Identification and Estimation of Auction Models with Dual Risk-Averse Bidders

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Can firms be risk averse?

- Huge expenditures on (re-)insurance, hedging, “risk management”
- Compelling structural evidence, most notably in the auctions literature
  - Bolotnyy and Vasserman, 2023; Luo and Takahashi, Forthcoming; Kim, 2024
- Nevertheless, IO economists are often reluctant to relax risk neutrality

Risk aversion and profit maximization

- One reason: EU models require a departure from profit maximization
  - Profit maximization presupposes a constant marginal utility of wealth
- Yaari (1987) sought to address this tension with *dual risk aversion*
  - Risk aversion is generated by probability distortion (like in VaR/CVaR)
  - The marginal utility of wealth (read: profits) remains constant
- Widely known, but no prior applications in empirical Bayesian games

This paper: estimate an auction model with dual risk aversion

1. Characterize BNE of first- and second-price auctions

2. Dual counterpart to classic identification results (Guerre, Perrigne, Vuong, 2009)

3. Estimation framework leveraging parallel FPA/SPA bidding

4. Application to USFS timber auctions

Related literature

1. Empirical models of auctions relaxing risk-neutrality
   - **EU models:** Bajari and Hortacsu, 2005; Guerre Perrigne Vuong, 2009; Lu and Perrigne, 2008; Campo et al, 2011; Fang and Tang, 2014; Bolotnyy and Vasserman, 2023; Chen, Gentry, Li, Lu, 2025; Luo and Takahashi, Forthcoming. **Non-EU models:** Aryal et al, 2018

2. Mechanism design with constant risk aversion
   - Gershkov, Moldovanu, Strack, Zhang (2022, 2023a, 2023b)

3. Rank-dependent utility models
   - **Experiments:** Goeree, Holt, Palfrey, 2002; Armantier and Treich, 2009. **Consumer choice:** Ciccheti and Dubin, 1994; Barseghyan et al., 2013

Preferences

- x is a random variable on $[0, \bar{x}]$ drawn from $H_x(\cdot)$

- Begin with a rank-dependent utility model (Quiggan, 1982),

$$U(x) = \int_0^{\bar{x}} u'(s) \cdot g(1 - H_x(s))\, ds$$

  - $u(\cdot)$ is a standard vN-M utility function, while $g : [0,1] \to [0,1]$ is a cdf
  - Three special cases:

	$u(\cdot)$	$g(\cdot)$
Risk-neutrality	Identity	Identity
EU risk-aversion	Concave	Identity
Dual risk-aversion	**Identity**	**Convex**

Auction game: key results

- A seller is endowed with a single indivisible good
- I symmetric potential bidders with private values $v_i \sim F(\cdot)$
- We characterize monotone, symmetric BNE in the case of dual risk-aversion:

Proposition

1. *In a SPA with reserve price r, the optimal strategy is $\beta(v_i) = v_i$ if $v_i \geq r$.*
2. *In a FPA with reserve price r, the optimal strategy is to bid:*

$$\beta(v_i) = v_i - \frac{\int_r^{v_i} g\left(F^{I-1}(s)\right) ds}{g\left(F^{I-1}(v_i)\right)}$$

if $v_i \geq r$. Moreover, $\beta(v_i)$ is greater than the risk-neutral optimal bid.

Discussion

- Auction design implications broadly similar to the EU case
  - Failure of revenue equivalence in favor of FPA
  - Key difference: optimal mechanism entails full-insurance (Gershkov et al 2022)

- In **Li, Lu, O'Keefe (2025)**, we study optimal reserve prices
  - Risk-neutral seller with valuation v_0, dual risk averse buyers:

$$\text{SPA:} \quad v_0 = r^* - (r^*)^{-1}(1 - F(r^*))$$

$$\text{FPA:} \quad v_0 = r^* - \left[F(r^*)^{I-1} f(r^*)\right]^{-1} \int_{r^*}^{\bar{v}} \frac{g\left(F(r^*)^{I-1}\right)}{g\left(F(s)^{I-1}\right)} F(s)^{I-1} f(s)\, ds$$

Optimal bidding in FPA

- For a bidder with valuation v_i, the optimal bid b_i satisfies:

$$v_i = b_i + \frac{1}{I-1} \cdot \frac{Q(b_i)}{q(b_i)} \left\{ \frac{Q^{I-1}(b_i) \cdot g'\left(Q^{I-1}(b_i)\right)}{g\left(Q^{I-1}(b_i)\right)} \right\}^{-1}$$

  - Q is the equilibirum distribution of bids, and q its density

- Problem: $g(\cdot)$ and $F(\cdot)$ are not separately identified from FPA bids alone
  - Analagous to non-identification of $[u(\cdot), F(\cdot)]$ in the EU model (GPV09)

Two identification strategies

1. Exogenous variation in I (GPV09)
    - $b(\alpha)$ varies with I, but $v(\alpha)$ does not (similar to GPV09)
    - Can accommodate endogenous I with an instrument
2. Second price auction data (Lu and Perrigne, 2008)
    - F is identified from the distribution of (winning) SPA bids

Estimation with SPA bids

- By integrating over quantiles (Liu Lou 2017, Luo Wan 2018), we obtain:

$$\hat{g}(\alpha) = \exp\left(- \int_{\log \hat{b}\left(\alpha^{1/(I-1)}\right)}^{\log \hat{b}(1)} \left[\frac{1}{\hat{\psi}(u) - u} \right] \cdot du \right)$$

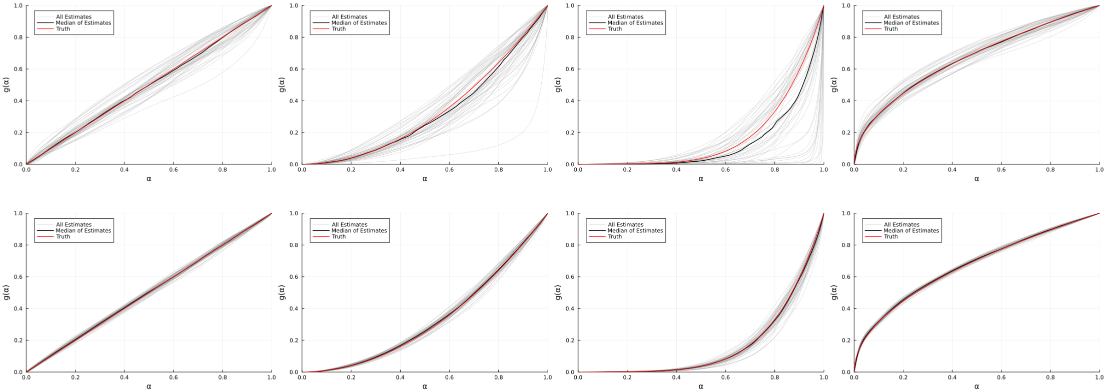
  - $\psi(\cdot) \equiv F^{-1}(Q(\cdot))$ is the statistical mapping from bids to values
  - **Tuning-parameter free** if bid ECDFs are used

- In practice, we use a sieve-based estimator (Chen, 2007) to ensure $g'(\cdot) > 0$

Monte Carlo simulations

$$g\left(x\right) = \left(ax + x^{\gamma}\right) / \left(1 + x\right), \quad S \in \{100, 1000\}$$

$a = 0, \gamma = 1$ | $a = 0, \gamma = 2$ | $a = 1, \gamma = 5$ | $a = 0, \gamma = 0.5$



Testing framework

1. **Specification tests**

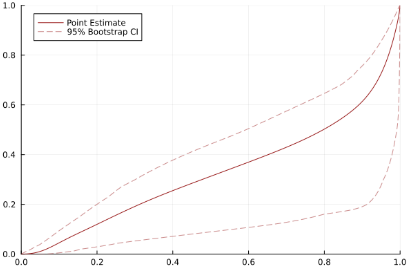
   - Dual utility $\Rightarrow$ pseudo-true $g(\cdot, I)$ is invariant to I for all $\alpha \in [0, 1]$
   - EU $\Rightarrow$ pseudo-true $u(\cdot, I)$ is invariant to I for all final wealths

2. **Testing for risk aversion**

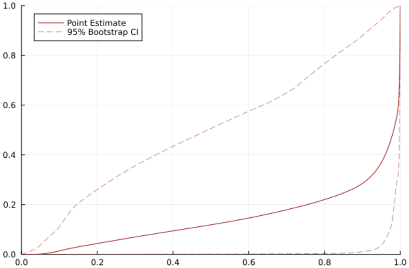
   - Dual utility $\Rightarrow$ test for convexity of $g(\cdot)$ using Fang and Seo (2021)

Estimated distortion functions

$I = 2$



$I = 3$



- Null of convexity not rejected in either sample by Feng and Seo (2021) test
- Complements prior work using EU models (Lu and Perrigne, 2008, Campo et al, 2011)

Revenue comparison and optimal reserves

	SPA		FPA: Risk Neutral		FPA: NEU		FPA: EU	
	$I = 2$	$I = 3$	$I = 2$	$I = 3$	$I = 2$	$I = 3$	$I = 2$	$I = 3$
Revenue ($r = 0$)	35,777	49,107	35,665	48,657	39,360	62,496	44,436	67,678
	(1,145)	(2,786)	(1,139)	(2,723)	(1,791)	(5,825)	(8,150)	(8,530)
Revenue ($r = r^*$)	42,553	54,598	42,048	54,452	44,648	64,611	-	-
	(2,024)	(3,598)	(2,053)	(3,583)	(1,718)	(4,569)	-	-
Optimal Reserve r^*	55,029	60,726	55,029	60,726	51,074	55,048	-	-
	(14,489)	(25,045)	(14,489)	(25,045)	(11,353)	(30,690)	-	-

- Risk-neutral model: (1) overstates r^* by $\approx$10%; (2) understates benefits of FPA

1. Theoretical framework

2. Identification

3. Estimation

4. Application: USFS Timber Sales

5. Conclusion

Conclusion

- We show how to estimate an auction model with dual risk averse bidders
- Appealing for applications in which bidders are profit-maximizing firms
- No more demanding than the EU case, and indeed more tractable
  - Closed form bidding strategies $\Rightarrow$ simplified counterfactuals
  - Optimal mechanism can be quantified in some cases (Gershkov et al 2022)

Thank you!

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