# ANALYSIS AND MODELLING OF TIME SERIES OF SURFACE WIND SPEED AND DIRECTION

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#### **ABSTRACT**

This work presents a climatological study of winds at a particular site and proposes a simple model to simulate data time series of horizontal surface wind. The model relies on three major hypotheses: (i) speed and direction are treated as independent variables; (ii) wind can be expressed as the sum of a deterministic component (including trend and cyclical variations) plus a probabilistic or stochastic component; and (iii) both components (deterministic and stochastic) are independent. An hourly based 7 year wind series measured at the meteorological station in Almazora, located at the Spanish Mediterranean coast, is analyzed. Only the deterministic component of this series is simulated in accordance with the model proposed. Results indicate that the model is especially suitable for predicting wind at sites in which some cyclic pattern is expected. Copyright © 1999 Royal Meteorological Society.

KEY WORDS: surface wind; deterministic model; time series; wind speed and direction; harmonic analysis; Mediterranean coast

#### 1. INTRODUCTION

Time series analyses of meteorological data have an increasing interest in many fields. Series are especially interesting for a better understanding of atmospheric phenomena in order to model them, to determine the climate of a geographical area, or to forecast possibilities of occurrence of some extreme situations. Therefore, the analysis of meteorological data series is useful in fields such as agriculture, air and sea traffic control, structural engineering calculations, global change studies, solar and wind resources estimation, etc.

Up to the present, this analysis has been carried out from several points of view: (i) simple characterization of the statistical distribution of the data series; (ii) harmonic analysis, modeling from the harmonic analysis; (iii) fitting to a model or probabilistic process such as AR, ARMA or ARIMA; (iv) adjustment to a relatively simple mathematical expression depending on physical magnitudes related to the data series; and (v) models that include differential equations establishing relations between the variables considered.

The aim of this paper is to present an approximation to the time series of wind speed and direction (treated as separated series), mainly studied by means of harmonic analysis, in order to identify trend (if any) and cyclic components. The final goal is to model wind data series, to reproduce the whole of the observed series and predict future values. Some particular applications are:

(i) To obtain estimations of wind speed and direction at a location where there are historical observations, but current measurements are not available. These estimations could be used as input data for a diagnostic model of air pollutants dispersion in real time.

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- (ii) To reduce the volume of data, because the information is summarized in one (or several) mathematical formulae (Coronas and Baldasano, 1984; Phillips, 1984).
- (iii) To check wind measurements obtained at the meteorological stations, in order to detect when observed data abnormally disagree from the expected behavior, hence making a recalibration of sensors necessary.

The proposed model does not directly depend on physical or geographical variables. Due to the complexity of factors and mechanisms that influence and steer the wind, which is highly variable both in space and time, the study has focused on getting a statistical model that describes the time variation of wind values in accordance with some cyclic patterns. This statistical model uses as input only the available wind data and assumes that future behavior should be the same as the one measured in the past. For this reason, the proposed model, apart from the suitability of the fitting obtained, is only valid for the site (meteorological station) to which the wind data series belong.

As an example of application, the methodology is applied to a 7 year wind data series of the surface meteorological station of Almazora (Castellón, Spain).

#### 2. BACKGROUND

Some other works (Ishida, 1990; Brett and Tuller, 1991; Frisch *et al.*, 1991; Gavaldà *et al.*, 1992; Schumann, 1992) tried to characterize the cyclic variations of wind speed and direction time series (not only to ground level data) by spectral analysis. Wickle *et al.* (1995) used the information given by the analysis to model the deterministic component of the series. Most of the authors filter the data to emphasize a particular range of frequencies. In this study, the authors work instead with hourly based series in search of all sorts of available cycles of the underlying process, only limited by the discretization of data.

Regarding the stochastic component of the series, Wickle *et al.* (1995) modeled this part by means of an autoregressive AR(1) process. Some other authors (Brown *et al.*, 1984; Breckling, 1989; Daniel and Chen, 1991; Fisher and Lee, 1994) take from the beginning the data as a realization of a stationary random process (AR, ARMA or ARIMA) or transform data to remove non-stationarity before modeling.

## 3. METHODOLOGY

The first decision taken at the beginning of this work was whether to deal either with a wind module and direction, assumed to be two different and independent time series, or with the components of the horizontal wind vector. One of the advantages of the former is that this is the usual way to record wind data by the traditional sensors (anemometer and vane). Furthermore, this way is more intuitive and it is easier to interpret speed and direction separately than with horizontal components. On the contrary, circular variables statistics (Mardia, 1972) are more complex than linear magnitudes statistics. However, because of its advantages, the first option was chosen.

Following the traditional approach, the model built for both separated series describes the variation of the observed variable as composed of a deterministic component (fully predictable) and a stochastic component (partially predictable). Trend and cyclic variations are included in the first component. Remaining irregular fluctuations are included in the second one. In this work, focus will just be on the deterministic component. Regarding the stochastic component, only stationarity is checked.

Figure 1 shows the scheme of the process followed in this work to analyze and model the wind data series. Previous steps include calculating some basic statistical parameters for both wind speed and direction and obtaining the distribution function from the full series of observations. The distribution function applied to wind speed data is widely used in wind energy assessment studies because it is not necessary to know when the different speeds occur, but the frequency of occurrence and whether they are

high enough. This in fact, (i) represents a static model that cannot describe the time evolution of wind and (ii) assumes that the observations are independent (although they are actually correlated), which implies that variances and confidence intervals of estimators calculated are erroneous. To surmount these difficulties, a dynamic model based on the statistical theory of time series analysis is needed.

Secondly, calculation of autocorrelation coefficients and different data plots allow us to identify patterns, cyclic or not, observed in the time series. Application of Fourier analysis (Jenkins and Watts, 1968) allows the identification of the cyclic patterns (i.e. the frequencies corresponding to the main

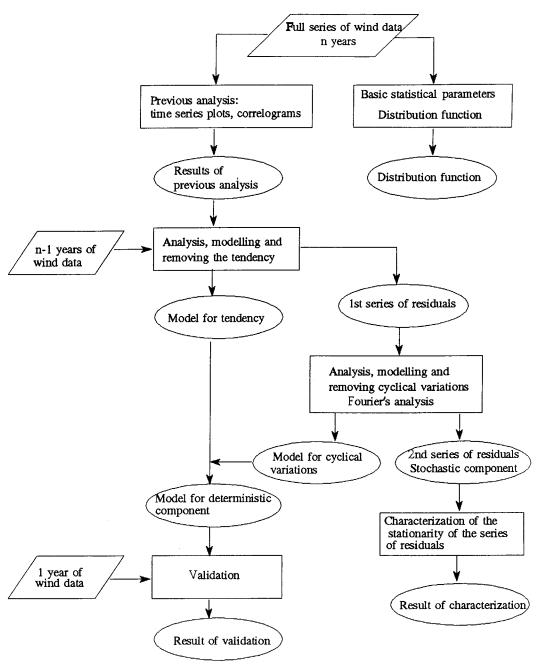


Figure 1. Scheme of the process followed in this work to analyse and model wind data series



Figure 2. Location of the meteorological station in Almazora (Castellón, Spain)

harmonics) in the individual annual series and in the full series. Relevant frequencies to be considered in the modeling should be the same for the whole set of observations and for the annual subsets.

Then, the authors try to model the deterministic component of the series, studying the speed and the direction separately; first the trend, if any, and after removing it, the cyclic variations. One unique expression for the cyclic variations (valid for all the years, as no cycle longer than 1 year is expected) is extracted from the previously obtained information, whose coefficients are fitted to the series of observations, except for 1 year of data, which is reserved to validate the model. The model obtained (trend + cyclic variations) estimates the most probable value of wind for any time at the site where the data series were measured. Finally, if the series calculated by the model is subtracted from the original data series, a series of residuals is obtained, the stationarity of which is analyzed.

### 4. EXAMPLE OF APPLICATION: ALMAZORA

Data used in this study consists of 7 years of hourly wind observations (61368 records) provided by the Instituto Nacional de Meteorología (INM) measured at Almazora (Castellón, Spain). Wind speed was measured as an hourly average in m s<sup>-1</sup> with a resolution of tenths, and dominant wind direction during every hour expressed as the central angle (°) of the sector (among 16 possible sectors). Measurement period is from January 1, 1983 to December 31, 1989. Sensors are at the height of 10 m asl.

Almazora town is located at 5 km from the Mediterranean shoreline (see Figure 2). It is in the middle of a great sedimentary plain (Castellón Plain) and close to the Mijares River. Its co-ordinates are 0°3′W longitude, 39°56′N latitude and 32 m altitude. The nearest mountains are 10 km away from the town: a coastal mountain range oriented NE–SW and heights up to 1000 m asl, and another range located inland, whose major orientation is NW–SE reaching 2000 m altitude. The Mijares River follows a NW–SE path down to the Castellón Plain.

Performance of the proposed model highly depends on the quality of the wind data series used in its development. Among the factors that influence the quality of data, the following can be mentioned: average period of measurements (frequency), percentage of useful available data over the potential data that could have been measured, and the quality of sensors. Furthermore, data show good agreement with wind climatology expected for that site as stated in the literature (Font Tullot, 1983; Bosque and Vilà, 1989, 1992; Carreras, 1992). Although the length of the series is shorter than desirable, the percentage of useful data is 100% of the total available data.

#### 4.1. Data characterization

Main winds blow from the second and fourth quadrants alternately throughout the year. Prevailing wind directions are NW and WNW, followed by ESE, E, SSE and SE, due to the channeling induced by the Mijares valley and orography. These six directions comprise 64.4% of the observations. Wind speed values  $< 1 \text{ m s}^{-1}$  are below the anemometer threshold, and are considered as calms. They represent 11.8% of the whole of available data.

Daily wind variation is very clear, especially from March through to September. It can be seen, for instance, in Figure 3(a), how the hourly mean speed during August is at a minimum at 21:00 h solar time (ST), increases to reach the maximum at 14:00 h ST, and then decreases. Simultaneously, the dominant

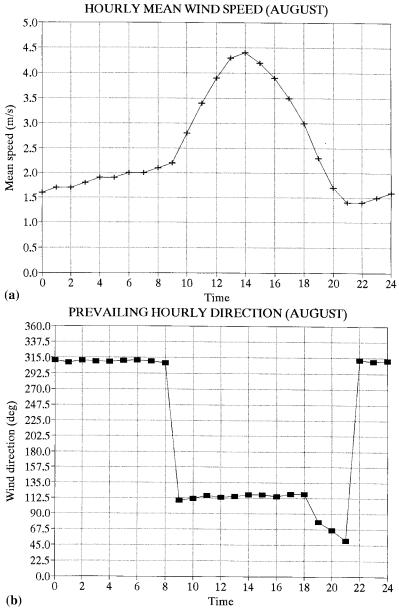


Figure 3. Hourly mean wind speed (a) and most probable direction (b) during the month of August. In (a), calm is considered as a 0.5 m s<sup>-1</sup> wind speed

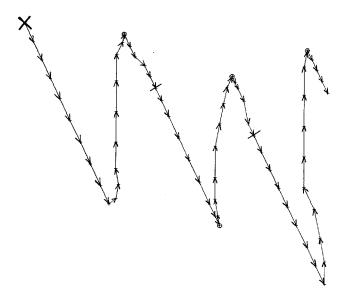


Figure 4. Representation of the hourly wind vector throughout 3 consecutive days from Almazora data series. Consecutive vectors are plotted one after the other. The series starts at the upper left corner (X) following the direction marked by the arrows. The change of day is indicated by a line perpendicular to vectors. Calms are represented by a circle. Vector length is proportional to wind speed

direction is ESE (112.5°) in the second quadrant, from 9:00 h ST until 18:00 h ST. Then it changes to be NNW (315°) in the fourth quadrant, from 22:00 h ST until 8:00 h ST (next day) (see Figure 3(b) and Figure 4). The same mean pattern is observed during most of the year, although the maximum speed and the time vary when the second quadrant winds are dominating. There is strong evidence that wind speed and direction are strongly correlated, as expected with a land—sea breeze situation.

Finally, Figure 5(a) and (b) show the annual and monthly mean speeds for the whole period of record. There is a clear decreasing trend, probably due to the growth of the number of buildings around the meteorological station through the years. These buildings belong to an industrial estate and their heights are lower than the anemometer height. The nearest buildings are about 30 m away from the mast. Nowadays, there are very few free building plots, and the surroundings of the meteorological station can be considered as an isotrope.

The presence of calms leads to the appearance of gaps in the speed and direction data series, which makes the calculation of the statistical properties of the series and the modeling difficult. To solve it, the authors assumed that a calm had an associated wind speed of  $0.5 \text{ m s}^{-1}$ . Taking into account this hypothesis, mean wind speed from the whole data series is  $2.7 \text{ m s}^{-1}$  and the standard deviation (S.D.) is  $1.8 \text{ m s}^{-1}$ . The histogram of the speed series appears in Figure 6, with data grouped in intervals of  $1 \text{ m s}^{-1}$ . It can be seen that the distribution of frequencies is unimodal and asymmetric, which is in agreement with the Weibull distribution:

$$f(v) = \frac{k}{A} \left(\frac{v}{A}\right)^{k-1} \exp(-(v/A)^k),\tag{1}$$

with A = 3.3 m s<sup>-1</sup> and k = 1.5, found by least-square fitting of the cumulative distribution function. The problem of calms in the case of wind direction is solved by linear interpolation between valid values. However, the direction value that replaces the calm in the series is not the one calculated as before, but the central direction corresponding to the sector to which the calculated direction belongs. In this way, coherence with the rest of the series is kept. In this case, the frequency distribution is bimodal, where the most frequent sectors are ESE (112.5°) and NW (315°) (Figure 7). Since the gap between both modes is 202.5°, the authors have considered this bimodal distribution as two unimodal distributions separated

by 180° (Mardia, 1972). Taking into account that wind direction is a circular variable and that the frequency distribution is bimodal, the mean direction calculated for the first modal component is 121.9°, while for the second one it is 301.9°. For both, the S.D. is 42.1°. A theoretical bimodal Von Mises' distribution has been fitted to the direction data series, making  $\mu_{\theta} = 121.9^{\circ}$  and calculating  $\lambda$  and  $\kappa$  by means of minimizing graphically the squared differences between the theoretical distribution and the frequency one, i.e.

$$g(\theta) = \frac{1}{2\pi I_0(\kappa)} \left[ \lambda \exp(\kappa \cos(\theta - \mu_\theta)) + (1 - \lambda) \exp(-\kappa \cos(\theta - \mu_\theta)) \right], \tag{2}$$

with  $\kappa=3$  and  $\lambda=0.36$ ;  $I_0(\kappa)$  is the zero-order modified Bessel function.

Comparing observed frequencies with those obtained by Equations (1) and (2), by using the Pearson  $\chi^2$  test, the hypotheses that these equations describe the observed wind speed and direction, respectively, are fulfilled.

MEAN WIND SPEED AND STANDARD DEVIATION

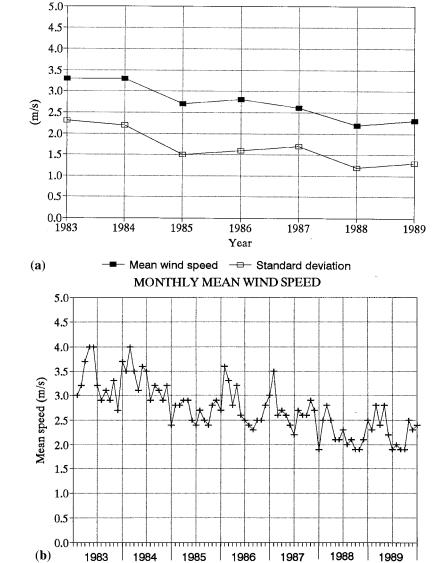


Figure 5. Annual (a) and monthly (b) mean wind speed from January 1983 through to December 1989

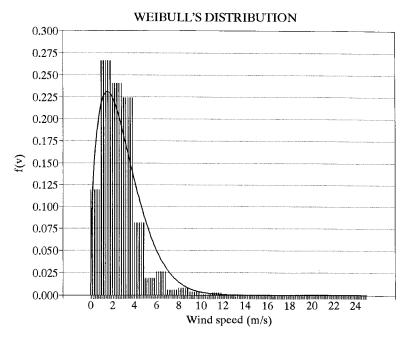


Figure 6. Histogram of wind speed and Weibull distribution that fits the data

Autocorrelation coefficients plots (correlograms) of speed and direction series are shown in Figure 8. The calculation of the line spectrum and the autocorrelation function for the direction series implies that the meteorological wind angle  $\theta$  as a complex number, in which the x-component of the unit vector with angle  $\theta$  is the real part and the y-component, the imaginary part. The transformation used is the following:

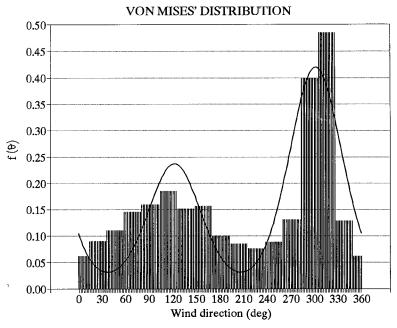


Figure 7. Histogram of wind direction and Von Mises distribution that fits the data

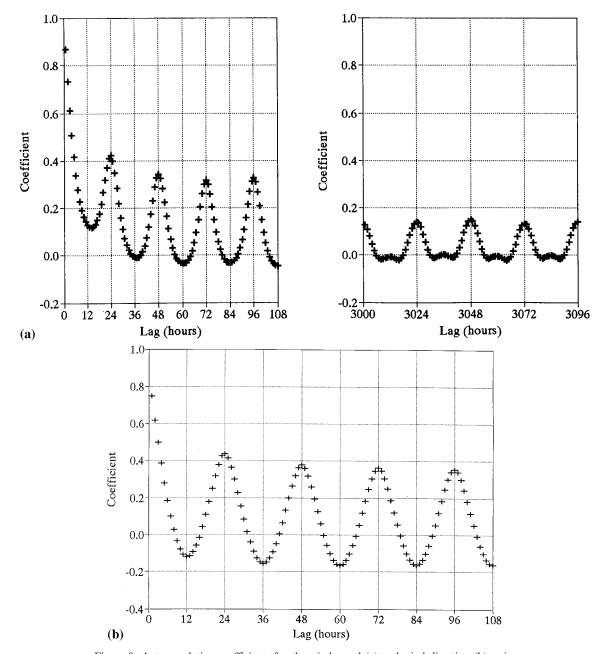


Figure 8. Autocorrelation coefficients for the wind speed (a) and wind direction (b) series

$$Re(\theta) \equiv \theta_x = -\sin \theta,$$

$$Im(\theta) \equiv \theta_y = -\cos \theta.$$
(3)

Then, the autocorrelation function and the line spectrum are computed using the discrete Fourier transform, the former supported on the autocorrelation theorem (Press et al., 1989).

First autocorrelation coefficients are large, which indicates the existence of an important short-term correlation. This correlation is usually due to short-span processes of a stochastic nature. After the 12th coefficient, correlograms for both speed and direction have a sinusoidal path and a 24-h period. This

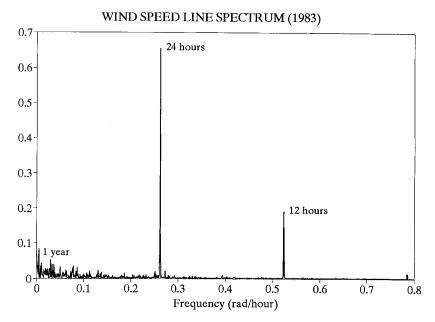


Figure 9. Line spectrum of the wind speed series (1983)

cyclic behaviour is also observed for high time lag values, which proves that the oscillation is well-established and deterministic. In the case of the wind speed, a cycle with a 12-h period is also observed.

Harmonic analysis applied to wind speed and direction series shows that the main frequencies correspond to the 1-year, 24-h and 12-h periods (Figures 9 and 10). The same cycles have also been observed by other authors in surface wind studies (Brett and Tuller, 1991; Gavaldà *et al.*, 1992). The 1-year and 24-h periods are the natural earth cycles. The 12-h period for wind speed series is well-defined and corresponds to the daytime and night-time maxima due to the full development of the land—sea

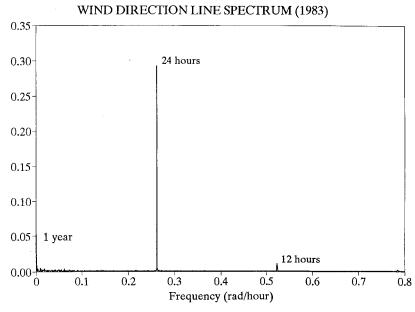


Figure 10. Line spectrum of the wind direction series (1983)

Table I. Verification and validation (bold) for the trend term of the wind speed model

t (year)	1983	1984	1985	1986	1987	1988	1989
Mean speed observed (m s <sup>-1</sup> )		3.3	2.7	2.8	2.6	2.2	2.3
Mean speed calculated (m s <sup>-1</sup> )		3.4	3.1	2.9	2.7	2.5	2.3

Table II. Model coefficients  $c_j$  for the cyclical term of the wind speed series (Equation (6))

Period	j	Frequency (rad h <sup>-1</sup> )	$c_j^{\text{a}}$ (m s <sup>-1</sup> )	
1 year	1	0.0007	0.086-0.061i	
24 h	2	0.2618	-0.389 + 0.090i	
12 h	3	0.5236	0.087 - 0.232i	

<sup>&</sup>lt;sup>a</sup>  $i = \sqrt{-1}$ .

breezes. In the case of wind direction, this 12-h period does not have this physical meaning and actually the peak in the line spectrum is much lower.

## 4.2. Proposed model

To calibrate the model, just the first 6 years of the data series are used (1983–1988). The last year (1989) is reserved for validation purposes.

In the case of wind speed,  $v_t$ , the proposed model is the following:

$$v_{t} = m_{v,t} + S_{v,t} + z_{v,t}, \tag{4}$$

where  $m_{v,t}$  is the trend term,  $S_{v,t}$  is the seasonal or cyclical term, and  $z_{v,t}$  is the stochastisc component. The trend has been calculated by fitting the mean annual speed values from 1983 to 1988 to a straight line. The expression obtained is:

$$m_{v,t} = 21 - 0.21(t - 1900),$$
 (5)

where t is the year. Regression coefficient is rather low  $(r^2 = 0.88)$ , but the model is able to predict the mean speed for 1989 (Table I). In order to obtain valid predictions for more than 1–2 years beyond the period of record, it is necessary to recalculate the model coefficients using the new available data.

The cyclical term,  $S_{v,t}$  is obtained through harmonic analysis as follows:

$$S_{v,t} = \sum_{j} \left[ c_j \exp(i\omega_j t) + c_{-j} \exp(-i\omega_j t) \right], \tag{6}$$

where  $\omega_j$  with j = 1, 2, 3, are the three main frequencies, t is time in h,  $c_j$ ,  $c_{-j}$  are indeterminate coefficients,  $\exp(i\omega_j t) = \cos(\omega_j t) + i\sin(\omega_j t)$  and i is the imaginary unit  $(\sqrt{-1})$ .

Fitting Equation (6), to the wind speed series, excluding trend, by a non-linear least-square technique, the resulting coefficients  $c_j$  are those appearing in Table II. Coefficients  $c_{-j}$  are not included in this table, since they are complex conjugates of  $c_j$ . If this fitted model is evaluated by the percentage of values of the simulated wind speed series that differ by 1 m s<sup>-1</sup> or less with regard to the actual series, one obtains the

Table III. Verification and validation (bold) for the cyclical term of the wind speed model

Year	1983	1984	1985	1986	1987	1988	1989
Percentage of values with $\Delta v \le 1 \text{ m s}^{-1}$	54.1	53.2	62.4	63.6	66.2	77.1	72.5

(Equation (0))							
Period	j	Frequency (rad h <sup>-1</sup> )	$\mathcal{C}_j^{-\mathbf{a}}$	$c_{-j}^{-a}$			
c <sub>0</sub> 1 year 24 h	0 1 2	0 0.0007 0.2618	0.135 - 0.121i 0.207 - 0.036i 0.058 - 0.351i				

Table IV. Model coefficients  $c_j$  for the cyclical term of the wind direction series (Equation (8))

values shown in Table III. Excluding the first 2 years, the rest of the years have percentages over 60%, which are acceptable if one considers that the proposed model does not take into account the stochastic component of the series. It is remarkable that the simulation improves with time. The reason is that the wind speed data dispersion is decreasing with time, in a similar way to what occurs with the annual mean speed (see Figure 5(a)).

In the case of wind direction,  $\theta_t$ , no trend was observed in the series analyzed, but a cyclical term exists similar to the one observed for the wind speed. Furthermore, the remaining fluctuations do not depend on the daily variation point, i.e. random fluctuations are additive. So, the model proposed is (remember that  $\theta$  is expressed as a complex number):

$$\theta_t = S_{\theta,t} + z_{\theta,t},\tag{7}$$

where the cyclical term,  $S_{\theta,t}$ , is expressed as follows:

$$S_{\theta,t} = c_0 + \sum_{j} \left[ c_j \exp(i\omega_j t) + c_{-j} \exp(-i\omega_j t) \right], \tag{8}$$

where  $\omega_i$  with j = 1, 2, are the two main frequencies.

The model coefficients for the deterministic seasonal term of the direction series, obtained by using a similar technique to that used for the speed series, appear in Table IV. As an evaluation of the ability of the proposed model to predict the actual wind direction series, Table V shows the percentages of the difference values between the simulated and observed series, less than or equal to one and two direction sectors. In the case of  $\Delta\theta \leq 45^{\circ}$ , all the values, except for 1984, are around 70%. This is a good result if the fact that the model only considers the deterministic component of the series is taken into account.

Finally, the study of the correlograms for the series of residuals shows that the trend and cyclical terms have been successfully removed. Furthermore, computed correlograms from subsets of the series of residuals have the same overall behavior as those for the entire series. This suggests that the autocorrelations of the underlying process are time-independent. Therefore, the remaining stochastic component can be reasonably considered as stationary (at least in a weak sense) and ergodic. Provided that stationarity and ergodicity conditions are met, the autocorrelation function may be used to make inferences about the underlying process of which the residual series is a sample (Chatfield, 1989; Brett and Tuller, 1991; Peña, 1994).

Table V. Verification and validation (bold) for the cyclical term of the wind direction model

Year	1983	1984	1985	1986	1987	1988	1989
Percentage of values with $\Delta\theta \le 22.5^{\circ}$ Percentage of values with $\Delta\theta \le 45^{\circ}$	55.1 73.4	37.7 60.6		55.8 72.0		59.9 72.3	

 $i = \sqrt{-1}$ .

#### 5. CONCLUSIONS

In principle, it is not possible to assert that wind is produced by only either deterministic or probabilistic phenomena. In accordance with Alvarez (1986), the deterministic hypothesis supports the existence of certain systematic and regular cycles whose behaviour it is intended to establish, but not the cause that originates them. That is why the validity of the model proposed (that only includes the deterministic component) is restricted to the point site of the meteorological station where the wind measurements were collected. The existence of well-defined oscillations related to natural cycles has been established. This confirms the deterministic hypothesis.

The proposed model is intended to reproduce the deterministic component of the wind data series. So, the methodology followed would be specially suitable for describing and predicting wind at sites in which some cyclic patterns are expected, such as land-sea breezes, katabatic-anabatic winds, etc. In the case studied, the deterministic component of the model is enough to produce useful results (within an acceptable error) ca. 60-70% of time, for both wind speed and direction.

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