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LINKING PROPERTIES OF DESIGN PROBLEMS TO OPTIMAL TEAM CHARACTERISTICS

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ABSTRACT

A team with the right characteristics can exceed the sum of their individual efforts. However, a team having the wrong characteristics may perform more poorly than the sum of its individuals. Therefore, it is crucial that teams are assembled and managed properly in order to maximize performance. This work examines how the properties of a design problem can be used to select the best values for team characteristics. Two characteristics are considered: team size and interaction frequency. A computational model of design teams that has been shown to effectively emulate human team behavior is leveraged to pinpoint optimized team characteristics for solving a variety of fluid and structural design problems. The nature of each design problem is characterized with respect to local and global behavior of the design space, alignment between objective functions, and the resources allotted for solving the problem. Regression analysis is used to create equations for predicting optimized team characteristics based on problem properties. These equations, which enable the informed design of design teams based on those characteristics, describe statistically significant relationships and are found to have useful levels of accuracy. Further analysis reveals insights about how the properties of a design problem can influence a team's search for solutions.

1 INTRODUCTION

Teams with the right characteristics perform more effectively than the mere sum of the constituent team members [1]. However, teams with the wrong characteristics may function much *less* effectively than the sum of individuals in certain situations [2,3]. It is therefore crucial for a team to have the right characteristics in order to achieve maximum effectiveness. This work investigates how the properties of a design problem can be used to inform the selection of the best team characteristics for solving it. Developing this relationship requires team performance to be assessed on a variety of different design problems, with respect to a variety of different team characteristics. This quickly compounds the number of conditions that would need to be evaluated, resulting in a research study requiring an unmanageable number of participant-hours. For that reason this work simulates the performance of engineering design teams using a computational model.

A variety of definitions for the word *team* have been supplied in the literature [4–7], but two concepts are pervasive across these definitions: multi-agency (the composition of a team from two of more individuals) and communication (the ability of those individuals to share information with one another). These two concepts are central to the nature of teamwork, and investigating their relationship to team effectiveness should provide fundamental insights. This work operationalizes the concepts of multi-agency and communication by specifically investigating the impact of ***team size*** and ***frequency of interaction***. These two characteristics are of specific interest for engineering design because they help to highlight the tension that exists between breadth and depth of search in the design space. Larger teams can enhance the breadth of search for solutions, but extremely divergent search may be unnecessary for some problems [8]. More frequent interaction leads to deeper and more focused search, but it can also lead to design fixation [9]. The appropriate selection of these characteristics ensures that teams are able to diverge and converge in a way that is appropriate for the task at hand. The results of this research provide a means for selecting appropriate values for these characteristics before work begins on a design problem.

Team size plays a role in the search for solutions, but prior findings are mixed. Studies that report negative results for larger teams typically find that those teams are plagued by low efficiency and coordination issues [10–12]. In contrast, other work has shown that larger teams may benefit from concurrent team work and a greater breadth of experience and opinions [13–15]. A meta-analysis of team characteristics showed that there is a small positive relationship between team size and performance [16]. However, a more detailed analysis indicated that the relationship between team size and performance depends on the type of team – project and management teams benefit from larger team sizes, but production teams do not [16]. This aligns with other work indicating that optimal team size may be task- or at least domain-specific [17,18].

In practice, many design tasks are limited by constraints on humanpower, (e.g., a fixed number of billable hours available for a project). This raises a perennial question: should resources be concentrated within a small team, spread among many individuals in a larger team, or something in between? With respect to software development, research has shown that the answer to this question depends in part on how easily the project can be partitioned into subtasks, and whether or not significant communication overhead is necessary after partitioning [19,20]. Analysis of completed projects has shown that while increasingly complex projects demand larger teams, larger teams also tend to be less efficient [21]. This implies that there exists an optimal team size that depends on project properties. This possibility led to the creation of a theoretical model relating the optimal size of software development teams to the predicted size of the project [22].

The impact of team size has also been explored in other domains. Computational work in social choice theory has shown that smaller teams are capable of making decisions which more fairly represent the preferences of the team [23]. In addition, work on team-inspired agent-based optimization algorithms has shown that the optimal agent team size depends on the objective function [24].

The ***frequency of interaction*** is a common measure of communication within a team [25–28]. It has been shown that relationship between frequency of interaction and task performance in cross-functional teams is approximately quadratic [29]. High and low interaction frequencies result in lower performance, with a well-defined optimal interaction frequency [29]. A computational model was developed to investigate this phenomenon further, and it indicated that higher interaction frequency tended to decrease the quality of communication [30]. Further computational work demonstrated a relationship between optimal interaction frequency and project complexity [31].

With respect to design, the benefit derived from interaction generally arises from the ability of individuals in a team to explore a variety of options, but then to collaboratively focus their attention on a shrinking set of the most promising alternatives [32]. However, interaction is not always beneficial when it is allowed. Computational simulations indicate that excessively frequent interaction can be detrimental to teams, resulting in the implicit prioritization of consensus over the search for good solutions [33]. This shift in priorities within a team is similar to a psychological phenomenon known as groupthink [3], which can be harmful to decision-making teams [34]. In contrast, less frequent interaction may lead to the formation of weak ties between members of a team [33]. Weak ties can be beneficial because they facilitate the transmission of diverse perspectives between individuals [35]. Other research has examined how team interaction can be structured to make teams more resilient to change [36]. Results suggested that encouraging interaction between individuals with diverse opinions could weaken confirmation bias, thus improving performance [36]. These studies indicate the importance of selecting an appropriate level of interaction to encourage independence and exploration while still allowing for the exchange of beneficial ideas.

The methodology employed in this work is similar in some respects to that used by researchers to study the connection between problem formulation and creative ideation outcomes [37]. In that work, participants were presented individually with a conceptual design problem. Characteristics of their problem formulation process were tracked using the P-map framework [38], and the outcomes of their work were quantified with respect to the ideation effectiveness metrics developed by Shah et al. [39]. Regression analysis was then used to relate the P-map variables to those ideation metrics, providing a predictive relationship [37].

The current work also uses regression to provide predictive equations, but focuses on predicting the team characteristics that lead to the best solutions. Another difference is that the current work does not involve human studies, but instead utilizes the Cognitively-Inspired Simulated Annealing Teams (CISAT) modeling framework [33] (discussed in more detail in Section 2). This framework provides simulation capabilities for predicting the performance of human design teams. CISAT allows the user to accurately control the activity of simulated engineering design teams and also makes it possible to rapidly and efficiently evaluate the performance of large numbers of teams with different characteristics.

The primary objective of this work is to establish a relationship between design problem properties and the best team characteristics. Two cases are specifically addressed. Case 1 considers the scenario in which a manager must determine the best team size as well as the best interaction frequency for that team. This scenario is most likely in larger design firms that permit the flexibility necessary to assemble ad hoc teams. Case 2 addresses the scenario in which a design team with a fixed size must determine how frequently to interact while solving the problem at hand. This is a more likely scenario in smaller firms that have less flexibility. Several stages of work are necessary to produce predictive relationships for these two cases:

1. A set of design problems is first defined (see Section 3).
2. The properties of these design problems are then computed (see Section 4).

3. The CISAT framework is used to find the best team characteristics for solving each problem (see Section 5).
4. Regression analysis is used to define equations that allow optimal team characteristics to be predicted based on problem properties (see Section 6).

The end result of this process is a tool that can be used to inform the selection of team size and interaction frequency for solving a given design problem. Although team size and interaction frequency are the focus of this paper, the method can be used to examine and design other team characteristics that can be tested within the CISAT framework.

2 THE CISAT MODELING FRAMEWORK

The Cognitively-Inspired Simulated Annealing Teams (CISAT) modeling framework is used here in the place of human studies to evaluate the performance of engineering design teams because of the large number of conditions involved in this research (on order of 100,000 problems solved). CISAT is an agent-based platform that is intended to simulate the process that teams go through in solving engineering design problems [33]. Every CISAT agent represents a human team member. These agents utilize simulated annealing [40] constructs in their search for solutions that optimize given objective functions. CISAT agents navigate the design space by selecting and applying move operators that are defined for the given design problem.

The individual behavior of CISAT agents is based on simulated annealing [40], a stochastic optimization algorithm that has previously been used as a model for human problem-solvers and engineers [41,42]. Simulated annealing concepts are used to structure agents' search pattern so that they progressively transition from stochastic search to deterministic search. Within this framework CISAT agents are allowed to adaptively employ different search strategies [43], learn the best order in which to apply move operators to the current solution [44], and change their breadth of search according to their proximity to target design values [45]. This allows each agent to develop its own solution concept independently from other agents in the team.

When interaction occurs, agents have an opportunity to share their current solution concepts. The way in which CISAT agents interact and collaborate is structured in order to reflect behavior that has been observed in human design teams. At the beginning of every iteration, each agent probabilistically decides whether or not to interact with the other agents in its team [46]. If an agent chooses not to interact, it continues to work on its current solution. If it does choose to interact, then it evaluates the solutions currently being pursued within the team and probabilistically selects one to adopt as its own. The weights used in this selection process are initially computed according to solution quality, with higher weight being placed on higher quality solutions. Weights are then modified so that agents do not greedily pursue the solutions with highest apparent quality [8]. The weights are further modified so that an agent has a greater preference for their own solution [47,48].

The CISAT framework has been used in previous work to effectively replicate the results of a cognitive study [8,33] and to investigate the effects of operation sequencing in engineering design [49]. These prior studies used CISAT to simulate the design of truss structures with well-defined rules dictating how structures could be modified. This work uses the CISAT framework in order to quickly and efficiently evaluate the performance of large numbers of different teams on several different design problems, each of which has a rule-based description similar to the prior CISAT studies.

Without CISAT, the work presented in this paper would have been excessively time consuming and prohibitively expensive. The CISAT framework is used here to simulate the performance of 1,120 conditions, with 100 teams in each. On average, each simulated team conducts 1,250 operations during solving, and human participants perform operations at a rate of approximately 1,200 operations per hour [33]. Therefore, the simulations in this work represent more than 100,000 participant-hours of human studies.

3 DESIGN PROBLEM DEFINITIONS

The relationship between design problem properties and optimal team characteristics is studied using both fluid network and structural design problems. These problem classes are used because their solutions are dictated by dissimilar physical phenomena, guaranteeing a broad range of problem characteristics. The problems also lend themselves well to computational design since the quality of potential solutions can be readily quantified. Within each problem class, four design problems are defined, with the intent of providing a variety of different problems within the class.

Example solutions for the structural design problems are shown in Figure 1. The problem types, all trusses, include two tower-style problems with both vertical and lateral loads (Figures 1a and 1b), a single-span bridge problem (Figure 1c), and a double-span bridge problem (Figure 1d). Pin supports (which resist both vertical and horizontal translation) are denoted by a solid triangle, while roller supports (which only resist vertical translation) are denoted by a black triangle on top of two circles. Loads are denoted by arrows.

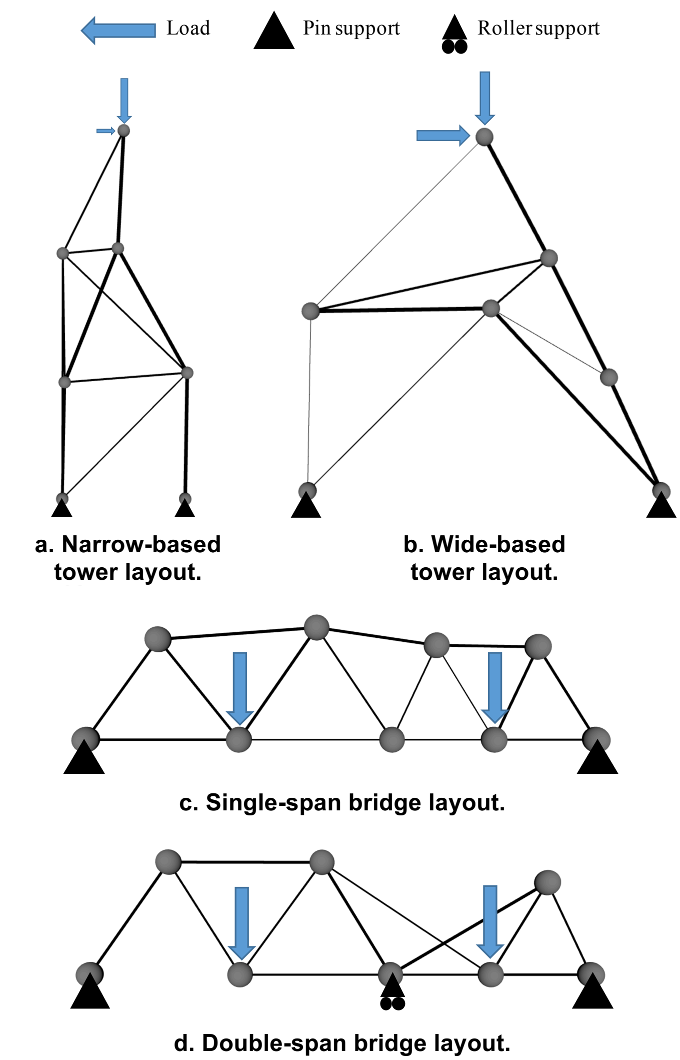


FIGURE 1. EXAMPLE SOLUTIONS TO STRUCTURAL DESIGN PROBLEMS (SHOWING REQUIRED LOADS AND SUPPORTS).

These structural design problems charge CISAT simulated teams with maximizing the factor-of-safety of their solutions while minimizing the mass. Support location and type are specified for each of the problems, and cannot be modified by CISAT agents. The location, magnitude, and direction of loads are similarly specified and immutable. CISAT agents are permitted to act upon solutions by adding and removing joints, adding and removing structural members, changing the size of members, and changing the location of joints (provided that the joints are neither supports nor loaded).

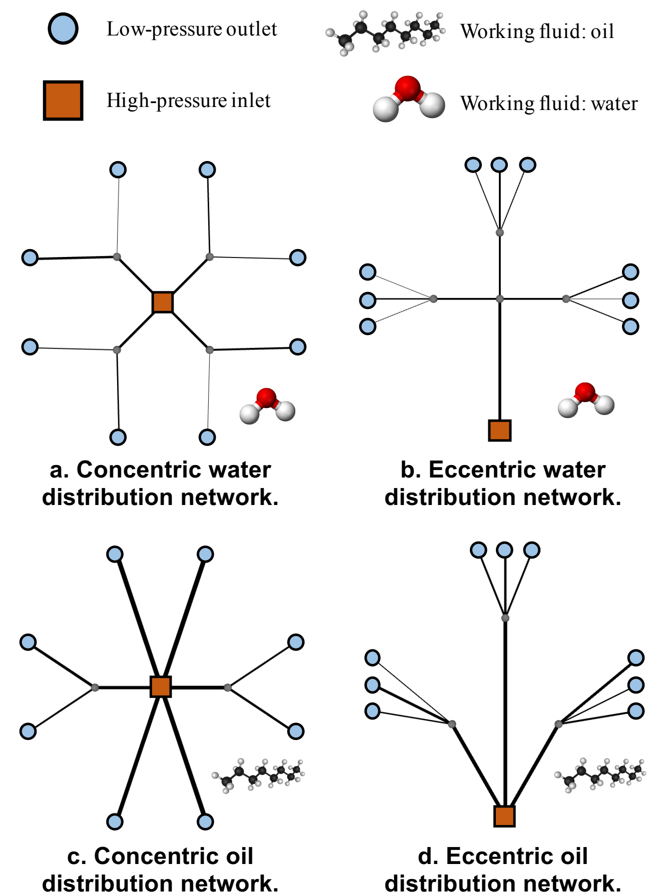


FIGURE 2. EXAMPLE SOLUTIONS TO FLUID CHANNEL DESIGN PROBLEMS (SHOWING PRESSURES AT REQUIRED INLETS AND OUTLETS).

Example solutions to the fluid network design problems are shown in Figure 2. The arrangement of inlets and outlets is specified as either concentric or eccentric. A concentric layout indicates that the high-pressure inlet is placed near the geometric center of the low-pressure outlets (Figures 2a and 2c). An eccentric layout denotes that the high-pressure inlet is placed far away from the center of the low-pressure outlets (Figures 2b and 2d). The working fluid for the network is also specified as either water (Figures 2a and 2b) or oil (Figures 2c and 2d). These fluids differ greatly in viscosity, impacting the behavior of the design problem. Laminar flow was assumed in order to afford quick closed-form evaluations of solution quality.

These fluid design problems require CISAT simulated teams to maximize the flow rate at each of the outlets while minimizing the total length of pipe used for the solution. The location and pressure of inlets and outlets are specified as part of the problem statement and cannot be changed by CISAT agents. Agents are permitted to modify solutions by adding or removing junctions, adding or removing lengths of pipe, and changing the diameter of pipes.

4 CHARACTERIZATION OF DESIGN PROBLEMS

The nature of each design problem is quantified with respect to three properties, each of which provides information that is informative for selecting team characteristics. These properties are:

1. the alignment between the objective functions of the problem (c_A), which increases the importance of search breadth (particularly if objective functions disagree);
2. the global behavior of the design space (c_G), which bears on the extent to which a team has to coordinate their search of the space; and
3. the local behavior of the design space (c_L), which can limit how efficiently an individual is able to search for local minima.

To perfectly assess these properties, it would be necessary to sample and analyze a vast number of solutions from within the design space. This presents many practical issues, but chief among them is the fact that such an extensive assessment would be only a few steps removed from solving the problem by brute force. Therefore, to be of any practical value the above properties must be mathematically defined in such a way that they may be computed to inform optimal solving without implicitly solving the problem at hand. To that end, the definitions of these properties are based on information that can be obtained from a random walk through the design space. This makes it possible for a manager to select appropriate team characteristics (team size and interaction frequency) before the team begins work, increasing the efficiency of the team's search and the final quality of their solution.

Particularly, the three properties can be computed by taking a random walk of finite length through valid solutions in the design space. The solutions traversed during the walk are then evaluated with respect to each of the objective functions associated with the current design problem, with the results stored in separate vectors. This result is a set of vectors $\{\boldsymbol{Y}^1, \boldsymbol{Y}^2, \ldots, \boldsymbol{Y}^N\}$, where N is the total number of objective functions. The vector $\boldsymbol{Y}^i$ contains values along the i^{th} objective function for the solutions traversed during the random walk. The $\boldsymbol{Y}^i$ vectors will be used below to offer mathematical definitions of the three properties.

4.1 Objective Function Alignment

Engineering design often necessitates the consideration of multiple objectives for a given design problem [50]. One can imagine a scenario in which a machine part is being designed with the objectives of minimizing mass while simultaneously minimizing cost. If the total cost of a part is driven by the cost of bulk material, then these objectives are aligned – they may even be related by a constant value (the per-mass cost of material). To illustrate the other extreme, imagine a scenario in which the mass of the part may only be decreased by machining away material. In this case, the cost of the part would likely be dominated by machining costs, so the objectives of minimum mass and minimum cost would be opposed. If objective functions disagree considerably, a team may need to search more divergently in order to discover a region of the design space in which objective functions are more aligned or a region in which all objective functions reach acceptable values. Less frequent interaction enables team members to pursue their own solutions in detail, potentially allowing team members to search the space divergently.

Objective function alignment can be quantified as the average pairwise Spearman correlation between the sampled values for each combination of objective functions. The characteristic value describing this quantity is computed as

$$c_A = \underset{\forall i,j:i\neq j}{\operatorname{mean}} \rho_S(\boldsymbol{Y}^i, \boldsymbol{Y}^j), \tag{1}$$

where ρ_S denotes the Spearman rank correlation between objective functions i and j for the sampled solutions. Spearman's rank correlation coefficient is a non-parametric measure of the correlation between two samples [51]. Rank correlation is used in lieu of linear correlation because an ordinal relationship is sufficient to indicate alignment between objective functions.

A value of $c_A < 0$ indicates that the objective functions show some level of misalignment or opposition. As noted above, this would necessitate a higher level of divergence, which could be facilitated by infrequent interaction. A value of $c_A > 0$ indicates that the objective functions show a meaningful degree of alignment ($c_A \equiv 1$ if there is only one objective function). In this case, a team could benefit from frequent interaction and enabling a quicker, convergent search for the solution.

4.2 Global Behavior

The Hurst exponent, H, expresses the long-term memory of a time series [52]. A Hurst exponent near 1 indicates that a high value is likely to be followed by another high value, while an exponent near 0 indicates that a high value is likely to be followed by a low value. Computing the Hurst exponent of a random walk can be indicative of the global roughness of the landscape – a value of $H = 1$ indicates that the time landscape is globally smooth, and $H = 0$ indicates global roughness. This also correlates approximately to the modality of the function – a lower value of H reveals a multimodal landscape (see [53] for examples of how function topology changes with H). For this work, the global behavior property is defined as the minimum Hurst exponent observed across objective functions:

$$c_G = \min_i H(\boldsymbol{Y}^i). \tag{2}$$

In other words, c_G is the Hurst exponent of the most multimodal objective function. A value of c_G near zero indicates a design problem with at least one objective function that is highly multimodal. A value of c_G near one indicates that all objective functions have few local optima. As a function becomes more multimodal, the team has to search broadly to find and evaluate local minima. Breadth of search like this could be enhanced by a low level of interaction between team members. This would encourage independent search of the design space, which in turn would delay convergence to a common solution.

4.3 Local Behavior

A fractal is a pattern that exhibits local self-similarity, meaning that similar patterns emerge across scales. Fractal-like patterns have been identified in a number of fields [54,55], and have been noted as a distinguishing characteristic of layout problems [56,57]. Computing the fractal dimension, D, of a random walk along a function reveals the local scaling relationship that the function follows. In general, a low fractal dimension indicates a locally smooth curve, while a higher value indicates roughness (see [53] for examples of how function topology changes with fractal dimension). This is in contrast to the Hurst exponent, which reveals *global* behavior. For this work, the local behavior property is defined as the maximum fractal dimension observed across objective functions:

$$c_L = \max_i D(\boldsymbol{Y}^i). \tag{3}$$

In other words, c_L is the fractal dimension of the roughest objective function. A large value of c_L indicates a design problem with at least one locally rough objective function, while a lower value of c_L indicates that all objective functions are locally smooth (and perhaps traversable with gradient methods). When a design space is locally rough, the local minima are not easy to find since gradient methods cannot be used. Therefore, infrequent interaction within a team could be beneficial, allowing individuals to thoroughly search different neighborhoods in the design space.

4.4 Example Characterization

Figure 3 shows an example of a random walk taken through a design space with two objective functions. Based on this random walk, the first objective function yields a Hurst exponent of $H_1 = 0.38$ and a fractal dimension of $D_1 = 1.50$, while the second function yields a Hurst exponent of $H_2 = 0.49$ and a fractal dimension of $D_2 = 1.10$.

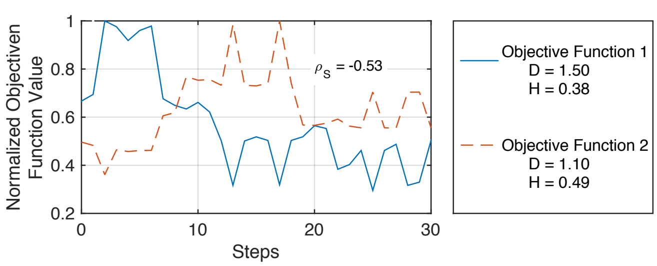


FIGURE 3. RANDOM WALK EXAMPLE.

Based on the above values, the global behavior property can be computed as $c_G = \min(H_1, H_2) = 0.38$ and the local behavior property can be computed as $c_L = \max(D_1, D_2) = 1.50$. Further, the alignment property can be determined by computing the Spearman correlation coefficients between the two objective functions, $c_A = \rho_S(\boldsymbol{Y}^1, \boldsymbol{Y}^2) = -0.53$.

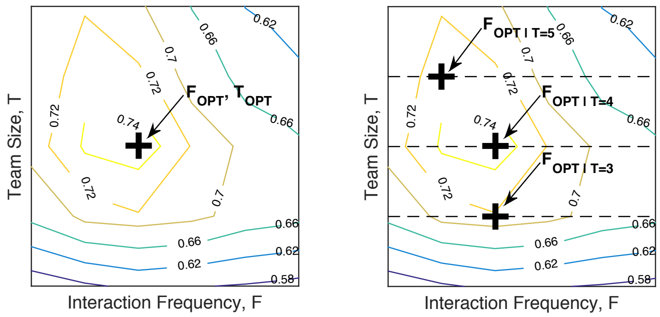
The properties of each design problem in this work are determined by computing the mean values each of c_A, c_L, and c_G obtained from 100 separate random walks. The repetition of the random walks ensures that the properties are estimated with high accuracy, thus reducing a possible source of error in the subsequent regression analysis.

5 FINDING OPTIMAL TEAM CHARACTERISTICS

This sections details how the optimal team characteristics are found for both Case 1 (in which both team size and interaction frequency must be chosen) and Case 2 (in which the team size is fixed). First, team performance is assessed with the CISAT modeling framework for every combination of design problem (8 problems defined in Section 3), team size T (from 2 to 6), interaction frequency F (values of 0, $\frac{1}{32}$, $\frac{1}{16}$, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, and 1, indicating the fraction during which teams interact), and total number of solution evaluations R (values of 500, 100, 1500, and 2000). The variable R is analogous to the number of billable hours available for a design project, and provides a critical limitation on the resolution with which the space can be searched. R will be referred to as resource availability in the remainder of this paper.

For every combination of the above variables, the CISAT modeling framework is used to simulate 100 design teams. A post-processing step is used to determine the fraction of teams that were able to achieve at least one solution that met the target values for all objective functions. This fraction is the criterion for selecting the best team characteristics. Further post-processing (outlined in the next two paragraphs) is used to extract sets of data for the regression analyses.

Case 1 corresponds to a situation in which both team size and interaction frequency must be chosen. To do so, resource availability and the design problem are held constant, and all combinations of team size and interaction frequency are examined to find the optimal combination. This process is illustrated visually in Figure 4a. Contours show the fraction of teams that were able to meet all design targets. The optimal interaction frequency (F_{OPT}) and the optimal team size (T_{OPT}) are chosen so that they maximize this fraction. When applied across all simulations this procedure results in a data set of 32 observations (8 design problems × 4 values of R). Every observation in this data set consists of values for two dependent variables (the team characteristics, T_{OPT} and F_{OPT}) and four independent variables (c_A, c_G, c_L, and R). This data set forms the basis for the regression analysis for Case 1.



a. Case 1 example **b. Case 2 example**

FIGURE 4. DETERMINING THE OPTIMAL TEAM CHARACTERISTICS.

Case 2 only requires an interaction frequency to be chosen – the team size is fixed. This allows team size to be used as a predictor variable since its value is specified a priori. Case 2 uses the same information as Case 1, but multiple values of optimal interaction frequency are chosen for every combination of design problem and resource availability, one for each value of team size that was simulated (see Figure 4b). This optimal interaction frequency is denoted by $F_{OPT|T}$ (optimal interaction frequency given team size) to differentiate it from F_{OPT} from Case 1. When applied across all simulations this procedure results in a data set of 160 samples (8 design problems × 4 values of R × 5 team sizes). Every observation consists of a single dependent variable ($F_{OPT|T}$) and five independent variables (c_A, c_G, c_L, R, and the given team size, T). This data set forms the basis of the regression analysis for Case 2.

6 REGRESSION ANALYSIS

Regression analysis can be used to create equations that relate the properties of the design problems (computed in Section 4 for the design problems defined in Section 3) to the best team characteristics for solving those problems (elucidated in Section 5). The resulting regression equations are tools that can be used to organize a team to most efficiently solve a design problem.

6.1 Case 1: Team Size Unspecified

Case 1 addresses a scenario in which a manager or principal engineer must select both the size of the team and the frequency with which the team members interact. The task of predicting the optimal team size and the optimal frequency of interaction based on problem properties and resource limits is given mathematically by:

$$\hat{T}_{OPT} = f(c_A,\ c_G,\ c_L,\ R), \tag{4}$$

and

$$\hat{F}_{OPT} = f(c_A,\ c_G,\ c_L,\ R), \tag{5}$$

where $\hat{T}_{OPT}$ is the predicted optimal team size and $\hat{F}_{OPT}$ is the predicted optimal frequency of interaction. The functional relationship between these predicted variables and the problem properties can be determined using regression techniques. The Pearson correlation between the measured values of P_{OPT} and T_{OPT} is small and not statistically significant (Pearson $\rho = 0.167$, $p > 0.25$). Therefore a true multivariate method like reduced rank regression [58] is not required. Equations to predict optimal interaction and team size can be found separately using ordinary least-squares regression.

Main effects models were computed for predicting both team size and interaction frequency, based on the local behavior property (c_L), global behavior property (c_G), alignment property (c_A), and availability of resources (R). The model for optimal team size explains only 14% of the variance in T_{OPT} ($R^2_{adj} = 0.140$, $F = 2.26$, $p < 0.1$), indicating that linear main effects alone cannot effectively predict optimal team size. The model for optimal interaction frequency explains more than 70% of the variance ($R^2_{adj} = 0.729$, $F = 21.9$, $p < 0.001$), indicating that it is possible to predict optimal interaction frequency using a simple linear model.

The contribution of a term to the accuracy of the model can be assessed by defining a new model that omits that term. Comparing the accuracy of the new model to that of the complete model indicates the contribution of the omitted term. Figures 5 and 6 show the contributions from each term in the two models, computed in this fashion. The significance level of the term, computed by a t-test comparing the coefficient to 0, is indicated by asterisks. A single asterisk indicates $p < 0.1$, two asterisks indicate $p < 0.05$, and three asterisks indicate $p < 0.01$. Bars that are yellow/light indicate terms that have a positive relationship with the dependent variable (increasing one will increase the other), and bars that are blue/dark indicate a negative relationship (decreasing one will increase the other).

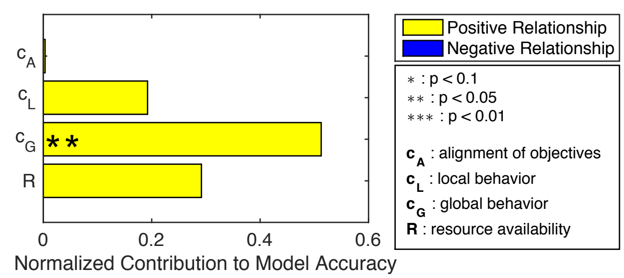


FIGURE 5. CONTRIBUTION TO TEAM SIZE MODEL FOR CASE 1, MAIN EFFECTS ONLY.

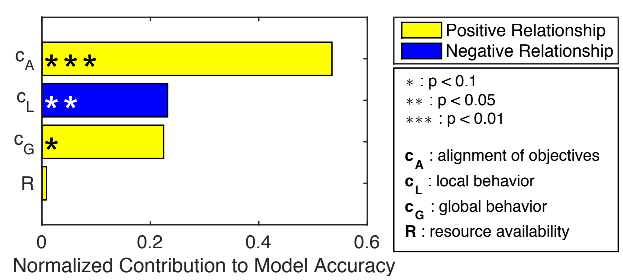


FIGURE 6. CONTRIBUTION TO INTERACTION FREQUENCY MODEL FOR CASE 1, MAIN EFFECTS ONLY.

The contributions in Figure 5 are generally less significant than those in Figure 6 because the team size model has lower overall accuracy. Together these figures indicate that both models receive much of their accuracy from the local and global behavior properties. Further, the team size model owes much of its accuracy to the resource availability variable (R), while the interaction frequency model receives a substantial contribution from the alignment variable (c_A).

The accuracy of the main effects models can be improved by adding terms to account for interaction between variables. Therefore, models were computed that incorporated both main effects and interaction effects. The new and extended model for optimal team size explains over 50% of the variance in T_{OPT} ($R^2_{adj} = 0.513$, $F = 4.27$, $p < 0.01$), representing a significant improvement in accuracy over the main effects model. The new model for optimal interaction frequency explains 80% of the variance ($R^2_{adj} = 0.820$, $F = 15.1$, $p < 0.001$). This only slightly improves on the main effects model, which explained 70% of the variance. Figures 7 and 8 shows the contribution to model accuracy from each term in these new models.

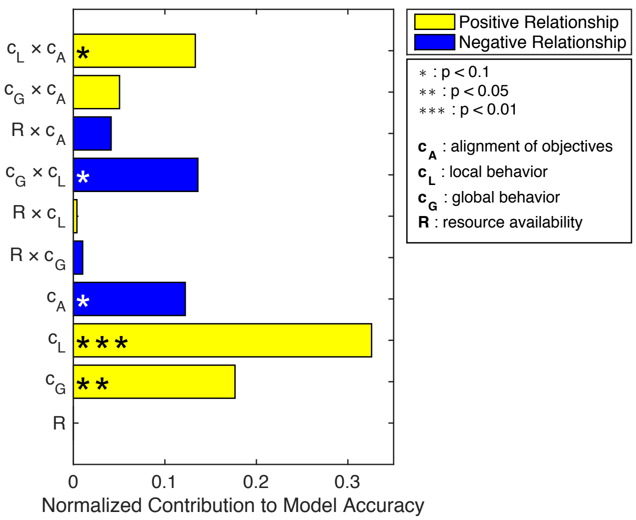


FIGURE 7. CONTRIBUTION TO TEAM SIZE MODEL FOR CASE 1, MAIN EFFECTS + INTERACTIONS.

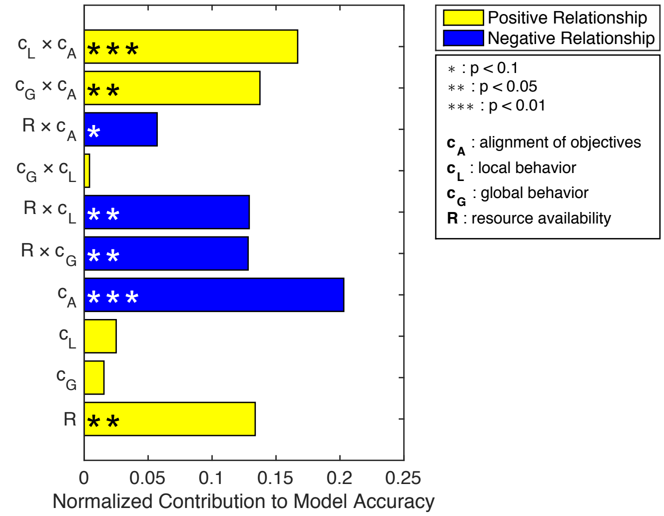


FIGURE 8. CONTRIBUTION TO INTERACTION FREQUENCY MODEL FOR CASE 1, MAIN EFFECTS + INTERACTIONS.

Figure **7** indicates that the increase in accuracy of the team size model is primarily due to the interaction between local and global behavior ($c_G \times c_L$) and the interaction between local behavior and objective alignment ($c_L \times c_A$). Main effects of local and global behavior still contribute substantially to the model's accuracy as well. Although the resource availability variable had little effect in the main effects model for interaction frequency, Figure 8 shows that the inclusion of interaction terms increased the contribution from that variable (both in the form of interaction terms and as a main effect).

6.2 Case 2: Team Size Fixed

Case 2 addresses situations in which an existing team must address a design problem. In this situation, the team size is fixed, but the frequency with which the design team interacts can be chosen by the design team manager. The task of predicting the optimal interaction frequency in this situation is given mathematically as:

$$\widehat{F}_{OPT|T} = f(c_A,\ c_G,\ c_L,\ R,\ T), \tag{6}$$

where T is the given size of the team, which is now known a priori. An equation to predict optimal interaction frequency can be found using least-squares regression. As in Case 1, a main effects model is first considered. This regression model explains over 70% of the observed variance ($R^2_{adj} = 0.726$, $F = 85.4$, $p < 0.001$), indicating once again that the best interaction frequency for solving a problem can be predicted well using only main effects, specifically the three problem properties describing local behavior, global behavior, and objective alignment. The contributions from each term in the model are shown in Figure 9. These results are very similar to the main effects model for interaction frequency in Case 1 (see Figure 5) with only a small contribution from the known team size.

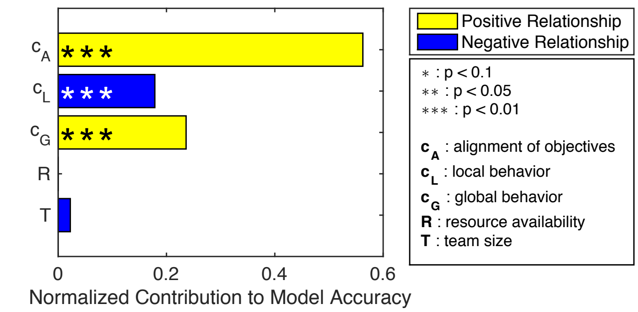


FIGURE 9. CONTRIBUTION TO FINAL MODEL FOR CASE 2, MAIN EFFECTS ONLY.

A model with both main effects and interaction effects was also computed to predict optimal interaction frequency for situations when team size is fixed. This regression model explains over 80% of the observed variance in P_{OPT} ($R^2_{adj} = 0.825$, $F = 51.1$, $p < 0.001$). Although the accuracy increases significantly with the inclusion of interaction terms, the model complexity also increases. The contribution from each term in the model is provided in Figure 10.

Including interaction terms increases the accuracy of this model by approximately 10%. While many of the interaction terms contribute to this boost in accuracy, the primary increase results from the interaction of local behavior with objective alignment ($c_L \times c_A$), and global behavior with objective alignment ($c_G \times c_A$). These interaction terms also contribute significantly to the interaction model for interaction frequency in Case 1 (see Figure **7**).

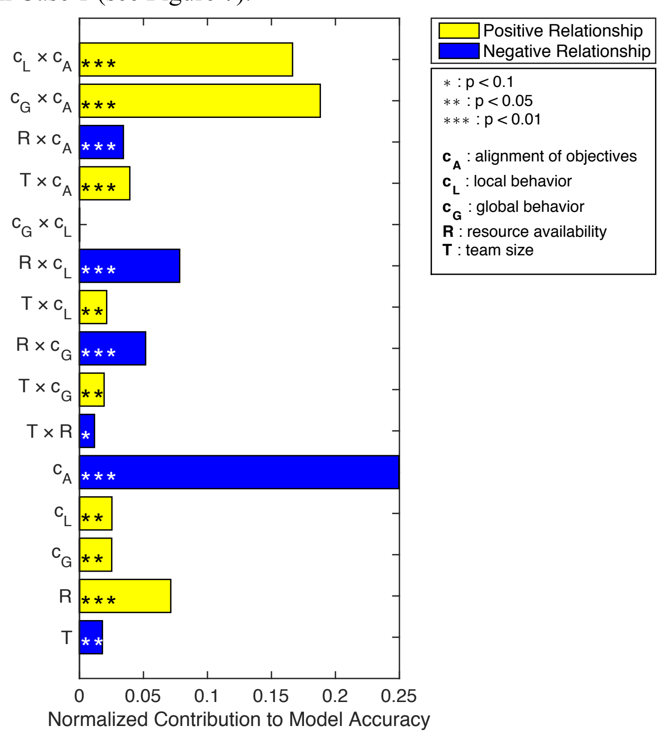


FIGURE 10. CONTRIBUTION TO FINAL MODEL FOR CASE 2, MAIN EFFECTS + INTERACTIONS.

7 DISCUSSION

The six models from the regression analysis are summarized in Table 1. The two models for optimal team size, T_{OPT}, differ greatly in quality. The model that includes interaction terms is preferred over the simpler model because it offers higher accuracy and higher statistical significance.

All models for predicting optimal interaction frequency (both in Case 1 and Case 2) achieve high statistical significance and good accuracy. Including interaction effects adds 6 additional terms to the regression equation for F_{OPT} and 10 additional terms for $F_{OPT|T}$, more than doubling the total number of terms in both equations (the relevant main effects models are shown in Figures 6 and 9, and the models with interaction effects in Figures 8 and 10). This large increase in complexity only results in a modest 10% increase in accuracy for either. Therefore, the simpler main effects models are preferred when predicting interaction frequency. The preferred models are highlighted in Table 1, and the implications of these models are discussed below.

TABLE 1. SUMMARY OF REGRESSION MODELS (PREFERRED MODELS SHADED).

Case	Independent variable	Terms	R^2_{adj}	p-value
1	T_{OPT}	main only	0.140	< 0.1
1	T_{OPT}	main + inter.	0.513	< 0.01
1	F_{OPT}	main only	0.729	< 0.001
1	F_{OPT}	main + inter.	0.820	< 0.001
2	$F_{OPT\|T}$	main only	0.726	< 0.001
2	$F_{OPT\|T}$	main + inter.	0.825	< 0.001

7.1 Predicting Optimal Team Size

For predicting optimal team size, the preferred model contains both main effects and interactions. This preferred model explains approximately 50% of the observed variance, indicating moderate accuracy.

Figure 7 indicates that the terms corresponding to the global behavior (c_G) and the interaction between local and global behavior ($c_L \times c_G$) were both statistically significant and contributed substantially to the accuracy of the model. The global behavior term (c_G) is positively related to optimal team size, while the interaction term ($c_G \times c_L$) is negatively related to optimal team size. These terms indicate that for low values of c_L, optimal team size increases quickly for small changes in c_G. However, for larger values of c_L, the optimal team size is less sensitive to changes in c_G. In other words, if the design space is locally smooth, then decreasing the modality of the design space increases the optimal team size. When the design space is locally rough, a change in the modality of the design space has a smaller effect on the optimal team size.

An interesting trend is that the availability of resources (R) contributed substantially to the accuracy of both team size models. In the main effects model, which has the highest relative contribution from this term, availability of resources is positively related to optimal team size. In other words, when more resources are available it is beneficial to increase team size, spreading resources amongst a greater number of individuals to increase the extent to which work can be done concurrently. This corresponds to the conventional wisdom for smaller software development teams [19,20,22]. This indicates that it may be possible to successfully apply some of the best practices from software development to the design of mechanical systems, especially if design tasks are similar to those used in this work.

7.2 Predicting Optimal Interaction Frequency

An inspection of Figures 6 and 9 indicates that the preferred models for interaction frequency are very similar. This is expected since the models only differ in the addition of team size as a predictor, and team size was shown to only have a small correlation with interaction frequency for the team sizes investigated in this work. In both of the preferred models the design space properties (c_A, c_G, and c_L) are the most impactful terms, while the resource availability variable (R) has no significant predictive value.

The objective alignment property is positively related to interaction frequency. In other words, less frequent interaction is preferred for design problems in which objective functions are not aligned. If objective functions are not aligned, then infrequent interaction allows for divergent search to take place as members of the team look for regions of the design space in which all objective functions reach suitable values.

The local behavior property is negatively related to optimal interaction frequency, which indicates that more frequent interaction is only beneficial if the design space is locally smooth. If the design space is locally rough, infrequent interaction enables individuals to perform diligent local search before considering trade-offs between solutions. Interacting too often could cause premature convergence in a portion of the design space that does not contain satisfactory solutions.

Finally, the global behavior property is positively correlated with optimal interaction frequency, indicating that multimodal design spaces require less frequent interaction. The reasoning for this relationship is much the same as that for the objective alignment and local behavior properties. Infrequent interaction allows individuals to act independently as they individually find local minima. Once individuals in a team have found a diverse set of local minima, interacting allows them to select a shrinking set of alternatives to pursue.

Considering the extreme values of the design space properties provides two illustrative examples of their relationship to optimal interaction frequency. On one hand, infrequent interaction is preferred for design problems in which objective functions are unaligned, exhibit rough local behavior, and are highly multimodal. This enables individuals to spend time independently refining solutions (essentially finding a set of local minima within the team) before the team interacts to consider trade-offs between the solutions in the set. In this scenario, frequent interaction could lead to premature convergence on a poor local minimum. On the other hand, frequent interaction is desirable for a design space in which objectives are aligned, there are few local minima, and the objective functions are locally smooth. This would enable a team of individuals to rapidly converge on a solution without spending undue time on divergent search.

Other work has studied the relationship between project complexity and task performance [31]. In that work, project complexity was measured through ambiguity (comparable to the inverse of this work's alignment variable, c_A) and multiplicity (similar to the inverse of this work's global behavior variable, c_G). That work identified nontrivial interaction between multiplicity and ambiguity, a result that is echoed in the interaction between the global behavior and objective alignment variables ($c_A \times c_G$) in this work (see Figures 8 and 10).

7.3 General Applicability

Care was taken to ensure that each of the design problem properties could be computed before solving begins using a random walk procedure. This procedure makes two important assumptions about how the design problem is formulated. These two assumptions must be true in order to compute the design problem properties defined in this paper, and these properties are in turn prerequisite for computing optimal team characteristics. More broadly, these two assumptions must be true to simulate a problem using the CISAT framework.

The first assumption is that the quality of a solution can be quantified with one or more well-defined objective functions. It may not always be feasible to define numerical objective functions for a design problem, especially if the objectives of the design are subjective in nature. However, it is possible to employ user surveys to robustly quantify subjective criteria like elegance or sportiness [59]. Such ratings-based data could be used to compute properties of the design problem, and from those properties the optimal team characteristics could be estimated.

The second assumption is that existing solutions can be modified efficiently, making it possible to produce a random walk through the solution space with little effort. This assumption was addressed here by ensuring that well-defined rules for modifying solutions were associated with each design problem, allowing the random walk procedure to be automated. It is straightforward to produce a random walk for any design problem for which such rules exist (such as problems defined by design grammars).

The CISAT modeling framework is necessary given the broad scope of the current work, but brings with it several limitations. For instance, CISAT models a limited (albeit important) set of phenomena. The influence on the performance of engineering design teams of phenomena that are not modeled could be significant. Further, this work only uses the CISAT model to examine the impact of team size and interaction frequency, while holding other characteristics constant (like self-bias and satisficing behavior, two characteristics that can be modulated in the CISAT framework). It is possible that these characteristics might interact with team size and interaction frequency to influence team performance.

8 CONCLUSIONS AND FUTURE WORK

This work defined the relationship between the properties of design problems and the team characteristics that lead to the best solutions to those problems. Rather than conducting human studies, this work used the CISAT modeling framework to simulate the performance of human engineering design teams. The simulations presented in this paper represent over 100,000 hours of equivalent human design studies. This research could not have been conducted at this breadth without the kind of capabilities provided by CISAT.

First, three design problem properties were defined and used to quantify design problems. These properties were the local behavior of the design space, the global behavior, and the extent to which objectives are aligned. These design properties can be computed before solving begins using a random walk procedure. The CISAT modeling framework was then used to simulate the performance of engineering design teams with a broad variety of team characteristics on several different engineering design problems. Post-processing of these results provided a set of optimal team characteristics for each of the design problems. Finally, regression analysis was used to define equations relating the optimal team characteristics to properties describing the design problems. These equations make it possible to predict optimal design team characteristics (team size and interaction frequency) based on design problem properties. Because properties of the design problem can be computed before solving begins, these predictive equations are powerful tools that facilitate the optimal design of design teams.

The selection of the optimal number of individuals in a team is a complicated relationship, depending greatly on the interaction of design space properties (particularly local smoothness and modality) with the availability of resources. The selection of an optimal interaction frequency can be predicted with high accuracy based on the properties of the design space. If a design problem has unaligned objectives, rough local behavior, and is highly multimodal, then teams should interact infrequently and spend time working independently to avoid premature convergence on unacceptable solutions. In contrast, if a design problem has aligned objectives, smooth local behavior, and fewer local minima, then frequent interaction within the team can yield a quick search that converges on an acceptable solution.

This paper focused on the selection of optimal team size and optimal interaction frequency, but the approach used here of exploring team characteristics using computational simulations of human teams could be applied to a variety of additional team characteristics as long as they are manipulable through the CISAT framework. The approach used here of informing team characteristics based on design problem properties could also be used to develop guidelines for the selection of optimized parameters for computational design algorithms.

Future work will continue to refine the relationships defined here by simulating design team performance on additional problems and explore new design space properties. Future work could also explore the development of similar relationships for teams employing hierarchical structures or consisting of individuals with highly-specialized skills or knowledge. The range of design problem types (conceptual design, topology design, and detailed design) to which these results apply will be examined. The relationships uncovered in this paper should also be further validated through studies with human teams of designers or engineers.

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