

# Identification and Estimation of Auction Models with Dual Risk-Averse Bidders

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May 2025

## Can firms be risk averse?

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- Huge expenditures on (re-)insurance, hedging, “risk management”
- Growing body of structural evidence, most notably in auctions
  - Bolotnyy and Vasserman, 2023; Luo and Takahashi, Forthcoming; Kim, 2024
- Nevertheless, IO economists are often reluctant to relax risk neutrality

## Risk aversion and profit maximization

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- One reason: EU models require a departure from profit maximization
  - Profit maximization presupposes a constant marginal utility of wealth
- Yaari (1987) developed *dual risk aversion* to address this tension
  - Risk aversion is generated by probability distortion (like in VaR/CVaR)
  - Marginal utility of wealth remains constant
- Widely known, but rarely (never?) applied in empirical Bayesian games

## **This paper: estimate an auction model with dual risk aversion**

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1. Characterize BNE of first- and second-price auctions
2. Dual counterpart to classic identification results (GPV09)
3. Estimation framework for parallel FPA/SPA bidding
4. Application to USFS timber auctions

## Related literature

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### 1. Empirical models of auctions that relax risk-neutrality

- **EU models:** Bajari and Hortacsu, 2005; Guerre Perrigne Vuong, 2009; Lu and Perrigne, 2008; Campo et al, 2011; Fang and Tang, 2014; Bolotnyy and Vasserman, 2023; Chen, Gentry, Li, Lu, 2025; Luo and Takahashi, Forthcoming. **Non-EU models:** Aryal et al, 2018

### 2. Empirical models of rank-dependent utility

- **Experiments:** Goeree, Holt, Palfrey, 2002; Armantier and Treich, 2009. **Consumer choice:** Cicchetti and Dubin, 1994; Barseghyan et al., 2013; Barseghyan et al., 2016

### 3. Mechanism design with constant risk aversion

- Gershkov, Moldovanu, Strack, Zhang (2022, 2023a, 2023b)

**1. Theoretical framework**

2. Identification

3. Estimation

4. Application: USFS Timber Sales

5. Conclusion

# Preferences

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- $x$  is a random variable on  $[0, \bar{x}]$  drawn from  $H(\cdot)$
- Begin with a **rank-dependent utility** model (Quiggan, 1982),

$$U(x) = \int_0^{\bar{x}} u(s) \cdot g'(1 - H(s)) \cdot dH(s)$$

- $u(\cdot)$  is a standard vN-M utility function, while  $g : [0, 1] \rightarrow [0, 1]$  is a cdf
- Three special cases:

	$u(\cdot)$	$g(\cdot)$
Risk-neutrality	Identity	Identity
EU risk-aversion	Concave	Identity
<b>Dual risk-aversion</b>	<b>Identity</b>	<b>Convex</b>

## Auction game: overview

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- A risk-neutral seller is endowed with a single indivisible good
- $I$  symmetric potential bidders with independent private values  $v_i \sim F(\cdot|I)$
- All bidders are symmetrically equipped with some convex distortion  $g(\cdot)$
- We characterize monotone, symmetric BNE of the SPA and FPA



## Optimal bidding in the FPA

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- Optimal bidding problem for a bidder with value  $v$

$$\max_b \{v - b\} \cdot g(1 - P\{b \text{ loses}\})$$

- The equilibrium bid function satisfies:

$$\beta'(v) = \{v - \beta(v)\} \cdot (1 - I) \cdot \frac{f(v|I)}{F(v|I)} \cdot \left\{ \frac{g'(F(v|I)^{I-1}) \cdot F(v|I)^{I-1}}{g(F(v|I)^{I-1})} \right\}$$

- Under risk-neutrality, term in brackets collapses to one
- This differential equation admits a closed form solution (unlike EU case)

## Proposition

1. In a FPA with reserve price  $r$ , the optimal strategy is to bid:

$$\beta(v_i) = v_i - \frac{\int_r^{v_i} g(F^{I-1}(s)) ds}{g(F^{I-1}(v_i))}$$

if  $v_i \geq r$ . Moreover,  $\beta(v_i)$  is greater than the risk-neutral optimal bid.

2. In a SPA with reserve price  $r$ , the optimal strategy is  $\beta(v_i) = v_i$  if  $v_i \geq r$ .

- In **Li, Lu, O'Keefe (2025)**, we characterize optimal reserve prices

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3. Estimation

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# Summary

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- $[g, F]$  is not identified in first price auction data alone
  - Analogous to non-identification of  $[u, F]$  in the EU model (GPV09)
- We consider two identification strategies:
  1. (Exogenous) variation in  $I$
  2. Parallel second-price auction data

# Non-identification

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- Let  $\mathbf{Q}(b_1, \dots, b_I | I) \equiv \prod_{i \leq I} Q(b_i | I)$  denote the joint distribution of FPA bids
- If  $[g, F]$  rationalizes  $\mathbf{Q}(\cdot | I)$ , then so does  $[\tilde{g}, \tilde{F}]$ 
  - $\tilde{g}(\alpha) = g(\alpha)^\delta$  for  $\delta > 1$
  - $\tilde{F}(\cdot | I)$  is the distribution of  $\tilde{\xi}(b)$ , where:

$$\tilde{\xi}(b) = b + \frac{1}{I-1} \cdot \frac{Q(b|I)}{q(b|I)} \left\{ \frac{Q^{I-1}(b|I) \cdot \tilde{g}'(Q^{I-1}(b|I))}{\tilde{g}(Q^{I-1}(b|I))} \right\}^{-1}$$

## Identification with exogenous participation

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- Suppose  $F(\cdot|I) = F(\cdot) \Rightarrow v(\cdot|I) = v(\cdot)$  for all  $I \in \mathcal{I}$
- FOC for bidder with  $\alpha$ -quantile valuation:

$$v(\alpha) = b(\alpha|I) + b'_R(\alpha|I) \cdot z(\alpha^{I-1})$$

- $b_R(\cdot|I)$  is the quantile function of  $Q_R(\cdot|I) \equiv Q(\cdot|I)^{I-1}$
- $z(\cdot) = g(\cdot)/g'(\cdot)$
- **Compatibility conditions:** because  $v(\alpha)$  does not vary with  $I$ ,

$$b(\alpha|I_1) + b'_R(\alpha|I_1) \cdot z(\alpha^{I_1-1}) = b(\alpha|I_2) + b'_R(\alpha|I_2) \cdot z(\alpha^{I_2-1})$$

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- $z(\cdot) = g(\cdot) / g'(\cdot)$
- **Compatibility conditions:** because  $v(\alpha)$  does not vary with  $I$ ,

$$z(\alpha) = \frac{b(\alpha^{1/(I_1-1)}|I_2) - b(\alpha^{1/(I_1-1)}|I_1)}{b'_R(\alpha^{1/(I_1-1)}|I_1)} + \frac{b'_R(\alpha^{1/(I_1-1)}|I_2)}{b'_R(\alpha^{1/(I_1-1)}|I_1)} \cdot z(\alpha^{(I_2-1)/(I_1-1)})$$

## Identification with exogenous participation (cont)

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- $K$  recursive applications gives

$$z(\alpha) = \overbrace{\sum_{0 \leq k \leq K} \left\{ \frac{b(\alpha_k | I_2) - b(\alpha_k | I_1)}{b'_R(\alpha_k | I_1)} \cdot \prod_{l=0}^{k-1} \frac{b'_R(\alpha_l | I_2)}{b'_R(\alpha_l | I_1)} \right\}}^{\tilde{z}_K(\alpha)} + \overbrace{z(\alpha_K) \cdot \prod_{l=0}^{K-1} \frac{b'_R(\alpha_l | I_2)}{b'_R(\alpha_l | I_1)}}^{R_K(\alpha)}$$

where  $\alpha_k = \alpha_{k-1}^{(I_2-1)/(I_1-1)}$  and  $\alpha_0 = \alpha^{1/(I_1-1)}$

- Because  $\tilde{z}_K(\alpha)$  is observable,  $z(\alpha)$  is identified if  $\lim_{K \rightarrow \infty} R_K(\alpha) = 0$
- Then  $g(\alpha) = \exp \left\{ - \int_{\alpha}^1 [z(s)]^{-1} ds \right\}$  is identified from  $g(1) = 1$



## Identification with second-price auction data

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- In some settings, second-price and first-price auctions are held in parallel
- Re-arranging the quantile FOC:

$$z(\alpha) = \left\{ b'_R \left( \alpha^{\frac{1}{I-1}} | I \right) \right\}^{-1} \cdot \left[ v \left( \alpha^{\frac{1}{I-1}} | I \right) - b \left( \alpha^{\frac{1}{I-1}} | I \right) \right]$$

- $\beta(v) = v \Rightarrow v(\cdot | I) \equiv F^{-1}(\cdot | I)$  is identified from the winning bids in SPA

## Can the data distinguish EU and DU?

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- With FPA bids alone, **no**
- Suppose  $I$  varies exogenously **and** we have SPA bids. Then:
  1. The dual utility (DU) and expected utility (EU) models are testable
  2. The rank-dependent utility model  $[u, g, F]$  itself is not identified
    - Without a restriction,  $u(\cdot) / u'(\cdot)$  and  $g(\cdot) / g'(\cdot)$  only known up to scale

1. Theoretical framework

2. Identification

**3. Estimation**

4. Application: USFS Timber Sales

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## Estimation with SPA winning bids

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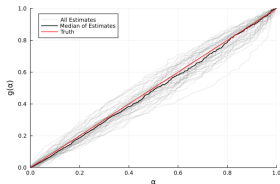
- By integrating over quantiles (Liu Lou 2017, Luo Wan 2018), we obtain:

$$g(\alpha) = \exp \left( - \int_{b(\alpha^{1/(I-1)}|I)}^{b(1|I)} \left[ \frac{1}{\psi(u|I) - u} \right] \cdot du \right)$$

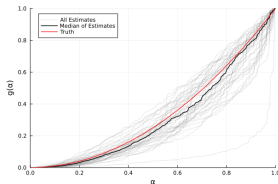
- $\psi(\cdot|I) \equiv F^{-1}(Q(\cdot|I)|I)$  is the statistical mapping from bids to values
- A **tuning-parameter free** estimator  $\hat{g}(\cdot)$  can be constructed from bid ECDFs
- Can impose shape restrictions via post-processing (Chen et al 2021) or by constrained sieve estimation (Chen, 2007) e.g., with convex splines (Meyer, 2008)

# Simulated point estimates, $n \in \{100, 1000\}$

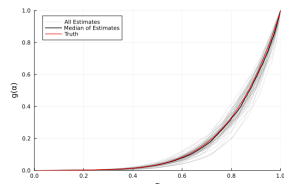
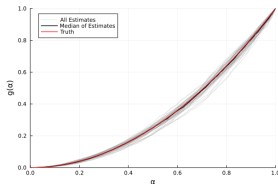
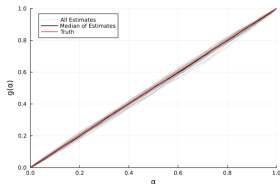
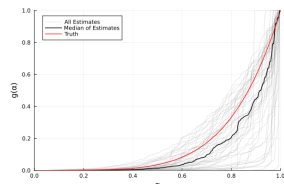
**Model 1:**  $g(x) = x$



**Model 2:**  $g(x) = x^2$



**Model 3:**  $g(x) = \frac{x+x^5}{1+x}$



*Note:* simulations assume  $I = 3$ ,  $v \sim \text{LogNormal}(1, 1)$  truncated at  $\bar{v} = 10$ .

## Summary of simulated point estimates

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		$\ g(\cdot) - \hat{g}(\cdot)\ $		FS Test Rej. Rate	
		50%	95%	Affine	Convex
$n = 100$	Model 1	0.038	0.075	0.00	0.00
	Model 2	0.057	0.133	0.94	0.00
	Model 3	0.081	0.246	0.90	0.00
$n = 1000$	Model 1	0.009	0.022	0.00	0.00
	Model 2	0.013	0.031	1.00	0.00
	Model 3	0.018	0.053	1.00	0.00

*Note:* Based on 50 simulations; FS Test is Feng and Seo (2021) test.

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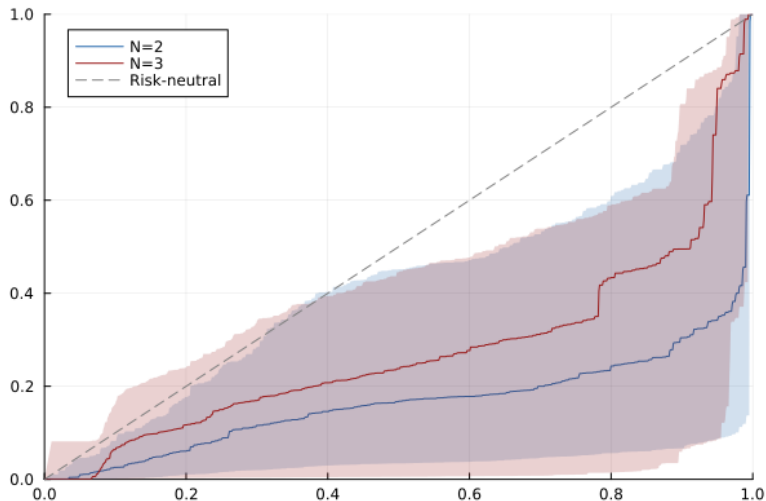
## 1979 USFS Timber Sales

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- In the late 1970s, the US Forest Service switched from SPA to FPA
- Non-binding reserve, pre-qualification of bidders  $\Rightarrow I$  is observed
- We implement our estimator on 1979 auctions having  $I \in \{2, 3\}$
- Prior EU-based work has found mixed evidence for risk aversion
  - In favor (Lu and Perrigne, 2008, Campo et al, 2011); against (Gimenes and Guerre, 2022)



## Estimated distortion functions



*Note:* Reports pointwise bootstrap confidence bands.

## Evidence of (dual) risk aversion

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	FS Test $p$ -value	
	Affine	Convex
$I = 2$	0.148	0.848
$I = 3$	0.136	0.888

## Revenue comparison and optimal reserves

	SPA		FPA		$\Delta$ SPA to FPA	
	$I = 2$	$I = 3$	$I = 2$	$I = 3$	$I = 2$	$I = 3$
Optimal Reserve $r^*$	55,029 (13,033)	60,726 (14,882)	49,852 (14,453)	54,878 (12,294)	-5,177 (10,712)	-5,848 (16,219)
Revenue ( $r = r^*$ )	42,708 (2,663)	54,565 (2,946)	46,163 (2,476)	63,207 (5,584)	3,455 (1,408)	8,641 (4,586)
Revenue ( $r = 0$ )	35,773 (1,474)	48,909 (2,394)	41,563 (1,778)	61,774 (5,876)	5,790 (1,820)	12,865 (5,510)

- Risk-neutral model: (1) overstates  $r^*$  by  $\approx 10\%$ ; (2) understates FPA revenue

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- 5. Conclusion**

## Conclusion

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- We study identification of auction models with dual risk averse bidders
- Appealing for applications in which bidders are profit-maximizing firms
- Application to timber sales (1) demonstrates plausibility of the dual utility model; (2) illustrates practical benefit of closed-form bidding strategies
- Next steps: quantification of the optimal mechanism? (Gershkov et al 2022)

# Thank you!

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