

Unilateral Market Power in Financial Transmission Rights Auctions

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Abstract

This paper explores market participants’ incentives to exercise unilateral market power in financial transmission rights (FTR) auctions. I present a model of strategic bidding in FTR auctions that illustrates how the standard market clearing mechanism for FTR auctions can facilitate substantial cross-path competition, limiting the profitability of demand reduction. Simulation evidence suggests that this force dramatically reduces rents from unilateral market power relative to other sources of auction revenue shortfalls. Proposed market reforms such as decentralized FTR sales and bidding-path restrictions that weaken cross-path competition could potentially worsen shortfalls.

Keywords: restructured electricity markets; congestion revenue; financial transmission rights; imperfect competition; multiunit auctions

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1 Introduction

Financial transmission rights (FTRs) are an important element of restructured electricity markets. Operators of restructured electricity markets in the United States (known as independent system operators or ISOs) collect congestion revenues of roughly \$4B per year, or about 4% of wholesale electricity costs (Parsons, 2023). FTRs are forward claims on these revenues. While many FTRs are directly allocated to energy market participants, a large share are sold in complex auctions in which participants – including financial speculators – can potentially profit from strategic behavior. A robust empirical literature has established that purely financial participants consistently earn large trading profits in FTR auctions, raising concerns that strategic distortions could be significant.¹

In this paper, I use simple economic theory to investigate the contribution of unilateral market power to so-called “auction revenue shortfalls.” Building on recent developments in the theory of imperfectly competitive financial markets (Rostek and Yoon, 2023), I show that the market clearing mechanism used in FTR auctions results in significant competitive pressure even when bidding activity is thinly dispersed across individual FTRs, weakening bidders’ incentives to shade their bids. The main results suggest that in the context of a dense, real-world transmission network, rents from unilateral market power could be relatively insignificant, despite apparent market imperfections such as limited participation for individual FTRs and a lack of explicit reserve prices. Well-documented auction revenue shortfalls may reflect the intrinsic costs of holding illiquid FTRs rather than insufficient competition.

In a typical FTR auction, bidders submit downward-sloping demand schedules for quantities of transmission capacity between points (nodes) in the network. After receiving bids,

¹Estimates in the literature suggest that financial participants in FTR auctions earn trading profits on the order of \$0.5B per year across all US ISOs. This includes approximately \$60M/year in MISO (O’Keefe, 2024), \$220M/year in PJM (Leslie, 2021), \$50M/year in NYISO (Leslie, 2021), and \$55M/year in CAISO (CAISO, 2019). To obtain the \$0.5B dollar figure, I divide the sum of these figures by the average share of congestion revenues in these four markets among the seven ISOs (as reported in Parsons (2023)). FTRs are also referred to as congestion revenue rights (CRRs) or transmission congestion contracts (TCCs).

the market operator allocates capacity across “paths” to maximize net revenue, subject to the constraint that final allocation of FTRs is *simultaneously feasible* with respect to the physical transmission constraints of the network.² Due to this constraint, which is enforced to ensure that the market operator remains budget-balanced, the residual supply curves faced by each bidder depend on the strategic behavior of bidders *throughout* the network. Bidders compete for capacity on common transmission elements when competing for disparate FTRs, and the *effective* level of competition for any individual FTR can be high even if few bids are submitted on it.

Section 2 captures this intuition in a stylized model of an imperfectly competitive FTR auction. Because the market clearing mechanism used in the FTR auction generalizes the familiar uniform price auction, the well-known characterization of price impacts in uniform price auctions can be adapted to the FTR auction setting. This shows how bidders’ incentives to exercise unilateral market power depend the structure of the transmission network, which mediates the strategic interaction of bidders on different paths. In a uniform price auction, bidders’ price impacts depend on the level of competition and the concavity of rival bidders’ preferences; in the FTR auction, bidders’ price impacts depend on the level of competition and the concavity of rival bidders’ preferences on all *strategically connected* FTRs – those that are related via equilibrium binding constraints – with greater weight given to bidding on electrically similar FTRs. In real-world FTR auctions, nearly all FTRs will be strategically connected, suggesting that effective levels of competition are generally high except possibly in the most peripheral regions of the transmission network.

Section 3 illustrates the implications of the model numerically in the context of the same six-node test network previously analyzed in Chao et al. (2000) and Deng et al. (2010). In this network, a “North” region with excess generation is linked to a “South” region with excess demand, resulting in congestion. Uncertainty over the level of future demand creates

²I use the term “path” to refer to particular point-to-point FTRs. This terminology is consistent with the ISOs’ own usage, but should not be confused with the older concept of a “contract path” referring to transmission rights on specific combinations of flowgates.

uncertainty over congestion, and hence FTR payouts. Risk averse speculators compete to purchase FTRs while accounting for the behavior of bidders throughout the entire transmission network. I compare the standard FTR auction to a benchmark mechanism with uniform price auctions for individual FTRs. With three bidders on every FTR, participants in the FTR auction bid as aggressively as if they faced 11.8 rival bidders in the benchmark mechanism.

Empirically validating the model is challenging because detailed information on real-world transmission networks is generally unavailable to researchers. For instance, the key object governing the extent of cross-auction strategic interaction is the power transfer distribution factor (PTDF) matrix, which summarizes the structure of the transmission network. This matrix cannot be reliably constructed from publicly available information.³ Lack of information on network constraints and the PTDF matrix precludes the use of structural methods to recover bidders' marginal valuations (e.g., Kastl, 2011), as such methods require complete knowledge of the market clearing procedure to simulate supply elasticities.

Nevertheless, the analysis provides several clear implications for potential market reforms to address auction revenue shortfalls. If market power rents are small, then auction revenue shortfalls may primarily reflect concavity in bidders' preferences (from risk aversion, inventory costs, or other sources). Proposed reforms that would decentralize FTR sales could significantly harm competition without addressing potentially more fundamental sources of underpricing. Likewise, reforms that maintain a centralized auction while prohibiting bidding on certain types of FTRs could have ambiguous effects on auction revenue shortfalls. I discuss the policy implications of the analysis in greater detail in the conclusion.

³Detailed transmission network information is considered Critical Energy Infrastructure Information by the US federal government and is subject to strict non-disclosure policies. Consequently, research on large-scale power systems is typically limited to "synthetic" grids of varying complexity.

1.1 Related Literature

ISOs accrue congestion revenue due to the use of locational marginal pricing (Schweppe et al., 1988). Hogan (1992) proposed that market operators issue FTRs funded with congestion revenue. Doing so enables the market operator to distribute congestion revenue among market participants in a manner that may help facilitate long-term contracting, among other potential benefits. Operators of all seven restructured energy markets in the United States issue FTRs, typically through a combination of direct allocation and auctions.

As FTR auction markets have matured, a significant empirical literature has documented that auction prices for FTRs tend to be “underpriced” relative to their ex post value (Bartholomew et al., 2003; Adamson and Englander, 2005; CAISO, 2016; Olmstead, 2018; Opgrand et al., 2022). Some authors have emphasized the potential for risk premia to explain underpricing (e.g., Bartholomew et al., 2003; Adamson and Englander, 2005; Baltadounis et al., 2017), while others have focused on private information (Leslie, 2021) or cross-market effects (Opgrand et al., 2022). Relative to this literature, my primary contribution is a novel explanation for why unilateral market power specifically may contribute little to underpricing. Moreover, while one might always question the significance of market power rents on the grounds that profitable opportunities tend to attract new entrants, the explanation I offer does not rely on the assertion that entry barriers are low.

Evidence of persistent underpricing has motivated a robust policy debate in recent years, with some observers asserting that FTR auctions should be eliminated in favor of more extensive direct allocation (CAISO, 2017; Parsons, 2020; Monitoring Analytics, 2022) and others arguing that existing auction mechanisms provide important benefits due to the unique benefits of FTRs for hedging congestion risk (Bushnell et al., 2018; FERC, 2022). My analysis is broadly consistent with the latter perspective, but provides insights into how specific policy proposals motivated by the former (such as bidding restrictions) may affect competition.

In order to analytically characterize market power in an FTR auction, I adapt a workhorse model of supply function competition from the literature on imperfectly competitive finan-

cial markets (Vives, 2008; Malamud and Rostek, 2017; Rostek and Yoon, 2023). While this approach is effective for the purpose of this paper, the use of supply function equilibria has long been considered intractable for many applications in nodal power markets (including FTR markets), leading to the use of Cournot equilibria and related approaches such as conjectured supply function equilibria (Day et al., 2002). For instance, Bautista Alderete (2005) builds a model of conjectured supply function equilibria in FTR auctions. In comparison to Bautista Alderete (2005), my primary focus is the microfoundations of market power rather than computational tractability, making a supply function approach essential. Deng et al. (2010) also studies the implications of simultaneous feasibility constraints for prices in FTR auctions, but in an environment with perfectly competitive bidding and flat demands.

Imperfect competition in restructured electricity markets has attracted significant attention both theoretically (e.g., Joskow and Tirole, 2000; Borenstein et al., 2000) and empirically (e.g., Wolfram, 1999; Borenstein et al., 2002; Hortaçsu and Puller, 2008), but relatively little work has examined market microstructure in realistic nodal power markets. One notable exception is Mercadal (2022), who extends of Hortaçsu and Puller (2008)’s empirical bidding model to the context of nodal power markets with transmission constraints. In comparison, I characterize bidders’ residual demand curves analytically, but without simplifying the transmission network to the same extent as Joskow and Tirole (2000).

More generally, Mercadal (2022) and several other authors have analyzed the role of financial participants in electricity markets (e.g., Birge et al., 2018; Jha and Wolak, 2023). Notably, Ledgerwood and Pfeifenberger (2013) and Lo Prete et al. (2019) discuss the possibility that financial speculators may submit virtual day-ahead bids strategically in order to influence FTR values. I do not consider this possibility here. However, this and other forms of potential market manipulation could be relevant to observed auction revenue shortfalls.

2 Market power in FTR auctions

Consider a market operator (ISO) that manages a grid with N nodes linked by M transmission constraints. There are $K = \frac{1}{2}N(N-1)$ distinct pairs of nodes. Each year, a congestion price vector $\zeta \in \mathbb{R}^M$ and a power transfer vector $Q \in \mathbb{R}^K$ are realized from a distribution F . There are K distinct FTRs, one for each pair of nodes. 1MW of FTR k is a forward claim on the difference in realized congestion prices incurred when injecting power at the source node $src_k \in N$ and withdrawing it at the sink node $snk_k \in N$.⁴ In particular, FTR k is a forward claim on $Z'_k \zeta$, where $Z \in \mathbb{R}^{K \times M}$ is the power transfer distribution factor (PTDF) matrix. For simplicity, ζ and Q are taken to be exogenous. Put differently, market participants view FTRs as purely financial assets and do not anticipate that FTR allocations will affect prices or allocations in the day ahead market (where settlement prices are determined).

The ISO may directly allocate FTRs to market participants such as load serving entities (LSEs), sometimes in the form of auction revenue rights (ARRs). After any directly allocated FTRs $\tilde{Q} \in \mathbb{R}^K$ are funded, the ISO collects a residual congestion revenue $(Q - \tilde{Q})' Z \zeta$. The FTR auction is a mechanism for allocating this residual congestion revenue.

In a standard FTR auction, bidders submit downward-sloping demand schedules for FTR capacity. Suppose bidder j submits bids on a subset of FTR paths $K_j \subseteq K$. Let $q_{jk}^{-1} : \mathbb{R} \rightarrow \mathbb{R}$ denote j 's inverse demand function for FTR k .⁵ After receiving all bids, the ISO awards FTRs in order to maximize the “economic value” of cleared bids (i.e., the willingness-to-pay

⁴The identity of the source and sink nodes is not important as negative FTR capacity (“counterflow”) is generally permitted. Note that I consider FTR obligations only. The holder of an FTR obligation is required to pay the ISO when realized congestion is negative. Some ISOs also issue FTR options, which do not require such payments. FTR options are jointly cleared with FTR obligations using a mechanism similar to (1), but with additional constraints (Hogan, 2013). As such, the cross-path competitive forces identified in this paper would also be present in a market that included FTR options.

⁵For the purpose of exposition, I assume that q_{jk} is strictly downward sloping (and that q_{jk}^{-1} is integrable). In practice, bids are step functions. As noted above, negative FTR quantities are permitted.

expressed by the inverse demands), subject to transmission constraints:

$$\begin{aligned} \max_{\{q_{jk}\}} \quad & \sum_{j \in J} \sum_{k \in K_j} \int_0^{q_{jk}} q_{jk}^{-1}(s) ds \\ \text{s.t.} \quad & P_m^L \leq \sum_{j \in J} \sum_{k \in K_j} z_{km} q_{jk} \leq P_m^U \quad \forall m \in M \end{aligned} \tag{1}$$

An allocation of FTRs which satisfies the constraints in (1) is said to be *simultaneously feasible*.⁶ In this expression, $P_m^U \in \mathbb{R}$ denotes the maximum powerflow on transmission element m and $P_m^L \in \mathbb{R}$ the minimum (i.e., the maximum powerflow in the reverse direction) after accounting for any directly allocated FTRs. z_{km} is an element of the PTDF matrix introduced above. The PTDF matrix is determined by the physical characteristics of the network; its entries take values between -1 and 1 depending on the powerflow induced by injecting and withdrawing 1MW of power at the source and sink nodes of the FTR. Due to the nature of powerflow, the PTDF matrix is generally dense.⁷

Lemma 1 establishes a well known fact regarding market clearing in the FTR auction. In particular, the price of FTR k is a linear combination of the shadow prices associated with any binding constraints in (1).

Lemma 1. *The clearing price for FTR k in (1) is $p_k = Z'_k \xi = Z'_k (\xi^L - \xi^U)$, where $\xi^L \geq 0$ and $\xi^U \geq 0$ are the vectors of Lagrange multipliers in (1).*

The FTR auction generalizes the familiar uniform price auction. For example, in a degenerate transmission network with two nodes and a single transmission constraint m having $P_m^L = 0$, (1) coincides with the auctioneer's problem in a standard uniform price auction with a reserve price of zero. In a more complex transmission network, (1) implies that the ISO's problem is equivalent to choosing a subset of binding constraints, and then

⁶Simultaneous feasibility is enforced by the ISO in order to sure that congestion revenues are sufficient to fund FTR payouts. In practice, FTR payouts are often underfunded due to discrepancies between the network model used in the FTR auction and the network model that is ultimately used in the physical market (see, e.g., Parsons, 2020).

⁷To illustrate, Appendix B constructs a PTDF matrix for a simplified network.

conducting simultaneous uniform price auctions for capacity on those constraints. This conceptualization will be useful below. An FTR bid expresses many implicit contingent bids for constraint-level capacity, and the price of FTR k is the sum of the clearing prices that must be paid in each “virtual” constraint auction to obtain sufficient network capacity to inject 1MW of power at src_k and withdraw it at snk_k .

In the next section, I analyze optimal bidding in the FTR auction. Before doing so, it is helpful to introduce additional notation. In the solution to (1), let \tilde{M}^U denote the subset of constraints that bind “upwards,” \tilde{M}^L the subset of constraints that bind “downwards,” and \tilde{M} the union of \tilde{M}^U and \tilde{M}^L . By complementary slackness, $p_k = \tilde{Z}'_k \tilde{\xi}$ where \tilde{Z}_k and $\tilde{\xi}$ are the subvectors of Z_k and ξ corresponding to \tilde{M} .

2.1 Price impacts under simultaneous feasibility

In a uniform price auction, the market clearing price is the price that exhausts the auctioneer’s supply of the good (provided that demand exceeds supply at the reserve price). In an FTR auction, by contrast, the market clearing price is determined by the solution to (1). Bidders with market power anticipate the effects of their bids on market clearing prices, or *price impacts* (Rostek and Yoon, 2023). In this section, I analyze bidders’ price impacts when market prices are determined according to (1).

For simplicity, suppose that bidders’ participation decisions, preferences, and the distribution of FTR payouts are common knowledge.⁸ Bidder j ’s expected utility is

$$u_j(q_j; p) = v_j(q_j) - p'_j q_j \quad (2)$$

where $q_j \in \mathbb{R}^{|K_j|}$ is the portfolio of FTRs awarded to bidder j , $v_j : \mathbb{R}^{|K_j|} \rightarrow \mathbb{R}$ is a twice-differentiable, concave valuation function having $\frac{\partial v_j}{\partial q_j \partial q_j} = -C_j$ for some positive definite

⁸In reality, some bidders may have superior information for forecasting FTR payouts (Leslie, 2021). It appears that private information can be incorporated into the model by extending prior work on uniform price auctions (Rostek and Yoon, 2023), at the cost of considerable technical complication.

matrix C_j , and p_j is the subvector of prices for K_j . A leading example of a suitable valuation function is the mean-variance utility obtained under constant absolute risk aversion (CARA) preferences, in which C_j corresponds to the covariance matrix of FTR payouts scaled by a risk aversion parameter.⁹ In what follows, it will be useful to let E_j denote the $K_j \times K$ matrix of zeros and ones such that $p_j = E_j p$.

Optimal bidding requires that the gradient of v_j with respect to q_j must be equal to the gradient of j 's auction expenditure with respect to q_j . In other words, marginal benefit to j of each FTR in K_j must be equal to its marginal acquisition cost in the auction. Under imperfect competition, the latter depends on bidder j 's *price impact*, which is summarized by the matrix $\Lambda_j = \frac{\partial p_j}{\partial q_j}$. In particular,

$$\frac{\partial v_j(q_j)}{\partial q_j} = p_j + \Lambda_j q_j \quad (3)$$

When $\Lambda_j = 0$, bidder j bids her marginal valuation for each FTR in K_j – j has no incentive to exercise unilateral market power. For elements of $\Lambda_j q_j$ that are greater than zero, optimal bidding exhibits *demand reduction* – bidder j requests inefficiently few units at each price level, shifting demand inwards and reducing prices.

In a Bayesian Nash equilibrium (BNE) of the FTR auction game with linear bid schedules, each bidder j submits an inverse demand function that satisfies (3), and prices are given by Lemma 1. It is clear from the auctioneer's problem (1) that bidders on any particular FTR must account for the strategies of bidders on other FTRs due to simultaneous feasibility. The dependence induced by simultaneous feasibility varies depending on the equilibrium configuration of binding constraints and is captured in the price impact matrix Λ_j .

In order to describe the cross-path strategic interactions induced by simultaneous feasibility, I introduce a notion of connectivity from graph theory.

⁹Since C_j can differ across bidders, the model incorporates heterogeneous risk aversion in the CARA framework. In the CARA framework, ex ante congestion exposure (from long-term contracts, for instance) is captured by shifts in v_j that leave the second derivative matrix unchanged.

Definition 1. Let $\tilde{M} \subseteq M$. Bidders j and j' are *direct competitors under \tilde{M}* if there exists some $m \in \tilde{M}$ such that $|Z_{km}| |Z_{k'm}| > 0$ for some $k \in K_j$ and $k' \in K_{j'}$. Let G denote the set of components of the undirected graph with vertices for each bidder and edges linking any two direct competitors. Bidders j and j' are *quasi-competitors under \tilde{M}* if they belong to the same component $g \in G$. FTRs k and k' are *strategically connected under \tilde{M}* if there exist quasi-competitors j and j' (possibly the same) such that $k \in K_j$ and $k' \in K_{j'}$.

Proposition 1 uses this definition to relate bidder j 's price impact Λ_j to the strategies of competing bidders. First, some additional remarks are useful. Observe that the set of bidders J can be partitioned into disjoint sets $\{J_1, \dots, J_{|G|}\}$, such that all bidders in J_g are quasi-competitors. Interaction between quasi-competitors in J_g is mediated through the virtual constraint auctions associated with \tilde{M}_g , where $\{\tilde{M}_1, \dots, \tilde{M}_{|G|}\}$ is an appropriate partition of the set of binding constraints \tilde{M} . All FTRs k and k' having $|Z_{km}| > 0$ and $|Z_{k'm'}| > 0$ for m and m' contained in some \tilde{M}_g are strategically connected. Finally, let \tilde{Z}_g denote the submatrix of \tilde{Z} containing the columns corresponding to \tilde{M}_g .

For convenience, I assume that Λ_j is positive semi-definite for all j .¹⁰ The following assumption ensures that Λ_j exists.¹¹

Assumption 1. Let $\tilde{M} \subseteq M$. There exist conformable positive definite matrices $\{\Psi_1, \dots, \Psi_G\}$ and conformable matrices $\{Y_1, \dots, Y_J\}$ such that:

$$Y_j' \Psi_g^{-1} Y_j + (\Psi_g^{-1} - 2W_j) Y_j - W_j = 0 \quad \forall j \in J_g, \quad \forall g \in G \quad (4)$$

¹⁰I do not attempt to provide conditions under which price impacts are positive semi-definite; however, previous work in market microstructure has regarded positive semi-definiteness as the natural generalization of the (often implicit) assumption that scalar price impacts are positive (e.g., in Kyle, 1985). The close link between the problem studied here and that of Malamud and Rostek (2017), where price impacts are shown to be positive semi-definite, suggests that this assumption may be reasonable.

¹¹Given the linear structure of payoffs specified above, it may be possible to further extend the results in Malamud and Rostek (2017) to establish uniqueness of the price impact matrix under the same conditions. I leave this for future work.

and

$$\Psi_g^{-1} = \sum_{j \in J_g} (I + Y_j')^{-1} W_j \quad \forall g \in G \quad (5)$$

where $W_j = \tilde{Z}_g' E_j' C_j^{-1} E_j \tilde{Z}_g$. Furthermore, $\Psi_g^{-1} - (I + Y_j')^{-1} W_j$ is invertible for each j .

In a degenerate network with a single FTR and a single binding constraint, Assumption 1 is satisfied whenever there are three or more bidders, mirroring a well-known sufficient condition for the existence of an equilibrium in uniform price auctions with common information (Rostek and Yoon, 2023). Assumption 1 generalizes this idea to the FTR auction, in which different bidders may submit bids on different portfolios of FTRs, and some pairs of FTRs may not be strategically connected. For example, if fewer than three bids are submitted on an FTR that is not strategically connected to any other FTR under \tilde{M} , then Assumption 1 is violated. In the proof of Proposition 1 below, I show that Y_j can be interpreted as the product of j 's constraint-level (rather than FTR-level) price impacts and a scaling matrix $C_j^{-1} E_j \tilde{Z}_g$. I discuss the interpretation of Ψ_g below.

We can now state the main result of the paper:

Proposition 1. *Suppose $\tilde{M} \subseteq M$ is the subset of binding transmission constraints in a BNE of the FTR auction game. Under Assumption 1, the equilibrium price impact of bidder $j \in J_g$ is given by:*

$$\Lambda_j = E_j \tilde{Z}_g \left(\Psi_g^{-1} - (I + Y_j')^{-1} W_j \right)^{-1} \tilde{Z}_g' E_j' \quad (6)$$

where Y_j is a solution to (4) for some Ψ_g satisfying (4).

Proofs of Proposition 1 and all other results are provided in Appendix A.

The significance of (6) is to clarify the manner in which bidder j accounts for the relevance of particular competing bids.¹² Specifically, (6) implies that bidder j 's equilibrium bidding

¹²For comparison, Malamud and Rostek (2017) consider an environment where $E_j \tilde{Z}_g$ is the identity matrix

strategy depends on the strategies of all j 's quasi-competitors, but no others. In reality, Z is dense and bidders tend to submit bids on many FTR paths, suggesting that almost all FTRs will be strategically connected, and almost all bidders will be quasi-competitors. Ψ_g is the matrix equivalent of a harmonic mean of the slopes of the bids in the virtual auctions associated with \tilde{M}_g . Similarly, the middle term on the right hand side of (6) is the matrix equivalent of a harmonic mean of the slopes of the “virtual” bids submitted by j 's quasi-competitors – in other words, it represents the slope of the residual supply of constraint capacity. FTR-level price impacts are related to constraint-level residual supply curves through the PTDF matrix—price impacts are lower when bidding on an FTR with large magnitude PTDF values concentrated on more competitive constraints.

To more clearly illustrate the unique features of FTR auctions, it is useful to restrict attention to the special case in which K_j is a singleton $K_j = \{k_j\}$ for each j (henceforth, “single-path bidding”). Define $\tilde{Z}_{k_j} = \left(E_j \tilde{Z}_g\right)'$ as the PTDF vector associated with k_j . In this case, individual bidders submit one and only one bid and the matrices Λ_j and C_j can be replaced with scalars λ_j and c_j . To demonstrate the implications of Proposition 1, Example 1 considers a simple equilibrium in which a single constraint binds and bidders' preferences take a special form that admits an explicit solution for λ_j .

Example 1. There are $J \geq 3$ bidders. Suppose a single constraint m^* binds in equilibrium and all bidders are strategically connected, with $G = \{1\}$. Assume that $c_j = cZ_{k_j m^*}^2$ for some constant $c > 0$. Since $\left|\tilde{M}_1\right| = 1$, Ψ_1^{-1} and Y_j are scalars. Then the mean slope is given by:

$$\Psi_1 = \frac{1}{J} \left(\frac{J-1}{J-2} \right) c$$

and $C_j = c_j \Sigma$ for a known matrix Σ and scalar c_j . They show that $\Lambda_i = c_j Y_j \Sigma$ and $Y_j = \beta_j I$ for a scalar β_j . In this setting, heterogeneity in K_j and the uncertain rank of \tilde{Z} prevents a simpler expression.

While for each bidder j ,

$$Y_j = \left(\frac{1}{J} \left(\frac{J-1}{J-2} \right) \right) - \frac{1}{2} + \sqrt{\left(\frac{1}{J} \left(\frac{J-1}{J-2} \right) \right)^2 + \frac{1}{4}}$$

Proposition 1 implies that $\lambda_j = cZ_{k_j m^*}^2 Y_j$. Clearly, λ_j is increasing in the magnitude $|Z_{k_j m^*}|$ of the shift factor between k and the binding constraint m^* , increasing in the concavity of preferences c , and decreasing in the number of bidders J .

Inspection of Proposition 1 suggests that an FTR auction produces more competitive bidding for FTR k than an equivalent uniform price auction. To see this, observe that uniform price auctions for each FTR could be implemented via (1) by replacing Z with the identity matrix and P^U and P^L with fixed volumes of FTR capacity for each path. Holding fixed participation, such a mechanism eliminates all cross-path competition and can therefore be expected to produce weakly larger price impacts λ_j^{Unif} for each j . Although I am unable to provide a general result, Example 2 shows that price impacts are strictly greater under the uniform price auction for the example introduced in Example 1.

Example 2. Consider the setting from Example 1, but now suppose there are $J_k \geq 3$ bidders on each path. In the FTR auction, we have already shown that:

$$\lambda_j = cZ_{k_j m^*}^2 \left(\left(\frac{1}{J} \left(\frac{J-1}{J-2} \right) \right) - \frac{1}{2} + \sqrt{\left(\frac{1}{J} \left(\frac{J-1}{J-2} \right) \right)^2 + \frac{1}{4}} \right)$$

where $J = \sum_{k \in K} J_k$. In a uniform price auction for FTR k , we would instead have:

$$\lambda_j^{Unif} = cZ_{k_j m^*}^2 \left(\left(\frac{1}{J_k} \left(\frac{J_k-1}{J_k-2} \right) \right) - \frac{1}{2} + \sqrt{\left(\frac{1}{J_k} \left(\frac{J_k-1}{J_k-2} \right) \right)^2 + \frac{1}{4}} \right)$$

Since $J_k < J$, price impacts are strictly lower in the FTR auction.

With single-path bidding, it may still be the case that there is more than one binding constraint for some component g , such that $|\tilde{M}_g| > 1$. In this case, Ψ_g^{-1} and Y_j are matrices

of dimension $|\tilde{M}_g| \times |\tilde{M}_g|$ for all $j \in J_g$, even though all price impacts are scalar. As the solution to a matrix quadratic equation, Y_j generally lacks a closed form. Given this, it is helpful for the purpose of deriving comparative statics to consider the following necessary condition for λ_j :

$$\lambda_j = \frac{2x_j - c_j + \sqrt{c_j^2 + 4x_j^2}}{2}$$

where $x_j = \tilde{Z}'_{k_j} \Psi_g \tilde{Z}_{k_j}$ represents j 's exposure to the mean slope Ψ_g among bidders in J_g . From this expression, it is clear that λ_j is increasing as x_j increases (holding c_j fixed).

This fact can be used to establish comparative statics for the general single-path bidding case, summarized in Corollary 1 below. In a typical uniform price auction, price impacts are declining in the number of bidders. In the FTR auction, the effects of a marginal bidder are more nuanced. If the set of binding constraints were fixed in advance, then λ_j would not only decline in the number of bids on FTR k_j but also in the number of bidders on strategically connected FTRs. In this sense, cross-path strategic interaction tends to reduce market power. In practice, however, the set of binding constraints in (1) can adjust in response to entry, potentially resulting in local increases in price impacts.

Corollary 1. *Consider an auction with single-path bidding. Let $J^0 = (J_1, \dots, J_K)$ denote an entry vector, where J_k is the number of bidders having $K_j = \{k\}$, and $J^1 = (J_1, \dots, J_{k'} + 1, \dots, J_K)$ the entry vector when an additional bidder path k' bidder. Let \tilde{M}^0 and \tilde{M}^1 denote the induced binding constraints under J^0 and J^1 , respectively.*

1. $\lambda_j(J^1; \tilde{M}^0) - \lambda_j(J^0; \tilde{M}^0)$ is positive if k' is strategically connected to k_j and zero otherwise
2. $\lambda_j(J^1; \tilde{M}^1) - \lambda_j(J^0; \tilde{M}^0)$ can be positive or negative

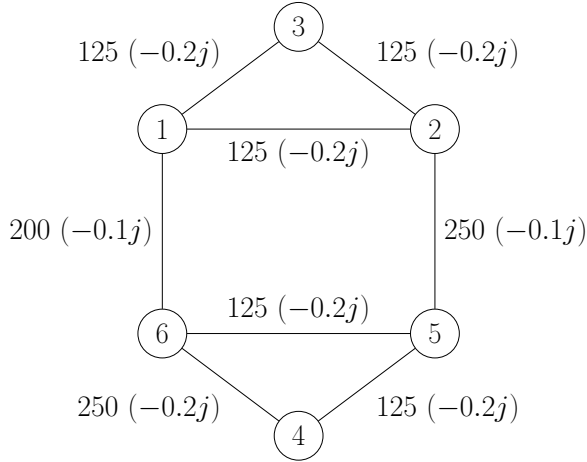
where $\lambda_j(\cdot; \cdot)$ expresses price impact as a function of J and \tilde{M} .

Since one would expect the PTDF matrix Z to be relatively dense in a full-scale power network, Corollary 1 suggests that bidders in real-world FTR auctions likely face a high level of effective competition, even if marginal impacts are difficult to sign due to possible adjustments in \tilde{M} . Although I do not attempt to perform a comprehensive empirical validation of the model in this paper, this prediction is consistent with the stylized fact that bidders in FTR auctions who face no direct competition frequently make losses.¹³ Broadly, the model suggests that the relationship between FTR-level participation and auction revenue shortfalls will tend to be weak except perhaps in peripheral parts of the network.¹⁴

3 Numerical illustration

In this section, I investigate the market design implications of the model in Section 2 by comparing equilibrium outcomes of a simulated FTR auction to equilibrium outcomes of an benchmark mechanism in which FTR capacity is sold in parallel uniform price auctions.

Figure 1: Six-Node Test Case



Wholesale Bids (Normal Scenario)

Node	Type	Supply	Demand
1	Gen	$10 + 0.05q$	—
2	Gen	$15 + 0.05q$	—
3	Load	—	$37.5 - 0.05q$
4	Gen	$42.5 + 0.025q$	—
5	Load	—	$75 - 0.1q$
6	Load	—	$80 - 0.1q$

I focus on the six-node test case analyzed in Chao et al. (2000) and Deng et al. (2010).

Figure 1 presents a schematic depiction of the network along with node-level supply and

¹³For instance, data from the Midcontinent ISO FTR auction analyzed in O’Keefe (2024) indicates that roughly half of bids on FTR paths with only a single bidder are loss-making.

¹⁴Interestingly, Olmstead (2018) analyzes data from Ontario and finds that auction prices are less predictive of FTR payouts when there are two or fewer bidders on an FTR than when there are three or more.

Table 1: Six-Node Test Case LMPs and Congestion Revenue

Scenario	Prob.	LMP-1	LMP-2	LMP-3	LMP-4	LMP-5	LMP-6	Revenue
Normal	0.6	23.03	33.55	28.29	46.71	41.45	51.97	\$7368.42
Load +25%	0.2	22.92	35.42	36.04	48.96	57.08	65.83	\$18203.12
Load -25%	0.2	21.20	28.25	24.73	37.07	33.54	40.59	\$4935.90

demand bids expressed as a function of the quantity of power q . Nodes 1, 2, and 3 constitute a “North” region with excess generation capacity, while nodes 4, 5, and 6 constitute a “South” region with excess demand. The network diagram indicates the branch capacities (in MW) and admittances (per unit).¹⁵ The supply and demand bids reflect exogenous supply and demand for power in the wholesale market.

Following Deng et al. (2010), I assume that the intercepts of the wholesale demand bids can be 25% higher or 25% lower with known probability, resulting in uncertainty over future congestion. Table 1 indicates the assumed probability of each demand scenario and resulting node-level locational marginal prices (LMPs), which determine FTR payoffs, along with the net congestion revenue earned by the market operator under each scenario. Expected congestion revenue is \$9,048.86.

The main purpose of this exercise is to compare the FTR auction described in Section 2 with a benchmark mechanism in which the market operator holds parallel uniform price auctions for each distinct FTR. To implement the latter, the market operator must first determine a quantity of FTRs to be sold on each path. To facilitate comparison, I focus on the allocation in which the quantity of FTRs sold on each path coincides with the quantity allocated on that path in the equilibrium of the FTR auction game.¹⁶

Rights are purchased by symmetric, risk averse financial speculators.¹⁷ Each speculator

¹⁵I do not impose the additional 340MW North-South flowgate constraint discussed in Chao et al. (2000). It appears that this constraint is imposed in Deng et al. (2010), at least for the “Normal” scenario, although this is not explicitly discussed in the text.

¹⁶In practice this particular allocation would be unknown and hence infeasible for a market operator seeking to implement path-wise uniform price auctions; however, the particular assumption made here does not affect the slope of the resulting bids.

¹⁷In a richer environment with both financial speculators and “hedgers” (firms with significant ex ante congestion exposure and possibly greater risk aversion), the choice of mechanism could have important implications for the surplus of different groups. I thank an anonymous referee for this insight.

Table 2: Auction Results by Selling Mechanism

	Standard Auction	Pathwise Uniform- Price Auctions	
Expected Value of FTRs	9048.86	9048.86	9048.86
Risk Premia	369.08	369.08	272.59
Market Power Rent	36.34	369.08	132.83
Operator Revenue	8643.44	8310.70	8643.44
Operator Revenue / Expected Value	0.9552	0.9184	0.9552
Participation Model	$J_k = 3$	$J_k = 3$	See text.

bids on one and only one FTR, and there are $J_k = 3$ distinct bidders on each FTR path.¹⁸ In (2), I assume that the marginal valuation function is $v_j(q) = E[\pi_k]q - \frac{\alpha}{2}q^2 \text{Var}(\pi_k)$ where π_k is the per-unit payoff of a path k FTR and $\alpha > 0$ is a risk preference parameter common to all bidders. π_k is defined the realized difference between the LMP at the sink node and the LMP at the source node. I assume $\alpha = 0.5 \times 10^{-3}$. Equilibria are obtained numerically. For the uniform price auctions, existence and uniqueness are standard given common knowledge of payoffs and $J_k \geq 3$ (Malamud and Rostek, 2017). For the FTR auction, I verify numerically that an equilibrium exists for one and only one combination of binding constraints, and that the associated price impacts appear to uniquely satisfy (6).

The first two columns of Table 2 present the market operator’s revenue under each mechanism. The standard FTR auction recovers 95.5% of expected congestion revenue, while the uniform price benchmark recovers only 91.8%. The table decomposes auction revenue shortfalls into risk premia demanded by the bidders and market power rents. Since bidders hold identical portfolios in either case, risk premia are the same for each mechanism. However, the FTR auction reduces market power rents by 90% as compared with the uniform price benchmark. In a larger and more realistic network, this effect could be even larger.

The striking difference in market power rents in Table 2 is a direct result of the cross-

¹⁸The main qualitative conclusions should remain unchanged for alternative exogenous specifications of participation. However, even in the polar case of full participation ($K_j = K$ for each j), one must repeatedly solve a challenging matrix-valued fixed point problem to find an equilibrium, which represents a significant increase in computational difficulty over the case of single path bidding.

path competitive linkages induced by the constraints in (1). In this network, there are fifteen distinct FTRs. All are strategically connected for the relevant range of parameters. The first two columns of Table 3 indicate that the price impact λ_j is uniformly lower for bids in the FTR auction as compared with bids in the uniform price benchmark. To illustrate the significance of this difference, the third column indicates the number of bidders that would be required for the relevant uniform price auction to deliver the same price impact as the FTR auction (relaxing the natural integer constraint on the number of bidders). This shows that, on average, bidders in the FTR auction bid as competitively as if they faced 11.8 directly competing bidders in a uniform price auction. This statistic describes the degree of competitiveness among bidders in terms of the slope of the equilibrium bid functions—because new entry dilutes risk, fewer additional bidders would be required to eliminate price differences across mechanisms. The fifth column shows that even after accounting for the dilution of risk, replicating the FTR auction clearing prices with uniform price auctions would require strictly more bidders on every path, and 31.5% more participation overall. In this case, total revenues are the same across mechanisms by construction; risk premia paid to speculators are now \$96.49 lower, but total market power rents remain substantially larger \$96.49 compared to the FTR auction. This is summarized in the final column of Table 2.

For any level of participation, the difference in market power rents across mechanisms crucially on the concavity of bidders' valuations, which is determined by bidders' risk preference α and the variance of each FTR $Var(\pi_k)$. In the limit as $\alpha \rightarrow 0$ or as $Var(\pi_k) \rightarrow 0$ for all k , bidders' marginal valuations converge towards the expected value of each FTR, and the difference in market power rents collapses to zero. (For either mechanism, there are no auction shortfalls.) Conversely, when α is larger or the variance of FTR payoffs is greater, bidders' demand greater risk premia, flattening the residual supply curves that bidders face and thereby enabling more aggressive demand reduction. This effect is relatively stronger under the uniform price mechanism, resulting in relatively greater market power rents.

These findings suggest that for meaningful levels of concavity in bidders' preferences the

Table 3: Price Impacts by Path

FTR	Price Impact λ_j		J_k for Equal:	
	Path-wise	Standard	Price Impact	Prices
1-2	0.00155	0.00058	4.67	3.57
1-3	0.00573	0.00075	9.64	3.99
1-4	0.00607	0.00070	10.62	4.02
1-5	0.02662	0.00111	26.09	4.19
1-6	0.02826	0.00111	27.37	4.20
2-3	0.00261	0.00060	6.36	3.80
2-4	0.00159	0.00053	4.98	3.63
2-5	0.01715	0.00095	19.99	4.16
2-6	0.01739	0.00096	20.06	4.16
3-4	0.00404	0.00067	8.00	3.91
3-5	0.00785	0.00098	9.99	4.00
3-6	0.00978	0.00098	11.96	4.05
4-5	0.01365	0.00100	15.66	4.12
4-6	0.01184	0.00102	13.66	4.09
5-6	0.00097	0.00060	3.63	3.29

FTR auction delivers lower market power rents for the same level of participation on the one hand, and for the same level of auction shortfalls on the other. It is natural to question the sensitivity of these findings to the maintained participation assumptions. Given the seeming complexity of bidders' potential participation strategies and the likely multiplicity of equilibria, I leave the development of a model of FTR auctions with endogenous entry to future work. Like any other analysis of imperfect competition, the results are predicated on the existence of entry barriers. With sufficient competition, there is no difference across mechanisms. As $J_k \rightarrow \infty$ for all k , bidders' marginal valuations converge towards the expected value of each FTR, and both risk premia and market power rents fall to zero.

4 Conclusion

Persistent underpricing in FTR auctions raises important questions for the design of restructured electricity markets. In this paper, I investigate the contribution of unilateral market power to underpricing. I use a simple theoretical model to demonstrate that the market

clearing mechanism used in FTR auctions can facilitate high levels of cross-path competition, which can significantly dampen bidders’ incentives to exercise market power. The numerical model presented in Section 3 suggests that in an FTR auction, market power rents may contribute relatively little to auction revenue shortfalls in comparison to any sources of concavity in FTR bidders’ valuations (such as risk premia or inventory costs).

Today, most ISOs directly allocate some FTRs to LSEs, while selling only “residual” FTRs in the FTR auction. Some observers have suggested eliminating the FTR auction and instead directly allocating 100% of FTR capacity to LSEs. Individual LSEs could then choose to sell their FTRs in a decentralized fashion. The analysis suggests that by breaking the strategic linkages across auctions, decentralization could enable speculative buyers to extract significant market power rents at current participation levels. Anticipating this, LSEs would be incentivized to hold larger FTR portfolios, which could be inefficient.

Another potential policy response to persistent auction revenue shortfalls is to eliminate bidding on FTRs that are perceived to be more vulnerable to strategic bidding. This approach was taken by the California ISO (CAISO) in 2018, which banned bidding on so-called “non-deliverable” FTRs. The framework in this paper suggests that this policy could have increased market power rents on the remaining “deliverable” FTRs, since bids on non-deliverable FTR paths exert competitive pressure elsewhere. However, the model presented here sets aside many details relevant to the CAISO’s intervention. I leave an empirical evaluation of the CAISO’s reforms for future work.

Apart from these specific policies, my findings reinforce the importance of separating the question of market power in FTR auctions from broader consideration of the efficiency of ISOs’ congestion revenue management practices. Even if trading profits earned by financial speculators do not reflect the exercise of market power, the central question remains whether the benefits that LSEs derive from FTR auctions exceed observed shortfalls. Conversely, the existence of trading profits (even those earned via market power) does not in itself imply that LSEs would be better off under proposed alternatives.

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A Model details and proofs

Proof of Lemma 1

The Lagrangian of the auctioneer's problem is:

$$L = \sum_{j \in J} \sum_{k \in K_j} \int_0^{q_{kj}} q_{kj}^{-1}(s) ds - \sum_{m \in M} \xi_m^U \left(K_m^U - \sum_{j \in J} \sum_{k \in K_j} z_{km} q_{kj} \right) - \sum_{m \in M} \xi_m^L \left(\sum_{j \in J} \sum_{k \in K_j} z_{km} q_{kj} - K_m^L \right)$$

In this expression, the first order condition with respect to q_{kj} is:

$$q_{kj}^{-1}(q_{kj}) + \sum_{m \in M} z_{km} (\xi_m^U - \xi_m^L) = 0$$

This implies that bidder j is awarded units of FTR k for which she bids strictly greater than $\sum_{m \in M} z_{km} (\xi_m^L - \xi_m^U)$. Thus, $p_k = \sum_{m \in M} z_{km} (\xi_m^L - \xi_m^U)$ is analogous to the stop out price of a uniform price auction for FTR k . Since bids assumed to be strictly downward sloping, rationing never occurs, and p_k is the market clearing price.

Proof of Proposition 1

Fix $g \in G$ and $j \in J_g$. From Lemma 1, we know that

$$p_j = E_j \tilde{Z} \tilde{\xi}$$

where $\tilde{\xi}$ is the subvector of constraint shadow prices corresponding to \tilde{M} . Since $j \in J_g$, the rows of $E_j \tilde{Z}$ corresponding to constraints not contained in \tilde{M}_g are identically zero. We can instead write:

$$p_j = E_j \tilde{Z}_g \tilde{\xi}_g$$

where $\tilde{\xi}_g$ and \tilde{Z}_g are, respectively, the subvector and submatrix of $\tilde{\xi}$ and \tilde{Z} corresponding to \tilde{M}_g .

$\tilde{\xi}_g$ must be a solution to the system of equations defined by $\sum_{j \in J_g} \sum_{k \in K_j} z_{km} q_{jk}(p_j) = \tilde{P}_{g,m}$ for each $m \in \tilde{M}_g$, where $\tilde{P}_{g,m} = P_m^U$ if $m \in \tilde{M}_g^U$ and $\tilde{P}_{g,m} = P_m^L$ if $m \in \tilde{M}_g^L$. In matrix form,

$$\tilde{Z}_g' \left(E_j' q_j + \sum_{j' \neq j} E_{j'}' q_{j'}(p_{j'}) \right) = \tilde{P}_g$$

Differentiating with respect to q_j gives:

$$\tilde{Z}_g' \left(E_j' + \sum_{j' \neq j} E_{j'}' \frac{\partial q_{j'}}{\partial p_{j'}} \frac{\partial p_{j'}}{\partial q_j} \right) = 0$$

Re-arranging and substituting $p_{j'} = E_{j'} \tilde{Z}_g \tilde{\xi}_g$ gives:

$$\left(\sum_{j' \neq j} \tilde{Z}_g' E_{j'}' \frac{\partial q_{j'}}{\partial p_{j'}} E_{j'} \tilde{Z}_g \frac{\partial \tilde{\xi}_g}{\partial q_j} \right) = -\tilde{Z}_g' E_j' \quad (7)$$

Since C_j is positive definite and Λ_j is assumed to be positive semi-definite, $C_j + \Lambda_j$ is positive definite and hence invertible. By the implicit function theorem, it follows that:

$$\frac{\partial q_{j'}}{\partial p_{j'}} = -(C_{j'} + \Lambda_{j'})^{-1}$$

Define $\Lambda_j^\xi = \frac{\partial \tilde{\xi}_g}{\partial q_j}$, and note that $\Lambda_j = \frac{\partial p_j}{\partial q_j} = E_j \tilde{Z}_g \Lambda_j^\xi$. Hence,

$$\frac{\partial q_{j'}}{\partial p_{j'}} = - \left(C_{j'} + E_{j'} \tilde{Z}_g \Lambda_{j'}^\xi \right)^{-1}$$

By the Woodbury matrix identity,

$$\frac{\partial q_{j'}}{\partial p_{j'}} = -C_{j'}^{-1} + C_{j'}^{-1} E_{j'} \tilde{Z}_g \left(I + \Lambda_{j'}^\xi C_{j'}^{-1} E_{j'} \tilde{Z}_g \right)^{-1} \Lambda_{j'}^\xi C_{j'}^{-1}$$

Pre- and post-multiplying by $\tilde{Z}'_g E'_{j'}$ and $E_{j'} \tilde{Z}_g$ gives:

$$\tilde{Z}'_g E'_{j'} \frac{\partial q_{j'}}{\partial p_{j'}} E_{j'} \tilde{Z}_g = -W_{j'} (I + Y_{j'})^{-1}$$

where $W_j = \tilde{Z}'_g E'_j C_j^{-1} E_j \tilde{Z}_g$ and $Y_j = \Lambda_j^\xi C_j^{-1} E_j \tilde{Z}_g$, and we make use of the fact that $I - (I + Y_{j'})^{-1} Y_j = (I + Y_{j'})^{-1}$. The LHS matrix and W_j are both positive definite and thus symmetric, therefore:

$$\tilde{Z}'_g E'_{j'} \frac{\partial q_{j'}}{\partial p_{j'}} E_{j'} \tilde{Z}_g = - (I + Y_{j'})^{-1} W_{j'}$$

Returning to (7), it follows that:

$$\left(\sum_{j' \neq j} (I + Y_{j'})^{-1} W_{j'} \right) \Lambda_j^\xi = \tilde{Z}'_g E'_j$$

Post-multiplying by $C_j^{-1} E_j \tilde{Z}_g$ gives:

$$\left(\Psi_g^{-1} - (I + Y_{j'})^{-1} W_j \right) Y_j = W_j$$

where $\Psi_g^{-1} = \sum_{j \in J_g} (I + Y_{j'})^{-1} W_j$. Some algebra shows that:

$$Y'_{j'} \Psi_g^{-1} Y_j + (\Psi_g^{-1} - 2W_j) Y_j - W_j = 0 \quad (8)$$

and:

$$\Psi_g = \left(\sum_{j \in J_g} (I + Y_{j'})^{-1} W_j \right)^{-1} \quad (9)$$

where we make use of the fact that $W_j Y_j$ must be symmetric (although in general Y_j will not be). Under Assumption 1, conformable matrices $\{\Psi_1, \dots, \Psi_G\}$ and $\{Y_1, \dots, Y_J\}$ solving (8) for all $j \in J_g$ and (9) for all $g \in G$ are guaranteed to exist. Assuming this is the case, it

follows from (7) that:

$$\Lambda_j = E_j \tilde{Z}_g \left(\Psi_g^{-1} - (I + Y_j')^{-1} W_j \right)^{-1} \tilde{Z}_g' E_j'$$

where we make use of the invertibility stipulation in Assumption 1.

Proof of Corollary 1

For 1., the case in which k' and k_j are not strategically connected is immediate. For the case in which k' and k_j are strategically connected, monotonicity of λ_j in x_j implies that it will suffice to show that $x_j^1 < x_j^0$. To this end, define the function $F_g : \mathbb{S}_+^{|\tilde{M}_g^0|} \times \mathbb{Z}_+ \rightarrow \mathbb{S}_+^{|\tilde{M}_g^0|}$

$$F_g(\Psi; J) = \Psi - h_g(\Psi; J)$$

where $h_g : \mathbb{S}_+^{|\tilde{M}_g^0|} \times \mathbb{Z}_+ \rightarrow \mathbb{S}_+^{|\tilde{M}_g^0|}$ is given by:

$$h_g(\Psi; J) = \left(\sum_{j \in J_g} \left\{ \frac{2\tilde{Z}_{k_j}' \Psi \tilde{Z}_{k_j} + c_j + \sqrt{c_j^2 + 4(\tilde{Z}_{k_j}' \Psi \tilde{Z}_{k_j})^2}}{2} \right\}^{-1} \tilde{Z}_{k_j} \tilde{Z}_{k_j}' \right)^{-1}$$

Inspection of h_g reveals two useful properties, summarized in the Lemma below.

Lemma 2. *[Monotonicity of h_g] h_g satisfies the following properties:*

1. $h_g(\Psi; J_{k'} + 1) < h_g(\Psi; J_{k'})$ in the positive semidefinite (psd) order
2. $h_g(\Psi'; J_{k'}) < h_g(\Psi; J_{k'})$ in the psd order if $\Psi' > \Psi$ in the psd order

By construction, $F_g(\Psi^0; J^0) = 0$ and $F_g(\Psi^1; J^1) = 0$. Observe that:

$$F_g(\Psi^1; J^1) = \{F_g(\Psi^1, J^1) - F_g(\Psi^1, J^0)\} + \{F_g(\Psi^1, J^0) - F_g(\Psi^0, J^0)\} + F_g(\Psi^0, J^0)$$

and therefore:

$$0 = \{F_g(\Psi^1, J^1) - F_g(\Psi^1, J^0)\} + \{F_g(\Psi^1, J^0) - F_g(\Psi^0, J^0)\}$$

The first property in Lemma 2 implies that $F_g(\Psi^1; J^1) - F_g(\Psi^1; J^0)$ is positive definite. But then $F_g(\Psi^1, J^0) - F_g(\Psi^0, J^0)$ must be negative definite. But the second property in Lemma 2 implies that $F_g(\Psi, J_{k'})$ is strictly increasing in the psd order as Ψ increases in the psd order; hence, $\Psi^1 < \Psi^0$ in the psd order. It is easy to see that $x_j^1 < x_j^0$ if $\Psi^1 < \Psi^0$ in the psd order.

For 2., consider the following example. Figure 2 presents a simple three-node network derived from the example in Section 3. As in Section 3, suppose that the intercepts of the demand bids can be 25% higher or 25% lower, with the same probabilities as those given in Table 1. Suppose there are three FTR paths: one linking nodes 1 and 2, another linking nodes 1 and 3, and a third linking nodes 2 and 3. In this network, the price impact λ_{12} generally decreases in J_{12} except when new entry induces a change in the set of equilibrium binding constraints, in which case the price impact λ_{12} increases.

To illustrate this pattern, Figure 3 plots the price impacts on each path as a function of the number of bidders J_{12} , holding fixed the numbers of bidders $J_{13} = 2$. The blue solid and red dashed lines in each panel correspond to $J_{23} = 5$ and $J_{23} = 6$, respectively. In the unshaded regions, only the branch between 1 and 2 binds in equilibrium. In the shaded regions, both of the branches between 1 and 2 and between 1 and 3 bind in equilibrium. The lines in the leftmost panel indicate that price impacts on 1-2 can increase when further entry on 1-2 induces a change in \tilde{M} ; otherwise, price impacts are decreasing in entry. The lines in the center and right panels indicate that price impacts on paths 1-3 and 2-3 typically decrease but can also increase as a result of entry on path 1-2.

Figure 2: Three-Node Test Case

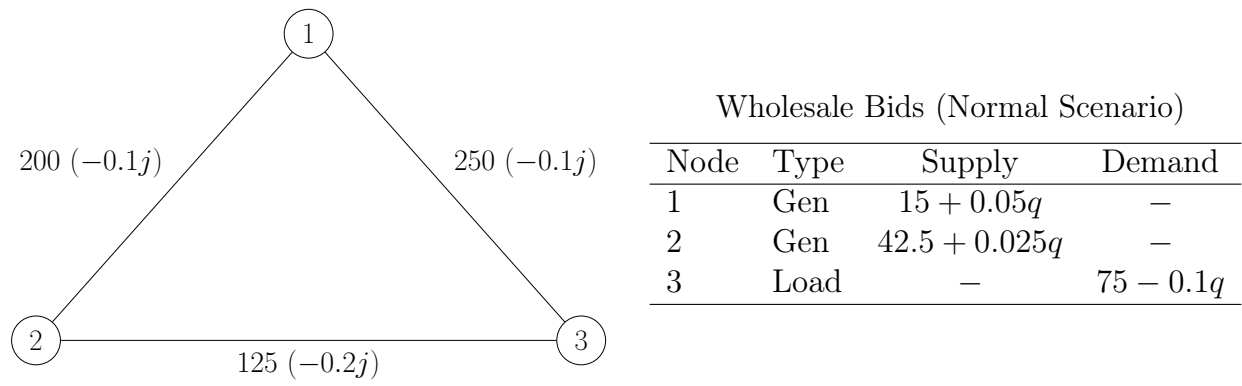
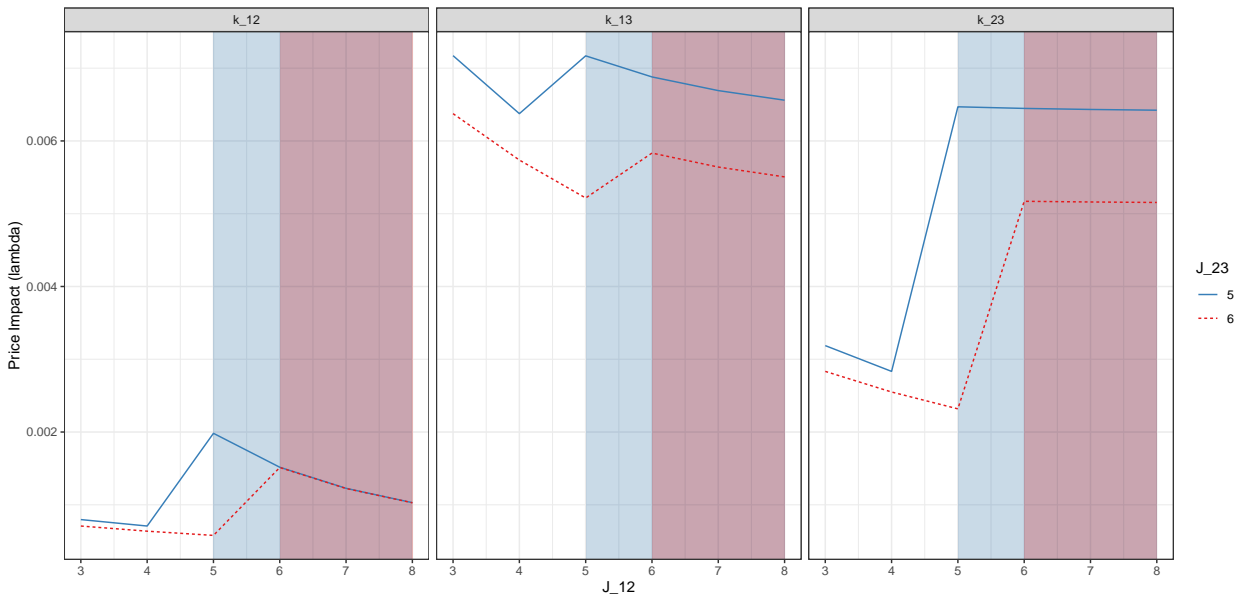


Figure 3: Price Impacts vs. FTR 1-3 Entry



B Additional Material

Construction of the PTDF Matrix Consider a three-node network ($N = 3$) having three branches ($M = 3$; in order: 1-2, 1-3, 2-3) with arbitrary branch admittances $\phi = (\phi_{12}, \phi_{13}, \phi_{23})$. As an illustration, I construct the PTDF matrix following Liu et al. (2009). First, let $T = HA\tilde{B}$ where H is a diagonal matrix of the branch susceptances, A is the node-arc incident matrix, and \tilde{B} is a block-diagonal matrix that also depends on the branch susceptances. T is a matrix of generation shift factors for each node. To construct \tilde{B} , we must designate a reference node. Letting Node 1 be the reference node, we have:

$$H = \begin{bmatrix} \phi_{12}^{-1} & 0 & 0 \\ 0 & \phi_{13}^{-1} & 0 \\ 0 & 0 & \phi_{23}^{-1} \end{bmatrix} \quad A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \quad \tilde{B} = \beta^{-1} \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\phi_{13}^{-1} - \phi_{23}^{-1} & -\phi_{23}^{-1} \\ 0 & -\phi_{23}^{-1} & -\phi_{12}^{-1} - \phi_{23}^{-1} \end{bmatrix}$$

where $\beta = \phi_{12}^{-1}\phi_{13}^{-1} + \phi_{12}^{-1}\phi_{23}^{-1} + \phi_{13}^{-1}\phi_{23}^{-1}$ is a scaling factor. In general, $Z = A_{FTR}T$ where A_{FTR} is the FTR design matrix. If there is one FTR for each branch (1-2, 1-3, 2-3), then the FTR design matrix coincides with the node-arc matrix. Substitution gives:

$$Z = \beta^{-1} \begin{bmatrix} -\phi_{12}^{-1}(\phi_{13}^{-1} + \phi_{23}^{-1}) & -\phi_{13}^{-1}\phi_{23}^{-1} & \phi_{13}^{-1}\phi_{23}^{-1} \\ -\phi_{12}^{-1}\phi_{23}^{-1} & -\phi_{13}^{-1}(\phi_{12}^{-1} + \phi_{23}^{-1}) & -\phi_{12}^{-1}\phi_{23}^{-1} \\ \phi_{12}^{-1}\phi_{13}^{-1} & -\phi_{12}^{-1}\phi_{13}^{-1} & -\phi_{23}^{-1}(\phi_{12}^{-1} + \phi_{13}^{-1}) \end{bmatrix}$$

In the special case that $\phi_{12} = \phi_{13} = \phi_{23}$, Z simplifies to:

$$Z = \begin{bmatrix} -2/3 & -1/3 & 1/3 \\ -1/3 & -2/3 & -1/3 \\ 1/3 & -1/3 & -2/3 \end{bmatrix}$$

As this example suggests, the PTDF matrix is generally dense, and the shift factors associated with any particular FTR depend on physical characteristics of transmission lines

throughout the network as well as the overall structure of the network.