Deflation results with MCMC

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1 A handful of constant preconditioners

Constant preconditioners are considered for the realization $\xi = \mathbf{1}$, denoted by $\mathbf{A}_0$.

1.1 Block Jacobi (bJ)

We first consider non-overlapping diagonal bJ preconditioners of the form

$$\mathbf{M}^{-1} = \sum_{j=1}^{n_b} \boldsymbol{\mathcal{R}}_j \mathbf{M}_j^{-1} \boldsymbol{\mathcal{R}}_j^T \tag{1}$$

with canonical restrictions $\boldsymbol{\mathcal{R}}_j$ from a global vector to each block $j \in [1, n_b]$. For each j, $\mathbf{M}_j^{-1}$ is applied with a Cholesky factorization $\mathbf{M}_j = \mathbf{L}_j \mathbf{L}_j^T = \boldsymbol{\mathcal{R}}_j^T \mathbf{A} \boldsymbol{\mathcal{R}}_j$.

1.2 Geometric domain decomposition

Let us consider a n_d-partition of Ω into subdomains $\{\Omega_d\}_{d=1}^{n_d}$ where each Ω_d is a contiguous union of elements, with a (non-Dirichlet) boundary denoted by $\partial\Omega_d$. The canonical restriction $\boldsymbol{\mathcal{R}}_d$ maps global (non-Dirichlet) degrees of freedom (DoFs) to mesh nodes contained in Ω_d. We assume local orderings such that $\boldsymbol{\mathcal{R}}_d = [\boldsymbol{\mathcal{R}}_{\mathrm{I},d}, \boldsymbol{\mathcal{R}}_{\Gamma,d}]$, where $\boldsymbol{\mathcal{R}}_{\mathrm{I},d}$ and $\boldsymbol{\mathcal{R}}_{\Gamma,d}$ map nodes from $\Omega_d \setminus \partial\Omega_d$ and $\partial\Omega_d$, respectively. Then, we write

$$\boldsymbol{\mathcal{R}}_d^T \mathbf{A} \boldsymbol{\mathcal{R}}_d = \begin{bmatrix} \boldsymbol{\mathcal{R}}_{\mathrm{I},d}^T \\ \boldsymbol{\mathcal{R}}_{\Gamma,d}^T \end{bmatrix} \mathbf{A} \begin{bmatrix} \boldsymbol{\mathcal{R}}_{\mathrm{I},d} & \boldsymbol{\mathcal{R}}_{\Gamma,d} \end{bmatrix} =: \begin{bmatrix} \mathbf{A}_{\mathrm{II},d} & \mathbf{A}_{\mathrm{I}\Gamma,d} \\ \mathbf{A}_{\Gamma\mathrm{I},d} & \mathbf{A}_{\Gamma\Gamma,d} \end{bmatrix} \tag{2}$$

and

$$\boldsymbol{\mathcal{R}}_d^T \mathbf{b} = \begin{bmatrix} \boldsymbol{\mathcal{R}}_{\mathrm{I},d}^T \\ \boldsymbol{\mathcal{R}}_{\Gamma,d}^T \end{bmatrix} \mathbf{b} =: \begin{bmatrix} \mathbf{b}_{\mathrm{I},d} \\ \mathbf{b}_{\Gamma,d} \end{bmatrix}. \tag{3}$$

After elimination of $\mathbf{u}_\mathrm{I}$ from $\mathbf{A}\mathbf{u} = \mathbf{b}$, we have

$$\mathbf{S}\mathbf{u}_\Gamma = \mathbf{b}_S \tag{4}$$

where $\mathbf{b}_S =$ and the Schur complement, $\mathbf{S}$, is applied

$$\mathbf{S}\mathbf{u}_\Gamma = \sum_{d=1}^{n_d} \mathbf{S}_d \mathbf{u}_{\Gamma,d} \tag{5}$$

with

$$\mathbf{S}_d \mathbf{u}_{\Gamma,d} = \mathbf{A}_{\Gamma\Gamma,d}\mathbf{u}_{\Gamma,d} - \mathbf{A}_{\Gamma\mathrm{I},d}\mathbf{A}_{\mathrm{II},d}^{-1}\mathbf{A}_{\mathrm{I}\Gamma,d}\mathbf{u}_{\Gamma,d}. \tag{6}$$

For non-overlapping partitions, $\mathbf{A}_{\mathrm{II},d}$ is SPD but $\mathbf{S}_d$ is semi-definite.

1.2.1 Neumann-Neumann (NN)

The NN preconditioner $\mathbf{M}_{NN}^{-1}$, see [], is defined as follows for Eq. (**??**),

$$\mathbf{M}_{NN}^{-1}\mathbf{u}_\Gamma = \sum_{d=1}^{n_d} \boldsymbol{\mathcal{R}}_{\Gamma,d} \tag{7}$$

where $\mathbf{S}_d^\dagger$ denotes the pseudo-inverse of $\mathbf{S}_d$.

1.2.2 Balanced domain decomposition (BDD)

1.2.3 Low rank Schur (LORASC)

$$\mathbf{S}\mathbf{u} = \lambda \mathbf{A}_{\Gamma\Gamma}\mathbf{u} \tag{8}$$

$$\tilde{\mathbf{S}}^{-1} = \mathbf{A}_{\Gamma\Gamma}^{-1} + \mathbf{E}\boldsymbol{\Sigma}\mathbf{E}^T \tag{9}$$

where $\mathbf{E} = [\mathbf{v}_1, \ldots, \mathbf{v}_i]$

1.3 Algebraic multigrid (AMG)

2 Deflation results

2.1 Weak scaling

Results in Tab. 1 are *.

2.2 Strong scaling results

2.3 Scaling of AMG

Table 1: Weak scaling results of deflation for a constant bJ preconditioner with $nvec = \lfloor 1.25 \times nb \rfloor$ and $spdim = 3 \times nb$.

n	nb	pcg iter	nvec	spdim	eigdefpcg iter	defpcg iter
20,208	8	828.6 ± 166.5	10	24	488.6 ± 93.4	xxx.xx ± xx.xx
40,406	16	1,435.2 ± 357.2	20	48	715.3 ± 154.6	xxx.xx ± xx.xx
80,598	32	2,241.5 ± 528.4	40	96	1,043.2 ± 294.2	xxx.xx ± xx.xx
160,941	64	3,321.4 ± 559.3	80	192	1,340.8 ± 370.6	xxx.xx ± xx.xx
321,386	128	5,698.0 ± 862.7	160	384	2,996.1 ± 728.3	xxx.xx ± xx.xx

Table 2: Weak scaling results of deflation for a constant LORASC preconditioner with $\varepsilon = 0$, $nvec = \lfloor 1.25 \times ndom \rfloor$ and $spdim = 3 \times ndom$.

n	ndom	pcg iter	nvec	spdim	eigdefpcg iter	defpcg iter
20,208	8	243.9 ± 49.9	10	24	158.6 ± 35.7	xxx.x ± xx.x
40,406	16	348.3 ± 89.6	20	48	181.7 ± 43.0	xxx.x ± xx.x
80,598	32	453.9 ± 115.9	40	96	201.6 ± 51.1	xxx.x ± xx.x
160,941	64	603.2 ± 134.7	80	192	239.5 ± 47.0	xxx.x ± xx.x
321,386	128	781.0 ± 000.0	160	384	280.0 ± 00.0	xxx.x ± xx.x

Table 3: Weak scaling results of deflation for a constant LORASC preconditioner with $\varepsilon = 0.01$, $nvec = \lfloor 1.25 \times ndom \rfloor$ and $spdim = 3 \times ndom$.

n	ndom	pcg iter	nvec	spdim	eigdefpcg iter	defpcg iter
20,208	8	231.6 ± 47.8	10	24	160.0 ± 36.3	xxx.x ± xx.x
40,406	16	324.5 ± 84.0	20	48	182.3 ± 44.2	xxx.x ± xx.x
80,598	32	388.7 ± 98.3	40	96	202.5 ± 53.7	xxx.x ± xx.x
160,941	64	498.3 ± 116.6	80	192	236.6 ± 45.4	xxx.x ± xx.x
321,386	128	661.0 ± 000.0	160	384	283.0 ± 00.0	xxx.x ± xx.x

Table 4: Weak scaling results of deflation for a constant Neumann-Neumann preconditioner with $nvec = \lfloor 1.25 \times ndom \rfloor$ and $spdim = 3 \times ndom$.

n	n_Γ	ndom	pcg iter	nvec	spdim	eigdefpcg iter	defpcg iter
20,208	641	8	77.6 ± 19.9	10	24	71.3 ± 17.8	xxx.x ± xx.x
40,406	1,422	16	107.3 ± 28.3	20	48	89.7 ± 27.3	xxx.x ± xx.x
80,598	2,800	32	157.2 ± 29.8	40	96	94.9 ± 27.2	xxx.x ± xx.x
160,941	5,600	64	533.9 ± 88.0	80	192	273.1 ± 84.8	xxx.x ± xx.x
321,386	11,200	128	794.0 ± 129.3	160	384	313.5 ± 83.0	xxx.x ± xx.x

Table 5: Strong scaling results of deflation for a constant bJ preconditioner with $nvec = \lfloor 1.25 \times nb \rfloor$ and $spdim = 3 \times nb$.

n	nb	pcg iter	nvec	spdim	eigdefpcg iter	defpcg iter
321,386	8	x,xxx.x ± xx.xx	10	24	xxx.xx ± xx.xx	xxx.xx ± xx.xx
321,386	16	x,xxx.x ± xx.xx	20	48	xxx.xx ± xx.xx	xxx.xx ± xx.xx
321,386	32	x,xxx.x ± xx.xx	40	96	xxx.xx ± xx.xx	xxx.xx ± xx.xx
321,386	64	4,256.9 ± 452.4	80	192	1,959.0 ± 550.7	xxx.xx ± xx.xx
321,386	128	5,698.0 ± 862.7	160	384	2,996.1 ± 728.3	xxx.xx ± xx.xx

Table 6: Strong scaling results of deflation for a constant LORASC preconditioner with $\varepsilon = 0$, $nvec = \lfloor 1.25 \times nb \rfloor$ and $spdim = 3 \times nb$.

n	nb	pcg iter	nvec	spdim	eigdefpcg iter	defpcg iter
321,386	8	xxx.xx ± xx.xx	10	24	xxx.xx ± xx.xx	xxx.xx ± xx.xx
321,386	16	xxx.xx ± xx.xx	20	48	xxx.xx ± xx.xx	xxx.xx ± xx.xx
321,386	32	xxx.xx ± xx.xx	40	96	xxx.xx ± xx.xx	xxx.xx ± xx.xx
321,386	64	xxx.xx ± xx.xx	80	192	xxx.xx ± xx.xx	xxx.xx ± xx.xx
321,386	128	xxx.xx ± xx.xx	160	384	xxx.xx ± xx.xx	xxx.xx ± xx.xx

Table 7: Strong scaling results of deflation for a constant Neumann-Neumann preconditioner with $nvec = \lfloor 1.25 \times ndom \rfloor$ and $spdim = 3 \times ndom$.

n_Γ	ndom	pcg iter	nvec	spdim	eigdefpcg iter	defpcg iter
xx	8	xxx.x ± xx.x	10	24	xxx.x ± xx.x	xxx.x ± xx.x
xx	16	xxx.x ± xx.x	20	48	xxx.x ± xx.x	xxx.x ± xx.x
xx	32	xxx.x ± xx.x	40	96	xxx.x ± xx.x	xxx.x ± xx.x
xx	64	xxx.x ± xx.x	80	192	xxx.x ± xx.x	xxx.x ± xx.x
xx	128	xxx.x ± xx.x	160	384	xxx.x ± xx.x	xxx.x ± xx.x

Table 8: Scaling results of the constant AMG preconditioner.

n	pcg iter
20,208	125.9 ± 35.0
40,406	136.5 ± 39.2
80,598	144.7 ± 51.7
160,941	150.4 ± 48.3
321,386	xxx.x ± xx.x