The Probability Distribution of Wind Velocity and Direction.

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Abstract

A simple model is presented for the joint distribution of wind velocity and direction. The results of the model are compared with real data. The agreement between model and observations is striking.

Introduction

The probability distribution of wind speed is commonly assumed to be a member of the Weibull family of distributions. This assumption is empirically based and is motivated by the hope that a two parameter distribution will give a suitable model to fit data from sites which exhibit a prevailing wind direction [1]. When there is no prevailing wind direction the Rayleigh distribution, the one parameter member of the family, is generally found to give a good fit to the data.

While the Weibull distribution may give a reasonable fit to the marginal distribution of wind speeds it does not give any information on the distribution of wind directions. A more realistic model is proposed which accounts also for the directional properties of wind.

The new model is based on the assumption that the fundamental process of interest is the component of wind speed along a given direction, so in principle the general problem of fitting wind data consists of obtaining the joint density function of wind speeds and directions. The marginal distributions of wind speed and wind direction are then readily obtained by integration of this joint density function. It is obvious that the reverse method of fitting the marginal distributions first, is unlikely to lead to a realistic model as joint distributions are not generally defined from knowledge of their marginal distributions.

It has been assumed in the past [1,2] that the distribution of the component of the wind velocity for a given direction follows a normal distribution. However, where the wind has a favoured direction, wind speeds along different directions may be statistically dependent, nonetheless wind speeds along the prevailing wind direction and the direction orthogonal to it must be independent; it is this factor which permits the derivation of the joint distribution of wind speed and direction in the general case.

The simplest model for the case where there is a prevailing wind direction is when the component of wind speed along the favoured wind direction is normally distributed with non-zero mean and a given variance while the component of wind speed along a direction at right angles is independent and normally distributed with zero mean and the same variance.

The Joint Distribution of Wind Velocity and Direction

Let

V_y = Wind velocity along the predominant wind direction. It is assumed normally distributed, mean μ, variance σ^2

V_x = Wind velocity along the perpendicular to the predominant wind direction. It is assumed normally distributed, mean zero and variance σ^2.

The joint density function of V_x and V_y is

$$f_{V_xV_y}(v_x,v_y) = \frac{1}{2\pi\sigma^2} \exp\left\{-\frac{v_x^2 + (v_y-\mu)^2}{2\sigma^2}\right\}, \quad -\infty < V_x,V_y < \infty$$

To obtain the joint density of wind speed and direction one needs the transformation

$$v_x = v\cos\theta$$
$$v_y = v\sin\theta$$

with

$$v = \sqrt{(v_x^2 + v_y^2)}\ .$$

Then, the joint distribution of V and Θ becomes

$$f_{V\Theta}(v,\theta) = \frac{v}{2\pi\sigma^2} \exp\left\{-\frac{\mu^2}{2\sigma^2}\right\} \exp\left\{-\frac{v^2 - 2\mu v\sin\theta}{2\sigma^2}\right\},$$

$$0 < \theta < 2\pi, \quad 0 < v < \infty\ .$$

Integration over v gives the marginal distribution Θ

$$f_\theta(\theta) = \frac{1}{2\pi}\exp(-\tfrac{1}{2}(\frac{\mu}{\sigma})^2)\left\{1 + \frac{\mu}{\sigma}\sqrt{2\pi}\sin\theta\cdot\Phi(-\frac{\mu}{\sigma}\sin\theta) \cdot\exp\tfrac{1}{2}(\frac{\mu}{\sigma}\sin\theta)^2\right\}, \quad 0 < \theta < 2\pi$$

where

$$\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{x^2}{2}}\,dx$$

The density function of wind directions reduces to $\frac{1}{2\pi}$, i.e. the uniform distribution in (0.2π), when $\mu = 0$.

The marginal distribution of wind velocity is obtained by integrating the joint distribution over θ, that is

$$f_V(v) = \left[\frac{1}{\sigma^2}\, v\exp(-\frac{v^2}{2\sigma^2})\right] \exp(-\frac{\mu^2}{2\sigma^2})\, I_0(\frac{\mu}{\sigma^2}v), \quad 0 < v < \infty\ .$$

where $I_0(x)$ is the modified Bessel function of the first kind and order zero. It is interesting that the term in square brackets is actually the density function of the Rayleigh distribution, so the extra terms act as a correction to the Rayleigh density, this correction tends to one as μ tends to zero.

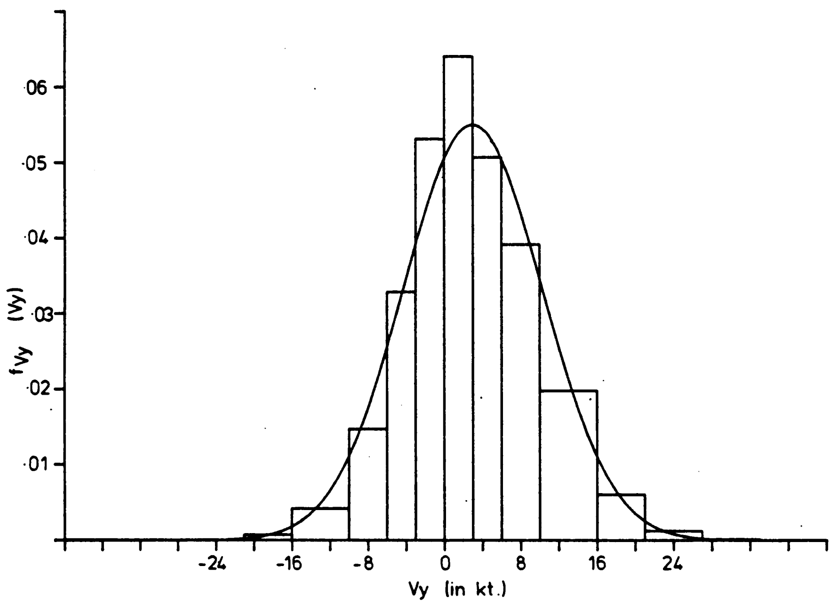

Figure 1 Probability distribution for wind speed along the predominant wind direction. The histogram of the data is compared with a fitted normal distribution, mean $\mu = 2.99$ variance $\sigma^2 = 52.7$

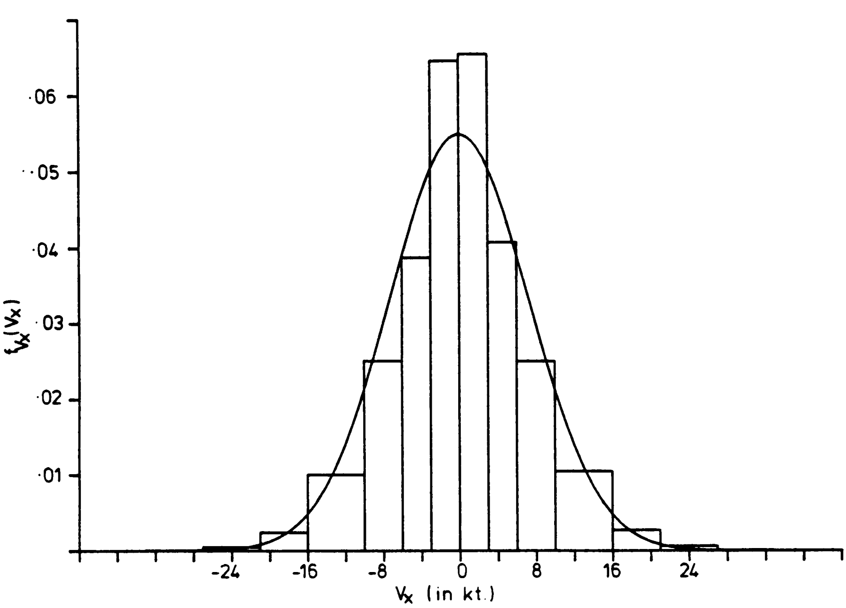

Figure 2 Probability distribution for wind speed along the perpendicular to the predominant wind direction. The histogram of the data is compared with a fitted normal distribution, mean $\mu = 0$, variance $\sigma^2 = 52.7$

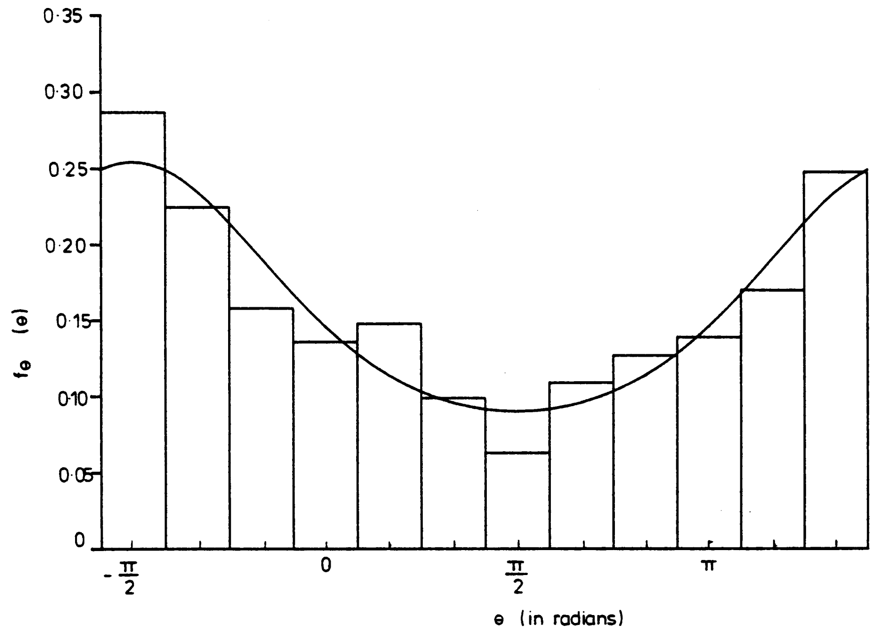

Figure 3 Angular distribution of wind. The histogram of the data is compared with the distribution derived from the theoretical model

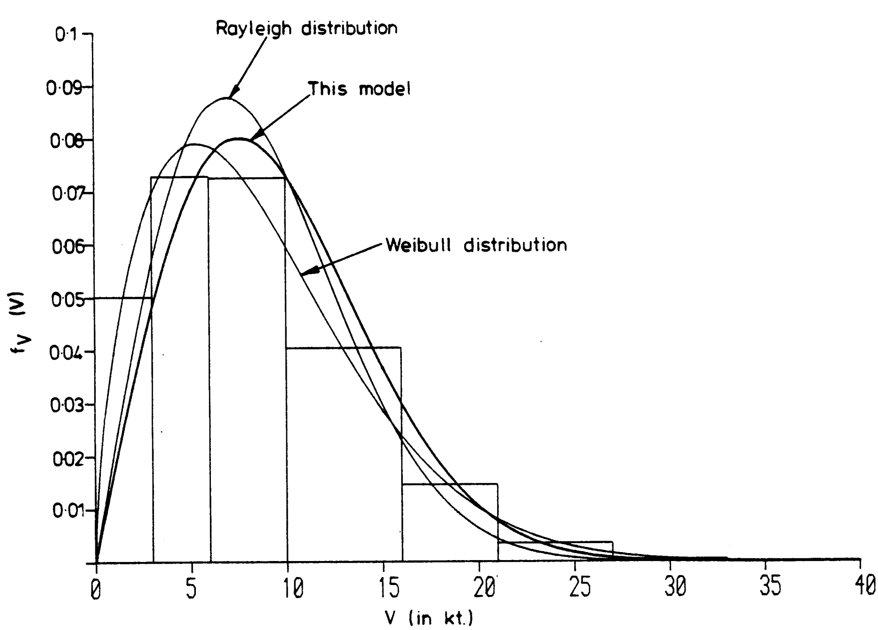

Figure 4 Distribution of wind velocity. The histogram of the data is compared with the distribution derived from the theoretical model. Fitted Rayleigh and Weibull distributions are included for comparison

Comparison with Experimental Results

Wind data from Aldergrove (Northern Ireland) [4] has been used to test the proposed model. Figures 1 and 2 show the histograms and fitted normal distributions for the predominant wind direction and its orthogonal projection. The histograms were derived from the observed distribution of wind speed and direction. It should be noted that the histogram for V_y was obtained by giving the appropriate weights to the projections to account for the larger probability of observing wind speeds along the prevailing wind direction. The original data contains a proportion of calm periods (no wind). This proportion is treated separately as a finite probability of zero wind [5].

Figures 3 and 4 show the marginal distributions of wind directions and speeds respectively. The agreement between theory and observation is excellent. It has been pointed out by the Meteorological Office [6] that due to the inertia of the anemometer the frequencies for the very low wind speeds are exaggerated at the expense of the wind speeds in the next higher range. This effect would explain the discrepancies with our model at low wind speeds.

Figure 4 also includes a comparison with the Rayleigh and Weibull distributions which were fitted to the observed data on wind speeds.

It is felt that standard statistical tests of goodness of fit are not very meaningful in this problem. The number of observations is so large that the sampling fluctuations are much smaller than the experimental errors in the data, for example, in addition to the inertia of the anemometer, the trace of the anemograph is accurate to about 1 knot [6]. Goodness of fit can then only be assessed in a subjective way. All three models give a reasonable fit to the wind speed data. However, our model is the only one which gives the complete picture with regard to directional data and the joint distribution of wind speeds and directions.

Conclusions

A very simple model, depending only on two parameters, for the joint distribution of wind speed and direction is presented. The main feature of the model is the assumption of normally distributed components of wind speed along any given direction. However, unlike existing theories, this model recognizes the fact that two independent components of wind speed lie along the direction of prevailing winds and its orthogonal direction, and also that the mean speed along the predominant direction is different from zero. This theory, in contrast with other existing models provides the joint distribution of the wind speed and direction from which both the angular distribution of winds and the distribution of wind speeds are obtained.

The derived distributions are in remarkable agreement with the observations.

In addition the simplicity of the model makes it suitable for use in the derivation of stochastic models for wind behaviour.

References

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