APPM 5600 — HOMEWORK # 1

1. (15 points) How would you perform the following calculations to avoid cancellation? Justify your answers.

   i. (5 pts) Evaluate $\sqrt{x+1}-1$ for $x \simeq 0$.

   ii. (5 pts) Evaluate $\sin(x)-\sin(y)$ for $x \simeq y$.

   iii. (5 pts) Evaluate $\frac{1-\cos(x)}{\sin(x)}$ for $x \simeq 0$.

2. (20 points) Consider the polynomial
   $p(x) = (x-2)^9 = x^9 - 18x^8 + 144x^7 - 672x^6 + 2016x^5 - 4032x^4 + 5376x^3 - 4608x^2 + 2304x - 512.$

   i. (5 pts) Plot $p(x)$ for $x = 1.920, 1.921, 1.922, \ldots, 2.080$ (i.e. $x = [1.920 : 0.001 : 2.080];$) evaluating p via its coefficients.

   ii. (5 pts) Produce the same plot again, now evaluating p via the expression $(x-2)^9$.

   iii. (10 pts) What is the difference? What is causing the discrepancy? Which plot is correct?

3. (25 points)**Cancellation of terms**. Consider computing $y = x_1 - x_2$ with $\tilde{x}_1 = x_1 + \Delta x_1$ and $\tilde{x}_2 = x_2 + \Delta x_2$ being approximations to the exact values. If the operation $x_1 - x_2$ is carried out exactly we have $\tilde{y} = y + \underbrace{(\Delta x_1 - \Delta x_2)}_{\Delta y}$.

   i. (10 pts) Find upper bounds on the absolute error $|\Delta y|$ and the relative error $|\Delta y|/|y|$, when is the relative error large?

   ii. (5 pts) First manipulate $\cos(x+\delta) - \cos(x)$ into an expression without subtraction. Then, tabulate or plot the difference between your expression and $\cos(x+\delta) - \cos(x)$ for $\delta = 10^{-16}, 10^{-15}, \cdots, 10^{-2}, 10^{-1}, 10^{0}$ (note that you can get an logarithmically equispaced vector in matlab by using `logspace`).

   iii. (10 pts) Taylor expansion yields $f(x+\delta) - f(x) = \delta f^{'}(x) + \frac{\delta^2}{2!} f^{''}(\xi), \quad \xi \in [x, x+\delta]$. Use this expression to approximate $\cos(x+\delta) - \cos(x)$ for the same values of δ as in (b). Determine for what values of δ each method is better.

4. (5 points) Show that $(1+x)^n = 1 + nx + o(x)$ as $x \to 0$ where $n \in \mathbb{Z}$.

5. (5 points) Show that $x \sin\sqrt{x} = O(x^{3/2})$ as $x \to 0^+$.

6. (30 points) The function $f(x) = (x-5)^9$ has a root (with multiplicity 9) at $x = 5$ and is monotonically increasing (decreasing) for $x > 5$ ($x < 5$) and should thus be a suitable candidate for your function above. Set `a=4.8` and `b=5.31` and `tol = 1e-4` and use `bisection` with: (10 points for bisection code)

   i. (5 pts) $f(x) = (x-5)^9$.

   ii. (5 pts) The expanded expanded version of $(x-5)^9$, that is,
   $f(x) = x^9 - 45x^8 + \ldots - 1953125$.

   iii. (10 pts) Explain what is happening.