Beamspace ESPRIT

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***Abstract*— Most high-resolution algorithms for sensor array processing require an eigendecomposition, which is a computation that is difficult to implement in parallel and requires $O(M^3)$ multiplications for an $M \times M$ matrix, corresponding to M sensors. Beamspace transformation is one way of reducing computation and sometimes improving the estimation accuracy. As a consequence of the beamspace transformation performed, however, arrays such as uniform linear arrays commonly used in direction finding lose their *displacement invariance* structure. As a result, computational complexity may actually *increase* since the computationally efficient ESPRIT algorithm cannot be applied directly. In this paper, a method for restoring the invariance structure resulting in a beamspace ESPRIT algorithm is described. Asymptotic performance analysis of beamspace ESPRIT and simulation results are presented as well.**

I. Introduction

The demand for high spatial resolution in direction finding requires large arrays with many elements. Since the computational complexity of most high-resolution direction-of-arrival (DOA) estimation algorithms is dominated by $O(M^3)$ eigendecomposition, increasing M leads to a cubic increase in complexity. Conventional eigendecomposition algorithms are also difficult to implement in parallel. Therefore, the high computational complexity may present a fundamental barrier to implementing these high-performance algorithms in *real time*. To achieve a significant reduction in computational time, a number of researchers such as Forster and Vezzosi [1], Lee and Wengrovitz [2], [3], Xu and Buckley [4], and Van Veen and Williams [5] proposed a so-called beamspace approach, which first projects the original data into a subspace of lower dimension (i.e., the beamspace) and then processes the beamspace data by using well-known direction finding algorithms such as MUSIC [6]. Since the beamspace data are effectively received from a synthetic or pseudoarray of size L, the computational complexity is reduced to $O(L^3)$. Lee and Wengrovitz [3] also analyzed the beamspace MUSIC and Min-Norm solutions and found performance improvement of the beamspace algorithms in some scenarios. Later Silverstein *et al.* [7] proposed a parallel scheme for implementing several beamspace processing procedures simultaneously. Recently, a fast algorithm for signal subspace decomposition (FSD) was proposed [8] to replace the burdensome eigendecomposition, resulting in almost another order of magnitude of reduction in the computation of each processor, i.e., from $O(L^3)$ to $O(L^2)$.

The remaining, potentially significant, component of the computational complexity of DOA estimation is the parameter estimation based on the signal subspace. In the uniform linear array (ULA) case, we can take advantage of the Vandermonde structure of ULA manifold vectors to transform the DOA estimation into a polynomial rooting problem, leading to the so-called Root-MUSIC algorithm [6]. Clearly, Root-MUSIC is usually much less computationally expensive than the well-known MUSIC algorithm [6] since the latter requires an exhaustive search, and the former only needs to root an order $2M-2$ polynomial, where M is the size of the real or synthetic array. However, ESPRIT [9], [10] can be even more efficient in DOA estimation as it requires only one $2d \times 2d$ and one $d \times d$ eigendecomposition, where $d(<M)$ denotes the number of sources. Unfortunately, Root-MUSIC and ESPRIT cannot be applied directly to beamspace-transformed data for DOA estimation because the ULA structure is lost after beamspace transformation. The reason is that the beamspace transformation is a linear transformation on the manifold vectors, and such a row operation alters the Vandermonde structure of the ULA manifold, which is the basis of Root-MUSIC. It is not difficult to see that a more general structural property of ULA than Vandermonde, *viz.* shift-invariance structure, is also lost. Nevertheless, Zoltowski *et al.* [11] successfully developed an efficient beamspace Root-MUSIC algorithm. Their approach first inflates the (L-dimensional) beamspace back to the (M-dimensional) element space, resulting in an order $2M - 2$ polynomial; then, by efficiently deflating the known $2M - 2L$ roots, it yields an $2L - 2$ polynomial, whose d roots are the signal poles. Here, a different approach is used to restore the shift-invariance structure lost in the beamspace transformation, resulting in a new beamspace ESPRIT algorithm.

II. Problem Statement

Assume that there are d sources impinging on a uniform linear array (ULA) of M sensors with separation distance D.

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Let $\boldsymbol{x}(t)$ be the data vector collected at time t. Then

$$\boldsymbol{x}(t) = \sum_{k=1}^{d} \boldsymbol{a}(\theta_k)s_k(t) + \boldsymbol{n}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \qquad (1)$$

where $\boldsymbol{A} = [\boldsymbol{a}(\theta_1)\vdots \cdots \vdots \boldsymbol{a}(\theta_d)], \boldsymbol{s}(t) = [s_1(t), \cdots, s_d(t)]^T$ and the array manifold vector corresponding to the kth source can be expressed as

$$\boldsymbol{a}(\theta_k) = [1, e^{j2\pi D \sin \theta_k/\lambda}, \cdots, e^{j2\pi(M-1)D \sin \theta_k/\lambda}]^T$$

where λ is the propagation wavelength. Assume that the noise $\boldsymbol{n}(t)$ is white with variance σ_n^2 and uncorrelated with the signals $s_k(t)$. Then, the covariance of $\boldsymbol{x}(t)$ becomes

$$\boldsymbol{R}_{xx} = \boldsymbol{A}\boldsymbol{S}\boldsymbol{A}^* + \sigma_n^2 \boldsymbol{I} \qquad (2)$$

where $\boldsymbol{S} = E\{\boldsymbol{s}(t)\boldsymbol{s}(t)^*\}$ is of rank d.

The problem is to estimate the DOA's θ_k given the data covariance matrix $\boldsymbol{R}_{xx}$ or an estimate thereof. Most signal subspace algorithms are comprised of two steps: 1) signal subspace decomposition and 2) parameter estimation based on the signal subspace estimate. It is easily shown that the principal eigenvectors corresponding to the d largest eigenvalues $\boldsymbol{E}_s = [\boldsymbol{e}_1, \cdots, \boldsymbol{e}_d]$ span the signal subspace $\mathcal{R}\{\boldsymbol{A}\}$, i.e., $\mathcal{R}\{\boldsymbol{E}_s\} = \mathcal{R}\{\boldsymbol{A}\}$. In practice, however, the sample covariance matrix $\hat{\boldsymbol{R}}_{xx}$ is an estimate of $\boldsymbol{R}_{xx}$ based on a finite number N of snapshots. In this case, the principal eigenvectors $\hat{\boldsymbol{E}}_s$ of the sample covariance matrix, which are consistent estimates of $\boldsymbol{E}_s$, span the estimated signal subspace.

III. Beamspace Processing

A. Subband or Beamspace Separation

For large arrays (i.e., large M), real-time implementation of signal subspace-based algorithms is impeded by the requirement for an $O(M^3)$ eigendecomposition. Recently, motivated by the beamspace approaches proposed in earlier papers, e.g., [2], [4], [5], Silverstein and *et al.* [7] proposed a parallel scheme, wherein the original data vector is decomposed into several lower-dimensional *subbands or beamspaces* via a transformation, e.g., a Fourier transform, and then, the DOA estimation is carried out based on the subband data. Since the data processing of each subband is independent, it can be carried out in parallel. If the dimension of each subband data is $L < M$, the computational time per subband can be reduced from $O(M^3)$ to $O(L^3)$. Since the beamspace transformation can be done in $O(ML)$ flops for each data vector, the beamspace sample covariance matrix can be accumulated in real time. Of course, for a ULA, the FFT can be employed to further reduce the computational effort.

B. Beamspace Signal Subspace Algorithms

The implementation diagram of beamspace processing is shown in Fig. 1.

Let $\boldsymbol{T}_i$ be an $M \times L$ matrix with orthogonal columns, i.e., $\boldsymbol{T}_i^*\boldsymbol{T}_i = \boldsymbol{I}$. Then, the beamspace transformation is defined by applying $\boldsymbol{T}_i$ to the snapshot $\boldsymbol{x}(t)$. The corresponding

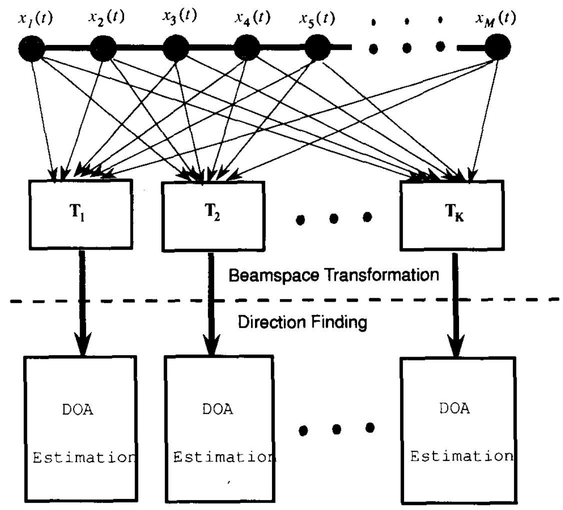

Fig. 1. Implementation diagram of beamspace transformation and direction finding.

transformed covariance matrix will have the form

$$\begin{aligned}\boldsymbol{R}_i &= \boldsymbol{T}_i^*\boldsymbol{R}_{xx}\boldsymbol{T}_i = \boldsymbol{T}_i^*\boldsymbol{A}\boldsymbol{S}\boldsymbol{A}^*\boldsymbol{T}_i + \sigma_n^2\boldsymbol{T}_i^*\boldsymbol{T}_i \\ &= \boldsymbol{A}_i\boldsymbol{S}\boldsymbol{A}_i^* + \sigma_n^2\boldsymbol{I} \end{aligned} \qquad (3)$$

where $\boldsymbol{A}_i = \boldsymbol{T}_i^*\boldsymbol{A} = [\boldsymbol{T}_i^*\boldsymbol{a}(\theta_1), \cdots, \boldsymbol{T}_i^*\boldsymbol{a}(\theta_d)]$ and $i = 1, \cdots, M/L$. The signal subspace span $\{\boldsymbol{A}_i\}$ can be formed as the range space of the eigenvectors $\boldsymbol{E}_i$ corresponding to the d largest eigenvalues of $\boldsymbol{R}_i$ since it can be seen that $\mathcal{R}\{E_i\} = \mathcal{R}\{\boldsymbol{T}_i^*\boldsymbol{E}_s\}$. In practice, since only the sample covariance $\hat{\boldsymbol{R}}_i = \boldsymbol{T}_i^*\hat{\boldsymbol{R}}_{xx}\boldsymbol{T}_i$ is available, its principal eigenvectors $\hat{\boldsymbol{E}}_i$ are used as basis vectors for the signal subspace estimate.

Observing that $\hat{\boldsymbol{R}}_i$ is $L \times L$, which can be much smaller than the dimension of the original $\hat{\boldsymbol{R}}_{xx}$, only $O(L^3)$ computational complexity is incurred in each beamspace. Since the processing in each beamspace can be carried out in parallel, the total computation time is also $O(L^3)$. In fact, using the recently proposed fast signal-subspace decomposition (FSD) approach [8] instead of the standard eigendecomposition, the computation time can be reduced to $O(L^2d)$. If further parallelization is incorporated, i.e., L or L^2 simple array processors are used, the computation time can be cut down to $O(Ld)$ or $O(\log Ld)$, respectively.

With the signal subspace estimate, various high-resolution algorithms, e.g., MUSIC, Root-MUSIC, and most notably ESPRIT, can be used to find the DOA estimates. It is easily seen that MUSIC can be trivially extended from element space to beamspace since it does not rely on any special structure of the array. Root-MUSIC (for ULA's) and ESPRIT, however, rely on the *Vandermonde* and *displacement invariance* structures, which are destroyed by the beamspace transformation. A beamspace Root-MUSIC algorithm was recently proposed in [11]. Herein, a beamspace ESPRIT algorithm is described.

IV. Beamspace ESPRIT

A. Review of Element-space ESPRIT

In the class of high-resolution direction-finding algorithms,

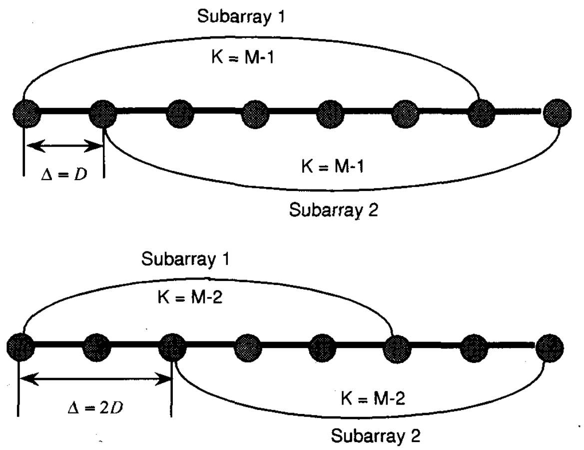

Fig. 2. Two possible subarrays from an eight-element ULA.

ESPRIT [9] is attractive for real-time applications due to its computational and implementational advantages. The basic idea behind the ESPRIT algorithm is to exploit the so-called array displacement invariance structure, i.e., two identical subarrays are separated by a common displacement Δ.

For simplicity, the discussion herein will be restricted to ULA's. As shown in Fig. 2, the first K and last K elements can be used to form two identical subarrays with displacement $\Delta = D(M-K), 1 \leq K \leq M-1$. Thus, the array manifold vectors for these two subarrays can be expressed as $\boldsymbol{A}_1 = \boldsymbol{J}_1\boldsymbol{A}$ and $\boldsymbol{A}_2 = \boldsymbol{J}_2\boldsymbol{A}$, where $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$ are selection matrices

$$\boldsymbol{J}_1 = [\overbrace{\boldsymbol{I}_K}^{K} \; \overbrace{\boldsymbol{0}}^{M-K}], \; \boldsymbol{J}_2 = [\overbrace{\boldsymbol{0}}^{M-K} \; \overbrace{\boldsymbol{I}_K}^{K}]. \tag{4}$$

Clearly, $\boldsymbol{J}_1\boldsymbol{A}$ picks the first K rows of $\boldsymbol{A}$, whereas $\boldsymbol{J}_2\boldsymbol{A}$ chooses the last K rows of $\boldsymbol{A}$

Since $\boldsymbol{A}$ is a Vandermonde matrix, it can be shown that

$$\boldsymbol{J}_1\boldsymbol{A} = \boldsymbol{J}_2\boldsymbol{A}\boldsymbol{\Phi}^* \tag{5}$$

where $\Phi = \text{diag}\,\{e^{j2\pi\Delta K \sin\theta_1/\lambda}, \cdots, e^{j2\pi\Delta K \sin\theta_d/\lambda}\}$.

Using $\boldsymbol{R}_{xx} = \boldsymbol{ASA}^* + \sigma_n^2\boldsymbol{I}$, it follows that $\mathcal{R}\{\boldsymbol{E}_s\} = \mathcal{R}\{\boldsymbol{A}\}$ or $\boldsymbol{E}_s = \boldsymbol{AP}$, where $\boldsymbol{P}$ is a full-rank $d \times d$ matrix. Then

$$\boldsymbol{J}_1\boldsymbol{E}_s = \boldsymbol{J}_1\boldsymbol{AP} = \boldsymbol{J}_2\boldsymbol{A}\boldsymbol{\Phi}^*\boldsymbol{P}. \tag{6}$$

Since $\boldsymbol{A} = \boldsymbol{E}_s\boldsymbol{P}^{-1}$

$$\boldsymbol{J}_1\boldsymbol{E}_s = \boldsymbol{J}_2\boldsymbol{E}_s\boldsymbol{P}^{-1}\boldsymbol{\Phi}^*\boldsymbol{P} = \boldsymbol{J}_2\boldsymbol{E}_s\boldsymbol{\Psi}. \tag{7}$$

From (7), we see that $(J)_1\boldsymbol{E}_s$ and $\boldsymbol{J}_2\boldsymbol{E}_s$ share the same column space and $\boldsymbol{\Psi} = \boldsymbol{P}^{-1}\boldsymbol{\Phi}^*\boldsymbol{P}$ can be found by solving a set of overdetermined equations. Since the eigenvalues of $\boldsymbol{\Psi}$ are the diagonal elements of $\boldsymbol{\Phi}$, and eigenvalue decomposition of $\boldsymbol{\Psi}$ yields $e^{j2\pi\Delta \sin\theta_k/\lambda}$ and, eventually, the DOA's $\theta_k, k = 1, \cdots, d$.

In the sample covariance matrix case, all the equalities in (6) and (7) are replaced by *approximate* equalities. Instead of solving a set of linear equations to find $\boldsymbol{M}$ in (7), a $2d \times 2d$ eigendecomposition is performed to find the total least squares (TLS) estimate of $\boldsymbol{\Psi}$ followed by a $d \times d$ eigendecomposition to find the DOA's [10]. This briefly summarizes the basic idea behind the TLS-ESPRIT approach to parameter (DOA) estimation based on exploiting sensor array invariances. The complete TLS-ESPRIT algorithm is listed below.

Summary of the TLS-ESPRIT Algorithm

1) Obtain the sample covariance $\hat{\boldsymbol{R}}_{xx}$, an estimate of $\boldsymbol{R}_{xx}$.
2) Compute the eigendecomposition of $\hat{\boldsymbol{R}}_{xx}$

$$\hat{\boldsymbol{R}}_{xx} = \hat{\boldsymbol{E}}\hat{\Lambda}\hat{\boldsymbol{E}}^*, \quad \hat{\boldsymbol{E}} = [\underbrace{\hat{\boldsymbol{E}}_s}_{d} \,|\hat{\boldsymbol{E}}_n].$$

3) If necessary, estimate the number of sources d.
4) Partition $\hat{\boldsymbol{E}}_s$ as

$$[\hat{\boldsymbol{E}}_x \, \hat{\boldsymbol{E}}_y] = [\boldsymbol{J}_1\hat{\boldsymbol{E}} \, \boldsymbol{J}_2\hat{\boldsymbol{E}}].$$

5) Compute the eigendecomposition of the $2d \times 2d$ matrix

$$\begin{bmatrix} \hat{\boldsymbol{E}}_x^* \\ \hat{\boldsymbol{E}}_y^* \end{bmatrix} [\hat{\boldsymbol{E}}_x\hat{\boldsymbol{E}}_y] = \boldsymbol{E}\boldsymbol{\Lambda}\boldsymbol{E}^*.$$

6) Partition $\boldsymbol{E}$ into $d \times d$ submatrices

$$\boldsymbol{E} \stackrel{\text{set}}{=} \begin{bmatrix} \boldsymbol{E}_{11} & \boldsymbol{E}_{12} \\ \boldsymbol{E}_{21} & \boldsymbol{E}_{22} \end{bmatrix}.$$

7) Calculate the eigenvalues $\lambda_k(-\boldsymbol{E}_{12}\boldsymbol{E}_{22}^{-1})$.
8) Then

$$\hat{\theta}_k = \sin^{-1}\left(-\frac{\lambda_k}{2\pi\Delta_K} \cdot \arg \lambda_k\right).$$

B. Beamspace ESPRIT

In beamspace processing, however, the translational invariance structure in the array manifold is altered by the row transformation $\boldsymbol{T}_i^*$, i.e., $\boldsymbol{A}_i = \boldsymbol{T}_i^*\boldsymbol{A}$, and consequently, $\boldsymbol{J}_1\boldsymbol{A}_i \neq \boldsymbol{J}_2\boldsymbol{A}_i\boldsymbol{\Phi}^*$. In general, it is very difficult to restore this property since the beamspace transformation is not invertible (from a high-dimension space to a low-dimension space). However, if $\boldsymbol{T}_i$ has the same shift invariance structure, the lost shift invariance structure can be restored, as is shown by the following theorem.

Theorem IV.1: Let $\boldsymbol{T}_i$ be an $M \times L (L < M)$ unitary matrix. Assume that its first $K(<L)$ rows and its last K rows share the same column span, *viz.*

$$\boldsymbol{J}_1\boldsymbol{T}_i = \boldsymbol{J}_2\boldsymbol{T}_i\boldsymbol{F} \tag{8}$$

where $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$ are defined as in (4), and $\boldsymbol{F}$ is a nonsingular $L \times L$ matrix. Let $\boldsymbol{T}_i^* = [\boldsymbol{t}_1, \boldsymbol{t}_2, \cdots, \boldsymbol{t}_M]$. If there exists an $L \times L$ matrix $\boldsymbol{Q}$ such that $\boldsymbol{Qt_k} = 0, K+1 \leq k \leq M$ and $\boldsymbol{QF^*t_j} = 0, 1 \leq j \leq M-K$, then

$$\boldsymbol{QA}_i = \boldsymbol{QF}^*\boldsymbol{A}_i\boldsymbol{\Phi}^* \tag{9}$$

where $\boldsymbol{A}_i = \boldsymbol{T}_i^*\boldsymbol{A}$, and $\boldsymbol{\Phi}$ is given in (5).

Proof: By the condition on $\boldsymbol{Q}$

$$\boldsymbol{QT}_i^* = [\overbrace{\boldsymbol{Q}t_1, \cdots, \boldsymbol{Q}t_K}^{K}, \overbrace{0 \cdots 0}^{M-K}] = \boldsymbol{QT}_i^* \boldsymbol{J}_1^* \boldsymbol{J}_1 \quad (10)$$

$$\begin{aligned}\boldsymbol{QF}^*\boldsymbol{T}_i^* &= [\overbrace{0 \cdots 0}^{M-K}\ \overbrace{\boldsymbol{QF}^* t_{M-K+1}, \cdots, \boldsymbol{QF}^* t_M}^{K}] \\ &= \boldsymbol{QF}^*\boldsymbol{T}_i^*\boldsymbol{J}_2^*\boldsymbol{J}_2. \end{aligned} \quad (11)$$

Therefore

$$\boldsymbol{QA}_i = \boldsymbol{QT}_i^* \boldsymbol{A} = \boldsymbol{QT}_i^* \boldsymbol{J}_1^* \boldsymbol{J}_1 \boldsymbol{A}. \quad (12)$$

By using (5), *viz.* $\boldsymbol{J}_1\boldsymbol{A} = \boldsymbol{J}_2\boldsymbol{A\Phi}^*$ and the transpose of (8), *viz.*, $\boldsymbol{T}_i^*\boldsymbol{J}_1^* = \boldsymbol{F}^*\boldsymbol{T}_i^*\boldsymbol{J}_2^*$, we see that

$$\boldsymbol{QA}_i = \boldsymbol{Q}\ \underbrace{\boldsymbol{T}_i^*\boldsymbol{J}_1^*}\ \underbrace{\boldsymbol{J}_1\boldsymbol{A}} = \boldsymbol{Q}\ \underbrace{\boldsymbol{F}^*\boldsymbol{T}_i^*\boldsymbol{J}_2^*}\ \underbrace{\boldsymbol{J}_2\boldsymbol{A\Phi}^*}. \quad (13)$$

By (11)

$$\boldsymbol{QF}^*\boldsymbol{A}_i = \underbrace{\boldsymbol{QF}^*\boldsymbol{T}_i}\ \boldsymbol{A} = \underbrace{\boldsymbol{QF}^*\boldsymbol{T}_i\boldsymbol{J}_2^*\boldsymbol{J}_2}\ \boldsymbol{A}. \quad (14)$$

Comparing (13) and (14), we can conclude that

$$\boldsymbol{QA}_i = \boldsymbol{QF}^*\boldsymbol{A}_i\boldsymbol{\Phi}^*. \quad (15)$$

Theorem IV.1 illustrates a way of restoring the lost array shift-invariance structure. Comparing (5) and (9) and realizing that $\mathcal{R}\{\boldsymbol{T}_i^*\boldsymbol{A}\} = \mathcal{R}\{\boldsymbol{T}_i^*\boldsymbol{E}_s\}$, $\boldsymbol{J}_1$ **and** $\boldsymbol{J}_2$ in (7) can be replaced by $\boldsymbol{Q}$ and $\boldsymbol{QF}^*$ respectively, and ESPRIT can be employed using the eigenvectors of the beamspace covariance matrix $\boldsymbol{R}_i$. It is worth noting that $\boldsymbol{Q}$ in Theorem IV.1 can be found by forming a projection matrix corresponding to the orthogonal subspace of $\mathcal{R}\{\boldsymbol{t}_{K+1}, \cdots, \boldsymbol{t}_M, \boldsymbol{F}^*t_1, \cdots, \boldsymbol{F}^*t_{M-K}\}$. Since, in general, $\boldsymbol{Q}$ is of rank $L - 2(M-K)$, only $L - 2(M-K)$ independent rows need be chosen from $\boldsymbol{Q}$ to form a new $\boldsymbol{Q}$. However, if standard Fourier basis vectors are used to form $\boldsymbol{T}_i$, $\boldsymbol{T}_i$ is a Vandermonde matrix, and $\boldsymbol{F}^*$ is diagonal. Moreover, it can be shown that $\mathcal{R}\{\boldsymbol{t}_{K+1}, \cdots, \boldsymbol{t}_M\} = \mathcal{R}\{\boldsymbol{F}^*\boldsymbol{t}_1, \cdots, \boldsymbol{F}^*\boldsymbol{t}_{M-K}\}$. Therefore, $\boldsymbol{Q}$ is only required to be orthogonal to $\{\boldsymbol{t}_{K+1}, \cdots, \boldsymbol{t}_M\}$ and is rank $L-K$. This special case will be analyzed in the following corollary.

Corollary IV.1.1: Let $\boldsymbol{T}_i$ be a collection of standard Fourier basis vectors, i.e.

$$\boldsymbol{T}_i = [\boldsymbol{T}_1^{(i)}, \cdots, \boldsymbol{T}_L^{(i)}] \quad (16)$$

where $\boldsymbol{T}_i^{(i)} = [1, e^{j2\pi k_i^{(i)}/M}, \cdots, e^{j2\pi k_i^{(i)}(M-1)/M}]^T$ and $\{k_l^{(i)}\}_{l=1}^L$ are integers. Then, for the same $\boldsymbol{F}$ as in (8)

$$[\boldsymbol{t}_{K+1}, \cdots, \boldsymbol{t}_M] = [\boldsymbol{F}^*\boldsymbol{t}_1, \cdots, \boldsymbol{F}^*\boldsymbol{t}_{M-K}] \quad (17)$$

or more concisely

$$\tilde{\boldsymbol{J}}_2\boldsymbol{T}_i = \tilde{\boldsymbol{J}}_1\boldsymbol{T}_i\boldsymbol{F} \quad (18)$$

where

$$\tilde{\boldsymbol{J}}_1 = [\overbrace{\ \boldsymbol{0}\ }^{K}\ \overbrace{\boldsymbol{I}_{M-K}}^{M-K}],\ \tilde{\boldsymbol{J}}_2 = [\overbrace{\boldsymbol{I}_{M-K}}^{M-K}\ \overbrace{\ \boldsymbol{0}\ }^{K}]. \quad (19)$$

Proof: Given the special Vandermonde matrix $\boldsymbol{T}_i$ as in (16) since the $\boldsymbol{J}_1\boldsymbol{T}_i$ and $\boldsymbol{J}_2\boldsymbol{T}_i$ are separated by K elements, it is easily seen that $\boldsymbol{F} = \boldsymbol{D}^{-(M-K)} = \boldsymbol{D}^{-M+K}$, where $\boldsymbol{D} = \mathrm{diag}\,\{e^{j2\pi k_1^{(i)}/M}, \cdots, e^{j2\pi k_L^{(i)}/M}\}$. Now, since $\tilde{\boldsymbol{J}}_2\boldsymbol{T}_i$ and $\tilde{\boldsymbol{J}}_1\boldsymbol{T}_i$ are displaced by $M-K$ elements, by the same reasoning

$$\tilde{\boldsymbol{J}}_1\boldsymbol{T}_i = \tilde{\boldsymbol{J}}_2\boldsymbol{T}_i\tilde{\boldsymbol{F}} \quad (20)$$

where $\tilde{\boldsymbol{F}} = \boldsymbol{D}^K$, Clearly it is easy to see that $\boldsymbol{D}^{-M} = \mathrm{diag}\,\{1, \cdots, 1\} = \boldsymbol{I}_L$. Therefore, $\boldsymbol{F} = \tilde{\boldsymbol{F}}$ and

$$\tilde{\boldsymbol{J}}_2\boldsymbol{T}_i = \tilde{\boldsymbol{J}}_1\boldsymbol{T}_i\boldsymbol{F}. \quad (21)$$

Taking the Hermitian conjugate of the above equation and appropriately reorganizing the rows, it is easily seen that (17) holds.

It is worth noting that the computation of $\boldsymbol{Q}\hat{\boldsymbol{E}}_i$ and $\boldsymbol{QF}^*\hat{\boldsymbol{E}}_i$ takes about $O(L^2d)$ flops, although $\boldsymbol{Q}$ and $\boldsymbol{QF}^*$ can be evaluated beforehand. In fact, this can be simplified. First of all, since $\boldsymbol{Q}$ is usually rank $L - 2(M-K)$ and in the standard case, it is rank $L - (M-K)$, we can delete $2(M-K)$ or $M-K$ columns, and the size of the matrix $\boldsymbol{Q}$ will be smaller. However, usually, $M-K$ is a small number, e.g., $M-K = 1$ or 2, and the $\boldsymbol{T}_i$ is a collection of standard Fourier vectors. In this case, we can just find $M-K$ orthonormal vectors $\boldsymbol{b}_1, \cdots, \boldsymbol{b}_{M-K}$ such that $\mathcal{R}\{\boldsymbol{b}_1, \cdots, \boldsymbol{b}_{M-K}\} = \mathcal{R}\{\boldsymbol{t}_1, \cdots, \boldsymbol{t}_{M-K}\}$; then, by Theorem IV.1 or Corollary IV.1.1, $\boldsymbol{Q} = \boldsymbol{I}_L - \sum_{k=1}^{M-K} \boldsymbol{b}_k\boldsymbol{b}_k^*$. Thus

$$\boldsymbol{Q}\hat{\boldsymbol{E}}_i = \hat{\boldsymbol{E}}_i - \sum_{k=1}^{M-K} \boldsymbol{b}_k\boldsymbol{b}_k^*\hat{\boldsymbol{E}}_i \quad (22)$$

$$\boldsymbol{QF}^*\hat{\boldsymbol{E}}_i = \boldsymbol{F}^*\hat{\boldsymbol{E}}_i - \sum_{k=1}^{M-K} (\boldsymbol{F}^*\boldsymbol{b}_k)(\boldsymbol{b}_k^*\hat{\boldsymbol{E}}_i). \quad (23)$$

It is not hard to see that the computational complexity in (22) is $O(L(M-K)d)$. Since $\boldsymbol{F}^*$ is a diagonal matrix, the computation of (23) requires $O(L(M-K)d)$. Therefore, if $L \gg (M-K)$, which is usually the case, the computational savings is quite significant. The basic beamspace ESPRIT algorithm is summarized below.

The Beamspace ESPRIT Algorithm

1) For the ith subband, obtain the beamspace covariance matrix $\hat{\boldsymbol{R}}_i = \boldsymbol{T}_i^*\hat{\boldsymbol{R}}_{xx}\boldsymbol{T}_i$.
2) Estimate the (beamspace) signal subspace (via an eigendecomposition or FSD) $\mathcal{R}\{\hat{\boldsymbol{E}}_i\}$.
3) If necessary, estimate the number of sources in the ith subband.
4) Find $\boldsymbol{Q}$ as in Theorem IV.1 or Corollary IV.1.1
5) Apply ESPRIT on $(\boldsymbol{Q}\hat{\boldsymbol{E}}_i, \boldsymbol{QF}^*\hat{\boldsymbol{E}}_i)$ to obtain the parameter (DOA) estimates θ_i, $i = 1, \cdots, d$.

C. Discussion

It should be clarified that the beamspace processing can be done separately on two subarrays, i.e., $\boldsymbol{T}_i^*\boldsymbol{J}_1\hat{\boldsymbol{R}}_{xx}\boldsymbol{J}_1^*\boldsymbol{T}_i$ and $\boldsymbol{T}_i^*\boldsymbol{J}_2\hat{\boldsymbol{R}}_{xx}\boldsymbol{J}_2^*\boldsymbol{T}_i$, where $\boldsymbol{T}_i$ is an $K \times L$ matrix (instead of an $M \times L$ matrix), where K is the number of overlapping elements in two subarrays, L the number of beams, and M the total number of array elements. In this case, after performing two eigendecompositions on these two matrices, the ESPRIT algorithm can be applied in a straightforward manner. Nevertheless, this simple approach does not use the full array aperture in the beamspace transformation, and it results in greater computational complexity as it requires two beamspace transformations instead of one. In comparison, the proposed beamspace ESPRIT algorithm is more efficient. However, its applicability requires that the beamspace transformation $\boldsymbol{T}_i$ satisfy the condition of Theorem IV.1. Fortunately, a ULA is a common array structure in sensor array processing, and in this case, $\boldsymbol{T}_i$ is very likely composed of standard FFT vectors, which clearly satisfies the condition of Theorem IV.1. For arrays with other displacement invariance properties, *viz.* satisfying $\boldsymbol{J}_1\boldsymbol{A} = \boldsymbol{J}_2\boldsymbol{A}\boldsymbol{\Phi}^*$, $\boldsymbol{T}_i$ is usually formed as follows: 1) choose L independent array manifold vectors $\boldsymbol{A}_L$ corresponding to DOA's in a sector or in the ith subband $(\theta_1^{(i)}, \theta_2^{(i)})$; 2) obtain an orthogonal basis of the range space spanned by the L manifold vectors. It is not too difficult to see that $\boldsymbol{T}_i = \boldsymbol{A}_L\boldsymbol{Q}_L$, where $\boldsymbol{Q}_L$ is an $L \times L$ nonsingular matrix. Then

$$\begin{aligned} \boldsymbol{J}_1\boldsymbol{T}_i &= \boldsymbol{J}_1\boldsymbol{A}_L\boldsymbol{Q}_L = \boldsymbol{J}_2\boldsymbol{A}_L\boldsymbol{\Phi}^*\boldsymbol{Q}_L \\ &= \underbrace{\boldsymbol{J}_2\boldsymbol{A}_L\boldsymbol{Q}_L}_{T_i}\,\underbrace{\boldsymbol{Q}_L^{-1}\boldsymbol{\Phi}^*\boldsymbol{Q}_L}_{F} = \boldsymbol{J}_2\boldsymbol{T}_i\boldsymbol{F}. \end{aligned} \tag{24}$$

Hence, in this case, the beamspace ESPRIT algorithm can be employed for DOA estimation.

V. Performance Analysis

The problem solved by ESPRIT can be asymptotically equivalently formulated as an optimization problem [12]

$$\min_{\boldsymbol{\eta}, \boldsymbol{T}} \|\boldsymbol{J}\hat{\boldsymbol{E}}_s - \boldsymbol{G}(\boldsymbol{\eta})\boldsymbol{T}\|^2 \tag{25}$$

where

$$\boldsymbol{J} = \begin{bmatrix} \boldsymbol{J}_1 \\ \boldsymbol{J}_2 \end{bmatrix}, \ \boldsymbol{G}(\boldsymbol{\eta}) = \begin{bmatrix} \boldsymbol{J}_1\boldsymbol{A} \\ \boldsymbol{J}_2\boldsymbol{A}\boldsymbol{\Phi} \end{bmatrix}, \tag{26}$$

$$\boldsymbol{\Phi} = \mathrm{diag}\,\{\rho_1 e^{j2\pi\Delta K \sin\theta_1/\lambda}, \cdots, \rho_d e^{j2\pi\Delta K \sin\theta_d/\lambda}\}. \tag{27}$$

The unknown parameters in $\boldsymbol{G}(\boldsymbol{\eta})$ can be collected in the $2Md$-dimensional vector $\boldsymbol{\eta}$

$$\begin{aligned} \boldsymbol{\eta} = [&\theta_1, \cdots, \theta_d, \rho_1, \cdots, \rho_d, \\ &vec(\tilde{\boldsymbol{I}}\,\mathrm{Re}\,(\boldsymbol{A})), vec(\tilde{\boldsymbol{I}}\,\mathrm{Im}\,(\boldsymbol{A}))] \end{aligned} \tag{28}$$

where $\tilde{\boldsymbol{I}} = [\boldsymbol{0}_{(\mathbf{M}-\mathbf{1})\times\mathbf{1}}\boldsymbol{I}]$. As shown in [12], the minimizer $\hat{\boldsymbol{\eta}}$ of (25) is the same as the maximizer of the following function $V_N(\boldsymbol{\eta})$

$$\begin{aligned} \hat{\boldsymbol{\eta}} &= \arg\max_{\boldsymbol{\eta}} V_N(\boldsymbol{\eta}) \\ &= \arg\max_{\boldsymbol{\eta}} \mathrm{Tr}\,\{\boldsymbol{P}_G(\boldsymbol{\eta})\boldsymbol{J}\hat{\boldsymbol{E}}_s\hat{\boldsymbol{E}}_s^*\boldsymbol{J}^*\} \end{aligned} \tag{29}$$

where $\boldsymbol{P}_G(\boldsymbol{\eta})$ is the projection matrix corresponding to the subspace $\mathcal{R}\{\boldsymbol{G}(\boldsymbol{\eta})\}$. It was shown in [12] that the estimate of first d parameters of $\hat{\boldsymbol{\eta}}$ i.e., $\{\theta_i\}_{i=1}^d$ are the same as the DOA estimates from the ESPRIT algorithm. Although it is not computationally efficient to compute the DOA estimates by optimizing (29), it is much easier to analyze the asymptotic variance of $\hat{\boldsymbol{\eta}}$ than to analyze the ESPRIT algorithm directly. The results of the performance analysis are summarized in the following theorem, which is a simple extension of the derivations in [12].

Theorem V.1: Let $\hat{\boldsymbol{\eta}}$ be obtained from (29) and $\boldsymbol{\eta}_0$ consist of the true parameters. Then, $\sqrt{N}(\hat{\boldsymbol{\eta}} - \boldsymbol{\eta}_0)$ is asymptotically Gaussian distributed with zero-mean and covariance matrix $\boldsymbol{C} = (\boldsymbol{V})^{-1}\boldsymbol{U}(\boldsymbol{V})^{-1}$, where the (i, j)th element of V and $\boldsymbol{U}$ are given by

$$\{\boldsymbol{V}\}_{ij} = -2\,\mathrm{Re}\,[\mathrm{Tr}\,(\boldsymbol{G}_j^*\boldsymbol{P}_G^\perp\boldsymbol{G}_i\boldsymbol{\Gamma}_V)] \tag{30}$$

$$\boldsymbol{U}_{ij} = 2\sigma_n^2\,\mathrm{Re}\,[\mathrm{Tr}\,(\boldsymbol{G}_j^*\boldsymbol{P}_G^\perp\boldsymbol{J}\boldsymbol{J}^*\boldsymbol{P}_G^\perp\boldsymbol{G}_i^*\boldsymbol{\Gamma}_U)] \tag{31}$$

where

$$\boldsymbol{G}_k = \frac{\partial}{\partial\boldsymbol{\eta}_k}\boldsymbol{G}(\boldsymbol{\eta}), \boldsymbol{P}_G^\perp = \boldsymbol{I} - \boldsymbol{P}_G$$

and

$$\boldsymbol{\Gamma}_V = \boldsymbol{G}^\dagger\boldsymbol{J}\boldsymbol{E}_s^*\boldsymbol{J}^*\boldsymbol{G}^\dagger, \tag{32}$$

$$\boldsymbol{\Gamma}_U = \boldsymbol{G}^\dagger\boldsymbol{J}\boldsymbol{E}_s\boldsymbol{\Lambda}_s(\boldsymbol{\Lambda}_s - \sigma_n^2\boldsymbol{I})\boldsymbol{E}_s^*\boldsymbol{J}^*\boldsymbol{G}^\dagger. \tag{33}$$

The variances of the ESPRIT (DOA) estimates $\{\hat{\boldsymbol{\theta}}_i\}_{i=1}^d$ are the first d diagonal elements of $\boldsymbol{C}$.

With Theorem V.1, the asymptotic performance bounds of the beamspace ESPRIT algorithm can be obtained quite easily. Defining

$$\boldsymbol{J} \overset{\text{def}}{=} \begin{bmatrix} \boldsymbol{Q} \\ \boldsymbol{Q}\boldsymbol{F} \end{bmatrix}, \quad \boldsymbol{G} \overset{\text{def}}{=} \begin{bmatrix} \boldsymbol{Q}\boldsymbol{T}_i^*\boldsymbol{A} \\ \boldsymbol{Q}\boldsymbol{F}\boldsymbol{T}_i^*\boldsymbol{A}\boldsymbol{\Phi} \end{bmatrix} \tag{34}$$

and $\hat{\boldsymbol{E}}_s \overset{\text{def}}{=} \hat{\boldsymbol{E}}_i, \boldsymbol{C}$ in Theorem V.1 gives the performance bound of the beamspace ESPRIT algorithm directly.

VI. Simulations and Discussions

To assess the performance of beamspace ESPRIT and to verify the asymptotic performance analysis, the following simulations were conducted. Two uncorrelated sources at 10 and 12° impinged on a 32-element ULA with half-wavelength element spacing. The signal-to-noise-ratios (SNR's) of these two sources were −5 dB. For each of 500 trials, 100 snapshots were taken. Twenty-seven beamspace transformations centered at 11°, ranging from 6 to 32 beams, were investigated. Using the real part[1] of each beamspace sample covariance matrix,

[1] Zoltowski et al. [11] showed that working with the real part of the beamspace covariance matrix does not affect the asymptotic performance of an algorithm in the case of uncorrelated sources but reduces the computational complexity significantly.

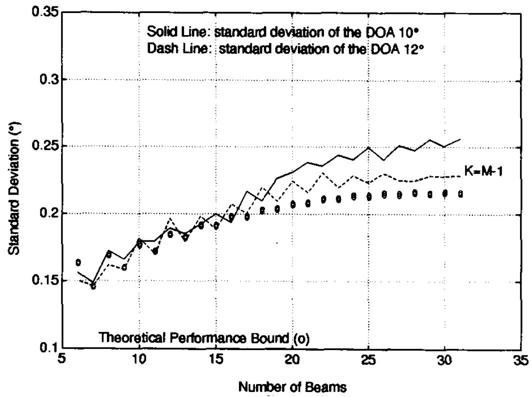

Fig. 3. Performance of beamspace ESPRIT ($K = M - 1$).

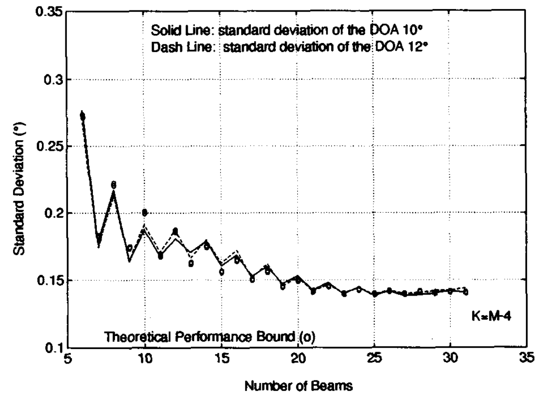

Fig. 5. Performance of beamspace ESPRIT ($K = M - 3$).

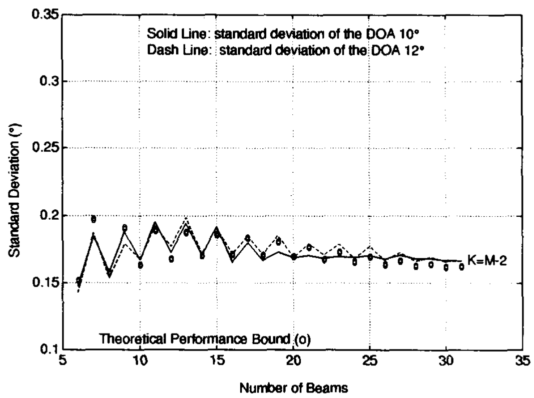

Fig. 4. Performance of beamspace ESPRIT ($K = M - 2$).

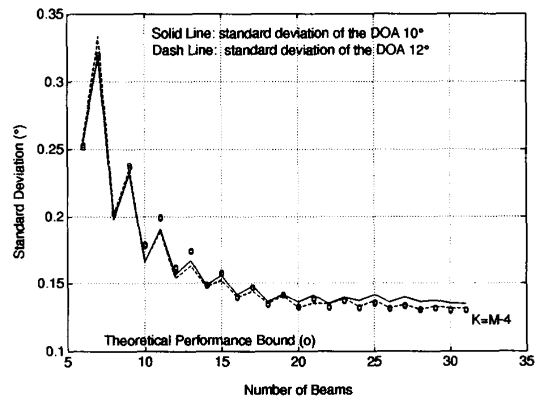

Fig. 6. Performance of beamspace ESPRIT ($K = M - 4$).

the beamspace Root-MUSIC [11] and ESPRIT ($K = M - 1, \cdots, M - 4$) algorithms were used to obtain DOA estimates. To simplify the performance comparison, the number of sources was assumed to be known. Simulation results are presented in Figs. 3–6, which show the standard deviations of the DOA estimates for beamspace ESPRIT. In each figure, two curves denote the standard deviations of two estimates corresponding to the DOA's 10° and 12°, whereas overlapping circles represent the performance bounds for the same DOA estimates.

Interestingly, numerical instability was observed in the beamspace Root-MUSIC algorithm perhaps due to the fact that it requires a procedure for deflating roots of a large polynomial. On the other hand, beamspace ESPRIT was numerically robust. This numerical problem can be overcome when M/L is an integer [11], and in this case, beamspace Root-MUSIC yielded comparable results to beamspace ESPRIT. Since we cannot explain the source of this numerical problem satisfactorily, the results of the beamspace Root-MUSIC are not shown here.

Since Root-MUSIC and ESPRIT are suboptimal algorithms, a variation in performance corresponding to various beamspace transformations was expected. From Figs. 3 and 6, it is interesting to note that beamspace ESPRIT for $K = M - 1$ generally performs better as the beamspace dimension L decreases, whereas the opposite is true if $K = M - 2, \cdots, M - 4$. This can be explained by a combination of two opposing factors. The increase in L enlarges the effective array aperture and thus improves the performance of beamspace ESPRIT, whereas reducing the number of beams incorporates more *a priori* information about the rough DOA locations and exploitation of more information usually results in better performance. Finally, it should be noted that the theoretical asymptotic performance bounds of Theorem V.1 were also plotted as small circles on the same figure and they are quite close to the simulation results.

In Table I, the beamspace Root-MUSIC and ESPRIT are compared in terms of computational complexity. The only difference is in the last step, i.e., the DOA estimation. In the simulation, we also counted the flops required by the beamspace Root-MUSIC and ESPRIT. From the simulation results, we found that the computational savings of beamspace ESPRIT is *significant* since it requires a factor of $11 - 30$ less computation (corresponding to $L = 6 - 32$) than beamspace Root-MUSIC.

TABLE I
COMPARISON OF COMPUTATIONAL COMPLEXITIES OF BEAMSPACE ROOT-MUSIC AND ESPRIT

Algorithms	Beamspace Transformation	Subspace Decomposition ED	Subspace Decomposition FSD	DOA Estimation
Beamspace Root-MUSIC	$O(ML^2)$	$O(L^3)$	$O(L^2d)$	$O(L^3)$
Beamspace ESPRIT	$O(ML^2)$	$O(L^3)$	$O(L^2d)$	$O(Ld^2)$

VII. CONCLUSIONS

A beamspace ESPRIT algorithm has been presented. The algorithm achieves a significant computational savings over element-space ESPRIT and beamspace Root-MUSIC. In a typical direction-finding scenario, asymptotic analysis and simulations also demonstrated a comparable or better performance of beamspace ESPRIT in comparison with its computationally more-expensive alternatives.

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REFERENCES

[1] P. Forster and G. Vezzosi, "Application of spheroidal sequences to array processing," in *Proc. IEEE ICASSP* (Dallas, TX), 1987, pp. 2268–2271.
[2] H. B. Lee and M. S. Wengrovitz, "Improved high-resolution direction-finding through use of homogeneous constraints," in *Proc. IEEE ASSP Workshop Spectrum Estimation and Modeling* (Minneapolis, MN), Aug. 3–5 1988, pp. 152–157.
[3] H. Lee and M. Mengrovitz, "Resolution threshold of beamspace root-MUSIC for two closely-spaced emitters," *IEEE Trans. Acoust. Speech, Signal Processing*, vol. 38, pp. 1545–1559, Sept. 1990.
[4] X. L. Xu and K. Buckley, "Reduced-dimension beamspace broadband source localization: Preprocessor design and evaluation," in *Proc. IEEE ASSP 4th Workshop Spectrum Estimation Modeling*, Aug. 1988, pp. 22–27.
[5] B. Van Veen and B. Williams, "Structured covariance matrices and dimensionality reduction in array processing," in *Proc. IEEE ASSP Workshop Spectrum Estimation Modeling* (Minneapolis, MN), Aug. 3–5 1988, pp. 168–171.
[6] R. O. Schmidt, "A signal subspace approach to multiple emitter location and spectral estimation," Ph.D. thesis, Stanford Univ., Stanford, CA, Nov. 1981.
[7] S. D. Silverstein, W. E. Engeler, and J. A. Taridif, "Parallel architectures for multirate superresolution spectrum analyzers," *IEEE Trans. Circuits Syst.*, vol. 38, no. 4, pp. 449–453, Apr. 1991.
[8] G. Xu and T. Kailath, "A fast algorithm for signal subspace decomposition and its performance analysis," in *Proc. ICASSP* (Toronto, Canada), May 1991, pp. 3069–3072, vol. 5.
[9] A. Paulraj, R. Roy, and T. Kailath, "A subspace rotation approach to signal parameter estimation," *Proc. IEEE*, vol. 74, no. 7, pp. 1044–1045, July 1986.
[10] R. Roy and T. Kailath, "ESPRIT—Estimation of signal parameters via rotational invariance techniques," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 32, no. 7, pp. 984–995, July 1989.
[11] M. D. Zoltowski, G. M. Kautz, and S. D. Silverstein, "Beamspace ROOT-MUSIC," *IEEE Trans. Signal Processing*, vol. 41, no. 1, pp. 344–364, Feb. 1993.
[12] B. Ottersten, M. Viberg, and T. Kailath, "Performance analysis of the total least squares ESPRIT algorithm," *IEEE Trans. Signal Processing*, vol. 39, no. 5, pp. 1122–1135, May 1991.

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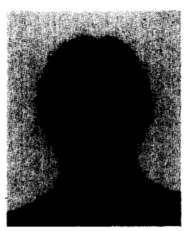

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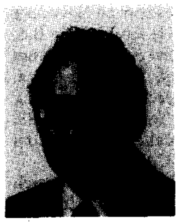

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