ECEN 5244 - Environmental Signal Processing

Homework #1

Issued 9/5/23
Due 9/21/23

1.1 (A) Let x and y be two zero-mean correlated Gaussian random variables. Find the following statistical quantities in terms of the expectations $E\left[x^2\right] = \sigma_x^2$, $E\left[y^2\right] = \sigma_y^2$, and $E\left[xy\right] = \sigma_x \sigma_y \rho$:

(i) $E\left[x^3\right]$ (ii) $E\left[x^4\right]$ (iii) $E\left[x^6\right]$ (iv) $E\left[x^2 y^4\right]$

(B) How many numbers are required to fully characterize all of the statistical moments of a real (i.e., non-complex) zero-mean $N-$dimensional Gaussian random variable?

(C) How would your answer to part (B) change if the random variables were complex?

(D) Find an expression for the joint characteristic function $M\left(j\nu_x, j\nu_y\right) = E\left[e^{j\nu_x x + j\nu_y y}\right]$ of the above pair of zero mean Gaussian random variables.

1.2 A positive definite covariance matrix $\overline{\overline{R}}_{xx}$ for the variable $\overline{x}$ can be factored into the form:

$$\overline{\overline{R}}_{xx} = \overline{\overline{E}}\,\overline{\overline{\Lambda}}\,\overline{\overline{E}}^T$$

where $\overline{\overline{E}}$ is a column eigenvector matrix and $\overline{\overline{\Lambda}}$ is a diagonal matrix of positive eigenvalues.

(A) Using this decomposition (which can be performed using Matlab using the "eig" command), synthesize a pseudorandom set of 500 three-dimensional vectors $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ with the following correlations and standard deviations:

$$\begin{aligned} \sigma_1 &= 1 \\ \sigma_2 &= 2 \\ \sigma_3 &= 3 \\ \rho_{12} &= 0.5 \\ \rho_{23} &= -0.5 \\ \rho_{13} &= 0 \end{aligned}$$

Use scatter plots to show the resulting statistical behavior of your set.

(B)What is the computed covariance matrix for your resulting set? Is it different than what you prescribed? Discuss reasons for any differences.

1.3 Derive the iterative expression in Brent's method for minimization of a one-dimensional nonlinear model.

1.4 Consider the associated .mat or .txt file (HW1_4.xxx, both available on Canvas) containing 500 points of measured data. Assume that the measurement noise on the data has a standard deviation of $\sigma = 0.5$.

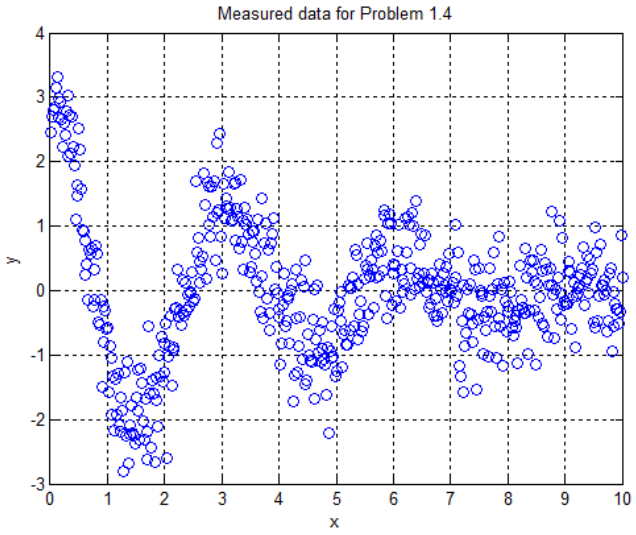
(A) Propose a suitable linear or nonlinear model for this data.

(B) Develop and demonstrate a parametric fit to the data using a gradient search method.

(C) Develop and demonstrate a parametric fit to this data using simulated annealing.

(D) Develop and demonstrate a parametric fit to this data using Prony's method.

For parts (B) - (D) compute χ^2 and use this criteria to determine whether or not your resulting fit is "good".


Measured data for Problem 1.4
y
x
4
3
2
1
0
-1
-2
-3
0
1
2
3
4
5
6
7
8
9
10