] of the samples of the chip-matched filter outputs within a symbol interval $[(n-1)T,\ nT]$ as

$$\begin{aligned} \mathbf{r} &= [\, r_1 \quad r_2 \quad \ldots \quad r_L \,]^T \\ &= \sum_{k=1}^{K} A_k b_k \mathbf{s}_k + \mathbf{n} \\ &= \mathbf{SAb} + \mathbf{n} \end{aligned} \tag{2}$$

where $\mathbf{A} = \mathrm{diag}\{[\, A_1\ A_2\ \ldots\ A_K \,]\}$ is the received amplitude diagonal matrix, $\mathbf{S} = [\mathbf{s}_1\ \mathbf{s}_2\ \ldots\ \mathbf{s}_K]$ is the $L \times K$ signature matrix with the kth column $\mathbf{s}_k$ being the signature vector of the kth user, $\mathbf{b} = [b_1\ b_2\ \ldots\ b_K]^T = [b_1\ \tilde{\mathbf{b}}^T]^T$ is the information vector sent by all the K users at time $t = n$ and b_1 is the bit sent by the first user at time $t = n$, and $\mathbf{n}$ is an L-dimensional Gaussian vector with independent σ^2-variance components. We maintain the restriction that $L \geq K$.

Most of the linear multiuser detectors for demodulating the kth user's data bit in (2) is in the form of a correlator followed by a hard limiter, which could be expressed as

$$\hat{b}_k = \mathrm{sign}\{\mathbf{w}_k^T \mathbf{r}\} \tag{3}$$

where $\mathbf{w}_k \in \mathbb{R}^{L \times 1}$ is the linear representation of multiuser detector. Linear multiuser detectors can be implemented in a decentralized fashion where only the user or users of interest need be demodulated.

3 New Blind Channel Model Using Blind Signature Matrix

Most classic multiuser detection schemes assume knowledge of the spreading codes and/or signal-to-noise ratios (SNR) of all active users that contribute to the received signal. These receivers then exploit this knowledge to easily achieve optimal or sub-optimal performance. In practical situations, e.g. at mobile station of CDMA systems, it is difficult for multiuser receiver to known other existing users' information. So many blind multiuser detectors are developed to operate without prior knowledge regarding other users but use statistic signal processing techniques to estimate other's information. In this paper, instead of estimating statistic channel information, we construct a "faked" blind spreading matrix for the desired user. We then show that it is possible to detect the desired user's information bits using this blind spreading matrix. The details are revealed in the following. Without loss of the generality, assume only the bits b_1 sent by the 1st user is considered here.

3.1 Blind Signature Matrix $\boldsymbol{\mathcal{S}}$

At first we construct a new $L \times M$ blind signature matrix $\boldsymbol{\mathcal{S}}$. It is defined by

$$\begin{aligned} \boldsymbol{\mathcal{S}} &= [\, \bar{\mathbf{s}}_1 \quad \bar{\mathbf{s}}_2 \quad \bar{\mathbf{s}}_3 \quad \ldots \quad \bar{\mathbf{s}}_M \,] \\ &= [\, \mathbf{s}_1 \quad \mathbf{r}_1 \quad \mathbf{r}_2 \quad \ldots \quad \mathbf{r}_{M-1} \,] \\ &= [\, A_1^{-1}\mathbf{SAe}_1 \quad \mathbf{SAb}_1 \quad \mathbf{SAb}_2 \quad \ldots \quad \mathbf{SAb}_{M-1} \,] + \mathbf{N} \\ &= \mathbf{SA}\,[\, A_1^{-1}\mathbf{e}_1 \quad \mathbf{b}_1 \quad \mathbf{b}_2 \quad \ldots \quad \mathbf{b}_{M-1} \,] + \mathbf{N} \\ &= \mathbf{SA}\,[\, A_1\mathbf{e}_1 \quad \mathbf{D} \,] + \mathbf{N} \\ &= \mathbf{SAB} + \mathbf{N} \end{aligned} \tag{4}$$

[1] Without the loss of generality, we drop parameter n and denote $\mathbf{r} = \mathbf{r}[n]$. The same to b_k

where $L \geq M \geq K$, $\mathbf{r}_i$, $i = 1,\ 2,\ \ldots,\ M-1$, are the arbitrary previously received independent vectors and the $K \times 1$ vector $\mathbf{b}_i$ is the corresponding bit vector for the information sent by all K users. $\mathbf{D} = [\mathbf{d}\ \tilde{\mathbf{D}}^{\mathrm{T}}]^{\mathrm{T}}$, The $(K-1) \times 1$ vector $\mathbf{d}$ is the bit vector consisting of the known bits previously sent for the desired user. $\mathrm{rank}\{\tilde{\mathbf{D}}\} = K-1$, $\mathbf{N} = [\mathbf{0}\ \tilde{\mathbf{N}}]$ and

$$\begin{aligned} \mathbf{B} &= \left[A_1^{-1}\mathbf{e}_1 \quad \mathbf{D} \right] \\ &= \begin{bmatrix} A_1^{-1}\mathbf{e}_1 & \begin{matrix} \mathbf{d}^{\mathrm{T}} \\ \tilde{\mathbf{D}} \end{matrix} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{c}^{\mathrm{T}} \\ \mathbf{0} \quad \tilde{\mathbf{D}} \end{bmatrix} \\ &= \begin{bmatrix} A_1^{-1} & \mathbf{d}^{\mathrm{T}} \\ \mathbf{0} & \tilde{\mathbf{D}} \end{bmatrix} \end{aligned} \tag{5}$$

and $\mathrm{rank}\{\mathbf{B}\} = K$.

Using the received signal vector definition (2) and the proposed blind signature matrix $\boldsymbol{\mathcal{S}}$ in (4), the received signal vector $\mathbf{r}$ can be expressed as

$$\begin{aligned} \mathbf{r} &= \mathbf{SAb} + \mathbf{n} \\ &= \mathbf{SABB}^{+}\mathbf{b} + \mathbf{n} \\ &= (\boldsymbol{\mathcal{S}} - \mathbf{N})\mathbf{B}^{+}\mathbf{b} + \mathbf{n} \\ &= \boldsymbol{\mathcal{S}}\mathbf{B}^{+}\mathbf{b} - \mathbf{NB}^{+}\mathbf{b} + \mathbf{n} \\ &= \boldsymbol{\mathcal{S}}\mathbf{f} + \bar{\mathbf{n}} \end{aligned} \tag{6}$$

where $\mathbf{B}^{+} = \mathbf{B}^{\mathrm{T}}(\mathbf{BB}^{\mathrm{T}})^{-1}$ is Moor-Penrose general inverse of $\mathbf{B}$ and $\mathbf{f}$ denotes the new $K \times 1$ detection vector defined by

$$\begin{aligned} \mathbf{f} &= \begin{bmatrix} f^1 \\ \tilde{\mathbf{f}} \end{bmatrix} \\ &= \left[A_1^{-1}\mathbf{e}_1 \quad \mathbf{D} \right]^{+} \mathbf{b} \\ &= \begin{bmatrix} A_1^{-1} & \mathbf{d}^{T} \\ \mathbf{0} & \tilde{\mathbf{D}} \end{bmatrix}^{+} \begin{bmatrix} b_1 \\ \tilde{\mathbf{b}} \end{bmatrix} \end{aligned} \tag{7}$$

with mean vector $\mathbf{m}_{\mathbf{f}} = \mathbf{0}$ and covariance matrix

$$\mathbf{C}_{\mathbf{f}} = E\left\{\mathbf{ff}^{\mathrm{T}}\right\}. \tag{8}$$

And $\bar{\mathbf{n}}$ is the new noise vector defined by

$$\bar{\mathbf{n}} = \mathbf{n} - \mathbf{NB}^{+}\mathbf{b} \tag{9}$$

with mean vector $\mathbf{m}_{\bar{\mathbf{n}}} = \mathbf{0}$ and covariance matrix

$$\begin{aligned} \mathbf{C}_{\bar{\mathbf{n}}} &= E\left\{(\mathbf{n} - \mathbf{NB}^{+}\mathbf{b})(\mathbf{n} - \mathbf{NB}^{+}\mathbf{b})^{\mathrm{T}}\right\} \\ &= \sigma^2\mathbf{I} + E\left\{\mathbf{Nff}^{\mathrm{T}}\mathbf{N}^{\mathrm{T}}\right\} \\ &= \sigma^2\left(1 - E\|\tilde{\mathbf{f}}\|_2^2\right)\mathbf{I}\ . \end{aligned} \tag{10}$$

4 Blind Multiuser Detections

Before giving blind multiuser detection solutions, we firstly show a semiblind multiuser detection scheme providing the amplitude, A_1, of the first user is already known. At this time, b_1 can be estimated using following theorem.

Lemma 1. *The bit b_1 sent for the first user at time $t = n$ can be detected using the following equation.*

$$b_1 \quad = \quad sign\left\{\mathbf{c}^{\mathrm{T}}\mathbf{f}\right\} \ . \tag{11}$$

Proof. With (5), b_1 can be estimated by

$$\begin{aligned} b_1 &= \operatorname{sign}\left\{b_1 - \mathbf{d}^{\mathrm{T}}\tilde{\mathbf{D}}^{+}\tilde{\mathbf{b}} + \mathbf{d}^{\mathrm{T}}\tilde{\mathbf{D}}^{+}\tilde{\mathbf{b}}\right\} \\ &= \operatorname{sign}\left\{\left[\, A_1^{-1} \quad \mathbf{d}^{\mathrm{T}}\,\right]\begin{bmatrix} A_1 b_1 - A_1\mathbf{d}^{\mathrm{T}}\tilde{\mathbf{D}}^{+}\tilde{\mathbf{b}} \\ \tilde{\mathbf{D}}^{+}\tilde{\mathbf{b}} \end{bmatrix}\right\} \\ &= \operatorname{sign}\left\{\mathbf{c}^{\mathrm{T}}\begin{bmatrix} A_1 & -A_1\mathbf{d}^{\mathrm{T}}\tilde{\mathbf{D}}^{+} \\ \mathbf{0} & \tilde{\mathbf{D}}^{+} \end{bmatrix}\begin{bmatrix} b_1 \\ \tilde{\mathbf{b}} \end{bmatrix}\right\} \\ &= \operatorname{sign}\left\{\mathbf{c}^{\mathrm{T}}\begin{bmatrix} A_1 & -A_1\mathbf{d}^{\mathrm{T}}\tilde{\mathbf{D}}^{+} \\ \mathbf{0} & \tilde{\mathbf{D}}^{+} \end{bmatrix}\mathbf{b}\right\} \end{aligned} \tag{12}$$

On the other hand, we know that

$$\mathbf{B}\begin{bmatrix} A_1\mathbf{e}_1 & \begin{matrix} -A_1\mathbf{d}^{\mathrm{T}}\tilde{\mathbf{D}}^{+} \\ \tilde{\mathbf{D}}^{+} \end{matrix} \end{bmatrix} = \begin{bmatrix} A_1^{-1} & \mathbf{d}^{\mathrm{T}} \\ \mathbf{0} & \tilde{\mathbf{D}} \end{bmatrix}\begin{bmatrix} A_1 & -A_1\mathbf{d}^{\mathrm{T}}\tilde{\mathbf{D}}^{+} \\ \mathbf{0} & \tilde{\mathbf{D}}^{+} \end{bmatrix} = \mathbf{I} \tag{13}$$

Using (13) and (5), (12) can then be re-written by

$$\begin{aligned} b_1 &= \operatorname{sign}\left\{\mathbf{c}^{\mathrm{T}}\mathbf{B}^{+}\mathbf{b}\right\} \\ &= \operatorname{sign}\left\{\mathbf{c}^{\mathrm{T}}\mathbf{f}\right\} \end{aligned} \tag{14}$$

and the detection vector can be re-written by

$$\mathbf{f} \quad = \quad \begin{bmatrix} A_1 b_1 - A_1\mathbf{d}^{\mathrm{T}}\tilde{\mathbf{D}}^{+}\tilde{\mathbf{b}} \\ \tilde{\mathbf{D}}^{+}\tilde{\mathbf{b}} \end{bmatrix} \tag{15}$$

Hence, this lemma is proven. □

We see that the classic multiuser detection model is transferred into (6) with the information bit vector $\mathbf{b}$ being replaced by the detection vector $\mathbf{f}$ and the original AWGN vector $\mathbf{n}$ being replaced by $\bar{\mathbf{n}}$ in (6). With Lemma 1, the bit b_1 sent for user 1 can still be detected with the detection vector $\mathbf{f}$ and the previously detected bits vector $\mathbf{c}$. In the following, we show that how to estimate A_1 proving we know how to estimate $\mathbf{f}$.

Before estimating A_1, we construct another blind spreading matrix $\boldsymbol{\mathcal{S}}'$ using another set of received vectors, $\mathbf{r}'_m$. If we have already known how to estimate the detection vector, we can get two detection vectors, $\mathbf{f}$ and $\mathbf{f}'$, corresponding to these two blind spreading matrices $\boldsymbol{\mathcal{S}}$ and $\boldsymbol{\mathcal{S}}'$. With $\mathbf{f}$ and $\mathbf{f}'$, A_1 can be estimated using the following theorem.

Lemma 2. *A_1 is the solution to the following equation*

$$\mathbf{c}^{\mathrm{T}}\mathbf{f} - \mathbf{c}'^{\mathrm{T}}\mathbf{f}' \quad = \quad 0 \ . \tag{16}$$

and it can be estimated by

$$A_1 \quad = \quad \frac{f_1 - f_1'}{\mathbf{d}'^{\mathrm{T}}\tilde{\mathbf{f}}' - \mathbf{d}^{\mathrm{T}}\tilde{\mathbf{f}}} \tag{17}$$

We can see that the performance of this blind multiuser detection frame work highly depends on the estimation of $\mathbf{f}$ and A_1 can actually be estimated from $\mathbf{f}$ too. In the following, various algorithms are proposed for estimating $\mathbf{f}$ based different criteria. With estimating $\mathbf{f}$, the formulations of different blind multiuser detectors are proposed too.

4.1 Least-Squares-Based Blind Detections

Let us start from minimizing least squares errors. Least-squares-based algorithms are widely used in practices because they are simple and easy to be implemented. They only use signal models without any probabilistic assumptions about data. The negative side, no claims about optimality can be made and the statistical performance cannot be assessed without some specific assumption about the probabilistic structure of the data.

4.1.1 Least-Squares Detection

At first, we assume the measurements of $\boldsymbol{\mathcal{S}}$ is assumed to be free of error. All errors are confined to the received vector $\mathbf{r}$. Hence, the detection vector can be estimated with solving the following equation

$$\mathbf{f}_{\mathrm{LS}} = \arg\min_{\mathbf{x}} \|\mathbf{r} - \boldsymbol{\mathcal{S}}\mathbf{x}\|_2 \quad \text{subject to} \quad \mathbf{r} \subseteq \mathbb{R}(\boldsymbol{\mathcal{S}}) \tag{18}$$

Suppose $\mathbf{U}^T\boldsymbol{\mathcal{S}}\mathbf{V} = \boldsymbol{\Sigma}$ is the SVD of $\boldsymbol{\mathcal{S}} \in \mathbb{R}^{L\times K}$ with $r = \mathrm{rank}(\boldsymbol{\mathcal{S}})$. And if $\mathbf{U} = [\,\mathbf{u}_1 \quad \mathbf{u}_2 \quad \ldots \quad \mathbf{u}_L\,]$, $\mathbf{V} = [\,\mathbf{v}_1 \quad \mathbf{v}_2 \quad \ldots \quad \mathbf{v}_K\,]$, $\boldsymbol{\Sigma} = \mathrm{diag}\{[\,\sigma_1 \quad \ldots \sigma_r \quad 0 \quad \ldots \quad 0\,]\}$ and $\mathbf{r} \in \mathbb{R}^{L\times 1}$, then LS estimation of $\mathbf{f}$ is

$$\mathbf{f}_{\mathrm{LS}} \;=\; \sum_{i=1}^{r} \frac{\mathbf{u}_i^T\mathbf{r}}{\sigma_i}\mathbf{v}_i \;=\; \boldsymbol{\mathcal{S}}^{+}\mathbf{r} \;=\; \mathbf{f} + \boldsymbol{\mathcal{S}}^{+}\bar{\mathbf{n}} \tag{19}$$

which minimizes $\|\boldsymbol{\mathcal{S}}\mathbf{d} - \mathbf{r}\|_2$ and has the smallest 2-norm of all minimizers. Moreover

$$\varepsilon_{\mathrm{LS}}^2 \;=\; \min_{\mathbf{x}\in\mathbb{R}} \|\boldsymbol{\mathcal{S}}\mathbf{x} - \mathbf{r}\|_2^2 \;=\; \sum_{i=r+1}^{L} (\mathbf{u}_i^T\mathbf{r})^2 \tag{20}$$

With the estimated amplitude $\hat{A}_1$, the linear filter representation of the least squares detector can be written by

$$\mathbf{w}_{\mathrm{LS}} \;=\; \boldsymbol{\mathcal{S}}^{+\mathrm{T}}\mathbf{c} \;. \tag{21}$$

4.1.2 Total Least-Squares Estimation

It assume $\boldsymbol{\mathcal{S}}$ to be error-free in the previous LS estimate of the detection vector $\mathbf{f}$. However, this assumption is not entirely accurate according to the definition of $\boldsymbol{\mathcal{S}}$ in (4) since there is a noise term, $\mathbf{N}$. On the other hand, $\mathbf{r}$ can also be expressed as

$$\begin{aligned} \mathbf{r} &= (\boldsymbol{\mathcal{S}} - \mathbf{N})\mathbf{B}^{+}\mathbf{b} + \mathbf{n} \\ &= \hat{\boldsymbol{\mathcal{S}}}\mathbf{d} + \mathbf{n} \end{aligned} \tag{22}$$

where $\hat{\boldsymbol{\mathcal{S}}} = \boldsymbol{\mathcal{S}} - \mathbf{N} = \mathbf{SAB}$. The minimization problem of (18) can then be transformed into the following TLS problem:

$$[\boldsymbol{\mathcal{S}}_{\mathrm{TLS}},\ \mathbf{f}_{\mathrm{TLS}}] \;=\; \arg\min_{\bar{\boldsymbol{\mathcal{S}}},\,\mathbf{x}} \|[\,\boldsymbol{\mathcal{S}} \quad \mathbf{r}\,] - [\,\bar{\boldsymbol{\mathcal{S}}} \quad \bar{\boldsymbol{\mathcal{S}}}\mathbf{x}\,]\|_2 \;, \tag{23}$$

subject to $\mathbf{r} \subseteq \mathbb{R}(\bar{\boldsymbol{\mathcal{S}}})$.

Let $\boldsymbol{\mathcal{S}} = \mathbf{U}'\mathbf{\Sigma}'\mathbf{V}'^T$ and $[\boldsymbol{\mathcal{S}}\ \mathbf{r}] = \mathbf{U\Sigma V}^T$ be the SVD of $\boldsymbol{\mathcal{S}}$ and $[\boldsymbol{\mathcal{S}}\ \mathbf{r}]$, respectively. If $\sigma'_K > \sigma_{K+1}$, TLS estimation of $\mathbf{f}$ then is

$$\mathbf{f}_{\mathrm{TLS}} = \left(\boldsymbol{\mathcal{S}}^T\boldsymbol{\mathcal{S}} - \sigma_{K+1}^2\mathbf{I}\right)^{-1}\boldsymbol{\mathcal{S}}^T\mathbf{r} \tag{24}$$

and

$$\begin{aligned} \varepsilon_{\mathrm{TLS}}^2 &= \min_{\mathbf{x}\in\mathbb{R}^{K\times 1}} \|\boldsymbol{\mathcal{S}}\mathbf{x} - \mathbf{r}\|_2^2 \\ &= \sigma_{K+1}^2\left[1 + \sum_{i+1}^{K} \frac{\mathbf{u}_i'^T\mathbf{r}^{\ 2}}{\sigma_i'^2 - \sigma_{K+1}^2}\right] \end{aligned} \tag{25}$$

where $\mathbf{U} = [\mathbf{u}_1\ \mathbf{u}_2\ \ldots\ \mathbf{u}_L]$, $\mathbf{V} = [\mathbf{v}_1\ \mathbf{v}_2\ \ldots\ \mathbf{v}_{K+1}]$, $\mathbf{\Sigma} = \mathrm{diag}\{[\sigma_1\ \sigma_2 \ldots\ \sigma_{K+\min\{L-K,\ 1\}}]\}$ and $\mathbf{U}' = [\mathbf{u}_1'\ \mathbf{u}_2'\ \ldots\ \mathbf{u}_L']$, $\mathbf{V}' = [\mathbf{v}_1'\ \mathbf{v}_2'\ \ldots\ \mathbf{v}_K']$, $\mathbf{\Sigma}' = \mathrm{diag}\{[\sigma_1'\ \sigma_2'\ \ldots\ \sigma_K']\}$. And the linear filter $\mathbf{w}_{\mathrm{TLS}}$ is

$$\mathbf{w}_{\mathrm{TLS}} = \boldsymbol{\mathcal{S}}(\boldsymbol{\mathcal{S}}^{\mathrm{T}}\boldsymbol{\mathcal{S}} - \sigma_{K+1}^2\mathbf{I})^{-1}\mathbf{c} \tag{26}$$

4.1.3 Mixed LS/TLS Estimation

In the LS problem of (18), it assumed the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$ is error-free. Again, this assumption is not completely accurate. In the TLS problem of (23), it assumed that in each column of the semi-blind signature matrix, $\boldsymbol{\mathcal{S}}$, some noise or error exists. This assumption also is not complete. Though there exists a noise or error matrix $\mathbf{N}$ in $\boldsymbol{\mathcal{S}}$ from (4), its first column is exactly known to be noise-free or error-free. Hence, to maximize the estimation accuracy of the detection vector $\mathbf{d}$, it is natural to require that the corresponding columns of $\boldsymbol{\mathcal{S}}$ be unperturbed since they are known exactly. The problem of estimating the detection vector $\mathbf{d}$ can then be transformed into the following MLS problem by considering (18) and (23):

$$[\boldsymbol{\mathcal{S}}_{\mathrm{MLS}},\ \mathbf{f}_{\mathrm{MLS}}] = \arg\min_{\bar{\boldsymbol{\mathcal{S}}},\ \mathbf{x}} \|[\,\tilde{\boldsymbol{\mathcal{S}}}\quad \mathbf{r}\,] - [\,\bar{\boldsymbol{\mathcal{S}}}\quad [A_1\mathbf{s}_1\ \bar{\boldsymbol{\mathcal{S}}}]\mathbf{x}\,]\|_2 \tag{27}$$

subject to $\mathbf{r} \subseteq \mathbb{R}([A_1\mathbf{s}_1\ \bar{\boldsymbol{\mathcal{S}}}])$. The following lemma outlines the MLS solution.

Consider the MLS problem in (27) and perform the Householder transformation Q on the matrix $[\,\boldsymbol{\mathcal{S}}\quad \mathbf{r}\,]$ so that

$$Q^{\mathrm{T}}[\,A_1\mathbf{s}_1\quad \bar{\boldsymbol{\mathcal{S}}}\quad \mathbf{r}\,] = \begin{bmatrix} R_{11} & \mathbf{R}_{12} & R_{1r} \\ \mathbf{0} & \mathbf{R}_{22} & \mathbf{R}_{2r} \end{bmatrix} \tag{28}$$

where $R_{11} \neq 0$, $\mathbf{R}_{12}$ is a $1 \times (M-1)$ vector, $\mathbf{R}_{22}$ is a $(L-1) \times (M-1)$ matrix and $\mathbf{R}_{2r}$ is a $(L-1) \times 1$ vector.

Denote σ' as the smallest singular value of $\mathbf{R}_{22}$ and σ as the smallest singular value of $[\,\mathbf{R}_{22}\quad \mathbf{R}_{2r}\,]$. If $\sigma' > \sigma$, then the MLS solution uniquely exists and is given by

$$\mathbf{f}_{\mathrm{MLS}} = \left(\boldsymbol{\mathcal{S}}^T\boldsymbol{\mathcal{S}} - \sigma^2\begin{bmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{M-1} \end{bmatrix}\right)^{-1}\boldsymbol{\mathcal{S}}^T\mathbf{r}\ . \tag{29}$$

4.2 Best Linear Unbiased Estimation

We begin by assuming the following linear structure $\mathbf{d}_{BLU} = \mathbf{W}_{\mathbf{f}}\mathbf{r}$ for this so-called best linear unbiased estimator (BLUE). Matrix $\mathbf{W}_{\mathbf{f}}$ is designed such that: (1) $\boldsymbol{\mathcal{S}}$ must be deterministic, and (2) $\bar{\mathbf{n}}$ must be zero mean with positive definite known covariance matrix $\mathbf{C}_{\bar{\mathbf{n}}}$. (3) $\mathbf{f}_{BLU}$ is an unbiased estimator of $\mathbf{f}$, and (4) the error variance for each of the M parameters is minimized as

$$\mathbf{W}_{\mathbf{f}} = \min_{\mathbf{W}} \mathrm{var}(\mathbf{W}\mathbf{r}) \tag{30}$$

In this way, $\mathbf{f}_{BLU}$ will be unbiased and efficient (within the class of linear estimators) by design. The resulting best linear unbiased estimator is (Gauss-Markov Theorem):

$$\mathbf{f}_{\mathrm{BLU}}=(\boldsymbol{\mathcal{S}}^{\mathrm{T}}\mathbf{C}_{\bar{\mathbf{n}}}^{-1}\boldsymbol{\mathcal{S}})^{-1}\boldsymbol{\mathcal{S}}^{\mathrm{T}}\mathbf{C}_{\bar{\mathbf{n}}}^{-1}\mathbf{r} \tag{31}$$

with the covariance matrix of $\mathbf{f}_{BLU}$ is

$$\mathbf{C}_{\mathbf{d}_{\mathrm{BLU}}}=(\boldsymbol{\mathcal{S}}^{\mathrm{T}}\mathbf{C}_{\bar{\mathbf{n}}}^{-1}\boldsymbol{\mathcal{S}})^{-1} \ . \tag{32}$$

Since the above data are of Gaussian distributions, the BLUE in (31) is also the minimum variance unbiased estimation.

4.3 Minimum Mean-Squared Estimation

Now we propose a linear estimator based on MMSE criterion. This class of estimators are generically termed Wiener filter. Given measurements $\mathbf{r}$, the MSE estimator of $\mathbf{d}$, $\mathbf{f}_{\mathrm{MS}} = f(\mathbf{r})$, minimizes the mean-squared error $J_{\mathrm{MS}} = E\{||\mathbf{f} - \hat{\mathbf{f}}||_2^2\}$. The function $f(\mathbf{r})$ may be nonlinear or linear and its exact structure is determined by minimizing J_{MS}. When $\mathbf{f}$ and $\mathbf{r}$ are jointly Gaussian, the linear estimator that minimizes the mean-sqared error is (Bayesian Gauss-Markov Theorem)

$$\mathbf{f}_{\mathrm{MS}} = (\mathbf{C}_{\mathbf{f}}^{-1} + \boldsymbol{\mathcal{S}}^{\mathrm{T}}\mathbf{C}_{\bar{\mathbf{n}}}^{-1}\boldsymbol{\mathcal{S}})^{-1}\boldsymbol{\mathcal{S}}^{\mathrm{T}}\mathbf{C}_{\bar{\mathbf{n}}}^{-1}\mathbf{r} \tag{33}$$

with the variance matrix of $\mathbf{f}_{\mathrm{MS}}$ is

$$\mathbf{C}_{\mathbf{f}_{\mathrm{MS}}} = (\mathbf{C}_{\mathbf{f}}^{-1} + \boldsymbol{\mathcal{S}}^{\mathrm{T}}\mathbf{C}_{\bar{\mathbf{n}}}^{-1}\boldsymbol{\mathcal{S}})^{-1} \ . \tag{34}$$

4.4 Kalman Filter Estimation

Kalman filter can be taken as an important generalization of Wiener filter with the ability to accommodate vector signals and noise which additionally may be nonstationary. It may also be thought of as a sequential MMSE estimator of a signal embedded in noise and this signal can be characterized by a state model. If the signal and noise are jointly Gaussian, then the Kalman filter is an optimal MMSE estimator, and if not, it is the optimal linear MMSE estimator.

$$\mathbf{x}[n] = \mathbf{P}\mathbf{x}[n-1] + \mathbf{Q}\mathbf{u}[n] \tag{35}$$

$$\mathbf{y}[n] = \mathbf{H}\mathbf{x}[n] + \mathbf{w}[n] \tag{36}$$

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