# APPM 5600 — HOMEWORK # 12

1. (20 point)

   (a) (6 points) Write a code to approximate $\int_{-5}^{5} \frac{1}{1+s^2} ds$ using a composite Trapezoidal rule. To do this, partition the interval $[-5, 5]$ into equally spaced points $t_0, t_1, \ldots, t_n$.

   Write another code to approximate $\int_{-5}^{5} \frac{1}{1+s^2} ds$ using a composite Simpson's rule. To do this, partition the interval $[-5, 5]$ into equally spaced points $t_0, t_1, \ldots, t_n$ where $n = 2k$ is even. The even indexed points should be the endpoints of your subintervals.

   You may combine the two into one code that selects the desired method if you wish.

   Turn in a listing of your code(s).

   (b) (7 points) Use the error estimates derived in class to choose $n$ so that

   $$\left| \int_{-5}^{5} \frac{1}{1+s^2} ds - T_n \right| < 10^{-4} \quad \text{and} \quad \left| \int_{-5}^{5} \frac{1}{1+s^2} ds - S_n \right| < 10^{-4},$$

   where $T_n$ is the result of the composite Trapezoidal rule and where $S_n$ is the result of the composite Simpson's rule. Be sure to explain your reasoning for choosing $n$ in both cases (these $n$ values will be different in the two cases).

   c) (7 points) Run your code with the predicted values of $n$ and compare your computed values $S_n$ and $T_n$ with that of the Matlab (or Scipy) function `quad` on the same problem. Run tt quad twice, once with the default tolerance of $10^{-6}$ and another time with the set tolerance of $10^{-4}$. Report the number of function evaluations required in both cases and compare these to the number of function values your codes (both $S_n$ and $T_n$) required to meet the tolerance

   Turn in your codes and the results of this test.

2. (20 points) Apply the composite midpoint rule, composite Trapezoidal rule, and composite Simpson's rule to approximate the integral

   $$-4 \int_0^1 x \ln(x) dx = 1.$$

   Use $n = 2, 4, 8, 16, \ldots, 512$. Plot the absolute value of the error versus the stepsize $h$ on a single log-log plot. Discuss the relationship of your results with the error formulas for these quadratures.

3. (30 points) Derive a quadrature based on the cubic Hermite interpolating polynomial with data $f(a)$, $f(b)$, $f'(a)$, and $f'(b)$. Derive an upper bound on the error.

4. (30 points) Assume the error in an integration formula has the asymptotic expansion

   $$I - I_n = \frac{C_1}{n\sqrt{n}} + \frac{C_2}{n^2} + \frac{C_3}{n^2\sqrt{n}} + \frac{C_4}{n^3} + \cdots$$

   Generalize the Richardson extrapolation process to obtain an estimate of $I$ with an error of order $\frac{1}{n^2\sqrt{n}}$. Assume that three values $I_n$, $I_{n/2}$ and $I_{n/4}$ have been computed.