GENERALIZED CIRCULAR AUTOREGRESSIVE MODELS FOR MODELING ISOTROPIC AND ANISOTROPIC TEXTURES

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ABSTRACT

A new class of random field models, called generalized circular autoregressive (GCAR) models, is introduced. GCAR models have non-causal neighbors which have same autoregressive parameter values if they are on the same circle or ellipse, and have circular or elliptical correlation structure. Parameter estimation is also considered, and multi-step estimation algorithm is presented. The efficacy of GCAR models in modeling real textures is demonstrated by synthesizing images resembling real textures.

1. INTRODUCTION

In this paper, a new class of models, called generalized circular autoregressive (GCAR) model, is introduced by generalizing CAR model to model isotropic and elliptic textures having complex patterns. GCAR models are defined with larger neighborhoods than eight pixels, and its neighbors are not interpolated. GCAR models have many advantages over earlier CAR modeling approach [6]. Since they can include larger neighborhood without interpolation, they can represent more diverse textures with smaller number of parameters than traditional noncausal autoregressive models, such as simultaneous autoregressive (SAR) model [1] or Markov random field (MRF) model [3]. Further, the GCAR model can also represent anisotropic textures by allowing elliptical neighbors. A set of elliptical neighbors are defined by a geometric transform of circular neighbors. The parameter estimation in GCAR models is done in three steps: In the first step, spatial interaction parameters are estimated assuming model order and geometric transform parameters are known. Then geometric transform parameters and model order are determined by a maximum-likelihood method using estimated spatial interaction parameters.

The ability of GCAR models is demonstrated by synthesizing images using manually selected parameters as well as parameters estimated from real textures selected from Brodatz texture album [8]. In the experiment with manually selected parameters, it is shown that diverse images of different orientation and elongation can be synthesized from the same set of spatial interaction parameters. For majority of textures tested, except structured or inhomogeneous textures, the synthesized images using estimated parameters resemble original textures.

2. GENERALIZED CIRCULAR AUTOREGRESSIVE MODEL

Generalized circular autoregressive (GCAR) models are defined with circular or elliptic neighbors, where the autoregressive parameter values are equal if the neighbors are on the same circle or ellipse. Many natural textures, such as cloud or grass lawn, have circular or elliptical correlation structures. In this section, isotropic GCAR model is first introduced then anisotropic GCAR model is introduced as a generalization of isotropic GCAR model.

2.1. Isotropic GCAR Model

Let $\{y(s) \mid s \in \Omega\}$, where $\Omega = \{(s_1, s_2) \in \mathbf{Z} \times \mathbf{Z} \mid 0 \leq s_1, s_2 < N\}$, be an image of size $N \times N$ following a 2-D GCAR model. Then $y(s)$ is represented by

$$y(s) = \sum_{r \in K_g} a_r y(s \ominus r) + \sqrt{\rho} w(s), \qquad (1)$$

where $s, r \in \mathbf{Z} \times \mathbf{Z}$, K_g is a finite index set where a_r is defined, $\ominus$ is a $modulo - (N, N)$ subtraction, and $w(s)$ is a 2-D white Gaussian process with zero mean and unit variance. It is also assumed that

$$a_r = a_t, \quad \text{if } ||r|| = ||t||. \qquad (2)$$

i.e., the values of the parameters for neighbors at equal distance are equal.

After observing neighbors at equal distances, it can be found that neighbors at equidistances contain four, eight or twelve pixels, and can be decomposed into one or more sets of four neighbors separated by $\pi/2$. For example, the neighbors at distance $\sqrt{3}$ are {(2,1), (1,2), (-1,2), (-2,1), (2,-1),(-1,-2),(1,-2),(2,-1)}, and are decomposed into the following two sets: $N_1 = \{(2,1), (-1,2), (-2,-1), (1,-2)\}$ and $N_2 = \{(1,2), (-2,1), (-1,-2), (2,-1)\}$. Further, each set containing four neighbors can be specified by the distance d and the angle θ to the neighbor at the first quadrant

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as illustrated in Figure ??. For example, N_1 is represented by $d = \sqrt{3}$ and $\theta = \tan^{-1}(\frac{1}{2})$.

2.2. Anisotropic GCAR Model

The isotropic GCAR model has circular symmetric neighbors, and is good for modeling isotropic textures without directional features. However, many natural textures, such as wood grain texture, show strong directional patterns, and isotropic GCAR model is not suitable for modeling such textures. To model anisotropic directional textures, the neighbors also need be elongated and directional. The anisotropic GCAR model is defined as a generalization of the isotropic GCAR model, and has elliptical neighbors. The anisotropic GCAR model is defined as

$$y(s) = \sum_{r \in K_g} a_r y(s \ominus r) + \sqrt{\rho} w(s), \tag{3}$$

and the parameter a_r is constant for ellipsoidal neighbors. i.e.,

$$a_r = a_t, \text{ if } ||Ar|| = ||At||, \text{ where} \tag{4}$$

$$A = \begin{bmatrix} \alpha \cos\theta & -\alpha \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \tag{5}$$

and α and θ are elongation and orientation parameters, respectively. It is equivalent that every neighbor pixels r intersecting with an ellipsoid with elongation parameter α and orientation parameter θ have the same parameter value a_r. It can also be shown that there exists at least a pair of symmetric neighbors having same parameter value, which are represented as a linear geometric transform of two pixels (0,1) and (0,-1).

2.3. Synthesis of GCAR Textures

Let $\mathbf{Y}$ and $\mathbf{W}$ be discrete Fourier transforms (DFT) of $\mathbf{y}$ and $\mathbf{w}$. The 2-D discrete Fourier transform (DFT) and inverse discrete Fourier transform (IDFT) pair is defined as

$$\begin{aligned} Y(u) &= \sum_{s \in \Omega} y(s) \exp\left(\frac{-j2\pi s^T u}{N}\right), \\ y(s) &= \frac{1}{N^2} \sum_{u \in \Omega} Y(u) \exp\left(\frac{j2\pi s^T u}{N}\right). \end{aligned} \tag{6}$$

By Theorems 1 and 2, both isotropic and anisotropic GCAR models can be written in the frequency domain as

$$\Lambda \mathbf{Y} = \sqrt{\rho} \mathbf{W}, \tag{7}$$

where $\Lambda = diag.[\lambda(0,0) \cdots \lambda(N-1, N-1)]$, and $\{\lambda(k,l)\}$ are defined in (8) for isotropic GCAR model and (9) for anisotropic GCAR models. Therefore, an image following a GCAR model can be synthesized in the frequency domain using (7) as summarized in the following synthesis algorithm:

Synthesis Algorithm:

1. Assume that the parameters $\{a_i, i = 1, \cdots, p\}$ are given, and the orientation and elongation parameters θ and α are assumed to be known. Generate $N \times N$ image $\mathbf{w}$ of white Gaussian noise having zero mean and unit variance.

2. Apply DFT to the white Gaussian random field $\mathbf{w}$ and obtain $\mathbf{W}$.

3. Compute the eigenvalues $\{\lambda(k,l), 0 \le k, l < N\}$ using (8) for isotropic model or (9) for anisotropic model.

$$\lambda_{ij}(k,l) = \begin{cases} 2\cos(2\pi r_i k/N) + \\ 2\cos(2\pi r_i l/N), \\ \quad \text{if } \theta_i = 0 \\ \\ 4\cos(2\pi r_i k \cos\theta_{ij}/N) \cdot \\ \cos(2\pi r_i l \sin\theta_{ij}/N), \\ \quad \text{otherwise} \end{cases} \tag{8}$$

$$\lambda_{ij}(k,l) = \begin{cases} 2\cos(2\pi r_{ij} k/N), \\ \quad \text{if } \theta_{ij} = 0 \\ \\ 2\cos\,(2\pi r_{ij} \\ [k\cos\theta_{ij} + l \sin\theta_{ij}]/N)\,, \\ \quad \text{otherwise} \end{cases} \tag{9}$$

4. An image $\mathbf{Y}$ in the frequency domain is synthesized by multiplying the inverse of eigenvalues to the DFT of white Gaussian process $\mathbf{W}$. i.e.,

$$Y(k,l) = \sqrt{\rho} W(k,l)/\lambda(k,l), \;\; k,l = 0, 1, \cdots, N-1. \tag{10}$$

5. The final synthesized image is obtained by applying IDFT to the frequency domain image $\mathbf{Y}$.

3. PARAMETER ESTIMATION

For the analysis of a texture using an image model, the model parameters need to be estimated from the observed texture. For an image following an anisotropic GCAR model, we need to estimate geometric transform parameters α and θ as well as spatial interaction parameters $\mathbf{a}$ and ρ.

In 1-D and causal 2-D AR models [11], a least squares (LS) estimators [13] are widely used under the Gaussian assumption of the noise process $w(s)$. However, the LS estimator is not consistent for noncausal models, such as GCAR model, because the noise process $w(s)$ is correlated to non-causal neighbors of $w(s)$. The maximum-likelihood (ML) estimators are consistent and efficient [13], but generally requires computationally expensive numerical optimization method. In this paper, we presented an estimation algorithm which yields estimates close to ML estimates with a faster convergence rate.

3.1. Estimation of Spatial Interaction Parameters

Define the parameter vector $\mathbf{a}$ and the neighbor vector $\mathbf{z}(s)$ by

$$\mathbf{a} = [a_1\ a_2\ \cdots\ a_p]^T \text{ and} \tag{11}$$

$$\mathbf{z}(s) = [z_1(s)\ z_2(s)\ \cdots\ z_p(s)]^T, \text{ where} \tag{12}$$

$$z_i(s) = \begin{cases} \sum_{j=1}^{q_i} [y(s + r_i(\cos\theta_{ij}, \sin\theta_{ij})) \\ +y(s - r_i(\cos\theta_{ij}, \sin\theta_{ij})) \\ +y(s + r_i(\cos\theta_{ij}, -\sin\theta_{ij})) \\ +\ y(s - r_i(\cos\theta_{ij}, -\sin\theta_{ij}))], \\ \quad \text{Isotropic GCAR} \\ \\ \sum_{j=1}^{q_i} [y(s + r_{ij}(\cos\theta_{ij}, \sin\theta_{ij})) \\ +\ y(s - r_{ij}(\cos\theta_{ij}, \sin\theta_{ij}))], \\ \quad \text{Anisotropic GCAR} \end{cases} \tag{13}$$

Then the GCAR model can be rewritten as

$$y(s) = \mathbf{a}^T \mathbf{z}(s) + \sqrt{\rho} w(s). \tag{14}$$

The ML estimator can be computed efficiently by the following iterations.

$$\begin{aligned} \hat{\mathbf{a}}_{t+1} = & \left[\sum_{s\in\Omega} \left(\mathbf{z}(s)\mathbf{z}^T(s) + \hat{\rho}_t \psi(s)\psi^T(s)\right)\right]^{-1} \\ & \left[\sum_{s\in\Omega} \left(\mathbf{z}(s)y(s) - \hat{\rho}_t \psi(s)\right)\right] \end{aligned} \tag{15}$$

$$\hat{\rho}_t = \frac{1}{N^2} \sum_{s\in\Omega} \left(y(s) - \hat{\mathbf{a}}_t^T \mathbf{z}(s)\right)^2, \tag{16}$$

where $\psi(s)$ is defined as

$$\psi(s) = [\sum_{j=1}^{q_1} \lambda_{1j}(s),\ \cdots,\ \sum_{j=1}^{q_p} \lambda_{pj}(s)]^T. \tag{17}$$

3.2. Selection of Model Order and Geometric Parameters

The model order p of the GCAR model needs to be correctly selected for good modeling performance, and the elongation parameter α and the orientation parameter θ also needs be determined for anisotropic GCAR models. They are considered as a maximum-likelihood decision problem in this paper.

The elongation and the orientation of a texture can be represented by a set of discrete values. For example, four different orientations are sufficient for many applications, and the orientation parameter θ can be discretized to the finite set $\Theta = \{\frac{k\pi}{4}, k = 0, \cdots, 3\}$. Similarly, four different elongation parameter values are sufficient for many applications, and the elongation parameter α can be discretized to the finite set $\mathcal{A} = \{1, 2, 4, 8\}$. We need to consider only for $\alpha \geq 1$ since values lower than 1 can be achieved by rotation. Thus, the geometric parameters α and θ can be estimated by minimizing the following cost function $J(p, \theta, \alpha)$ over the set $\Theta \times \mathcal{A}$.

$$\begin{aligned} J(p, \theta, \alpha) &= \sum_{s\in\Omega} \log\left(1 - \hat{\mathbf{a}}^T \psi(s)\right) - \frac{p}{2} \log(2\pi) \\ &\quad +(\frac{N^2}{2} - p - 1) \log \hat{\rho} + \frac{1}{2} \log |S| \end{aligned} \tag{18}$$

4. EXPERIMENTAL RESULTS

The validity of GCAR model in modeling textures is demonstrated by synthesizing textures from parameters estimated from real textures.

The geometric parameters of GCAR model correspond well with attributes of textures. For example, the estimated values of geometric parameters match well with elongation and orientation of patterns in the original texture. Similarly, a texture of arbitrary elongation and orientation can be synthesized from the same set of parameters. If a model is valid in modeling real textures, the synthesized texture should be similar to the original textures.

Figure 1 shows the results with textures with elongated patterns. The original textures are shown in the left column of Figure 1, and they are from the top, pressed cork (D04), grass lawn (D09), pressed calf leather (D24), and wood grain (D68). Using estimated parameters from each original in Figure 1, an image of double size (256 × 256) is synthesized using the synthesis algorithm given in Section 2, and shown in the right column of in Figure 1. Each synthesized image is similar to the original even though the image size is doubled. This result suggests that an image of arbitrary size can be synthesized from a small example texture if the original image follows a GCAR model.

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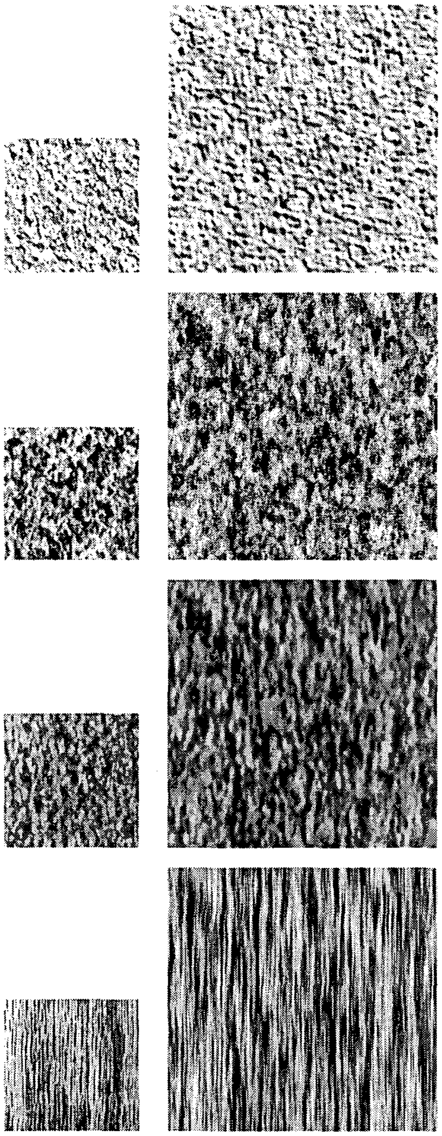

Figure 1: Original textures (left) with directional patterns, and synthesized textures (right) of double size using the parameters estimated from the originals. The original textures used are from the top, pressed cork (D04), grass lawn (D09), pressed calf leather (D24), and wood grain (D68).