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Joint distributions of Wind/Waves/Current in West Africa and derivation of multivariate extreme I-FORM contours

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ABSTRACT

In-situ measurements over a three year period have been extensively analysed from a statistical point of view. Distributions of wind and current (intensity and direction), together with waves systems - main swell, secondary swell and wind sea - (peak period, significant wave height, spectral width, direction) are analysed jointly, looking for possible correlation between the variables (principal component analysis, statistical tests, ...). When correlation was proven, joint distributions have been estimated and modelled by analytical distribution laws. As distribution of extreme values may differ from distribution for intermediate values, matching of the two solutions is performed. Finally, I-FORM contours are derived based on the analytical laws. For coherence with extreme value prediction, some constraints are imposed so that 100 year extreme value estimation for the significant wave height, current velocity and wind velocity is consistent

KEY WORDS: joint probabilities, multivariate extremes, I-FORM, West Africa

INTRODUCTION

The traditional approach to the design of offshore structures has been to quantitatively assess the loads created by simultaneous and co-linear extreme return values of wind, wave and current. It has now long been recognised that this represents an overly conservative assessment of the environmental forces, and during the past fifteen to twenty years, Response-Based Design has been proposed as a more realistic and less conservative alternative because it aims at:

- Achieving 1- to 100-year return periods for design parameters of interest (such as vessel excursions, mooring or riser design loads) rather than environmental parameters;
- Accounting for joint probabilities of wind, wave and current;
- Accounting for structural loading and response characteristics.

The work presented in this publication is a part of a study of "Joint Probabilities of Wind / Wave / Current and Response-Based Design of FPSO, Moorings and Risers" funded by TOTAL S.A., R&D division, with the support of TOTAL E&P Angola. This work aimed at the analysis of joint distributions of wind / wave/ current and at the derivation of multivariate extremes which were required to calculate extreme responses. Then, the extreme responses calculated on this basis were compared to the results of the response based design. The complete study is presented in parent papers presented at ISOPE 2007, from Orsero et al, François et al., Fontaine et al., which contain a discussion of the two different philosophies for the calculation of extreme responses.
The Angola Block 17 metocean site records have been used as the main database for this study. The paper presents successively the key ideas for the analysis of extreme values and joint probabilities, the metocean data, the analysis of these data and particularly their joint analysis and the derivation of joint extremes based on I-FORM contours.

KEY IDEAS FOR ANALYSIS OF JOINT PROBABILITY AND JOINT EXTREME METOCEAN DATA

Traditionally, structures were often designed to sustain loads created by the combination of 100-year return values for wind, wave and currents.
Current knowledge of the climate offshore West Africa suggests that wind, waves and current are generated by different and un-correlated sources and therefore may have different directions and intensities. In other words, the 100-year current is not likely to be observed at the same time as the 100-year wind and waves. This now starts to be accounted for in design codes as the 100-year condition for a given type of solicitation is generally associated to 1- or 10-year return period (or 95% quantile) for the others.
However, it should be noted that there are a number of environmental combinations that have the same probability of occurrence as the one currently recommended in metocean specifications and guidelines, and these other combinations could induce a higher structural response.
As far as directionality is concerned, co-linearity is imposed in most cases unless otherwise stated. It should be noted that due to coupled (non linear) effects, co-linear environments may not always yield to the extreme response.
As a result, a more rational design should account for joint probability distributions of the different environmental loading, both in intensity and direction, so as to give more confidence in the nominal design parameters to be chosen.

METOCEAN DATA AND ANALYSIS

Metocean instrumentation

Angola Block 17 is located approximately 7°40S - 11°43E, i.e. 210km away from Luanda, by water depths ranging from 1250 to 1400 m, as shown in Fig. 1.

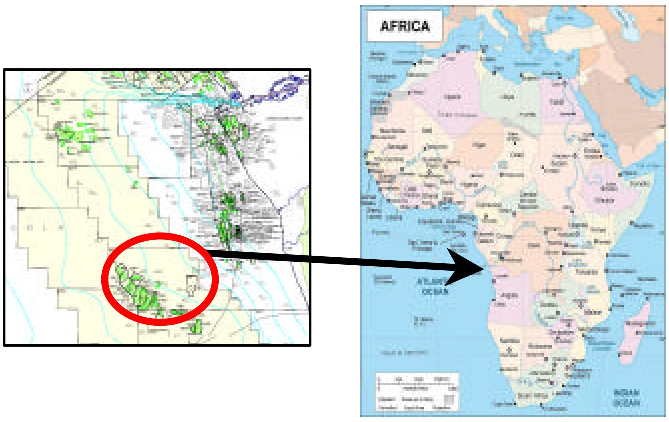

Fig. 1. Angola Block 17 location.

The metocean instrumentation of Angola Block 17 consists of:

- a Datawell Waverider Buoy
- a Vaisala Maws anemometer and met station
- a mooring line supporting two Acoustic Doppler Current Profilers (ADCPs) by RD Instruments (RDI) – a 300 kHz Workhorse and a 75 kHz LongRanger

The instrumentation has been maintained on site from May 2002 until July 2004, thus offering 2 years of simultaneous wind / wave / current data. The wave buoy was kept in place for an additional year.

The analysis of recorded metocean data showed that even if the maritime climate is mild, the wave, wind and current conditions are complex and need to be analysed carefully.

Wave analysis

Although the wave climate is mild offshore West Africa, it is a key aspect of floating structures design because of the presence of crossed seas: multiple swells and a local wind sea usually act simultaneously, with various periods and directions, and may have a significant effect on floater behaviour.
During the project, much work has been done on spectral shapes and sea-state partitioning into swells and wind sea systems.

The frequency-direction wave spectra were calculated using an Iterated Maximum Likelihood Method with MATLAB tools, from successive half-hour raw data and then averaged over one hour.
Several spectral parameters were calculated from the raw data or from the wave spectra: spectral density, main direction and directional spreading as a function of wave frequency. In addition, the maximum wave height was also calculated.
Most of the time, multi-modal wave spectra are observed, as shown in Fig. 2 below, exhibiting simultaneous swells and often a local wind sea.
Sea state systems were initially derived from a partitioning of sea-state spectra based on APL Waves Analysis software by SubChem Systems, Version 3.3.
An example of partitioning is presented in Fig. 3

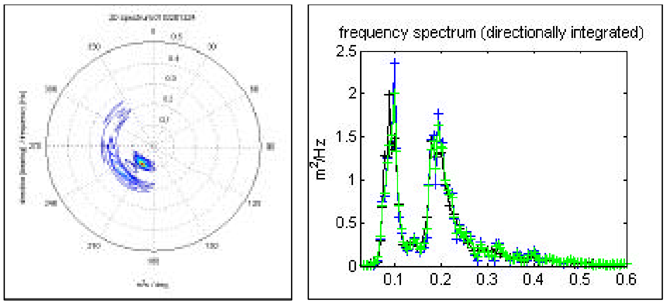

Fig. 2. A typical multi-modal wave spectrum measured offshore Angola.

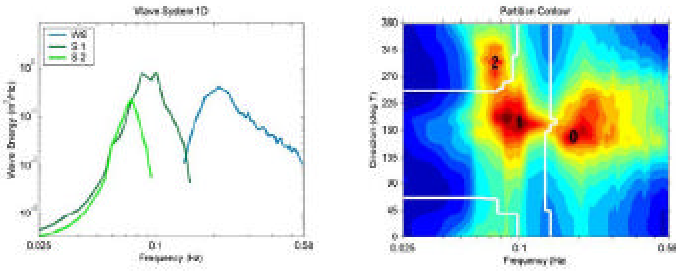

Fig. 3. Example APL WAVES spectral partition plot from 17 Nov. 2001

A Quality Control was carried out and some adjustments were made in particular as regards the secondary swell system. After this QC, APL wave systems were characterised with a maximum of 3 components:

- Main Swell (characterised by the highest Hs among the components)
- Secondary Swell (characterised from the second or the third system with Tp > 9s)
- Wind Sea (characterised with Tp < 9s)

The second swell and the wind sea were not present in every sea state: the partitioning presented in Table 1 shows that the sea-states are mainly composed of a main swell alone or a main swell with a wind sea component, about 84% of occurrences.

Table 1 : Statistics of APL Wave Systems.

Main Swell alone	46.10%
Main Swell & Second Swell	8.20%
Main Swell & Second Swell & Wind Sea	7.70%
Main Swell & Wind Sea	38.00%

Wave partitioning resulted in the production of time series of reduced parameters: significant height Hsi / peak period Tpi / peak direction ?i for each of the three systems – main and secondary swells, and wind sea – identified during the three and a half years of wave records.
This data, as well as for the total sea-state parameters, was then analysed to produce tables of statistics (min, mean, max, std and quantiles), empirical distributions (Hsi, Tpi) and the joint distributions (Hsi-Tpi, Hsi-?i, Tpi-?i, Hsi-Tpi per directional sectors).
This statistical analysis is illustrated in Figs 4~5. This last one highlights the different order of magnitude of main swells and wind seas significant heights.

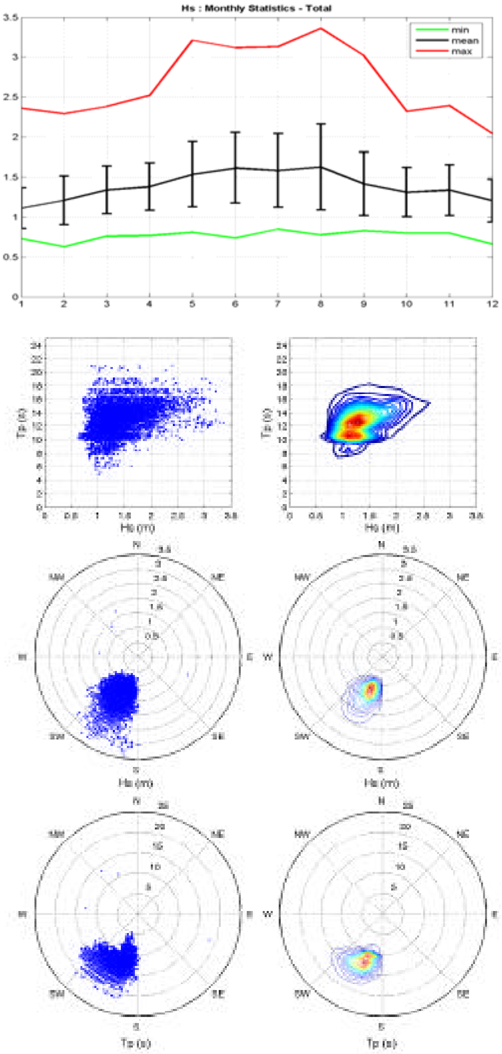

Fig. 4. Statistics of total sea-state parameters.

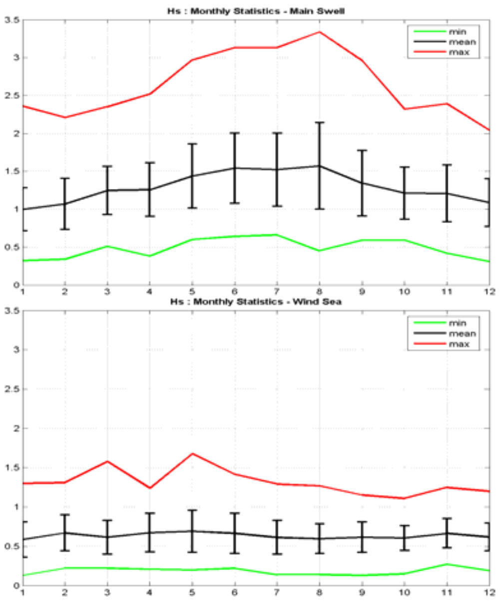

Fig. 5. Statistics of main swells and wind seas significant heights.

Wind analysis

The wind records have also been processed, by analysing separately two different statistical populations, respectively due to trade winds and squalls. Squalls were detected during a previous project (Nerzic, 2004), which also benefited from TOTAL experience acquired during the West Africa Gust Joint Industry Project. Squall events were removed from the data base for the purpose of this study, because they are rare although extreme phenomena and, therefore, they can not be accounted for in the joint probability analysis.
Tables of statistics (min, mean, max, std and quantiles), empirical distributions (Vm) and joint distributions (Vm-?) of wind were prepared are illustrated in Fig. 6.
Note also that a modified Kaimal wind spectrum was adjusted to the data during the CLAROM-CEP&M M6410/03 project, and it was used for further analyses of the moored FPSO behaviour.

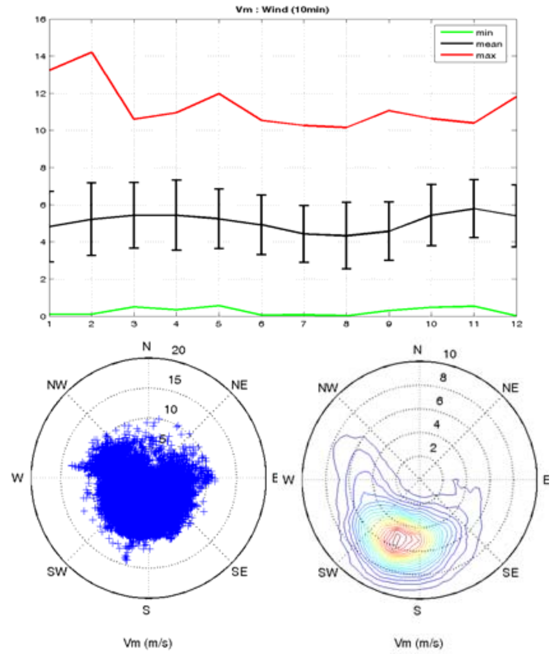

Fig. 6: Statistics of wind parameters.

Current analysis

First, a sub-surface current profile reduction was carried out. The aim was to provide the project team with time series of sub-surface current profiles: first to make the data easily useable in the calculation of drift forces on the FPSO hull, and secondly to lead to a statistical representation of the current profile climatology which could be used in reliability methods.
Following a quality control and consolidation of the current database, a visual inspection of the current profiles showed the complexity and variety of the sub-surface currents: the data often exhibits relatively strong currents (up to 0.8m/s), with sheared or rotating speeds, as illustrated in Fig. 7:
Then, various methods for reducing sub-surface profiles to a reduced number of parameters were analysed and their efficiency compared:

- a uniform profile (with the same direction over the FPSO draft),
- a sheared profile (with the same direction over the FPSO draft),
- an EOF decomposition (Empirical Orthogonal Function, also know as PCA Principal Component Analysis).

The latter method basically consists in a linear transformation of the data to a new coordinate system such that the greatest variance by any projection of the data comes to lie on the first coordinate / EOF mode, the second variance on the second EOF mode etc.

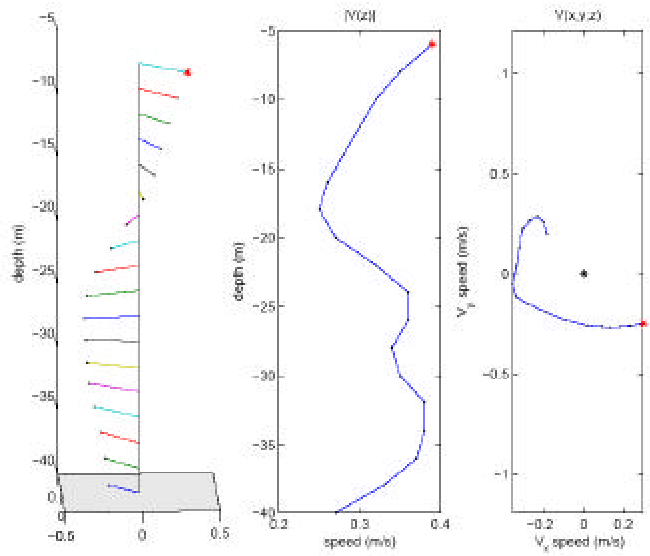

Fig. 7. Example of measured sub-surface current profiles.

Although the use of an EOF decomposition with 3 modes would have minimised the error, it is eventually the uniform profile that has been considered for the rest of the study, as the use of more complicated current profiles is laborious in standard hydrodynamic and structural models. Unfortunately, this assumption adds some uncertainty to the analysis.

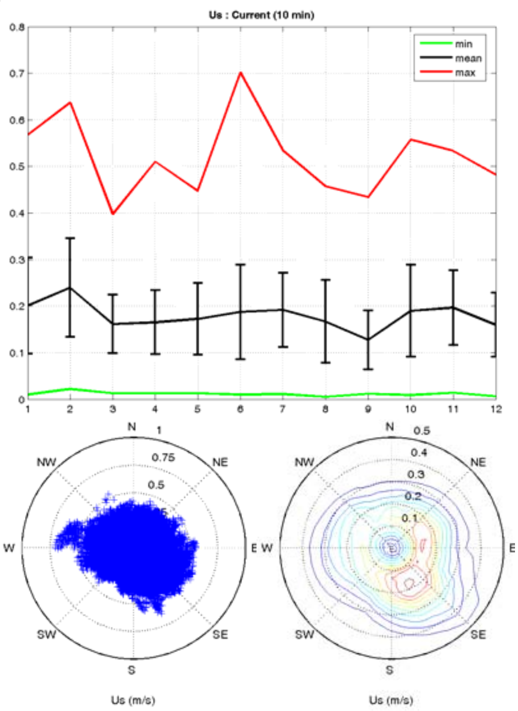

Fig. 8. Statistics of current parameters.

DEPENDENCY ANALYSIS AND JOINT PROBABILITIES

The main objective of this task was the analysis of the dependence between metocean variables, in order to derive their joint distribution models.
If two variables X_1 & X_2 have marginal probability density functions (pdf) $f_1(x_1)$ & $f_2(x_2)$, their joint pdf is:

- $f_{1,2}(x_1,x_2) = f_1(x_1) * f_2(x_2)$, if they are independent
- $f_{1,2}(x_1,x_2) = f_1(x_1) * f_{2/1}(x_2,x_1)$, where $f_{2/1}(x_2|x_1)$ is the conditional distribution of X_1 vs. X_2 , if they are dependent

The first step of the analysis was to describe the marginal pdf of the "main" variables i.e. the variables representative of the intensity of each metocean component: wave system significant heights Hs_i, wind and current speeds Vm and Us. Their empirical distributions were fitted by probability distribution models, Gumbel and Weibull, using a least square method that minimises the error between the empirical cumulative distribution and the fitted model. The analysis highlighted the need for two types of marginal distribution models, one giving the best fit of the bulk of the distribution and another for adjusting the highest values.

The second step was to analyse the dependency between variables, using three methods described in Nerzic & Frelin (2006):

- Statistical test of significance of correlation: Bravais-Pearson and Spearman
- Scatter plot between two variables (X_1 and X_2) and analysis of main conditional statistical parameters (mean and standard deviation) of one variable (X_2) vs. the other (X_1).
- Principal Component Analysis (PCA) that allows for the analysis of dependency between more than 2 variables.

The dependency analysis resulted in following conclusions:

- Independence of sub-surface currents vs. wind and sea states.
- Independence between sea state systems
- Independence of wind and swell systems
- Dependence of wind and wind sea (intensity and direction)
- Dependence of Hs and Tp in every sea state system.

Finally, joint distributions were derived. In case of dependency, conditional distributions were built using standard models such as Weibull or Log-Normal. A few examples are illustrated in Figs. 9 ~ 10.

Similar analyses were carried out for each directional sector and the derivation of joint directional distributions stressed the lack of suitable models for such application.
To overcome the difficulty, P. Orsero (Compiègne University of Technologies, UTC) has proposed an analytical model of directional directions: it enables to achieve a complete Inverse-FORM computation, although it works mainly because the design points are close to the dominant directions of the metocean variables, which means that the possible issue of extrapolating to “extremes” the directional distributions was not raised.
As an example for current, UTC empirical solution consisted in fitting, by least squares method, a simple Gaussian model to the current direction probability distribution function: this model is actually the combination of a constant uniform pdf and 3 normal Gauss pdfs.
The current velocity distribution is then defined as conditional to the current direction: typically, a Gumbel model is adjusted to each current speed pdf per directional sector. The empirical directional Gumbel parameters are numerically adjusted using the same analytical representation as for the current direction pdf, i.e., a combination of a constant uniform pdf and 3 normal Gauss pdfs.

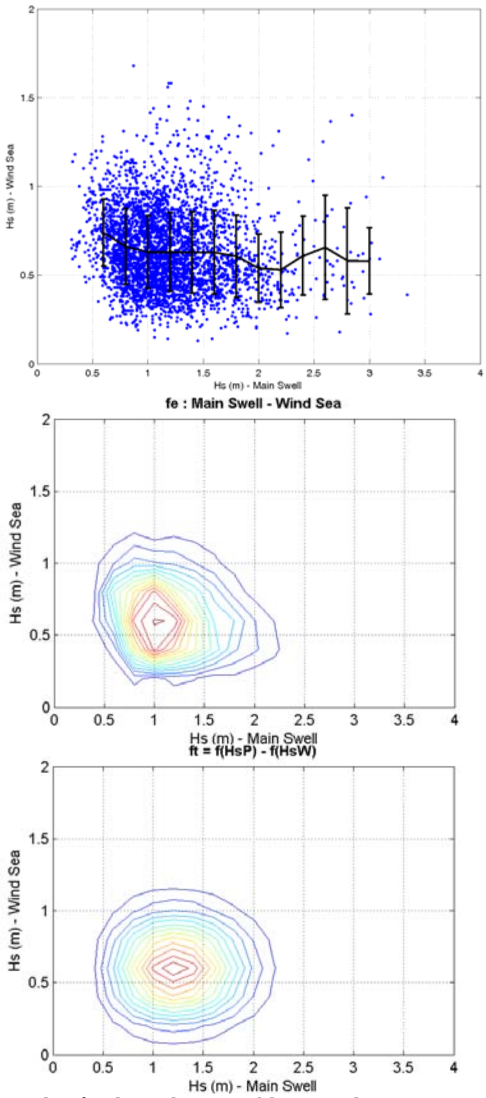

Fig. 9. Example of independent variables: Wind Sea – Main Swell Scatter-plot, empirical joint distribution, model joint distribution.

As a consequence, a joint distribution of current speed and current direction is available in the form:

$$p_{U,\Theta}(u,\theta_u) = p_{U|\Theta}(u,\theta_u).\ p_{\Theta}(\theta_u)$$

A fairly similar methodology has been applied to wind direction and wind speed conditional to direction.
On the other hand, due to the narrowness of the main swell direction distribution, the swell wave height has been assumed as independent from swell direction. The swell direction distribution is taken zero out of a narrow angular sector around the dominant direction.

JOINT EXTREMES

"Corrective extremal factors"

In the Inverse-FORM methodology, the distribution model shall be representative of the global distribution of each variable, including the distribution of high values (tail distribution). The extreme contours shall be able to represent extremes of a given variable associated to "normal" conditions of another variable. Furthermore, the extreme contours shall be consistent with the extremes calculated for each variable, which may lead to some difficulties in the application of a standard Inverse-FORM methodology, because the extremes are often calculated on the basis of distributions of peak values (Peak over Threshold method), or distributions of maximal values over given periods of time (e.g. extrapolation of annual maxima).

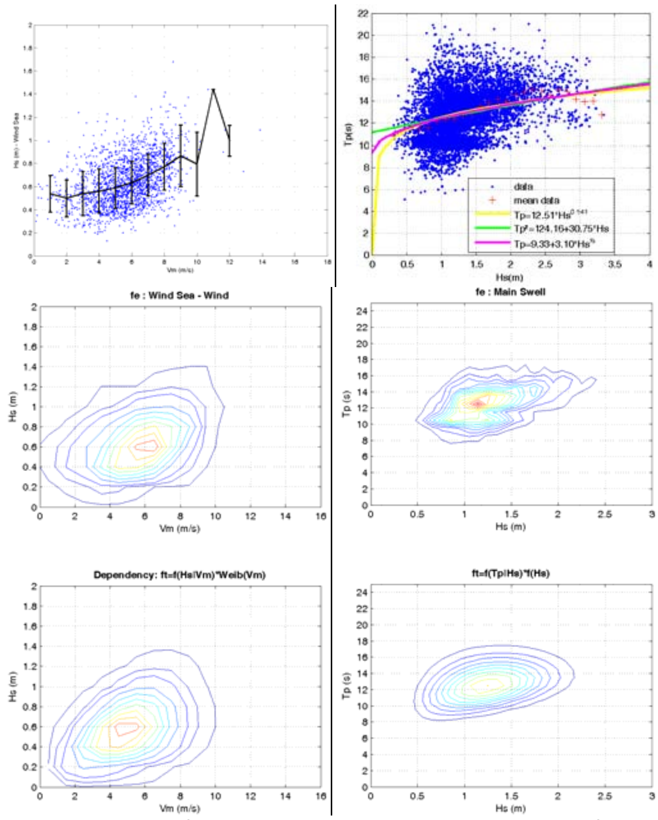

Fig. 10. Examples of dependent variables Wind Sea – Wind (left) and Hs – Tp (right). Scatter-plot, empirical joint distribution, model joint distribution.

To overcome these difficulties, the following practical/empirical methodology has been applied:

1) Two types of marginal distribution models of main variables (Hsi, Vm, Us) were considered to lead to a good representation of the global distribution but also of the severe conditions, because this is required for the derivation of extreme contours:

- the best fits of the bulk of the data cumulative distribution function (cdf), F(X)
- the fits of the 10% highest values, i.e. the tail of the distribution, $F_{10\%}(X)$, for modelling of severe environmental conditions. It should be noted that the selection of a 10% threshold is somewhat arbitrary.

For each main variable, a new distribution model was set up (cf. Figure 11) so that,

if x_t defined as $F(X<x_t)= F_{10\%}(X<x_t)$

$\bar{F}$ $(X<x)=F(X<x),\ x< x_t$

$\bar{F}$ $(X<x)=F_{10\%}(X<x),\ x>= x_t$

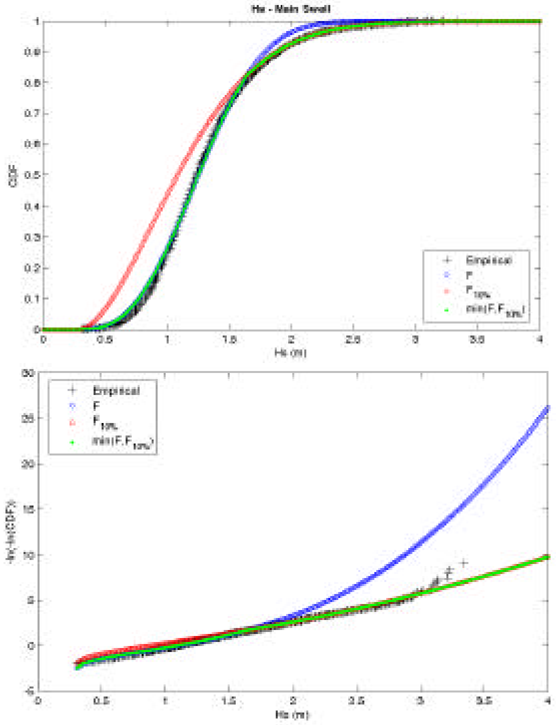

Fig. 11. New distribution model of Hs (Main Swell).

2) A "corrective extremal factor" was associated to each main variable, so that the extremes calculated from the distribution $\underline{F}$ coincide with the considered return value calculated for each variable separately:
If F(X) is the revised distribution of the variable X, if the 100-y extreme (for instance) is x_{100}, and if the number of events is N (e.g. 100*365.25*8 for Hs at a time step of 3 hours), then a corrective factor k is applied so that

$$F(X > x_{100}) = 1 - 1/(k.\ N)$$

this leads to a correction of the cumulative distribution F(X) to

$$F_c(X>x)=1-k(1-F(X>x)) \text{ or}$$

$$F_c(X<x)=kF(X<x)$$

Defined like this, Fc is no more a cumulative distribution, but over the levels we are interested in, k could be supposed to be x dependent and to tend to 1.
When the time steps of two variables are not the same (e.g Hs at time steps of 3 hours and Vm at time steps of 10 minutes), the cdf must be adapted in order to get extreme contours that are consistent with extremes calculated on different time bases.
If X1 is the variable defined on the longer basis, say a duration D1, a new set of variables is derived from the second variable: X2m = max (X2 over D1) then, a new distribution model of X2m is built, and the same approach as above is applied.

3) Then, the standard I-FORM methodology can be applied on the basis of the 2 cdf, Fc1(X1) and Fc2(X2). With the methodology presented above, the two cdf were built to get conservative contours consistent with extreme values (at least the 100-y contour with the 100-y extremes).

Inverse-FORM contours

The Inverse First Order Reliability Method allows to calculate extreme environmental contours for a given return period (i.e. for a probability of non-exceedance p, related to the specified return period) from the joint distributions of several variables. Contours are then searched for the points which maximize some response function. An introduction to the method is given in Winterstein et al. (1993, 1995).
In the space of the standard normal variables, the extreme environmental contour with a p-probability is the hypersphere (a circle in 2D) with a radius $b = \Phi^{-1}(1-p)$, where ? is the Gauss function.
The standard normal variables are calculated by the Rosenblatt transformation: if X is a physical variable, the standard normal variable is $U = \Phi^{-1}(F_x(X))$, where F is the marginal distribution of X.
The variables on the contour of p-probability in the standard normal space are $U_c, \sum_i u_{ci}^2 = b^2$. To get the values of these variables in the physical space, the inverse Rosenblatt transformation is applied: $X_c = F_x^{-1}(\Phi(U_c))$.

Based on the comprehensive analyses described in the previous sections, the Inverse-FORM methodology was applied. This led to joint models for several couples of variables. Some Inverse-FORM contours resulting from this analysis are presented in the following Figures 12 and 13.

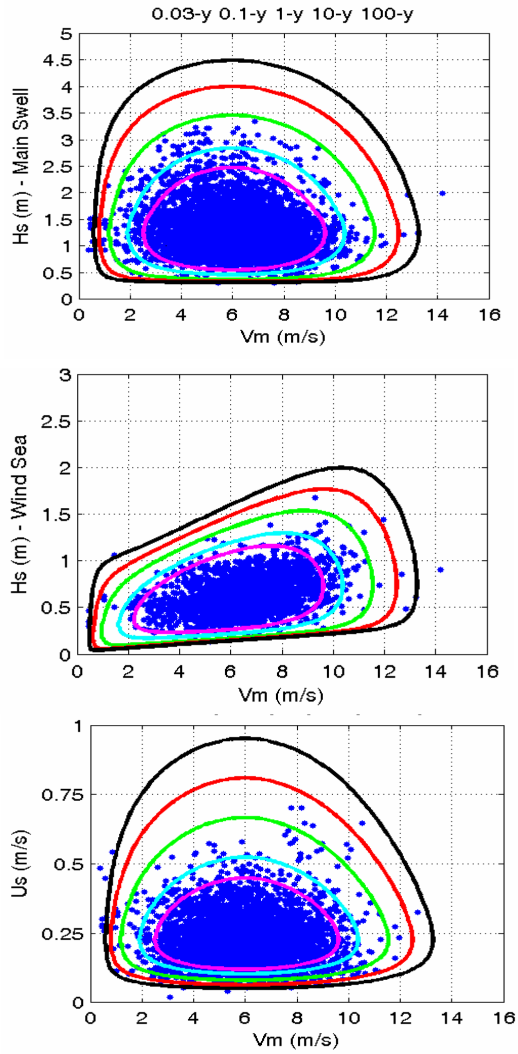

Fig. 12. Examples of Inverse-FORM contours for Hs Main Swell, Hs Wind Sea and Current with Wind.

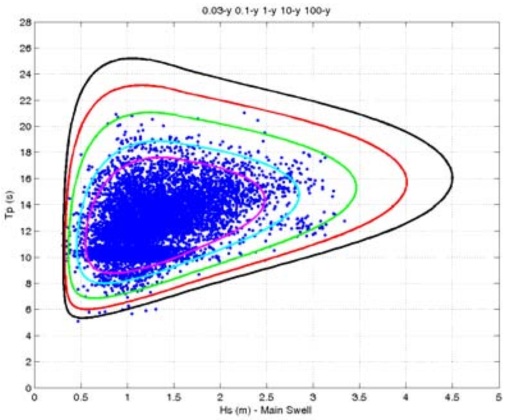

Fig. 13. Examples of Inverse-FORM contours Hs-Tp Main Swell.

LIMITATIONS

This study has led to joint extreme criteria that can be used for design. However, some limitations in the method and some uncertainties must be stressed. In particular, squall winds were excluded from the study and currents representation was simplified. But the main difficulties came from the application of I-FORM methodology, in particular on following topics:

- The necessity to use two distribution models to fit the empirical data, one for the bulk of the distribution and another for adjusting the highest values.
- The necessity to use corrective extremal factors in the distribution models, in order to remain consistent with extreme values.
- The extrapolation to 100 years from only 3 years of data and the choice of the law $F_{10\%}$
- The potential influence of seasonal dependency between parameters.
- The method to get directional contours, which is still being debated.
- The need to simplify the contours for design purposes
- Etc.

In addition, we must emphasize the fact that the I-FORM methodology was applied to pairs of variables only. It must be stressed that it is more difficult to deal with multivariate statistics in higher dimensions than the two considered in this study, particularly when the variables are dependent.

CONCLUSIONS

The example presented in this paper shows the application of a robust methodology to derive joint extremes of metocean parameters (wave, wind, current) for use in design of offshore structures. The methodology can be transposed to any offshore location provided that simultaneous records of wave, wind and current are available. However, the methodology can still be improved and the authors and project partners are currently discussing for further developments.

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