

Index-Based Price Adjustment in US Strategic Petroleum Reserve Drawdowns

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Abstract

Sales from the US Strategic Petroleum Reserve (SPR) are conducted using basis auctions, in which winning bidders’ final payments are indexed to spot prices. This mechanism can eliminate potential inefficiencies arising from private information about the evolution of oil prices between auction and delivery, benefiting the seller (the US government). Reduced form evidence suggests that bids in SPR sales are consistent with pure private values. In a stylized model calibrated to the 2022 SPR drawdown, the use of basis auctions plausibly generates tens of millions of dollars in savings.

Keywords: Strategic petroleum reserve; long-term contracts; contingent payment auctions; private information; oil markets

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1 Introduction

In 2022, the Department of Energy (DOE) auctioned 180 million barrels of crude oil from the United States Strategic Petroleum Reserve (SPR). A distinctive feature of the auction mechanism used by DOE is *price indexing*: a winning bidder’s final payment is linked to the value of an oil price index at delivery, weeks or months after the auction.

The auctions literature has identified two key mechanisms by which ex post price adjustment may benefit a seller. First, making payments contingent upon information revealed after the auction can reduce information rents (Hansen, 1985; Riley, 1988). Second, contingent payments can mitigate the winner’s curse: the harm from overestimating the value of a good is reduced if the final payment is adjusted downwards when it turns out to be less valuable (Skrzypacz, 2013). Whether these effects are large in practice depends on the structure of valuations and sources of surplus in a given setting, the nature of bidders’ information, and the particular properties of the auction protocol adopted by the seller.

This paper analyzes the performance of DOE’s form of index-based price adjustment during the 2022 drawdown. I present empirical evidence consistent with large revenue gains through the channels identified above. The paper begins with a stylized model of single-unit auctions capturing both forces: for a well-chosen price index, private information about the common component of valuations becomes less important, compressing the distribution of types and weakening adverse selection to the seller’s benefit. In the model, the benefits of price indexing are greatest when the price index exactly coincides with the common component of valuations, in which case adverse selection is eliminated altogether. In the next part of the paper, I test for violations of this condition in bidding data from the 2022 drawdown. I find no evidence of adverse selection after price indexing. The key question is therefore whether private information would have been a more significant factor had DOE used a different format, such as a fixed price auction. Absent direct evidence of private information, I conclude by showing that price indexing could have generated tens of millions of dollars in additional revenue during 2022 for a large range of plausible information structures.

To guide the empirical analysis, I begin by analyzing the impact of price indexing in the context of Goeree and Offerman (2003)’s model of a first price auction in which common and private components enter additively into bidders’ valuations. This framing is useful to capture the fact that in a drawdown, potential buyers differ both in their beliefs about future crude oil prices and in their opportunity costs of purchasing SPR crude. The latter generally differ from publicly reported spot prices because of transportation costs, long-term supply contracts, inventories, and other idiosyncratic factors. I use this basic model to show how price indexing can benefit the seller along several dimensions, including expected revenue

and default risk. I then extend the model to address additional features common to crude oil and other commodities markets, including the possibility of hedging outside the auction mechanism and the sale of multiple units via uniform price auctions.

Empirically, the model predicts that there should be no evidence of common values if the price index coincides with the common component of bidders' valuations (up to a shift of location). I test this prediction using two complementary strategies. First, I show that greater valuation uncertainty is not associated with greater bid shading, consistent with the absence of private information (Nyborg et al., 2002).¹ To quantify uncertainty, I exploit the fact that the time elapsed between the auction and delivery is lengthy and varies considerably within the sample. Second, I show that there is no statistically significant within-auction correlation in bids after controlling for participation and other observables. This result is consistent with independent private values and hence the absence of private information (Hickman et al., 2021). Although both results are negative, they together provide suggestive evidence that adverse selection is weak after price indexing. Furthermore, they imply that DOE could not have significantly improved revenue by choosing a different price index. Because the results may reflect sampling variability, I provide numerical evidence that the second test has favorable power in multiunit settings even for modestly sized samples.

While the test results are consistent with the absence of common values after price indexing, they do not establish that common values would have been present had DOE adopted a different auction protocol. Thus, complementary evidence of private information is necessary to establish a revenue impact from price indexing. There are some reasons to think that firms bidding for SPR crude oil, primarily large domestic oil refiners, may be privately informed about (local) crude oil prices. While it is unlikely that individual firms hold much information about oil prices in general, contract pricing for physical delivery can deviate significantly from popular financial market benchmarks depending on time, location, and crude oil specifications. Pricing in the physical market is not fully transparent as parties generally have no reporting obligations (Fattouh, 2011). Moreover, the US oil refining sector has become relatively concentrated in recent decades. Taking this into account, I argue that individual buyers are plausibly informed about future supply and demand shocks relevant to pricing US-delivered physical contracts, including their own future refinery operations or (in the case of vertically integrated firms) crude oil production or storage decisions.

Detecting the presence and significance of this type of private information is difficult for two reasons. First, the theoretical analysis suggests that the bidding data itself is not informative about bidders' underlying beliefs about oil prices. This interpretation is consistent with the empirical results above. Second, there is limited policy variation as DOE

¹A similar approach was considered in Hortaçsu (2002).

has used similar price adjustment mechanisms since the creation of the SPR in 1975. Given these constraints, I take an indirect approach and (roughly) bound the amount of private information for which price indexing could have significantly increased revenue relative to a fixed price counterfactual during the 2022 drawdown. There is greater reason to believe that the impact of price indexing was significant if this bound is small than if it were large.

The main challenge in constructing such a bound is the difficulty of characterizing equilibria in pay-as-bid auctions under realistic assumptions on bidder valuations and supply. To avoid this challenge, I instead analyze a hypothetical uniform price auction with price indexing calibrated to the 2022 drawdown. Building on Vives (2010), I show that the revenue benefit of price indexing in the uniform price setting depends on two sufficient statistics: the slope of bidders' marginal valuation functions and the ratio between the precision of bidders' private information about oil prices and the precision of the idiosyncratic cost distribution. I calibrate the slope parameter from observed bidding data under the assumption that revenue in pay-as-bid and uniform price auctions is similar on average (an approximation which I argue is reasonable based on well-known results from the multiunit auctions literature). This allows me to approximate the minimum ratio of precisions consistent with revenue impacts of a given size. I conclude that private information amounting to 5-15% of the total variation in bidder's total ex ante information would suffice to increase DOE revenue by \$0.25-\$1.00 per barrel, corresponding to an impact of tens of millions of dollars in 2022.

The results of this paper are potentially relevant for sellers in other contexts and for recent debates surrounding the potential use of fixed prices in DOE's crude oil purchases to fill the SPR.² The analysis suggests that price indexing can be useful when three conditions are met. First, buyers' valuations must have an important time-varying common component (in this case, the comovement of opportunity costs as oil prices change over time). Second, the time between auction and delivery must be long enough that valuations might change significantly through this component. Third, it must be possible to construct a suitable price index. These conditions plausibly obtain in many commodity markets. As discussed below, price indexing is a common feature of many privately-negotiated long-term contracts, especially in the commodities sector. Perhaps surprisingly, there are relatively few examples of formal basis auctions. Among instances of government participation in commodities markets through formal auctions, for example, fixed price formats appear to be the norm.³ Future research

²DOE has historically used index-based price adjustment in crude oil procurements. In 2022, DOE sought the ability to make fixed price procurements. I discuss this policy change in Section 3.

³Examples include sales of gold by the UK treasury (Wetherilt and Young, 2003), bulk grain tenders by governments such as Egypt and Turkey (e.g., Heigermoser et al., 2020), and bulk electricity procurement (e.g., Cleary and Bishop Ratz, 2021). The scarcity of examples partly reflects the limited role of government entities as active buyers and sellers in the commodities sector.

may be useful in further clarifying the tradeoffs associated with price indexing and the circumstances in which sellers might benefit (or not) from switching to formal basis auctions.

Related literature Price indexing is distinguished from other forms of price adjustment by the use of pre-specified formulae to adjust final payments based on publicly available information (often spot market prices). This contrasts with cost-plus contracting, for example, where final payments are adjusted according to one party’s reported costs.⁴ Price adjustment mechanisms of various forms are ubiquitous in long-term contracting. In a classic series of empirical papers (Joskow, 1985, 1988, 1990), Paul Joskow describes the extensive use of both price indexing and cost-plus clauses in electric utility coal procurement contracts. Building on earlier work by Goldberg (1985) and Goldberg and Erickson (1987), Joskow argues that price adjustment mitigates ex post opportunism, keeps moral hazard in check, and preserves incentives for cost-minimization over the life of a contract.⁵ Price indexing (also called formula pricing) is also typical of contracts for physical delivery of crude oil and refined products (Fattouh, 2011), among other commodities such as natural gas and grain. The use of price indexing is not limited to commodities markets. In another classic paper, Crocker and Reynolds (1993) document the use of so-called “Economic Price Adjustment” clauses in the procurement of Air Force engines. Acquatella et al. (2023) analyze price indexing in reimbursement for physician-administered drugs under Medicare Part B.

Price indexing specifically has received limited attention in the empirical auctions literature. One notable exception is Kosmopoulou and Zhou (2014), who find that the introduction of escalation clauses for bitumen (asphalt) costs in Oklahoma highway procurement contracts was associated with more aggressive bidding. Howell (2020) finds that similar clauses in Kansas highway procurement reduced the sensitivity of bids to oil prices. It is not obvious why more work of this kind has not emerged given that auctions are often used to allocate long-term contracts, and long-term contracts often feature price indexing. Indeed, Kosmopoulou and Zhou (2014) report that nearly all states use some form of price adjustment for bitumen and fuel costs in highway procurement, a fact which has largely gone unremarked in studies of highway procurement. Relative to this literature, the main contribution of this paper is to formulate and empirically evaluate the prediction that price indexing reduces the significance of private information about common value elements.⁶

⁴Bajari and Tadelis (2001) rationalize cost-plus contracting as an optimal response to ex post adaptation costs arising from project complexity (e.g., in construction). In contrast, price indexing tends to be used for routine or predictable transactions where market prices constitute the main source of uncertainty.

⁵It can also prevent the dissipation of rents through costly information acquisition (Goldberg, 1985).

⁶Both Kosmopoulou and Zhou (2014) and Howell (2020) emphasize the effect of price adjustment on the allocation of risk between buyers and sellers. Polinsky (1987) provides an early theoretical treatment of this issue. In contrast, I maintain the assumption of risk neutrality.

Royalties and related forms of securitization constitute a different form of price adjustment (DeMarzo et al., 2005; Che and Kim, 2010). Kong et al. (2022) estimate a model of multiattribute (cash-royalty) auctions in which bidders receive affiliated, bidimensional private signals. The information structure in this paper can be viewed as a simplification of their more general structure: bidders receive two signals, but “types” are one-dimensional and all information enters additively into bidders’ payoffs (Goeree and Offerman, 2003). Another key difference with Kong et al. (2022), and related work such as Bhattacharya et al. (2022), is the absence of moral hazard: setting aside default, bidders’ ex post auctions have no impact on surplus or bidding strategies, and hence do not inform auction design.

One factor that may have constrained prior empirical work on price indexing is the relative scarcity of data on bidding for long-term contracts outside formal auctions in regulated settings. From this perspective, bidding data from the 2022 SPR drawdown might be viewed as a window into the broader market for long-term bilateral commodities contracts. The information contained in the data is potentially illuminating to the extent that SPR sale procedures and contracts resemble the informal negotiations and bilateral contracts by which oil refiners and other bidders maintain their supply portfolios in the regular course of business.

The management of the SPR has become an important domain of public debate in recent years. This paper provides the first microeconomic analysis of the sales procedures used in the historic 2022 SPR drawdown. In doing so, this paper complements prior work in macroeconomics analyzing whether SPR releases stabilize oil prices (Newell and Prest, 2017; Kilian and Zhou, 2020; Stevens and Zhang, 2021). In the only prior retrospective analysis of the 2022 drawdown that I am aware of, Razeq et al. (2023) argue that the 2022 release may have inadvertently contributed to gasoline price increases. Earlier work in operations research addresses optimal procurement and drawdown policies for petroleum reserves while abstracting from mechanism design (Teisberg, 1981; Oren and Wang, 1986).

Structure of the paper Section 2 presents the model and extensions. Section 3 introduces the data and describes the SPR drawdown procedure in detail. Section 4 presents the tests for common values in the 2022 bidding data. Section 5 presents the bounding exercise. Section 6 concludes. All proofs are contained in the Appendix.

2 Model

This section presents a generic model of auctions with price indexing highlighting potential benefits to the seller. I use the model to develop intuition for the empirical analysis of the 2022 SPR drawdown, which constitutes the remaining sections of the paper.

A seller is endowed with a single, indivisible good which cannot be immediately delivered to a buyer. In order to sell the good, the seller will hold an auction at time $t = 0$ for delivery at time $t = 1$. There are N symmetric, risk neutral potential buyers who differ in their opportunity costs of purchasing the good. Buyer i 's valuation is the opportunity cost of purchasing a substitute good at time $t = 1$. This opportunity cost is given by $w_1 + u_i$, where w_1 is the realization of a stochastic price index W_t at $t = 1$ and u_i is a *cost basis*.^{7,8}

Each potential buyer i privately observes his own cost basis u_i . In addition, i receives a private signal ω_i of the true but not-yet-known ex post index price W_1 . I impose the following technical conditions on the distributions of u_i and ω_i , which allow potential buyers to be ordered according to a scalar “type” $s_i = \omega_i/N + u_i$ (Goeree and Offerman, 2003).

Assumption 1. (i) u_i is drawn independently across bidders from a distribution F_u having log-concave density f_u with support on a bounded interval $[\underline{u}, \bar{u}]$; (ii) ω_i is drawn independently across bidders from a distribution F_ω having log-concave density f_ω with support on a bounded interval $[\underline{\omega}, \bar{\omega}]$ and satisfying $E[\omega_i] = w_1$; (iii) ω_i is independent of u_i .

(ii) amounts to the assumption that bidders lack an informative prior for the distribution of W_1 before observing their private signals $\omega_1, \dots, \omega_n$.⁹ Because the private signals are distributed independently, the mean signal $\bar{\omega} = \frac{1}{N} \sum_{i \leq N} \omega_i$ therefore represents the best available estimate of w_1 when the auction is held at $t = 0$.¹⁰ Hence, $\bar{\omega}$ can be viewed as the common component of bidders’ valuations at the time of the auction.

The seller chooses an auction format. One natural candidate is a first price auction in fixed prices. Suppose there is no reserve price and that ex post default is impossible. Goeree and Offerman (2003) characterize a pure strategy Bayes Nash equilibrium of this game:

Lemma 1. Let $y_i = \max_{j \neq i} s_j$. The N -tuple of strategies $(\beta_1(\cdot), \dots, \beta_N(\cdot))$, where

$$\beta_i(x) = E[\bar{\omega} + u_i | s_i = x, y_i \leq x] - E[x - y_i | s_i = x, y_i \leq x], \quad (1)$$

is an equilibrium of the first price auction in fixed prices.

The strategy described by equation (1) has a standard interpretation. The first term represents i 's interim assessment of his opportunity cost conditional on winning the auction.

⁷Throughout the paper I assume that u_i is independent of the realization of the index price W_t . Hence, buyer i 's cost basis does not vary with the oil price level. This assumption is potentially restrictive. Below, I allow W_t to differ from the index price chosen by the seller. In this way, the model can accommodate additively separable common shocks to buyers’ costs bases that are correlated with changes in oil prices. However, the model cannot accommodate buyer-specific cost shocks driven by changing oil prices.

⁸Upper case letters are used to indicate random variables and lower case letters their realizations.

⁹This structure is similar to that of the “Wallet Game” (Bulow and Klemperer, 2002).

¹⁰ $\bar{\omega}$ is the “best” estimate in the sense that $\bar{\omega}$ is the lowest-variance unbiased estimator of W that can be constructed from the signals w_1, \dots, w_n .

This term reflects a potential buyer's anticipation of the winner's curse: the auction winner learns that other bidders had lower types, and hence that $\bar{\omega}$ is less than ω_i . The second term represents i 's information rent. This is the surplus bidder i earns on account of being privately informed about his own type (in this case the scalar $s_i = \omega_i/N + u_i$).

An alternative to a fixed price auction is a *basis auction*. In a basis auction, bids are expressed as a differential to a floating index price chosen by the seller. In a first price basis auction, the highest differential wins, and the winner's final payment is the sum of his or her bid and the realization of the index price upon delivery at $t = 1$. If W_t is the price index, the first price basis auction corresponds to a first price auction with independent private values (IPV). The pure-strategy Bayes Nash equilibrium of such a game is well-known:

Lemma 2. *Let $y_i = \max_{j \neq i} u_j$. The N -tuple of strategies $(\delta_1(\cdot), \dots, \delta_N(\cdot))$, where*

$$\delta_i(x) = E[y_i | y_i \leq x], \quad (2)$$

is an equilibrium of the first price basis auction with index price W_t .

Unlike (1), the strategy (2) does not depend on bidders' private signals $\omega_1, \dots, \omega_N$. As a result, the seller's expected revenue improves, for two reasons. On one hand, bidders' "types" (now simply the private costs u_1, \dots, u_N) are less dispersed. On the other, adverse selection is eliminated. Total surplus also increases because the private signals no longer affect the allocation of the good: the highest cost basis u_i wins.¹¹

Proposition 1. *Expected revenue and total surplus are greater in the first price basis auction.*

I prove Proposition 1 using properties of the order statistics of samples from log-concave distributions. Milgrom and Weber (1982)'s linkage principle provides familiar intuition: price indexing tightens the statistical linkage between a bidder's information (the signal ω_i) and his expected payment, increasing the seller's expected revenue.

In addition to these benefits, the basis auction also reduces buyers' incentive to default or seek renegotiation after low realizations of W_1 . After a fixed price auction, winning bidders are incentivized to default if $w_1 + u_i$ is lower than $\beta_i(\omega_i/n + u_i)$ at delivery. After a basis auction, there is no such incentive: because $\delta(u_i) \leq u_i$, the winning bidder always benefits from taking delivery. The ability of price indexing to reduce opportunistic ex post behavior is a key emphasis of the long-term contracting literature (Goldberg, 1985; Joskow, 1988).

¹¹Efficiency need not hold in the case of bidder asymmetry (Maskin and Riley, 2000).

Imperfect price indices In practice, it may not be possible to index bids using the “ideal” index price W_t .¹² Suppose the seller chooses some other index M_t . Buyers information ω_i may consist of some information about the index M_t and some information about the residual common value $Z_t := W_t - M_t$. In place of the signal ω_i , I now suppose potential buyers observe two distinct signals μ_i and ζ_i corresponding to m_1 and z_1 , respectively.

Assumption 2. (i) μ_i is drawn independently across bidders from a distribution F_μ having log-concave density f_μ with support on a bounded interval $[\underline{\mu}, \bar{\mu}]$ and satisfying $E[\mu_i] = m_1$; (ii) ζ_i is drawn independently across bidders from a distribution F_ζ having log-concave density f_ζ with support on a bounded interval $[\underline{\zeta}, \bar{\zeta}]$ and satisfying $E[\zeta_i] = z_1$; (iii) ζ_i and μ_i are mutually independent and independent from u_i .

In this case, the equilibrium bidding strategy in the first price basis auction is:

$$\delta_i(x) = E[\bar{z} + u_i | \tilde{s}_i = x, y_i \leq x] - E[x - y_i | \tilde{s}_i = x, y_i \leq x] \quad (3)$$

where $\tilde{s}_i = \zeta_i/N + u_i$ and $y_i = \max_{j \neq i} \tilde{s}_j$. Thus, the basis auction will not fully eliminate the winner’s curse or the information rents from private information about W_t unless Z_t is non-stochastic (in which case M_t must coincide with W_t up to a shift of location). Moreover, the bidder with the highest private cost basis u_i is no longer certain to win, reducing total surplus. Nevertheless, the use of the basis auction with an imperfect price index will benefit the seller so long as the distribution of the private information about Z_t is stochastically smaller than the distribution of information about $W_t \equiv M_t + Z_t$.

Hedging In general, the ability to hedge outside the auction mechanism could affect equilibrium bidding strategies and hence the comparison of auction formats. Here I present a brief argument that the existence of a futures market specifically will not necessarily affect the seller’s comparison of auction formats above provided that buyers are risk neutral.¹³

Suppose there is a single futures contract that settles at time $t = 1$, and let F_1^s denote the price of this contract at time s . The settlement price of the futures contract is assumed to equal the realization of the index price, $F_1^1 = w_1$ (as would be the case under no arbitrage). Buying and selling the futures contract is costless, including short-selling.

I first consider the case of the basis auction. Let F_1^{0-} denote the market price of the futures contract immediately prior to the auction at $t = 0$ and F_1^{0+} the market price immediately afterwards. These prices may contain information about W_1 . If the interim price F_1^{0+} is

¹²A winner in the basis auction obtains what Joskow (1988) describes as a “market price” contract. He writes: “The primary problem with a market price contract is defining an appropriate market price norm[.]”

¹³The same argument applies to other hedging instruments, such as over-the-counter forwards.

less than a winning bidder's conditional expectation of W_1 , it is profitable to purchase a contract. But under the assumption that opportunity costs also float with the index price, a *losing* bidder would also profit from purchasing a contract given the same information. Thus, bidder i 's ex ante expected gain conditional on winning the auction with bid d_i is

$$\left(E \left[\min \{ F_1^{0+}, W_1 \} \middle| f_1^{0-}, \omega_i \right] + u_i \right) - \left(E \left[\min \{ F_1^{0+}, W_1 \} \middle| f_1^{0-}, \omega_i \right] + d_i \right)$$

which simplifies to $u_i - d_i$. As in the original basis auction, bidders choose their strategies to maximize this gain. Hence, the equilibrium bidding strategy will be the same.

A similar argument can be made for the case of the fixed price auction. It is profitable for all bidders to take short positions whenever their interim conditional expectations of W_1 are below the futures price. Bidder i 's ex ante expected gain conditional on winning the auction with bid b_i is the difference between the interim opportunity cost

$$E \left[\min \{ W_1, F_1^{0+} \} \middle| f_1^{0-}, \omega_i, \max_{j \neq i} s_j \leq \beta^{-1}(b_i) \right] + u_i$$

and expected payment

$$b_i + E \left[\min \{ 0, F_1^{0+} - W_1 \} \middle| f_1^{0-}, \omega_i, \max_{j \neq i} s_j \leq \beta^{-1}(b_i) \right]$$

where the last term represents the expected profit from the short position. This difference simplifies to $E \left[W_1 \middle| f_1^{0-}, \omega_i, \max_{j \neq i} s_j \leq \beta^{-1}(b_i) \right] + u_i - b_i$ which is identical to the expected gain in the original model except that the expectation of W_1 now depends explicitly on the pre-auction futures price f_1^{0-} . Thus, the equilibrium bidding strategy is essentially unchanged from the original fixed price auction. If the original model is modified to make explicit that f_1^{0-} is known to all bidders (as would be true in reality), there is no change.

The revenue benefit of the basis auction stems from the concentration of the type distribution and the attenuation of adverse selection. The existence of a futures market does not directly affect the distribution of the private signals $\omega_1, \dots, \omega_N$, and therefore does not affect these mechanisms. However, two important caveats should be made. First, the role of financial contracts could be significantly different if bidders were risk averse.¹⁴ Second, the futures market provides another venue in which bidders can profit from private information about oil prices. ω_i should therefore be interpreted as the part of a bidder's private information which is not already aggregated into futures prices at the time of the auction.

¹⁴While risk aversion provides a natural motivation for hedging, it also affects how bidders form their bids in the auction

Multiple units Price indexing is often used when transacting commodities in bulk. The uniform price and pay-as-bid auctions are two common generalizations of the first price auction to the case of multiple or divisible goods. The uniform price auction admits well-behaved symmetric linear equilibria for many Gaussian information structures. I therefore focus on the uniform price case, comparing the basis auction and the fixed price formats.¹⁵

Suppose there are exactly Q identical (and divisible) goods for sale. Departing from the model above, I now assume that buyer i 's marginal opportunity cost takes the form:

$$v_i(q) = w_1 + u_i - \lambda q \quad (4)$$

where $\lambda > 0$ captures the diminishing marginal benefits from purchasing a larger number of units. The assumption of diminishing marginal benefits is standard in the multiunit auction literature and is typically often motivated by opportunity costs, inventory costs, or risk aversion, among other possibilities. The assumption that λ is the same for all bidders simplifies the equilibrium analysis below but is not essential.

For this section only, I parameterize the general log-concave distributions F_u and F_ω in Assumption 1 while dropping the bounded support assumptions:

Assumption 3. (i) u_i is drawn independently across bidders from a Normal distribution with mean θ_u and variance σ_u^2 ; (ii) ω_i is drawn independently across bidders from a Normal distribution with mean w_1 and variance σ_ω^2 ; (iii) ω_i is independent of u_i .

Models combining linear marginal valuations and Gaussian information structures are widely used in theoretical papers on imperfectly competitive financial markets (Rostek and Yoon, 2023). The case of uniform price auctions with fixed supply is analyzed in Vives (2010) and Vives (2011). The model presented here differs from Vives (2010) by assuming that bidders' private information is the sum of distinct common and private value shocks.¹⁶

The main goal of this section is to demonstrate that the revenue advantage of the basis auction in Proposition 1 persists in the uniform price environment. Part (i) of the following result shows that this is the case whenever N is sufficiently large. In addition, I analytically characterize the difference in expected revenue. Part (ii) shows that this revenue advantage becomes smaller when bidders have less private information about W_t .

¹⁵Equilibrium analysis of the pay-as-bid auction has proven challenging despite its wide usage in sovereign debt auctions and other settings (Hortacsu and McAdams, 2018; Pycia and Woodward, 2025). Wittwer (2018) and Allen and Wittwer (2025) present an interesting closed form model of a pay-as-bid auction with private information which could serve as a starting point for extending the analysis in this paper, at the cost of a stochastic supply assumption. (I thank a referee for this reference.)

¹⁶In Vives (2010) and Vives (2011) bidders observe one signal of a composite type encompassing both common and private values. This assumption simplifies the equilibrium characterization.

Proposition 2. (i) Fix $r = \sigma_u^2/\sigma_\omega^2 > 0$. For large N , the seller's expected revenue is greater in the uniform price basis auction in the uniform price auction in fixed prices. The difference per unit is:

$$\Delta = \lambda \left(\frac{Q}{N} \right) \Psi(r, N) \quad (5)$$

for a known $\Psi(r, N) > 0$ (see Appendix A.1). (ii) For fixed N , $\lim_{s \rightarrow \infty} \Psi(s, N) = 0$.

In a uniform price basis auction indexed to W_t , bidders compete by submitting downward-sloping demand schedules. The optimal demand schedule maximizes profits at each possible clearing price (in this case, a basis to W_t). The interim expected profit of a bidder who wins q_i units when d is the basis price that clears the auction is:

$$E[\pi_i | \omega_i, u_i, d] = (E[W_1 + u_i | \omega_i, u_i, d] - E[W_1 | \omega_i, u_i, d] - d) q_i - \frac{\lambda}{2} q_i^2$$

which immediately simplifies to:

$$E[\pi_i | \omega_i, u_i, d] = (u_i - d) q_i - \frac{\lambda}{2} q_i^2$$

Hence, bidder i infers no useful information from the auction-clearing basis price. There is no adverse selection. In the unique symmetric linear equilibrium, bidder i submits a demand schedule

$$X(d, \omega_i, u_i) = \left(\frac{1}{\lambda + \psi_{basis}} \right) \cdot (u_i - d) \quad (6)$$

where $\psi_{basis} = \frac{1}{N-1} \left(\frac{N-1}{N-2} \right) \lambda > 0$ is referred to as price impact.

In contrast, the uniform price auction in fixed prices does not eliminate adverse selection, leading bidders to behave more cautiously. In this case, interim expected profits are:

$$E[\pi_i | \omega_i, u_i, p] = (E[W_1 | \omega_i, u_i, p] + u_i - p) q_i - \frac{\lambda}{2} q_i^2$$

Assumption 3 implies that $E[W_1 | \omega_i, u_i, p]$ is linear in the conditioning variables. Under the hypotheses of Proposition 2, I show in Appendix A.1 that there is a symmetric linear equilibrium in which each bidder i submits a demand function

$$X(p, \omega_i, u_i) = \left(\frac{1}{\lambda + \psi_{fixed}} \right) \cdot \{E[W_1 | \omega_i, u_i, p] + u_i - p\} \quad (7)$$

where the price impact ψ_{fixed} is greater than ψ_{basis} above. A comparison of expected revenue

per unit given the equilibrium strategies (6) and (7) gives (5).

Proposition 2 does not directly speak to whether price indexing would improve the seller’s expected revenue in the pay-as-bid context. However, a significant body of work in the empirical auctions literature points to what Pycia and Woodward (2025) describe as a “rough revenue equivalence” between the pay-as-bid and uniform price formats – at least in the case of private values, seller revenue is often similar across formats.¹⁷ If one expected the pay-as-bid basis and fixed price auctions to generate similar revenue as their uniform price counterparts, Proposition 2 could then be interpreted as suggestive evidence that the basis auction would also be likely to improve seller revenue in the pay-as-bid case.

2.1 Connection to SPR drawdowns

The model suggests that price indexing will tend to reduce the importance of private information about the common component of valuations. Empirically, the bidding data ought to exhibit little evidence of adverse selection unless bidders hold significant private information about the residual common value Z_t not captured by the price index M_t .¹⁸ This section briefly discusses potential sources of private information in SPR drawdowns.

The SPR was created by the 1975 Energy Policy and Conservation Act (ECPA) in response to the 1973 oil crisis. Currently, the SPR stores up to 713.5 million barrels (MMbbl) of crude oil at four distinct locations along the US Gulf Coast, shown in Figure 1. Each location stores a combination of sweet (i.e., low sulfur) and sour (i.e., high sulfur) crude oils in underground salt caverns. Locations differ in their capacities, crude oil characteristics, access to pipelines and shipping terminals, and proximity to major Gulf Coast refineries.

During an SPR drawdown, DOE uses pay-as-bid basis auctions to sell millions of barrels of oil at once. In recent sales, two distinct price indices have been used depending on the type of oil. For sweet crude, the price index is the average over the five days surrounding delivery of the Argus WTI Houston price; for sour crude, Argus Mars is used instead.¹⁹

The bases between Argus WTI Houston, Argus Mars, and WTI Cushing (a key benchmark in futures markets) can be significant.²⁰ These differences primarily reflect regional

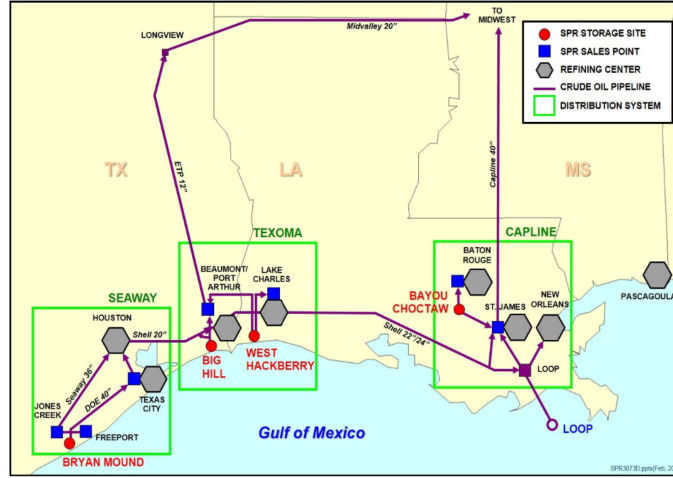
¹⁷Marszalec (2017) Table 10 compiles a summary of point estimates from the literature. Revenue differences of less than 5% have been found in most prior studies, albeit typically in the sovereign debt context where the number of bidders is often large. Ausubel et al. (2014) prove that there is no general revenue ranking.

¹⁸I emphasize that the predictions I draw from the model are heuristic in nature for the SPR drawdown specifically: I have not provided a model of a pay-as-bid auction with imperfect price indices.

¹⁹WTI Houston Weighted Average Month 1, Houston Close and Mars Weighted Average Month 1, Houston Close. The names refer to the daily average reported physical transaction price for front month contracts.

²⁰During 2022, the basis between WTI Houston and WTI Cushing ranged from \$0.54 to \$2.50 per barrel. The basis between Mars and WTI Cushing ranged from −\$6.16 to \$0.26 per barrel. Source: closing prices for WTI Houston (Argus) vs. WTI Trade Month Futures (Bloomberg: HRT1 Comdty) and Mars (Argus)

Figure 1: SPR System Map



Source: DOE.

heterogeneity in supply and demand for different types of crude oil. Issues related to measurement also play an important role. Although Argus WTI Houston and Argus Mars are widely accepted as benchmarks for sweet and sour crude on the US Gulf Coast, it is important to understand that these (and other) oil price benchmarks represent “assessed” prices. Buyers and sellers of physical crude are not required to report transaction prices. Assessed prices are typically based on the average prices of transactions voluntarily reported to a price reporting agency (PRA), but can also incorporate information from non-price sources such as bids and offers (or even interviews with market participants). Assessed prices are potentially vulnerable to manipulation and may not accurately reflect market conditions if there is reporting bias or if the PRA’s modeling assumptions are incorrect (Fattouh, 2011).

Bidders in this environment may be privately informed both about the price index M_t and the residual common value Z_t . Buyers are typically large oil refiners or commodities marketing firms. Within regional markets for physical crude, large buyers and sellers may be able to anticipate the effects of their own strategic decisions (for example, refinery utilization) on the index price M_t . Many oil refiners are vertically integrated into marketing (including storage) or even production, which could generate additional information about M_t . The limitations of commercial price indices also introduce a source of potential private information about the residual common value Z_t : a buyer that does not report its transactions to the PRA knows something about (local) crude prices that other buyers do not.

Transportation costs and crude oil quality can also contribute to the residual common value Z_t . In order to take delivery of SPR crude, a buyer must typically secure capacity on vs WTI Spread Trade Month Swap (Bloomberg: YVA1 Comdty).

one or more commercial pipelines. Most pipeline capacity is allocated via firm, long-term contracts. Winning bidders that do not already hold firm capacity on pipelines connecting to SPR facilities must quickly purchase capacity in the secondary markets, which are prone to scarcity pricing.²¹ Buyers may differ in their beliefs about future prices in this market, which can shift the value of SPR crude for many firms. Buyers may also differ in their beliefs about the quality of SPR crude. Buyers who have previously purchased crude from a particular SPR site may have more information about the sulfur content, API gravity, and other characteristics of SPR crude. As discussed below, DOE attempts to mitigate quality concerns through additional price adjustments, but these price adjustments are not tied to market prices and may not accurately reflect buyers’ valuations for crude oil characteristics.

The cost basis u_i can reflect many factors. One key factor is the cost of transportation. As discussed above, obtaining pipeline capacity in secondary markets can be costly for buyers that do not already hold firm capacity reservations. (Maritime shipment via Jones Act vessels is subject to similar dynamics.) Another potentially important factor is a buyer’s portfolio of non-SPR supply contracts. A buyer that holds lower-cost supply contracts benefits less from purchasing SPR crude at the margin.²² For an oil refiner, the value of SPR crude will also differ depending on output mix (gasoline vs. other distillates) and the demand for those outputs as well as the refinery’s input mix and the supply of any inputs complementary to SPR crude.²³ In the multiunit context, a positive λ value is reasonable if the opportunity cost at which a buyer obtains SPR crude declines with purchase quantity. For example, shipping costs may increase non-linearly with quantity, reducing the benefit of larger purchases.

3 The 2022 Emergency Drawdown

In 2022, United States initiated seven large sales (or “drawdowns”) from the SPR. This section provides further background on SPR sales and introduce the data.

Historical background The main purpose of the SPR is to “protect the U.S. economy from severe petroleum supply interruptions” by distributing stockpiled oil in times of crisis (DOE, 2024). Prior to 2015, emergency sales occurred infrequently and did not exceed more than about 5% of SPR inventory.²⁴ Beginning in 2015, Congress mandated additional sales in

²¹The necessity of allowing buyers time to arrange shipment is one important reason that delivery does not occur immediately.

²²Pre-existing inventories can play a similar role.

²³SPR crude would typically be blended with crude from other sources before distillation. Refineries are engineered to process unique blends (“slates”) of crude oils having specific chemical properties.

²⁴Notable “emergency” releases occurred after Operation Desert Storm (17.3 MMbbl), Hurricane Katrina (11 MMbbl), and the 2011 Libyan crisis (30.6 MMbbl).

Figure 2: Notice of Sale Example

Crude Oil Stream MLI	DLI - Mode of Delivery	Delivery Period	MLI Qty (MB)	MBD Avg	DLI Qty (MB)	MIN Qty (MB)
Bayou Choctaw Sweet 007		6/21-6/30	1,100			
	DLI-A Pipeline St. James Pipeline	6/21-6/30		300	1,100	350
	DLI-H Pipeline Baton Rouge (Bourre)	6/21-6/30		200	225	225

Crude Oil Stream MLI	DLI - Mode of Delivery	Delivery Period	MLI Qty (MB)	MBD Avg	DLI Qty (MB)	MIN Qty (MB)
Bayou Choctaw Sour 008		7/1-8/15	6,000			
	DLI-A Pipeline St. James Pipeline	7/1-8/15		300	4,300	350
	DLI-B Vessel Saint James Dock	7/1-8/15		200	800	250
	DLI-H Pipeline Baton Rouge (Bourre)	7/1-8/15		200	900	225

Source: DOE Notice of Sale (Emergency Drawdown No. 2B, May 24, 2022).

order to raise revenue for budgetary purposes. These sales were dwarfed in size by a series of seven emergency sales totaling 180 MMbbl (roughly one third of SPR inventory) conducted in 2022 in response to oil market disruptions following the Russia-Ukraine conflict. I focus on these sales. Detailed bidding data is publicly available from DOE.

The 2022 drawdown occurred during a period of high prices and elevated uncertainty in global oil markets. Figure 5 plots the time series of daily WTI Cushing prices (panel a) and 30-day realized volatility (panel b) from 2000 to present, both in 2024 dollars. The time period spanning the 2022 drawdown is shaded in gray. WTI Cushing prices increased rapidly early in 2022, with prices reaching \$135 per barrel (2024 dollars) and 30-day realized volatility peaking at 82% on an annualized basis.²⁵ As discussed below, price indexing might be particularly useful during times of high volatility.

3.1 The mechanics of SPR sales

DOE has used its current sales procedures with only minor modification since 1983.²⁶ Each sale begins with the issuance of a Notice of Sale. In the Notice of Sale, DOE stipulates target volumes of sweet and sour crude oil to be sold from designated storage sites, as well as crude specifications, logistical details concerning delivery, and additional contract terms.

Figure 2 provides an example sales specification taken from the third sale in 2022. In this

²⁵Although significant, this episode was not particularly extreme by historical standards: prices had been higher in real terms over much of the period spanning 2008 through 2015, and 30-day realized volatility was greater in 2003, 2008, 2015, and 2020.

²⁶<https://www.spr.doe.gov/reports/SSPs/PART625.html>

sale, which occurred on June 1, DOE sought to release both sweet and sour crude from the Bayou Choctaw storage site near Baton Rouge, LA. For the sweet crude “master line item” (MLI), 1,100 thousand of barrels (Mbbl) were offered for delivery between June 21 and June 30. For the sour crude MLI, 6,000 Mbbl of sour crude were offered for delivery between July 1 and August 15. The rate at which crude oil can be transferred from storage to particular “points of sale” is limited by physical transfer capacity and scheduling constraints. The “delivery line items” (DLIs) in Figure 2 indicate that pipeline deliveries of sweet crude to the Baton Rouge (Bourre) point of sale were limited to 225 Mbbl of the total available 1,100 Mbbl. There were no limitations on pipeline delivery at the St. James point of sale. For sour crude, the same two points of sale were made available for 900 Mbbl and 4,300 Mbbl, respectively, while 800 Mbbl was only made available for vessel delivery (e.g., to oil barges).

A bid specifies a quantity demanded and price for delivery of oil at a particular point of sale. Firms are permitted to submit multiple bids (i.e., demand schedules). However, because of the logistical costs inherent to the physical transfer of oil, the minimum allowed bid size is large relative to the total quantity available (the 225-350 Mbbl “MIN Qty” in Figure 2). For each MLI (crude oil stream), the auction is cleared as a pay-as-bid auction subject to DLI (point of sale) constraints: bids are fulfilled from high price to low price until the total MLI quantity is exhausted, unless a DLI constraint becomes binding, in which case any remaining MLI quantity is allocated to unconstrained DLIs.²⁷

As discussed above, an important feature of the SPR auction is that bids are not expressed in fixed prices, but rather as a basis (referred to as a price adjustment factor or PAF) relative to one of the two index prices introduced above. The window for taking delivery of purchased crude typically does not begin until a few weeks after the auction, and can last more than a month (as in the Bayou Choctaw example above). During this time, the index price can move considerably. Table 1 shows the average value of the WTI Houston and Mars crude oil price indices during the delivery window in comparison to the prevailing index price on the date the Notice of Sale was issued (the “base reference price” or BRP).²⁸ After the first sale (March 8), the sweet and source crude price indices were 12% higher than the BRP on the average day during the delivery window. After the fourth sale (June 28), price indices were

²⁷In the previous example, if the high bidder for Bayou Choctaw Sweet requested 225 Mbbl at Baton Rouge (Bourre), leaving no remaining supply, then any further bids for delivery at Baton Rouge (Bourre) would be discarded and the remaining 875 Mbbl could only be sold to bidders requesting delivery at St. James. On the other hand, if the high bidder requested 1,100 Mbbl at St. James, no further supply would be available for bidders requesting delivery at Baton Rouge (Bourre).

²⁸The reported crude oil indices correspond to the Bloomberg tickers USCRMEHC (Crude Oil WTI Houston) and USCRMARS (Crude Oil Mars). These indices are not identical to the specific price indices used by DOE in practice, which are published exclusively by Argus Americas Crude and could not be obtained for this study. It is reasonable to expect that these series are highly correlated.

more than 25% lower on average.

Reserve prices One notable feature of the auction environment absent from the model in Section 2 is a reserve price fixed at 95% of DOE’s estimate of the market value of crude. DOE reserves the right to reject bids that are “below 95 percent of the sales price, as estimated by the Government, of comparable crude oil being sold in the same area at the same time.” I describe this reserve price as *nominal* because bids below it may still be accepted “if the Contracting Officer determines such action is necessary to achieve SPR crude oil supply objectives and such offered prices are reasonable.” As explained below, the nominal reserve price appears not to have been binding during the 2022 SPR drawdown.

Price adjustments for quality SPR crude is segregated into eight crude oil streams (sweet and sour at each location) spread across sixty salt caverns. Within each cavern is a mixture of crude oils accumulated from different sources over time. Crude oil within a single cavern eventually becomes well-mixed due to temperature gradients within the salt caverns (Department of Energy, 2024). Nevertheless, the physical characteristics of crude oil delivered to a buyer may differ from stream-level specifications disclosed in the Notice of Sale depending on the specific caverns used. When this occurs, DOE adjusts final payments according to tables of pre-specified price differentials that scale with each 0.1 degree difference in API gravity and 0.01 percentage point difference in sulfur content from announced levels. In the framework of the model, these adjustments can be viewed as partially “indexing” for one potentially important component of the residual common value Z_t . However, price increments are fixed in advance rather than being tied to market benchmarks. There are no adjustments for discrepancies in other crude oil characteristics.²⁹

Use of fixed prices in acquisitions Prior to 2022, a price indexing mechanism was also used for purchases to fill the SPR. In late 2022, DOE modified its acquisition procedures to allow fixed price purchases. After the change of administration in 2025, DOE again modified its acquisition procedures, this time in order to prohibit fixed price purchases.³⁰

Approximately 58 MMbbl of sour crude has been purchased by DOE since the conclusion

²⁹This is not to say that other crude oil characteristics cannot affect buyer valuations. Some potentially important characteristics include acidity, viscosity, and metal content.

³⁰The stated rationale for the 2022 rule change was to “increase flexibility” in “circumstances in which a fixed-price acquisition would better meet [...] statutory objectives.” A press release from the White House on October 18, 2022 explained that, “This repurchase approach will protect taxpayers and help create certainty around future demand for crude oil. That will encourage firms to invest in production right now[.]” The justification for the 2025 recision was that changes to permit the use of fixed-price contracts had “only served to unnecessarily create confusion in industry, which uses index-price contracts[.]”

of the 2022 emergency sales.³¹ It is not clear that either the 2022 rule change or its 2025 rescission substantively affected DOE acquisition practice during this period. The initial rule change in 2022 permitted but did not obligate the use of fixed prices. Based on available documentation, it appears that relatively rigid forms of price-indexing have been used in all tenders since the 2022 rule change, including the most recent tender which occurred after the 2025 rescission. In particular, DOE has indexed prices to the award date (generally occurring within two weeks of offer submission, but weeks or months before delivery).³²

DOE’s purchases during this time were criticized by some after 16 of 27 tenders were undersubscribed or cancelled (including 7 outright failures). In light of this paper’s analysis, it is plausible that a more flexible indexing procedure could have attracted a greater number of offers, perhaps at lower prices. Yet the key factor explaining undersubscription was likely DOE’s aggressive price targets (initially presented as guidance, later formalized with reserve prices). A limitation of the present paper is that I do not model reserve prices.

3.2 Bidding data

This section briefly describes the bidding data. To begin, Table 2 presents summary statistics on participation, demand, and winning bids for each crude oil stream (MLI) and delivery location (DLI). On average, 10.9 bidders submitted 3.7 bids each for a given sweet crude MLI, often submitting bids on multiple DLIs at once.³³ The highest bid was \$1.19 greater than the index price and \$4.08 greater than the lowest winning bid. The lowest winning bid was \$2.89 less than the index price, but \$2.24 greater than the nominal reserve. Clearing prices were generally higher for pipeline delivery in comparison to vessel delivery. Similar patterns are present for the sour crude sales. Two key differences are that auctions for sour crude had fewer bidders on average, and higher prices relative to the price index.

Figure 6 presents firm-level bid curves as well as the total demand curve among bids submitted for one specific DLI in a particular sale.³⁴ The figure illustrates that there is significant variation across bidders both in terms of maximum expressed willingness-to-pay (the bid intercepts) as well as in terms of bid slope. In this example, no single bidder

³¹Of the 180 million barrels sold in the 2022 drawdown, 140 million barrels were “secured” via cancellation of previously scheduled sales that would have occurred between 2024 and 2027.

³²27 tenders occurred between 2022 and 2024 (after the initial rule change), and 1 occurred in 2025 (after the rescission). For the first 17 tenders, DOE Requests for Proposal (RFPs) indicate that payments were indexed to market prices over three days following a notice of award. For the remaining 10 tenders, the specific price index was modified but remained fixed to the date of the notice of award. The 2025 RFP appears to revert to the identical pricing language as the first 17 tenders. I have not been able to obtain documentation for tenders that occurred prior to 2022.

³³Like in other multiunit auction settings, the multiple bids submitted by an individual bidder together constitute a downward-sloping demand schedule. Each bid is associated with a particular quantity demanded.

³⁴For this DLI there was effectively a fixed supply of crude, simplifying the visual presentation.

demanded more than half of the available supply (indicated by the vertical red line), while total demand at any price exceeded supply by a factor of three.

Domestic oil refiners were by far the largest purchasers of crude oil in the 2022 SPR sales. 74% of the crude oil sold in the 2022 SPR releases was purchased by one of the four largest US refiners (Marathon, Valero, ExxonMobil, and Phillips 66) or by Motiva (a Saudi Aramco subsidiary which owns a large refinery in Texas). Most of the remaining crude oil was purchased by widely known commodities trading firms.³⁵

Interestingly, more than 15.2% of bids by quantity were submitted below the nominal reserve price, indicating that bidders may not have expected the reserve to be enforced. Consistent with this belief, there were no auctions in which the DOE rejected otherwise competitive bids for being lower than the reserve price. However, the vast majority of these low bids were unsuccessful. Only 1.4% of accepted bids were below the nominal reserve.

Exclusive tranches Empirical methods for auctions typically assume that the unit of observation is a single, independent auction with a fixed number of (potential) bidders. The latter assumption can be violated in SPR sales due to DLI constraints: with respect to a single DLI, competition may change discontinuously with quantity.³⁶

For the case of sweet crude in the example in Figure 2, bidders who submitted bids for the St. James DLI faced more competition for quantities above than below 875 Mbbl because quantities in excess of 875 Mbbl were also contested by bidders who submitted bids for the Baton Rouge (Bourre) DLI.

To abstract from complications arising from DLI constraints, the primary unit of observation for the empirical analysis below is an *exclusive tranche*. An exclusive tranche is defined as a portion of crude oil that accessible only to bidders on a particular DLI. In the example, there is one exclusive tranche of 875 Mbbl associated with St. James, but no exclusive tranche associated with Baton Rouge (Bourre). For the case of sour crude in Figure 2, there were three exclusive tranches: one for each DLI. In total, there were 54 exclusive tranches (2.16 per MLI per sale). Each exclusive tranche can be viewed as a distinct pay-as-bid auction with a fixed number of bidders. Restricting attention to exclusive tranches involves little loss of data: 89% of crude oil made available during the 2022 SPR drawdown and 93% of revenue can be attributed to an exclusive tranche.³⁷

³⁵In practice the distinction between refiner and marketer is not sharp. Some refiners operate trading desks, and some marketers buy and sell on behalf of refiners.

³⁶A similar effect occurs in competitive wholesale electricity markets due to transmission congestion.

³⁷Revenue calculation based on base reference prices.

4 Tests of common and private values

In this section, I test a key prediction from the theoretical analysis: if the price indices used to determine final payments are sufficiently similar to the common component of valuations, there should be no evidence of adverse selection. In other words, the absence of evidence of adverse selection would be consistent with a model in which price indexing benefits DOE through the channels described in Section 2. Conversely, positive evidence of adverse selection would suggest DOE’s choice of index could be improved.

An important caveat is that the mere absence of evidence of adverse selection would not imply that price indexing generated material benefits for DOE during the 2022 drawdown. To draw this conclusion, it would be necessary to further prove that bidders are privately informed about the index price M_t . I defer further discussion of this issue to Section 5.

4.1 Test of common values

Few formal tests of common values have been proposed for the pay-as-bid auction.³⁸ One exception is a reduced form test due to Nyborg et al. (2002).³⁹ This test is based on the prediction that bidders in a pay-as-bid auction will tend to reduce the level and steepen the slope of their bids as common value uncertainty becomes more severe (Ausubel, 2004).⁴⁰ Like in the first price auction, a winning bidder learns that other bidders received lower signals of the common value. In addition, a bidder who wins a larger number of units learns that other bidders received relatively worse signals (the “champion’s plague”). A negative correlation between bid levels and uncertainty, or a positive correlation between bid slope and uncertainty, is therefore consistent with common values (Nyborg et al., 2002).

I implement this test using two distinct measures of uncertainty. The first is the CBOE Crude Oil Volatility Index (or OVX). OVX is a measure of implied volatility constructed from the price of options on the USO ETF, which tracks WTI Cushing prices.⁴¹ WTI Cushing prices are highly correlated with, but not identical to, the index prices used by DOE. Hence, OVX should correlate with uncertainty about the index price M_t . Because

³⁸Hortaçsu and Kastl (2012) exploit the availability of an instrument shifting some bidders’ information in the context of Canadian treasury auctions. Bonaldi et al. (2024) estimate a parametric model of a uniform price auction in which bidders have both private and common values, enabling a decomposition exercise. I lack the data to implement the test of Hortaçsu and Kastl (2012), while the lack of convenient and flexible parametric models for pay-as-bid auctions precludes an approach similar to Bonaldi et al. (2024).

³⁹Nyborg et al. (2002) study pay-as-bid sovereign debt auctions. Prior applications of the test include Bjørnnes (2001), Elsinger and Zulehner (2007) and Mariño and Marszalec (2023).

⁴⁰An additional prediction of Nyborg et al. (2002) is that bidders will reduce their maximum quantity demanded if the seller imposes a reserve price. I do not evaluate this prediction.

⁴¹The OVX is similar to the better-known CBOE VIX, which approximates stock market volatility based on S&P 500 index options. See Chen et al., 2018 for further discussion of the OVX.

price indexing reduces the value of information about M_t , one would not expect to find a strong correlation between the OVX and observed bid slopes or levels unless uncertainty about WTI Cushing is correlated with uncertainty about the basis between M_t and WTI Cushing or with uncertainty about time-varying elements of the residual common value Z_t . The latter could occur if, for example, pipeline shipping costs were correlated with oil prices.

The second measure of uncertainty is defined as the number of days elapsed between the sale date and the midpoint of the DLI delivery window indicated in the Notice of Sale (see Figure 2 for an example). If uncertainty increases over longer time horizons, as one might expect, time-to-delivery correlates both with index price uncertainty and with uncertainty about time-varying elements of the residual common value Z_t . The test can detect the latter. For example, a longer time-to-delivery plausibly correlates with greater uncertainty about the crack spread (the difference between refined product and crude oil prices), which lacks a simple relationship with the oil price level but which affects refiner profits. On the other hand, time-to-delivery would not correlate with time-invariant elements of Z_t such as those stemming from differences in crude oil characteristics not addressed by DOE’s quality adjustments, unless the common value of those characteristics also changed over time.

Results Table 3 summarizes regressions of bid levels and slopes on both measures of uncertainty. Bid levels are measured using a firm’s quantity-weighted average PAF. As in Nyborg et al. (2002), the steepness of bid slopes is measured by the quantity-weighted standard deviation in PAFs across a firm’s bids. Observations are weighted by the measure of uncertainty, and two specifications for the bid level are included: one using bidder-tranche level average bids, and one using the tranche-level average of bidders’ within-tranche average bids.⁴² I also control for sulfur content, delivery method, and SPR site. Together, these characteristics can be interpreted as a control for potential entry.⁴³ To complement the regression results, Figure 7 depicts the correlations between bid levels, bid slopes and each measure of uncertainty. I present two versions in each panel: the raw correlation (left) and the correlation obtained after residualizing on the controls included in the main regressions (right).

The coefficients on uncertainty in the bid level regressions in columns (1), (2), (4), and (5) of Table 3 are all positive. Thus, bid levels appear to increase on average when uncertainty is greater. The point estimate in (4) implies that shifting the delivery window back by a week increases average bids by about \$0.16 per barrel. The point estimate in (1) indicates that an increase of similar magnitude is implied by a 5 point increase in OVX (corresponding to a 5 percentage point increase in annualized implied volatility). These results can be viewed

⁴²The latter helps to address bias that may arise from correlation in the error terms within an auction.

⁴³Bjønnes (2001) argues that controlling for participation is important for distinguishing between bidding behavior driven by common values and bidding behavior driven by risk aversion.

as inconsistent with the presence of residual private information, although the effects are small in magnitude and none is statistically significant. Correspondingly, there is little visual evidence of strong correlation in panels (a) or (c) of Figure 7. The coefficients on uncertainty in the slope regressions in columns (3) and (6) are statistically significant, but differ in sign. A steeper slope corresponds to larger standard deviation in bid prices. Hence, the estimate in (3) implies that bids become steeper when the OVX is greater, while the estimate in (6) implies that bids become flatter when time-to-delivery is longer. The first result is consistent with the presence of private information; the second is not. Taken together, the results in Table 3 do not provide clear evidence of adverse selection.

4.2 Test of private values

In this section I consider a direct test of independent private values (IPV) due to Hickman et al. (2021). The central idea underlying this test is that, under the null hypothesis of IPV with no unobserved heterogeneity, there should be no residual correlation in bids after controlling for the number of bidders N and other observables.

In a single-unit context, this idea can be implemented by regressing observed bids on the mean of within-auction rival bids, observables, and a flexible function of N . IPV is rejected if the coefficient on within-auction rival bids is different from zero. This idea can be extended to the context of a multiunit auction by restricting attention to quantity levels that are common to all submitted bids. For example, if all bidders are willing to accept a single unit of the good, one might use the highest price bid submitted by each bidder (the “bid intercept”). I take the bid intercept approach here.

Hickman et al. (2021) report that the single-unit test has favorable power even in small samples. It rejects not only common values, but also various alternative private values models such as the affiliated private value (APV) model, the conditional independent private values (CIPV) model, and IPV models with auction-level unobserved heterogeneity. In Appendix B, I verify the finite-sample power of the multiunit bid intercept test using simulated bidding data from a calibration of Vives (2011)’s uniform price auction model. The test rejects CIPV and pure common values at high rates in sample sizes comparable to the bidding data.

DOE allows bidders to stipulate a minimum bid quantity.⁴⁴ When a bidder’s highest price bid has a non-trivial minimum quantity, it is not clear that the associated bid price should be treated like an intercept: the bidder is not willing to accept a smaller number of units at the indicated price. To address this issue, I implement the bid intercept test separately for all exclusive tranches and for the subsample of exclusive tranches in which no

⁴⁴In Figure 2, note the “Min Qty” values which range from 225-350 Mbbl across DLIs.

such bids were submitted. In the former case, I retain bids with high minimum quantities when calculating N but exclude them from the estimation sample and from the calculation of the mean of rivals' bids.

Table 4 reports the results. The main specification is reported in column (3). As in Richert (2024), this model includes a cubic polynomial of N . The coefficient on the mean rival bid is negative and not significantly different from zero. Thus, the test fails to reject the null of IPV. A similar result is shown column (5), which retains the tranches excluded due to bids with high minimum quantities in the manner described above.⁴⁵ The remaining specifications test the robustness of the result in column (3). Column (1) omits all covariates. Column (2) includes all covariates, but omits the polynomial in N . In both cases, I obtain positive but insignificant coefficients. For the main results in column (3), standard errors are clustered at the tranche level. Column (4) reports standard errors calculated without clustering. This change does not cause the main estimate to become statistically significant at conventional levels. Together with the results above, these findings provide suggestive evidence against the presence of adverse selection in the bidding data.

5 Gains from price adjustment

The analysis thus far suggests, but does not prove, that price indexing could have generated significant revenue gains for DOE during the 2022 drawdown. As emphasized above, evidence against adverse selection does not imply that adverse selection was eliminated by price indexing. The key question is whether bidders are privately informed about the index price M_t . Because the model suggests that bidding strategies do not depend on this type of information, the bidding data itself cannot resolve this question.

In this section I take an indirect approach and attempt to quantify the magnitude of private information for which DOE's revenue gains from price indexing would have been large. If only a small amount of private information would be needed to generate large gains, then it is more likely that the quantitative impact of price indexing was significant. Such an exercise is inherently counterfactual and hence requires a model for which equilibrium analysis is tractable. I therefore adopt the uniform price framework from Section 2, which is similar but not identical to DOE's pay-as-bid format. Although this entails a loss of realism, I can obtain useful bounds on bidders' information by calibrating the closed form representation of the revenue gain from price indexing presented in Proposition 2.

I proceed in two steps. First, I exploit the empirical regularity that uniform price and

⁴⁵Both samples further exclude auctions with fewer than two bidders, since in this case the mean of rivals' bids cannot be calculated.

pay-as-bid auctions tend to generate similar seller revenue to calibrate the slope parameter λ in bidders' valuation functions (4). Second, I use Proposition 2 to calculate the minimum amount of private information necessary for price indexing to generate a given increase in expected revenue across all seven sales that occurred in 2022. I find that relatively little private information would be necessary to obtain a \$1 per barrel increase in expected revenue from the use of the basis auction versus a fixed price auction.

Calibration of slope λ The observed bidding data is generated by DOE's pay-as-bid basis auction. The expected auction-clearing basis price in the uniform price basis auction is

$$E[d] = \theta_u - \lambda \cdot \left(\frac{Q}{N}\right) \cdot \left(\frac{N-2}{N-1}\right)$$

If expected revenue in a pay-as-bid basis auction is similar to the expected revenue in a uniform price basis auction, then the quantity-weighted average winning basis d_τ observed in exclusive tranche τ can be predicted by the linear model

$$x'_\tau \beta_{pred} - \lambda_{pred} \cdot \left(\frac{Q_\tau}{N_\tau}\right) \cdot \left(\frac{N_\tau-2}{N_\tau-1}\right)$$

where x_τ is a vector of tranche characteristics and λ_{pred} is the slope parameter. I calibrate β and λ_{pred} to minimize the within-sample prediction error of this model. To do so, I fit a linear regression weighting tranches by quantity. I obtain a point estimate $\hat{\lambda}_{pred} = 0.0022$ for the best predictor of λ_{pred} with 95% bootstrap confidence interval (0.0005, 0.0051). The point estimate implies that bidders' marginal valuations decline by \$1.10 per barrel for an additional 0.5 million barrels of quantity (the average size of observed awards).⁴⁶

Information structure analysis Within the context of the uniform price model, the equilibrium expected revenue benefit Δ of the basis auction is described by (5) for a given slope parameter λ and precision ratio $r = \sigma_u^2/\sigma_\omega^2$. These two parameters are sufficient to determine the revenue impact of price indexing conditional on Q and N .

The precision ratio r is difficult to calibrate. If bidders in DOE's basis auction have no private information, as suggested both by the analysis in Section 2 and the empirical results in Section 4, then the observed bidding data contains no information that can be used to pin down σ_ω . Nevertheless, the relationship (5) can be used to derive bounds on the amount of private information necessary to obtain revenue effects of a particular size. To this end, I

⁴⁶For statistical purposes, the calibration assumes that λ is the same for all bidders.

let T denote the collection of exclusive tranches in the bidding data and define

$$\bar{\Delta}(\lambda, r) := \lambda \left(\sum_{\tau \in T} Q_{\tau} \right)^{-1} \left(\sum_{\tau \in T} \frac{Q_{\tau}}{N_{\tau}} \Psi(r, N_{\tau}) \right)$$

to approximate the hypothetical per barrel increase in expected revenue from price indexing if DOE had used a uniform price basis auction (rather than the true pay-as-bid format). If the pay-as-bid auctions are believed to generate similar revenues as uniform price auctions, this quantity approximates the revenue increase from price indexing in the 2022 SPR drawdown.

Figure 8 plots $\log \bar{\Delta}(\lambda_{pred}, r)$ versus the ratio $\frac{1}{1+r} = \sigma_{\omega}^2 / (\sigma_u^2 + \sigma_{\omega}^2)$. The latter is interpreted as the fraction of variance in bidders' ex ante marginal valuations attributable to private information about the common value W_t .⁴⁷ A value of $\log \bar{\Delta}(\lambda, r)$ equal to zero implies that expected revenue increases by \$1.00 per barrel due to price indexing. DOE sold 180 million barrels of crude during the 2022 drawdown; hence, an increase of \$1.00 per barrel would imply a total gain of \$180M. This occurs for $\frac{1}{1+r} = 0.15$ (95% bootstrap CI: (0.07, 0.36)), where private information constitutes a relatively small share of the total variance in valuations. For a sense of scale, one would expect that the standard deviation of private costs σ_u is unlikely to exceed a few dollars per barrel. $\frac{1}{1+r} = 0.15$ occurs when σ_{ω} is less than half as large.⁴⁸ The expected revenue gain remains economically meaningful even for substantially smaller amounts of private information. For $\frac{1}{1+r} = 0.04$ (bootstrap CI: (0.02, 0.20)), expected revenue increases by \$0.25 per barrel, implying a total gain of \$45M.

Market volatility If potential buyers' private information about oil prices becomes more dispersed during times of elevated uncertainty, one would expect adverse selection in the fixed price auction to worsen, making the basis auction more attractive. If this is the case, the benefit from using a basis auction may be greater during emergency sales like those in 2022 than during non-emergency sales (such as budgetary sales during periods of greater stability). During 2022, uncertainty about the residual common value Z_t was plausibly elevated for sour crude in particular: because Russia was previously a major exporter of sour crude to Europe, the market for sour crude changed rapidly in 2022, potentially compromising the accuracy of commercial sour crude price indices like Argus Mars. 55% of the total 180 MMbbl sold was sour crude.

⁴⁷A confidence band is obtained using the reported bootstrap confidence interval for λ_{pred} .

⁴⁸For larger amounts of private information, the revenue benefit is greater. When there is too much private information, however, equilibrium existence can fail. A similar phenomenon occurs in Vives (2010) and Vives (2011) when adverse selection is too severe. See also Rostek and Yoon (2023).

Crack spreads In practice, oil refiners often manage price risk by attempting to lock in a “crack spread” – a differential between forward crude oil and refined product prices corresponding to a firm’s profit margin. Refined products are typically sold in bulk on a floating price basis (for example, as a differential to WTI Cushing). In this situation, the basis auction allows winning bidders to hedge the crack spread *without* entering into a futures position. Although futures markets for crude oil are among the deepest financial markets in existence, taking futures positions is not costless due to basis costs, maturity mismatch, and the costs associated with margin requirements. Bidders who anticipate hedging the crack spread after winning may have higher valuations under the basis auction than under the fixed price auction, further contributing to any revenue and efficiency advantages.

6 Conclusion

This paper analyzes index-based price adjustment during DOE’s historic 2022 SPR drawdown. Drawing on the auctions literature, I use an illustrative model to show how price indexing can reduce information rents, limit adverse selection, improve efficiency, and reduce the risk of default. In the extreme case that the common component of bidders’ valuations coincides with the price index, bidders act as if they compete in a private values environment even if they are informed about future oil prices. Empirically, I fail to find evidence of common values in bidding data from the 2022 drawdown, consistent with this implication of the model. If bidders are believed to be privately informed about future oil prices, this suggests that index-based price adjustment is an effective auction design tool in the context of SPR drawdowns. Absent direct evidence of private information, which is difficult to measure in this context, I argue that relatively little private information would be required in order for the use of price indexing to have had a significant impact.

These results highlight the potential value of price indexing for sellers in settings where buyers’ valuations have important time-varying components but goods cannot be delivered immediately. This perspective complements the typical motivations for price indexing in the long-term contracting literature (Goldberg, 1985; Goldberg and Erickson, 1987; Joskow, 1988), which has emphasized contract performance within bilateral relationships rather than ex ante competition. At the same time, the apparently limited use of formal basis auctions in practice raises a question for future research.

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A Proofs

Proposition 1

The argument is made in two parts. First, I show that expected information rents in the basis auction must be smaller than expected information rents in the first price auction. This follows from the following general property of order statistics for independent random variables drawn from log-concave distributions.

Lemma 3. *Suppose that Y_1, \dots, Y_N and X_1, \dots, X_N are samples of independent random variables, with $Y_i = X_i + U_i$ for $U_i \perp X_i$ and X_i having a log-concave density. Then:*

$$E [Y_{(n:n)} - Y_{(n-1:n)}] \geq E [X_{(n:n)} - X_{(n-1:n)}]$$

where $Y_{(n:n)}$ and $Y_{(n-1:n)}$ are the n th and $(n-1)$ th order statistics.

Proof. By Theorem 3.B.7 of Shaked and Shanthikumar (2007), it follows that $X \leq_{disp} X + U = Y$ (that is, Y dominates X in the dispersive order). Theorem 3.B.31 says that if $X \leq_{disp} Y$, then $V_{i:n}$ dominates $U_{i:n}$ in the stochastic order for $i \geq 2$, where $U_{i:n} = X_{i:n} - X_{i-1:n}$ and $V_{i:n} = Y_{i:n} - Y_{i-1:n}$. \square

Recall that the expected information rent is equal to the expected difference between the first and second order statistics of bidders' types. Under the maintained assumptions, bidders types are drawn from log-concave distributions.

Next, the winning bidder's interim valuation in the fixed price auction is lower in expectation than in the basis price auction. This is because:

$$\begin{aligned} E [S_{(n:n)}] + \left(\frac{N-1}{N} \right) E [\omega_j | s_j \leq S_{(n:n)}] &= \frac{1}{N} E [\omega_i | s_i = S_{(n:n)}] + \left(\frac{N-1}{N} \right) E [\omega_i | s_j \leq S_{(n:n)}] \\ &\quad + E [u_i | s_i = S_{(n:n)}] \\ &= E [\omega_i] + E [u_i | s_i = S_{(n:n)}] \\ &\leq w_1 + E [U_{(n:n)}] \end{aligned}$$

where the second equality follows from the observation that:

$$\begin{aligned} E [\omega_i] &= E [\omega_i | s_i = S_{(n:n)}] P (s_i = S_{(n:n)}) + E [\omega_i | s_i \leq S_{(n:n)}] \{1 - P (s_i = S_{(n:n)})\} \\ &= E [\omega_i | s_i = S_{(n:n)}] \frac{1}{N} + E [\omega_i | s_i \leq S_{(n:n)}] \cdot \left(\frac{N-1}{N} \right) \end{aligned}$$

Combining the previous results gives that the basis price auction generates greater expected surplus, while reducing expected information rents. Thus, expected revenue is improved.

A.1 Uniform price auction model

I first characterize equilibrium of the fixed price auction before establishing the revenue comparison result. Consider a bidder who observes signals ω_i and u_i . The optimal quantity bid at clearing price p is the solution to

$$\max_{q_i} (E[W_1|\omega_i, u_i, p] + u_i - p) q_i - \frac{\lambda}{2} q_i^2$$

By standard arguments, a necessary condition of equilibrium is that:

$$q_i = \left(\frac{1}{\lambda + \psi_i} \right) \cdot \{E[W_1|\omega_i, u_i, p] + u_i - p\} \quad (8)$$

where $\psi_i = \frac{dp}{dq_i}$ is bidder i 's price impact. In a symmetric equilibrium, $\psi_i = \psi_j = \psi$ for all bidders i and j . We conjecture that (8) is linear: $q_i = a + b\omega_i + cu_i - Dp$. The market clearing price p is therefore a linear function of Gaussian random variables conditional on ω_i, u_i . Moreover, W_1 is Gaussian conditional on ω_i, u_i . By the projection theorem,

$$E[W_1|\omega_i, u_i, p] = \omega_i + \frac{Cov(W_1, p|\omega_i, u_i)}{Var(p|\omega_i, u_i)} \cdot (p - E[p|\omega_i, u_i])$$

The right hand side is linear in the conditioning variables:

$$E[W_1|\omega_i, u_i, p] = \gamma_0 + \gamma_w \omega_i + \gamma_u u_i + \gamma_p p$$

where the coefficients $(\gamma_0, \gamma_1, \gamma_u, \gamma_p)$ can be expressed in terms of (a, b, c, D) and the model primitives. In particular, the slope coefficient is $D = \frac{1-\gamma_p}{\lambda+\psi}$. Differentiation of the market clearing condition gives $\psi = \frac{1}{(N-1)D}$, eliminating ψ . Matching the coefficient on ω_i then gives $b = D$. Matching the coefficient on u_i yields the cubic equation:

$$-\left(\frac{N-1}{N\sqrt{N}}\right)^2 \alpha^3 + \left(3N + \frac{1}{N} - 4\right) \alpha^2 + ((2-3N) - r) \alpha + N = 0 \quad (9)$$

where $\alpha := \left(1 + \left(\frac{c}{b}\right)^2 \frac{r}{N}\right)^{-1}$. The discriminant of this equation is

$$\Delta(N, r) = - \left(\frac{N-1}{N}\right)^2 (4Nr^2 + (9N(3N-2) - 1)r + 4) r \sigma_w^8$$

which is strictly negative for $N \geq 2$ and $\sigma_u, \sigma_w > 0$. Hence, (9) has one real root. This root can be expressed algebraically using Cardano's formula. We then obtain c by inverting the definition of α . Finally, we solve for a by again matching coefficients. This completes the characterization of the equilibrium bidding strategy.

It follows that the expected price in the fixed price auction is

$$E[p] = E[W_1] + \theta_u - \lambda \cdot \frac{Q}{N} \left(\frac{N-1}{N-2} + \Psi(N, r) \right) \quad (10)$$

where

$$\Psi(N, r) = \frac{4Y(N, r)^{1/3} - 2^{2/3}Y(N, r)^{2/3} + 6\sqrt[3]{2}N \cdot r - 2\sqrt[3]{2}}{10Y(N, r)^{1/3} - 2^{2/3}Y(N, r)^{2/3} + 6\sqrt[3]{2}N \cdot r - 2\sqrt[3]{2}} - \frac{N-1}{N-2}$$

for $Y(N, r) = 2 + 9N(3N-1)r - Z(N, r)$ where $Z(N, r) = 3\sqrt{3}N \left(\frac{N}{N-1}\right) \sqrt{-\sigma_w^{-8}\Delta(N, r)}$. In comparison, the expected price in the basis auction is

$$E[d] = \theta_u - \lambda \cdot \frac{Q}{N} \left(\frac{N-1}{N-2} \right) \quad (11)$$

The difference between $E[W_1] + E[d]$ and (10) is (5).

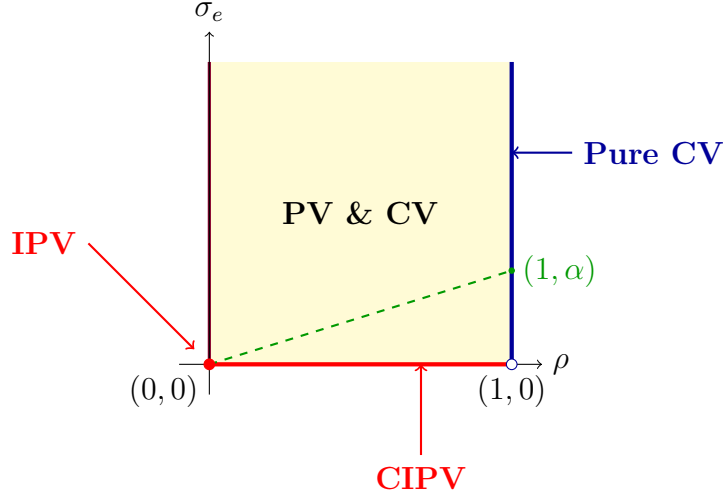
It remains to characterize the limiting behavior of $\Psi(\cdot, \cdot)$. It is not difficult to show that $\Psi(N, r)$ is strictly positive when $Y(N, r)$ is sufficiently negative. It therefore suffices to prove that $\lim_{N \rightarrow \infty} Y(N, r) \rightarrow -\infty$ with $\frac{d}{dN}[Y(N, r)] < 0$ for sufficiently large N . This must be true because when r is fixed

$$Z(N, r) = 27r \cdot N^2 + (2r - 9)rN + O(1)$$

and therefore $Y(N, r) = -2r^2N + O(1)$.

Next, consider a fixed N . For large r the numerator of the first fraction of $\Psi(N, r)$ is approximated by $\sqrt[3]{3} \cdot \sqrt{2N} (4 - \sqrt[3]{4} \cdot (9N - 3)) r^{1/2}$ while the denominator is approximated by $\sqrt[3]{3} \cdot \sqrt{2N} (10 - \sqrt[3]{4} \cdot (9N - 3)) r^{1/2}$. The ratio equals $\frac{N-1}{N-2}$; hence $\lim_{s \rightarrow \infty} \Psi(N, s) = 0$.

Figure 3: Information Structures in the Vives (2011) Model



B Power analysis for Hickman et al. (2021) test

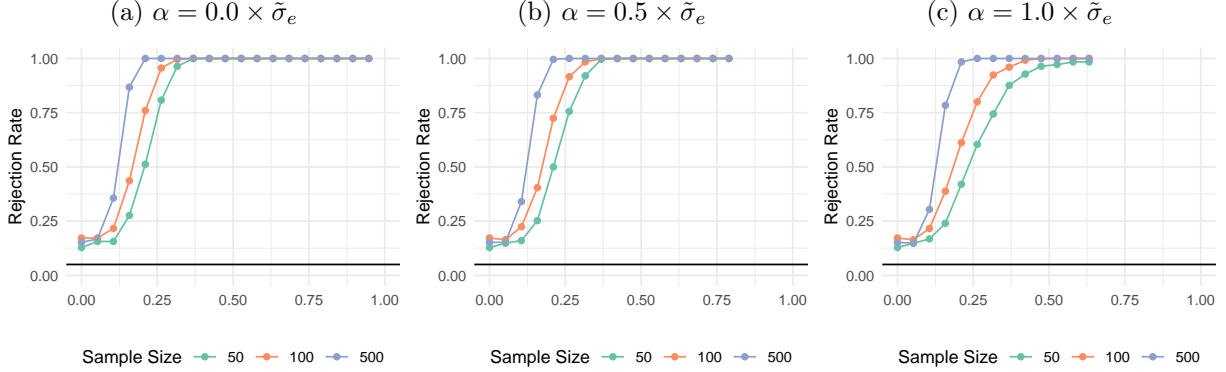
This section evaluates the power of the Hickman et al. (2021) test in small samples.

In order to generate simulated bid data, I adopt the uniform price auction model of Vives (2011). Bidders are assumed to have marginal valuations of the form $v_i(q) = \theta_i q - \lambda q$, where $\theta_i = W + u_i$. The private cost bases u_i are distributed $u_i \sim N(0, \sigma_u^2)$ and the oil price signals w_i are distributed $w_i \sim N(0, \sigma_w^2)$. Both u_i and w_i are assumed to be independent from each other and independent across bidders. I assume $(\lambda, \sigma_u, \sigma_w) = (100, 10, 20)$ based on typical calibrated values from Section 5.⁴⁹ Let $\tilde{\sigma}_e^2$ denote the inverse of $\frac{1}{\sigma_w^2} - \frac{1}{\sigma_w^2 + \sigma_u^2}$, which implies that $\text{Var}(\theta_i | s_i) = \sigma_w$ when $\rho = \frac{\sigma_w^2}{\sigma_w^2 + \sigma_u^2}$ (as in Section 5). In the exercises below I vary ρ and σ_e without restricting the posterior variance $\text{Var}(\theta_i | s_i)$.

Figure 3 depicts the information structure in the space of ρ and σ_e , where ρ denotes the correlation $\text{cor}(\theta_i, \theta_j) = \rho$ for all $i \neq j$. The origin corresponds to independent private values. The vertical line that begins at $(0,0)$ encompasses IPV models in which bidders do not perfectly observe their own valuations. The x -axis, where $\sigma_e = 0$, coincides the family of conditional IPV models with increasingly greater correlation between bidder values. The limit of perfect correlation in the CIPV model is a model of pure common values in which all bidders are perfectly informed about the value of the good. The vertical line that begins at the point $(1,0)$ encompasses classical pure common values with uncertain valuations, with the severity of the winner's curse increasing in σ_e . Finally, the interior shaded region encompasses a class of mixed information structures with both private and common values.

⁴⁹Quantities are denominated in thousands of barrels (Mbbl).

Figure 4: Simulated Rejection Rates for $(\rho, \sigma_e) = (t, \alpha t)$



Notes: (1) As t increases (x-axis), the information structure is further from IPV. See Appendix B for details. (2) Approximate rejection rates from 250 simulations.

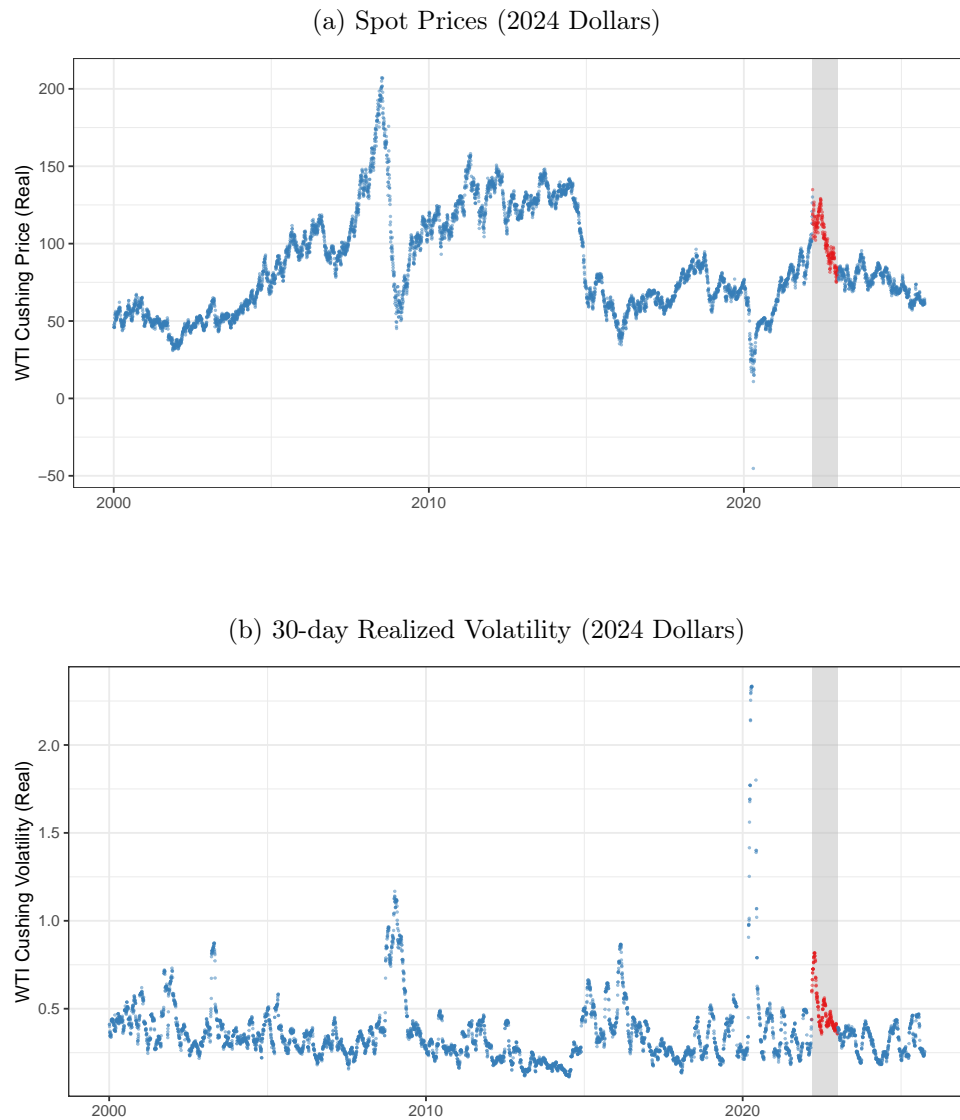
The simulation design is as follows. To evaluate the power of the model, I consider the ability of the Hickman et al. (2021) test to reject sequences of models that correspond to rays through the origin like the dashed green line in Figure 3. For instance, the ray defined by $(t, \alpha t)$ for $\alpha = 0$ encompasses the family of CIPV models described above. Motivated by the data, I generate samples of S auctions for $S \in \{50, 100, 500\}$. The total quantity sold in each auction i is fixed at $Q = 1000$ Mbbl. The number of bidders in each simulated auction is fixed at $N = 5$, the average number observed in the data.⁵⁰

Figure 4 presents simulated power curves for a 5% test for each sample size along the rays $\alpha \in \{0, 0.5\tilde{\sigma}_e, \tilde{\sigma}_e\}$. Even for the case of $S = 50$ auctions, which is slightly less than the number of exclusive tranches in the data (54), the test attains rejection rates above 75% for $\rho > 0.25$ in the CIPV model. As σ_e increases for the same level of ρ , corresponding to the introduction of private information, rejection rates decrease slightly but remain high for modest deviations from IPV. One apparent limitation of the test implied by the results is that the rejection rate at $t = 0$ is around 10% for all designs, above the nominal level.⁵¹

⁵⁰Note that when N is small (relative to the amount of private information), a symmetric linear BNE may not exist. I drop auctions for which no such BNE exists.

⁵¹In larger sample sizes the rejection rate appears to converge to the nominal level.

Figure 5: WTI Cushing Spot Price and 30-Day Realized Volatility



Notes: (1) Spot price calculated as EIA Cushing, OK WTI Spot Price FOB price deflated by BLS Consumer Price Index for all goods less energy, all urban consumers, not seasonally adjusted (Series ID CUUR0000SA0LE); (2) Gray-shaded region spans March 8, 2022 (publication date of the first emergency sale Notice-of-Sale) through December 31, 2022 (end of delivery period for last emergency sale); (3) 30-day realized volatility calculated as the annualized 30-day standard deviation of the log price return.

Table 1: Mean Delivery Index Price (DIP) vs. Base Reference Price (BRP)

Date of Sale	Delivery Window	Sweet Crude			Sour Crude		
		BRP	DIP	% Δ	BRP	DIP	% Δ
2022-03-08	24-84 days	95.15	106.99	12.44	92.60	103.76	12.06
2022-04-12	33-79 days	111.00	116.57	5.02	106.28	108.34	1.93
2022-06-01	20-75 days	113.39	110.51	-2.54	108.81	94.54	-13.11
2022-06-28	49-94 days	121.42	89.15	-26.58	114.21	85.47	-25.16
2022-08-02	45-80 days	103.68	87.39	-15.71	97.61	82.77	-15.20
2022-09-27	35-64 days	89.45	87.56	-2.12			
2022-10-25	37-67 days	89.91	77.07	-14.28	83.86	71.39	-14.87

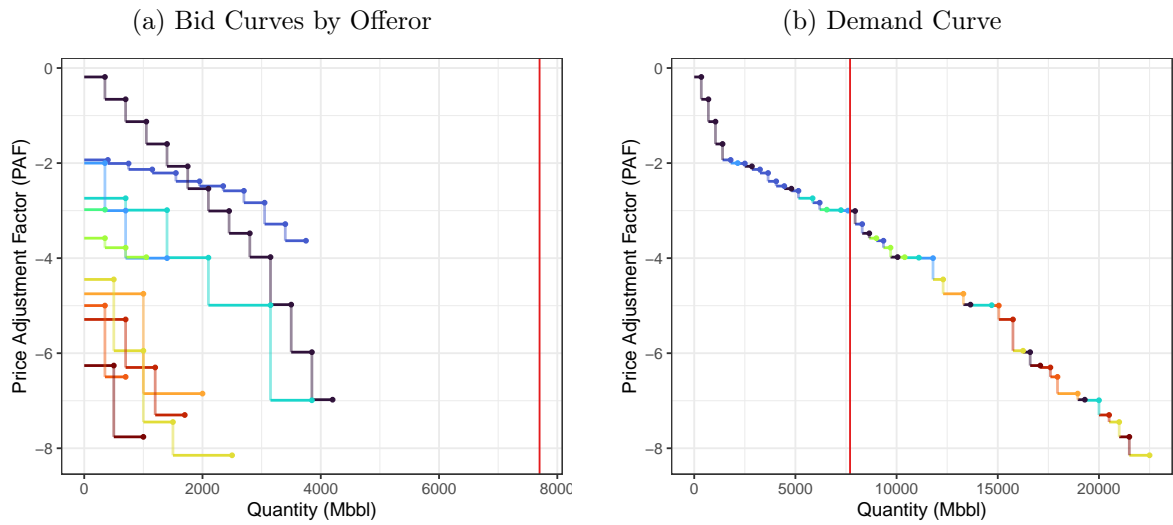
Notes: (1) Delivery index price calculated from daily closing prices of Bloomberg tickers USCRMEHC (Crude Oil WTI Houston) and USCRMARS (Crude Oil Mars), see main text for discussion; (2) Base reference prices are obtained from auction-specific Notices of Sale; (3) Delivery windows vary within sale by crude oil stream (MLI) and delivery location (DLI), the maximum range is recorded.

Table 2: Summary of Bidding Data (Sample Means)

	Sweet Crude			Sour Crude		
	Per DLI (Pipeline)	Per DLI (Vessel)	Per MLI	Per DLI (Pipeline)	Per DLI (Vessel)	Per MLI
Number of bidders	6.10	4.89	10.91	4.05	3.69	8.00
Number of bids	19.65	5.67	40.36	13.82	6.31	27.57
Number of winning bidders	3.30	1.11	6.00	2.36	1.23	4.36
Number of winning bids	8.55	1.25	16.45	7.71	3.20	13.86
Total quantity demanded	9906.75	2961.11	23962.27	7559.77	3880.38	16272.14
Total quantity sold	3825.00	643.75	7422.73	3927.14	1654.00	7072.14
Maximum winning PAF	0.42	-2.21	1.19	1.28	0.90	1.85
Average winning PAF	-0.76	-2.23	-0.93	0.23	0.06	-0.08
Minimum winning PAF	-2.03	-2.24	-2.89	-0.56	-0.58	-1.44
Nominal reserve PAF (5% BRP)	-5.12	-5.06	-5.13	-5.18	-5.21	-5.15

Notes: 2022 SPR drawdown bidding data. Includes all bids. (1) PAF refers to price adjustment factor; (2) Bid quantities are expressed in thousands of barrels (Mbbbl); (3) MLI refers to Master Line Item, designating the crude oil stream. DLI refers to Delivery Line Item, designating the place of delivery. Crude oil from a single stream is typically available for delivery at 2-3 locations. Each bid must specify both a crude oil stream (MLI) and delivery location (DLI); (4) The significance of the nominal reserve is discussed in Section 3.

Figure 6: Example Bid and Demand Curves



Notes: (1) Bids for FY22 Emergency Drawdown No. 3B, West Hackberry Sweet Crude MLI, Pipeline DLI @ Sun (Non-SPR Tank). In this sale, the maximum and minimum DLI quantities coincide at 7,700 thousand barrels. The red line is located at 7,700 thousand barrels. Each color corresponds to a different bidder. (2) PAF refers to price adjustment factor; (3) Bid quantities are expressed in thousands of barrels (Mbbl).

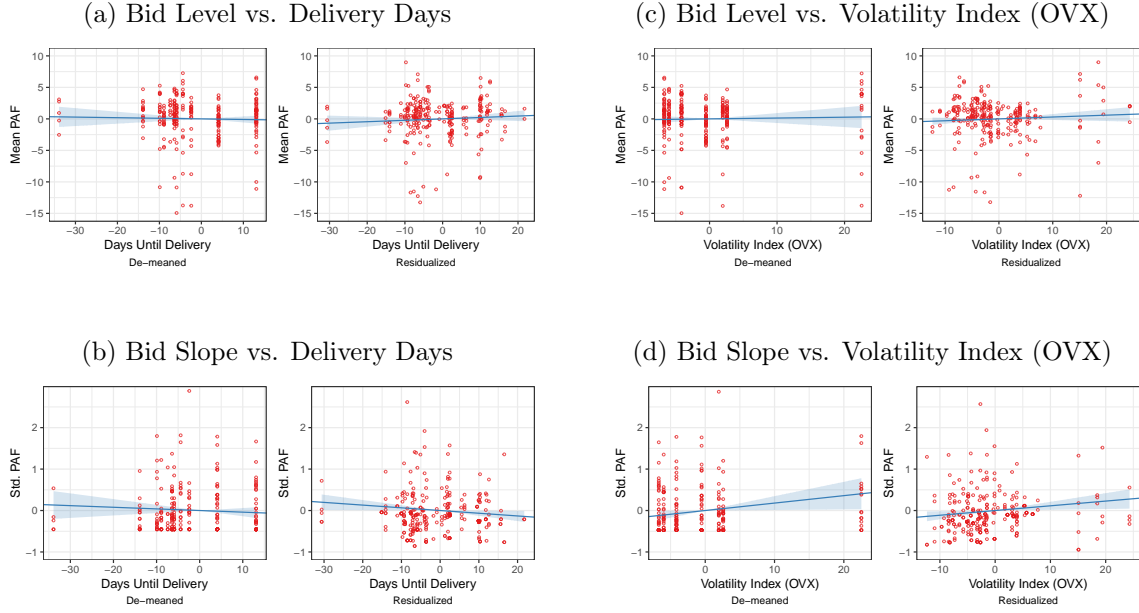
Table 3: Test for Private Information (Nyborg et al., 2002)

	<i>Dependent variable:</i>					
	Mean PAF		Std. PAF	Mean PAF		Std. PAF
	(1)	(2)	(3)	(4)	(5)	(6)
Volatility Index (OVX)	0.030 (0.024)	0.048 (0.037)	0.011** (0.005)			
Days Until Delivery				0.022 (0.020)	0.034 (0.024)	−0.007** (0.003)
Sweet Crude	−1.121** (0.492)	−1.111** (0.503)	0.003 (0.154)	−1.106** (0.479)	−1.072** (0.461)	0.025 (0.155)
Vessel DLI	−2.823*** (0.446)	−2.691*** (0.364)	0.091 (0.154)	−2.837*** (0.485)	−2.690*** (0.379)	0.109 (0.155)
Log Tranche Size (MMbbl)	−1.251*** (0.273)	−1.324*** (0.384)	0.249*** (0.043)	−1.210*** (0.272)	−1.262*** (0.338)	0.273*** (0.051)
Location FE	Yes	Yes	Yes	Yes	Yes	Yes
Dependent Variable	Level	Level	Slope	Level	Level	Slope
Unit of Observation	Bidder-Tranche	Tranche	Bidder-Tranche	Bidder-Tranche	Tranche	Bidder-Tranche
SE Clustering	Tranche	Event-Type	Tranche	Tranche	Event-Type	Tranche
Observations	268	54	268	268	54	268
Adjusted R ²	0.088	0.346	0.076	0.080	0.306	0.071

*p<0.1; **p<0.05; ***p<0.01

Notes: (1) Tranche refers to exclusive tranche, defined in Section 3.2. (2) Means and standard deviations of price adjustment factors (PAFs) are weighted by bid quantity; (3) Days Until Delivery is the number of days from the auction date to the midpoint of the delivery window; (3) Volatility Index (OVX) is the closing price of the CBOE Crude Oil Volatility Index on the auction date; (4) Observations are weighted by uncertainty (Days Until Delivery in (1)-(3), Volatility Index (OVX) in (4)-(6); (5) Heteroskedasticity-robust standard errors clustered by sale-crude oil stream (MLI)-delivery location (DLI) for columns (1), (3), (4), (6) and sale-crude oil type (sweet/sour) for columns (2) and (5).

Figure 7: Correlations Between Bidding Strategy and Uncertainty



Notes: (1) Red circles indicate bidder-tranche quantity-weighted mean price adjustment factor (PAF) or standard deviation PAF. (2) For de-meaned panels, the regression line is obtained via bidder-tranche level regression weighted by uncertainty. (3) For residualized panels, the slope and regression line correspond to those reported in Table 3. The y- and x-axis variables are residualized on the dependent variables described in Table 3 (4) Confidence bands are heteroskedasticity-robust and clustered by sale-crude oil stream (MLI)-delivery location (DLI); (5) Days Until Delivery is the number of days from the auction date to the midpoint of the delivery window; (6) Volatility Index (OVX) is the closing price of the CBOE Crude Oil Volatility Index on the auction date.

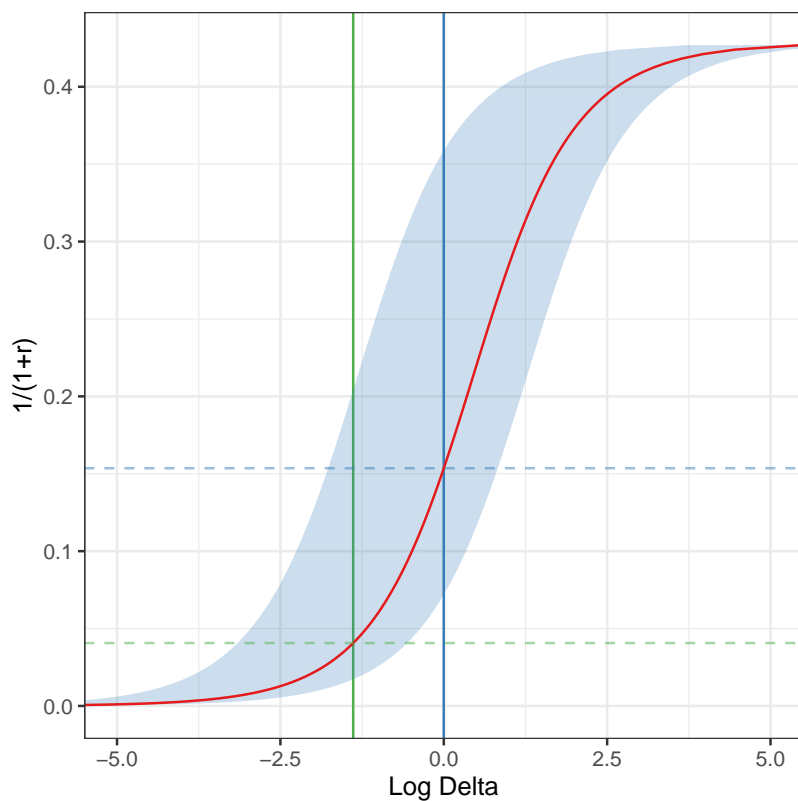
Table 4: Test of Independent Private Values (Hickman et al., 2021)

	(1)	(2)	(3)	(4)	(5)
Mean Rival PAF	0.074 (0.169)	0.008 (0.167)	-0.075 (0.135)	-0.075 (0.099)	-0.095 (0.131)
Days Until Delivery		0.010 (0.032)	0.005 (0.030)	0.005 (0.029)	0.002 (0.021)
Sweet Crude		-0.694 (0.546)	-0.172 (0.567)	-0.172 (0.670)	-0.688 (0.502)
Vessel DLI		-1.640*** (0.623)	-2.415*** (0.745)	-2.415*** (0.744)	-1.607*** (0.507)
Location FE	No	Yes	Yes	Yes	Yes
Polynomial in N	Cubic	None	Cubic	Cubic	Cubic
Restricted Sample	Yes	Yes	Yes	Yes	No
SE Clustering	Tranche	Tranche	Tranche	None	Tranche
Observations	144	144	144	144	217
R^2	0.070	0.090	0.115	0.115	0.101

*p<0.1; **p<0.05; ***p<0.01

Notes: (1) Tranche refers to exclusive tranche, defined in Section 3.2; (2) Excludes tranches not having at least two bidders willing to accept delivery location (DLI) minimum quantity; (3) Restricted sample excludes auctions with any bids stipulating minimum quantities greater than the delivery location (DLI) minimum quantity. Unrestricted sample retains auctions but omits such bids from the sample and from Mean Rival PAF, but not from calculation of N ; (4) Days Until Delivery is the number of days from the auction date to the midpoint of the delivery window; (5) Heteroskedasticity-robust standard errors clustered by sale-crude oil stream (MLI)-DLI except for column (4), which is unclustered

Figure 8: Private Information Thresholds for Large Revenue Gains in Uniform Price Model



Notes: (1) $1/(1+r) = \sigma_{\omega}^2/(\sigma_u^2 + \sigma_{\omega}^2)$ is the fraction of bidders' ex ante marginal valuations attributable to private information; (2) the blue (green) vertical line corresponds to a \$1/barrel (\$0.25/barrel) expected revenue gain from price-indexing; (3) 95% confidence band obtained via bootstrap estimation of the calibrated slope of bidder marginal valuations λ_{pred} .