# On MIMO Relay Network with Finite-Rate Feedback and Imperfect Channel Estimation


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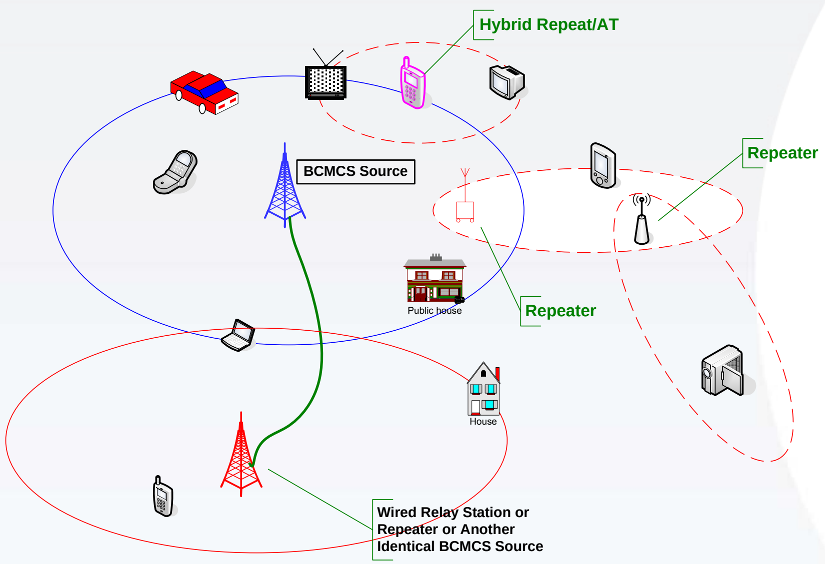
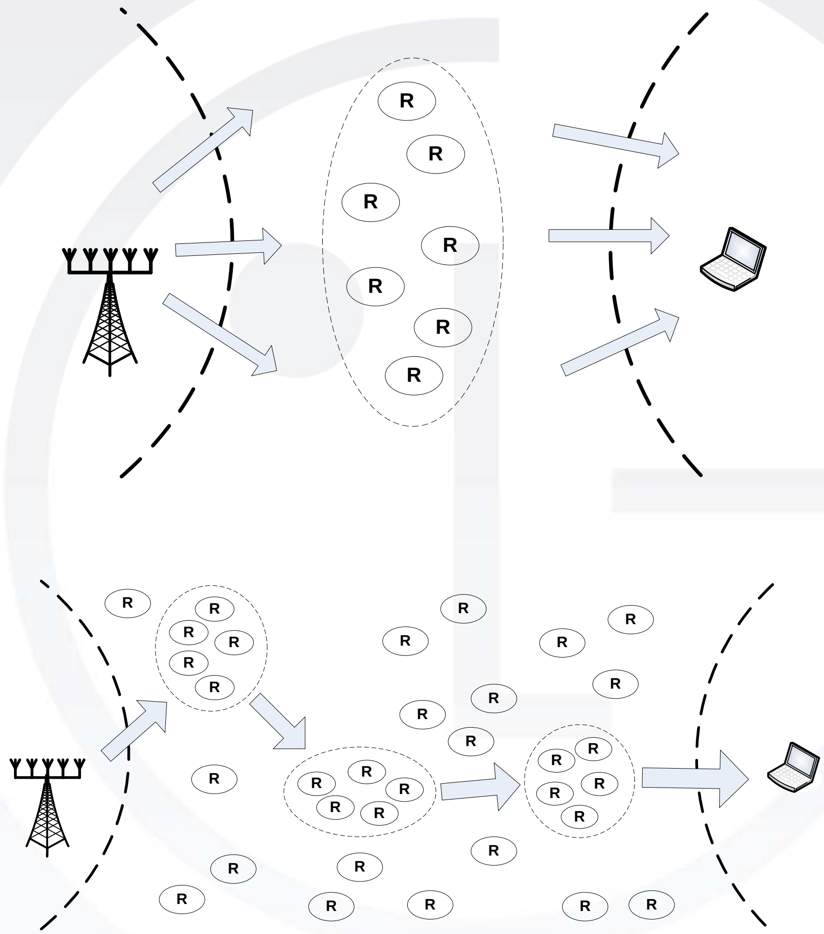
# Introduction

- MIMO brings an additional dimension as well as new challenges into next generation wireless system design. Relay network with MIMO is believed to be the next frontier for relieving network from capacity and coverage limits.
- Many existing discussions of MIMO precoding mismatch with finite-rate feedback are based on the assumptions such as: [Mukkavilli et al., 03; Xia and Giannakis, 04; Roh and Rao 04 ]
  - The forward-link channel estimation or reverse-link feedback channel is perfect;
  - Beamforming distortion is mostly decided by channel quantization error.
- In the current system development, a more accurate closed-loop MIMO beamforming model is desired to include
  - Forward-link channel parameter estimation errors,
  - Reverse-link channel feedback errors, and
  - Lloyd vector quantization errors
- In this discussion, we want to understand how imperfect channel estimation/quantization and channel feedback affect MIMO beamforming, such as
  - some performance bounds: channel estimation/quantization, etc.
  - some scalability results in terms of codebook size, pilot size, power etc.

# MIMO Relay Communication Scenario



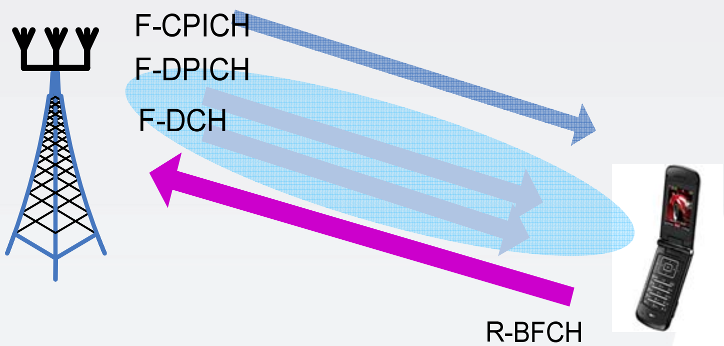
# MIMO Channel Capacity

- Noncoherent MIMO:
  - Channel **H** is unknown to Tx/Rx.
    **$C = K(1 - K/T)\log(SNR) + O(1)$**, where $K = \min\{M, N, T/2\}$.
    [ Zhang and Tse, 2000 ]

- Coherent MIMO:
  - **H** is known to Rx only: **$C = \min(M, N)\log(SNR) + O(1)$**.
  - **H** is known to Tx/Rx: **$C = \min(M, N)\log(SNR) + O(1)$**.
    [Blankenburg and Wyner, 1970's; Foschini and Telatar 1995]

- In higher SNR region, that transmitter knows the channel may not help much in terms of achievable channel capacity.
- However, CQI at Tx can help simplify receiver design.
  - Perfect CQI feedback is too expensive. It is not reasonable to feedback *M*N* complex values
  - Finite-rate feedback with channel quantization is the more practical approach.

# MIMO BF Model with Feedback

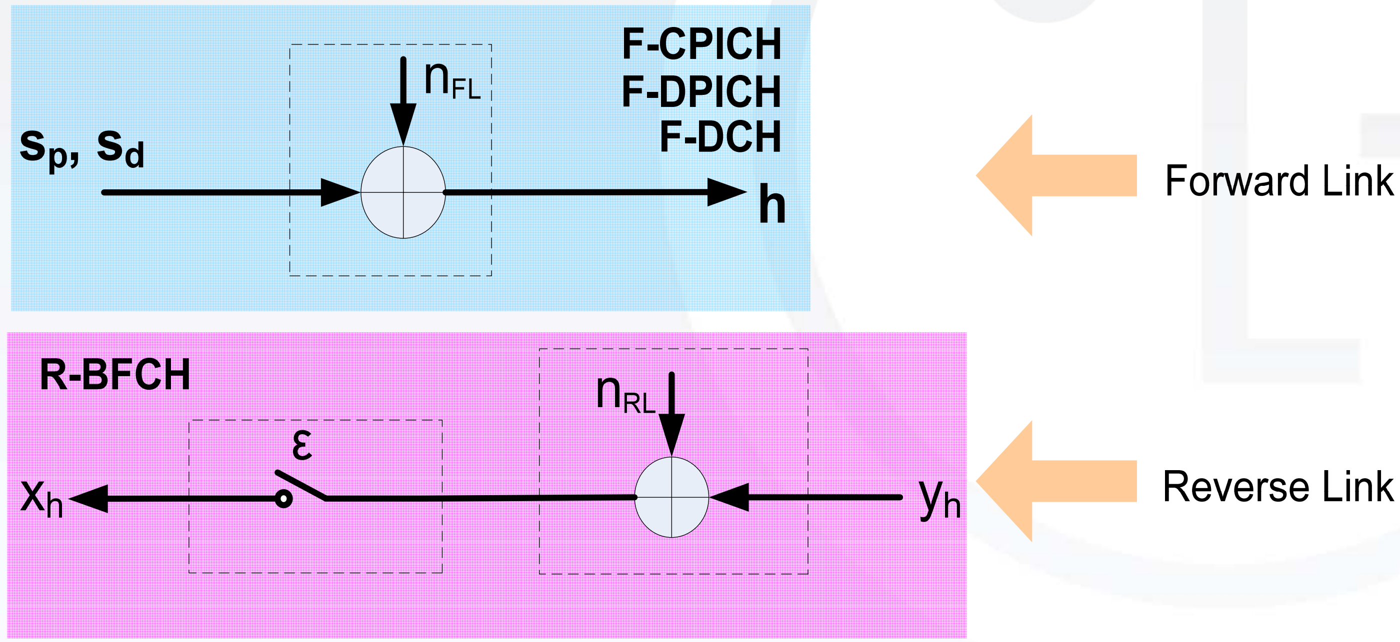



F-CPICH: Forwardlink Common Pilot Channel

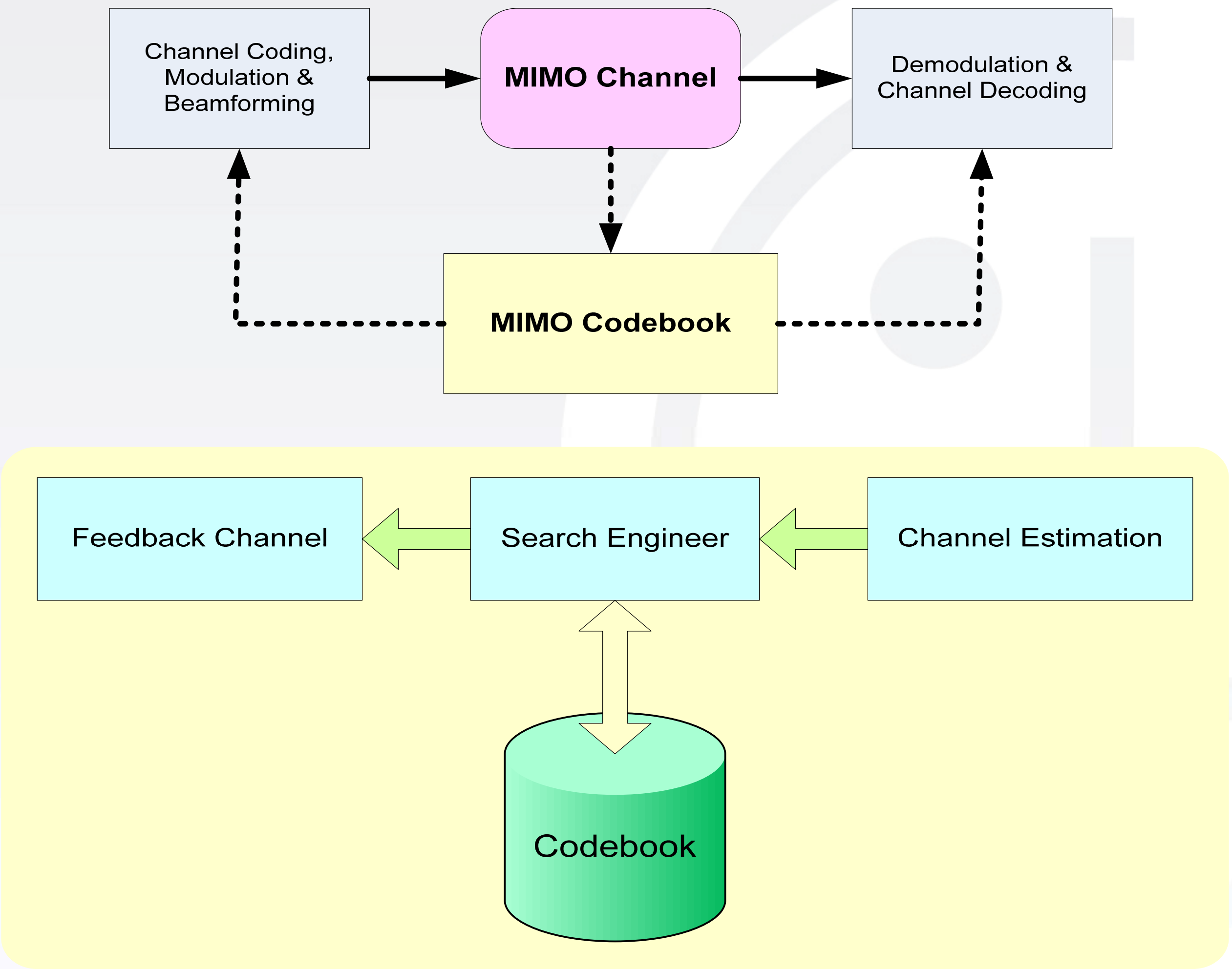
F-DPICH: Forwardlink Dedicate Pilot Channel

F-DCH: Forwardlink Traffic Channel

R-BFCH: Reverselink CQI Channel



# MIMO BF with Limited-Rate Feedback



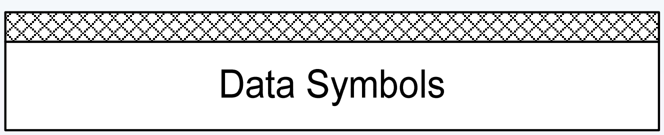
# Forwardlink Pilot/Data Channel Design

Pilot Symbols


(a) Time multiplexed pilot and data symbols

$$s_k(l) = \begin{cases} s_{pk}(l) & 1 \le l \le Q \\ s_{dk}(l-Q) & Q+1 \le l \le L \end{cases}$$

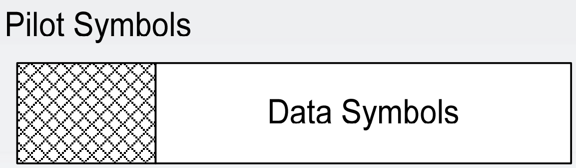
Pilot Symbols




(b) Superimposed pilot and data symbols

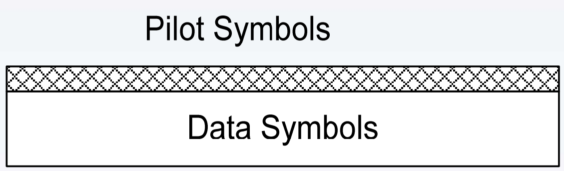
$$s_k(l) = \frac{\sigma_p}{P} s_{pk}(l) + \frac{\sigma_d}{P} s_{dk}(l)$$

# Cramer-Rao Bound on Channel Estimation




(a) Time multiplexed pilot
and data symbols

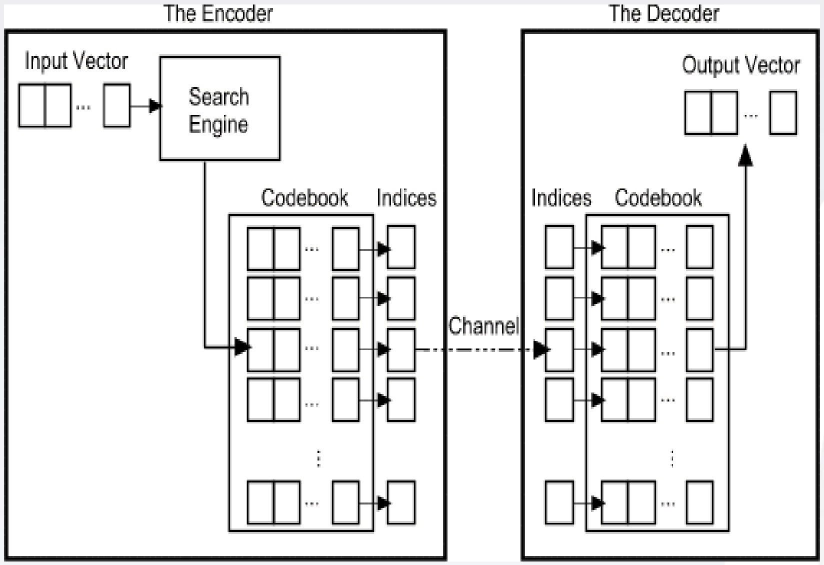
$$\varepsilon_h^{SIP} \geq \left[\rho_h^2 + L\left(\rho_p^2 + \rho_d^2\right)\right]^{-1}$$

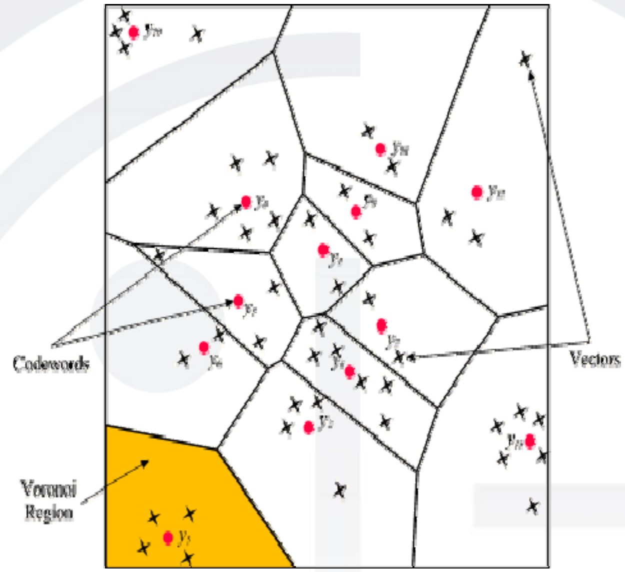



(b) Superimposed pilot
and data symbols

$$\varepsilon_h^{TMP} \geq \left[\rho_h^2 + Q\rho_p^2 + (L - Q)\,\rho_d^2\right]^{-1}$$

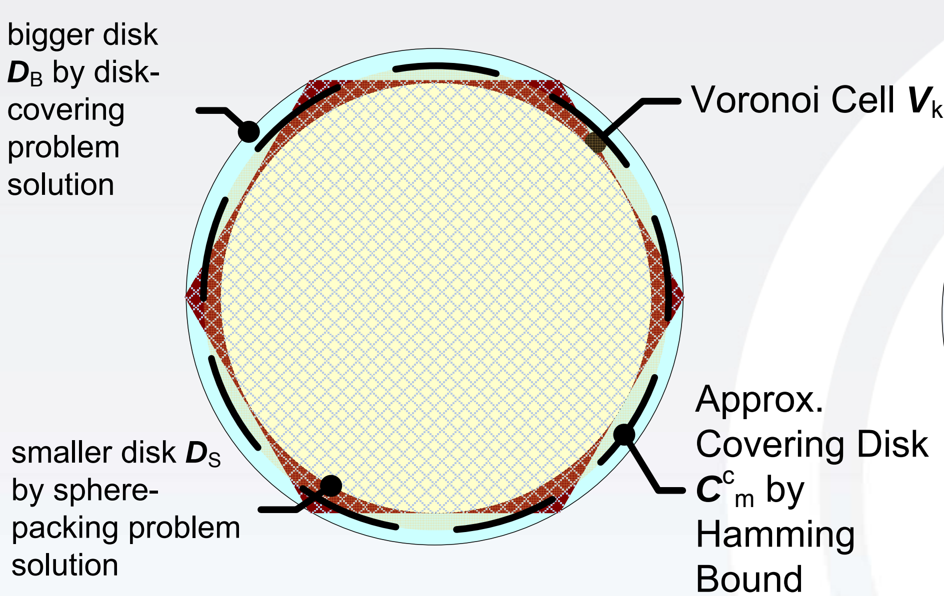
# Vector Quantization and Codebook

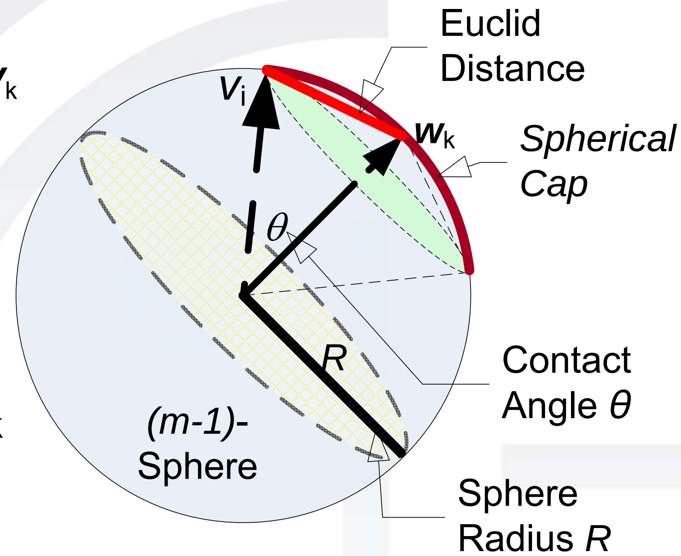



- Generally, the quantizer maps the input vector to the index of a codeword in the codebook. The decode will do the reverse.
  - Speech coding. VQ in EVRC, 4GV, AMR, etc.
  - Video/Audio Compression. E.g., TwinVQ in MPEG-4
  - MIMO CQI Generation. HS-DPCCH in HSDPA Rel. 6, R-CQI in UMB,
- Designing a best codebook as well as finding the general boundary of Voronoi cell is *NP-hard*.

# Codebook Voronoi Cell Boundary




- Upper Bound: finding the bigger disk $D_U$ – Disk Covering Problem:
  - what is the smallest disk for covering Voronoi cell in a Voronoi diagram?
- Lower Bound: finding the smaller disk $D_L$ – Sphere Packing Problem:
  - What is the largest disk covered by the Voronoi cell in a given Voronoi diagram?

# Hamming Bound on Codebook

**Proposition 1.** *(Hamming Boundary) The boundary of the uniform complex Voronoi cell $\boldsymbol{\mathcal{V}}_k$ can be approximated by a $(M-1)$-unit complex sphere or a closed complex space curve.*

$$\begin{aligned} \mathrm{B}\left(\boldsymbol{\mathcal{V}}_k\right) &\approx \boldsymbol{\mathcal{S}}_M^{\mathrm{c}}(1) \bigcap \boldsymbol{\mathcal{L}}_M^{\mathrm{c}}(\mathbf{w}_k,\ \cos(\theta)) \\ &= \left\{\mathbf{v}:\ \|\mathbf{v}\| = 1,\ \angle\left(\mathbf{v},\ \mathbf{w}_k\right) = \theta\right\}\ , \end{aligned} \tag{17}$$

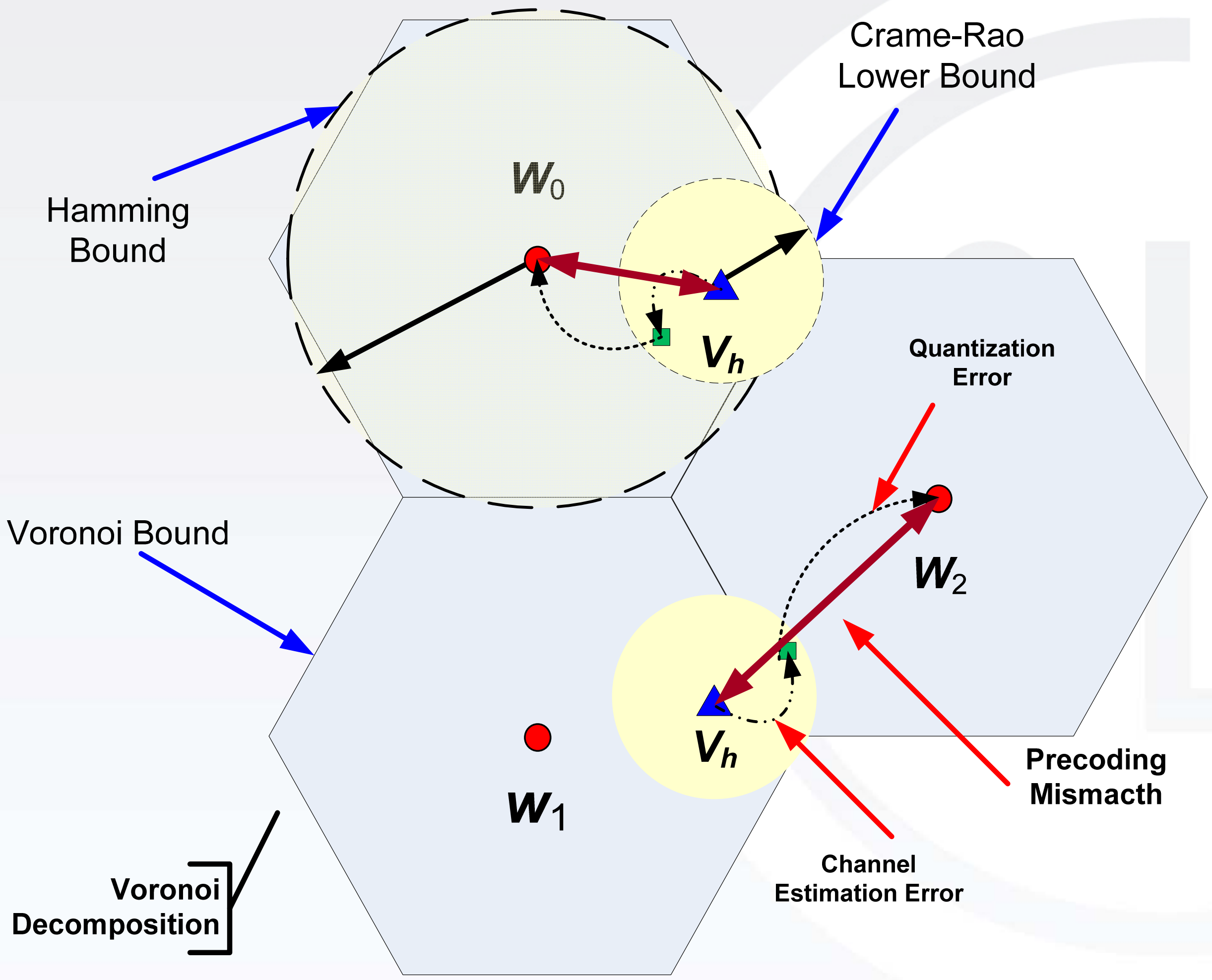
*where* $\boldsymbol{\mathcal{L}}_M^{\mathrm{c}}\left(\mathbf{w}_k,\ \cos(\theta)\right) = \left\{\mathbf{v}:\ \mathbf{v}^{\mathrm{H}}\mathbf{w}_k = \cos(\theta)\right\}$ *denotes a complex space curve and* $\theta$ *is*

$$\theta = \arccos\left(\alpha_0\right) \tag{18}$$

*with*

$$\alpha_0 = \left(\frac{2^B-1}{2^B}\right)^{\frac{1}{2M-2}}\ . \tag{19}$$

# MIMO Precoding Mismatching – A Signal Processing Perspective



# SINR with Precoding Mismatch

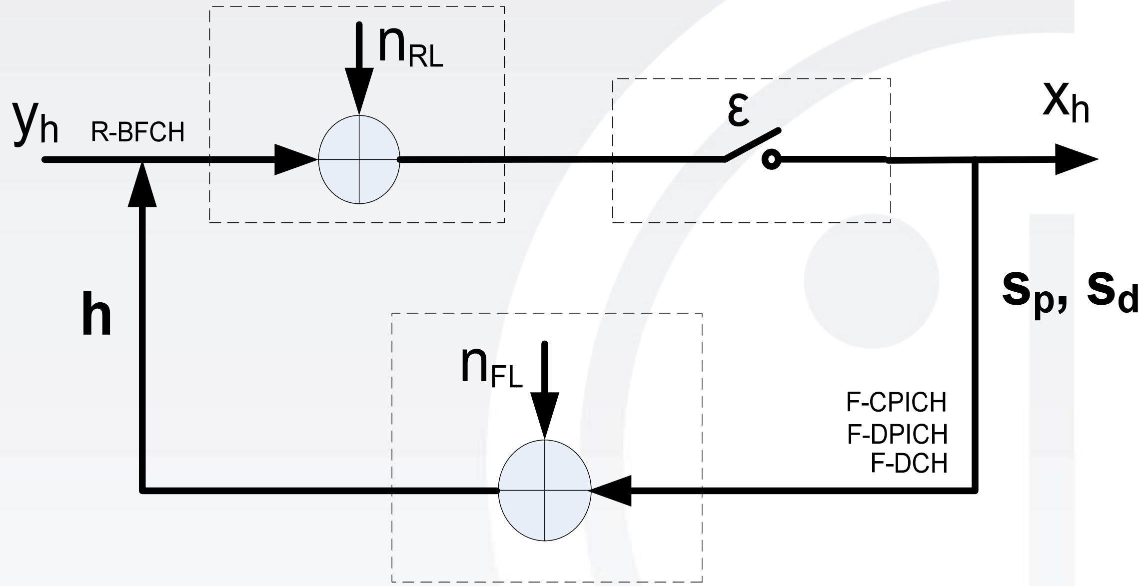
**Lemma 1.** *A heuristic mean of $\rho_i$ can be expressed by*

$$\bar{\rho}_i \approx \frac{\sigma_\alpha^2 \rho_i}{1+\left(\frac{1-\sigma_\alpha}{M-1}\right)^2 \sum\limits_{j\neq i} \frac{\lambda_i}{\lambda_j}\rho_j}, \tag{22}$$

*where $\sigma_\alpha$ denotes the standard deviation of $\alpha$ and $\sigma_\alpha^2$ is given by*

$$\sigma_\alpha^2 = E\left\{\alpha_i^2\right\} = \frac{1}{M} + \frac{M-1}{M}\left(\frac{2^B-1}{2^B}\right)^{\frac{1}{M-1}}. \tag{23}$$

# MIMO FB Chan. Model with Chan. Esti.




- Feedforward Channel (R-BFCH):
  - $n_{RL}$: CQICH Gaussian Noise
  - ε: CQICH erasure rate
- Feedback Channel (F-CPICH + F-DPICH + F-DCH)
  - $n_{RL}$: Forwardlink Gaussian Noise

## MIMO Precoding Mismatching – A Coding Perspective

$$\mathrm{E}\{SINR_{\mathrm{Tx}}\} = (1 - P_{e\mathrm{BLK}})\mathrm{E}\{SINR_{\mathrm{Tx}}\}_{\mathrm{perfect}} + P_{e\mathrm{BLK}}\,\mathrm{E}\{SINR_{\mathrm{Tx}}\}_{\mathrm{imperfect}}$$

- What is the $P_{e\mathrm{BLK}}$ here? R-BFCH error rate.
- For a codebook of size $2^B$, the relationship between $P_{e\mathrm{BLK}}$ and feedback channel reliability roughly is

forwardlink channel coherence time

codeword length

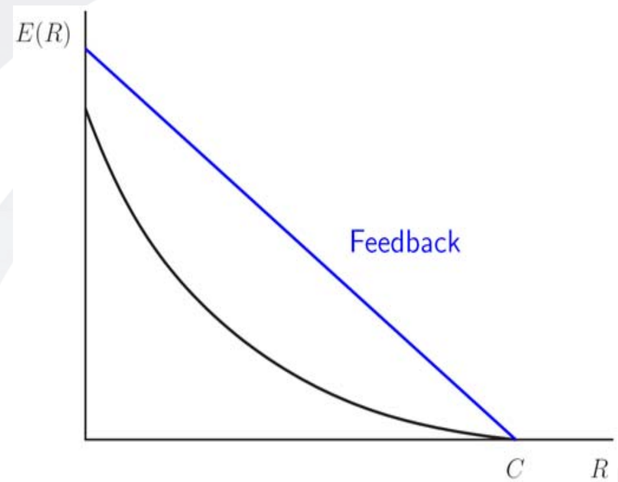
$$P_{e\mathrm{BLK}} = B \cdot P_e^{(n)} \approx B \cdot 2^{-(RT_c)E(R;P,\sigma^2)}$$

codeword feedback error rate

Feedback channel rate

# Reliability of Feedback Channel

- The error probability of Gaussian Channel decays exponentially with the reliability E(R).
- With feedback, high reliability possibly is achievable.
- Noisy Feedback is much worse than perfect feedback.
- Less forward-link channel estimation errors will possibly help improve R-BFCH reliability.

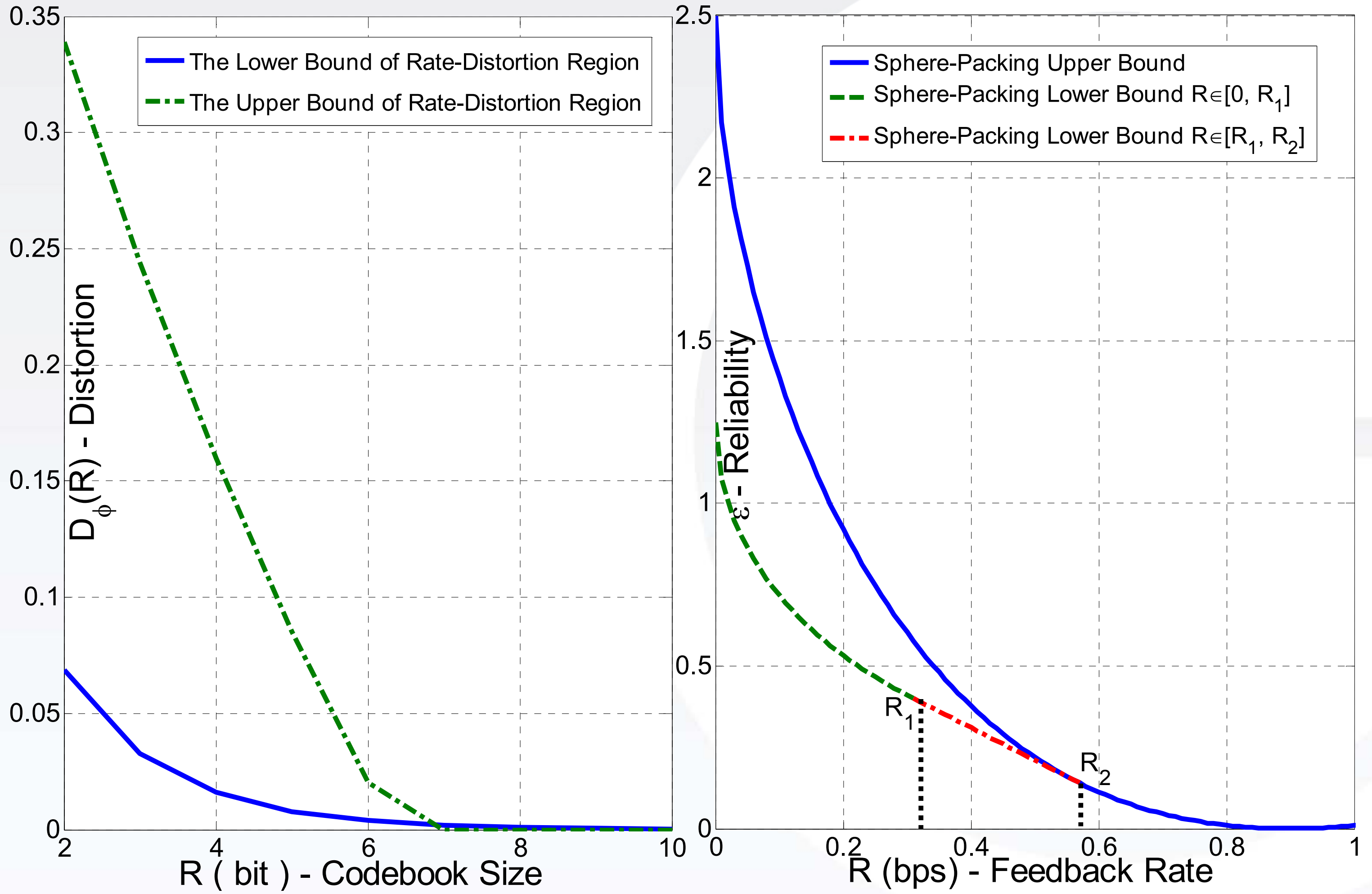



$$E(R) = \limsup_{n\to\infty} -\frac{1}{n}\log P_{e,\mathrm{opt}}^{(n)}(R)$$

$$P_e^{(n)} \doteq 2^{-nE(R)} \quad R < C$$

$$E_{\mathrm{FB}}\left(R; P_{\mathrm{RL}}, \sigma_{\mathrm{RL}}^2\right) \le E\left(R; P_{\mathrm{RL}}\left(1+\sigma_{\mathrm{FL}}^{-2}\right)\right)$$

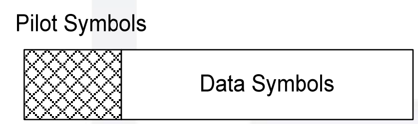
# Rate, Distortion and Reliability



# Spectral Efficiency of MIMO Relay Link (1/3)

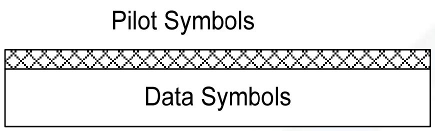
MIMO Relay Assumptions:
- Amplify-and-forward relay
- No macro-diversity or cooperation between transmissions or groups.

$$\bar{\eta}^{TMP} \approx \frac{L-Q}{L} K \log\left[1+\frac{\rho}{M}\left(\frac{\sigma_{\alpha}^{2}}{1+\frac{(K-1)(1-\sigma_{\alpha})^{2}\rho}{(M-1)^{2}}}\right)\right]^{G}$$



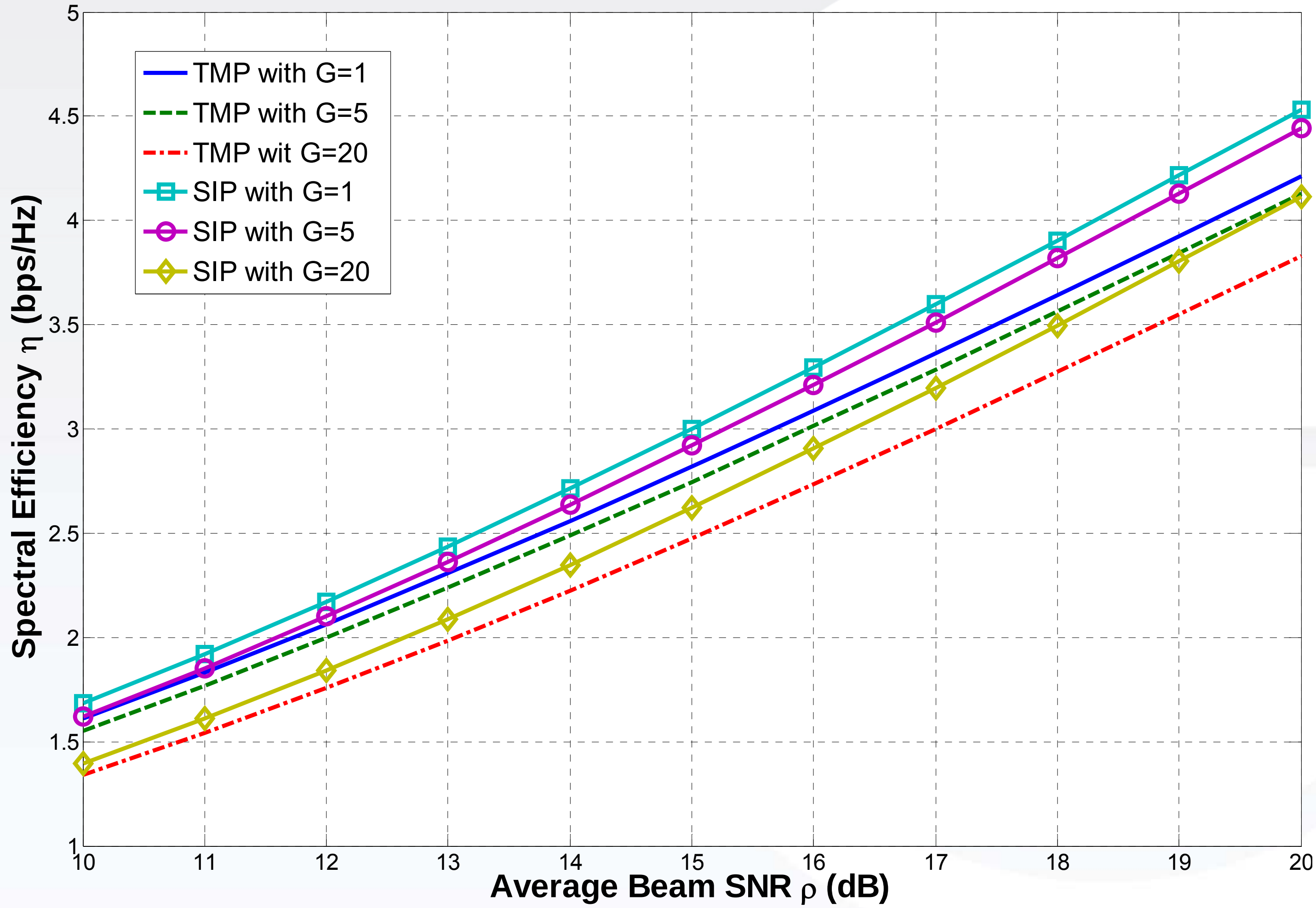
(a) Time multiplexed pilot and data symbols

$$\bar{\eta}^{SIP} \approx K \log\left[1+\frac{\sigma_{d}^{2}\rho}{PM}\left(\frac{\sigma_{\alpha}^{2}}{1+\frac{(K-1)(1-\sigma_{\alpha})^{2}\rho}{(M-1)^{2}}}\right)\right]^{G}$$

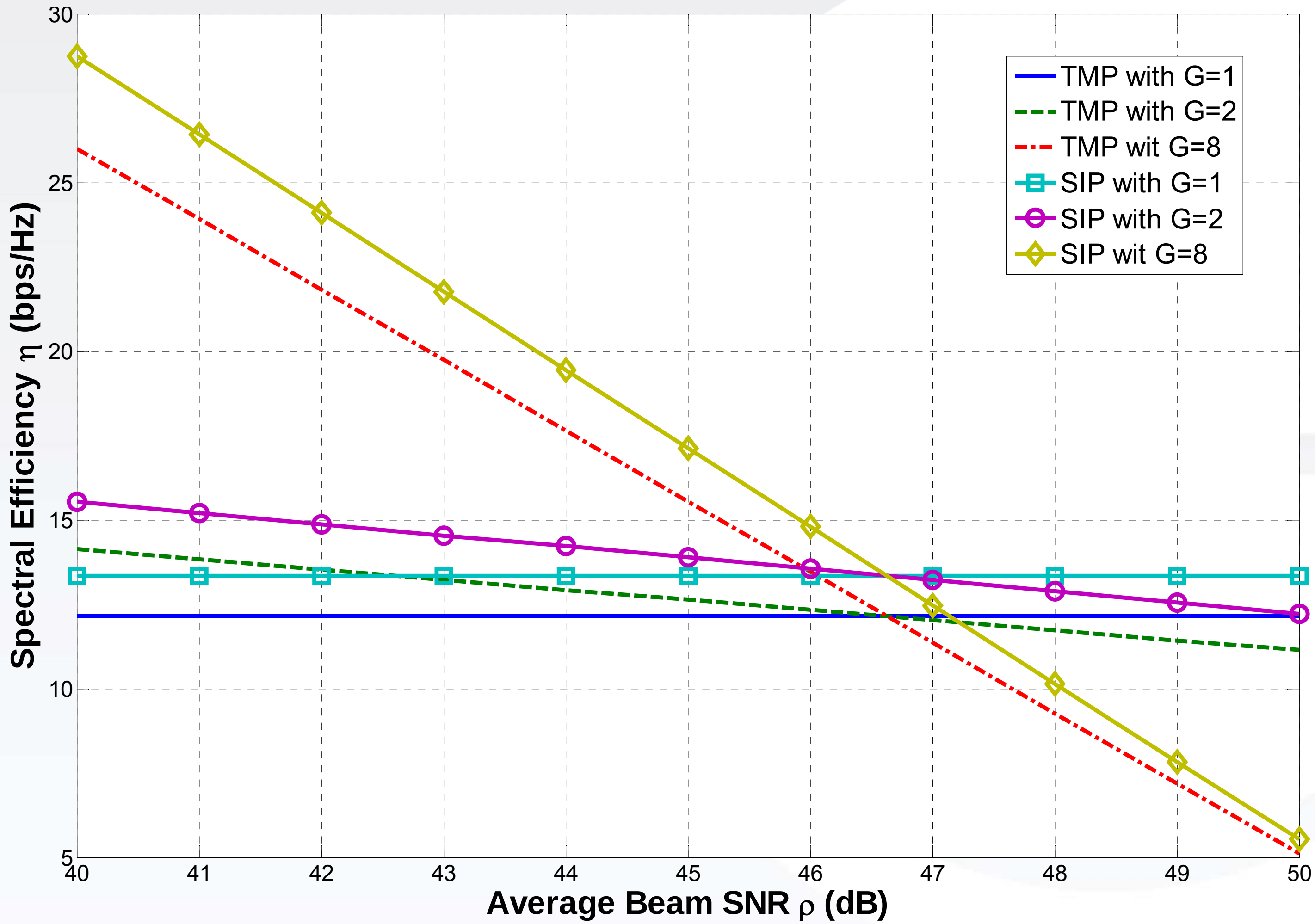


(b) Superimposed pilot and data symbols

# Spectral Efficiency of MIMO Relay Link (2/3)



# Spectral Efficiency of MIMO Relay Link (3/3)



# Conclusions

- MIMO precoding mismatch is a function of
  - Codebook design:  Voronoi cell/boundary
  - The reliability of R-BFCH:
- Both FL channel and RL channel design can affect the achievable rate **R** of R-BFCH.
  - When FL with high SNR, the benefits of increasing RL reliability is similar as increasing FL power (**FL/RL coupling**)
  - When FL with low SNR, RL design is more important than FL.
- MIMO relay link spectral efficiency is a function of
  - Pilot design: TMP/SIP
  - Codebook size: larger codebook size is good in general
  - The number of hops: more hops may not be good, but..
  - More hops or higher Tx Power?
    - In high SNR region, more Tx power helps more.
    - In high IBI region, more hops helps more.

# Thank you!