Semi-Blind Adaptive Multiuser Detection For Asynchronous CDMA

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***Abstract*—In this paper, we propose a semiblind multiuser detection framework for asynchronous CDMA. Compared with most existing semiblind/blind detectors, the proposed framework requires a minimum number of previously received signals, which is about the number of interfering signals, and no detection filter converging or subspace separation procedure. The computational complexity and detection delay are therefore much lower. In this framework, a semiblind multiuser signal model is used instead of the widely-discussed conventional multiuser model or subspace-based parametric multiuser signal model. Following this framework, several semiblind linear detectors are developed using least squares (LS), minimum variance unbiased estimation (MVU) and minimum mean squared error (MMSE) estimation criteria. Meanwhile, a multi-window scheme is proposed for simultaneously detecting several bits and a recursively adaptive procedure is developed for further lowering the complexity. After these, the asymptotic multiuser efficiency (AME) of the proposed framework, the comparison between the employed semiblind multiuser signal model and the conventional signal model, and several estimation bounds are discussed. Computer simulation results are presented to support the performance of the proposed semi-blind multiuser detection schemes.**

I. Introduction

Multiuser detection strategy is a method for minimizing the effect of multiple access interference (MAI) and solving the near-far problem without a significant reduction in the signal energies of the strong users in order for the weaker users to achieve reliable communication. It has been extensively investigated over the past decades or so, since MAI is the dominant impairment for CDMA systems and exists even in perfect power-controlled CDMA systems [1, 2, 3, 4, 5, 6, 7, 8]. Most multiuser detection schemes are based on the conventional multiuser signal model and then detect desired users' bits using statistical signal estimation techniques, which include the minimum bit-error rate (MBER) [1, 2], least-square errors [3], MMSE [3, 4, 5] and minimum output energy (MOE) [5] criteria. In the conventional signal model, received signals and multiuser receivers are represented by using users' amplitudes, timing and spreading signatures, which are difficult to be known a prior by most semiblind/blind detectors. On the other hand, the converging and training procedure employed by many semiblind/blind multiuser detectors for discovering interference structure normally cost multiuser receivers lots of time and computation resources. Recently, there have been lots of attentions focused on subspace-based signal models, in which each received signal is taken as a combination of the bases of signal and noise subspaces [9, 10, 11], and subspace-based multiuser detectors [6, 8]. Subspace-based multiuser detection essentially is a method for blindly reconstructing existing conventional detectors. Although their performance can be well above many previous semiblind/blind approaches, subspace-based multiuser detectors need compute the covariance matrix of received signals and separate signal/noise subspace bases. This makes it difficult to be implemented in many practical situations, especially where the wireless channel and the number of users experience fast dynamic changes. Hence, recent blind multiuser detection research is focused on reducing detection complexity and delay.

Although both the conventional multiuser signal model and subspace-based multiuser signal model provide a natural and straightforward description of received signals, most semiblind or blind detectors based on them are hard to be implemented in many practical applications since the signal bases used in these two models are unknown beforehand and it is nontrivial to estimate them by receivers. In order to detect desired user's information bits with minimum computational complexity and prior knowledge, we propose a semiblind multiuser signal model and a novel multiuser detection framework, which require only desired users' spreading sequence, amplitude, several previously received signals and possible channel noise variance, for asynchronous CDMA channel. Based on this framework, we develop several semiblind linear multiuser detectors using least squares, minimum variance unbiased and minimum mean squared estimation criteria. In the proposed signal model and framework, each received asynchronous signal and semiblind receiver is taken as a combination of the desired user's spreading sequence, several previously received signals and noise. Therefore, there is no detection filter convergence or subspace separation procedure required for interference signal structure discovery. The proposed semiblind schemes are simple and direct. They require a minimum number of previously received signals, which is about the number of interfering signals. A recursively adaptive implementation is developed for further reducing the complexity. Finallt, the comparison between the proposed semiblind signal model and the conventional model, the asymptotic multiuser efficiency of the proposed framework, several estimation bounds and also computer simulations are presented to demonstrate their performance.

II. Data Model and Problem Description

The conventional K-user asynchronous multiuser model is presented [7]. The baseband representation of the received signal due to the kth user is given by

$$r_k(t) \quad = \quad \sum_{i=-\infty}^{+\infty} A_k b_k[i] s_k(t - iT_c - \tau_k) \qquad (1)$$

where $b_k[i]$ is the ith bit sent by the kth user. We assume that $\{b_k[i]\}$ are independent and identically distributed random variables with $\mathrm{E}\{b_k[i]\} = 0$ and $\mathrm{E}\left\{|b_k[i]|^2\right\} = 1$. $s_k(t)$ denotes the normalized signal waveform of the kth user during the interval $[(n-1)T,\ nT]$, $\tau_k \in [0,\ T_c)$ is a uniform random variable which denotes the transmission delay from the kth user to the base station and A_k is the amplitude of the received signal of the kth user. The baseband signal at the input of the receiver at the mobile station is

$$r(t) \quad = \quad \sum_{k=1}^{K} r_k(t) + n(t) \qquad (2)$$

where $n(t)$ is additive white Gaussian noise (AWGN) with power spectral density σ_n^2.

The received signal is synchronized for each user individually, passed through the corresponding chip matched filter (CMF) and sampled at least at the chip rate $1/T_c$. The vector of the output samples of the CMF for kth user in the nth symbol interval can be expressed as

$$\mathbf{r}_k[n] = \left[\, r_k(nT + T_c + \tau_k) \quad \ldots \quad r_k(nT + LT_c + \tau_k) \,\right]^{\mathrm{T}} \qquad (3)$$

where L denotes the spreading gain.

Prior to developing our semi-blind decorrelating detectors, we review the classic single-truncated-window decorrelating detector, in which the system is assumed to be chip-synchronous and the observation window is restricted to one symbol interval [7]. Without loss of generality, we consider the detection of the first user while the other users' signals are treated as interference. A typical interferer has two consecutive symbols interfering with the symbol of user 1 so that the received signal $\mathbf{r}$ can be conventionally and straightforwardly expressed by

$$\begin{aligned} \mathbf{r}_1 &= A_1 b_1[n]\mathbf{s}_1 + \sum_{\tau_k \geq \tau_1}^{K} A_k \left\{ b_k[n-1]\mathbf{s}_{k-} + b_k[n]\mathbf{s}_{k+} \right\} \\ &\quad + \sum_{\tau_k < \tau_1} A_k \left\{ b_k[n]\mathbf{s}_{k-} + b_k[n+1]\mathbf{s}_{k+} \right\} + \mathbf{n} \\ &= \mathbf{S}_1 \mathbf{A}_1 \mathbf{b}_1 + \mathbf{n} \end{aligned} \qquad (4)$$

where $\mathbf{s}_{k-}$ and $\mathbf{s}_{k+}$ are effective signature sequences or partial signature sequences that are completely determined by the spreading sequences $\mathbf{s}_k$ and the delays relative to the first user $\tau_{k1} = \tau_k - \tau_1$, $\mathbf{n}$ is an L-dimension Gaussian vector with independent σ_n^2-variance components, where $L \geq 2K - 1$, and

$$\mathbf{S}_1 = \left[\, \mathbf{s}_1 \quad \mathbf{s}_{2-} \quad \mathbf{s}_{2+} \quad \ldots \quad \mathbf{s}_{K-} \quad \mathbf{s}_{K+} \,\right] \,, \qquad (5)$$

$$\mathbf{A}_1 = \mathrm{diag}\left\{\, A_1 \quad A_2 \quad A_2 \quad \ldots \quad A_K \quad A_K \,\right\} \,, \qquad (6)$$

$$\mathbf{b}_1 = \left[\, b_1[n] \quad b_2[n-1]\; b_2[n] \quad \ldots \quad b_K[n]\; b_K[n+1] \,\right]^{\mathrm{T}} . \qquad (7)$$

The classic single-truncated-window decorrelating detector actually performs the following operation

$$\hat{\mathbf{b}}_1 \quad = \quad \mathrm{sgn}\left\{ \mathbf{S}_1^{+} \mathbf{r}_1 \right\} \qquad (8)$$

where $[\cdot]^{+}$ denotes generalized inverse. The single-truncated-window decorrelating detector is designed to completely eliminate MAI caused by other users, which is achieved at the expense of enhancing the ambient noise. There are some desirable features of this multiuser detector. It can readily be decentralized in the sense that the demodulation of each user can be implemented completely independently. It does not require knowledge of the received amplitudes of all users but $\mathbf{S}_1^{+}$, which makes it hard to be directly implemented in many practical situations. Also since each interfering signal's spreading sequence is separated into two subsequences in $\mathbf{S}$, the sequence energy or values are splitted into a double-size matrix $\mathbf{S}_1$, where $\eta_1 = K/(2K-1)$, which is about one half when K is large enough. Therefore, $\mathbf{S}_1^{+}$ is prone to be singular and the performance of the single-window decorrelating detector is less than the compelled decorrelating detector [7].

III. Semi-blind Signal Model And Framework

In order to perform multiuser detection without estimating the received signal bases, we extend the synchronous semiblind detection idea in [12, 13] and define a new asynchronous $PL \times M$ semi-blind signal signature matrix $\boldsymbol{\mathcal{S}}$ for user 1. The signature matrix $\boldsymbol{\mathcal{S}}$ will be used as a set of signal bases for representing received signals and given by

$$\begin{aligned} \boldsymbol{\mathcal{S}} \quad &= \quad \left[\, \mathbf{I}_P \otimes (A_1 \mathbf{s}_1) \quad \bar{\mathbf{r}}_1 \quad \bar{\mathbf{r}}_2 \quad \ldots \quad \bar{\mathbf{r}}_{M-P} \,\right] \\ &= \quad \mathbf{S}\bar{\mathbf{A}}\mathbf{B} + \bar{\mathbf{N}} \end{aligned} \qquad (9)$$

where the $PL \times (PK+K-1)$ matrix

$$\mathbf{S} = [\,\bar{\mathbf{S}}_1 \quad \bar{\mathbf{S}}_2 \quad \bar{\mathbf{S}}_3 \quad \ldots \quad \bar{\mathbf{S}}_K\,] \tag{10}$$

is a multi-window version of the original signature matrix $\mathbf{S}_1$ with

$$\bar{\mathbf{S}}_1 = \mathrm{diag}\{\,\mathbf{s}_1 \quad \mathbf{s}_1 \quad \ldots \quad \mathbf{s}_1\,\}_{PL\times P} \tag{11}$$

and

$$\bar{\mathbf{S}}_k = \mathrm{diag}\{\,\mathbf{s}_{k_-} \quad \mathbf{s}_k \quad \ldots \quad \mathbf{s}_k \quad \mathbf{s}_{k_+}\,\}_{PL\times(P+1)} \ , \tag{12}$$

the $(PK+K-1)\times(PK+K-1)$ diagonal matrix

$$\bar{\mathbf{A}} = \mathrm{diag}\{\,\bar{\mathbf{A}}_1 \quad \bar{\mathbf{A}}_2 \quad \ldots \quad \bar{\mathbf{A}}_K\,\} \tag{13}$$

denotes a multi-window amplitude diagonal matrix with

$$\bar{\mathbf{A}}_1 = \mathrm{diag}\{\,A_1 \quad A_1 \quad \ldots \quad A_1\,\}_{P\times P} \tag{14}$$

and

$$\bar{\mathbf{A}}_k = \mathrm{diag}\{\,A_k \quad A_k \quad \ldots \quad A_k\,\}_{(P+1)\times(P+1)} \ , \tag{15}$$

$\bar{\mathbf{r}}_i$ are several previously received vectors of length P symbols,

$$\mathbf{B} = \begin{bmatrix} & \mathbf{G} & \\ \mathbf{0} & & \tilde{\mathbf{D}} \end{bmatrix} = \begin{bmatrix} \mathbf{I}_P & \bar{\mathbf{D}} \\ \mathbf{0} & \tilde{\mathbf{D}} \end{bmatrix}, \tag{16}$$

with the $PL \times (M-P)$ multi-window bits matrix $\mathbf{D} = [\bar{\mathbf{D}}^{\mathrm{T}}\ \tilde{\mathbf{D}}^{\mathrm{T}}]^{\mathrm{T}}$, in which $\bar{\mathbf{D}}$ is a known $(M-P)\times P$ vector consisting of previously detected bits for the desired user. $\mathbf{G} = \left[\mathbf{I}_P\ \bar{\mathbf{D}}\right]$. rank$\{\mathbf{B}\} \le PK+K-1$ and rank$\{\tilde{\mathbf{D}}\} \le PK+K-P-1$. The $PL\times M$ matrix $\bar{\mathbf{N}} = [\mathbf{0}\ \tilde{\mathbf{N}}]$ denotes the multi-window noise matrix. $\otimes$ denotes the Kronecker product and we maintain $PK+K-1 \le M \le PL$. $M = PK+K-1$ is the minimum number for identifying $\mathbf{S}$ from $\boldsymbol{\mathcal{S}}$.

Using (4) and (9), the semiblind representation of the current received signal vector $\mathbf{r}$, which is of length P symbols, is given by

$$\begin{aligned}\mathbf{r} &= [\,\mathbf{r}_1[n]^{\mathrm{T}} \quad \mathbf{r}_1[n-1]^{\mathrm{T}} \quad \ldots \quad \mathbf{r}_1[n-P+1]^{\mathrm{T}}\,]^{\mathrm{T}} \\ &= \boldsymbol{\mathcal{S}}\mathbf{d} + \bar{\mathbf{n}}\end{aligned} \tag{17}$$

where the $M\times 1$ vector $\mathbf{d}$ denotes the new detection vector and is defined as

$$\mathbf{d} = \mathbf{B}^{+}\bar{\mathbf{b}}_1 \tag{18}$$

and $\bar{\mathbf{n}}$ is the new noise vector and defined as

$$\bar{\mathbf{n}} = \mathbf{n} - \bar{\mathbf{N}}\mathbf{B}^{+}\bar{\mathbf{b}}_1 \tag{19}$$

With (16) and (18), the bits vector $\bar{\mathbf{b}}$ which consists of P bits sent by user 1 at the consecutive symbol intervals $t = n-P+1, n-P+2, \ldots, n$ can be obtained by

$$\bar{\mathbf{b}}_1 = \mathbf{G}\mathbf{d} \ . \tag{20}$$

The proposed semiblind detection framework can be illustrated in Fig. 1.

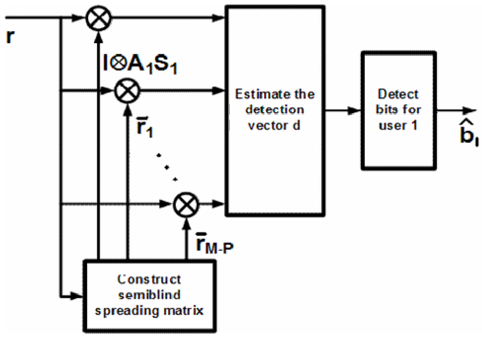

Fig. 1. The proposed semiblind Multiuser detection model and framework.

IV. Semi-Blind Multiuser Detection

After defining the known semi-blind signature matrix $\boldsymbol{\mathcal{S}}$ in (9), the conventional signal model (4) is transformed into (17) with the desired bits vector $\mathbf{b}_1$ replaced by the detection vector $\mathbf{d}$ and the original noise vector $\mathbf{n}$ replaced by $\bar{\mathbf{n}}$. On the other hand, the desired bits vector $\mathbf{b}$ can be detected using $\mathbf{d}$, $\mathbf{G}$ and (20). Now the question is how to efficiently estimate $\mathbf{d}$ in (17). In the following, we develop new estimation schemes based on LS, MUV and MMSE criteria.

A. *Least Square Estimation*

We first assume that the received measurements in $\boldsymbol{\mathcal{S}}$ are assumed to be free of error. Hence, all errors are confined to the received vector $\mathbf{r}$ due to $\bar{\mathbf{n}}$ in (17) in the nth bit interval. The following LS estimator is proposed along the lines of the classic decorrelating detector. Suppose $\mathbf{U}^{\mathrm{T}}\boldsymbol{\mathcal{S}}\mathbf{V} = \boldsymbol{\Sigma}$ is the SVD of $\boldsymbol{\mathcal{S}} \in \mathbb{R}^{PL\times M}$ with $r = \mathrm{rank}\{\boldsymbol{\mathcal{S}}\}$. If $\mathbf{U} = [\mathbf{u}_1\ \mathbf{u}_2\ \ldots\ \mathbf{u}_{PL}]$, $\mathbf{V} = [\mathbf{v}_1\ \mathbf{v}_2\ \ldots\ \mathbf{v}_M]$, $\boldsymbol{\Sigma} = \mathrm{diag}\{[\sigma_1\ \ldots\ \sigma_r\ 0\ \ldots\ 0]\}$ and $\mathbf{r} \in \mathbb{R}^{L\times 1}$, then [14]

$$\mathbf{d}_{\mathrm{LS}} = \textstyle\sum_{i=1}^{r} \frac{\mathbf{u}_i^{\mathrm{T}}\mathbf{r}}{\sigma_i}\mathbf{v}_i = \boldsymbol{\mathcal{S}}^{+}\mathbf{r} \tag{21}$$

minimizes $\|\boldsymbol{\mathcal{S}}\mathbf{d} - \mathbf{r}\|_2$ and has the smallest 2-norm of all minimizers. Moreover, the minimum squared error achieved is

$$\varepsilon_{\mathrm{LS}}^2 = \min_{\mathbf{x}\in\mathbb{R}} \|\boldsymbol{\mathcal{S}}\mathbf{x} - \mathbf{r}\|_2^2 = \sum_{i=r+1}^{L} \left(\mathbf{u}_i^{\mathrm{T}}\mathbf{r}\right)^2 \ . \tag{22}$$

so that the bit sent by the first user, b_1, in the nth signaling interval is detected with

$$\hat{\mathbf{b}}_{1\mathrm{LS}} = \mathrm{sign}\left\{\mathbf{G}^{\mathrm{T}}\boldsymbol{\mathcal{S}}^{+}\mathbf{r}\right\} \ . \tag{23}$$

B. *Total Least-Square Estimation*

The previous LS estimate of the detection vector $\mathbf{d}$ from (17), (18) and (21) is the solution to

$$\hat{\mathbf{d}} = \min_{\mathbf{x}} \|\mathbf{r} - \boldsymbol{\mathcal{S}}\mathbf{x}\|_2 \quad \text{subject to} \quad \mathbf{r} \subseteq \mathbb{R}(\boldsymbol{\mathcal{S}}) \tag{24}$$

where the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$ is assumed to be error-free. However, this assumption is not entirely accurate according to the definition of $\boldsymbol{\mathcal{S}}$ in (9) since there is a noise term, $\mathbf{N}$.

Furthermore, $\mathbf{r}$ can also be expressed as

$$\begin{aligned} \mathbf{r} &= (\boldsymbol{\mathcal{S}} - \bar{\mathbf{N}})\mathbf{B}^{+}\mathbf{b} + \mathbf{n} \\ &= \hat{\boldsymbol{\mathcal{S}}}\mathbf{d} + \bar{\mathbf{n}} \end{aligned} \tag{25}$$

where $\hat{\boldsymbol{\mathcal{S}}} = \boldsymbol{\mathcal{S}} - \mathbf{N} = \mathbf{SAB}$. The minimization problem of (24) can then be transformed into the following TLS problem:

$$\left[\hat{\boldsymbol{\mathcal{S}}},\ \mathbf{x}\right] = \min_{\bar{\boldsymbol{\mathcal{S}}},\ \mathbf{x}} \|[\,\boldsymbol{\mathcal{S}} \quad \mathbf{r}\,] - [\,\bar{\boldsymbol{\mathcal{S}}} \quad \bar{\boldsymbol{\mathcal{S}}}\mathbf{x}\,]\|_2 \ , \tag{26}$$

subject to $\mathbf{r} \subseteq \mathbb{R}(\bar{\boldsymbol{\mathcal{S}}})$. Let $\boldsymbol{\mathcal{S}} = \mathbf{U}^{'}\mathbf{\Sigma}^{'}\mathbf{V}^{'T}$ and $[\boldsymbol{\mathcal{S}}\ \mathbf{r}] = \mathbf{U\Sigma V}^{\mathrm{T}}$ be the SVD of $\boldsymbol{\mathcal{S}}$ and $[\boldsymbol{\mathcal{S}}\ \mathbf{r}]$, respectively. If $\sigma^{'}_{PK+K-1} > \sigma_{PK+K}$, then [15]

$$\mathbf{d}_{\mathrm{TLS}} = \left(\boldsymbol{\mathcal{S}}^{\mathrm{T}}\boldsymbol{\mathcal{S}} - \sigma^2_{PK+K}\mathbf{I}\right)^{-1}\boldsymbol{\mathcal{S}}^{\mathrm{T}}\mathbf{r} \tag{27}$$

and

$$\begin{aligned} \varepsilon^2_{\mathrm{TLS}} &= \min_{\mathbf{x}\in\mathbb{R}^{K\times 1}} \|\boldsymbol{\mathcal{S}}\mathbf{x} - \mathbf{r}\|^2_2 \\ &= \sigma^2_{PK+K}\left[1 + \sum_{i+1}^{K} \frac{\mathbf{u}_i^{'T}\mathbf{r}^{\ 2}}{\sigma_i^{'2} - \sigma^2_{PK+K}}\right] \end{aligned} \tag{28}$$

where $\mathbf{U} = [\mathbf{u}_1\ \mathbf{u}_2\ \ldots\ \mathbf{u}_{PL}]$, $\mathbf{V} = [\mathbf{v}_1\ \mathbf{v}_2\ \ldots\ \mathbf{v}_{M+1}]$, $\mathbf{\Sigma} = \mathrm{diag}\{[\sigma_1\ \sigma_2 \ldots\ \sigma_{M+\min\{PL-M,\,1\}}]\}$ and $\mathbf{U}^{'} = [\mathbf{u}^{'}_1\ \mathbf{u}^{'}_2\ \ldots\ \mathbf{u}^{'}_{PL}]$, $\mathbf{V}^{'} = [\mathbf{v}^{'}_1\ \mathbf{v}^{'}_2\ \ldots\ \mathbf{v}^{'}_{M}]$, $\mathbf{\Sigma}^{'} = \mathrm{diag}\{[\sigma^{'}_1\ \sigma^{'}_2\ \ldots\ \sigma^{'}_{M}]\}$.
The bit sent by the first user, b_1, in the nth signaling interval can be detected with

$$\hat{\mathbf{b}}_{1\mathrm{TLS}} = \mathrm{sign}\left\{\mathbf{G}\left(\boldsymbol{\mathcal{S}}^{\mathrm{T}}\boldsymbol{\mathcal{S}} - \sigma^2_{PK+K}\mathbf{I}\right)^{-1}\boldsymbol{\mathcal{S}}^{\mathrm{T}}\mathbf{r}\right\} . \tag{29}$$

C. *Mixed LS/TLS Estimation*

In the LS problem of (24), it assumed the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$ is error-free. Again, this assumption is not completely accurate. In the TLS problem of (26), it assumed that in each column of the semi-blind signature matrix, $\boldsymbol{\mathcal{S}}$, some noise or error exists. This assumption also is not complete. Though there exists a noise or error matrix $\mathbf{N}$ in $\boldsymbol{\mathcal{S}}$ from (9), its first column is exactly known to be noise-free or error-free. Hence, to maximize the estimation accuracy of the detection vector $\mathbf{d}$, it is natural to require that the corresponding columns of $\boldsymbol{\mathcal{S}}$ be unperturbed since they are known exactly. The problem of estimating the detection vector $\mathbf{d}$ can then be transformed into the following MLS problem by considering (24) and (26):

$$\min_{\tilde{\boldsymbol{\mathcal{S}}},\ \mathbf{x}} \|[\,\tilde{\boldsymbol{\mathcal{S}}} \quad \mathbf{r}\,] - [\,\bar{\boldsymbol{\mathcal{S}}} \quad [\mathbf{A}_1\mathbf{S}_1\ \bar{\boldsymbol{\mathcal{S}}}]\mathbf{x}\,]\|_2 \tag{30}$$

subject to $\mathbf{r} \subseteq \mathbb{R}([\mathbf{A}_1\mathbf{S}_1\ \bar{\boldsymbol{\mathcal{S}}}])$. Consider the MLS problem in (30) and perform the Householder transformation $\mathbf{Q}$ on the matrix $[\,\boldsymbol{\mathcal{S}} \quad \mathbf{r}\,]$ so that

$$\mathbf{Q}^{\mathrm{T}}[\,\mathbf{A}_1\mathbf{S}_1 \quad \bar{\boldsymbol{\mathcal{S}}} \quad \mathbf{r}\,] = \begin{bmatrix} \mathbf{R}_{11} & \mathbf{R}_{12} & R_{1r} \\ \mathbf{0} & \mathbf{R}_{22} & \mathbf{R}_{2r} \end{bmatrix} \tag{31}$$

where $\mathbf{R}_{12}$ is a $1 \times (M-1)$ vector, $\mathbf{R}_{22}$ is a $(L-1) \times (M-1)$ matrix and $\mathbf{R}_{2r}$ is a $(L-1) \times 1$ vector.

Denote σ' as the smallest singular value of $\mathbf{R}_{22}$ and σ as the smallest singular value of $[\,\mathbf{R}_{22} \quad \mathbf{R}_{2r}\,]$. If $\sigma' > \sigma$, then the MLS solution uniquely exists and is given by [15]

$$\mathbf{d}_{\mathrm{MLS}} = \left(\boldsymbol{\mathcal{S}}^{\mathrm{T}}\boldsymbol{\mathcal{S}} - \sigma^2\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{M-1} \end{bmatrix}\right)^{-1}\boldsymbol{\mathcal{S}}^{\mathrm{T}}\mathbf{r} \ . \tag{32}$$

The bit sent by the first user, b_1, during the nth signaling interval can be detected with

$$\hat{\mathbf{b}}_{1\mathrm{MLS}} = \mathrm{sign}\left\{\mathbf{G}\left(\boldsymbol{\mathcal{S}}^{\mathrm{T}}\boldsymbol{\mathcal{S}} - \sigma^2\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{M-1} \end{bmatrix}\right)^{-1}\boldsymbol{\mathcal{S}}^{\mathrm{T}}\mathbf{r}\right\} . \tag{33}$$

D. *Minimum Variance Unbiased Estimation*

The optimal estimator which constrains the bias to be zero and minimizes the variance is termed MVU estimator. When MVU estimator

$$\mathbf{d}_{\mathrm{MVU}} = \mathbf{f}(\mathbf{r}) \tag{34}$$

exists, it may be found that it attains the Cramer-Rao Lower Bound (CRLB) so that

$$\frac{\partial \ln Pr(\mathbf{r};\ \mathbf{d})}{\partial \mathbf{d}} = \mathbf{I}(\mathbf{d})\,[\mathbf{f}(\mathbf{r}) - \mathbf{d}] \ , \tag{35}$$

where $Pr(\mathbf{r};\ \mathbf{d})$ is the joint PDF of $\mathbf{r}$ and $\mathbf{d}$ and $\mathbf{I}(\mathbf{d})$ is the Fisher information matrix (FIM) defined by

$$\mathbf{I(d)} = \mathrm{E}\left\{\left(\frac{\partial \ln Pr(\mathbf{r};\ \mathbf{d})}{\partial \mathbf{d}}\right)\left(\frac{\partial \ln Pr(\mathbf{r};\ \mathbf{d})}{\partial \mathbf{d}}\right)^{\mathrm{H}}\right\} . \tag{36}$$

Though the determination of the optimal MVU estimator is generally difficult, it can be evident with linear constraint and the MVU estimator for $\mathbf{d}$ then is [16]

$$\mathbf{d}_{\mathrm{MVU}} = (\boldsymbol{\mathcal{S}}^{\mathrm{T}}\mathbf{C}_{\bar{\mathbf{n}}}^{-1}\boldsymbol{\mathcal{S}})^{-1}\boldsymbol{\mathcal{S}}^{\mathrm{T}}\mathbf{C}_{\bar{\mathbf{n}}}^{-1}\mathbf{r} \,. \tag{37}$$

The covariance matrix of $\mathbf{d}_{\mathrm{MVU}}$ given by

$$\mathbf{C}_{\mathbf{d}_{\mathrm{MVU}}} = (\boldsymbol{\mathcal{S}}^{\mathrm{T}}\mathbf{C}_{\bar{\mathbf{n}}}^{-1}\boldsymbol{\mathcal{S}})^{-1} \ . \tag{38}$$

Though the PDF of $\mathbf{B}$ may be determined, the PDF of $\mathbf{B}^{+}$ is largely unknown. This makes it hard to calculate the closed-form solution of $\mathbf{C}_{\mathbf{d}}$ and $\mathbf{C}_{\bar{\mathbf{n}}}$. However, with Girko's Law, when $\alpha = (PK+K-2)/(M-1)$ is fixed,

K, $M \to \infty$, the diagonal element of $\frac{1}{M-1}\tilde{\mathbf{D}}^{+}\tilde{\mathbf{b}}\tilde{\mathbf{b}}^{\mathrm{T}}\tilde{\mathbf{D}}^{+\mathrm{T}}$ may be approximated by [17, 18]

$$\lim_{k,M\leftarrow\infty} \frac{1}{M-1}\left[\tilde{\mathbf{D}}^{+}\tilde{\mathbf{b}}\tilde{\mathbf{b}}^{\mathrm{T}}\tilde{\mathbf{D}}^{+\mathrm{T}}\right]_{ii}^{-1} = 1-\alpha \ . \quad (39)$$

Hence $\mathbf{C}_{\mathbf{d}}$ can be decided by

$$\mathbf{C}_{\mathbf{d}} = \begin{bmatrix} \frac{2M-PK-K}{M-PK-K+1} & \mathbf{0}^{\mathrm{T}} \\ \mathbf{0} & \frac{1}{M-PK-K+1}\mathbf{I} \end{bmatrix} , \quad (40)$$

and

$$\mathbf{C}_{\bar{\mathbf{n}}} = \sigma^2 \frac{2M-PK-K}{M-PK-K+1}\mathbf{I} \quad (41)$$

and the bit vector for user 1 can be detected by

$$\hat{\mathbf{b}}_{1\mathrm{MVU}} = \mathrm{sign}\left\{\mathbf{G}(\boldsymbol{\mathcal{S}}^{\mathrm{T}}\boldsymbol{\mathcal{S}})^{-1}\boldsymbol{\mathcal{S}}^{\mathrm{T}}\mathbf{r}\right\} \quad (42)$$

Now we can see that the proposed MVU detector in (42) actually has almost the same as the LS detector proposed in [12, 13].

E. *Minimum Mean Squared Error Estimation*

Though the optimal Bayesian estimators are difficult to determine in closed form or too computationally intensive to implement in general, they can be found under the joint Gaussian assumption and linear constrain. This class of MMSE estimators are generically termed Wiener filter. Given measurements $\mathbf{r}$, the MMSE estimator of $\mathbf{d}$, $\mathbf{d}_{\mathrm{MMSE}} = f(\mathbf{r})$, minimizes the mean-squared error $J_{\mathrm{MSE}} = E\{||\mathbf{d}-\hat{\mathbf{d}}||_2^2\}$. The function $f(\mathbf{r})$ may be nonlinear or linear and its exact structure is determined by minimizing J_{MSE}. When $\mathbf{d}$ and $\mathbf{r}$ are jointly Gaussian, the linear estimator $\mathbf{W}_{\mathrm{MMSE}}$ that minimizes the mean-squared error is (Bayesian Gauss-Markov Theorem)

$$\mathbf{d}_{\mathrm{MMSE}} = (\mathbf{C}_{\mathbf{d}}^{-1} + \boldsymbol{\mathcal{S}}^{\mathrm{T}}\mathbf{C}_{\bar{\mathbf{n}}}^{-1}\boldsymbol{\mathcal{S}})^{-1}\boldsymbol{\mathcal{S}}^{\mathrm{T}}\mathbf{C}_{\bar{\mathbf{n}}}^{-1}\mathbf{r} \quad (43)$$

and its performance is measured by the covariance matrix of the error $\epsilon = \mathbf{d}-\hat{\mathbf{d}}$ given by

$$\mathbf{C}_{\epsilon} = \left(\mathbf{C}_{\mathbf{d}}^{-1} + \boldsymbol{\mathcal{S}}^{\mathrm{T}}\mathbf{C}_{\bar{\mathbf{n}}}^{-1}\boldsymbol{\mathcal{S}}\right)^{-1} . \quad (44)$$

The bit vector for user 1 can be detected by

$$\hat{\mathbf{b}}_1 = \mathrm{sign}\left\{\mathbf{G}(\mathbf{C}_{\mathbf{d}}^{-1} + \boldsymbol{\mathcal{S}}^{\mathrm{T}}\mathbf{C}_{\bar{\mathbf{n}}}^{-1}\boldsymbol{\mathcal{S}})^{-1}\boldsymbol{\mathcal{S}}^{\mathrm{T}}\mathbf{C}_{\bar{\mathbf{n}}}^{-1}\mathbf{r}\right\} . \quad (45)$$

Combined with (40) and (41), $\hat{\mathbf{b}}_1$ can be further simplified as

$$\hat{\mathbf{b}}_{1\mathrm{MMSE}} = \mathrm{sign}\left\{\mathbf{G}\left(\mathcal{C}\sigma^{\mathbf{2}} + \boldsymbol{\mathcal{S}}^{\mathrm{T}}\boldsymbol{\mathcal{S}}\right)^{-1}\boldsymbol{\mathcal{S}}^{\mathrm{T}}\mathbf{r}\right\} \quad (46)$$

where

$$\mathcal{C} = \begin{bmatrix} 1 & \mathbf{0}^{\mathrm{T}} \\ \mathbf{0} & (2M-PK-K)\mathbf{I} \end{bmatrix} . \quad (47)$$

V. Adaptive Implementation

Following the well-known Sherman-Morrison-Woodbury matrix inverse lemma [14], an adaptive implementation of the proposed MVU semiblind detector can be expressed by

$$\hat{\mathbf{b}}_1(n) = \mathrm{sign}\left\{\mathbf{G}(n)\boldsymbol{\mathcal{C}}_{\mathcal{S}}^{+}(n)\boldsymbol{\mathcal{S}}^{\mathrm{T}}(n)\mathbf{r}(n)\right\} \quad (48)$$

$$\boldsymbol{\mathcal{C}}_{\mathcal{S}}^{+}(n) = \boldsymbol{\mathcal{C}}_{\mathcal{S}}^{+}(n-1) - \left[\boldsymbol{\mathcal{C}}_{\mathcal{S}}^{+}(n-1)\mathbf{U}(n-1)\mathbf{U}^{\mathrm{T}}(n-1)\boldsymbol{\mathcal{C}}_{\mathcal{S}}^{+}(n-1)\right]\left[\mathbf{I} + \mathbf{U}^{\mathrm{T}}(n-1)\boldsymbol{\mathcal{C}}_{\mathcal{S}}^{+}(n-1)\mathbf{U}(n-1)\right]^{-1} \quad (49)$$

where

$$\boldsymbol{\mathcal{C}}_{\mathcal{S}}(n) = \boldsymbol{\mathcal{S}}(n)^{\mathrm{T}}\boldsymbol{\mathcal{S}}(n) \quad (50)$$

and $\mathbf{U}(n-1)$ is designed using SVD so that

$$\mathbf{U}(n-1)\mathbf{U}^{\mathrm{T}}(n-1) = \boldsymbol{\mathcal{C}}_{\mathcal{S}}(n) - \boldsymbol{\mathcal{C}}_{\mathcal{S}}(n-1) \quad (51)$$

VI. Performance Analysis

A. *Multiuser Signal Model Comparison*

The comparison between the proposed asynchronous multiuser signal models and the classic single-window signal model is given in Table 1.

B. *Comparison with The Classic Decorrelator*

When $P = 1$, there is no noise in $\boldsymbol{\mathcal{S}}$ and $\mathbf{G}$ is accurately known beforehand, there is the following relationship between the user 1's MVU detector $\mathbf{w}_{\mathrm{MVU}}$ and the decorrelator $\mathbf{w}_{\mathrm{DD}}$.

$$\mathbf{w}_{\mathrm{MVU}} = \mathbf{G}(\boldsymbol{\mathcal{S}}^{\mathrm{T}}\boldsymbol{\mathcal{S}})^{-1}\boldsymbol{\mathcal{S}}^{\mathrm{T}} = A_1^{-1}\mathbf{w}_{\mathrm{DD}} \quad (52)$$

On the other hand, $\mathbf{w}_{\mathrm{DD}}$ can be taken as a special case of $\mathbf{w}_{\mathrm{BLU}}$ with $\mathbf{B} = \mathbf{I}$ and $P = 1$.

C. *The New Noise Vector* $\bar{\mathbf{n}}$

The mean of the semi-blind noise term $\bar{\mathbf{n}}$ in (19) given by

$$\bar{\mathbf{m}} = \mathrm{E}\{\bar{\mathbf{n}}\} = 0 \quad (53)$$

The variance of $\tilde{\mathbf{n}}$ satisfies the following inequality

$$\left|\mathrm{E}\{(\bar{\mathbf{n}}-\bar{\mathbf{m}})^2\}\right|_{\infty} \le \sigma_n^2 + (P+1)(K-1)\|\tilde{\mathbf{D}}^{+}\|_2^2\sigma_{\bar{n}}^2 \quad (54)$$

where $\left|\star\right|_{\infty}$ denotes the infinity norm of vector $\star$ and $\sigma_{\bar{n}}^2$ is the power of the noise term $\bar{\mathbf{N}}$ in the semi-blind signature matrix $\boldsymbol{\mathcal{S}}$.

Parameters	The proposed model	The conventional model
Common/Shared	dedicated	shared
Required Timing Information	only user 1	all users
Required Amplitude Information	only user 1	all users
Required Spreading Sequences	only user 1	all users
Input Vector	$\mathbf{r}$ – $1 \times \mathrm{PL}$	$\mathbf{r_1}$ – $1 \times \mathrm{L}$
Output Vector	$\bar{\mathbf{b}}_1$ – $1 \times \mathrm{P}$	$\mathbf{b}_1$ – $1 \times \mathrm{K}$
Num. of Detected Bits	P	1
Spreading Matrix	$\boldsymbol{\mathcal{S}}$ – $\mathrm{PL} \times \mathrm{M}$	$\mathbf{S}$ – $\mathrm{L} \times \mathrm{K}$
Amplitude Matrix	N/A	$\mathbf{A}$ – $\mathrm{K} \times \mathrm{K}$
Noise Vector	$\bar{\mathbf{n}}$ – $1 \times \mathrm{PL}$	$\mathbf{n}$ – $1 \times \mathrm{L}$

Table 1. The comparison of the proposed semiblind multiuser signal model and the conventional signal model

D. *AME and Near-Far Resistance*

A commonly used performance measure for a multiuser detector is AME and near-far resistance [7]. Since the proposed algorithms converge to the conventional decorrelating detector as $\sigma \to 0$, their AME and near-far resistance for user 1 is

$$\bar{\eta}_1 = [\mathbf{R}^{+}]_{11}^{-1} = \left[(\mathbf{S}^{\mathrm{T}}\mathbf{S})^{+}\right]_{11}^{-1} . \tag{55}$$

E. *CRLB for* $\mathbf{d}$ *Estimation*

The CRLB is given by the inverse of the FIM. Providing the blind spreading matrix $\boldsymbol{\mathcal{S}}$ is known beforehand, we first define the parameter vector $\phi = \left[\bar{\sigma}^2 \ \mathbf{d}^{\mathrm{T}}\right]^{\mathrm{T}}$, where $\bar{\sigma}^2 = (1 + \frac{M-1}{M-PK-K+1})\sigma^2$, for computing the FIM, which is defined by

$$\mathbf{I}(\phi) = \mathrm{E}\left\{\left(\frac{\partial \ln \Pr}{\partial \phi}\right)\left(\frac{\partial \ln \Pr}{\partial \phi}\right)^{\mathrm{H}}\right\} \tag{56}$$

where $\ln \Pr$ is the log-likelihood function given by

$$\ln \Pr = C - L \ln \bar{\sigma}^2 - \frac{1}{2\bar{\sigma}^2} \parallel \mathbf{e} \parallel_2^2 , \tag{57}$$

C is a constant and $\mathbf{e} = \mathbf{r} - \boldsymbol{\mathcal{S}}\mathbf{d}$. Providing $\boldsymbol{\mathcal{S}}$ is known, the closed-form CRLB expression of $\mathbf{d}$ is then given by

$$\mathrm{CRLB}(\mathbf{d} \mid \boldsymbol{\mathcal{S}}) = (1 + \frac{M-1}{M-PK-K+1})\sigma^2(\boldsymbol{\mathcal{S}}^{\mathrm{T}}\boldsymbol{\mathcal{S}})^{+} . \tag{58}$$

From (58), it shows that the accuracy of estimating $\mathbf{d}$ may increase with increasing M.

VII. Computer Simulations

In this section, various computer simulation results are presented to demonstrate the performance of our proposed semiblind detectors. In our computer simulations, we assume a chip-level synchronized single base station system. In this system, there are $K = 10$ users sending asynchronous signals to the base station. The delays between interfering users and the first user are random variables between 1 and 63 chips. All spreading sequences are random sequences with the spreading gain

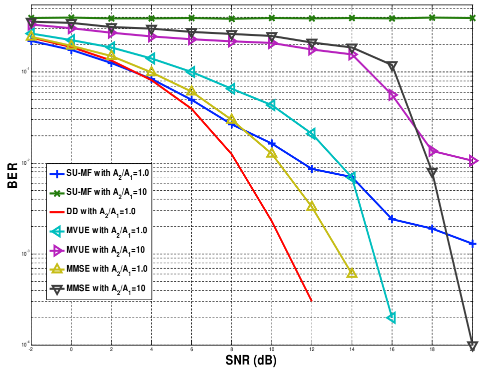

Fig. 2. The BER performance comparison of the single-user matched filter, decorrelating detector and the proposed detectors.

$G = 64$. The size of the semiblind spreading matrix $\boldsymbol{\mathcal{S}}$ is 64×30 with $M = 30$. In Fig. 2, we compare the BER performance of the proposed semiblind detectors with the single-user matched filter (SU-MF) and decorrelating detector against changing SNR. It shows that the performance of the conventional decorrelator and SU-MF is better than the proposed semiblind detectors when the near-far ratio is small, $A_2/A_1 = 1.0$. However, when MAI is strong and $A_2/A_1 = 10$, the SU-MF experiences the near-far problem and the performance of the proposed semiblind detectors is between the decorrelating detector and SU-MF. In Fig. 3, we examine the near-far resistance of the proposed semiblind detectors. We see that the near-far resistance of the MVU detector is close to the decorrelating detector and the near-far resistance of the MMSE semiblind detector is close to the SU-MF when A_2/A_1 is small. When A_2/A_1 becomes large, the near-far resistance of both the MVU and MMSE semiblind detectors are close the SU-MF. In Fig. 4 and 5, the BER performance of the proposed detectors is compared with the conventional detectors with changing the M and P individually. We see that the BER performance of the MVU semiblind detector is going down and the BER performance of the MMSE semiblind detector is going up

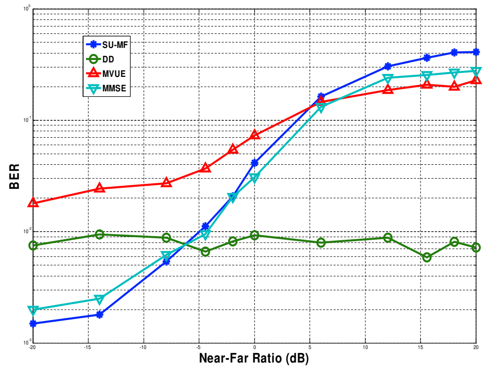

Fig. 3. The near-far resistance comparison of the single-user matched filter, decorrelating detector and the proposed detectors. $SNR = 6dB$

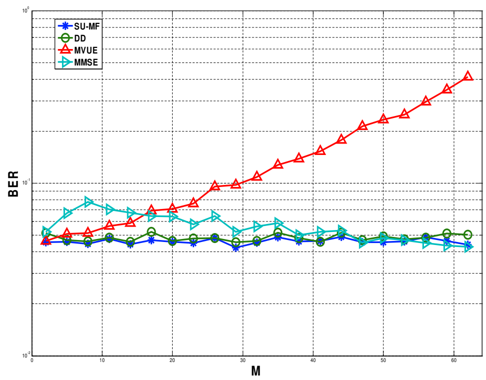

Fig. 4. The BER performance of the single-user matched filter, decorrelating detector and the proposed detectors with changing M. $P = 1$

when M is increased. However, the BER performance of the MMSE semiblind detector is going down while the BER performance of the MVU detector is going up when P is increased.

VIII. Conclusions

In this paper, a new semiblind multiuser detection framework and two semiblind detectors are proposed for asynchronous CDMA. Compared with most existing semiblind/blind multiuser detection schemes, the proposed schemes are simple and direct without any estimation or subspace separation operation and require a minimum number of previously received signals, as well as desired user's spreading sequence, timing and amplitude. Their performance are comparable with the conventional single-window decorrelating detector.

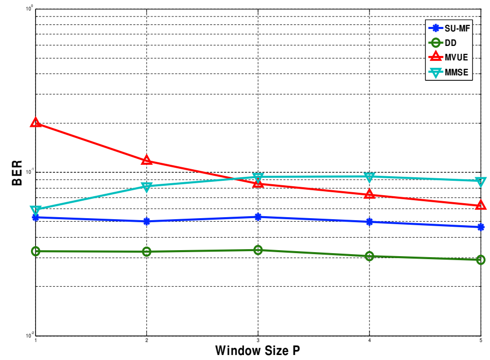

Fig. 5. The BER of the single-user matched filter, decorrelating detector and the proposed detectors with changing P. $M = 64$

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