1 B) Since X is zero-mean we need only the variance

C) we need three numbers: The variances of the real and imaginary parts, and the correlation coefficient

D) $M = \mathbb{E}[\exp(i\vec{v}\cdot\vec{x})] = 2\pi\, \mathcal{F}^{-1}[P_{xy}(x,y)]$

$$= \int_{\mathbb{R}^2} \exp(i\vec{v}\cdot\vec{x})\, P_{xy}(x,y)\, dx\, dy$$

$$= \int_{\mathbb{R}} \frac{\exp(i(v_x x + v_y y))}{2\pi\sqrt{|D|}} \exp\left(-\frac{1}{2D}\left(\sigma_y^2 x^2 - 2\sigma_x\sigma_y \rho x y + \sigma_x^2 y\right)\right) dx\, dy$$

Now we let $z = \vec{v}\cdot\binom{x}{y}$, and letting $\sigma_z^2 = \vec{v}^T \Sigma\, \vec{v}$, the integral becomes

$$\frac{1}{\sqrt{2\pi}\,\sigma_y} \int_{-\infty}^{\infty} \exp(iz) \exp\left(-\frac{1}{2}\frac{z^2}{\sigma_y^2}\right) dz$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma_y} \int_{-\infty}^{\infty} \exp\left(-\frac{1}{2}\left[\frac{z^2}{\sigma_z^2} - 2iz\right]\right) dz$$

By completing the square we see $\frac{1}{\sigma_z^2} z^2 - 2iz = \sigma_z^{-2}\left[z + \frac{i}{\sigma_z^2}\right]^2 + \frac{1}{\sigma_z^2}$.

So the integral becomes

$$\frac{1}{\sqrt{2\pi}\,\sigma_z} \int_{-\infty}^{\infty} \exp\left(\frac{-1}{2\sigma_z^2}\left(z + \frac{i}{\sigma_z^2}\right)^2 + 1\right)$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma_z} \exp\left(-\frac{1}{2}\sigma_z^{-2}\right) \int_{-\infty}^{\infty} \exp\left(\frac{-1}{2\sigma_z^2}\left(z + \frac{i}{\sigma_z^2}\right)\right) dz$$

$= \exp(-\frac{1}{2}\sigma_z^{-2})\, P_z(z)$ where $z \sim N(\frac{i}{\sigma_z^2}, \sigma_z^2)$. Thus, the integral is evaluated to $\sqrt{2\pi}\,\sigma_z$, and so

$$\mathbb{E}[i\vec{v}\cdot\vec{x}] = \exp\left(-\frac{1}{2\sigma_z^2}\right) = \exp\left(-\frac{1}{2}\vec{v}^T\Sigma^{-1}\vec{v}\right)$$ as