$$\frac{y^2}{s^2} = \frac{\langle a\rangle^2}{\sigma_a^2} + \vec{y}^* \Sigma_{nn}^{-1} \vec{y} - \left[(\vec{y} - \langle a\rangle \mathbb{1})^* (\Sigma_{nn}^{-1}) (\vec{y} - \langle a\rangle \mathbb{1}) \right.$$

$$\left. - \frac{\sigma_a^2}{\sigma_n^2} \left(\frac{1}{\sigma_n^2 + N\sigma_a^2} \right) (\vec{y} - \langle a\rangle \mathbb{1})^* (\mathbb{1}\mathbb{1}^*) (\vec{y} - \langle a\rangle \mathbb{1}) \right]$$

$$= \frac{\langle a\rangle^2}{\sigma_a^2} + y^* \Sigma_{nn}^{-1} y - \left[y^* \Sigma_{nn}^{-1} y - \langle a\rangle \mathbb{1}^* \Sigma_{nn}^{-1} \vec{y} - \vec{y}^* \Sigma_{nn}^{-1} \langle a\rangle \mathbb{1} + \langle a\rangle^2 \mathbb{1}^* \Sigma_{nn}^{-1} \mathbb{1} \right.$$

$$\left. - \frac{\sigma_a^2}{\sigma_n^2} \left(\frac{1}{\sigma_n^2 + N\sigma_a^2} \right) \left[(\vec{y} - \langle a\rangle \mathbb{1})^* (\mathbb{1}\mathbb{1}^*) (\vec{y} - \langle a\rangle \mathbb{1}) \right] \right]$$

$$= \frac{\langle a\rangle^2}{\sigma_a^2} + \frac{2\langle a\rangle \sum_{i=1}^{n} y_i}{\sigma_n^2} + \frac{N\langle a\rangle^2}{\sigma_n^2} + \frac{\sigma_a^2 (\vec{y} - \langle a\rangle \mathbb{1})^* (\mathbb{1}\mathbb{1}^*) (\vec{y} - \langle a\rangle \mathbb{1})}{\sigma_n^2 (\sigma_n^2 + N\sigma_a^2)} \Bigg]$$

$$= \frac{\langle a\rangle^2}{\sigma_a^2} + \frac{2\langle a\rangle \sum_{i=1}^{n} y_i}{\sigma_n^2} + \frac{N\langle a\rangle^2}{\sigma_n^2} + \frac{\sigma_a^2}{\sigma_n^2 (\sigma_n^2 + N\sigma_a^2)} \left(y^* \mathbb{1}\mathbb{1}^* y - \langle a\rangle \mathbb{1}^* \mathbb{1}\mathbb{1}^* y \right.$$

$$\left. - y^* \mathbb{1}\mathbb{1}^* \mathbb{1} \langle a\rangle - N^2 \langle a\rangle^2 \right)$$

$$= \frac{\langle a\rangle^2}{\sigma_a^2} + \frac{2\langle a\rangle \sum_{i=1}^{N} y_i}{\sigma_n^2} + \frac{N\langle a\rangle^2}{\sigma_n^2} + \frac{\sigma_a^2}{\sigma_n^2 (\sigma_n^2 + N\sigma_a^2)} \left[\left(\sum_{i=1}^{N} y_i \right)^2 - 2N\langle a\rangle \sum_{i=1}^{n} y_i + N^2 \langle a\rangle^2 \right]$$

Letting $z = \sum_{i=1}^{N} y_i$ this becomes

$$\frac{\langle a\rangle^2}{\sigma_a^2} + \frac{2\langle a\rangle z}{\sigma_n^2} + \frac{N\langle a\rangle^2}{\sigma_n^2} + \frac{\sigma_a^2}{\sigma_n^2 (\sigma_n^2 + N\sigma_a^2)} \left[z^2 - 2N\langle a\rangle z + N^2 \langle a\rangle^2 \right]$$

$$= \frac{\langle a\rangle^2}{\sigma_a^2} + \frac{2\langle a\rangle z}{\sigma_n^2} - \frac{N\langle a\rangle^2}{\sigma_n^2} + \frac{\sigma_a^2 (z - N\langle a\rangle)^2}{\sigma_n^2 (\sigma_n^2 + N\sigma_a^2)}$$

$$= \langle a\rangle^2 \left(\frac{1}{\sigma_a^2} + \frac{N}{\sigma_n^2} \right) + \frac{2\langle a\rangle z}{\sigma_n^2} + \frac{N^2 \sigma_a^4 \left(\frac{z}{N} - \langle a\rangle \right)^2}{\sigma_n^2 (\sigma_n^2 + N\sigma_a^2)}$$

$$\text{So } y^2 = \langle a\rangle^2 \left(\frac{\sigma_n^2 - N\sigma_a^2}{\sigma_n^2 + N\sigma_a^2} \right) + \frac{2N\sigma_a^2 \cdot \left(\frac{z}{N} \right) \langle a\rangle}{N\sigma_a^2 + \sigma_n^2} + \frac{N^2 \sigma_a^4 \left(\frac{z}{N} - \langle a\rangle \right)^2}{(\sigma_n^2 + N\sigma_a^2)^2}$$