3.5 i) We can find the correlation function as

$$R_{vv}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\sigma_n^2 \exp(i\omega t)}{1+(\omega\tau)^2}\, d\omega.$$ Now letting

$$k = \omega\tau,\quad R_{vv}(t) = \frac{\sigma_n^2}{2\pi}\int_{-\infty}^{\infty} \frac{\exp(ik\frac{t}{\tau})}{1+k^2}\frac{dk}{\tau}$$

$$= 2\pi i \operatorname*{Res}_{k=i}\left[\frac{\sigma_n^2}{2\pi}\frac{\exp(ik\frac{t}{\tau})}{(1+k^2)\tau}\right]$$

$$= \frac{\sigma_n^2 \exp(-\frac{t}{\tau})}{2\tau} = \frac{\sigma_n^2}{2\tau}\exp\left(-\frac{t}{\tau}\right),$$ for the $t>0$

part, and by symmetry the $t<0$ part will be

$\frac{\sigma_n^2}{2\tau}\exp\left(\frac{t}{\tau}\right)$, So $R_{vv}(t) = \frac{\sigma_n^2}{2\tau}\exp\left(-\frac{|t|}{\tau}\right)$, and

$$\boxed{R_{vv}[k] = \frac{\sigma_n^2}{2\tau}\exp\left(-|k|\frac{T}{\tau}\right).}$$

ii) This is $\mathbb{E}[V_i V_{i+1}] = \frac{1}{N^2}\mathbb{E}\left(\sum_{k=i}^{i+N-1} V[kT]\sum_{j=i+1}^{i+N} V[jT]\right)$

$$= \mathbb{E}\left[\sum_{k=i}^{i+N-1}\sum_{j=i+1}^{i+N} V[kT]V[jT]\right]\cdot\frac{1}{N^2}$$

$$= \sum_{k=i}^{i+N-1}\sum_{j=1}^{N}\mathbb{E}\left[V[kT]V[(k+j)T]\right]\cdot\frac{1}{N^2}$$

$$= \frac{\sigma_n^2}{2\tau}\sum_{k=i}^{i+N-1}\sum_{j=1}^{N}\exp\left(-j\frac{T}{\tau}\right)\cdot\frac{1}{N^2}$$

$$= \frac{\sigma_n^2}{2\tau N}\exp\left(-\frac{T}{\tau}\right)\sum_{j=0}^{N-1}\exp(-jT/\tau)$$

$$\boxed{= \frac{\sigma_n^2}{2\tau N}\exp\left(-\frac{T}{\tau}\right)\left[\frac{1-\exp(-N\frac{T}{\tau})}{1-\exp(-T/\tau)}\right]}$$