# DOA Estimation Algorithm Based on FFT in Switch Antenna Array

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**Abstract**

An algorithm for fast DOA estimation in single-channel antenna array is proposed for the high complexity and low timeliness of the present algorithms. This algorithm applies the spatial FFT of multi-channel antenna array to single-channel antenna array. It takes the switching time into operation to construct the new transformation function, which can be utilized to operate spatial FFT to the sampled data of single-channel antenna array directly. Based on this, the SAA-FFT algorithm fitted for single-channel antenna array is proposed. The theoretical analysis and simulation results demonstrate that the algorithm is effective.

**Keywords:** switch antenna array; DOA estimation; spatial FFT; array signal processing

## 1. Introduction

Conventional high resolution array signal processing are always based on simultaneously sampling of the whole multiple-sensor array, which requires that the number of receiving sensors should be equal to the number of receivers [1]. It, as the number of sensors increases, may render the whole system complex, bulky and costly. What's more, the difference in gain and phase among the receiving channels deteriorates the accuracy significantly, even makes the system noneffective [2].

For the above question, switch antenna array [3]-[4] is a promising solution, which contains only one receiver switching to receiving antennas for sampling.

For DOA estimation in switch antenna array, several high resolution algorithms were proposed [5]-[7]. However, these algorithms, which are based on multiple signal classification (MUSIC) algorithm, are all high in algorithm complexity. Following the above analysis, the estimation delay caused by the time of operation can not be ignored at all, especially when processing high-velocity moving targets. Moreover, these algorithms, which are able to process uncorrelated source only, deteriorate significantly with coherent sources.

In this paper, we propose a new algorithm for fast DOA estimation in switch antenna array (hereinafter SAA), which is able to process uncorrelated and coherent sources with low complexity. The proposed algorithm, which is based on spatial FFT [8], takes the switching time into operation to construct the new transforming function which can be utilized to operate FFT with the sampled data of SAA directly.

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## 2. Signal model

Consider the SAA architecture shown in Figure 1, which is composed of a uniform linear array with $L$ antennas, a single receiver and a DSP module. The single receiver sequentially samples the array outputs at a regular interval $\tau_0$, so that the time-diversity sampled signal $x_l$ is obtained, where $l = 0, 1, \cdots, L-1$.

Suppose there exist $K$ narrowband signal sources impinge the array from $\theta_1, \theta_2, \cdots, \theta_K$, the common carrier frequency is $f$. If antenna 0 is sampled at the moment $t_0$, antenna $l$ is sampled at the moment $(t_0 + l\tau_0)$, so $x_l$ is

$$x_l(t_0 + l\tau_0) = \sum_{k=1}^{K} s_k(t_0 + l\tau_0)\exp(\mathrm{j}2\pi f \tau_{lk}) + n_l(t_0 + l\tau_0) \quad (1)$$

$$\tau_{lk} = \frac{ld\sin\theta_k}{\mathrm{c}} \quad (2)$$

where, $s_k(t)$ is the receive signal of the reference antenna (antenna 0) from signal source $k$, $n_l$ is Gaussian white noise associated with the observation of antenna $l$, $\tau_{lk}$ is the time-delay of the wave propagation from the reference antenna to antenna $l$, $d$ is the interelement ULA spacing.

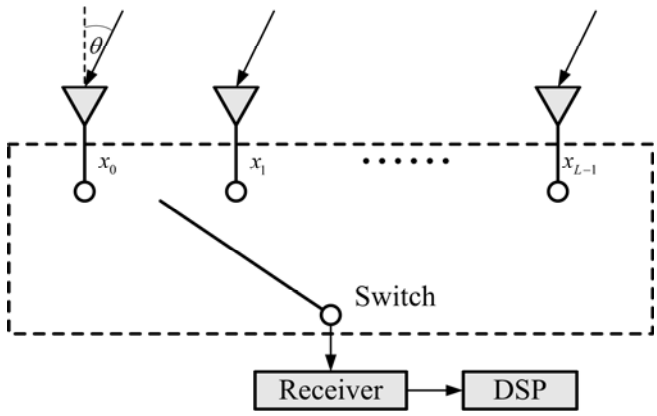


Fig.1. The system model of SAA

Since $s_k(t)$ is narrowband, $s_k(t_0 + l\tau_0)$ can be given by

$$s_k(t_0 + l\tau_0) \approx s_k(t_0)\exp(j2\pi f l\tau_0) \quad (3)$$

as a result, rewriting (1) in the following form

$$x_l(t_0 + l\tau_0) = \sum_{k=1}^{K} s_k(t_0)\exp(\mathrm{j}2\pi f \tilde{\tau}_{lk}) + n_l(t_0 + l\tau_0) \quad (4)$$

$$\tilde{\tau}_{lk} = \tau_{lk} + l\tau_0 = l\cdot\left(\frac{d\sin\varphi_k}{c} + \tau_0\right) \quad (5)$$

where $\tilde{\tau}_{lk}$ is the time-delay of $\tau_{lk}$ plus sampling interval $l\tau_0$.

In vector notation, (4) is of the form

$$\mathbf{x}(t_0) = \mathbf{A}\mathbf{s}(t_0) + \mathbf{n}(t_0) \quad (6)$$

$$\mathbf{x}(t_0)=[x_0(t_0),x_1(t_0+\tau_0),\cdots,x_{L-1}(t_0+(L-1)\tau_0)]^{\mathrm{T}} \quad (7)$$

$$\mathbf{A}=[\mathbf{a}(\theta_1),\mathbf{a}(\theta_2),\cdots,\mathbf{a}(\theta_K)] \quad (8)$$

$$\mathbf{a}(\theta_k)=[\exp(\mathrm{j}2\pi f\tilde{\tau}_{0k}),\exp(\mathrm{j}2\pi f\tilde{\tau}_{1k}),\cdots,\exp(\mathrm{j}2\pi f\tilde{\tau}_{(L-1)k})]^{\mathrm{T}} \quad (9)$$

$$\mathbf{s}(t_0)=[s_1(t_0),s_2(t_0),\cdots,s_K(t_0)]^{\mathrm{T}} \quad (10)$$

$$\mathbf{n}(t_0)=[n_0(t_0),n_1(t_0+\tau_0),\cdots,n_{L-1}(t_0+(L-1)\tau_0)]^{\mathrm{T}} \quad (11)$$

where, $\mathbf{A}$ is the steer matrix of SAA, whereas $\mathbf{A}_0$ is the one of conventional array [1] as follows:

$$\mathbf{A}_0=[\mathbf{a}_0(\theta_1),\mathbf{a}_0(\theta_2),\cdots,\mathbf{a}_0(\theta_K)] \quad (12)$$

$$\mathbf{a}_0(\theta_k)=[\exp(\mathrm{j}2\pi f\tau_{0k}),\exp(\mathrm{j}2\pi f\tau_{1k}),\cdots,\exp(\mathrm{j}2\pi f\tau_{(L-1)k})]^{\mathrm{T}} \quad (13)$$

Compare to $\mathbf{A}_0$, $\mathbf{A}$ with $\tilde{\tau}_{lk}$ can be rewritten as

$$\mathbf{A}=\mathbf{\Lambda}\cdot\mathbf{A}_0 \quad (14)$$

$$\mathbf{\Lambda}=\begin{bmatrix}1 & 0 & \cdots & 0\\ 0 & \exp(\mathrm{j}2\pi f\tau_0) & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \exp[\mathrm{j}2\pi f(L-1)\tau_0]\end{bmatrix} \quad (15)$$

according to (12) and (14), $\mathbf{a}(\theta)$ can also be rewritten as:

$$\mathbf{a}(\theta)=\mathbf{\Lambda}\cdot\mathbf{a}_0(\theta) \quad (16)$$

according to (15), when the value of $f\tau_0$ is a natural number, we have $\mathbf{\Lambda}=\mathrm{diag}[1,1,\cdots,1]$, which can be ignored actually in (14) and (16), that is to say $\mathbf{A}=\mathbf{A}_0$, $\mathbf{a}(\theta)=\mathbf{a}_0(\theta)$, thus we are able to process $\mathbf{y}(t)$ of SAA as the sampled data of conventional array at this moment.

## 3. Algorithm

The spatial FFT in conventional array transforms the sampled data $\mathbf{x}_0(t_0)=[x_0(t_0),x_1(t_0),\cdots,x_{L-1}(t_0)]^{\mathrm{T}}$ from one snapshot into $\mathbf{X}_0=[X_0(0),X_0(1),\cdots,X_0(L-1)]^{\mathrm{T}}$:

$$X_0(i)=\sum_{l=0}^{L-1}x_l(t_0)\exp(-\mathrm{j}2\pi li/L) \quad (17)$$

where $i=0,1,\cdots,L-1$. Recompose $X_0(i)$ into $X(i)$: when $i=0,1,\cdots,L/2-1$, $X(i)=X_0(i)$, when $i=L/2,L/2+1,\cdots L-1$, $X(i-L)=X_0(i)$; when $i=-L/2$, , $X(-L/2)=X(L/2)$. $X(i)$ are the outputs of FFT after recomposition, where $i=-L/2,-L/2+1,\cdots,L/2$. The angle of azimuth $\hat{\theta}_i$ corresponding to $X(i)$ is:

$$\hat{\theta}_i=\sin^{-1}(\frac{\lambda}{d}\cdot\frac{i}{L}) \quad (18)$$

We can regard $|X(-L/2)|^2,|X(-L/2+1)|^2,\cdots,|X(L/2)|^2$ as the discrete form of spatial spectrum, base on which, spatial spectrum is able to be estimated.

We are able to obtain transforming function $\exp(-\mathrm{j}2\pi li/L)$ from (17). Defining $\beta_0(i)=\exp(-j2\pi i/L)$, then (17) can be reformed as

$$X_0(i)=\left[1,\beta_0(i),\cdots,\beta_0^{\,L-1}(i)\right]\cdot\mathbf{x}_0(t_0) \quad (19)$$

based on (14), we can obtain

$$\mathbf{x}_0(t_0)=\mathbf{\Lambda}^{-1}\cdot\mathbf{x}(t_0) \quad (20)$$

substituting (20) in (19), (19) is written as

$$\begin{aligned}X_0(i)&=\left[1,\beta_0(i),\cdots,\beta_0^{\,L-1}(i)\right]\cdot\mathbf{\Lambda}^{-1}\cdot\mathbf{x}(t_0)\\&=\left[1,\beta_0(i)\exp(-\mathrm{j}2\pi f\tau_0),\cdots,[\beta_0(i)\exp(\mathrm{j}2\pi f\tau_0)]^{L-1}\right]\cdot\mathbf{x}(t_0)\end{aligned} \quad (21)$$

defining $\beta(i)=\beta_0(i)\exp(-\mathrm{j}2\pi f\tau_0)$, (21) can be reformed as

$$\begin{aligned}X_0(i)&=\left[1,\beta(i),\cdots,\beta^{L-1}(i)\right]\cdot\mathbf{x}(t_0)\\&=\sum_{l=0}^{L-1}x_l(t_0+l\tau_0)\exp[-\mathrm{j}2\pi l(i/L+f\tau_0)]\end{aligned} \quad (22)$$

The algorithm of Spatial FFT in SAA (hereinafter SAA-FFT) is shown in (22). Through a comparison between the classic Spatial FFT in (17) and SAA-FFT in (22), we can find that the former possesses $x_l(t_0)$ as its transformed function and $\beta_0^l(i)$ as its transforming function, while the latter possesses $x_l(t_0+l\tau_0)$ as its transformed function and $\beta^l(i)$ as its transforming function, the two algorithms can produce the same result.

Especially, when $f\tau_0$ is natural number, the two transforming functions are equal, at this moment; the right result is able to be obtained if we use the classic algorithm to operate with $\mathbf{x}(i)$.

We can pad $\mathbf{x}_0(i)$ with trailing zeros to improve the precision of spatial spectrum estimation [9]; furthermore, the estimation stabilization can be improved through averaging the results from FFT operations with multi-cycle sampled data [9].

Based on the above analysis, the implementation of the proposed algorithm is summarized as follows

Step_1 Obtain receive data vectors $\mathbf{x}_1,\mathbf{x}_2,\cdots,\mathbf{x}_N$ of $N$ cycles of sample in SAA.

Step_2 Pad each $\mathbf{x}_n$ with trailing zeros to length $Q$.

Step_3 According to (22), compute the $Q$-point FFT of each $\mathbf{x}_n$ to obtain $\mathbf{X}_1,\mathbf{X}_2,\cdots,\mathbf{X}_N$.

Step_4 Compute the mean of each $\mathbf{X}_n$ by $\bar{\mathbf{X}}=\frac{1}{N}\sum_{n=1}^{N}\bar{\mathbf{X}}_n$.

Step_5 Regard $|\bar{X}(-Q/2)|^2,|\bar{X}(-Q/2+1)|^2,\cdots,|\bar{X}(Q/2)|^2$ as the discrete form of spatial spectrum, obtain the corresponding azimuth to each point according to (23).

$$\hat{\theta}_i=\sin^{-1}(\frac{\lambda}{d}\cdot\frac{i}{Q})\ \ (i=-Q/2,-Q/2+1,\cdots,Q/2) \quad (23)$$

## 4. Simulation

In this section, we devise several simulation scenarios to verify the validity of the proposed algorithm. We also compare the proposed algorithm with conventional array's classical spatial FFT and the algorithm proposed in [7].

In first simulation, we assume a uniform linear SAA with $L=32$ half-wave-length spaced sensors with the switch interval $\tau_0=1\mu s$ and a conventional array with the same configuration at the same place. Two narrowband sources with identical power equal to SNR=10dB are assumed to impinge the two arrays from $\theta_1=9^\circ$ and $\theta_2=47^\circ$. The former array utilizes the proposed algorithm with $N=50$ cycles of sample, while the latter array utilizes classical spatial FFT with $N=50$ snapshots. The padded zeros numbers $Q$ of the two algorithms are both 128.

Figures 2 shows simulation results of DOA estimation for the two algorithms in two arrays respectively, which demonstrate that the proposed algorithm in SAA can obtain the same result as the classical spatial FFT algorithm in conventional array.

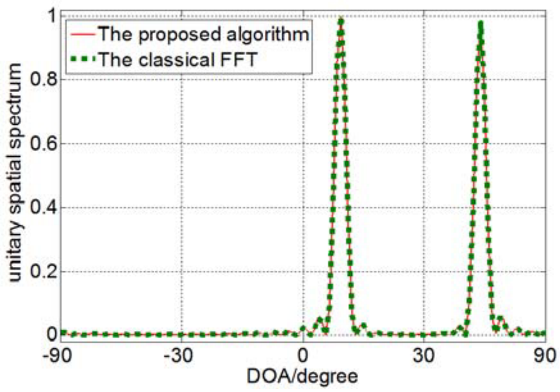

Fig.2. DOA estimation comparison of 2 kinds of FFT

In second simulation, two algorithms in SAA are compared with the assumption that $L=32$, $\tau_0 = 1\mu s$, $N=50$, $Q=128$, the number of search points in the algorithm proposed in [7] is also $Q$.

Figures 3 shows simulation results when the sources are uncorrelated with SNR=10dB Figures 4 shows the results when the sources are coherent with $\mathrm{SNR} = -10\mathrm{dB}$.

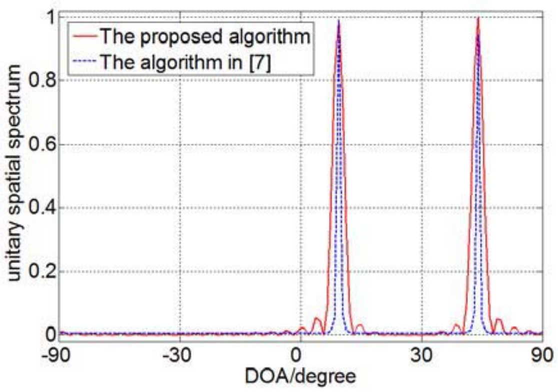

Fig.3. DOA estimation comparison with uncorrelated sources

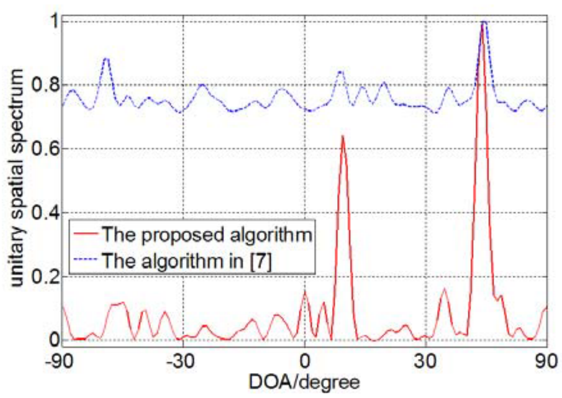

Fig.4. DOA estimation comparison with coherent source

For uncorrelated sources, the algorithm proposed in [7] performs better with narrower major lobes and lower side lobes, since it, as a subspace estimation algorithm, possesses super resolution. For coherent sources, the proposed algorithm has better performance with lower SNR while the other almost can't work.

Through statistical analysis method, we compare the performance of the above two algorithms in third simulation. The root-mean-square errors (RMSEs) of DOA estimations are computed using 1000 independent Monte Carlo simulation running under the SNR as -20~0dB. Figures 5 and 6 show the RMSEs of each algorithm with uncorrelated sources and coherent sources respectively. As the two figures show, the algorithm proposed in [7] performs better when processing uncorrelated sources, which, however, deteriorates significantly with coherent sources. The proposed algorithm has good performance in both cases if only $\mathrm{SNR} \geq -10\mathrm{dB}$.

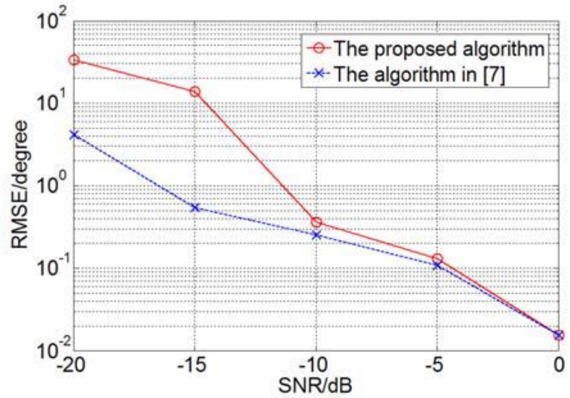

Fig.5. RMSE comparison with uncorrelated source

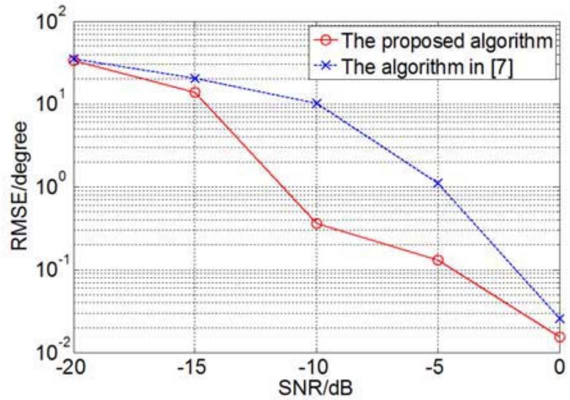

Fig.6. RMSE comparison with coherent source

For illustrating the proposed algorithm possesses low complexity, it is compared with the algorithm in [7] as follows.

The assumption here is same to the second simulation, thus, according to [9] and [10], the complexity $C_1$ of the proposed algorithm and $C_2$ of the algorithm in [7] are respectively obtained as

$$C_1 = O(NQ\log_2 Q) \tag{24}$$

$$C_2 = O(NL^2 + L^3 + QL^2) \tag{25}$$

where $NQ\log_2 Q \ll NQ^2$, thus $C_1 \ll C_2$.

Substituting $L=32$, $N=50$, $Q=128$ in (24) and (25), we can obtain that $C_1=44800$ and $C_2=215040$, which demonstrates that the proposed algorithm has lower complexity than the algorithm in [7].

## 5. Conclusions

In this paper, we propose a novel DOA estimation algorithm in SAA. The proposed algorithm is based on the spatial FFT algorithm in conventional array except in two aspects. First, the sampled data is obtained by SAA; second, the transforming function includes the switch time. The appealing advantage of the proposed algorithm lies in that it is able to process coherent signal sources with low complexity. Simulations have demonstrated validity and superiority of the novel algorithm.


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