DOA estimation based on combined unitary ESPRIT for coprime MIMO radar

Jianfeng Li,Defu Jiang,and Xiaofei Zhang

***Abstract*—Direction of arrival (DOA) estimation for coprime multiple-input multiple-output (MIMO) radar is studied, and a combined unitary estimation of signal parameters via rotational invariance technique (ESPRIT) based algorithm is proposed. The transmitter and receiver adopt coprime arrays, which are sparse but still uniform. So unitary ESPRIT is firstly used to obtain arbitrary ambiguous DOA estimations based on the rotational invariances of transmit and receive arrays, respectively. After recovering all the other estimations, unique DOA estimation is achieved by finding the coincide results from transmit and receive arrays based on the coprime-ness. The proposed algorithm obtains more accurate DOA estimation, achieves higher angle resolution and identifies more targets than conventional methods. Multiple simulations are conducted to verify the improvement of the proposed algorithm.**

***Index Terms*—MIMO radar, coprime array, DOA estimation, unitary ESPRIT.**

I. INTRODUCTION

MULTIPLE-input multiple-output (MIMO) radars utilize multiple antennas to transmit signals and receive the reflected signals also via multiple antennas, so they can improve target detection performance, enhance spatial resolution and achieve other advantages over traditional radar systems [1]-[2]. As a basic and important issue for MIMO radar, direction of arrival (DOA) estimation has attracted lots of attention [3]-[4]. Many effective DOA estimation algorithms have been proposed for MIMO radar, such as unitary reduced-dimension (URD) estimation of signal parameters via rotational invariance (ESPRIT) method [5], RD multiple signal classification (MUSIC) method [6], unitary root (UR) MUSIC method [7], tensor based method [8] and so on. However, they all require the inter-element spacing of the arrays not to be larger than half-wavelength to avoid the angle ambiguity problem, and the compact arrays limit the estimation performance and suffer from the mutual coupling problem [9].

Coprime array consists of two sparse uniform linear arrays (ULAs) with coprime antenna number and inter-element spacing, and it can achieve narrow beam pattern due to the large degrees of freedom (DOF) in the co-array domain [10]-[11]. Coprime array concept has been applied in MIMO system for beamforming to improve detection probability [12]. In Ref. [13], augment coprime array (ACA) was adopted in MIMO radar for DOA estimation, but the tensor decomposition requires high-computational iterations and has a limitation on the number of detectable targets.

In this paper, a DOA estimation algorithm based on combined unitary ESPRIT for coprime MIMO radar is proposed. The two sparse subarrays in coprime array are used as the transmit array and receive array, respectively. Unitary ESPRIT is extended to sparse array to obtain DOA estimations, which are ambiguous but can be used to recover all the other estimations including the correct ones. Finally, DOA is uniquely determined by choosing the coincide estimations from the transmit array and receive array based on their coprime relationship. The proposed algorithm obtains closed-form solution of DOA estimation with neither peak searches nor iterations. In contrast to URD ESPRIT method [5], UR MUSIC [7] and ACA based method [13], the proposed algorithm can achieve better DOA estimation performance and handle more targets.

Notation: $(.)^T$, $(.)^H$, $(.)^{-1}$ and $(.)^+$ denote transpose, conjugate-transpose, inverse, pseudo-inverse operations, respectively. diag($\mathbf{v}$) stands for diagonal matrix whose diagonal element is a vector $\mathbf{v}$. $\mathbf{I}_M$ and $\mathbf{\Pi}_M$ are $M{\times}M$ identity matrix and reverse identity matrix, respectively. $\mathbf{0}_{M\times M}$ is a $M{\times}M$ zero matrix,$\otimes$ is the Kronecker product, and E[.]is expectation operator. Re($\cdot$) and Im($\cdot$) mean to get real and imaginary parts of complex value, respectively. min(.) means to get minimum from multiple numbers.

II. DATA MODEL

Consider a monostatic MIMO radar shown in Fig.1, where coprime arrays are adopted to transmit and receive signals. The transmit array has M elements with adjacent interval being Nd and the receive array has N elements with adjacent interval being Md. M and N are co-prime integers, and d is the unit spacing which is generally set as d=λ/2, where λ denotes the signal wavelength. Assume that there are K far-field targets, then the output after the processing of matched filters at the receiver is [3]

$$\begin{aligned}\mathbf{x}(t) &= [\mathbf{a}_t(\theta_1)\otimes\mathbf{a}_r(\theta_1),\cdots,\mathbf{a}_t(\theta_K)\otimes\mathbf{a}_r(\theta_K)]\mathbf{s}(t)+\mathbf{n}(t)\\ &= \mathbf{A}\mathbf{s}(t)+\mathbf{n}(t)\end{aligned} \tag{1}$$

where θ_k is the DOA of the k-th target. $\mathbf{s}(t) = [s_1(t), s_2(t),\cdots,s_K(t)]^T$ is a signal vector which contains target information. $\mathbf{n}(t)$ is an additive white Gaussian noise (AWGN) vector, which has zero mean and covariance matrix $\sigma^2\mathbf{I}_{MN}$. $\mathbf{A} = [\mathbf{a}_t(\theta_1)\otimes\mathbf{a}_r(\theta_1),\cdots,\mathbf{a}_t(\theta_K)\otimes\mathbf{a}_r(\theta_K)]$ is direction matrix whose column $\mathbf{a}_t(\theta_k)\otimes\mathbf{a}_r(\theta_k)$ is the kronecker

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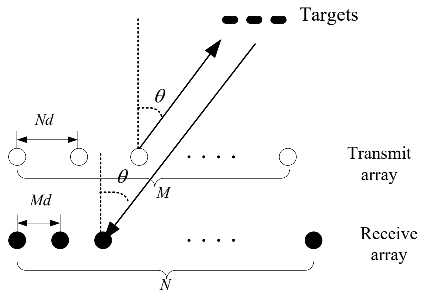

Fig. 1. Framework of coprime MIMO radar

product of the transmit and receive steering vectors for the k-th target,

$$\mathbf{a}_t(\theta_k) = [1, e^{-jN\pi\sin\theta_k}, \cdots, e^{-j(M-1)N\pi\sin\theta_k}]^T \tag{2}$$

$$\mathbf{a}_r(\theta_k) = [1, e^{-jM\pi\sin\theta_k}, \cdots, e^{-j(N-1)M\pi\sin\theta_k}]^T \tag{3}$$

The sparse arrays can avoid the mutual coupling problem, and the corresponding steering vectors still have vandermonde structures. The covariance matrix of the output is expressed as

$$\mathbf{R} = E[\mathbf{x}(t)\mathbf{x}^H(t)] = \mathbf{A}\mathbf{R}_s\mathbf{A}^H + \sigma^2\mathbf{I}_{MN} \tag{4}$$

where $\mathbf{R}_s = E[\mathbf{s}(t)\mathbf{s}^H(t)]$.

III. DOA ESTIMATION BASED ON COMBINED UNITARY ESPRIT

A. Ambiguous DOA estimation

To reduce the complexity of decomposition, the covariance matrix in Eq.(4) will be transformed to a real-valued one via unitary transformation. Define unitary matrix as

$$\mathbf{Q}_{2q} = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{I}_q & j\mathbf{I}_q \\ \mathbf{\Pi}_q & -j\mathbf{\Pi}_q \end{bmatrix}, \mathbf{Q}_{2q+1} = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{I}_q & 0 & j\mathbf{I}_q \\ 0 & \sqrt{2} & 0 \\ \mathbf{\Pi}_q & 0 & -j\mathbf{\Pi}_q \end{bmatrix} \tag{5}$$

Then the real-valued covariance matrix is obtained via

$$\mathbf{R}_r = \mathrm{Re}(\mathbf{Q}_{MN}^H\mathbf{R}\mathbf{Q}_{MN}) \tag{6}$$

Perform eigenvalue decomposition (EVD) of the covariance matrix in Eq.(6), and the K eigenvectors corresponding to the K largest eigenvalues form the signal subspace $\mathbf{E}_s$, which satisfies

$$\mathbf{E}_s = \mathbf{Q}_{MN}^H\mathbf{A}\mathbf{T}\text{=}\mathbf{B}\mathbf{T} \tag{7}$$

where $\mathbf{T}$ is a $K\times K$ non-singular matrix, and $\mathbf{B} = \mathbf{Q}_{MN}^H\mathbf{A}$ is the new direction matrix after the unitary transformation [5], [14].

Based on the uniformities of both transmit and receive arrays, the direction matrix satisfies rotational invariance

$$\mathbf{J}_1\mathbf{A}\mathbf{\Phi}_t = \mathbf{J}_2\mathbf{A}, \mathbf{J}_3\mathbf{A}\mathbf{\Phi}_r = \mathbf{J}_4\mathbf{A} \tag{8}$$

where $\mathbf{J}_1 = [\mathbf{I}_{(M-1)N}, \mathbf{0}_{(M-1)N\times N}]$, $\mathbf{J}_2 = [\mathbf{0}_{(M-1)N\times N}, \mathbf{I}_{(M-1)N}]$, $\mathbf{J}_3 = \mathbf{I}_M \otimes [\mathbf{I}_{(N-1)}, \mathbf{0}_{(N-1)\times 1}]$, and $\mathbf{J}_4 = \mathbf{I}_M \otimes [\mathbf{0}_{(N-1)\times 1}, \mathbf{I}_{(N-1)}]$ are selecting matrices. $\mathbf{\Phi}_t = diag(e^{-jN\pi\sin\theta_1}, e^{-jN\pi\sin\theta_2}, \cdots, e^{-jN\pi\sin\theta_K})$, $\mathbf{\Phi}_r = diag(e^{-jM\pi\sin\theta_1}, e^{-jM\pi\sin\theta_2}, \cdots, e^{-jM\pi\sin\theta_K})$ are diagonal matrices which contain DOA information. As now both the covariance matrix and signal subspace are real-valued, the rotational invariance is also transformed into real-valued one [14],

$$\mathbf{K}_1\mathbf{B}\mathbf{\Lambda}_t = \mathbf{K}_2\mathbf{B}, \mathbf{K}_3\mathbf{B}\mathbf{\Lambda}_r = \mathbf{K}_4\mathbf{B} \tag{9}$$

where selecting matrices $\mathbf{K}_1 = \mathrm{Re}(\mathbf{Q}_{(M-1)N}^H\mathbf{J}_2\mathbf{Q}_{MN})$, $\mathbf{K}_2 =$ Im $(\mathbf{Q}_{(M-1)N}^H\mathbf{J}_2\mathbf{Q}_{MN})$, $\mathbf{K}_3 =$ Re $(\mathbf{Q}_{(N-1)M}^H\mathbf{J}_4\mathbf{Q}_{MN})$, and $\mathbf{K}_4 =$ Im $(\mathbf{Q}_{(N-1)M}^H\mathbf{J}_4\mathbf{Q}_{MN})$ are real-valued. $\mathbf{\Lambda}_t = diag(\tan(-N\pi\sin\theta_1/2), \cdots, \tan(-N\pi\sin\theta_K/2))$ and $\mathbf{\Lambda}_r = diag(tan(-M\pi sin\theta_1/2), \cdots, \tan(-M\pi\sin\theta_K/2))$ are real-valued diagonal matrices.

Combine Eq.(7) and Eq.(9), then the signal subspace satisfies

$$\mathbf{K}_1\mathbf{E}_s\mathbf{\Sigma}_t = \mathbf{K}_2\mathbf{E}_s, \mathbf{K}_3\mathbf{E}_s\mathbf{\Sigma}_r = \mathbf{K}_4\mathbf{E}_s \tag{10}$$

where $\mathbf{\Sigma}_t = \mathbf{T}^{-1}\mathbf{\Lambda}_t\mathbf{T}$ and $\mathbf{\Sigma}_r = \mathbf{T}^{-1}\mathbf{\Lambda}_r\mathbf{T}$. The least squares solutions of Eq.(10) are

$$\mathbf{\Sigma}_t = (\mathbf{K}_1\mathbf{E}_s)^+\mathbf{K}_2\mathbf{E}_s, \mathbf{\Sigma}_r = (\mathbf{K}_3\mathbf{E}_s)^+\mathbf{K}_4\mathbf{E}_s \tag{11}$$

To make the estimations corresponding to transmit and receive arrays automatically paired, construct matrix

$$\mathbf{\Sigma}_z = \mathbf{\Sigma}_t + j\mathbf{\Sigma}_r = \mathbf{T}^{-1}(\mathbf{\Lambda}_t + j\mathbf{\Lambda}_r)\mathbf{T} \tag{12}$$

According to Eq.(12), the eigenvalues of $\mathbf{\Sigma}_z$ give the estimations of the diagonal elements in $\mathbf{\Lambda}_t + j\mathbf{\Lambda}_r$, whose real and imaginary parts give the DOA estimations from transmit and receive arrays, respectively. Use $\alpha_{tk}, k = 1, ..., K$ to denote the real parts of the eigenvalues, then DOA estimated from transmit array is

$$\sin\breve{\theta}_k^0 = -2\arctan(\alpha_{tk})/(N\pi), k = 1, ..., K \tag{13}$$

The value of $-N\pi\sin\theta_k/2$ within the tangent function satisfies$-N\pi/2 \le -N\pi\sin\theta_k/2 \le N\pi/2$, but the result of the arctangent function $\arctan(\cdot)$ in Eq.(13) is within the range $[-\pi/2, \pi/2]$. Therefore, the estimation obtained in Eq.(13) is an ambiguous one, and there are still other N-1 solutions in the rest N-1 ranges, which will be shown in the next section. Similarly, use $\alpha_{rk}, k = 1, ..., K$ to denote the imaginary parts of the eigenvalues of $\mathbf{\Sigma}_z$, then DOA estimated from receive array is

$$\sin\overset{\frown}{\theta}_k^0 = -2\arctan(\alpha_{rk})/(M\pi), k = 1, ..., K \tag{14}$$

Similarly, there are still other M-1 solutions except Eq.(14). Till now, the main reason of adopting unitary ESPRIT can be shown: 1. It fully exploits the uniformity of the array to reduce complexity based on real-valued de-composition; 2. It obtains closed-form solutions of DOA estimation, which are ambiguous but automatically paired, and they will be used to recover the rest estimations in the next section.

B. Unique DOA determination

Due to the large inter-element spacing Nd for transmit array, it is derived that there are N solutions $\breve{\theta}_k^n, n = 0, ..., N-1$ including the one obtained via Eq.(13), and they satisfy

$$\tan(\frac{-N\pi\sin\breve{\theta}_k^n}{2}) = \tan(\frac{-N\pi\sin\breve{\theta}_k^0}{2}), n = 1, ..., N-1 \tag{15.a}$$

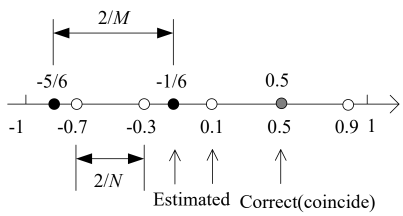

Fig. 2. Relationship within the estimations (DOA=30, *N*=5, *M*=3)

which is rewritten as [15]-[16]

$$\sin \breve{\theta}_k^{n} = \sin \breve{\theta}_k^{0} - \frac{2p}{N}, n = 1, ..., N-1 \qquad (15.b)$$

where *p* is an integer varies follow *n*. Eq.(15.b) indicates that the adjacent interval of the *N* solutions is 2/*N*. An example of their relationship is shown in Fig.2 (hollow circles), where *N*=5 and the correct DOA is 30. $\sin \breve{\theta}_k^{0}$=0.1 is the estimation obtained from Eq.(13) but 0.5 should be the correct solution. As Fig.2 shows, all the *N* estimations including the correct one are (-0.7, -0.3, 0.1, 0.5, 0.9), which follow uniform distribution with interval 2/*N*=0.4.

Similarly for the receive array, the *M* estimations $\widehat{\theta}_k^{m}, m = 0, ..., M-1$ including the one obtained via Eq.(14) satisfy

$$\sin \widehat{\theta}_k^{n} = \sin \widehat{\theta}_k^{0} - \frac{2q}{M}, m = 1, ..., M-1 \qquad (16)$$

where *q* is an integer varies follow *m*, and an example of *M*=3 is also shown in Fig.2, where $\sin \widehat{\theta}_k^{0} = -1/6$ is the estimated result via Eq.(14).
Based on the results from Eq.(13) and Eq.(14) and the relationships in Eq.(15) and Eq.(16), all the *N* and *M* estimations for transmit and receive arrays can be recovered, respectively. According to [15]-[16], the correct estimation can be achieved by finding the intersections of the *N* and *M* estimations based on the coprime-ness of *N* and *M* (see 0.5 in Fig.2).
In practice, the covariance matrix is estimated via finite snapshots,

$$\hat{\mathbf{R}} = \frac{1}{T}\sum_{t=1}^{T} \mathbf{x}(t)\mathbf{x}^H(t) \qquad (17)$$

where *T* is the snapshot number. Therefore, the correct estimations will not be strictly overlapped, and the DOA is estimated by averaging two closest solutions,

$$\hat{\theta}_k = \arcsin\left(\frac{\sin \breve{\theta}_k^{n_t} + \sin \widehat{\theta}_k^{m_r}}{2}\right), k = 1, ..., K \qquad (18)$$

where $\breve{\theta}_k^{n_t}$ and $\widehat{\theta}_k^{m_r}$ denote the corresponding angles of the two closest solutions. The estimations obtained via Eq.(13) and Eq.(14) are automatically paired (the same target), which can avoid the interference between targets and save complexity when searching the closet-solutions in Eq.(18).

Up to now, we have presented the combined unitary ESPRIT based DOA estimation algorithm for coprime MIMO radar. The major steps can be summarized below:
Step.1. Estimate the covariance matrix via Eq.(17);
Step.2. Transform the covariance matrix into real-valued one and perform EVD to obtain the signal subspace $\mathbf{E}_s$;
Step.3. Based on the real-valued rotational invariance, obtain automatically paired but ambiguous DOA estima-tions via Eq.(11)-Eq.(14);
Step.4. Recover all the estimations based on Eq.(15)-Eq.(16), and determine the unique DOA via Eq.(18).

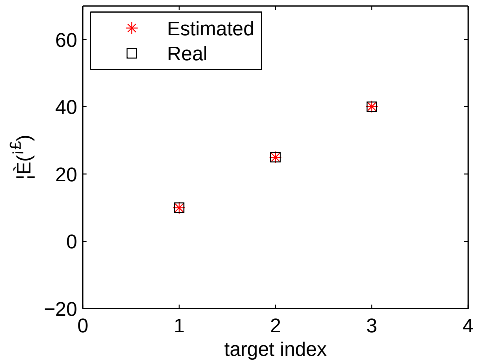

Fig. 3. DOA estimation result of the proposed algorithm (SNR=5dB)

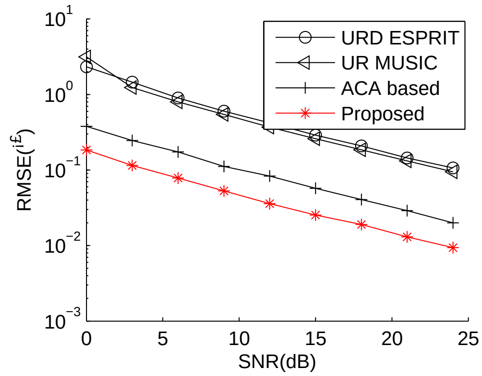

Fig. 4. DOA estimation performance comparison versus SNR

IV. SIMULATION RESULTS

Assume that there are $K = 3$ uncorrelated targets with the angles being $\theta_1 = 10°$, $\theta_2 = 25°$ and $\theta_3 = 40°$, respectively. The transmit and receive arrays have *M*=4 and *N*=3 antennas, respectively. *T*=128 samples are collected.

Fig.3 shows the estimation results of the proposed algorithm when signal to noise ratio (SNR) is 5dB. It can be indicated that the proposed algorithm can uniquely and correctly estimated all the DOAs.

Define root mean square error (RMSE) as

$$RMSE = \frac{1}{K}\sum_{k=1}^{K}\sqrt{\left(\frac{1}{L}\sum_{l=1}^{L}[(\hat{\theta}_{k,l} - \theta_k)^2]\right)} \qquad (19)$$

where $\hat{\theta}_{k,l}$ is the estimate of DOA θ_k of the lth Monte Carlo trial. The total trial number is *L*=200.

Under the measurement of RMSE, Fig.4 shows the DOA estimation performance comparison between URD ESPRIT method [5], UR MUSIC [7], ACA based method [13] (requires some adjustments to fit monostatic MIMO radar) and the proposed algorithm (all adopt the same number of antennas

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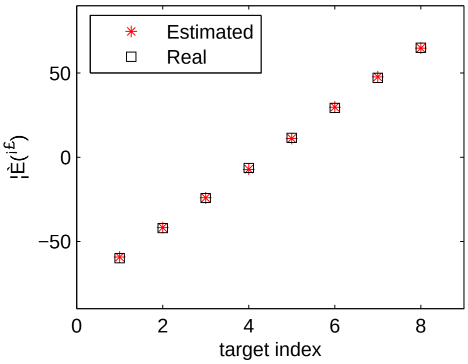

Fig. 5. DOA estimation resutls of the proposed algorithm with large number of targets (K=8, SNR=5dB)

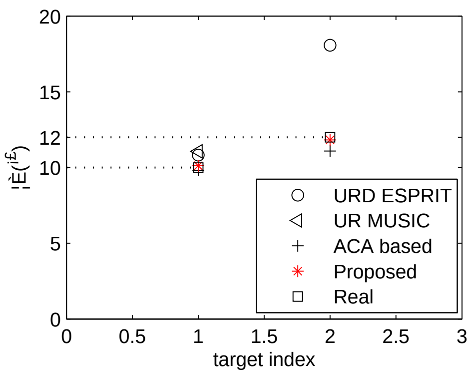

Fig. 6. DOA estimation results of closely-spaced targets (SNR=5dB)

for fair comparison and also assume no mutual coupling for compact arrays). It is shown that the proposed algorithm achieves much better DOA estimation performance than the other methods.

The maximum numbers of targets that URD ESPRIT, UR MUSIC and ACA based methods can handle are all M+N-2, and the proposed algorithm can identify min(M(N-1), N(M-1)) targets (determined by unitary ESPRIT). Fig.5 shows the estimation result of proposed algorithm when there are large number of targets (K= min(M(N-1), N(M-1))=8), and URD ESPRIT, UR MUSIC and ACA based methods all fail to work now. So the proposed algorithm can handle more targets than other methods.

We use two closely-spaced targets to test the angular resolutions of the algorithms in Fig.6, where two targets with angles $\theta_1 = 10^\circ$ and $\theta_2 = 12^\circ$ are adopted. It is shown that the URD ESPRIT, UR MUSIC and ACA based methods all have big estimation errors while the proposed can accurately distinguish the two targets. So the proposed algorithm can achieve higher angular resolution than other methods.

V. Conclusion

In this paper, a combined unitary ESPRIT based DOA estimation algorithm for coprime MIMO radar is proposed. The sparse transmit and receive arrays can avoid mutual coupling problem, and their uniformities can be exploited to obtain automatically paired but ambiguous DOA estimations via unitary ESPRIT. After recovering all solutions, the unique DOA estimation is achieved by finding the coincide solutions from transmit and receive arrays based on coprime-ness. The advantages of the proposed algorithm can be summarized as: 1) it avoids mutual coupling problem via sparse array; 2) it has low complexity for the re-al-valued EVD and closed-form solution of DOA estimation; 3) it has better DOA estimation performance, achieves higher resolution and handles more targets than URD ESPRIT, UR MUSIC and ACA based methods.

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