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Robust Modified Multiple Signal Classification Algorithm for Direction of Arrival Estimation

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Abstract
A modified robust multiple signal classification (MUSIC) algorithm for direction-of-arrival (DOA) estimation of coherent source signals is devised in this work. Classical and subspace-based methods require large antenna elements and huge computations for the estimation of intended signals. It also deviates from its performance in low signal-to-noise ratio (SNR) conditions. Hence efficient and robust DOA estimators are required to tackle the real-time problems in wireless communication applications. To overcome these problems, the classical MUSIC algorithm is modified by assimilating Jordon canonical matrix in the covariance matrix for the reconstruction of data. With this modification, it is possible to make an accurate estimation of coherent sources even under an extremely low SNR environment. Furthermore, the proposed method requires fewer antenna elements, fewer snapshots, and less computation as compared to the classical MUSIC algorithm. The experimental results signify the effectiveness of the new method.

Keywords Coherent signals · Direction-of-arrival · FOSS · MUSIC · Toeplitz matrix

1 Introduction

DOA estimation of coherent sources using antenna arrays plays a remarkable role in wireless communication applications [1–3], especially in sonar [4], radar [5], and mobile commutations [5], etc. DOA algorithms are responsible for the estimation of desired signals. Basically, there are two types of DOA algorithms, namely, classical and subspace method based algorithms. The beam scanning concept is used in the classical method for direction finding [7]. In this approach, each direction power is calculated by scanning a beam through space. The direction which has the highest power is considered as an angle of arrival of the signal [8, 9].

Subspace method based algorithms have high resolution in DOA estimation [11]. In this approach, the autocorrelation of the user signal and noise model is formed and then they are converted into matrix [11]. The Eigen structures of this matrix are formed and are

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sorted to give signal and noise subspace. Hence subspace-based methods are more accurate and effective for direction finding as compared to classical methods as signal and noise spaces are separated in this method [12]. MUSIC [13] algorithm was put forth by R.O. Schmidt in the year 1986. This method provides high-resolution signal estimation by isolating signal and noise subspaces. This is a popular eigenstructure based and one of the most studied methods. This algorithm has many variants. Some of its most popular variants are spectral- MUSIC [14] and root-MUSIC [14]. Some of the significant work for improving the MUSIC algorithm is presented in the literature. The complexity of the MUSIC algorithm is reduced by forming the covariance matrix without decomposing the eigenvalues in [15].

Z. Ying and N. Boon have proposed the estimation of the signal using the MUSIC-like algorithm which does not require a priori knowledge of the number of sources [16]. Improved MUSIC algorithm for high-resolution DOA estimation is reported in [17]. This method uses double orthogonal-triangular (QR) decomposition instead of singular value decomposition (SVD) [18]. This method significantly reduces the computational load as compared to normal MUSIC. W.J. Zeng et al. in [19] have devised the ***l*****p**-MUSIC DOA estimation algorithm for an impulsive noise environment. Hanna Becker et al. have devised the 2Q-MUSIC DOA algorithm in [18]. This method optimizes the compromise between the maximum number of sources and system performance.

In the above methods, importance was given to reduce the complexity of MUSIC. In this work, the proposed MUSIC algorithm not only reduces the computational complexity but also works accurately and makes high-resolution estimation even under extremely noisy environments.

Furthermore, this work also presents the optimization of several parameters suitable for practical applications.

2 Array Signal Model

A geometry of uniform linear antenna array (ULA) for the DOA problem is shown in Fig. 1. Let, a ULA with L antenna elements, $L = (1, 2, \cdots, L)$, $M(M < L)$ number of source signals. Let the spacing between each antenna element be 'd' and the value for inert element spacing be $\lambda/2$. Consider $s(t)$ as a baseband signal received at each antenna element at the different instant of time. Hence there exists a time delay for each signal arrival. This time delay can be composed as:

$$\Delta\tau = \frac{k\,M \sin\psi}{c} \tag{1}$$

where, k = integer is an integer in the range of $0 \le k \le L$, 'c' is the speed of light. M = number of incoming source signals. ψ = angle of each plane wave impinging on antenna elements.

The received narrowband signal can be modulated digitally using a low pass equivalent $s_l(t)$. The expression of the modulated $s(t)$ can be computed as

$$s(t) = \mathrm{Re}\left[s_l(t) \exp\left(j2\pi f_c t\right)\right] \tag{2}$$

Here f_c represents the carrier signal.

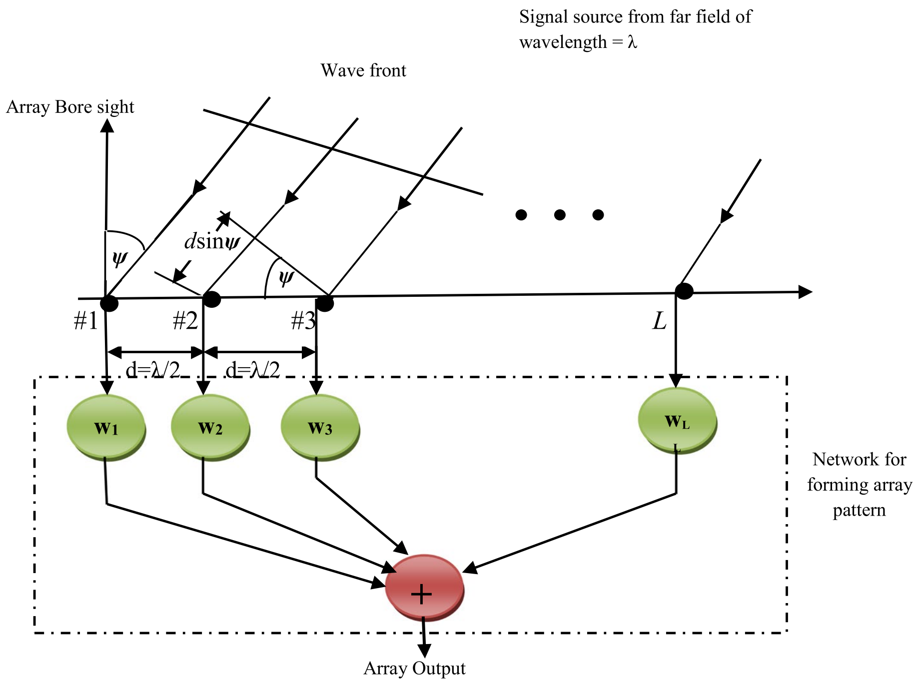

Fig. 1 Geometry of uniform linear array

The source received at kth antenna element can be obtained by substituting t by $(t - \Delta\tau)$ in the above equation

$$x_k(t) = \mathrm{Re}\left[s_l(t - \Delta\tau)\exp\left(j2\pi f_c(t - \Delta\tau)\right)\right] \tag{3}$$

Here $x_k(t)$ is the desired signal received by ULA.

The above expression at the kth array antenna is represented as

$$x_k(t) = \left[s_l(t - \Delta\tau)\exp\left(-j2\pi f_c(t - \Delta\tau)\right)\right] \tag{4}$$

Now, this received signal can be sampled for the period of 'T' seconds as

$$x_k(nT) \cong \left[s_l(nT - \Delta\tau)\exp\left(-j2\pi f_c(nT - \Delta\tau)\right)\right] \tag{5}$$

In practice, the value of the sampling period (T), is much greater than propagation delay $(\Delta\tau)$ across each antenna element. That is, $T >> \Delta\tau$. Thus (5) implies as

$$x_k(nT) \cong s_l(nT)\left[\exp\left(-j2\pi f_c\Delta\tau\right)\right] \tag{6}$$

We know that the relationship between f_c and c is

$$c = f_c\lambda \tag{7}$$

Here 'λ' is the wavelength of a radio wave. Thus the inter-element antenna spacing in ULA can be computed as

$$d = \frac{M}{\lambda} \tag{8}$$

Let us substitute the values of $\Delta\tau$ and f_c from Eqs. (1) and (7) in Eq. (6)

$$x_k(nT) \cong s_l(nT)\left[\exp\left(-j2\pi \frac{c}{\lambda}\frac{kM\sin\psi}{c}\right)\right] \tag{9}$$

$$\begin{aligned} &\cong s_l(nT)\left[\exp\left(-j2\pi \frac{kM\sin\psi}{\lambda}\right)\right] \\ &\cong s_l(nT)\left[\exp\left(-j2\pi\, k\, d\sin\psi\right)\right] \quad \because \text{from (8)} \end{aligned} \tag{10}$$

In the above equation, the value of 'nT' represents discrete, hence the discrete-time notation with an index 'n' can be used in place of 'nT'. Thus (10) can be written as

$$x(n) \approx s_l(n)\left[\exp\left(-j2\pi kd\sin\psi\right)\right], \quad \text{Here } x(n) = x_k(n) \tag{11}$$

Now the above equation is rewritten in vector form as

$$\mathbf{x}(n) = \mathbf{s}_l(n)\,\mathbf{a}(\psi) \tag{12}$$

here, $a(\psi) = \left[\exp\left(-j2\pi kd\sin\psi\right)\right]$ denotes the *steering vector,* varying between $0 \le k \le L-1$.

This sampled baseband signal at the kth element for 'M' number of sources is represented as

$$\mathbf{x}(n) = \sum_{i=0}^{M-1} \mathbf{s}_i(n)\,\mathbf{a}\left(\psi_i\right), \quad \text{where } i = 0, 1, 2 \ldots M-1 \tag{13}$$

Here $\mathbf{s}_i(n)$ is the nth source symbol of ith signal.

2.1 Formulation of array data matrix

Array data matrix for L antenna elements can be constructed by representing the above equation in matrix form as

$$\begin{bmatrix} x_0(n) \\ x_1(n) \\ \cdot \\ \cdot \\ \cdot \\ x_{L-1}(n) \end{bmatrix} = \begin{bmatrix} a_0(\psi_0) & a_0(\psi_1) & \cdots & a_0(\psi_{M-1}) \\ a_1(\psi_0) & a_1(\psi_1) & \cdots & a_1(\psi_{M-1}) \\ \vdots & \vdots & \ddots & \vdots \\ a_{M-1}(\psi_0) & a_{M-1}(\psi_1) & \cdots & a_{M-1}(\psi_{M-1}) \end{bmatrix} \begin{bmatrix} s_0(n) \\ s_1(n) \\ \cdot \\ \cdot \\ \cdot \\ s_{M-1}(n) \end{bmatrix} \tag{14}$$

The expression of the induced signal $x(n)$ does not include the contribution of noise. Let us assume that, there is an uncorrelated noise that exists at each antenna element. Let this noise vector $n_k(n)$ be the white Gaussian at each antenna element. Let, the noise has σ_n^2 variance and zero mean.

$$\begin{bmatrix} x_0(n) \\ x_1(n) \\ \cdot \\ \cdot \\ \cdot \\ x_{L-1}(n) \end{bmatrix} = \begin{bmatrix} a_0(\psi_0) & a_0(\psi_1) & \cdots & a_0(\psi_{M-1}) \\ a_1(\psi_0) & a_1(\psi_1) & \cdots & a_1(\psi_{M-1}) \\ \vdots & \vdots & \ddots & \vdots \\ a_{M-1}(\psi_0) & a_{M-1}(\psi_1) & \cdots & a_{M-1}(\psi_{M-1}) \end{bmatrix} \begin{bmatrix} s_0(n) \\ s_1(n) \\ \cdot \\ \cdot \\ \cdot \\ s_{M-1}(n) \end{bmatrix} + \begin{bmatrix} n_0(n) \\ n_1(n) \\ \cdot \\ \cdot \\ \cdot \\ n(n)_{L-1} \end{bmatrix} \tag{15}$$

The above expression can also be represented as

$$\mathbf{x}(n) = \mathbf{s}(n)\left[\mathbf{a}\left(\psi_o\right), \mathbf{a}\left(\psi_1\right), \mathbf{a}a\left(\psi_2\right), \cdots, \mathbf{a}a\left(\psi_{L-1}\right)\right] + \mathbf{n}(n) \tag{16}$$

The above expression in matrix–vector form is

$$\mathbf{X} = \mathbf{A} \cdot \mathbf{S} + \mathbf{N} \tag{17}$$

Here $\mathbf{a}\left(\psi_i\right)$=steering vector of $L-$ element for the $\left(\psi_i\right)$ directions.$\mathbf{S}$=induced signal vector, which is complex micro chromatic at time $n\mathbf{N}$=vector of noise at each antenna element, it has zero mean and σ_n^2 variance.$\mathbf{A}$ =$\left[\mathbf{a}\left(\psi_o\right), \mathbf{a}\left(\psi_1\right), \mathbf{a}\left(\psi_2\right), \cdots, \mathbf{a}\left(\psi_{L-1}\right)\right]$, L x L array manifold.

A. Formulation of **spatial** covariance matrix or Array Correlation Matrix (ACM)

We assume that the antenna elements in ULA are always more than the number of received signals. That is $L > M$. The induced signals are time-variant and our approximations relay on time snapshots of arriving signals. Now, let us define $L \times L$ array correlation matrix $\mathbf{R}_{xx}$ as

$$\mathbf{R}_{xx} = \mathrm{E}\left[\mathbf{x}(n) \cdot \mathbf{x}^H(n)\right]$$

The above equation can be simplified by exploiting the expectation operator. Thus

$$\mathbf{R}_{xx} = \mathbf{A}\mathrm{E}\left[\mathbf{s}(n) \cdot \mathbf{s}^H(n)\right]\mathbf{A}^H + \mathrm{E}\left[\mathbf{N} \cdot \mathbf{N}^H\right] \quad i.e., \tag{18}$$

$$\mathbf{R}_{xx} = \mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H + \mathbf{R}_{nn}$$

where $\mathbf{R}_{ss} = M \times M$ source correlation matrix (SCM).$\mathbf{R}_{nn} = \sigma_n^2\mathbf{I}_L = L \times L$ noise correlation matrix.$I = L \times L$ identity matrix.$\mathbf{A}^H$=Hermitian transpose of $\mathbf{A}$

Equation (18) can be expressed as

$$\mathbf{R}_{xx} = \mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H + \sigma_n^2\mathbf{I}_L \tag{19}$$

If the exact statics for signals and noise is unknown, then we can consider that the process is ergodic. Thus we can use a time-average correlation for approximating the correlation. In this case, the matrices of correlation are obtained by

$$\mathbf{R}_{xx} = \frac{1}{K}\sum_{n=1}^{\mathrm{K}}\mathbf{X}\mathbf{X}^H \quad \mathbf{R}_{ss} = \frac{1}{K}\sum_{n=1}^{\mathrm{K}}\mathbf{S}\mathbf{S}^H \quad \mathbf{R}_{nn} = \frac{1}{K}\mathbf{N}\mathbf{N}^H$$

Here, $\mathbf{X}=\mathbf{x}(\mathrm{n})$, $\mathbf{S}=\mathbf{s}(\mathrm{n})$ and $\mathbf{N}=\mathbf{n}(\mathrm{n})$

Often, the array correlation matrix is referred to as the covariance matrix in the literature. This is possible only when the mean values of noise and signals are zero. In this case, the correlation and covariance matrices are identical. The mean value of arriving signals must be zero because; the direct current (DC) signals cannot be received by antenna elements [2].

The meaning of 'Eigen' is peculiar or appropriate in German word. Important and useful information of a matrix can be obtained by the use of eigenvalues and eigenvectors. Thus, they are very useful parameters in adaptive filtering and spectrum estimation problems. The following characteristics equation can be solved to obtain the eigenvalues of $L \times L$ array covariance matrix R.

$$\therefore |\mathbf{R} - \lambda \mathbf{I} = 0|$$

'L' eigenvalues $\left(\lambda_1, \lambda_2, \lambda_3, \cdots, \lambda_L\right)$ can be obtained by solving the above equation.

The above expression represents the noise part of the subspace. This expression has L Eigen values and M Eigenvectors. $\mathbf{E}_N = \left[\, \bar{e}_1 \ \bar{e}_2 \ \ldots \ \bar{e}_P \,\right]$

Now the expression for pseudo spectrum of MUSIC algorithm can be given as

$$P(\psi) = \left[\frac{1}{\left| \mathbf{a}(\psi)^H \mathbf{E}_N \ \ \mathbf{E}_N^H \mathbf{a}(\psi) \right|} \right] \tag{20}$$

3 The Proposed Algorithm

In this section, a new coherent DOA estimator by modifying the classical MUSIC algorithm is devised. This method presents high stability, high accuracy, and high resolution. This method has a great capability for coherent DOA estimation using fewer samples (snapshots) and fewer antennas. This reduces the computational complexity and enhances the speed of execution. Also, it shows superior performance in low SNR conditions, this makes the algorithm more immune to noise and robust. Hence proposed modified MUSIC algorithm is the most suitable candidate for a performance-driven communication system.

We consider $\mathbf{y}_1(n), \cdots, \mathbf{y}_G(n)$ as G random array outputs from $\mathbf{x}(n)$. Thus the G x G array correlation matrix $\mathbf{R}_{yy}$ is defined as

$$\mathbf{R}_{yy} = \mathbf{A}_y \mathbf{R}_{ss} \mathbf{A}_y^H + \sigma_n^2 \mathbf{I}_G \tag{21}$$

where A_y is the direction matrix of array observation y(n).Now, we obtain the $L \times G$ equivalent array covariance matrix as

$$\tilde{\mathbf{R}} = \mathbf{R}_{xy} = \mathbf{A}_y \mathbf{R}_{ss} \mathbf{A}^H + \sigma_n^2 \mathbf{I}_{L \times G} \tag{22}$$

The major downside of the conventional MUSIC algorithm is that it deteriorates from its performance in low SNR scenarios. Hence DOA of the desired signal may not possible accurately. In this approach, the classical MUSIC algorithm is improved by incorporating Jordon canonical matrix. This matrix is used in the equivalent covariance matrix $\tilde{\mathbf{R}}$ for the successful recovery of the data. Let us represent Jordon canonical matrix as:

$$\mathbf{T} = \begin{bmatrix} 0 & 0 & - & 1 \\ 0 & 0 & - & 0 \\ - & - & - & - \\ 1 & 0 & - & 0 \end{bmatrix} \tag{23}$$

$$\hat{\mathbf{R}} = \tilde{\mathbf{R}} + \mathbf{T}\,\tilde{\mathbf{R}}^{*}\,\mathbf{T} \tag{24}$$

In practical wireless communication systems, most of the time there might occur coherent correlated sources. These sources might be partially correlated or perfectly correlated with each other. In a scenario, where only partially correlated and uncorrelated sources are co-existed, our new method can resolve such signals effectively. However, perfectly correlated sources reduce the rank of the correlation matrix. This results in more than (L − M) noise eigenvalues.

To resolve coherent sources, we divide the L-elements of the ULA into N overlapping each with P antennas. That is, the subarray 0 would comprise antenna elements from 0 to $P-1$, the subarray 1 would include antenna elements 1 to P, etc. Thus, $N = L - P + 1$. The L correlation matrices can be estimated by exploiting the data from each subarray. Now, the modified correlation matrix to resolve coherent correlated sources is expressed as

$$\hat{\mathbf{R}}_N = \frac{1}{N}\sum_{n=0}^{N-1}\mathbf{R}_n \tag{25}$$

This mathematical modeling can estimate the DOAs up to N-1 coherent sources. This is in view of the fact that the correlation matrix $\hat{\mathbf{R}}_N$ becomes full rank.

4 Results and Discussion

The parameters used for the simulation of the proposed modified MUSIC algorithm are tabulated in Table 1.

Let us consider four mobile users coming from different directions. The detailed information is tabulated in Table 2. The simulated modified MUSIC spectrum is shown in Fig. 1.

From Fig. 2, we can note that the four coherent sources having directions −20°, −10°, 10°, and 20° are estimated accurately. The pseudo spectrum of the modified MUSIC algorithm shown here has very sharp peaks representing high directivity. The performance of this proposed algorithm can be evaluated by studying it for various scenarios.

Table 1 Parameters used for the proposed modified MUSIC algorithm

S. no.	Parameters	Values
1	Array configuration	ULA
2	No. of Antennas, L	10–50
3	Inter element spacing	λ/2
4	K values	100–800

Table 2 Data of induced signals

Signal	DOA (°)	SNR(dB)	K	D (λ)
1	– 20	20	400	0.45
2	– 10	20	400	0.45
3	10	20	400	0.45
4	20	20	400	0.45

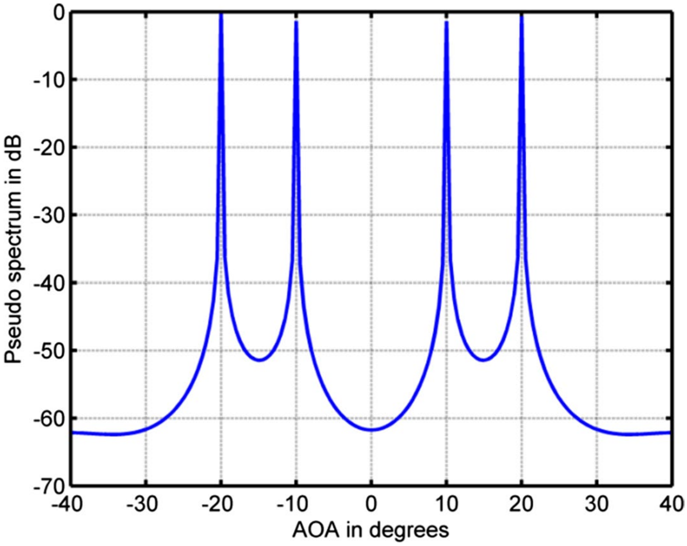

Fig. 2 Pseudo spectrum of Modified MUSIC algorithm

Table 3 Data of induced signals for widely spaced targets

Signal	DOA (°)	SNR (dB)	D (λ)	K
1	– 20	20	0.45	400
2	20	20	0.45	400

4.1 Modified MUSIC Algorithm for Various Antenna Elements

Let us consider two coherent source signals impinges ULA from directions – 20° and 20° with SNR = 20 dB. Complete data is tabulated in Table 3. Let the number of antenna elements be L = 3, 6, 10, and 15. Figure 3a and b respectively denote the pseudo spectrum of the proposed method for widely spaced and closely spaced targets for different antenna elements.

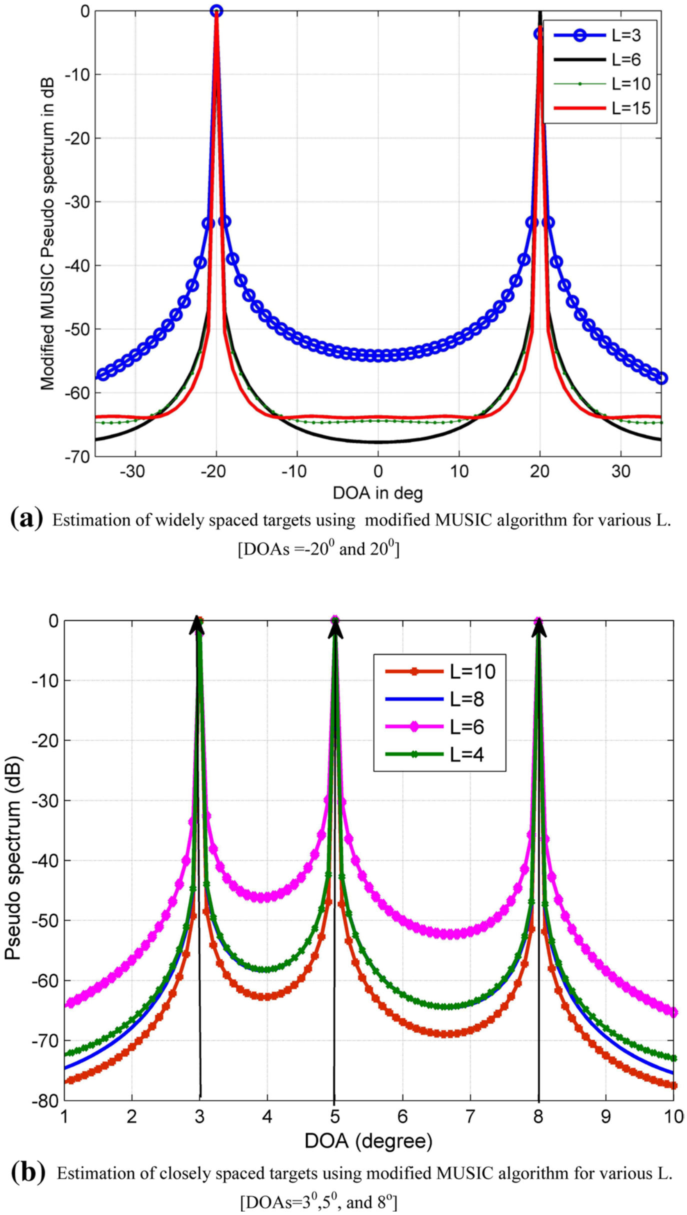

Fig. 3 **a** Estimation of widely spaced targets using modified MUSIC algorithm for various L. [DOAs = – 20° and 20°]. **b** Estimation of closely spaced targets using modified MUSIC algorithm for various L. [DOAs = 3°, 5°, and 8°]

Table 4 Data of induced signals

Signal	DOA (°)	SNR (dB)	K
1	0	20	400
2	40	20	400

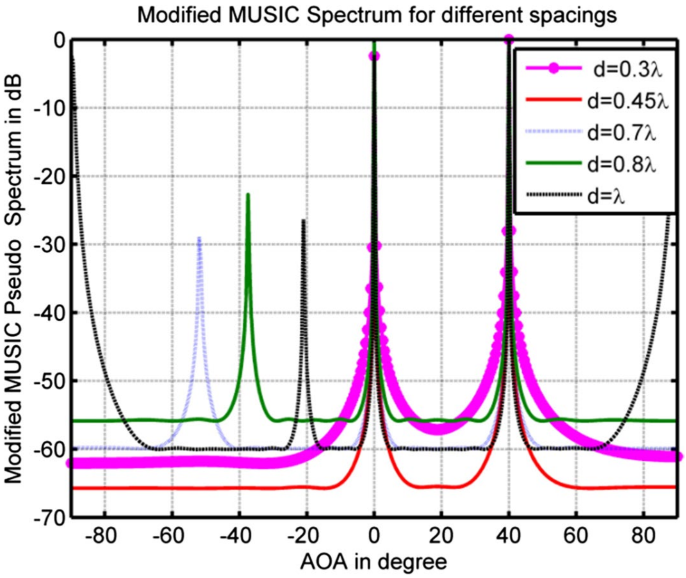

Fig. 4 The pseudo spectrum of modified MUSIC algorithm for various d

Table 5 Data of induced signals

Signal	DOA (°)	SNR(dB)	*D* (λ)
1	– 20	20	0.45
2	20	20	0.45

4.2 Modified MUSIC Algorithm for Various Inter Element Spacing

Let us assume that the two coherent sources with angles 0° and 40° impinge ULA with SNR = 20 dB and $L = 10$. Data of induced signals are tabulated in Table 4. Let the number of inter-element spacing be $d = 0.3$ λ, 0.45, 0.7 λ, 0.8 λ, and λ. Figure 4 shows the pseudo spectrum of the proposed method for different inter-element spacing values.

The following aspects are noted from Fig. 4.

- More than one main lobe(called as grating lobe) will be produced when the *d* values are 0.7 λ, 0.8 λ and λ
- Good DOA estimation is observed when the d value is between 0.45 and 0.55 λ
- Hence it can be concluded that the more than more required lobes will result if the inter-element spacing value becomes large than the 0.6 λ.

4.3 Modified MUSIC Algorithm for Different Snapshots

We assume that the two coherent sources with angles – 200 and 200 impinge ULA with SNR = 20 dB and $L = 10$. Data of induced signals are tabulated in Table 5. Let the

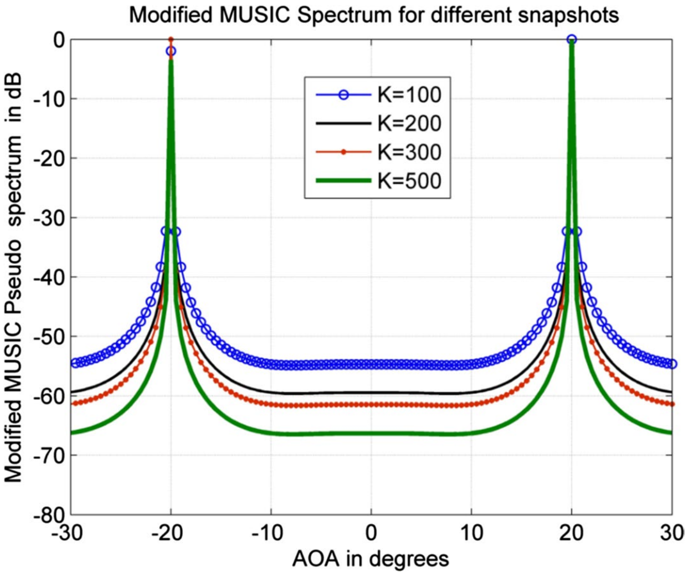

Fig. 5 The pseudo spectrum of modified MUSIC algorithm for various *K* values

number of snapshots (K) be 100, 200, 300, and 500. Figure 5 shows the pseudo spectrum of the proposed method for different snapshot values.

The following aspects are noted from Fig. 5.

- The number of snapshots is directly propositional to the resolution of the algorithm.
- Hence larger the snapshot value more will be the resolution of the algorithm.
- Figure 4 shows the high-resolution DOA estimation even for 200 samples, which is very less as compared to the traditional MUSIC algorithm.

4.4 Modified MUSIC Algorithm for Different SNR Scenarios

Table 6 tabulates the data of two narrowband signals impinging on the smart antenna system with two different directions.

Let us consider the various SNR scenarios, let SNRs be −20 dB, −10 dB, 0 dB, 20 dB and 40 dB. The simulated modified MUSIC spectrum for various SNR values is shown in Fig. 6.

Figure 7 shows the pseudo spectrum of the classical MUSIC algorithm for the same parameters with DOA = 20° and 60°.

The following aspects are the important observations noted from Figs. 6 and 7.

Table 6 Data of induced signals

Signal	DOA (°)	L	*D* (λ)	K
1	−20	10	0.45	400
2	20	10	0.45	400

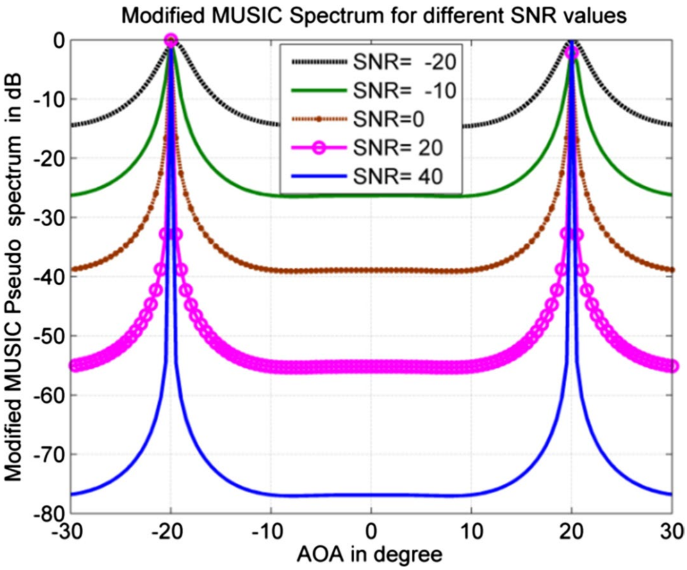

Fig. 6 Pseudo spectrum of modified MUSIC method

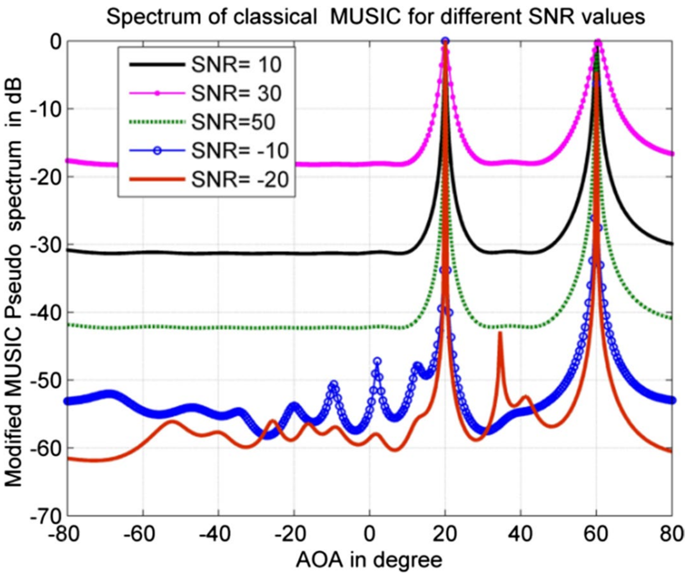

Fig. 7 The pseudo spectrum of the classical MUSIC algorithm for various SNR values

- The classical MUSIC algorithm deviates from its performance and makes bad estimation when SNR becomes below 0 dB.
- The modified MUSIC algorithm makes a good estimation for the low SNR environment.

4.5 Coherent DOA Estimation

In this experiment, we assume a 6-element antenna array separated by half-wavelength spacing receives two uncorrelated signals from − 20° and 20° and a group of two coherent sources from − 10° to − 15°. We employ 100 snapshots and fix the SNR to 5 dB. The root mean square error is evaluated using the following expression [20–22].

$$\mathrm{RMSE}(\psi) = \sqrt{\frac{1}{T}\sum_{n=0}^{T}\left(\hat{\psi}_n - \psi\right)^2} \tag{26}$$

Here, T denotes the Monte Carlo trials and, ψ and $\hat{\psi}_i$ respectively denote the true and the estimated DOAs. Figures 8 illustrates the RMSE performance against the SNR of the proposed robust MUSIC algorithm in comparison with the MUSIC [13] and the FOSS [23] methods over 1000 Monte Carlo trails. In Fig. 8, we perceive that the MUSIC algorithm completely deviates from its RMSE performance for the case of coherent DOA estimation. Hence it has the worst performance. It is also noted that the proposed method has a great accuracy of coherent estimation, especially in the low SNR. However, the FOSS method outperforms the new method by about 1 dB in the high SNRs. Figure 9 demonstrates the performance of the DOA estimators over several snapshots. Here the snapshots are varied from 100 to 1000 keeping other parameters the same. It is seen that the new method outperforms the classical MUSIC and the FOSS methods for the regime of snapshots.

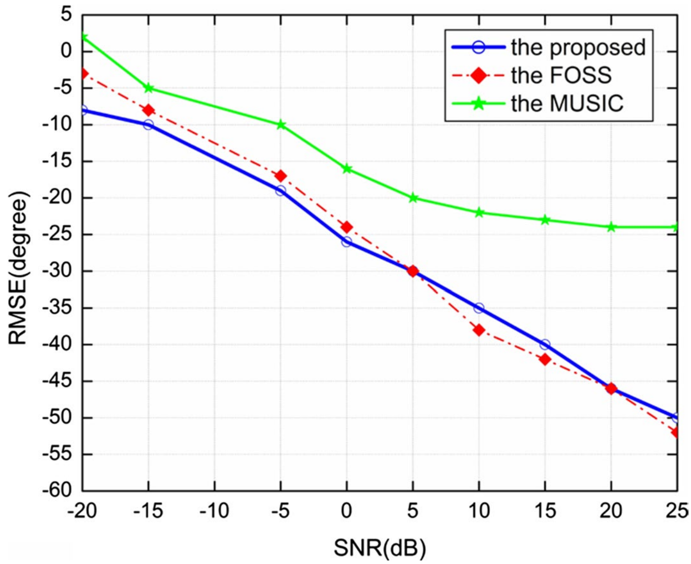

Fig. 8 Analysis of RMSE performance against SNR

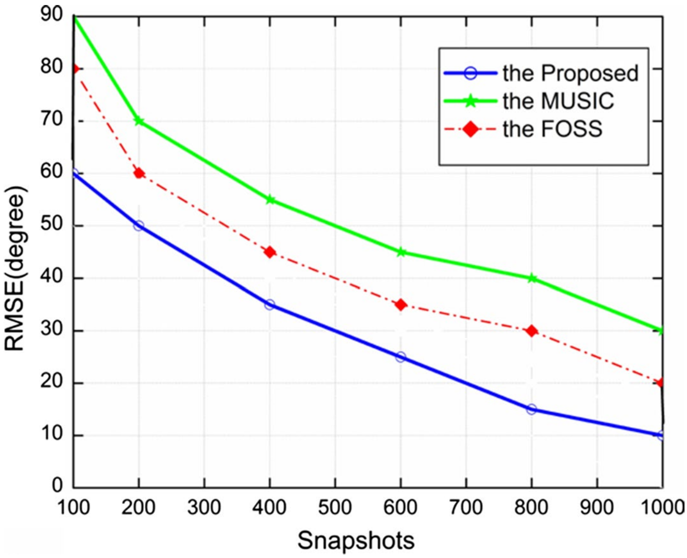

Fig. 9 Analysis of RMSE performance against snapshots

5 Conclusion

In this work, a modified MUSIC method for DOA estimation of coherence for various conditions is studied. The performance of this algorithm is analyzed in detail by varying L, K, d, and SNR. It is discovered that, as L is increased, beamwidth reduces, and directivity increases. The number of snapshots (K) used in the algorithm is directly propositional to the computation complexity of the algorithm. This algorithm makes good DOA estimation even when > 200. Classical MUSIC algorithm requires at least 800 snapshots for the good estimation, due to this huge computations are required which puts lots of burner on the processor. The proposed DOA estimator uses 100–200 snapshots to do the same job. Inter-element spacing (d) plays an important role in the antenna array configuration. If the value of $d < 0.5\ \lambda$, it causes the mutual coupling effect, and if $d > 0.5\ \lambda$, grating lobes (more than one main beam) will be produced. Hence from the experimental results, it is clear that the value of d should be strictly 0.5 λ to avoid the mutual coupling and grating lobes effect. Also, the proposed estimator works accurately in very low SNR scenarios. The simulation results show that the proposed method is superior compared to the classical MUSIC algorithm in the sense of computation complexity and robustness.

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