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# Influencing generative design through continuous evaluation: Associating costs with the coffeemaker shape grammar

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**Abstract**

A grammatical approach to product design is demonstrated. In particular, shape grammars are shown to be especially useful for products that are differentiated primarily on the basis of form yet driven by function; they allow products to be designed as a sequence of well-defined steps. However, it is not always clear how to choose the sequence of rules that should be applied to generate the final shape. In this paper we demonstrate that at each stage during the process, partial designs of the final product can be used to provide feedback to the designer based on specific design objectives and thus suggest possible rule choices. We take advantage of the shape grammar for the generation of coffeemakers introduced by Agarwal and Cagan, and associate with the grammar rules expressions that model manufacturing costs. With each application of a shape grammar rule, an understanding of the overall cost of manufacturing the product is incrementally improved. Thus, at each stage of the design process the designer has an indication of what the overall cost of the product will be and how the selection of one grammar rule over another influences the final cost. Once the complete product is generated, an appraisal of its manufacturing cost is given to the designer. This evaluation methodology helps the designer understand the implications of decisions made early on in the design process. We have also verified the accuracy of this approach through the costs of some commercially available coffeemakers, generated by this method, which are comparable to the costs for those designs listed in the literature.

**Keywords:** Coffeemakers; Shape Grammars; Manufacturing Cost

## 1. INTRODUCTION

Shape grammar-based systems have been successfully used for generative design in architecture and recently for product design. However, there are no formal techniques that help the designer in selecting which rule to apply at any given stage. In this work, we argue that using performance metrics along with a grammar-based generative system will create a powerful feedback mechanism for the designer during the design generation process. Additionally, in the generative design of products, those designs that fare the best in terms of performance metrics are often the ones that are the most successful in the marketplace. However, the association of such metrics within shape grammars for engineering applications has received little attention. One approach to the evaluation of designs created by grammars is to use external analysis after the generation sequence is complete. The drawback to such an approach is that since the evaluation is carried out after the design is completed, no information can be provided to the designer during the design generation phase. In contrast, our approach is to associate performance evaluation directly with the grammar rules themselves. We illustrate the power of such an approach by associating manufacturing costs with the coffeemaker shape grammar of Agarwal and Cagan (1998).

Product cost is one of the most important design constraints during the design and redesign of products; a product will not succeed in the marketplace if it is not properly

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priced for its intended market. Studies of product design and product manufacture suggest that a significant portion of a product's cost is determined by the decisions made early in the design process (e.g., Nevins & Whitney, 1989; Ulrich & Pearson, 1996). Thus, the designers need to be made aware of not only how their later design decisions affect the cost of a product, but also (and perhaps more critically) how their early decisions affect the manufacturing cost. Further, it is important that this information be made available as soon as a design change is made so that a more extensive redesign iteration can potentially be avoided. To be able to make this information available, it is necessary that the manufacturing costs are correlated with the product configurations that exist at any given stage of the design creation cycle and not just with the final finished product. Such a technique will enable the designer to gauge the effect of a change when it is made as well as ensure that an accurate appraisal of the final manufacturing cost is provided to the designer as soon as the design is completed. An immediate feedback of this nature will enable the designer to explore a variety of design alternatives that may otherwise have been too time-intensive to consider. However, to achieve this, it is important that the design and costing methodology allow for rapid generation of results and, thereby, enable the designer to create designs with a small turnaround time.

We propose that a shape grammar-based design paradigm will meet all the requirements of such a system. Grammars not only allow the rapid generation of a wide variety of feasible designs by the application of different rules in a rule set, but they also maintain representations of the partial designs at each stage. We claim that based on the rules applied to reach a particular design stage, it is possible to estimate the manufacturing cost at that stage. In this paper we argue that by associating manufacturing cost with the grammar rules, one can identify design changes that have positive and negative effects on the product cost. The designer can then receive immediate feedback on how the various changes will affect the cost of the product, thereby allowing informed decisions to be made about whether to accept or reject those changes. In addition, because partial design costs are available at each stage, the final cost is obtained as soon as the design is completed. As an example of the usability of such a strategy, we apply this technique to the generation and costing of coffeemakers based on a grammar developed by Agarwal and Cagan (1998) by associating manufacturing cost expressions with the shape rules in the grammar. This then provides the designer with an integrated methodology for the design and costing of coffeemakers and allows various trade-offs to be studied from a manufacturing cost perspective. It is important to note that, while this paper discusses the strategy for associating expressions modeling cost with the coffeemaker grammar rules, the underlying technique is general and can be applied to a variety of grammars and performance metrics.

Some commercially available coffeemakers are designed and costed using our method and the results compared to those reported by Ulrich and Pearson (1993) using a disassembly-based costing approach. In spite of the completely different costing strategy used in the two studies, the costs estimated by both methods turn out to be similar, validating our approach. Before we discuss the details of our method, it is worthwhile to examine the traditional costing approaches and contrast them with our technique.

Most of the current costing methodologies are either based on parametric estimation models or require a bottom-up cost calculation. Bottom-up cost estimation techniques require either a complete computer aided design (CAD) model of the product [e.g., complexity theory (Hoult & Meador, 1997) or commercial software like Cost Advantage from Cognition Corporation] or a detailed step-by-step manufacturing breakdown of the product (e.g., commercial software like KAPES from PS Industries). Neither of these two bottom-up techniques can provide information about estimated cost early on in the design phase because they work with complete product designs. Also, to determine how a design change will affect the cost, the cost has to be reestimated for the new product and compared with the original cost. These techniques also require a significant amount of set-up time before actual estimation can be carried out. Parametric estimation techniques (e.g., commercial software like SEER H from GA SEER and PRICE H from Lockheed Martin PriceSystems), on the other hand, require little information about the product design. They use statistical methods to relate the product weight, volume, manufacturing process, and a few other parameters to the final product cost. These techniques require extensive calibration based on existing products before they can be used to estimate cost and they do not provide any information about the influence of design decisions on cost. Because they do not identify the key cost drivers, these techniques are unable to provide feedback regarding possible redesign directions. They also usually require an estimate of the final weight and volume of the product (which may not be readily available) before cost can be estimated. In summary, most traditional costing techniques are unable to provide information about the product cost until after the design is completed and therefore may require the products to be completely recosted if a design change is made. We believe that the technique proposed in this paper will address these concerns.

Next, we will briefly discuss the coffeemaker grammar. We will then define a manufacturing cost structure for injection molded parts, metal stamped parts, and product assembly. By associating various elements of this cost structure with the coffeemaker grammar rules, a methodology for estimating costs of the designs generated by this grammar will be obtained. We will demonstrate the technique by discussing an example and verify the results by comparing them to those in the literature. We will demonstrate how this method can be used to study design trade-offs and guide the design generation process. Finally, we will conclude with a brief discussion on how this technique can be applied within other shape grammars.

## 2. COFFEEMAKER SHAPE GRAMMAR

### 2.1. Shape grammars

A shape grammar (Stiny, 1980*a*, 1980*b)* derives designs in the language it specifies by successive application of shape transformation rules to some evolving shape, starting with an initial shape. It can be used to describe how complex shapes are built from simple entities and how a complex shape can be decomposed into simpler subshapes. Shape grammars have been successfully used for spatial design in the field of architecture including villas in the style of Palladio (Stiny & Mitchell, 1978), Mughul gardens (Stiny & Mitchell, 1980), prairie houses in the style of Frank Lloyd Wright (Koning & Eizenberg, 1981), Greek meander patterns (Knight, 1986), suburban Queen Anne Houses (Flemming, 1987), and windows in the style of Frank Lloyd Wright (Rollo, 1995).

Examples illustrating the ideas behind shape grammars can be found in Stiny (1980*a*, 1980*b)*. While there has been a limited application of shape grammars to engineering design, they had not been used for the generation of individual products until Agarwal and Cagan (1997, 1998) presented the coffeemaker grammar. Fitzhorn (1990) and Longenecker and Fitzhorn (1991) have presented shape grammars specifying the languages of constructive solid geometry and boundary representations (i.e., realizable solids). Brown, McMahon, and Sims Williams (1993) presented a manufacturing-oriented shape grammar that specifies the language of all axi-symmetric objects manufacturable on a given lathe. That work is particularly relevant here because, although they used a completely different strategy, they presented a technique for estimating the manufacturing time (which is an important component of manufacturing cost) for the various parts machined by the lathe. Reddy and Cagan (1995*a*, 1995*b*); Shea, Cagan, and Fenves (1997); and Shea and Cagan (1997) presented parametric shape grammars for the design of planar and geodesic dome truss structures that used the shape annealing technique of Cagan and Mitchell (1993) to generate optimal structures.

Stiny (1981) presented a general design description methodology that relied on associating description rules with the grammar rules much like we do here. However, the description rules themselves as well as their association with the grammar rules vary vastly based on the application domain. No formal techniques exist for creating these description rules or for associating them with the grammar rules. This work uses cost expressions along with the shape rules and thus applies grammars to the concurrent design and costing of a class of individual products.

### 2.2. Coffeemaker grammar

The coffeemaker grammar is a parametric, labeled 2D shape grammar consisting of 100 rules and can recreate a number of existing coffeemakers as well as create an infinite number of new designs. The rules in the shape grammar manipulate one or more of the three views of the product—top, side, front—to create a final 3D shape. The coffeemaker is considered to be made up of three main parts: the filter unit, the water storage unit, and the base unit. These three units are arranged around the space for the coffee pot, which acts as the initial shape for the grammar. The grammar creates a complete coffeemaker by first designing the base and the filter units and then blending them together using the water storage unit. Due to the similar functional breakdown of coffeemakers, the function drives the form in the product and in the application of the grammar rules; function labels are used to maintain the proper function-to-form sequence.

The designs generated by the grammar can satisfy a wide variety of functional requirements. For example, the design can be a single-heater or double-heater unit, have a conical or a flat filter (with or without a flow rate control mechanism), and can use a lid or a grating to cover the water storage unit. It should be pointed out, though, that the designs generated by the grammar do not incorporate all of the design details. For example, the number and position of screws, the power cord, the color of the product, and the form of the switch are not designed by the existing grammar rules. Thus, in this work, the cost corresponding to these components is added separately and is not obtained directly through the shape grammar.

## 3. ASSOCIATING COST EQUATIONS WITH THE GRAMMAR RULES

The first step in associating the manufacturing costs with the rules of the coffeemaker grammar involves breaking the cost into its components in a manner compatible with the shape rules, that is, such that each of the components can be associated with the rules. In this work we assume the cost of a coffeemaker to be made up of three main components—the cost of manufactured parts, which will form the focus of this work; the cost of purchased parts; and the cost of assembling all of the parts into a functional product. The cost of the manufactured parts can be further broken into five components—material cost, equipment operating cost, tooling cost, burden, and labor cost. Expressions for each of these components (for plastic and metal parts) are given in Appendix A. As mentioned in the appendix, each of these five cost components depends primarily on the part configuration and geometry and that is precisely the information obtained from the shapes representing the designs. This fact is crucial to the success of our methodology because it allows us to develop general parametric expressions from the shape rules that are then instantiated as the shape rules are applied. More specifically, we develop expressions for the areas and volumes of the shapes generated by the shape rules and then use them together with Eqs. (A1)–(A9) to determine the manufacturing costs. If the cost components involved an attribute that could not be determined from the

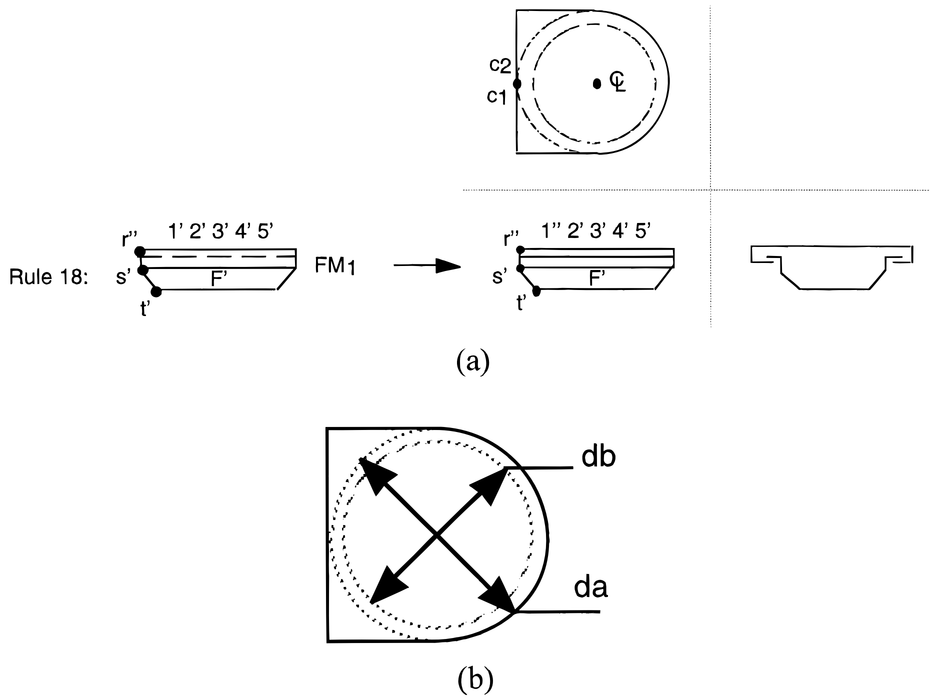


**Fig. 1.** (a) Shape rule creating a sliding filter unit; (b) top view created by the shape.

shape rules, then this technique could not have been directly applied.

Developing expressions for areas and volumes that are associated with the shape rules involves examining the geometric forms created by a rule. Note that the most general form of the grammar rule must be used, though frequently approximations need to be made when calculating areas and volumes to keep the expressions tractable. As an example consider the shape rule shown in Figure 1(a). This figure represents Rule 18 of the shape grammar of Agarwal and Cagan and is used to generate a sliding filter unit for the coffeemaker (the three views on the right-hand side of the shape rule—at the top, at the bottom left, and at the bottom right—correspond to the three views—top, side, and front—of the coffeemaker)

To determine the projected area of the filter unit created by this shape rule, the top view of the shape [Fig. 1(b)] is examined. The area of the top view is given as [$d_a$ and $d_b$ are defined by Eqs. (B3) and (B4)]:

$$A_{\text{top}} = \frac{d_a^2}{2} + \frac{\pi d_a^2}{8}. \tag{1}$$

The first term in the expression corresponds to the area of the half square (in the left part of the top view) and the second term corresponds to the semicircle (in the right part of the top view). Next, the volume of the filter unit at this stage is determined. Note that the volume at this stage corresponds to only the top part of the filter unit because only that part of the unit is designed by this rule. The volume is given as:

$$V_{\text{top}} = \left( \frac{d_a^2}{2} + \frac{\pi d_a^2}{8} - \frac{\pi d_b^2}{4} \right) \times 2. \tag{2}$$

The first two terms in the parentheses correspond to the total area of the filter unit. The third term in the parentheses corresponds to the hollow area of the filter unit (based on a 2-mm wall thickness). Thus, the quantity in the parentheses corresponds to the total solid area of the filter unit. This quantity multiplied by the height (again equal to 2 mm) is the volume of the top part of the filter unit at this stage. These two equations are the same as Eqs. (B15) and (B16).

As another example, consider the shape rule shown in Figure 2(a). This rule corresponds to Rule 29 of the coffeemaker grammar and is used to design an elliptical base unit (as opposed to a polygonal unit). To determine the projected area of the base unit generated by this rule, the top view shape (Fig. 2b) is examined. The projected area is given by [the diameters are defined by Eqs. (B37)–(B39)]:[1]

$$\text{Area} = \pi(d_{\text{major\_outer}} \times d_{\text{minor\_outer}} - d_{\text{plate}}^2). \tag{3}$$

[1]These expressions have been derived by assuming that the heater plate is circular. This assumption, while not necessary, was found to be valid in all commercially available designs.

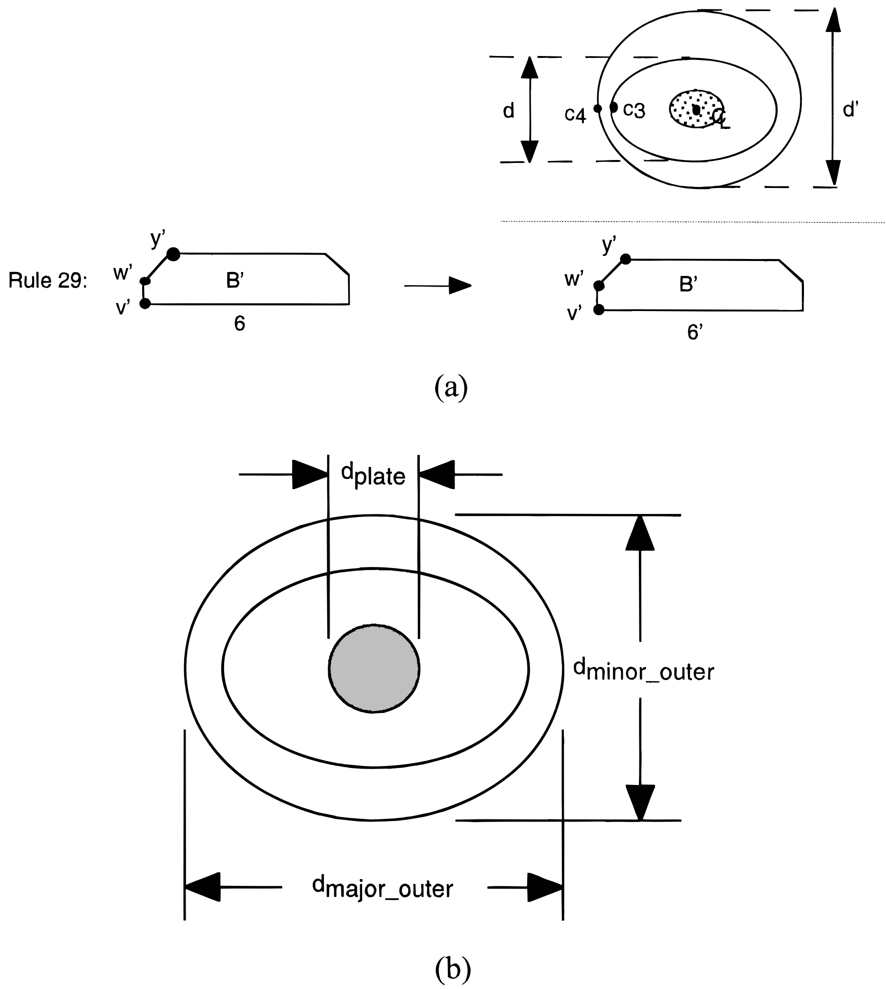


**Fig. 2.** (a) Rule generating a base unit; (b) top view of the unit generated by the rule.

This equation is the same as Eq. (B40). Equations corresponding to the other shape rules can similarly be derived and are listed in Appendix B.

## 4. USING COST EXPRESSIONS TO GUIDE THE DESIGN GENERATION PROCESS

This section demonstrates how the expressions derived in the previous section can be used to guide the design generation process for coffeemakers. The methodology will be illustrated by following an example generation sequence. The first set of rules that are applied to the initial shape shown in Figure 3(a) distinguish between the two main classes of coffeemakers, those with one heating element and those with two. They also break apart the space around the initial shape into three regions, corresponding to the filter, base, and water storage units, and design the basic cross-sectional shapes of the filter and the base units. The rule designing a one-heater unit (signified by the square label), for example, is shown in Figure 3(b); a similar rule designs a two-heater unit. Next the filter, the base, and the water storage units are designed separately. For each of these three units, the various shape rules ensure that form design is carried out within the context of function design, that is, only forms that do not violate any functional specifications can be created. This is done by first applying the function design rules that add labels to the shape based on the required functional specifications and then using the various form design rules, based on the labels, to create the actual shapes. Because the function design rules do not directly create 3D shapes, they add no cost to the design. Note that the decisions made during the function design will strongly influence the cost of the product; they just do not directly add cost to the design.

First, the form design of the filter unit is carried out based on the functional specifications. This step creates new shapes and modifies existing ones and thus cost equations are associated with these form design rules. The rule shown in Figure 4, for example, generates a rotating filter. This rule

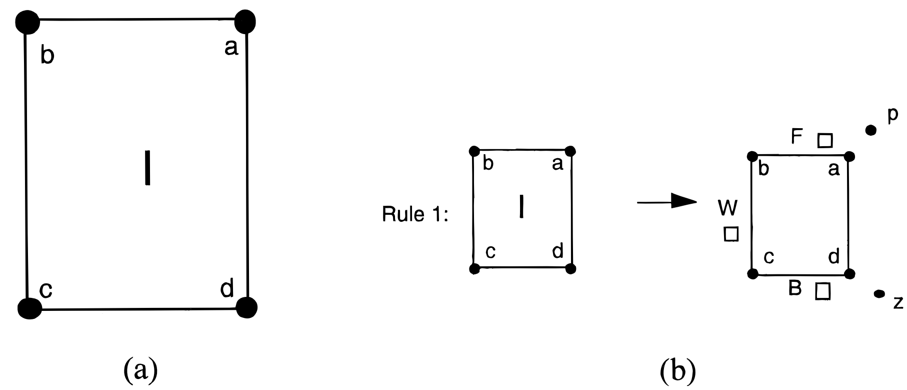


**Fig. 3.** (a) Initial shape for the example generation sequence; (b) rule designing a one-heater unit.

also creates a top view for the filter unit and thus converts it into a 3D object. The shape around the filter unit obtained after this rule is applied to the evolving shape is shown in Figure 5. Note that the shape generated has a top view and a side view (a single view is defaulted to be a side view; multiple views separated by a hairline are the top and the side views). The labels correspond to the various functional attributes of the filter unit: $FT_1$ signifies a conical filter, $FI$ corresponds to the inlet tube, $FF_1$ means that the coffee flow rate cannot be changed, and $FS_1$ signifies that a flow stop mechanism is present in the coffeemaker.

The first step in associating the manufacturing cost with this rule is to calculate the volume and projected area of the created shape. The area and volume calculated are then used along with Eqs. (A1), (A3), and (A4) to calculate the material and equipment operating cost. Note that the area and volume are calculated only for the top part of the filter unit (based on a 2-mm wall thickness mentioned earlier) because the rest of the unit has not been completely designed yet. The expressions for the area ($A$) and the volume ($V_{top}$) are given by Eqs. (B17) and (B18). It must be reiterated that these equations have been determined solely based on the geometry of the shapes created by this shape rule and are independent of the rest of the design. The incremental totals for the cost of the filter unit for the example coffeemaker are now determined. In addition to the material and equipment operating costs, the cost at this stage also includes tooling, burden, and labor costs. From the geometry of the shape, $d_a = 120$ mm and $d_b = 116$ mm. Using Eq. (B17), the area of the top of the filter is given by $A = 12081$ mm$^2$. The equipment size that is required is now calculated from Eq. (A3) and is equal to 3200 kN. The tooling cost based on an annual volume of 1,000,000 parts [Eq. (A2)] is thus \$0.015. The operating cost, calculated from Eq. (A4) based on a cycle time of 30 s, is \$0.165. The labor and burden costs [Eqs. (A5) and (A6), respectively] are \$0.015 and \$0.018. To calculate the material cost, first the volume of material used needs to be calculated. Using Eq. (B18), $V_{top} = 3027.96$ mm$^3$. The cost of the material required can now be calculated using Eq. (A1), and is equal to \$0.002 for this particular design. These costs can be used by the designer to get some indication of the final cost of the filter unit even before it is designed.

The design of the filter unit is completed by applying the other filter form design rules based on the function labels previously associated with the design. The filter design sequence is shown in Figure 6, where labels are omitted for

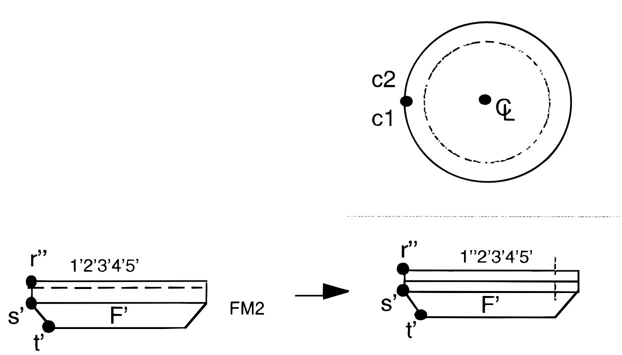


**Fig. 4.** Shape rule designing a rotating filter.

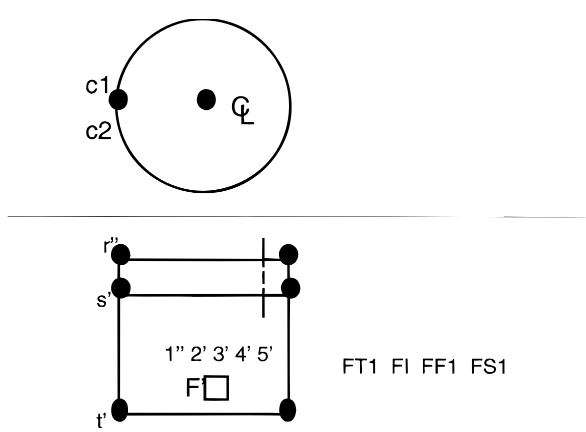


**Fig. 5.** Shape after the design of a rotating filter unit.

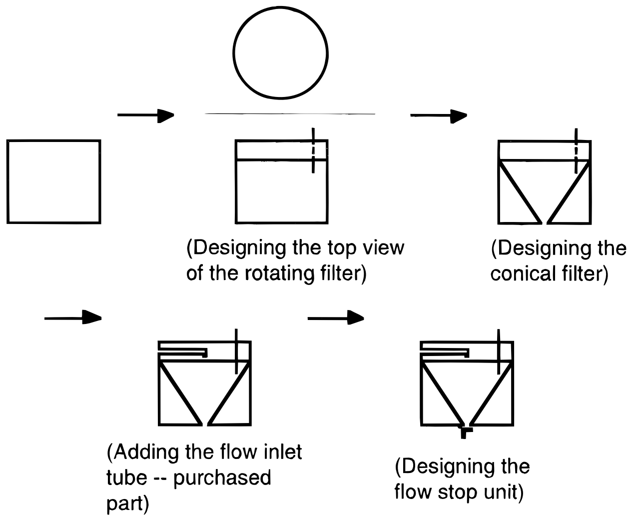


**Fig. 6.** Filter design sequence for the example coffeemaker.

**Table 1.** *Incremental cost (in $) of the filter unit for the example coffeemaker at each stage during the design sequence*

| Design Step | Material Cost | Equipment Operating Cost | Tooling Cost | Burden | Labor Cost |
|---|---|---|---|---|---|
| Top view | 0.003 | 0.165 | 0.015 | 0.018 | 0.015 |
| Conical filter | 0.115 | 0.033 | 0.013 | 0 | 0.003 |
| Flow stop | 0.027 | 0.045 | 0.009 | 0.003 | 0.008 |

clarity. The manufacturing costs associated with each design step are shown in Table 1. The first row corresponds to the design of the top of the filter (discussed above), whereas the second row corresponds to the design of the conical filter. Because the same machine is used both for manufacturing the top of the filter and the conical cup, the burden cost is added only once. The third row corresponds to the design of the flow stop mechanism. The costs in the three rows of Table 1 must be added together to determine the final cost of the filter unit and it is equal to $ 0.472. Note that because the inlet tube is a purchased part, no cost is added for that step.

The cost of $ 0.472 for the filter unit of the coffeemaker is derived independent of the rest of the design (which, in fact, has not even been completed at this stage). This provides useful feedback to the designer about the cost of the design and can be used to guide further design decisions and suggest directions for redesign. For example, if the designer decides (based on the feedback provided by the technique) that the cost of the unit is too high, one possible change might be to remove the flow stop mechanism. This would then bring down the cost of the filter unit to $ 0.38 (the first two rows of Table 1). Note that this change in the design (and therefore the cost) of the filter unit can be made at this stage itself, rather than at the end of the design cycle when redesign might be more expensive. While removing the flow stop mechanism from the filter unit is a rather obvious choice for reducing the cost of the unit, the same procedure can be applied to study more complex trade-offs.

Continuing with the design process, the next step is the design of the base unit, which follows the same procedure as the design of the filter unit. The base design sequence for the example coffeemaker is demonstrated in Figure 7. A smooth blend is assumed between the top and bottom planes of the base unit. The manufacturing costs are calculated using Eqs. (B37)–(B41), (B43), and (B69). The costs obtained are shown in Table 2.

Again, suppose the designer wants to make changes that will bring down the cost of the base unit. There are various possible changes that can be explored to determine their effect on the cost of the base unit (and, therefore, the cost of the coffeemaker). One alternative is to use a cylindrical base

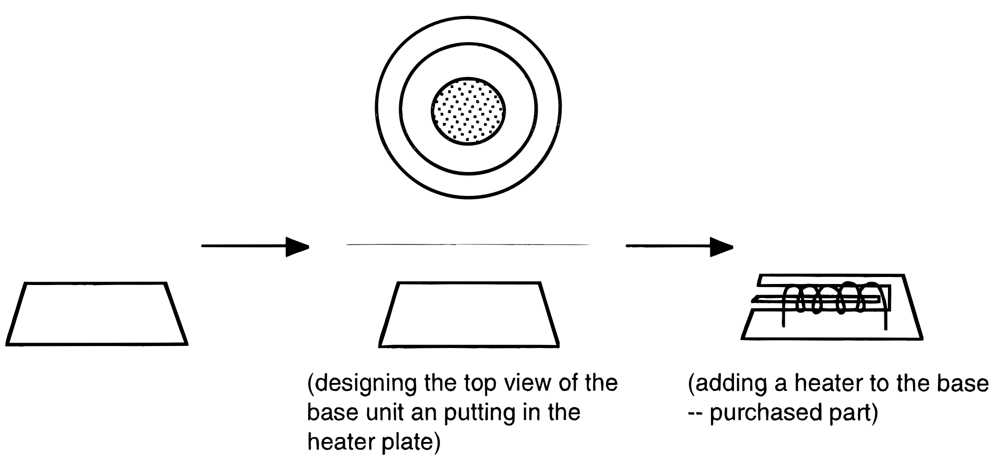


**Fig. 7.** Base design sequence for the example coffeemaker.

**Table 2.** *Incremental cost (in $) of the coffeemaker at each stage during the base design sequence*

| Design Step | Material Cost | Equipment Operating Cost | Tooling Cost | Burden | Labor Cost |
|---|---|---|---|---|---|
| Top view | 0.102 | 0.264 | 0.022 | 0.018 | 0.024 |
| Heater plate | 0.019 | 0.039 | 0.015 | 0.004 | 0.018 |

**Table 3.** *Incremental costs (in $) of the modified base unit*

| Design Step | Material Cost | Equipment Operating Cost | Tooling Cost | Burden | Labor Cost |
|---|---|---|---|---|---|
| Top view | 0.073 | 0.166 | 0.022 | 0.007 | 0.024 |
| Heater plate | 0.019 | 0.039 | 0.015 | 0.004 | 0.018 |

unit similar to that shown in the base design sequence of Figure 8. It is not immediately clear whether such a change will reduce the cost of the base unit or increase it. However, the equations discussed above can be used to determine the costs shown in Table 3. By examining the table, the designer can determine not only that the cost of the base unit will decrease due to the proposed change, but the exact magnitude of that change is also known ($0.387 instead of $0.525). If the designer feels that the reduction in cost is significant, then the change can be accepted; otherwise the original designer preference of a tapered base unit can be preserved. Note, however, that these decisions can be made at this stage, rather than waiting until the design of the entire coffeemaker is complete to determine their effect. It is information like this that helps the designer in making the appropriate choices with respect to rule selection, highlighting the value of this approach.

The last stage in the design of a coffeemaker is the creation of a water storage unit that satisfies all functional requirements and blends the three units together into a final product. To do this, the top view cross sections of the water storage unit are generated on four horizontal planes (at the top and bottom of the base and the filter units). The cross sections on these four planes are then blended together in the vertical direction to create the final 3D shape of the water storage unit, which also integrates together all the units of the product. The cross section on each plane is generated by merging together shapes created by sweeping a desired number of squares and circles in a designer-specified manner. One such rule that sweeps a square about the center of the filter is shown in Figure 9. The dimensions of the square as well as the distance from the center are specified by the designer and can change as a function of the sweep angle. This process imparts the grammar with an ability to generate shapes not commonly seen in commercial products as well as the more traditional ones.

For the example, the sweep and merge sequence generating the water storage unit cross sections is shown in Figure 10. The manufacturing costs at the various stages of the sweep sequence (on a plane) are shown in Table 4. It should be noted that each row corresponds to the cost of the water storage unit after the respective design step, and the costs at the end of step 2 must not be added to those at the end of design step 1 to obtain a total cost. This is because the shape at the end of step 1 is just an intermediate shape and the costs corresponding to that shape are relevant only at that step. Once a new shape is created by step 2, the costs must be updated. This is different than the design of the other units of the coffeemaker where, once a part is created, it is not modified and thus the costs calculated after the application of each rule remain valid throughout the process.

The 3D shape of the final product is shown in Figure 11. It is similar to a Rowenta FK26-S coffeemaker shown in Figure 12. The final cost of the coffeemaker estimated by using the method discussed above is $7.32 (cost of manu-

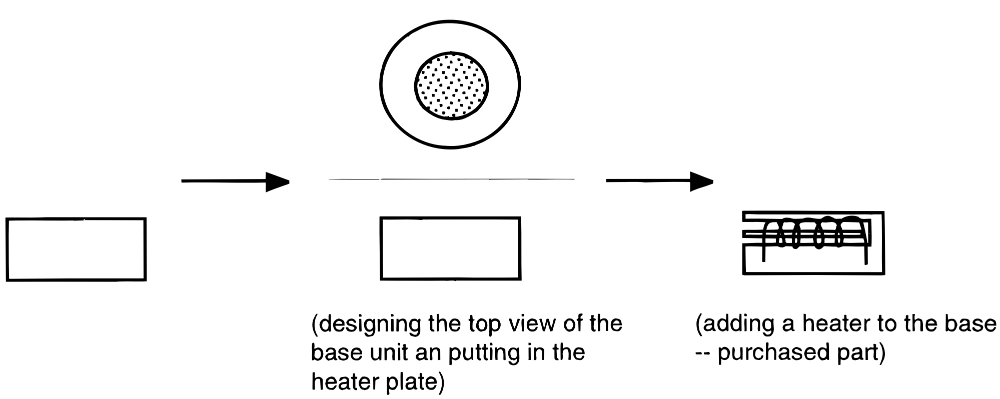


**Fig. 8.** A modified base design sequence.

**Table 4.** *Incremental cost (in $) of the water storage unit for the example coffeemaker at each stage during its generation*

| Design Step | Material Cost | Equipment Operating Cost | Tooling Cost | Burden | Labor Cost |
|---|---|---|---|---|---|
| Step 1 | 0.123 | 0.199 | 0.035 | 0.010 | 0.024 |
| Step 2 (Final design) | 0.111 | 0.199 | 0.035 | 0.010 | 0.024 |

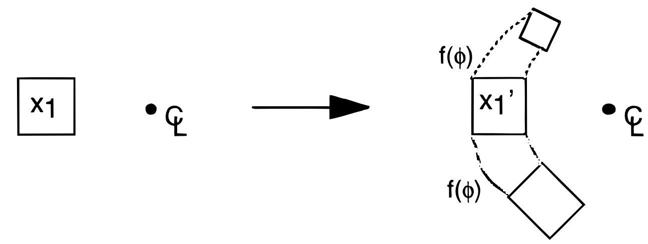


**Fig. 9.** Representative sweep rule.

**Fig. 12.** Rowenta FK26-S coffeemaker.

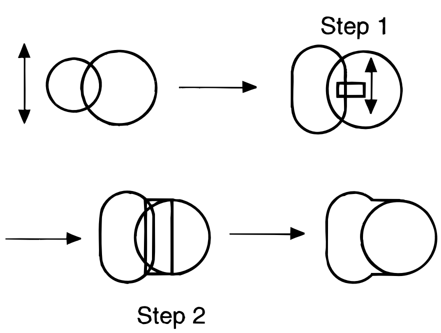


**Fig. 10.** Sweep sequence generating the water storage unit of the example coffeemaker.

factured and purchased parts and the cost of assembly). The cost of the Rowenta FK26-S estimated by Ulrich and Pearson (1993) is $7.09, which is within 3% of the cost generated by using the shape grammar. Further, and perhaps more critically, the shape grammar costing method also provides incremental costs at each stage of the design process in addition to estimating the cost of the final product.

Suppose, however, that the designer decides to use a new water storage sweep sequence shown in Figure 13. The costs resulting from this sequence are shown in Table 5 (again, only the last row must be used).

If the designer, based on the feedback received at each stage, chooses to accept all the design changes discussed above, then the design shown in Figure 14 results. It is similar to the Rowenta FG22-O coffeemaker shown in Fig-

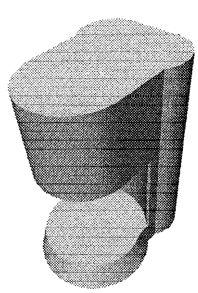

**Fig. 11.** Coffeemaker generated by the choice of shape rules.

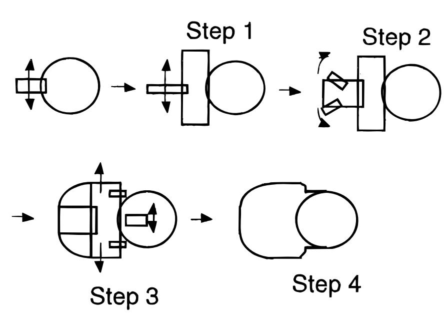


**Fig. 13.** Modified water storage design sequence.

**Table 5.** *Incremental cost of the modified water storage unit during each stage in its design*

| Design Step | Material Cost | Equipment Operating Cost | Tooling Cost | Burden | Labor Cost |
|---|---|---|---|---|---|
| Step 1 | 0.177 | 0.180 | 0.033 | 0.007 | 0.026 |
| Step 2 | 0.236 | 0.199 | 0.035 | 0.010 | 0.024 |
| Step 3 | 0.170 | 0.264 | 0.035 | 0.018 | 0.024 |
| Step 4 (Final design) | 0.163 | 0.264 | 0.035 | 0.018 | 0.024 |

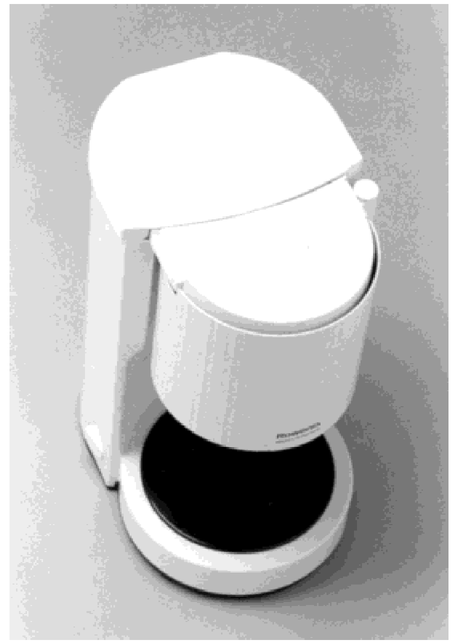

**Fig. 15.** Rowenta FG22-O coffeemaker.

ure 15. The manufacturing cost of the coffeemaker based on our costing scheme is \$5.87. The cost of this coffeemaker as reported by Ulrich and Pearson is \$5.92, which is within about 1% of the cost generated using the shape grammar costing method.

This method was also used to determine the cost of a Mr. Coffee coffeemaker as \$6.30 and the cost of a Proctor-Silex coffeemaker as \$6.12. Even though these costs could not be verified from the literature, they are in the same range as the costs determined above.

## 5. CONCLUDING REMARKS

This paper sets forth a new integrated product design and costing methodology using shape grammars. The validity of this approach is demonstrated by developing cost expressions that are associated with the shape rules of the coffeemaker grammar. By following the grammar, a variety of coffeemakers can be generated; by applying the associated cost equations, the designs can be costed during and after the design process. By modifying the rules or parameters along the generation sequence, new designs can be created. The cost expressions can be used to provide feedback about how those changes affect the cost of the design and thus aid in the generation process. These costs can also be used to guide the generation process by suggesting possible rule choices based on certain cost preferences. The various cost and manufacturing parameters (like labor rate, tool efficiency, etc.) can also be changed to simulate different kinds of production facilities.

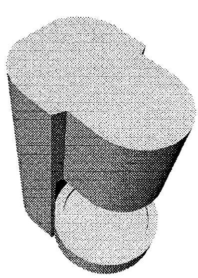

**Fig. 14.** Final shape of the modified coffeemaker generated by the grammar.

This method strikes a balance between the rough parametric estimation techniques based on a few parameters like weight and volume and the extremely detailed bottom-up estimation methods. Such an integrated design and costing methodology would help the designer to quickly identify the key cost drivers by recognizing the design steps that contribute the most to the cost. This information can then be used during the design of new products and the redesign of existing ones, for example, to accept or reject design changes during the generation process rather than after the design is completed.

To automatically generate designs and costs using this methodology, the shape grammar and the cost expressions must be implemented. The geometric representation of the coffeemaker grammar, however, has not been implemented computationally. The cost expressions discussed in this work, on the other hand, have been implemented in the computer package MAPLE. Thus, as the designer chooses the various design rules and parameters, the cost of the design can be updated automatically. It is also possible to optimize the design parameters, based on a designer specified objective like cost, once the designer chooses a sequence of grammar rules that result in a valid design. Once the shape grammar is implemented, the cost expressions could be called automatically depending upon the choice of shape rules, which could in turn be governed by the feedback obtained from the expressions. This strategy can then be used to explore the design space and optimize valid designs with a technique such as shape annealing (Cagan & Mitchell, 1993) that has been previously applied to truss and dome design (Reddy & Cagan, 1995*a*, 1995*b*; Shea & Cagan, 1997; Shea et al., 1997).

This paper discusses how manufacturing cost equations can be associated with the coffeemaker grammar rules; however, the underlying technique is more general and can be used for other applications as well. We believe that the proposed strategy would be even more useful for domains that are less constrained and thus require more parameters and shape rules to be chosen. Note, though, that to associate a performance metric with the grammar rules, it is critical that the metric depend only on the information provided by the shape rules. Given such a metric and a shape grammar, developing expressions associating the metric with the grammar rules is, in principle, straightforward. The left- and right-hand sides of a shape rule have to be examined and parametric expressions modeling the performance metric have to be written in terms of the parameters of the shapes. For example, expressions estimating the coffee brewing time can be developed and associated with the coffeemaker grammar rules. To use such a system, the expressions have to be instantiated along with the shape rules and an updated performance measure obtained as soon as a shape rule is applied. The feedback can then be used to accept or reject design changes or indicate directions for further explorations, as was done in the example discussed earlier. We believe that integrated systems like the one discussed in this paper would increase the usefulness of shape grammar-based generative techniques by helping the designer in making appropriate rule choices during the generation sequence based on a specified performance measure.

In summary, we argue that it is possible to obtain a measure of the performance of a design along with the creation of that design rather than at its completion. The coffeemaker grammar and the manufacturing costs were used to demonstrate the feasibility of such a system in this paper; other integrated systems like this one would make it easier for designers of consumer and other products with frequent but varying production runs to create new designs quickly as well as meet the various performance criteria.

## ACKNOWLEDGMENTS

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## APPENDIX A

### Manufacturing cost components for coffeemakers

The manufacturing cost of a coffeemaker consists of the cost of the parts and the cost of assembling them into a functional product. Some of the parts in the coffeemaker are manufactured in-house while others are purchased.[2] Further, the manufactured parts can either be plastic injection molded parts or metal stamped parts. The cost of manufactured parts

[2]The term *purchased parts* is loosely used to refer to all parts of the coffeemaker that are not designed by the shape grammar.

depends primarily on the material used, the part configuration and size, labor rates, equipment operating costs, tooling costs, burden, equipment depreciation, and the annual volume. Because the parts in this work are designed using the coffeemaker grammar, the rules of the grammar are used to calculate the part sizes and configurations. The part geometries also influence the equipment and tooling costs. Realistic values, based on standard industry practice, are used for the cost of the purchased parts, the material cost, and other financial variables (burden, depreciation, etc.). Industry information is also used to determine the affect of part geometry on manufacturing parameters like cycle time and size of machine required. Assembly cost, which varies among coffeemakers and depends on the overall size and complexity of the design, is calculated based on the approach outlined by Boothroyd and Dewhurst (1989). Note that the cost of tooling, manufacturing cycle times, and assembly times depend on the manufacturing facility used; we assume a medium cost, well-run facility for the calculations in this work, but other cost parameters can as easily be used. The final cost of the coffeemaker is obtained by summing the cost of all the parts and the assembly cost.

The focus of this work is to compare the costs between different product designs and illustrate how applying different shape grammar rules during the generation process results in different costs. Hence, to reduce the variations due to manufacturing processes, all external parameters are assumed to correspond to a well-run, medium-cost environment, as defined by Ulrich and Pearson (1996): a labor rate of \$11/h, useful tool life of 1 year,[3] and annual volume of 1,000,000 are used. In addition, all injection molded parts are assumed to be polypropylene, all metal bases are aluminum, and all heating plates are steel. Thus, the major variations in the cost of manufactured parts come from the part size and part configuration. Next, we will discuss how these variables affect the cost of the parts.

The manufacturing cost of the injection molded parts is generally obtained by adding the material cost, tooling cost, equipment operating cost, labor cost, and burden, which includes equipment depreciation. The cost of material depends on the market rate for polypropylene and the size of the component, that is, the total volume of material contained in the part. A constant market rate of \$0.84/kg is assumed for polypropylene. The density of polypropylene is assumed to be 0.00091 kg/1000 mm$^3$. Thus, the material cost of a part is obtained as:

$$\text{Material Cost} = (\$0.84/\text{kg}) \times (0.00091\ \text{kg}/1000\ \text{mm}^3) \times (\text{part volume}). \tag{A1}$$

The tooling cost for injection molded parts depends on the part size and configuration, number of cavities required in the tool, and the desired surface finish on the parts. The number of cavities in the tool depend on annual part volumes (assumed to be 1,000,000 per year) and quality requirements. Because the volume is relatively low and the parts are all "decorative," that is, the aesthetics of the part is an important quality component, it is assumed that two cavity molds will be used for all the plastic parts. Two cavity molds can easily support the required annual volume while allowing special part handling to avoid scratching and warping of the plastic. In general, parts with no undercuts result in simple two-part molds, whereas parts with undercuts result in more costly molds that require multiple moving parts. The tooling cost per part is then:

$$\text{Tooling Cost per part} = \frac{\text{Total tooling cost}}{\begin{array}{c}\text{Useful tool life based on the number}\\ \text{of parts produced by that tool}\end{array}}. \tag{A2}$$

The cost to operate injection molding equipment depends on the manufacturing cycle time and the size of the equipment. Manufacturing cycle time includes the time required to fill the mold cavities with plastic, the time required to cool the plastic, and the time required to open the mold, eject the parts, and close the mold. Of these, part cooling requires the largest proportion of the time. Total cycle time depends on the number of mold cavities, the material being injected, the ejection system, and part design. Because our focus is on cost variations occurring due to the changes in part design, the number of cavities, material, and ejection system are not considered variables while calculating the cycle time. Also, wall thickness and tolerances are assumed to be the same for all injection molded parts. Thus only the part configuration and size influence the cycle time. For example, parts that have thick sections require a longer cycle time to allow all the material to cool before the part is ejected from the mold. Similarly, parts with long unsupported walls also require a longer cycle time to cool the parts and avoid warping.

Equipment size depends on the projected area of the part and the number of cavities in the mold (which are held fixed at two cavities for each mold in this study). The plane upon which the area is projected for this calculation is the one that is normal to the direction of motion of the die. The projected area corresponds to only the solid areas in the part and excludes any hollow sections. A clamping force of 6.9 kN/cm$^2$ is required to hold the part in place. The equipment size is now given by:

$$\text{Equipment Size} = \text{Projected part area} \times \text{number of cavities} \times 6.9\ \text{kN/cm}^2. \tag{A3}$$

Once the appropriate equipment is selected (based on the equipment size), the operating cost is determined using the relative operating cost matrix shown in Table A1 [from Dewhurst (1988)].

[3] It is assumed that design changes would necessitate tool change about once each year.

**Table A1.** *Equipment size versus relative operating cost for plastic parts*

| Equipment Size (kN) | Relative Operating Cost |
|---|---|
| 0 < 500 | 1.00 |
| 501 < 1000 | 1.08 |
| 1501 < 1600 | 1.29 |
| 1601 < 3200 | 1.71 |

**Table A2.** *Equipment size versus relative operating cost for metal parts*

| Equipment Size (kN) | Relative Operating Cost |
|---|---|
| 0 < 190 | 1.00 |
| 191 < 285 | 1.06 |
| 286 < 400 | 1.18 |
| 401 < 535 | 1.25 |
| 536 < 670 | 1.92 |

The total operating cost for the equipment based on an operating rate of \$23/h (which corresponds to the costs of the facility and energy consumed) is now found as:

$$\text{Operating Cost per Part} = \frac{\$23/\text{h} \times \text{Relative Operating Cost} \times \text{Cycle Time}}{3600\ \text{s/h} \times \text{number of cavities} \times \text{tool efficiency}}. \quad \text{(A4)}$$

Tool efficiency accounts for scrap, downtime, and other miscellaneous losses. Efficiency is assumed to be a constant 99.5% for all pieces of injection molding equipment. This figure corresponds to a medium-cost, well-run facility defined by Ulrich and Pearson.

Labor cost per part depends on the labor rate, part cycle time, number of tool cavities, and the number of machines manned by one operator, that is, the percentage of an operator's time that is dedicated to each part. The number of machines required is estimated initially by determining the total number of plastic parts to be made and the collective base cycle time for these parts. The base cycle time for the filter, water storage unit, base unit, and lid is 156 s. Based on a 3-shift operation, running 235 days per year, three pieces of equipment are required. If additional pieces such as a flow stop are added, then more equipment will be required. The labor cost is calculated as:

$$\text{Labor Cost per Part} = \frac{\$11/\text{h} \times \text{Cycle Time}}{3600\ \text{s/h} \times \text{number of cavities} \times \text{number of machines}}. \quad \text{(A5)}$$

Although burden typically consists of multiple sources, the main component of burden cost is due to equipment depreciation. A straight line depreciation is assumed over 6 years. Thus, the burden depends on the original cost and the useful life of the equipment. Burden cost per part, as defined by Ulrich and Pearson, is now:

$$\text{Burden Cost per Part} = \frac{\$(21873 + 59 \times \text{Equipment Size in kN})}{6\ \text{years} \times 1{,}000{,}000\ \text{parts/year}}. \quad \text{(A6)}$$

It must be kept in mind that the burden cost must be added only once for each machine even if it is used to manufacture more than one part for a coffeemaker.

The cost of the metal stamped parts consists of the same five basic components—material cost, tooling cost, equipment operating cost, labor cost, and burden. Appropriate modifications must be made to certain parameters to account for the differences between the plastic injection molding process and the metal stamping process. The equipment size is determined from Eq. (A7) (where the clamping force is 125 N/mm$^2$):

$$\text{Equipment Size} = 0.7 \times \text{Part Thickness} \times \text{Part Perimeter} \times 125\ \text{N/mm}^2. \quad \text{(A7)}$$

The relative operating cost is obtained from Table A2 [from Dewhurst (1988)].

Cost expressions for the material cost and burden cost for the metal stamping process are now given as:

$$\text{Material Cost per Part} = (\$0.33/\text{kg}) \times (2.7 \times 10^{-6}\ \text{kg/mm}^3) \times (\text{Part Volume}), \quad \text{(A8)}$$

$$\text{Burden Cost per Part} = \frac{\$(30400 + 73 \times \text{Equipment Size in kN})}{6\ \text{years} \times 1{,}000{,}000\ \text{parts/year}}. \quad \text{(A9)}$$

The other cost components are the same as for the injection molding process except that the number of machines per operator for the metal stamping process is 1 and not 3 as for the injection molding process.

## APPENDIX B

### Area and volume expressions

This appendix lists the complete set of area and volume equations associated with the coffeemaker grammar that are needed to determine the manufacturing costs of a design. All the labels refer to the corresponding labels from the coffeemaker grammar of Agarwal and Cagan. The figures shown depict the representative rules from the grammar that are used to develop the expressions that follow the figures. Note

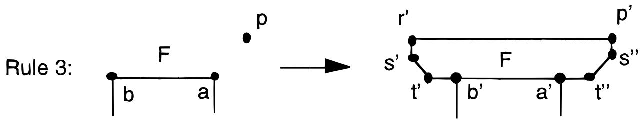



**Fig. B1.** Rule designing the basic cross section of the filter unit. This rule is applied after the initial form decomposition rules have been applied (see Fig. 3).

that these rules do not constitute the complete coffeemaker grammar, but rather only those that directly affect the manufacturing cost. See Agarwal and Cagan (1998) for the complete grammar and the context of these rules. In the figures, $x$ and the $y$ coordinates of a labeled point, say $r'$, are represented as $x'_r$, $y'_r$.

## Filter area and volume equations

$$d_{\text{top}} = |x'_r + x'_p| \tag{B1}$$

$$d_{\text{bot}} = |x'_s - x''_s| \tag{B2}$$

$$d_a = (|x'_r - x'_p| + |x'_s - x''_s|)/2 \tag{B3}$$

$$d_b = d_a - 4 \tag{B4}$$

$$d_c = |x'_t - x''_t| \tag{B5}$$

$$d_d = d_c - 4 \tag{B6}$$

$$d_e = |x'_s - x''_s| \tag{B7}$$

$$d_f = d_e - 4 \tag{B8}$$

$$x'_f = (x'_s + x'_r)/2 \tag{B9}$$

$$y'_f = (y'_s + y'_r)/2 \tag{B10}$$

$$h_a = y'_f - y'_t \tag{B11}$$

$$h_b = .5 \cdot |x''_s - x'_s| \cdot |y''_s - y''_t| / |x''_s - x''_t| \tag{B12}$$

$$h_c = h_b + y''_t - y''_s \tag{B13}$$

$$h_d = y'_f - y'_s \tag{B14}$$

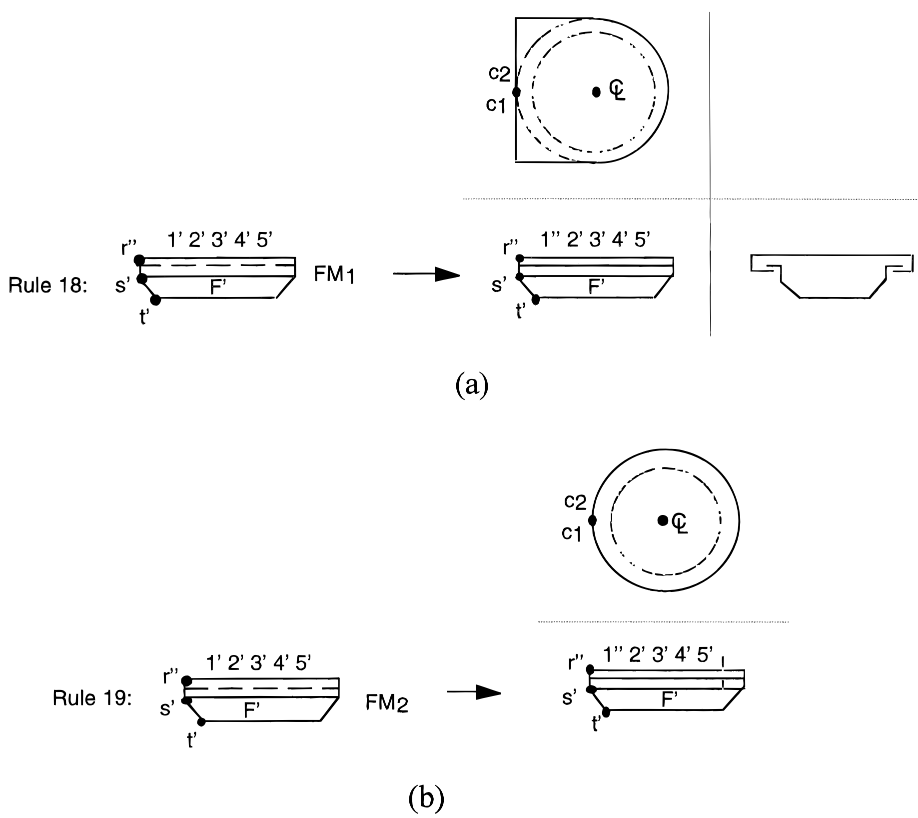



**Fig. B2.** (a) Rule designing a sliding filter; (b) rule designing a rotating filter.

$$\text{filter area} = .8927 \cdot d_a^2 \qquad \text{(sliding filter)} \tag{B15}$$

$$\text{filter top volume} = 1.7854 \cdot d_a^2 - 1.5708 \cdot d_b^2 \qquad \text{(sliding filter)} \tag{B16}$$

$$\text{filter area} = .8390 \cdot d_a^2 \qquad \text{(rotating filter)} \tag{B17}$$

$$\text{filter top volume} = 1.6781 \cdot d_a^2 - 1.5708 \cdot d_b^2 \qquad \text{(rotating filter)} \tag{B18}$$

$$\text{filter bottom volume} = .785 \cdot (d_c^2 - d_{\text{hole}}^2) \qquad \text{(flat filter)} \tag{B19}$$

$$\text{filter side volume} = 5.36 \cdot (d_a^2 - d_b^2) \qquad \text{(flat, sliding filter)} \tag{B20}$$

$$\text{filter cup volume} = .785 \cdot h_a \cdot (d_c^2 - d_d^2) \qquad \text{(flat, sliding filter, } x_s'' = x_t''\text{)} \tag{B21}$$

$$\text{filter cup volume} = .2618 \cdot (h_b \cdot (d_e^2 - d_f^2) - h_c \cdot (d_c^2 - d_d^2)) \qquad \text{(flat sliding filter, } x_s'' \neq x_t''\text{)} \tag{B22}$$

$$\text{filter side volume} = 0 \qquad \text{(flat, conical filter, } x_s'' = x_t''\text{)} \tag{B23}$$

$$\text{filter cup volume} = .785 \cdot h_a \cdot (d_a^2 - d_b^2) \qquad \text{(flat, rotating filter, } x_s'' = x_t''\text{)} \tag{B24}$$

$$\text{filter cup volume} = .785 \cdot (h_b \cdot (d_e^2 - d_f^2) - h_c \cdot (d_c^2 - d_d^2)) \qquad \text{(flat rotating filter, } x_s'' \neq x_t''\text{)} \tag{B25}$$

$$\text{filter bottom volume} = 0 \qquad \text{(conical filter } x_s'' = x_t''\text{)} \tag{B26}$$

$$\text{filter cup volume} = .2618 \cdot (h_b \cdot (d_e^2 - d_f^2) - h_c \cdot (d_{\text{hole}}^2 - (d_{\text{hole}} - 4)^2) \qquad \text{(conical filter, } x_s'' = x_t''\text{)} \tag{B27}$$

$$\text{filter side volume} = .785 \cdot h_a \cdot (d_e^2 - d_f^2) + 5.36 \cdot (d_a^2 - d_b^2) \qquad \text{(conical sliding filter } x_s'' = x_t''\text{)} \tag{B28}$$

$$\text{filter side volume} = .785 \cdot h_a \cdot (d_a^2 - d_b^2) \qquad \text{(conical rotating filter } x_s'' = x_t''\text{)} \tag{B29}$$

$$\text{filter cup volume} = .2618 \cdot (d_e^2 \cdot (h_b - h_a) - h_c \cdot d_c^2) \qquad \text{(conical filter } x_s'' \neq x_t''\text{)} \tag{B30}$$

$$\text{filter bottom volume} = .785 \cdot (d_c^2 - d_{\text{hole}}^2) \qquad \text{(conical sliding filter } x_s'' \neq x_t''\text{)} \tag{B31}$$

$$\text{filter side volume} = 5.36 \cdot (d_a^2 - d_b^2) \qquad \text{(conical sliding filter } x_s'' \neq x_t''\text{)} \tag{B32}$$

$$\text{filter bottom volume} = 0 \qquad \text{(conical rotating filter } x_s'' \neq x_t''\text{)} \tag{B33}$$

$$\text{filter side volume} = .785 \cdot h_d \cdot (d_a^2 - d_b^2) \qquad \text{(conical rotating filter } x_s'' \neq x_t''\text{)} \tag{B34}$$

$$\text{filter volume} = \text{filter top volume} + \text{filter side volume} + \text{filter cup volume} + \text{filter bottom volume} \qquad \text{(no flow stop)} \tag{B35}$$

$$\text{filter volume} = \text{filter top volume} + \text{filter side volume} + \text{filter cup volume} + \text{filter bottom volume} + 15 \cdot d_c + 20 \qquad \text{(flow stop)} \tag{B36}$$

## Initial base unit area and volume equations

$$d_{\text{plate}} = |x_c' - x_d'| \tag{B37}$$

$$d_{\text{major\_outer}} = |x_z' - x_v'| \tag{B38}$$

$$d_{\text{major\_inner}} = |x_y' - x_m'| \tag{B39}$$

$$\text{base area} = \pi \cdot (d_{\text{major\_outer}} \cdot d_{\text{minor\_outer}} - d_{\text{plate}}^2)/4 \tag{B40}$$

$$\text{base top volume} = 2 \cdot \text{base area} \tag{B41}$$

$$\text{base side volume} = |y_y' - y_w'| \cdot (((d_{\text{major\_outer}}^2 + d_{\text{minor\_outer}}^2)/2)^{.5} + ((d_{\text{major\_inner}}^2 + d_{\text{minor\_inner}}^2)/2)^{.5})(x_y' = x_w' \text{ or } y_w' = y_v') \tag{B42}$$

$$\begin{aligned} \text{base side volume} = {} & |y_y' - y_w'| \cdot ((d_{\text{major\_outer}}^2 + d_{\text{minor\_outer}}^2)^{.5}) + |y_w' - y_v'| \cdot ((d_{\text{major\_outer}}^2 + d_{\text{minor\_outer}}^2)/2)^{.5} \\ & + |y_w' - y_v'| \cdot (((d_{\text{major\_outer}}^2 + d_{\text{minor\_outer}}^2)/2)^{.5} + ((d_{\text{major\_inner}}^2 + d_{\text{minor\_inner}}^2)/2)^{.5})(x_y' \neq x_w' \text{ and } y_w' \neq y_v') \end{aligned} \tag{B43}$$

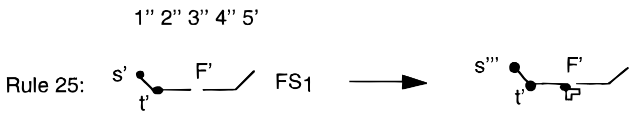



**Fig. B3.** Rule adding a flow stop unit to the filter unit.

$$a'_k = [(v'_{kx} - C_x)^2 + (v'_{kz} - C_z)^2]^{.5} \tag{B44}$$

$$a_k = [(v_{kx} - C_x)^2 + (v_{kz} - C_z)^2]^{.5} \tag{B45}$$

$$a'_n = [(v'_{nx} - C_x)^2 + (v'_{nz} - C_z)^2]^{.5} \tag{B46}$$

$$a_n = [(v_{nx} - C_x)^2 + (v_{nz} - C_z)^2]^{.5} \tag{B47}$$

$$b'_k = [(v'_{(k+1)x} - C_x)^2 + (v'_{(k+1)z} - C_z)^2]^{.5} \tag{B48}$$

$$b_k = [(v_{(k+1)x} - C_x)^2 + (v_{(k+1)z} - C_z)^2]^{.5} \tag{B49}$$

$$b'_n = [(v'_{1x} - C_x)^2 + (v'_{1z} - C_z)^2]^{.5} \tag{B50}$$

$$b_n = [(v_{1x} - C_x)^2 + (v_{1z} - C_z)^2]^{.5} \tag{B51}$$

$$c'_k = [(v'_{kx} - v'_{(k+1)x})^2 + (v'_{kz} - v'_{(k+1)z})^2]^{.5} \tag{B52}$$

$$c_k = [(v_{kx} - v_{(k+1)x})^2 + (v_{kz} - v_{(k+1)z})^2]^{.5} \tag{B53}$$

$$c'_n = [(v'_{1x} - v'_{nx})^2 + (v'_{1z} - v'_{nz})^2]^{.5} \tag{B54}$$

$$c_n = [(v_{1x} - v_{nx})^2 + (v_{1z} - v_{nz})^2]^{.5} \tag{B55}$$

$$s'_k = .5(a'_k + b'_k + c'_k) \tag{B56}$$

$$s_k = .5(a_k + b_k + c_k) \tag{B57}$$

$$s'_n = .5(a'_n + b'_n + c'_n) \tag{B58}$$

$$s_n = .5(a_n + b_n + c_n) \tag{B59}$$

$$A'_k = [s'_k(s'_k - a'_k)(s'_k - b'_k)(s'_k - c'_k)]^{.5} \tag{B60}$$

$$A_k = [s_k(s_k - a_k)(s_k - b_k)(s_k - c_k)]^{.5} \tag{B61}$$

$$A'_n = [s'_n(s'_n - a'_n)(s'_n - b'_n)(s'_n - c'_n)]^{.5} \tag{B62}$$

$$A_n = [s_n(s_n - a_n)(s_n - b_n)(s_n - c_n)]^{.5} \tag{B63}$$

$$\text{base area: } A' = \Sigma A'_k + A'_n - \pi \cdot \mathrm{d}^2_{\text{plate}}/4 \tag{B64}$$

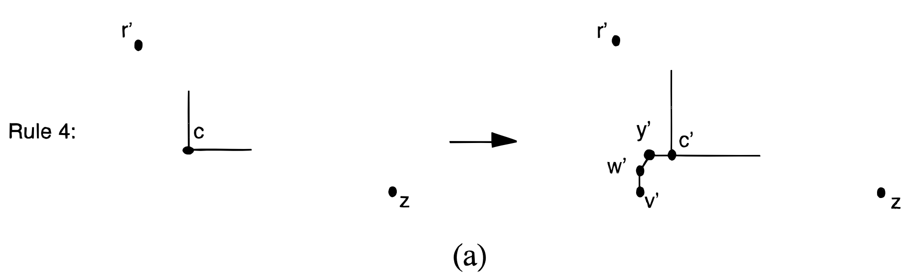
$$\text{base top area: } A = \Sigma A_k + A_n - \pi \cdot \mathrm{d}^2_{\text{plate}}/4 \tag{B65}$$

$$\text{base top volume} = 2 \cdot \text{base top area} - \pi \cdot d^2_{\text{plate}}/2 \tag{B66}$$

$$\begin{aligned}\text{base side volume} = {} & \Sigma\{(c_k + c'_k) \cdot |y'_y - y'_v|\} + (c_n + c'_n) \\ & \cdot |y'_y - y'_v| (x'_y = x'_w \text{ or } y'_w = y'_v)\end{aligned} \tag{B67}$$

$$\begin{aligned}\text{base side volume} = {} & \Sigma\{((c_k + c'_k) \cdot |y'_y - y'_w| + 2 \cdot c'_k \cdot |y'_w - y'_v|)\} \\ & + (c_n + c'_n) \cdot |y'_y - y'_v| + 2 \cdot c'_n \cdot |y'_w - y'_v|) \\ & (x'_y \neq x'_w \text{ and } y'_w \neq y'_v)\end{aligned} \tag{B68}$$

$$\text{base volume} = \text{base top volume} + \text{base side volume} \tag{B69}$$



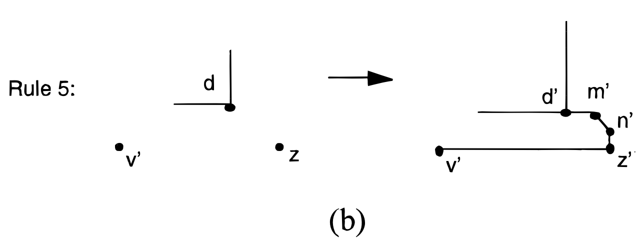




**Fig. B4.** (a) and (b) Rules designing the basic cross section of the base unit. These rules are also applied after the initial form decomposition rules have been applied (see Fig. 3).

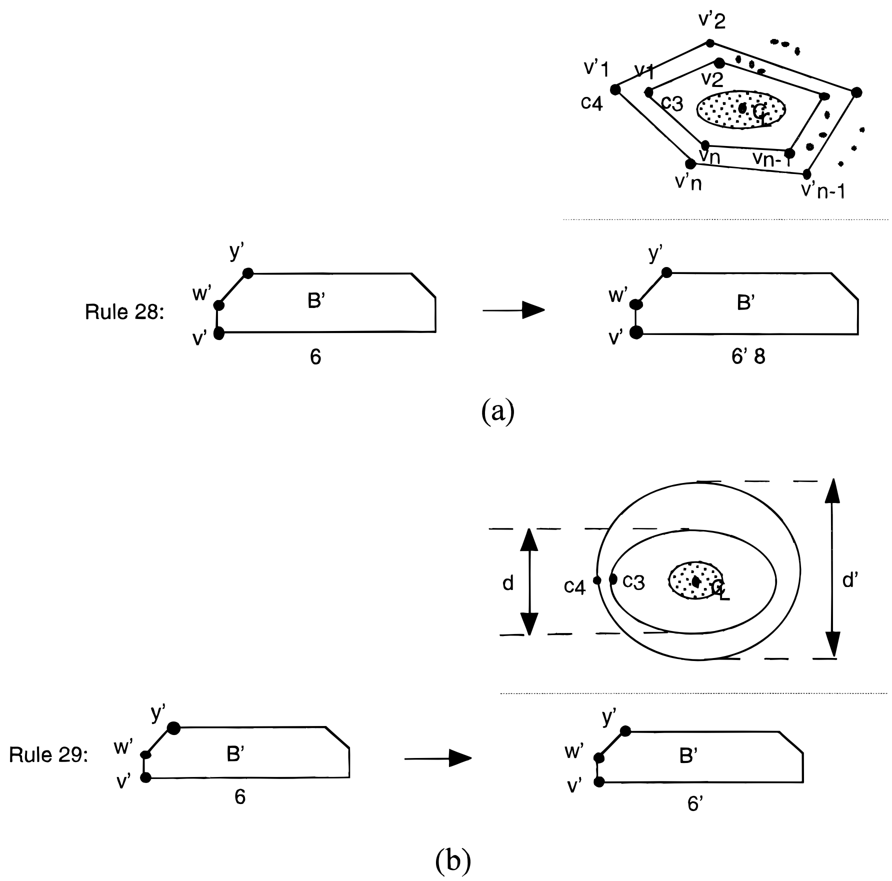



**Fig. B5.** (a) Rule designing a polygonal base unit; (b) rule designing an elliptical base unit.

## Sweep area and volume equations

Straight Square Sweep

$$h_{\text{first}} = h(z_{\text{first}}) \tag{B70}$$

$$d_{\text{first}} = d(z_{\text{first}}) \tag{B71}$$

$d(z)$ increasing slope (first corresponds to the smaller square and second to the larger)

$$z_{1\text{top}} = z_{\text{first}} - h_{\text{first}}/2 \tag{B72}$$

$$y_{1\text{top}} = r + d_{\text{first}}/2 \tag{B73}$$

$$z_{2\text{top}} = z_{\text{second}} - h_{\text{second}}/2 \tag{B74}$$

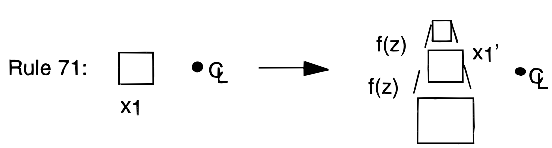



**Fig. B6.** Representative rule depicting a straight square sweep.

$$y_{2\text{top}} = r + d_{\text{second}}/2 \tag{B75}$$

$$z_{1\text{bot}} = z_{\text{first}} - h_{\text{first}}/2 \tag{B76}$$

$$y_{1\text{bot}} = r - d_{\text{first}}/2 \tag{B77}$$

$$z_{2\text{bot}} = z_{\text{second}} - h_{\text{second}}/2 \tag{B78}$$

$$y_{2bot} = r - d_{\text{second}}/2 \tag{B79}$$

$$\begin{aligned}\text{sweep area} = {} & \Sigma\{(z_{2\text{top}} - z_{1\text{top}})\cdot(y_{1\text{top}} - r + y_{2\text{top}} - r)\} \\ & + d(z_{\text{final}})\cdot h(z_{\text{final}})\end{aligned} \tag{B80}$$

$$\begin{aligned}\text{sweep perimeter} = {} & \Sigma\{2\cdot((y_{1\text{top}} - r - (y_{2\text{top}} - r))^2 \\ & + (z_{2\text{top}} - z_{1\text{top}})^2)^{.5}\} + 2\cdot h(z_{\text{final}}) \\ & + d(z_{\text{final}})\end{aligned} \tag{B81}$$

$d(z)$ decreasing slope (first corresponds to the larger square and second to the smaller)

$$z_{1\text{top}} = z_{\text{first}} + h_{\text{first}}/2 \tag{B82}$$

$$y_{1\text{top}} = r + d_{\text{first}}/2 \tag{B83}$$

$$z_{2top} = z_{second} + h_{second}/2 \tag{B84}$$

$$y_{2top} = r + d_{second}/2 \tag{B85}$$

$$z_{1bot} = z_{first} + h_{first}/2 \tag{B86}$$

$$y_{1bot} = r - d_{first}/2 \tag{B87}$$

$$z_{2bot} = z_{second} + h_{second}/2 \tag{B88}$$

$$y_{2bot} = r - d_{second}/2 \tag{B89}$$

$$\text{sweep area} = \Sigma\{(z_{2top} - z_{1top})\cdot(y_{1top} - r + y_{2top} - r)\} + h(z_{initial})\cdot d(z_{initial}) \tag{B90}$$

$$\text{sweep perimeter} = \Sigma\{2\cdot((y_{1top} - r - (y_{2top} - r))^2 + (z_{2top} - z_{1top})^2)^{.5}\} + 2\cdot h(z_{initial}) \tag{B91}$$

Straight Circle Sweep

$$ro_{first} = ro(z_{first}) \tag{B92}$$

$$ro_{second} = ro(z_{second}) \tag{B93}$$

$$c = ((z_{first} - z_{second})^2 - (ro_{first} - ro_{second})^2)^{.5} \tag{B94}$$

$$ro_{end} = ro(z_{max}) \text{ and } ro(z_{min}) \tag{B95}$$

$ro(z)$ decreasing or constant (first corresponds to the larger circle and second to the smaller)

$$\text{theta} = \cos^{-1}((ro_{first} - ro_{second})/(z_{second} - z_{first})) \tag{B96}$$

$$\text{theta}_{end} = \text{theta at } z_{second} = z_{max} \text{ and } z_{min} \tag{B97}$$

$$\text{sweep area} = \Sigma\{c\cdot(ro_{first} + ro_{second})\} + \text{theta}_{end}\cdot ro_{end}^2 \tag{B98}$$

$$\text{sweep perimeter} = \Sigma\{2\cdot c\} + 2\cdot\text{theta}_{end}\cdot ro_{end} \tag{B99}$$

$ro(z)$ increasing (first corresponds to the smaller circle and second to the larger)

$$\text{theta} = \cos^{-1}((ro_{second} - ro_{first})/(z_{second} - z_{first})) \tag{B100}$$

$$\text{sweep area} = \Sigma\{c\cdot(ro_{first} + ro_{second})\} + ro_{end}^2\cdot(\pi - \text{theta}_{end}) \tag{B101}$$

$$\text{sweep perimeter} = \Sigma\{2\cdot c\} + 2\cdot(\pi - \text{theta})\cdot ro_{end} \tag{B102}$$

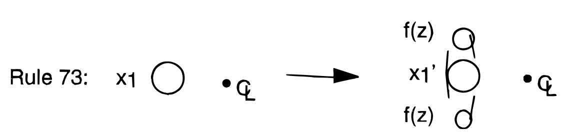



**Fig. B7.** Representative rule depicting a straight circle sweep.

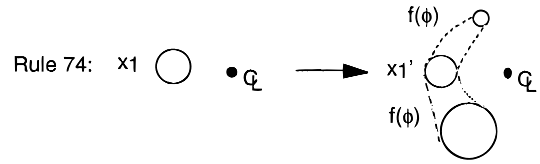



**Fig. B8.** Representative angular circle sweep rule.

Angular Circle Sweep

$$r_{first} = r(\phi_{first}) \tag{B103}$$

$$r_{second} = r(\phi_{second}) \tag{B104}$$

$$ro_{first} = r(\phi_{first}) \tag{B105}$$

$$ro_{second} = r(\phi_{second}) \tag{B106}$$

$$ro_{end} = ro(z_{max}) \text{ and } ro(z_{min}) \tag{B107}$$

$$c = (r_{first}^2 + r_{second}^2 - 2\cdot r_{first}\cdot r_{second}\cdot\cos(\phi_{second} - \phi_{first}))^{.5} \tag{B108}$$

$$L = (c^2 - (ro_{first} - ro_{second})2)^{.5} \tag{B109}$$

$$\text{theta}_{end} = \text{theta at } z_{max} \text{ and } z_{min} \tag{B110}$$

$ro(\phi)$ decreasing or constant

$$\text{sweep area} = \Sigma\{L\cdot(ro_{first} + ro_{second})\} + \text{theta}_{end}\cdot ro_{end}^2 \tag{B111}$$

$$\text{sweep perimeter} = \Sigma\{2\cdot L\} + 2\cdot\text{theta}_{end}\cdot ro_{end} \tag{B112}$$

$ro(\phi)$ increasing

$$\text{sweep area} = \Sigma\{L\cdot(ro_{first} + ro_{second})\} + (\pi - \text{theta}_{end})\cdot ro_{end}^2 \tag{B113}$$

$$\text{sweep perimeter} = \Sigma\{2\cdot L\} + 2\cdot(\pi - \text{theta}_{end})\cdot ro_{end} \tag{B114}$$

Angular Square Sweep

$$h_{initial} = h(\phi_{initial}) \tag{B115}$$

$$d_{initial} = d(\phi_{initial}) \tag{B116}$$

$$r_{initial} = r(\phi_{initial}) \tag{B117}$$

$$\text{diagonal}_{initial} = (h_{initial}^2 + d_{initial}^2)^{.5} \tag{B118}$$

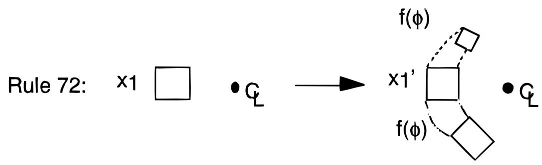



**Fig. B9.** Representative angular square sweep rule.

$$h_{\text{final}} = h(\phi_{\text{final}}) \quad \text{(B119)}$$

$$d_{\text{final}} = d(\phi_{\text{final}}) \quad \text{(B120)}$$

$$r_{\text{final}} = r(\phi_{\text{final}}) \quad \text{(B121)}$$

$$\text{diagonal}_{\text{final}} = (h_{\text{final}}^2 + d_{\text{final}}^2)^{.5} \quad \text{(B122)}$$

$$\text{angle}_1 = \tan^{-1}(.5\cdot h_{\text{initial}}/(r_{\text{initial}} + .5\cdot d_{\text{initial}})) \quad \text{(B123)}$$

$$\text{angle}_2 = \tan^{-1}(.5\cdot h_{\text{initial}}/(r_{\text{initial}} - .5\cdot d_{\text{initial}})) \quad \text{(B124)}$$

$$\text{angle}_3 = \tan^{-1}(.5\cdot h_{\text{final}}/(r_{\text{final}} + .5\cdot d_{\text{final}})) \quad \text{(B125)}$$

$$\text{angle}_4 = \tan^{-1}(.5\cdot h_{\text{final}}/(r_{\text{final}} - .5\cdot d_{\text{final}})) \quad \text{(B126)}$$

$$r_1 = (.5\cdot h_{\text{initial}})^2 + (r_{\text{initial}} + .5\cdot d_{\text{initial}})^2 \quad \text{(B127)}$$

$$r_2 = (.5\cdot h_{\text{initial}})^2 + (r_{\text{initial}} - .5\cdot d_{\text{initial}})^2 \quad \text{(B128)}$$

$$r_3 = (.5\cdot h_{\text{final}})^2 + (r_{\text{final}} + .5\cdot d_{\text{final}})^2 \quad \text{(B129)}$$

$$r_4 = (.5\cdot h_{\text{final}})^2 + (r_{\text{final}} - .5\cdot d_{\text{final}})^2 \quad \text{(B130)}$$

$$z_{\text{right1}} = r_1\cdot\sin(\phi_{\text{initial}} + \text{angle}_1) \quad \text{(B131)}$$

$$y_{\text{right1}} = r_1\cdot\cos(\phi_{\text{initial}} + \text{angle}_1) \quad \text{(B132)}$$

$$z_{\text{left1}} = r_1\cdot\sin(\phi_{\text{initial}} - \text{angle}_1) \quad \text{(B133)}$$

$$y_{\text{left1}} = r_1\cdot\cos(\phi_{\text{initial}} - \text{angle}_1) \quad \text{(B134)}$$

$$z_{\text{right2}} = r_2\cdot\sin(\phi_{\text{initial}} + \text{angle}_2) \quad \text{(B135)}$$

$$y_{\text{right2}} = r_2\cdot\cos(\phi_{\text{initial}} + \text{angle}_2) \quad \text{(B136)}$$

$$z_{\text{left2}} = r_2\cdot\sin(\phi_{\text{initial}} - \text{angle}_2) \quad \text{(B137)}$$

$$y_{\text{left2}} = r_2\cdot\cos(\phi_{\text{initial}} - \text{angle}_2) \quad \text{(B138)}$$

$$z_{\text{right3}} = r_3\cdot\sin(\phi_{\text{final}} + \text{angle}_3) \quad \text{(B139)}$$

$$y_{\text{right3}} = r_3\cdot\cos(\phi_{\text{final}} + \text{angle}_3) \quad \text{(B140)}$$

$$z_{\text{left3}} = r_3\cdot\sin(\phi_{\text{final}} - \text{angle}_3) \quad \text{(B141)}$$

$$y_{\text{left3}} = r_3\cdot\cos(\phi_{\text{final}} - \text{angle}_3) \quad \text{(B142)}$$

$$z_{\text{right4}} = r_4\cdot\sin(\phi_{\text{final}} + \text{angle}_4) \quad \text{(B143)}$$

$$y_{\text{right4}} = r_4\cdot\cos(\phi_{\text{final}} + \text{angle}_4) \quad \text{(B144)}$$

$$z_{\text{left4}} = r_4\cdot\sin(\phi_{\text{final}} - \text{angle}_4) \quad \text{(B145)}$$

$$y_{\text{left4}} = r_4\cdot\cos(\phi_{\text{final}} - \text{angle}_4) \quad \text{(B146)}$$

$$\text{slope}_{\text{top1}} = (y_{\text{left3}} - y_{\text{left1}})/(z_{\text{left3}} - z_{\text{left1}}) \quad \text{(B147)}$$

$$\text{slope}_{\text{top2}} = (y_{\text{right3}} - y_{\text{left1}})/(z_{\text{right3}} - z_{\text{left1}}) \quad \text{(B148)}$$

$$\text{slope}_{\text{top3}} = (y_{\text{left3}} - y_{\text{right1}})/(z_{\text{left3}} - z_{\text{right1}}) \quad \text{(B149)}$$

$$\text{slope}_{\text{top4}} = (y_{\text{right3}} - y_{\text{right1}})/(z_{\text{right3}} - z_{\text{right1}}) \quad \text{(B150)}$$

$$y_{\text{top1}} = y_{\text{left1}} + \text{slope}_{\text{top1}}(z - z_{\text{left1}}) \quad \text{(B151)}$$

$$y_{\text{top2}} = y_{\text{left1}} + \text{slope}_{\text{top2}}(z - z_{\text{left1}}) \quad \text{(B152)}$$

$$y_{\text{top3}} = y_{\text{right1}} + \text{slope}_{\text{top3}}(z - z_{\text{right1}}) \quad \text{(B153)}$$

$$y_{\text{top4}} = y_{\text{right1}} + \text{slope}_{\text{top4}}(z - z_{\text{right1}}) \quad \text{(B154)}$$

$$\text{slope}_{\text{bot1}} = (y_{\text{left4}} - y_{\text{left2}})/(z_{\text{left4}} - z_{\text{left2}}) \quad \text{(B155)}$$

$$\text{slope}_{\text{bot2}} = (y_{\text{right4}} - y_{\text{left2}})/(z_{\text{right4}} - z_{\text{left2}}) \quad \text{(B156)}$$

$$\text{slope}_{\text{bot3}} = (y_{\text{left4}} - y_{\text{right2}})/(z_{\text{left4}} - z_{\text{right2}}) \quad \text{(B157)}$$

$$\text{slope}_{\text{bot4}} = (y_{\text{right4}} - y_{\text{right2}})/(z_{\text{right4}} - z_{\text{right2}}) \quad \text{(B158)}$$

$$y_{\text{bot1}} = y_{\text{left2}} + \text{slope}_{\text{bot1}}(z - z_{\text{left2}}) \quad \text{(B159)}$$

$$y_{\text{bot2}} = y_{\text{left2}} + \text{slope}_{\text{bot2}}(z - z_{\text{left2}}) \quad \text{(B160)}$$

$$y_{\text{bot3}} = y_{\text{right2}} + \text{slope}_{\text{bot3}}(z - z_{\text{right2}}) \quad \text{(B161)}$$

$$y_{\text{bot4}} = y_{\text{right2}} + \text{slope}_{\text{bot4}}(z - z_{\text{right2}}) \quad \text{(B162)}$$

$$y_{\text{test1a}} = y_{\text{top1}}(z = z_{\text{right1}}) = y_{\text{left1}} + \text{slope}_{top1}(z_{\text{right1}} - z_{\text{left1}}) \quad \text{(B163)}$$

$$y_{\text{test1b}} = y_{\text{top1}}(z = z_{\text{right3}}) = y_{\text{left1}} + \text{slope}_{\text{top1}}(z_{\text{right3}} - z_{\text{left1}}) \quad \text{(B164)}$$

if $y_{\text{test1a}} \geqq y_{\text{right1}}$ and $y_{\text{test1b}} \geqq y_{\text{right3}}$

{

$$\text{lower}_{\text{top}} = -\text{angle}_1 \quad \text{(B165)}$$

$$\text{upper}_{\text{top}} = \phi_{\text{final}} - \text{angle}_3 \quad \text{(B166)}$$

$$\text{line}_{\text{top}} = 11 \quad \text{(B167)}$$

}

$$y_{\text{test2a}} = y_{\text{top2}}(z = z_{\text{right1}}) = y_{\text{left1}} + \text{slope}_{\text{top2}}(z_{\text{right1}} - z_{\text{left1}}) \tag{B168}$$

$$y_{\text{test2b}} = y_{\text{top2}}(z = z_{\text{left3}}) = y_{\text{left1}} + \text{slope}_{\text{top2}}(z_{\text{left3}} - z_{\text{left1}}) \tag{B169}$$

if $y_{\text{test2a}} \geqq y_{\text{right1}}$ and $y_{\text{test2b}} \geqq y_{\text{left3}}$

{

$$\text{lower}_{\text{top}} = -\text{angle}_1 \tag{B170}$$

$$\text{upper}_{\text{top}} = \phi_{\text{final}} + \text{angle}_3 \tag{B171}$$

$$\text{line}_{\text{top}} = 12 \tag{B172}$$

}

$$y_{\text{test3a}} = y_{\text{top3}}(z = z_{\text{left1}}) = y_{\text{right1}} + \text{slope}_{\text{top3}}(z_{\text{left1}} - z_{\text{right1}}) \tag{B173}$$

$$y_{\text{test3b}} = y_{\text{top3}}(z = z_{\text{right3}}) = y_{\text{right1}} + \text{slope}_{\text{top3}}(z_{\text{right3}} - z_{\text{right1}}) \tag{B174}$$

if $y_{\text{test3a}} \geqq y_{\text{left1}}$ and $y_{\text{test3b}} \geqq y_{\text{right3}}$

{

$$\text{lower}_{\text{top}} = \text{angle}_1 \tag{B175}$$

$$\text{upper}_{\text{top}} = \phi_{\text{final}} - \text{angle}_3 \tag{B176}$$

$$\text{line}_{\text{top}} = 21 \tag{B177}$$

}

$$y_{\text{test4a}} = y_{\text{top4}}(z = z_{\text{left1}}) = y_{\text{right1}} + \text{slope}_{\text{top4}}(z_{\text{left1}} - z_{\text{right1}}) \tag{B178}$$

$$y_{\text{test4b}} = y_{\text{top4}}(z = z_{\text{left3}}) = y_{\text{right1}} + \text{slope}_{\text{top4}}(z_{\text{left}} - z_{\text{right1}}) \tag{B179}$$

if $y_{\text{test4a}} \geqq y_{\text{left1}}$ and $y_{\text{test4b}} \geqq y_{\text{left3}}$

{

$$\text{lower}_{\text{top}} = \text{angle}_1 \tag{B180}$$

$$\text{upper}_{\text{top}} = \phi_{\text{final}} + \text{angle}_3 \tag{B181}$$

$$\text{line}_{\text{top}} = 22 \tag{B182}$$

}

$$y_{\text{test5a}} = y_{\text{bot1}}(z = z_{\text{right2}}) = y_{\text{left2}} + \text{slope}_{\text{bot1}}(z_{\text{right2}} - z_{\text{left2}}) \tag{B183}$$

$$y_{\text{test5b}} = y_{\text{bot1}}(z = z_{\text{right4}}) = y_{\text{left2}} + \text{slope}_{\text{bot1}}(z_{\text{right4}} - z_{\text{left2}}) \tag{B184}$$

if $y_{\text{test5a}} \geqq y_{\text{right2}}$ and $y_{\text{test5b}} \geqq y_{\text{right3}}$

{

$$\text{lower}_{\text{bot}} = -\text{angle}_2 \tag{B185}$$

$$\text{upper}_{\text{bot}} = \phi_{\text{final}} - \text{angle}_4 \tag{B186}$$

$$\text{line}_{\text{bot}} = 11 \tag{B187}$$

}

$$y_{\text{test6a}} = y_{\text{bot2}}(z = z_{\text{right2}}) = y_{\text{left2}} + \text{slope}_{\text{bot2}}(z_{\text{right2}} - z_{\text{left2}}) \tag{B188}$$

$$y_{\text{test6b}} = y_{\text{bot2}}(z = z_{\text{left4}}) = y_{\text{left2}} + \text{slope}_{\text{bot2}}(z_{\text{left4}} - z_{\text{left2}}) \tag{B189}$$

if $y_{\text{test6a}} \geqq y_{\text{right1}}$ and $y_{\text{test6b}} \geqq y_{\text{left3}}$

{

$$\text{lower}_{\text{bot}} = -\text{angle}_2 \tag{B190}$$

$$\text{upper}_{\text{bot}} = \phi_{\text{final}} + \text{angle}_4 \tag{B191}$$

$$\text{line}_{\text{bot}} = 12 \tag{B192}$$

}

$$y_{\text{test7a}} = y_{\text{bot3}}(z = z_{\text{left2}}) = y_{\text{right2}} + \text{slope}_{\text{bot3}}(z_{\text{left2}} - z_{\text{right2}}) \tag{B193}$$

$$y_{\text{test7b}} = y_{\text{bot3}}(z = z_{\text{right4}}) = y_{\text{right2}} + \text{slope}_{\text{bot3}}(z_{\text{right4}} - z_{\text{right2}}) \tag{B194}$$

if $y_{\text{test7a}} \geqq y_{\text{left1}}$ and $y_{\text{test7b}} \geqq y_{\text{right3}}$

{

$$\text{lower}_{\text{bot}} = \text{angle}_2 \tag{B195}$$

$$\text{upper}_{\text{bot}} = \phi_{\text{final}} - \text{angle}_4 \tag{B196}$$

$$\text{line}_{\text{bot}} = 21 \tag{B197}$$

}

$$y_{\text{test8a}} = y_{\text{bot4}}(z = z_{\text{left2}}) = y_{\text{right2}} + \text{slope}_{\text{bot4}}(z_{\text{left2}} - z_{\text{right2}}) \tag{B198}$$

$$y_{\text{test8b}} = y_{\text{bot4}}(z = z_{\text{left4}}) = y_{\text{right2}} + \text{slope}_{\text{bot4}}(z_{\text{left4}} - z_{\text{right2}}) \tag{B199}$$

if $y_{\text{test8a}} \geqq y_{\text{left1}}$ and $y_{\text{test8b}} \geqq y_{\text{left3}}$

{

$$\text{lower}_{\text{bot}} = \text{angle}_2 \tag{B200}$$

$$\text{upper}_{\text{bot}} = \phi_{\text{final}} + \text{angle}_4 \tag{B201}$$

$$\text{line}_{\text{bot}} = 22 \tag{B202}$$

}

$$\phi_{\text{temp\_top}} = (\phi_{\text{top}} - \text{lower}_{\text{top}})\cdot\phi_{\text{final}}/\text{upper}_{\text{top}} \tag{B203}$$

$$\phi_{\text{temp\_bot}} = (\phi_{\text{bot}} - \text{lower}_{\text{bot}})\cdot\phi_{\text{final}}/\text{upper}_{\text{bot}} \tag{B204}$$

$$\begin{aligned} r_{\text{top}} = {} & r(\phi = \phi_{\text{top}}) + .5\cdot d_{\text{final}} \\ & \cdot(\phi_{\text{top}} - \text{lower}_{\text{top}})/\text{upper}_{\text{top}} - .5\cdot d_{\text{initial}} \\ & \cdot(\text{upper}_{\text{top}} + \text{lower}_{\text{top}} - \phi_{\text{top}})/\text{upper}_{\text{top}} \end{aligned} \tag{B205}$$

$$r_{\text{top\_initial}} = r_{\text{top}}(\phi_{\text{top}} = \text{lower}_{\text{top}}) \tag{B206}$$

$$r_{\text{top\_final}} = r_{\text{top}}(\phi_{\text{top}} = \text{upper}_{\text{top}}) \tag{B207}$$

$$\begin{aligned} r_{\text{bot}} = {} & r(\phi = \phi_{\text{bot}}) - .5\cdot d_{\text{final}} \\ & \cdot(\phi_{\text{bot}} - \text{lower}_{\text{bot}})/\text{upper}_{\text{bot}} - .5\cdot d_{\text{initial}} \\ & \cdot(\text{upper}_{\text{bot}} + \text{lower}_{\text{bot}} - \phi_{\text{bot}})/\text{upper}_{\text{bot}} \end{aligned} \tag{B208}$$

$$r_{\text{bot\_initial}} = r_{\text{bot}}(\phi_{\text{bot}} = \text{lower}_{\text{bot}}) \tag{B209}$$

$$r_{\text{bot\_final}} = r_{\text{bot}}(\phi_{\text{bot}} = \text{upper}_{\text{bot}}) \tag{B210}$$

$$dr_{\text{top}} = d(r_{\text{top}})/d\phi_{\text{top}} \tag{B211}$$

$$dr_{\text{bot}} = d(r_{\text{bot}})/d\phi_{\text{bot}} \tag{B212}$$

$$ds_{\text{top}} = (r_{\text{top}}^2 + dr_{\text{top}}^2)^{.5} \tag{B213}$$

$$ds_{\text{bot}} = (r_{\text{bot}}^2 + dr_{\text{bot}}^2)^{.5} \tag{B214}$$

$$a_{\text{initial}} = r_{\text{bot\_initial}} \tag{B215}$$

$$b_{\text{initial}} = r_{\text{top\_initial}} \tag{B216}$$

$$a_{\text{final}} = r_{\text{bot\_final}} \tag{B217}$$

$$b_{\text{final}} = r_{\text{top\_final}} \tag{B218}$$

$$\text{integral}_{\text{top}} = \text{int}(r_{\text{top}}, \phi_{\text{top}} = \text{lower}_{\text{top}}..\text{upper}_{\text{top}}) \tag{B219}$$

$$\text{integral}_{\text{bot}} = \text{int}(r_{\text{bot}}, \phi_{\text{bot}} = \text{lower}_{\text{bot}}..\text{upper}_{\text{bot}}) \tag{B220}$$

if $\text{line}_{\text{top}} = 11$ and $\text{line}_{\text{bot}} = 11$

{

$$c_{\text{initial}} = d_{\text{initial}} \tag{B221}$$

$$c_{\text{final}} = d_{\text{final}} \tag{B222}$$

$$\text{end\_area} = d_{\text{final}}\cdot h_{\text{final}} \tag{B223}$$

}

if $\text{line}_{\text{top}} = 11$ and $\text{line}_{\text{bot}} = 12$

{

$$c_{\text{initial}} = d_{\text{initial}} \tag{B224}$$

$$c_{\text{final}} = \text{diagonal}_{\text{final}} \tag{B225}$$

$$\text{end\_area} = d_{\text{final}}\cdot h_{\text{final}}/2 \tag{B226}$$

}

if $\text{line}_{\text{top}} = 21$ and $\text{line}_{\text{bot}} = 11$ then

{

$$c_{\text{initial}} = \text{diagonal}_{\text{initial}} \tag{B227}$$

$$c_{\text{final}} = d_{\text{final}} \tag{B228}$$

$$\text{end\_area} = d_{\text{initial}}\cdot h_{\text{initial}}/2 + d_{\text{final}}\cdot h_{\text{final}} \tag{B229}$$

}

if $\text{line}_{\text{top}} = 21$ and $\text{line}_{\text{bot}} = 12$

{

$$c_{\text{initial}} = \text{diagonal}_{\text{initial}} \tag{B230}$$

$$c_{\text{final}} = \text{diagonal}_{\text{final}} \tag{B231}$$

$$\text{end\_area} = d_{\text{initial}}\cdot h_{\text{initial}}/2 + d_{\text{final}}\cdot h_{\text{final}}/2 \tag{B232}$$

}

if $\text{line}_{\text{top}} = 21$ and $\text{line}_{\text{bot}} = 22$

{

$$c_{\text{initial}} = d_{\text{initial}} \tag{B233}$$

$$c_{\text{final}} = \text{diagonal}_{\text{final}} \tag{B234}$$

$$\text{end\_area} = d_{\text{initial}}\cdot h_{\text{initial}} + d_{\text{final}}\cdot h_{\text{final}}/2 \tag{B235}$$

}

if $\text{line}_{\text{top}} = 22$ and $\text{line}_{\text{bot}} = 11$

{

$$c_{\text{initial}} = \text{diagonal}_{\text{initial}} \tag{B236}$$

$$c_{\text{final}} = \text{diagonal}_{\text{final}} \tag{B237}$$

$$\text{end\_area} = d_{\text{initial}}\cdot h_{\text{initial}}/2 + d_{\text{final}}\cdot h_{\text{final}}/2 \tag{B238}$$

}

if $\text{line}_{top} = 22$ and $\text{line}_{bot} = 12$

{

$$c_{initial} = \text{diagonal}_{initial} \tag{B239}$$

$$c_{final} = d_{final} \tag{B240}$$

$$\text{end\_area} = d_{initial} \cdot h_{initial}/2 \tag{B241}$$

}

if $\text{line}_{top} = 22$ and $\text{line}_{bot} = 22$

{

$$c_{initial} = d_{initial} \tag{B242}$$

$$c_{final} = d_{final} \tag{B243}$$

$$\text{end\_area} = d_{initial} \cdot h_{initial} \tag{B244}$$

}

$$s_{initial} = (a_{initial} + b_{initial} + c_{initial})/2 \tag{B245}$$

$$s_{final} = (a_{final} + b_{final} + c_{final})/2 \tag{B246}$$

$$\text{triangle}_{initial} = (s_{initial} \cdot (s_{initial} - a_{initial}) \cdot (s_{initial} - b_{initial}) \cdot (s_{initial} - c_{initial}))^{.5} \tag{B247}$$

$$\text{triangle}_{final} = (s_{final} \cdot (s_{final} - a_{final}) \cdot (s_{final} - b_{final}) \cdot (s_{final} - c_{final}))^{.5} \tag{B248}$$

if $\text{line}_{bot} = 11$

{

$$\text{sweep area} = \Sigma(\text{integral}_{top} - \text{integral}_{bot} + \text{triangle}_{initial} - \text{triangle}_{final} + \text{end\_area}) \tag{B249}$$

}

if $\text{line}_{bot} = 12$

{

$$\text{sweep area} = \Sigma(\text{integral}_{top} - \text{integral}_{bot} + \text{triangle}_{initial} + \text{triangle}_{final} + \text{end\_area}) \tag{B250}$$

}

if $\text{line}_{bot} = 22$

$$\text{sweep area} = \Sigma(\text{integral}_{top} - \text{integral}_{bot} - \text{triangle}_{initial} + \text{triangle}_{final} + \text{end\_area}) \tag{B251}$$

}

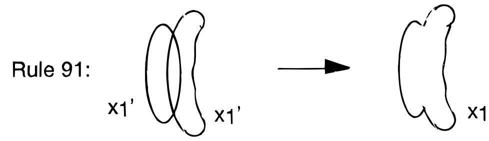



**Fig. B10.** Representative merge rule.

$$\text{sweep perimeter} = \Sigma\{\text{int}(ds_{top}, \phi_{top} = \text{lower}_{top}..\text{upper}_{top}) + \text{int}(ds_{bot}, \phi_{bot} = \text{lower}_{bot}..\text{upper}_{bot})\} \tag{B252}$$

## Water storage unit area and volume equations

$$\text{water storage unit area} = \Sigma\{\text{sweep area}_i\} - \Sigma\{\text{sweep area}_i \cap \text{sweep area}_k\} - \Sigma\{\text{sweep area}_i \cap \text{filter area}\} + \text{filter area} \tag{B253}$$

$$\text{water storage unit volume} = [2 \cdot \Sigma\{\text{sweep perimeter}_i\} - 2 \cdot \Sigma\{\text{sweep perimeter}_i \cap \text{sweep perimeter}_k\} - 2 \cdot \Sigma\{\text{sweep perimeter}_i \cap \text{filter perimeter}\}] \cdot |y'_b - y'_c| + 2 \cdot \text{filter area} + \text{filter perimeter} \cdot |y'_r - y'_s| \tag{B254}$$

($i$ from 1 to number of sweeps $-$ 1; $k$ from $i + 1$ to number of sweeps)

## Base unit area and volume equations

$$\text{base unit area} = 2 \cdot \Sigma\{\text{sweep perimeter}_i\} - 2 \cdot \Sigma\{\text{sweep perimeter}_i \cap \text{sweep perimeter}_k\} - 2 \cdot \Sigma\{\text{sweep perimeter}_i \cap \text{base perimeter}\} + \text{base area} \tag{B255}$$

$$\text{base unit volume} = [2 \cdot \Sigma\{\text{sweep perimeter}_i\} - 2 \cdot \Sigma\{\text{sweep perimeter}_i \cap \text{sweep perimeter}_k\} - 2 \cdot \Sigma\{\text{sweep perimeter}_i \cap \text{base perimeter}\}] \cdot |y'_c - y'_z| + \text{base volume} \tag{B256}$$

($i$ from 1 to number of sweeps $-$ 1; $k$ from $i + 1$ to number of sweeps)

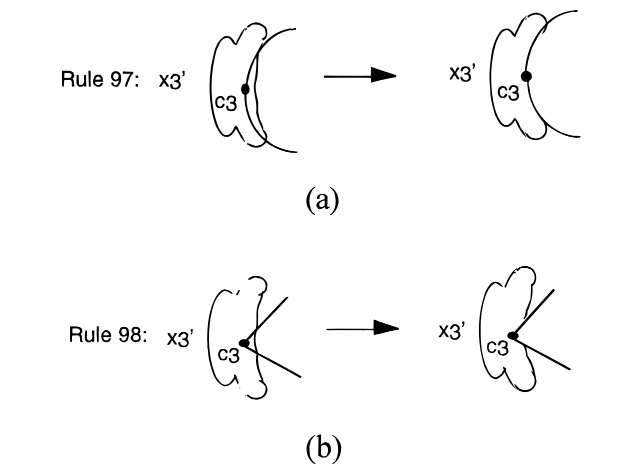



**Fig. B11.** (a) and (b) Representative rules depicting the merging of the base unit with the water storage unit.

## Lid area and volume equations

$$\text{grate area} = 0.5\cdot(\Sigma\{\text{sweep area}_i\} - \Sigma\{\text{sweep area}_i \cap \text{sweep area}_k\}) \tag{B257}$$

$$\begin{aligned}\text{lid area (grate)} = {} & \Sigma\{\text{sweep area}_i\} \\ & - \Sigma\{\text{sweep area}_i \cap \text{sweep area}_k\} \\ & - \Sigma\{\text{sweep area}_i \cap \text{filter area}\} \\ & + \text{filter area} - \text{grate area}\end{aligned} \tag{B258}$$

$$\begin{aligned}\text{lid volume (grate)} = {} & [2\cdot\Sigma\{\text{sweep perimeter}_i\} \\ & - 2\cdot\Sigma\{\text{sweep perimeter}_i \cap \text{sweep perimeter}_k\} \\ & - 2\cdot\Sigma\{\text{sweep perimeter}_i \cap \text{filter perimeter}\}] \\ & \cdot|y'_r - y'_f| + 2\cdot\text{lid area}\end{aligned} \tag{B259}$$

($i$ from 1 to number of sweeps $-$ 1; $k$ from $i + 1$ to number of sweeps)

$$\begin{aligned}\text{lid}_1\text{ area (hinged)} = {} & \Sigma\{\text{sweep area}_i\} \\ & - \Sigma\{\text{sweep area}_i \cap \text{sweep area}_k\} \\ & - \Sigma\{\text{sweep area}_i \cap \text{filter area}\}\end{aligned} \tag{B260}$$

$$\begin{aligned}\text{lid}_2\text{ area (hinged)} = {} & [2\cdot\Sigma\{\text{sweep perimeter}_i\} \\ & - 2\cdot\Sigma\{\text{sweep perimeter}_i \cap \text{sweep perimeter}_k\} \\ & - 2\cdot\Sigma\{\text{sweep perimeter}_i \\ & \quad \cap \text{filter perimeter}\}] + \text{filter area}\end{aligned} \tag{B261}$$

$$\text{lid}_1\text{ volume (hinged)} = 2\cdot(\text{lid}_1\text{ area}) \tag{B262}$$

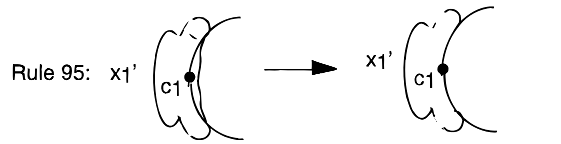



**Fig. B12.** Representative rule depicting the merging of the filter unit with the water storage unit.

$$\begin{aligned}\text{lid}_2\text{ volume (hinged)} = {} & [2\cdot\Sigma\{\text{sweep perimeter}_i\} \\ & - 2\cdot\Sigma\{\text{sweep perimeter}_i \cap \text{sweep perimeter}_k\} \\ & - 2\cdot\Sigma\{\text{sweep perimeter}_i \cap \text{filter perimeter}\}] \\ & \cdot|y'_r - y'_f| + 2\cdot\text{filter area}\end{aligned} \tag{B263}$$

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