On The Feedback Channel for MIMO Beamforming

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***Abstract*—The question on how much feedback we need for MIMO beamforming is addressed in terms of the distortion and reliability of MIMO beamforming feedback in this paper. We model MIMO beamforming feedback with noisy Gaussian binary erasure feedback channel, in which the MIMO forwardlink is modelled as a Gaussian channel and the reverselink is simplified as a Gaussian binary erasure channel. For the forwardlink channel quantization, an information-theoretic lower bound and a heuristic sphere-packing upper bound for the achievable rate-distortion region are given. For the reverselink channel feedback, the rate-reliability region is analyzed with Shannon bounds. Though a large codebook design with high-rate feedback is generally desired for less distortion, it may be unnecessary due to the limitation of channel estimation and feedback channel. Tradeoffs between codebook design and channel state information feedback are therefore revealed.**

I. INTRODUCTION

Multi-antenna systems have received much attention over the last decades, due to their promise of higher spectrum efficiency with no additional transmit power increase. For multiple-input multiple-output (MIMO) transmission, it is well-known that transmission delay, demodulation complexity and reliability can be improved by making channel state information (CSI) available at transmitter side. This is usually achieved through a reverselink CSI feedback channel from forwardlink receiver. In practice, the feeded back CSI is imperfect and suffers from various impairments including quantization with MIMO codebook, channel estimation deviation, feedback channel imperfection, etc. It results in increased interference, degraded forwardlink throughput and limited coverage. Therefore it is desirable to understand the limitation of feedback for MIMO beamforming and especially the tradeoffs between codebook design and feedback.

In reality, the performance of MIMO beamforming with feedback depends on many factors including the codebook design, forwardlink channel estimation, reverselink channel quality. MIMO beamforming systems with ideal Lloyd vector quantization (VQ) [1], different channel model [2] or different performance metrics [3], [4] were intensively investigated. Most of them are done without considering the effects of ”noisy” feedback due to forwardlink pilot structure and channel estimation, even though they are among the most important components of an actual multi-antenna system. In reality, MIMO CSI is estimated with forwardlink pilot channels sent from each active transmitter antenna. An overview of pilot-assisted transmission (PAT) including pilot placement and channel estimation can be found in [5]. Pilot channels are traditionally designed to be orthogonal to other channels. Nonorthogonal pilot design like superimposed pilots (SIP) has recently received much attention too [6]. Optimal pilot placement was discussed in [7]. And the feedback capacity and reliability have intensively been investigated over decades. Though feedback doesn’t increase the capacity of memoryless channels [8], [9], a feedback coding scheme with the decoding error probability decreasing more rapidly than the exponential of any order is achievable [10]. Since CSI feedback plays such a critical role in MIMO transmission, it is important to understand how codebook design, channel design and quality affect the distortion and reliability of feedback.

In general, the procedure for MIMO channel estimation, quantization and feedback is an example of joint source-channel coding problem. Though a codebook of large size can help minimize the distortion by quantization, the gain may be much limited by channel estimation deviation and feedback error. There always is the essential problem for achieving the beamforming capacity with a simple codebook from an engineering standpoint. In this paper, the distortion and reliability of MIMO feedback are discussed with considering forwardlink design, codebook limitation and reverselink reliability. The forwardlink is modelled as a Gaussian channel with multiplexed pilot and data signals for channel estimation. The Cramer-Rao lower bound (CRLB) for channel estimation and Shannon rate-distortion lower bound are derived for the forward link. Besides this, a heuristic sphere-packing upper bound is given for channel quantization with MIMO codebook. The achievable rate-distortion region is restricted between these two bounds. For reverselink, the MIMO feedback channel is modelled as a Gaussian binary erasure channel with additive Gaussian noise and binary erasure. The binary erasure part is a simplified but popular model for the channel fading in reality. With the erasure mechanism, the reverselink is roughly simplified into a Gaussian channel model with forwardlink feedback. The achievable rate-reliability region of feedback channel is typically limited by the Shannon reliability bounds presented in our discussion. With the analysis of feed channel distortion and reliability, tradeoffs between codebook design and feedback are revealed and the problem how much feedback is necessary is essentially addressed.

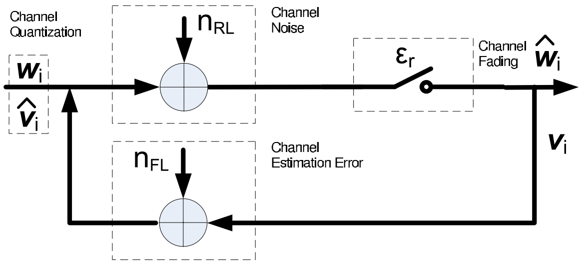

Fig. 1. Noisy Gaussian binary erasure feedback channel with channel quantization.

II. System Model And Problem Description

Consider a MIMO link consisting of a transmitter with M transmit antennas, a receiver with N receive antenna and a MIMO channel represented by the $N \times M$ matrix $\mathbf{H} = [\mathbf{h}_1\ \mathbf{h}_2\ \ldots\ \mathbf{h}_N]^{\mathrm{T}}$ with $\mathbf{h}_n = [h_{n,1}\ h_{n,2}\ \ldots\ h_{n,M}]^{\mathrm{T}}$ and $h_{nm} \sim \mathcal{CN}\left(0, \sigma_h^2\right)$. The $N \times 1$ received signal $\mathbf{y}$ is

$$\mathbf{y} \;=\; [y_1\ y_2\ \ldots\ y_N]^{\mathrm{T}} \;=\; \mathbf{HWx} + \mathbf{n} \tag{1}$$

where $\mathbf{x} = [x_1\ x_2\ \ldots\ x_M]^{\mathrm{T}}$ is the $M \times 1$ signal vector transmitted by the source with $\mathbf{R_x} = \mathrm{E}\left\{\mathbf{xx}^{\mathrm{H}}\right\} = \frac{P}{M}\mathbf{I}_M$, P is the total transmit power, $\mathbf{W} = [\mathbf{w}_1\ \mathbf{w}_2\ \ldots\ \mathbf{w}_M]$ is a $M \times M$ MIMO beamforming precoding matrix with $\|\mathbf{w}_m\|_2 = 1$, $\mathbf{n} \sim \mathcal{CN}\left(0, \sigma_n^2 \mathbf{I}_N\right)$ is a complex circular white Gaussian vector, $[*]^{\mathrm{T}}$ and $[*]^{\mathrm{H}}$ denotes the transpose operator and Hermitian conjugate operator, respectively. The MIMO channel achievable spectral efficiency is

$$\eta = \log\left|\mathbf{I} + \tfrac{P}{M}\mathbf{HWW}^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\right| = \sum_{k=1}^{K} \log\left(1 + \tfrac{\rho_i}{M}\right) \tag{2}$$

where ρ_i denotes the received signal-to-noise ratio (SNR) of the kth beam, which is given by

$$\rho_i \;=\; \frac{\mathrm{var}\{\mathbf{Hw}_i x_i\}}{\sigma^2} \;=\; \frac{\lambda_i P_i}{\sigma^2} \tag{3}$$

with λ_i denoting the antenna gain of the ith beam and also the ith eigenvalue of the MIMO channel autocorrelation matrix $\mathbf{R_H}$ defined by

$$\mathbf{R_H} = \mathbf{H}^{\mathrm{H}}\mathbf{H} = \sum_{m=1}^{M} \mathbf{h}_m \mathbf{h}_m^{\mathrm{H}} = \sum_{i=1}^{M} \lambda_i \mathbf{v}_i \mathbf{v}_i^{\mathrm{H}}\ , \tag{4}$$

$\mathbf{v}_i$ denotes the ith eigenvector, the operator $|*|$ denotes the determinant of matrix $*$, and $\mathrm{var}\{*\}$ denotes the variance of random variable $*$. (2) and (3) are the results from the assumption of perfect CSI available at transmitter, however this may not be consistent with practical applications.

MIMO beamforming with finite-rate feedback is modelled as a noisy Gaussian binary erasure feedback channel depicted in Fig. 1. In reality, the receivers estimate $\mathbf{H}$ or $\mathbf{R_H}$ with the pilots sent by transmitter. Accuracy of the channel estimation depends on both forwardlink design and receiver design. The pilot transmission is very helpful for receiver to estimate CSI. After channel estimation, the receiver chooses a beamforming vector from a shared MIMO precoding codebook. This is called channel quantization. It means the receiver actually feeds back the chosen precoding index(es) to transmitter(s) instead of actual channel response for the next transmitter precoding. With a MIMO codebook $\boldsymbol{\mathcal{W}}$ of the size 2^R consisting of M-dimensional normalized vectors $\{\mathbf{w}_1, ..., \mathbf{w}_{2^R}\}$, it usually takes the receiver to feedback R bits for each beam stream. The codebook is designed to quantize channel responses with certain distortion measures [1]. It is related to Grassmannian line packing, the spherical packing on unit sphere $\boldsymbol{\mathcal{S}}_n(1)$ [11] etc., where $\boldsymbol{\mathcal{S}}_n(r) = \{\mathbf{v} :\ \|\mathbf{v}\| = r\}$ denotes $(n-1)$-sphere of radius r. When a codeword $\mathbf{w}_k$ is chosen to precode signals for ith beam $\mathbf{v}_i$ at the transmitter side, degradation will happen on the signals received by receiver because of the imperfect channel estimation and finite-rate feedback. The degradation on received signals can be expressed by

$$\begin{aligned}\delta_i \;&=\; \min_{\mathbf{w}\in\boldsymbol{\mathcal{W}}} \|\mathbf{Hv}_i - \mathbf{Hw}\| \\ &=\; \lambda_i\left(1 - \mathbf{v}_i^{\mathrm{H}}\mathbf{w}_k\right) - \sum_{j\neq i} \lambda_j \mathbf{v}_j^{\mathrm{H}}\mathbf{w}_k\end{aligned} \tag{5}$$

where $\mathbf{v}_i$ is the ideal precoding vector for ith eigen-mode of $\mathbf{H}$. It is known that δ_i, which depends on $\mathbf{v}_i$, $\mathbf{w}_k$ and λ_i, etc., is one of the major factors limiting closed-loop MIMO beamforming throughput. The reverselink channel is modelled by concatenating a Gaussian channel and a binary erasure channel due to feedback redundancy and the employed erasure mechanism, which drops those CSI packets with low SNR.

In MIMO feedback channel design, the rate-distortion function and the rate-reliability function are of the most important concerns. For the rate-distortion function, the region of achievable rate distortion pair $(R,\ D)$ with the squared error distortion

$$D(R) \;=\; \max_i \sum_{m=1}^{M} \left(w_{i,m} - v_{i,m}\right)^2 \tag{6}$$

is of the most interests for understanding the effects of channel estimation and quantization. On the other hand, the achievable rate reliability pair $(R,\ \epsilon)$, with the error exponent ϵ denoting how fast the reverselink bit-error rate (BER) P_e decays for the transmit rate R and given by

$$\epsilon \;=\; \lim_{n\to\infty} \sup \epsilon_n \;=\; \lim_{n\to\infty} \sup -\tfrac{1}{n}\ln P_e\ , \tag{7}$$

where $\epsilon_n = -\frac{1}{n}\ln P_e$ denotes the reliability of n-bits block transmission and n is the minimum block-length needed in order to operate at rate R with the error probability P_e, is of the most importance to understand the performance of feedback design. In the following sections, we will discuss how the imperfect channel estimation, quantization and quality affect the achievable rate-distortion region and the achievable rate-reliability region of MIMO feedback channel.

III. The Rate-Distortion Region

A. Bemaforming Mismatch Lower Bound

In general, the channel quantization distortion is mostly decided by channel quality, channel estimation and codebook

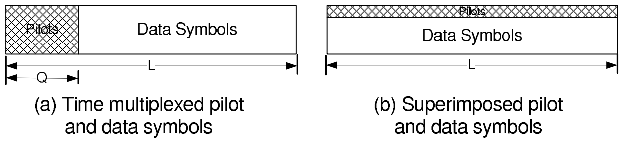

Fig. 2. Pilot patterns for channel estimation.

design. Given σ_{FL}^2 the MSE of forwardlink channel estimation, the minimum rate at distortion $D_{\mathbf{h}}$ is given by

$$\begin{aligned} R(D_{\mathbf{h}}) &= \min_{E\|\mathbf{h}-\hat{\mathbf{w}}\|_2^2 \le D_{\mathbf{h}}} I\left(\mathbf{h}_i;\ \hat{\mathbf{h}}_i\right) \\ &= \min_{E\|\mathbf{h}_i-\hat{\mathbf{w}}_i\|_2^2 \le D_{\mathbf{h}}} \left\{H(\mathbf{h}) - H\left(\mathbf{h}|\hat{\mathbf{h}}\right)\right\} \\ &\ge M\log_2\left(2\pi e\sigma_{\text{FL}}^2\right) - M\log_2\left(2\pi e E\|h_i-\hat{w}_i\|_2^2\right) \\ &= M\log_2\left(\frac{\sigma_h^2}{\frac{1}{M}D_{\mathbf{h}}+\sigma_{\text{FL}}^2}\right) \end{aligned} \tag{8}$$

with $D_{\mathbf{h}}$ the channel quantization distortion defined by

$$D_{\mathbf{h}} = \max \mathrm{E}\left\|\mathbf{h}-\hat{\mathbf{h}}\right\|_2^2 \tag{9}$$

and $0 \le \sigma_h^2 \le \frac{1}{M}D_{\mathbf{h}} + \sigma_{\text{FL}}^2$; otherwise, $R(D_{\mathbf{h}}) = 0$. Meanwhile, the lower bound to the mean-squared error (MSE) of unbiased channel estimates of $\mathbf{h}$ is given by CRLB, which is defined as the inverse of the Fisher Information Matrix (FIM),

$$\begin{aligned} \text{MSE}\left\{\mathbf{h}-\hat{\mathbf{h}}\right\} &\ge [\mathbf{F}(\vartheta)]^{-1} \\ = \mathrm{E}^{-1}&\left\{\left[\frac{\partial \ln \mathbf{Pr}(\mathbf{y}|\vartheta)}{\partial\vartheta^*}\right]\left[\frac{\partial \ln \mathbf{Pr}(\mathbf{y}|\vartheta)}{\partial\vartheta^*}\right]^{\mathrm{H}}\right\} \end{aligned} \tag{10}$$

with $\boldsymbol{\vartheta}$ the parameter vector to be estimated. (10) can be further detailed by the following lemma.

Lemma 1. *(Cramer-Rao Lower Bound) For the time-multiplexed pilot and data design shown in Fig. 2(a) and defined by*

$$s_k(l) = \begin{cases} s_{pk}(l) & 1 \le l \le Q \\ s_{dk}(l-Q) & Q+1 \le l \le L \end{cases}, \tag{11}$$

the CRLB of channel estimation is

$$\sigma_{FL}^2 \ge \sigma_{TMP}^2 = \left[\rho_h^2 + Q\rho_p^2 + (L-Q)\rho_d^2\right]^{-1}. \tag{12}$$

For the superimposed pilot and data design shown in Fig. 2(b) and defined by

$$s_k(l) = \frac{\sigma_p}{P}s_{pk}(l) + \frac{\sigma_d}{P}s_{dk}(l),\ 1 \le l \le L, \tag{13}$$

with $\sigma_p^2 + \sigma_d^2 = P$, the CRLB of channel estimation is

$$\sigma_{FL}^2 \ge \sigma_{SIP}^2 = \left[\rho_h^2 + L\left(\rho_p^2+\rho_d^2\right)\right]^{-1} \tag{14}$$

where $\rho_p^2 = \frac{\sigma_p^2}{\sigma_n^2}$ denotes the SNR of received pilot signals and $\rho_d^2 = \frac{\sigma_d^2}{\sigma_n^2}$ denotes the SNR of received data signals.

(8) shows that the existence of channel estimation error decreases the minimum codebook size necessitated for channel quantization. When the channel estimation is very ”noisy” with $\sigma_h^2 \le \sigma_{\text{FL}}^2$, the minimum codebook size is

$$R_{\text{L}} = 0, \tag{15}$$

which means channel quantization may unnecessary because the channel estimation is too inaccurate. When the channel estimation is not so ”noisy” but $\sigma_{\text{FL}}^2 \gg \frac{1}{M}D_{\mathbf{h}}$,

$$R_{\text{L}} \approx M\log_2\left(\frac{\sigma_h^2}{\sigma_{\text{FL}}^2}\right). \tag{16}$$

This means the channel quantization distortion σ_{FL}^2 shall be comparable to the channel estimation deviation σ_h^2; otherwise, the quantization is meaningless. When the channel estimation is good enough with $\sigma_{\text{FL}}^2 \ll \frac{1}{M}D_{\mathbf{h}}$, it requires that $D_{\mathbf{h}}$ the channel quantization error shall satisfy

$$\sigma_{\text{FL}}^2 \ll \tfrac{1}{M}D_{\mathbf{h}} \le \sigma_h^2 - \sigma_{\text{FL}}^2. \tag{17}$$

After $D_{\mathbf{h}}$ is properly chosen, the role of channel quantization becomes important and the squared error distortion $D_{\mathbf{h}}$ can be written by

$$D_{\mathbf{h}} = \max \mathrm{E}\left\|\mathbf{h} - w_r\mathbf{w}_\phi\right\|_2^2 \tag{18}$$

with w_r denoting the quantized vector channel norm and $\mathbf{w}_\phi$ denoting the quantized vector channel phase. In existing MIMO beamforming system design, the receiver usually tracks the channel norm information for adaptive modulation purpose and phase quantization instead of quantizing and feeding back the channel norm. In this case, (18) can be rewritten by

$$\begin{aligned} D_{\mathbf{h}} &= \max \mathrm{E}\left\|\mathbf{h} - r\mathbf{w}_\phi\right\|_2^2 \\ &= \max \mathrm{E}\left\|r(\mathbf{v}-\mathbf{w}_\phi)\right\|_2^2 \\ &= 2M\sigma_h^2\left(1-\sqrt{1-D_\phi}\right) \end{aligned} \tag{19}$$

with $r = \|\mathbf{h}\|$ denoting the channel gain and

$$D_\phi = \mathrm{var}\left\{\sin\angle(\mathbf{v},\ \mathbf{w})\right\} \tag{20}$$

denoting the phase quantization deviation. Therefore a lower bound for distortion rate of channel quantization for MIMO beamforming is

$$\begin{aligned} R(D_\phi) &= \min_{\mathrm{E}\|\mathbf{v}-\hat{\mathbf{v}}\|_2^2 \le D_\phi} I(\mathbf{v};\ \hat{\mathbf{v}}) \\ &\ge \max\left\{-M\log_2\left(2-2\sqrt{1-D_\phi}+\frac{\sigma_{\text{FL}}^2}{\sigma_h^2}\right),\ 0\right\} \end{aligned} \tag{21}$$

for each beam. With the feedback rate of R, (21) also tells us that the minimum precoding mismatch for forwardlink MIMO beamforming is

$$\begin{aligned} D_\phi &= \max \mathrm{E}\left\|\hat{\mathbf{v}}-\mathbf{v}\right\|_2^2 \\ &\le 1-\left(1-2^{-\frac{R}{M}-1}+\frac{\sigma_{\text{FL}}^2}{2\sigma_h^2}\right)^2. \end{aligned} \tag{22}$$

B. Bemaforming Mismatch Upper Bound

The beamforming mismatch upper bound depends on the codebook design. The maximum beamforming mismatch can be determined by the largest radius of the codebook Voronoi cell $\left\{\boldsymbol{\mathcal{V}}_i:\ 1 \le i \le 2^R\right\}$, which in general is the solution to the disk-covering problem that still is open. Instead of the exact boundary for the Voronoi cell $\boldsymbol{\mathcal{V}}_i$, we suggest a heuristic approach using sphere-packing bound and sphere cap to approximate the actual polytope boundary. It is an approximate of the sphere packing solution, in which all

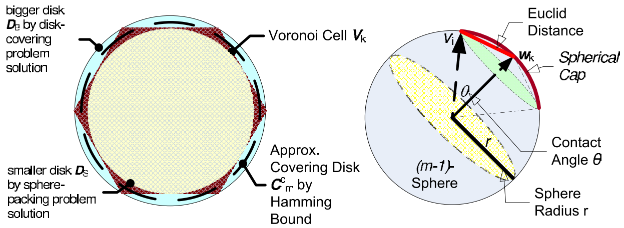

Fig. 3. Voronoi cell and various bounds

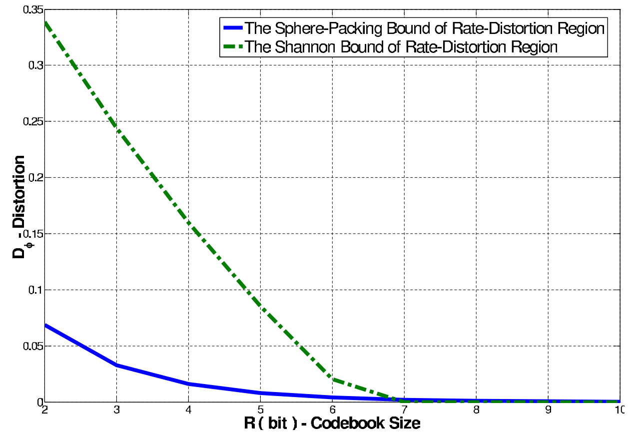

Fig. 4. The rate-distortion region with $M=4$ and $\frac{\sigma_h^2}{\sigma_{\rm FL}^2}=4.8$dB.

spheres are supposed to be non-overlappedly placed. With our approach, sphere caps are overlapped with each other in space but the interior of them has the same area as the Voronoi cell. The border of this sphere cap is termed sphere-packing boundary. The relationship between sphere-packing boundary and Voronoi cell is shown in Fig. 3. For an uniform random codebook of size 2^R in M-dimensional Euclid space, the area of a Voronoi cell is given by

$$A\left(\boldsymbol{\mathcal{V}}_k\right) \quad = \quad \frac{2\pi^M}{2^R\Gamma(M)} \tag{23}$$

where $\Gamma\left(*\right)$ denotes the gamma function. On the other hand, the area of $(m-1)$-complex sphere cap $\boldsymbol{\mathcal{C}}^{\rm c}_m\left(\psi,\ R\right)$ with contact angle ψ and radius r is

$$A\left(\boldsymbol{\mathcal{C}}^{\rm c}_m\left(\psi,\ r\right)\right) = \left[1-\cos^{2(m-1)}\left(\psi\right)\right] S^{\rm c}_m\left(r\right), \tag{24}$$

where $\boldsymbol{\mathcal{S}}^{\rm c}_m\left(r\right)$ denotes a M-dimension complex ball in Euclid space with the radius r. The relationship between $\boldsymbol{\mathcal{S}}^{\rm c}_m\left(r\right)$ and $\boldsymbol{\mathcal{C}}^{\rm c}_m\left(\psi,\ r\right)$ can be shown in Fig. 3 and it can be verified that

$$A\left(S^{\rm c}_m\left(r\right)\right) \quad = \quad A\left(\boldsymbol{\mathcal{C}}^{\rm c}_m\left(\pi,\ r\right)\right) \ . \tag{25}$$

With matching the sum area of the sphere-cap with the whole sphere area, the boundary of a Voronoi cell can be approximated by a hypershpere or a closed space curve defined in the following proposition.

Proposition 1. *(Sphere-Packing Boundary) The boundary of the uniform complex Voronoi cell $\boldsymbol{\mathcal{V}}_k$ can be approximated by a $(M-1)$-unit complex sphere or a closed complex space curve.*

$$\begin{aligned}\mathbf{B}\left(\boldsymbol{\mathcal{V}}_k\right) \quad &\approx \quad \boldsymbol{\mathcal{S}}^{\rm c}_M(1)\bigcap\boldsymbol{\mathcal{L}}^{\rm c}_M(\mathbf{w}_k,\ \cos(\theta)) \\ &= \quad \left\{\mathbf{v}:\ \|\mathbf{v}\|=1,\ \angle\left(\mathbf{v},\ \mathbf{w}_k\right)=\theta\right\}\ ,\end{aligned} \tag{26}$$

where $\boldsymbol{\mathcal{L}}^{\rm c}_M\left(\mathbf{w}_k,\ \cos(\theta)\right)=\left\{\mathbf{v}:\ \mathbf{v}^{\rm H}\mathbf{w}_k=\cos(\theta)\right\}$ denotes a complex space curve and θ is

$$\theta \quad = \quad \arccos\left(\alpha_0\right) \tag{27}$$

with

$$\alpha_0 \quad = \quad \left(\frac{2^R-1}{2^R}\right)^{\frac{1}{2M-2}} \ . \tag{28}$$

Lemma 2. *The standard deviation of α is given by*

$$\sigma^2_\alpha \quad = \quad {\rm E}\left\{\alpha^2\right\} \quad = \quad \frac{1}{M}+\frac{M-1}{M}\left(\frac{2^R-1}{2^R}\right)^{\frac{1}{M-1}} \ . \tag{29}$$

With Lemma 2, a heuristic upper bound of MIMO beamforming mismatch is given by

$$D_\phi \quad \geq \quad \left[\frac{M-1}{M}-\frac{M-1}{M}\left(\frac{2^R-1}{2^R}\right)^{\frac{1}{M-1}}\right]^{\frac{1}{2}} \ . \tag{30}$$

With (22) and (30), the rate-distortion region can heuristically be determined. One example of the rate-distortion region is shown in Fig. 4.

IV. The Rate-Reliability Region

The reverselink channel model is a concatenation of a Gaussian channel and binary erasure channel, which are independent to each other. In generally, the reliability of reverselink is controlled by both channel fading and received SNR. When the erasure rate ε_r is high, it means the amount of fading of reverselink is very high. Higher erasure rate also means it takes the forwardlink transmitter longer time to accurately filter out a proper forwardlink precoding word and it usually yields higher MIMO precoding mismatch given a certain channel coherent time. Since the unreliable symbols are erased based on their received SNR, the left symbols are more reliable and their reliability is mostly decided by $\gamma_{\rm RL}$. In this case, the well-known sphere-packing upper bound of Gaussian channel reliability function is

$$\epsilon\left(R\right) \quad \leq \quad \epsilon_{\rm sp}\left(R\right) \tag{31}$$

with $\epsilon_{\rm sp}\left(R\right)=\frac{\gamma_{\rm RL}}{2}-\frac{\sqrt{\gamma_{\rm RL}}\eta\cos\theta}{2}-\ln\left(\eta\sin\theta\right)$ and the lower bound is

$$\epsilon\left(R\right)\geq\begin{cases}\frac{\gamma_{\rm RL}}{4}\left(1-\cos\theta\right) & 0\leq R\leq R_1\\ \frac{\gamma_{\rm RL}}{4}\left(1-\cos\theta_1\right)+R_1-R & R_1\leq R\leq R_2\\ \epsilon_{\rm sp}\left(R\right) & R_2\leq R\leq C\end{cases} \tag{32}$$

where $\gamma_{\rm RL}=\frac{P_{\rm RL}}{\sigma^2_{\rm RL}}$ denotes the reverselink SNR, $\theta=\arcsin e^{-R}$ denotes the sphere-packing angle,

$$\eta \quad = \quad \frac{1}{2}\left(\sqrt{\gamma_{\rm RL}}\cos\theta+\sqrt{\gamma_{\rm RL}\cos^2\theta+4}\right)\ , \tag{33}$$

$$R_1 \quad = \quad \frac{1}{2}\ln\left(\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{\gamma_{\rm RL}}{4}}\right)\ , \tag{34}$$

$$R_2 \quad = \quad \frac{1}{2}\ln\left(\frac{1}{2}+\frac{\gamma_{\rm RL}}{4}+\frac{1}{2}\sqrt{1+\frac{\gamma_{\rm RL}}{4}}\right) \tag{35}$$

and $\theta_1=\arcsin e^{-R_1}$. The rate-reliability region with sphere-packing bounds is shown in Fig. 5.

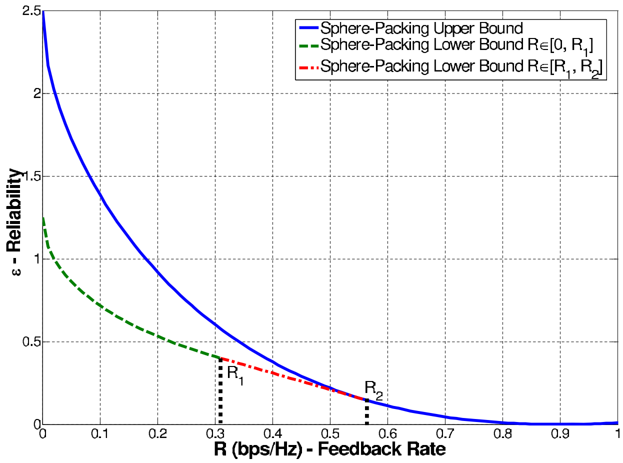

Fig. 5. The rate-reliability region with $\gamma_{\mathrm{RL}} = 7$dB.

V. Conclusions

In this paper, the distortion and reliability of MIMO beamforming feedback channel, which is modelled as a Gaussian binary erasure channel, are discussed for the problem how much feedback is necessary. The lower bound and upper bound for the rate-distortion region and the rate-reliability region of MIMO beamforming feedback are therefore derived. The tradeoffs between codebook design and channel structure are revealed.

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