

Automation and the Future of Work: Assessing the Role of Labor Flexibility

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Abstract

We study the economic incentives for automation when labor and machines are perfect substitutes. Labor may still be employed in production, even when it is a costlier input than robots on a productivity-adjusted basis. This occurs if firms face uninsurable idiosyncratic risk, adjusting the stock of machines is costly, and workers can be hired and fired quickly enough. Even though labor survives, jobs become less stable, as workers are hired in short-lived bursts to cope with shocks. We calibrate a general equilibrium, multi-industry version of our model to match data on robot adoption in US manufacturing sectors, and use it to compute the employment and labor share consequences of progress in automation technology. A fall in the relative price of robots leads to relatively few job losses, while reductions in adjustment costs, or improvements in relative robot productivity, can be far more disruptive. The model-implied semi-elasticity of aggregate employment to robot penetration (number of robots per thousand employees) ranges between 0.01% and 0.12%, depending on the underlying source of increased robot adoption, consistent with findings in the empirical literature. In an extension, we show that reduced-form hiring and firing costs unambiguously depress long-run employment.

Keywords: Automation, Investment, Capital-Labor Substitution, Factor Shares, Tasks, Technological Change.

JEL Classification: O33, J23, E22, E23, E24.

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1 Introduction

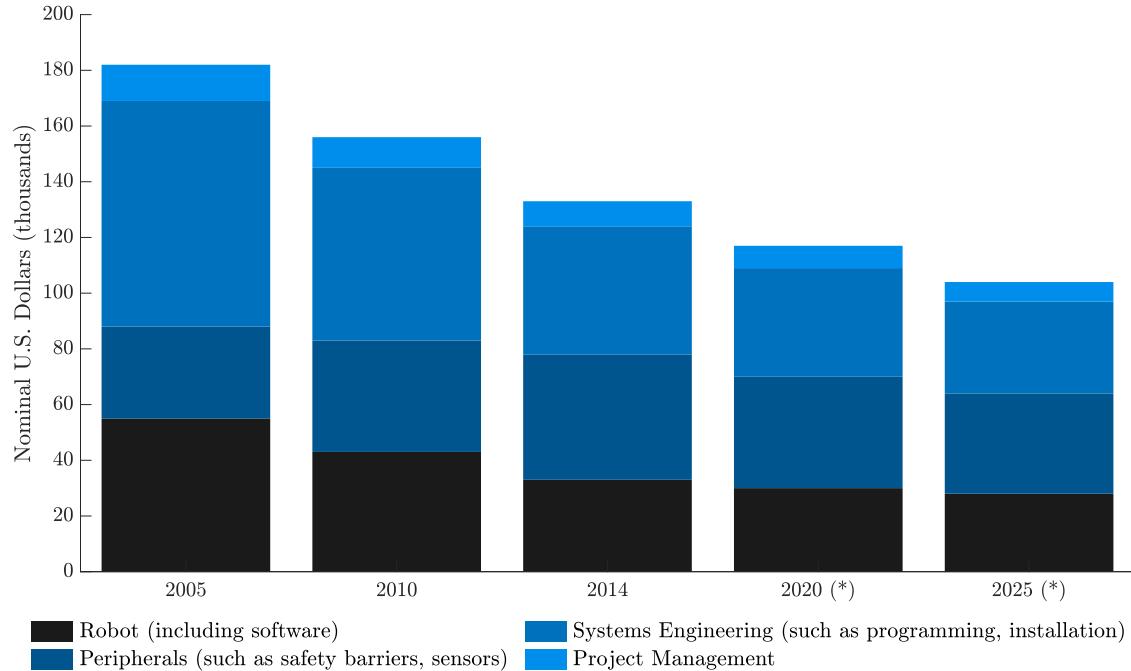
Over the last few years, the progress in robotics, software engineering, and AI, coupled with the secular decline in the labor share of output, has sparked a discussion on whether machines will progressively replace humans in performing tasks (Acemoglu and Restrepo, 2018c; Autor, 2015; Autor and Salomons, 2018; Berg et al., 2018; Graetz and Michaels, 2018; Sachs and Kotlikoff, 2012). In this paper, we investigate the long-run consequences of improvements in automation technology under the deliberately extreme assumption that robots and humans are perfect substitutes within tasks, to provide an upper bound for potential employment losses.

The recent literature has stressed a number of reasons why robots and human workers might not be perfect substitutes. Autor (2015) discusses the ability of workers to perform the multiple and differentiated tasks that typically constitute a job. Acemoglu and Restrepo (2018b,c, 2020) highlight the importance of human comparative advantage in carrying out specific tasks, a point that is also made by Berg et al. (2018) and Graetz and Michaels (2018). Finally, there might be some non-routine occupations, or categories of workers that are poised to benefit from automation (Berg et al., 2018; Sachs and Kotlikoff, 2012; Sachs et al., 2015). While we acknowledge the importance of these mechanisms, we abstract from them to focus on tasks where workers and robots can indeed be perfect substitutes. In this sense, we describe a worst-case scenario for workers employed in low-skilled, routine occupations.

Our baseline model follows the spirit of the task-based framework adopted by Acemoglu and Restrepo (2018b), and focuses on a firm choosing whether to automate a single task where humans and robots are perfect substitutes. We describe how human labor can survive thanks to its *flexibility as an input in production*, which we identify as its distinctive comparative advantage.¹ Labor survives alongside robots under three main assumptions that set our framework apart from the existing literature. First, we introduce demand shocks by assuming that firms face uninsurable idiosyncratic risk. Shocks create a source of demand for flexible inputs that can readily adjust to a volatile environment. Second, we model robots as having capital-like features that impair their rapid deployment in production, in contrast to the standard assumption of a rental market that allows for immediate adjustments of the robot stock. Finally, we assume that employees can be hired and fired more easily than robots can be bought, installed, and sold. Accordingly, firms respond to positive shocks using the more flexible factor, but generally employ *both* workers and machines. Notably, firms employ both workers and machines even if labor and robots are perfect substitutes, and even if robots are cheaper on a productivity-adjusted basis. Our model therefore presents a context where imperfect long-run substitutability between robots and workers *arises endogenously*.

¹Tesla's recent history is a real-world example of the mechanism we describe. After months of unsuccessful attempts at scaling up the production of the Tesla Model 3 by radical automation, Elon Musk tweeted: “Yes, *excessive automation at Tesla was a mistake. To be precise, my mistake. Humans are underrated.*” Installing and adapting robots to the various tasks turned out to be harder than expected, pushing the company to meet its demand backlog by hiring thousands of (human) workers. See the Forbes coverage in Muller (2018).

Figure 1: Total System Cost of a Typical Spot-Welding Robot in the U.S. Automotive Industry



Note: Authors' elaboration on data from [Sirkin et al. \(2015\)](#). Values are expressed in nominal terms. Asterisks denote BCG projections for the components of interest.

The dynamics implied by our model reproduce the empirical dichotomy between firm-level and aggregate effects of automation on employment. At the firm level, investment in automation technology has been associated to higher wages and employment ([Acemoglu et al., 2020](#); [Aghion et al., 2020](#); [Bonfiglioli et al., 2020](#); [Koch et al., 2019](#)). However, aggregate labor demand at the sector or geographic levels has been found to fall with increased robot penetration ([Acemoglu and Restrepo, 2020](#); [Dauth et al., 2019](#)). We also show that the survival of production-line employment comes at the cost of reduced job stability, which hearkens back to the findings in [Eggleston et al. \(2021\)](#); [Rutledge et al. \(2019\)](#). Simulated time series from our model highlight that, in an automated world, labor is only hired in short-lived “bursts” to cope with sudden increases in the desired production scale.

Our modeling choices are informed by data on robot costs. Figure 1 reports data from [Sirkin et al. \(2015\)](#) detailing the main cost items for the setup of a spot-welding robot in the U.S. automotive industry. The figure highlights three main facts that feature in our model and calibration. First, in line with the broader evidence in the report by the [International Federation of Robotics \(2017\)](#) and [Korus \(2019\)](#), the purchase price of robots has been trending down, and it is projected to keep doing so in the future. Second, purchase costs represent but a small fraction of the total cost of a robotic system, which is mostly made up of installation-related costs, reflected by the adjustment costs in our model. Third, the nature of these adjustment costs, and particularly those related to programming and "peripherals", suggest that robot systems have a firm-specific component that

might affect their redeployment to different contexts.

In order to gauge the quantitative implications of our theoretical findings, we develop a multi-industry, general equilibrium version of our baseline model, which we calibrate to match data on the adoption of robots between 2010 and 2014. This exercise reveals that, in line with the evidence in Figure 1, robot adjustment costs might indeed be sizable. This result stems from the low aggregate semi-elasticity of robot penetration—the number of robots per thousand employees—to purchase prices observed in the data. Under our calibration, we establish that even a dramatic reduction in the relative price of robots causes only a modest fall in aggregate employment, with changes in the technical substitutability and flexibility of robots posing a more substantial threat. Notably, general equilibrium effects mitigate employment losses, as they imply a fall in the equilibrium wage. Our model generates a semi-elasticity of aggregate employment to robot penetration which ranges between 0.01% and 0.12%, depending on the underlying source of increased robot adoption. These magnitudes are in line with the quantitative findings of [Acemoglu and Restrepo \(2020\)](#), who compute a semi-elasticity of 0.2%.

Our quantitative results are robust to three alternative calibration strategies. First, we account for uncertainty in robot price data—a crucial threat to the estimate of price elasticity. We do so by re-calibrating the multi-sector model for a wide range of potential robot price changes (between 50% and 150% of our baseline). Second, we assess the robustness of our results to account for the fact that the low price elasticity that informs our baseline calibration might arise from slow investment responses to current and anticipated future price changes. To this end, we propose an alternative calibration that matches the increase in robot penetration observed between 2004-2014, assuming that this change occurs along a perfect-foresight transition of robot prices to lower levels, announced in year 2000. Finally, we allow for uncertainty surrounding firms' growth prospects by introducing non-stationary shocks. All these alternative strategies broadly confirm our quantitative findings.

We further extend our theoretical results in two directions. First, we analyze an extension featuring labor adjustment costs, which we interpret as reduced-form labor market frictions. Second, we show that our qualitative findings are robust to using linear or fixed costs of adjustment. In line with our theory, we find that high hiring and firing costs dampen labor's flexibility comparative advantage, and increase the long-run displacement of production-line workers. This suggests that removing strict employment protection measures could be an effective policy to safeguard unskilled jobs in the long run, counter to what intuition might suggest. The rigid-labor extension also shows that when the transition to lower robot prices is gradual, stricter labor market regulations can induce firms to anticipate the adoption of robots to smooth out workforce adjustment costs. This result speaks to empirical evidence suggesting that higher unionization is associated with higher robot penetration at the current stage ([Acemoglu and Restrepo, 2018a](#)).

Related Literature Our modeling framework relates to two distinct strands of theoretical literature. The first deals with automation and its long-run impact on employment and factor shares. The second relates to modeling investment under uncertainty and costly reversibility. Our findings also speak to the recent empirical literature on the effects of automation on employment and wages.

In the theoretical literature, several papers view automation as a form of factor-augmenting innovation: labor-augmenting according to [Bessen \(2020\)](#); capital-augmenting according to [Sachs and Kotlikoff \(2012\)](#); [Sachs et al. \(2015\)](#); [Nordhaus \(2021\)](#); [Berg et al. \(2018\)](#). These studies either impose exogenous technological limits on the extent of automation, or conclude in favor of a long-run demise of labor, which sees its factor share falling to zero. Our stance is decidedly closer to the task-based approach pioneered by [Zeira \(1998\)](#) and later adopted by [Acemoglu and Restrepo \(2018b,c\)](#) and [Graetz and Michaels \(2018\)](#). In this setting, labor and robots are perfect substitutes within a subset of tasks. In our paper, we focus on this subset, and analyze the consequences of advances in automation when there are no technological limits to substitution.

All the above contributions make two common assumptions: robots are rented on the market; and firms operate in a deterministic environment. These features allow for bang-bang solutions in favor of either robots or labor when perfect substitution is possible. Our framework departs from both these assumptions, bridging the literature on automation with that on investment under uncertainty and costly reversibility ([Abel, 1983](#); [Pindyck, 1988, 1991](#); [Caballero, 1991](#); [Abel and Eberly, 1996, 1997](#); [Dixit and Pindyck, 1994](#); [Stokey, 2009](#)). Our theoretical model is closest to [Abel and Eberly \(1996\)](#), in that we assume decreasing returns to scale, perfectly flexible labor, and solve our model in continuous time. However, we depart from the standard investment literature by adopting a production function that features perfect substitutability between labor and capital. We rely on results summarized in [Achdou et al. \(2017, 2014\)](#) for our numerical solution method.

Our findings reproduce a number of features of the growing empirical literature on automation. In our framework, increased robot penetration leads to lower aggregate employment, even if investment in robots is positively correlated with productivity and employment at the level of the individual firm, consistent with recent studies. In particular, [Acemoglu et al. \(2020\)](#), [Bonfiglioli et al. \(2020\)](#), [Koch et al. \(2019\)](#), [Aghion et al. \(2020\)](#) find that employment generally increases at the level of the firm undertaking the automation effort. By contrast, [Acemoglu and Restrepo \(2020\)](#); [Acemoglu et al. \(2020\)](#) and [Dauth et al. \(2019\)](#) estimate negative employment effects of automation at various levels of aggregation. Finally, [Egginton et al. \(2021\)](#) show that an increase in robot adoption is associated with an increase in temporary work and employee headcount.²

²We refer the interested reader to [Bessen et al. \(2020\)](#) for an extensive review of recent contributions.

2 Model

This section presents the framework that we employ to model long-run labor demand. The main result is that human labor is employed in the long run even if it is a more expensive input than robots on a productivity-adjusted, flow-cost basis. This occurs because robots are a costly input to adjust in response to shocks, while labor can be readily hired and fired in this benchmark case.

Perfect technological substitutability implies that a robot is at most as valuable as the savings it realizes for the firm when it is used to substitute workers. As a result, firms install a limited quantity of robots, and hire labor to cope with shocks. We provide an analytical bound to the extent of automation that prevails in this environment, which we use to conduct comparative statics exercises on the parameters of interest. A reports formal derivations of all the results presented below.

2.1 Environment

Time is continuous and lasts forever, and there is no aggregate uncertainty. In this section, all prices are exogenous and constant in time. We postpone the discussion of the equilibrium to Section 3. There is a measure-one continuum of firms, each producing a homogeneous good. At each point in time, and for a given robot stock, R , and realization of the revenue shock, z , each firm solves a static profit-maximization problem by optimally selecting how much labor to hire, L , as well as the utilization rate of the robot stock, u . This gives the *operating profit function*:

$$\Pi(R, z) \equiv \max_{L \geq 0, 0 \leq u \leq 1} p z (\Gamma L + (1 - \Gamma) u R)^\theta - w L - m u R.$$

We let p denote the price of the good, m the flow cost of robots, and w the flow wage. Moreover, the parameter, $\Gamma = \text{MRTS}_{LR}/(1 + \text{MRTS}_{LR})$, is a normalized marginal rate of technical substitution that controls the fixed rate at which robots and labor can be technologically substituted for each other. In this sense, Γ captures how easily a task can be automated. An important point to note is that labor and robots are *perfect substitutes*. We make this extreme assumption to abstract from forms of labor-capital complementarity that the literature has already shown to act as a backstop to full automation. It is worth noting here that the substitutability assumption sets robots apart from traditional capital, which is usually taken as a complement to labor. By contrast, we see automation as enabling *perfect substitution*, and complete displacement of, e.g., low-skilled manufacturing workers. The parameter $\theta \in (0, 1)$ captures *decreasing returns to scale*, which can be motivated by a downward-sloping demand curve for the differentiated product of the firm, or by the presence of a fixed factor of production. Note that this unmodeled factor could encompass workers that are complemented by robots, as well as complementary capital. Finally, decreasing returns to scale allow for a well-defined notion of desired firm size, which plays a crucial role in our focus on idiosyncratic revenue volatility.

The firm's dynamic problem is

$$V_0(R_0, z_0) \equiv \max_{I: U \times [0, \infty) \rightarrow \mathbb{R}} \mathbb{E}_0 \int_0^\infty \exp\{-\rho t\} \left(\Pi(R_t, z_t) - p_R I_t - \frac{\psi_R}{2} (I_t)^2 \right) dt$$

s.t. $dR_t = (I_t - \delta R_t) dt,$
 $dz_t = \mu(z_t) dt + \sigma(z_t) dW,$

where we denote by $(U, \mathcal{B}, (\mathcal{F}_t)_{t \in \mathbb{R}_+}, \mathbb{P})$ the associated filtered probability space. The only source of risk in the model arises from the *idiosyncratic* revenue shock z_t , a standard Itô process which takes values in $\mathcal{Z} \subseteq \mathbb{R}_+$. We assume that the robot stock depreciates at rate δ , capturing physical depreciation or technological obsolescence. The parameter

$$\Omega \equiv \frac{1-\Gamma}{\Gamma} w - m,$$

which we will refer to as *flow labor savings*, denotes the productivity-adjusted flow cost savings from using robots instead of labor to produce a unit of output. To make the problem interesting, we assume throughout that $\Omega > 0$, and that the present discounted value of labor savings is larger than the purchase price of robots, $p_R < \Omega/(\rho + \delta)$. Both these conditions are required to ensure that firms want to purchase robots.

In our model, investment is subject to a quadratic adjustment cost, parameterized by ψ_R . We focus on this specification to derive an analytical characterization of the desired robot stock. We see convex, symmetric costs as a reasonable assumption. First, when buying robots, we envision that the firm will have to adjust the work environment or the production process to make it navigable by automated machines. This might involve sizable costs that are more than proportional to the upfront investment (Sirkin et al., 2015). Second, it is also reasonable to think that a seller will have to face similar adjustment costs to dismantle a robot system in place, related for example to the removal of barriers and sensors from the work space.^{3,4} We assess the robustness of our findings to different specifications of adjustment costs in Section 4.4.

We adopt a continuous-time setup, together with quadratic adjustment costs, for three reasons. First, this choice provides a *worst-case scenario for human labor*. Indeed, in this setting firms continuously make investment decisions, in contrast to both a discrete-time setup, and a lumpy-investment framework. In the former case, firms would mechanically be unable to adjust the stock of robots in the same time period, while in the latter case they optimally choose to stay idle unless shocks are sufficiently large. In both scenarios, firms have a strong incentive to respond to shocks by hiring labor. In our model, firms could in principle avoid hiring workers by setting a high enough investment rate.

³In practice, it is not uncommon for firms to rent advanced or complex machines. Accordingly, R_t could be interpreted as the size of leasing obligations that a firm has, which often involve penalties for early termination.

⁴Our results are qualitatively unchanged if we assume irreversible investment coupled with one-sided convex costs.

Second, these assumptions allow us to solve the investment problem very efficiently by leveraging the sparsity of the matrix encoding agents' policy functions (Achdou et al., 2017). Alternative specifications would be unfeasible to calibrate as done in Section 3. Third, our modeling choices allow us to obtain a series of analytical results that clarify the economics of the proposed mechanism.

2.2 Static Problem Solution: The Operating Profit Function

The solution of the static problem, detailed in A.2, yields

$$\Pi(R, z) = \begin{cases} (1-\theta)pz(1-\Gamma)^\theta \bar{R}(z)^\theta + \Omega R & R \leq \bar{R}(z) \\ pz(1-\Gamma)^\theta R^\theta - mR & \hat{R}(z) \geq R > \bar{R}(z) \\ pz(1-\Gamma)^\theta (\hat{R}(z))^\theta - m\hat{R}(z) & \hat{R}(z) < R, \end{cases}$$

where

$$\bar{R}(z) \equiv \frac{1}{1-\Gamma} \left(\frac{pz\theta\Gamma}{w} \right)^{\frac{1}{1-\theta}}, \quad \hat{R}(z) \equiv \frac{1}{1-\Gamma} \left(\frac{pz\theta(1-\Gamma)}{m} \right)^{\frac{1}{1-\theta}}$$

are the *full automation cutoff* and the *optimal rental-market scale*, respectively. To understand this solution, first note that decreasing returns to scale imply that the firm has a desired size associated with each value of the revenue shifter z . The quantity, $\hat{R}(z)$, represents the amount of robot capital that the firm would instantly install if robots were rented on the market at rate, m . However, since robots, R , are a state variable, the firm will generally be either oversized or undersized relative to this optimal rental-market scale. If the realized revenue shifter, z , is relatively small, the firm is oversized, $R > \hat{R}(z)$. Thus, it uses utilization to turn off robots in excess of $\hat{R}(z)$, and employs no labor in production. If instead the revenue shifter, z , is relatively large, the firm will find itself undersized relative to the optimal static scale, $R < \hat{R}(z)$. In this scenario, robots are fully utilized, and labor gets hired only if the shock is sufficiently large. This happens when the stock of installed robots is below the full automation cutoff, $R < \bar{R}(z)$. This cutoff results from the fact that, due to decreasing returns to scale, the marginal product of labor is decreasing in R , and it drops below the wage rate w for any $L > 0$ if $R > \bar{R}(z)$. When z is high enough, the optimal labor policy is given by

$$L^*(R, z) = \frac{1-\Gamma}{\Gamma} [\bar{R}(z) - R].$$

This expression suggests a ready interpretation of the economics at play in our model. Each firm inherits a stock of robots accumulated in previous instants, R . The revenue shifter, z , then determines the firm's labor demand, which is proportional to the gap between the full automation cutoff associated with z , $\bar{R}(z)$, and the fixed robot stock, R . The aggregate labor demand is then obtained as the integral of such gaps over the stationary distribution of R and z , and for any given stock R , the individual labor demand will be driven by relatively large revenue shocks. Indeed, for

intermediate realizations of z , the installed robot stock falls above the full automation cutoff, but still below the optimal rental-market scale, $\hat{R}(z) \geq R > \bar{R}(z)$. In this case, the firm fully utilizes its installed robot stock without hiring any labor. The solution of the dynamic problem shows that, under mild assumptions, the stationary distribution always features a mass of firms whose installed robot stocks are below the relevant full automation cutoff.

The above solution implies that, for a given revenue shock z , marginal operating profits are constant at Ω for $R \leq \bar{R}(z)$. In this region, each additional unit of robots reduces the number of labor hires, $((1-\Gamma)/\Gamma)(\bar{R}(z) - R)$, needed to achieve the same optimal level of revenues, $p_z(1-\Gamma)^\theta \bar{R}(z)^\theta$. In the intermediate region, marginal operating profits decrease in R . Finally, when $R \geq \hat{R}(z)$, the firm sets u to use only $\hat{R}(z)$ robots, so marginal operating profits are nil.

2.3 Dynamic Problem Solution: The Investment Policy Function

The optimal investment choice is given by

$$I^*(R, z) = \frac{1}{\psi_R} [V_R(R, z) - p_R],$$

arising from the equalization of the marginal benefit from holding an additional unit of capital to its net marginal cost. As investment is a linear function of the marginal value of robots, $V_R(R, z)$, it inherits its basic properties, which, in turn, stem from the properties of the instantaneous profit function. Importantly, since the marginal operating profit from a robot is bounded, we can show that the marginal value of a robot, $V_R(R, z)$, is also bounded from above by the present discounted value of labor savings, $\Omega/(\rho + \delta)$. The intuition for this result comes from a simple fact. As robots and workers are perfect substitutes, an additional robot can at most be worth as much as it is worth in the most favorable states of the world, that is, when the revenue shock, z , is relatively high. In these instances, a robot produces marginal profits given by the flow labor savings, Ω .

Given these properties of the marginal value, $V_R(R, z)$, we can immediately conclude that the investment policy is also bounded, and it is decreasing in the purchase price, p_R , and the adjustment cost parameter, ψ_R . As a result, the stock of robots that each firm is willing to install is also bounded. We now proceed to derive an expression for this upper bound, which allows us to characterize the stationary distribution of firms, and establish the conditions for the survival of labor in the long run.

We start by defining $R^*(z)$ as the stock of robots that the firm would optimally upkeep if it received the same revenue shock z forever, while operating in a stochastic environment. For any revenue shock z , $R^*(z)$ satisfies

$$I^*(R^*(z), z) = \delta R^*(z).$$

Since $I^*(R, z)$ is non-increasing in R , for any z , optimal net investment is negative if $R > R^*(z)$ and positive otherwise. That is, when the firm receives a given shock z , it starts adjusting towards the

corresponding desired steady state, $R^*(z)$. Under our assumptions, $R^*(z)$ is non-decreasing in z , and non-decreasing in w , for all z . Moreover, if $\delta > 0$, $R^*(z)$ is bounded from above by

$$R_{\max}^* \equiv \frac{1}{\delta \psi_R} \left[\frac{\Omega}{\rho + \delta} - p_R \right],$$

and tends to 0 as $\psi_R \rightarrow \infty$.

The fact that $R^*(z)$ is non-decreasing in w implies that the aggregate demand for labor is downward sloping. Moreover, the formula for R_{\max}^* clarifies the effect of many key parameters on the extent of automation. In particular, the bound is strictly decreasing in ψ_R , δ , ρ , and p_R . As robots become less flexible, more prone to obsolescence, or simply more costly, the investment opportunity they provide becomes less attractive for any given present discounted value of labor savings.

2.4 The Stationary Distribution and the Extent of Automation

Given a stationary process for z , the solution of the model gives rise to a stationary distribution of firms that solves the relevant Kolmogorov Forward Equation. In what follows, we lay out the conditions for partial automation, and characterize analytically the upper bound to the mass of fully automating firms. The intuition behind the results lies in a simple observation. While the maximum desirable stock of robots, R_{\max}^* , is fixed, the stock of robots needed to achieve full automation, $\tilde{R}(z)$, is increasing in the revenue shock z . Thus, firms receiving relatively large shocks will find themselves with a robot stock below the relevant $\tilde{R}(z)$, and optimally choose to hire labor. The higher the adjustment costs, the lower R_{\max}^* is, and the higher the mass of firms that hire workers in the stationary distribution. The following proposition formalizes this intuition.⁵

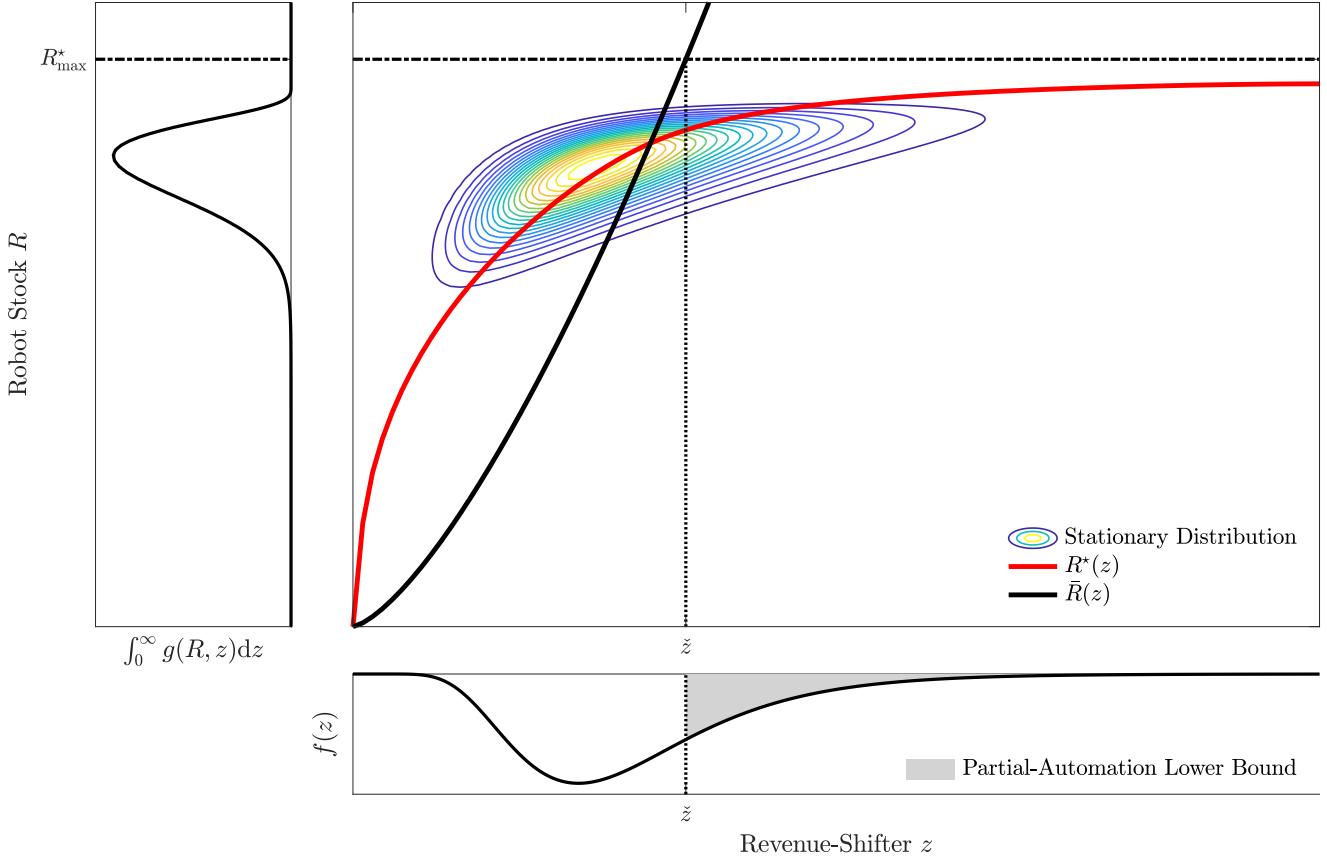
Proposition 1 (Conditions for Long-Run Partial Automation). *Consider a non-degenerate diffusion process or Continuous Time Markov Chain for z admitting a stationary distribution on \mathcal{Z} such that $z_1 \equiv \inf \mathcal{Z}$ and $z_N \equiv \sup \mathcal{Z}$, $z_1 \geq 0$. Define $G(\tilde{R}, \tilde{z}, t) \equiv \mathbb{P}\{R(t) \leq \tilde{R}, z(t) \leq \tilde{z}\}$, the CDF of the distribution of firms at time t , and $g(\tilde{R}, \tilde{z}, t)$ the associated PDF. For any purchase price of robots p_R , as long as $\delta > 0$, there exist a finite value of the adjustment cost parameter ψ_R such that the stationary distribution $G(R, z, \infty)$ does not feature full automation. If $z_N = \infty$, any $\psi_R > 0$ implies that the stationary distribution does not feature full automation.*

If robot reallocation involves sufficiently important frictions, captured by the parameter ψ_R , human labor will be saved *in the long run*. Importantly, this result obtains for any level of p_R , including 0, and for finite values of ψ_R . Figure 2 shows the stationary distribution for an illustrative calibration of the model together with the marginal distributions over R and z , and the schedules $\tilde{R}(z)$ and $R^*(z)$.⁶ The marginal distribution over z is highlighted to show the area corresponding to the

⁵We refer the reader to Proposition 4 in A for a discussion of the deterministic case.

⁶Figures 2 and 3 are produced using the calibration described in B.1. As in the following Section, we assume an

Figure 2: Stationary Distribution and Marginals over z and R



Note: This figure presents the stationary distribution of firms in the model, assuming an Exponential Ornstein-Uhlenbeck (EOU) process for the revenue shifter, and adopting the calibration in B.1. The upper-right panel displays the stationary distribution in the (z, R) space. Contour lines range from blue to yellow, with lighter colors denoting higher density. The two solid lines trace the loci corresponding to the desired stochastic steady state, $R^*(z)$ (in red) and the full-automation cutoff, $\bar{R}(z)$ (in black). By definition, all firms to the north-west of $\bar{R}(z)$ are fully automated, while firms to the south-east of $\bar{R}(z)$ are partially automated. The black dash-dotted line denotes the maximum amount of robots that firms would want to upkeep, R_{\max}^* . The threshold, \check{z} , solves $R_{\max}^* = \bar{R}(\check{z})$. All firms above \check{z} are partially automated. The lower and the upper-left panels report the marginals of the stationary distribution along its two dimensions. The lower panel displays the marginal distribution of shocks, z , highlighting the area corresponding to the partial-automation lower bound (the mass above \check{z}). The upper left panel depicts the marginal distribution of robot stocks.

partial-automation lower bound, the minimum mass of firms that hire labor. We obtain this bound observing that firms cannot be fully automated if they receive a sufficiently large revenue shock, \check{z} . The graph depicts the determination of the threshold \check{z} as the revenue shock corresponding to the intersection of R_{\max}^* with the $\bar{R}(z)$ schedule. By definition of R_{\max}^* and $\bar{R}(z)$

$$\check{z} \equiv \left[\frac{1-\Gamma}{\delta \psi_R} \left(\frac{\Omega}{\rho + \delta} - p_R \right) \right]^{1-\theta} \frac{w}{p \theta \Gamma}.$$

Exponential Ornstein-Uhlenbeck process for the shock z . This calibration uses parameter values that are consistent with our quantitative exercise. The only exception is the parameter ψ_R for which we choose a lower value to highlight the features of the model.

Importantly, \check{z} is unaffected by the properties of the stochastic process. For any stationary distribution $F(z)$ induced by the stochastic process for z , the lower bound to the mass of partially automated firms is given by $1 - F(\check{z})$. This bound is very conservative, as the whole mass of firms that are below the $\check{R}(z)$ schedule and to the left of \check{z} are actually hiring workers and therefore *not* fully automated.

We are unable to provide comparative statics results for objects of interest like the aggregate stock of robots, or labor demand, at this level of generality, with the exception of the negative effects of higher flow wages on aggregate labor.⁷ We can only be certain of the effects of parameters of interest on R_{\max}^* , but not on the shape of the whole $R^*(z)$ schedule. Thus, we can characterize analytically the *maximum* stock of robots in the economy, but not the *average* stock of robots. However, we verify numerically that the labor demand generated by the model,

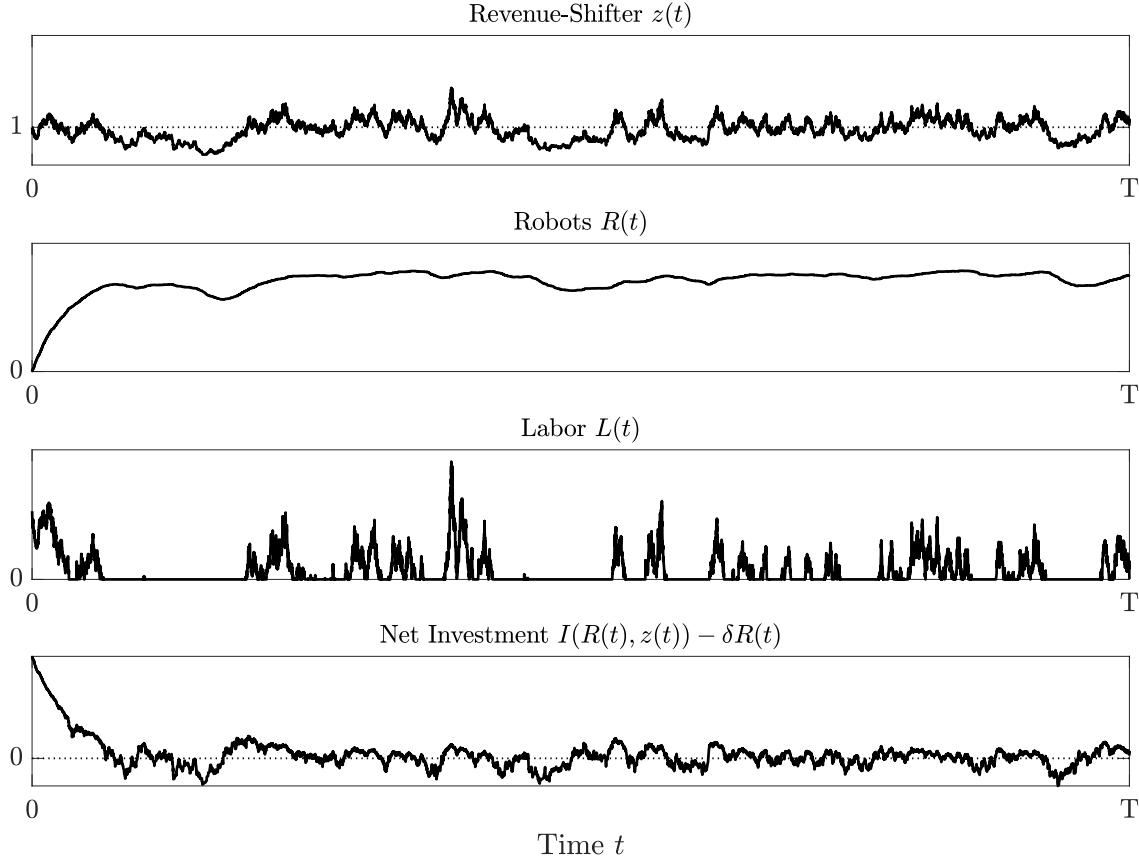
$$L^d(w) \equiv \int \left[\frac{1-\Gamma}{\Gamma} (\check{R}(z) - R) \mathbf{1}_{\{R \leq \check{R}(z)\}} \right] dG(R, z),$$

decreases with a fall in the price of robots; decreases if adjustment costs are lower; increases with a higher variance of the idiosyncratic shock; and increases with the good price.

While in the aggregate a higher robot adoption leads to lower employment, the firm-level dynamics shows that the demand for labor can increase in tandem with robot investment, and in the steady state, more productive firms install more robots. Another notable feature of firm-level dynamics is that, when automation is prevalent, labor is mostly hired in “bursts” to cope with positive shocks. These features are consistent with the evidence presented in [Acemoglu et al. \(2020\)](#); [Aghion et al. \(2020\)](#); [Bonfiglioli et al. \(2020\)](#); [Eggerton et al. \(2021\)](#); [Koch et al. \(2019\)](#), insofar as firms investing in automation technologies are found to be more productive, to employ more labor, and to pay higher wages. While for us the wages are held constant and all employees are homogeneous, it is generally true in our model that a more productive firm will both hire more labor, and install new robot capital at the same time. Figure 3 displays simulated time series for the main variables of interest, starting from a zero robot stock and a revenue shock equal to its unconditional mean. The path for the robot stock is substantially smoother than the stochastic revenue process, while labor is highly volatile in response to shocks, falling to zero when the revenue shifter is low relative to the installed robot stock. Interestingly, we find a mildly positive correlation between the robot stock and the labor series. To understand this result, consider the persistence of the stochastic process and the relative downside risk in higher states. When a large shock hits, robots are fixed, and labor is hired to meet heightened demand for the firm’s product. At the same time, the firm chooses to increase its robot stock, as shocks are persistent and will fade away only after some time. However, firms do not want to scale up their robot ownership substantially, as they are concerned that they will not be able to fully utilize them. These two facts explain the mild positive correlation that we find, and highlight a potential concern for the empirical literature trying to estimate capital-labor comple-

⁷We refer the reader to Proposition 6 in [A](#) for a formal statement.

Figure 3: Simulated Time Series



Note: This figure presents a simulated time series with 1000 simulation steps constructed as follows. At each step, we draw an arrival rate for each possible transition of the state variables in the row of the infinitesimal generator corresponding to the current state. We then choose the smallest realization (corresponding to the earliest event to occur) and sum this time increment to the cumulative time elapsed since time zero. We then update the state accordingly, and compute policy functions for investment and labor in the destination state. The time-zero state is that of a startup firm with no robots, $R_0 = 0$, and $z_0 = 1$. We adopt the calibration reported in [B.1](#).

mentarity from firm-level data. Through the lenses of our model, future low-skilled, routine jobs will be increasingly characterized by temporary employment coupled with higher turnover rates. Our model is therefore equipped to speak to the evidence that exposure to automation is associated with more “non-traditional” work arrangements, featuring a higher degree of hours volatility ([Eggleston et al., 2021](#); [Rutledge et al., 2019](#)).

3 Calibration and Quantitative Analysis

In this section, we develop a multi-sector equilibrium version of our model that we can calibrate to the data to gauge the quantitative implications of developments in robotic technology. We assess the labor market consequences of a fall in robot purchase prices and adjustment costs, or an increase in the relative productivity of robots.

[Section 3.2](#) presents a limit case of the model, and the associated analytical characterization of

the key comparative statics in equilibrium. Section 3.3 describes our calibration strategy. We find that the calibrated parameters imply reasonable values of two untargeted moments: the economy-wide adjustment costs to purchase price ratio; and the semi-elasticity of aggregate employment to robot penetration. Section 3.4 contains our main quantitative findings.

The model predicts that, *ceteris paribus*, substantial reductions in the purchase price of robots will cause only modest job losses. Taken in isolation, the effect of small reductions in adjustment costs, or small increases in relative robot productivity will be similarly negligible in most industries, with the exception of automotive. However, more sizeable changes in adjustment costs or relative productivity result in substantial employment losses.

3.1 Equilibrium with Multiple Sectors

In order to map our model to the data, we consider each task as a sector, so that parameters can be intended as sector-level aggregates and can be set to target sectoral characteristics. Accordingly, the revenues of each firm i in sector s can be written as

$$p_s Q_s(z_i, L_i, R_i, u_i) \equiv p_s z_i (\Gamma_s L_i + (1 - \Gamma_s) u_i R_i)^{\theta_s}.$$

The parameter Γ_s can be interpreted as the average relative MRTS across all the tasks that are aggregated to produce the output of sector s , which commands a price of p_s . The production technology and the stochastic process have the same parameters for all firms in the same sector, although the value of the stochastic shock varies across firms at each instant t . We choose an Exponential Ornstein–Uhlenbeck (EOU) process for the revenue-shifter,

$$dz_{it} = -\lambda_s (\log(z_{it}) - \mu_{zs}) z_{it} dt + \sigma_s z_{it} dW,$$

where W is a Wiener process. Equivalently, z_t follows an EOU process if and only if $\log(z_t)$ follows an Ornstein-Uhlenbeck process. Under this assumption, z_t admits a stationary distribution, $\text{Lognormal}(\mu_{zs} - \sigma_s^2 / 2\lambda_s, \sigma_s / \sqrt{2\lambda_s})$.⁸ We assess the robustness of our findings to the inclusion of non-stationary shocks in Section 4.2, following Reed and Jorgensen (2004).

The investment problem of the firm is unchanged, and we assume that all firms in all sectors face the same adjustment cost schedule and robot prices. While this assumption is restrictive, it is suitable for our calibration exercise as we use data for mechanical arms in all sectors, thus focusing on the same type of robot capital.

We leave robot producers outside of our general equilibrium model, so the robot price is exogenous. We do this for a number of reasons. First, we want to focus on relatively unskilled production-line employees, who are most directly threatened by automation. We believe that these workers

⁸Kondo et al. (2018) show that a lognormal approximates well the distribution of firm- and establishment-level productivity, and it is superior to the common alternative choice of a Pareto distribution.

would have a hard time reallocating to a robot-producing sector, which would likely require a higher-skilled workforce. Second, we want to look at the worst-case scenario for low-skilled workers, and we believe that this is best captured by an economy where these individuals cannot reallocate to the robot-producing sector. This scenario is consistent with our other modeling choices that exclude a built-in backstop to full automation, which would arise naturally if our workers had to be employed in the robot-producing sector. Finally, we want to conduct comparative statics with respect to a fall in the relative robot price, which can be most directly achieved by determining the robot price exogenously.⁹

In our equilibrium model, we are mainly interested in determining wages and relative prices. There is no aggregate uncertainty. Labor is homogeneous across sectors so there is a single labor market. Labor demand is the same as in the previous section, but now we have to sum over the N sectors composing our economy:

$$L^D(w, \mathbf{p}) = \sum_{s=1}^N \int_0^\infty L_s(w, p_s, R, z) dG_s(R, z).$$

Here \mathbf{p} is the vector collecting all the p_s sectoral prices, and the dependence of labor demand on w is explicit. Finally note that both the individual labor demand, L_s , and the stationary distribution, G_s , vary across sectors as a result of different stochastic processes, and different prices, which will generate sectoral heterogeneity in the relevant cutoffs \bar{R} and \hat{R} .

The equilibrium wage is determined by the intersection of labor demand and an isoelastic labor supply,

$$L^S(w) = \left(\frac{w}{\chi} \right)^\varphi,$$

where φ denotes the Frisch elasticity of labor supply. This supply schedule arises from a representative household endowed with GHH preferences over consumption and labor, where the final good is the numeraire. For simplicity, and since we are not concerned with the equilibrium interest rate, we assume that households are hand-to-mouth and own a differentiated portfolio of all the firms in the economy. As a result, they receive all the profits in the economy in addition to labor income.

Households consume a final good that is the CES aggregate of the Y_s intermediate goods,

$$Y \equiv A_F \left(\sum_{s=1}^N \xi_s Y_s^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}},$$

⁹In a previous version of our general equilibrium model, we experimented with a perfectly elastic supply of robots arising from a robot-producing sector operating a technology linear in labor, supplied by the same workers employed in the other sectors of the economy. In that version, the robot price is endogenized as the equilibrium wage over the productivity of the robot sector. Improvements in robot technology can then push up the equilibrium wage, as more workers are demanded to produce robots. This accelerates the demise of labor in manufacturing sectors, as workers quickly transition to the robot sector.

where A_F is an aggregate productivity term and η denotes the elasticity of substitution across industries. The demand for each intermediate good Y_s is then

$$Y_s^D = \left(\frac{\xi_s}{p_s} \right)^\eta \frac{Y}{A_F}.$$

The costs associated with robot maintenance, purchase and adjustment are rebated to the household. In essence, these goods and services are produced using the same aggregate of intermediates as the final good. The supply of each intermediate good is given by

$$Y_s^S = \int Q_s(z, L, R, u) dG_s(R, z).$$

An equilibrium is then a collection of prices $w, \{p_s\}$ and quantities $L, Y, \{Y_s\}$ such that the labor market and all goods markets clear. We provide a formal definition, together with the full system of equations, in C.

3.2 The Partial Automation Limit

We obtain closed-form expressions for the main aggregates and characterize some comparative statics in the special case where no firm is fully automated. This case is empirically relevant, as no firms are currently fully automated, and our calibration suggests that the data on robot penetration can only be matched by a scenario where almost all firms are partially automated. In our model, this scenario occurs when a sector has $\check{z}_s \rightarrow 0$, which implies that the lower bound to partial automation is given by $(1 - F_s(\check{z}_s)) \rightarrow 1$. In this limit case, all firms within each sector are partially automated and choose to install an amount of robots arbitrarily close to the the sector-level maximum, $R_{\max,s}^*$.¹⁰ Therefore, while not varying at the level of the individual firm, the installed stock of robots varies across sectors according to the relative MRTS parametrized by Γ_s . To understand this result, recall that \check{z} represents the lowest revenue shock that makes a firm with a stock of robots equal to $R_{\max,s}^*$ hire labor. Thus, this limit case does not feature any downside risk to installing the highest desired robot stock, as the firm will almost always utilize it fully to save on labor costs. The comparative statics are summarized in the following Proposition.

Proposition 2 (Comparative Statics in General Equilibrium). *Consider the general equilibrium model with elasticity of substitution across industries $\eta = 1$, $(1 - F_s(\check{z}_s)) \rightarrow 1$, and $\theta_s = \theta$ for all s . In the neighborhood of an equilibrium supported by prices (w, p) , aggregate equilibrium labor is increasing in m ,*

¹⁰The reader might wonder how $\check{z}_s \rightarrow 0$ without having $R_{\max,s}^* \rightarrow 0$. In the partial automation scenario,

$$\check{z}_s = \left[\frac{1 - \Gamma_s}{\Gamma_s} \frac{R_{\max,s}^*}{L_s} \int z^{\frac{1}{1-\theta_s}} dF(z) \right]^{1-\theta_s}.$$

Therefore, $\check{z}_s \rightarrow 0$ if the ratio of $R_{\max,s}^*$ to sectoral labor is low enough.

p_R and ψ_R , and aggregate robot penetration—defined as the ratio of the aggregate robot stock R to aggregate labor L —is decreasing in m , p_R and ψ_R .

Proposition 2 shows that price effects do not overturn partial equilibrium decisions by firms when p_R or ψ_R are changed. The assumption that decreasing returns are the same across all sectors allows us to exclude reallocation effects from changes in the equilibrium wage, regardless of heterogeneity in the relative productivity of robots. This also results from the Cobb-Douglas assumption for the final good aggregator, which prevents reallocation in expenditure shares.

3.3 Data, Calibration Strategy, and Untargeted Moments

Consistent with the structure of our model, a number of parameters are common to all sectors. We calibrate our model annually choosing a required rate of return of 4%, which pins down the discount rate of the firm ρ . We set p_R/w in both 2010 and 2014 to the corresponding ratio in the data. We obtain data on average annual wages for production employees in manufacturing from the Occupational Employment Statistics (OES code 51-0000) and time series data for unit robot costs from [Korus \(2019\)](#). We set the depreciation rate, δ , to target an average service life of 12 years ([International Federation of Robotics, 2017](#)). In our calibration, the flow robot cost is set to $m = 0$, so that all steady state maintenance costs are subsumed in the adjustment cost paid to replenish depreciation. We set $A_F = 1$, and the final good as the numeraire. Our baseline calibration also sets ψ_R to match the increase in aggregate robot penetration—the number of robots per thousand employees—observed between 2010 and 2014, given the observed fall in the relative price of robots. For computational feasibility, we consider both 2010 and 2014 as steady states. We explore the robustness to this assumption in Section 4.1. Data on average robot penetration for the 13 manufacturing sectors defined by the International Federation of Robotics (IFR) is from [Acemoglu and Restrepo \(2020\)](#). In our baseline calibration, we set the elasticity of labor supply to $\varphi = 1$, and choose χ to target a total aggregate labor of $L = 1$ in 2010.

The remaining parameters of the model are specific to each sector. We use NBER-CES data ([Becker et al., 2013](#)) to obtain plausible values for the scale parameters θ_s by targeting the average production workers' share of value added in each sector before 1980, after which industrial robots started being more widely employed ([Graetz and Michaels, 2018](#)). Consistent with the results on cross-industry substitution in [Oberfield and Raval \(2014\)](#), we assume that the production function is Cobb-Douglas ($\eta = 1$), which implies that ξ_s represents the share of each sector in manufacturing value added. We obtain these shares from the BEA GDP-by-industry data. The values of robot per thousand employees in 2014 provide a target for R_s/L_s in each sector of our model, which we use to calibrate the sector-specific normalized MRTS, Γ_s . We force each sector-specific process to have a mean of 1, using a procedure that we detail in [B.3](#). Under this normalization, the parameters of this process are fully determined by the diffusion parameter σ_s , and the mean-reversion parameter λ_s . We obtain estimates of these parameters fitting an Ornstein-Uhlenbeck process to

Table 1: Parameters Common to all Sectors

Description	Variable	Value	Source/Target
Discount factor	ρ	$\log(1 + 0.04)$	4% annual real interest rate
Relative robot price in 2010	$\left(\frac{p_R}{w}\right)_{2010}$	1.4348	Korus (2019) and OES
Relative robot price in 2014	$\left(\frac{p_R}{w}\right)_{2014}$	1.0209	Korus (2019) and OES
Relative flow robot cost	$\left(\frac{m}{w}\right)$	0	Negligible energy costs of robots
Robot depreciation	δ	$\log(1 + 1/12)$	International Federation of Robotics (2017)
Final good productivity	A_F	1	Final good is the numeraire
Adjustment cost parameter	ψ_R	1262.11	2010-2014 fall in value-added weighted R/L

firm-specific, revenue-based log-TFP residuals obtained from Compustat data. We first compute the residuals from the following regression at the IFR-sector-firm-year level:¹¹

$$\log(\text{Sales}_{ist}) = \gamma_i + f_{st} + \alpha_s \log(\text{Emp}_{ist}) + \beta_s \log(\text{K}_{i,s,t-1}) + \varepsilon_{ist},$$

where Emp_{ist} is firm employment, and $\text{K}_{i,s,t-1}$ is the lagged capital stock, constructed using a perpetual-inventory method. We include firm fixed effects, time-by-IFR-sector fixed effects and assume that the technological parameters α_s and β_s are the same for all firms within the same IFR sector. The estimated residual from this regression, $\hat{\varepsilon}_{ist}$ provides us with firm-specific estimates of log-TFP. Consistent with much of the literature on firm-specific productivity, these residuals can be thought of as the sum of firm-specific stochastic trends and a mean-reverting autoregressive component. Our theoretical model is concerned with the latter component, that we see as transient, unpredictable idiosyncratic shocks. We therefore isolate this component by applying the filter proposed by Hamilton (2018), firm by firm.¹² We then compute the maximum-likelihood estimators (MLE) for the parameters of the Ornstein-Uhlenbeck (OU) process, following Tang and Chen (2009). In this step, we assume that firms are sampled i.i.d. within each sector, so that we can compute the joint PDF of firm residuals as a product of individual firms' PDF's belonging to the same sector.¹³ We then transform the process to a mean-one exponential Ornstein-Uhlenbeck, using Itô's lemma to map the MLE estimators for the OU process obtained above to this context. In Section 4.2, we explore an alternative calibration featuring non-stationary shocks and firm entry and exit. Further details on our data, strategy, and numerical algorithm are reported in B.

Tables 1 and 2 report the values of the parameters together with the relevant targets. Our model requires a large value of the adjustment cost parameter ψ_R in order to match the existing robot penetration and, more importantly, the low aggregate elasticity of robot penetration to purchase prices that we observe in the data. Such low elasticity is a distinctive feature of our model, which could not arise in the absence of risk and adjustment costs. According to the calibrated model,

¹¹Details on the variable construction and the Compustat identifiers are reported in B.

¹²Hamilton (2018)'s criticism of the HP filter applies strongly to our context, where we have relatively short time series and are interested in the autoregressive properties of the filtered series.

¹³We verify that our results are essentially unaffected if we take the median or the mean of the full distribution of MLE estimators obtained by fitting the OU process to the each firm's residuals separately.

Table 2: Sector-Specific Targets and Calibrated Parameters

Sector	Targets		Calibrated Parameters				
	Robots per 1000 Employees		ξ_s	θ_s	Γ_s	σ_s	λ_s
	2010	2014					
Automotive	87.56	117.72	0.06	0.35	0.52	0.12	0.94
Electronics	10.34	13.11	0.15	0.28	0.77	0.15	0.92
Food and Beverages	4.86	6.17	0.12	0.21	0.85	0.17	0.77
Wood and Furniture	0.02	0.14	0.03	0.41	0.89	0.13	0.93
Miscellaneous Manufacturing	2.36	13.82	0.04	0.32	0.85	0.15	1.02
Basic Metals	5.40	7.17	0.03	0.38	0.87	0.13	0.99
Industrial Machinery	1.79	2.37	0.08	0.33	0.88	0.13	1.05
Metal Products	6.24	8.28	0.07	0.35	0.84	0.14	0.76
Clay Glass and Minerals	0.28	0.68	0.02	0.33	0.89	0.12	1.31
Paper and Publishing	0.01	0.11	0.05	0.28	0.89	0.10	0.79
Plastics Chemicals and Pharmaceuticals	7.95	9.90	0.27	0.18	0.78	0.18	0.79
Apparel and Textiles	0.01	0.05	0.01	0.43	0.89	0.08	0.81
Shipbuilding and Aerospace	0.15	0.54	0.07	0.32	0.89	0.14	0.83

Table 3: Semi-elasticity of Employment to Robot Penetration (Robots/1000 Employees)

	Relative Robot Price p_R	Adjustment Costs ψ_R	MRTS _{LR}
% ΔL	-0.0095	-0.0264	-0.1201
$\Delta(R/L \times 1000)$	[-0.0084, -0.0104]	[-0.0173, -0.0345]	[-0.0628, -0.1749]

Note: This table reports the value of the semi-elasticity of employment to a change in robot penetration around the 2014 steady state, for different underlying changes driving the increase in robot adoption. The intervals in brackets denote the range of semi-elasticities that is consistent with the range of calibrations used for Figures 5a and 5b.

the current data on robot penetration are matched by the partial-automation scenario described in Section 3.2, with two main implications. First, almost all firms *within each sector s* choose to install approximately the same level of robots, $R_{\max,s}^*$, which varies across sectors with the technological parameters Γ_s . Second, the stock of robots observed in the data in each sector is small relative to the full automation cutoff $\bar{R}(z)$ for most values of the revenue shock z .

Our calibration fits two untargeted moments. First, the value of the adjustment cost parameter implies that the economy-wide ratio of adjustment costs to purchase price in the 2014 steady state of the model is:

$$\frac{\sum_{s=1}^{13} \frac{\psi_R}{2} (\delta R_{\max,s}^*)^2}{\sum_{s=1}^{13} p_{R,2014} \delta R_{\max,s}^*} \approx 2.11.$$

This value is consistent with the data on spot-welding mechanical arms (Sirkin et al., 2015) reported in Figure 1, where the same ratio averages approximately 2.66 over the years 2005-2014.¹⁴ Second,

¹⁴Note that spot-welding mechanical arms are among the most widely used in the automotive sector. If we compute the same ratio for automotive only we obtain 3.28, which is around the corresponding value in the data for 2014, 3.03. We obtain adjustment costs in the data as the sum of the following components of the cost breakdown: project management; systems engineering, such as programming and installation; peripherals, such as safety barriers and sen-

our calibration compares favorably with estimates from [Acemoglu and Restrepo \(2020\)](#), who find that an additional robot per thousand employees reduces the employment to population ratio by 0.2%. Table 3 shows the corresponding semi-elasticity implied by our model, computed around the calibrated 2014 steady state. In our model, the semi-elasticity of employment to robot penetration depends on which underlying change causes a higher robot adoption in the first place. In particular, we focus on small changes occurring across all sectors in the three quantities, p_R , ψ_R , and $MRTS_{LR}$. For each of these scenarios, we compute the implied employment losses and increases in robots per thousand employees, which we use to construct the reported semi-elasticities. One more robot per thousand employees is associated with an employment fall between 0.0095%—if caused by a decline in p_R —and 0.1201%—if caused by a decline in $MRTS_{LR}$, the number of robotic arms needed to replace one worker. The brackets below each figure report the semi-elasticities that obtain if we calibrate the model assuming that actual robot prices have fallen between 50% and 150% of the value employed in our baseline, as detailed in the following section.

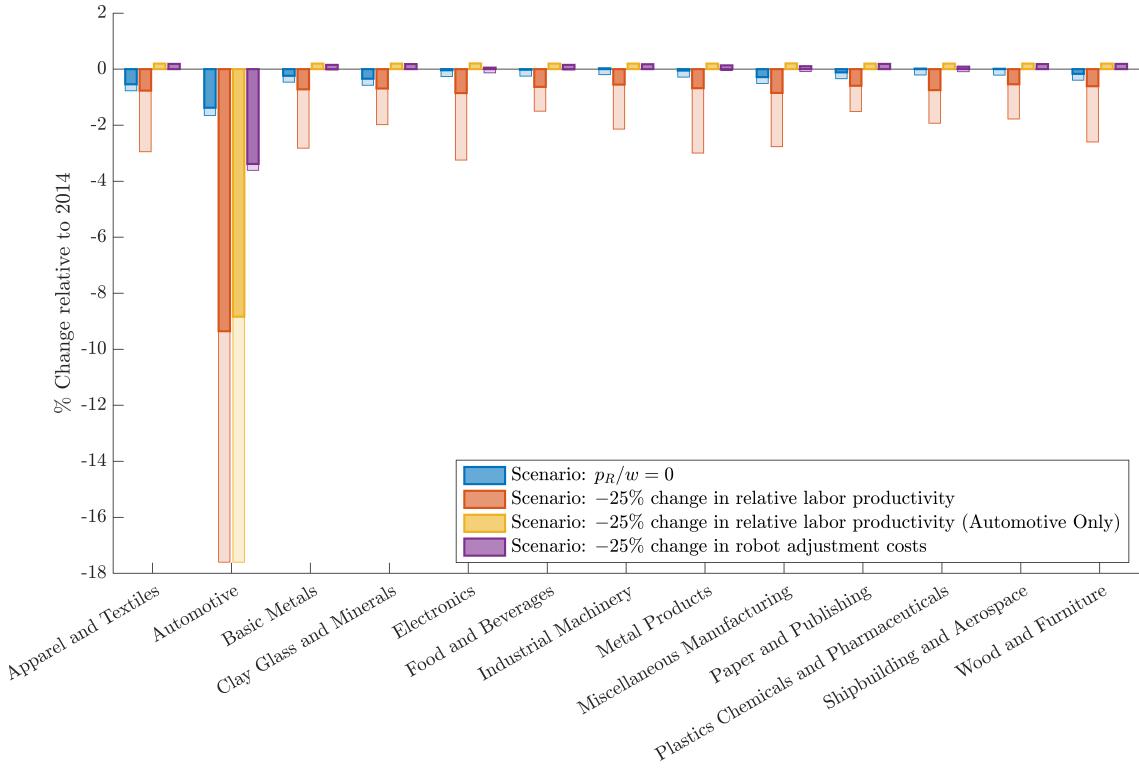
3.4 Quantitative Results

In Figure 4, we quantify the labor market consequences of four scenarios, relative to the calibrated 2014 equilibrium. First, we focus on a reduction of the purchase price of robots all the way down to $p_R = 0$. Second, we analyze a fall of 25% in the $MRTS_{LR}$, i.e., an increase in relative robot productivity across all sectors. Third, we examine a fall of 25% in the $MRTS_{LR}$ limited to the automotive sector. Finally, we consider a fall in adjustment costs, and thus in steady-state maintenance costs, of 25%. *Ceteris paribus*, a robot price reduction yields limited employment effects in the calibrated model. This result is hardly surprising, since the value of ψ_R that we find is high relative to the equilibrium wage, suggesting that robots are in fact relatively rigid. Consistent with the intuitions in our model, the only way to generate a high robot penetration is to have a low adjustment cost parameter and/or a low $MRTS_{LR}$, which signals that less robots are needed to replace one worker. For instance, the higher penetration of robots in the automotive sector is interpreted by our model as a low $MRTS_{LR}$. Figure 4 also shows how labor is threatened by technological improvements that render robots relatively more productive, or that reduce their installation and resale costs. While we present the results for changes of p_R , ψ_R , $MRTS_{LR}$ in isolation, it is crucial to note that changes in one variable have knock-on effects on the other variables. In particular, a fall in ψ_R or $MRTS_{LR}$ increases the elasticity of aggregate labor to the relative price of robots.

Figure 4 allows a decomposition of employment effects into partial and general equilibrium effects. The full length of the bars shows the partial equilibrium effect, while the dark segment highlights total employment effects in general equilibrium. As a result, the lighter portion of the bars represents the attenuation effect that comes from accounting for general equilibrium price feed-

sors. All of these components are akin to the adjustment costs in our model, as they generally are unrecoverable and firm-specific.

Figure 4: Total Sectoral Employment



Note: This figure presents comparative statics results for the percentage change in sectoral employment, starting from the 2014 steady state of the calibrated multi-sector model. For each sector, the bars report comparative statics for four scenarios. From left to right, we consider a fall in the relative price of robots to 0, a 25% fall in relative labor productivity in each sector, a 25% fall in relative labor productivity in automotive only, and a 25% reduction in adjustment costs. The full length of the bars gives the partial-equilibrium effect, obtained holding the wage and relative prices fixed, the darker portion of each bar highlights total employment effects in general equilibrium, where prices and wages are allowed to adjust.

back. A finite labor supply elasticity occasions a fall in the wage that dampens the adverse effects on labor demand. This attenuation effect is steepest in the case of a fall in $MRTS_{LR}$; the total general-equilibrium effect is half of its partial-equilibrium counterpart. Further, there is a reallocation effect at play, which depends on the suitability of various sectors to automation. This is evident from comparing the automotive sector—currently featuring the highest relative robot productivity—to other sectors. For example, a fall in ψ_R reduces employment in automotive, while it leads to such a large fall in the equilibrium wage that other sectors hire more workers. Our general equilibrium framework also allows us to investigate the consequences of technological advances in a single sector. To this end, Figure 4 also displays the employment effect of a 25% fall in $MRTS_{LR}$ in the automotive industry. Relative price and wage effects help labor survive. While there is still a substantial fall in the number of workers employed in the automotive sector, the reduced labor demand coming from this sector puts downward pressure on the real equilibrium wage, leading the other sectors to expand their employment.

Figure 5 displays the aggregate employment and labor share consequences of smoothly varying

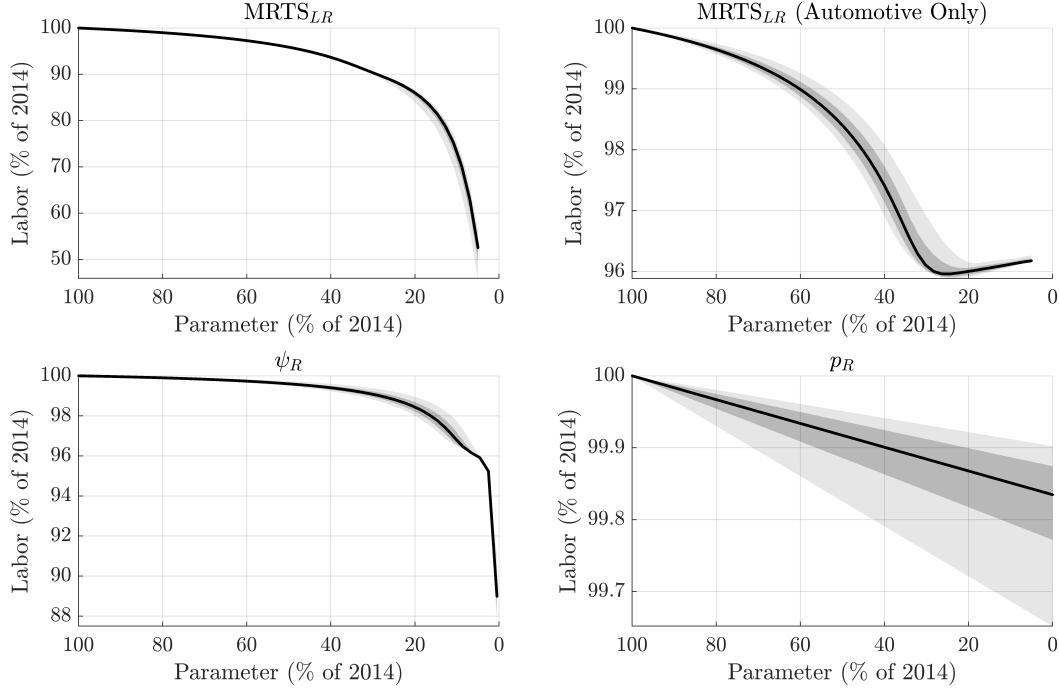
the parameters of interest one at a time. We also report bands around our baseline calibration, constructed to account for the uncertainty surrounding robot price data. Specifically, we re-calibrate our model assuming that the actual fall in relative robot prices is 50%, 75%, 125%, 150% of the value that we employ in the baseline. This gives a range of values for ψ_R and Γ_s consistent with observed increases in robot penetration. In each figure, the dark (light) gray bands display the comparative statics results when the percentage change in relative robot prices ranges between 75% (50%) and 125% (150%) of the baseline value. Lower changes in the robot price imply a lower estimated ψ_R , so that the lower bound of these bands considers the scenario where the robot price fall is 75% or 50% of the baseline, while the upper bound is given by the cases where we assume that the change in robot prices has been larger than the data we use for the baseline.

Turning to our results, Figure 5a shows that the employment effects are modest. The potential fall in $MRTS_{LR}$ across all sectors poses the most relevant threat to the survival of labor. Nevertheless, a 70% reduction in relative labor productivity would cause a relatively modest 20% reduction in aggregate manufacturing employment. As a comparison, a 70% fall in ψ_R only causes about a 1% fall in aggregate employment, and an equivalent reduction in p_R reduces total labor by a mere 0.11%.¹⁵ Once again, there is substantial attenuation coming from general equilibrium price effects. The labor share consequences, reported in Figure 5b, are similar.

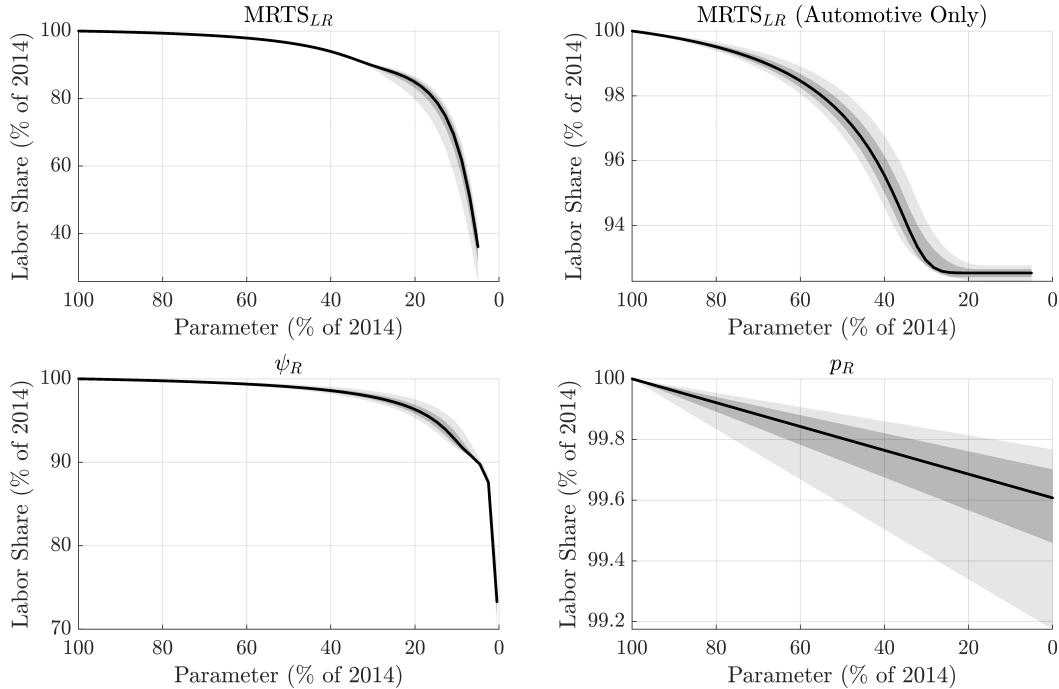
¹⁵The reader might wonder why the bands tighten around some values of the parameters (e.g. when ψ_R is 5% of the benchmark). We investigated this issue and concluded that automotive disproportionately drives the employment effect before these points, after which it becomes essentially fully automated and thus irrelevant for the labor response. After this point, the other sectors jointly drive the comparative statics.

Figure 5: Comparative Statics on the Calibrated Model

(a) Aggregate Employment



(b) Labor Share



Note: This figure presents comparative statics results for the percentage change in aggregate employment (panel (a)) and the labor share (panel (b)), relative to the 2014 steady state of the multi-sector model. Solid lines report general equilibrium effects. Shaded areas report the effects that obtain re-calibrating the model, assuming percentage robot price changes over 2010–2014 between 75–125% of our baseline (darker shades), or between 50–150% of our baseline (lighter shades).

4 Robustness and Extensions

In this section, we consider two alternative calibration strategies and two extensions to the theoretical model presented in Section 2. We first present a calibration exercise where manufacturing is considered as a single sector, and the change in robot penetration occurs along a perfect-foresight transition to lower robot prices. Second, we consider a version of our calibration featuring non-stationary shocks and firm entry and exit. Both these robustness exercises produce quantitative results that are largely in line with our baseline. Our first extension introduces labor market rigidity by means of convex adjustment costs. An illustrative numerical simulation shows that long-run employment decreases when hiring and firing costs are increased. In the second extension, we show that our theoretical findings are robust to both linear and fixed adjustment costs specifications.

4.1 Single-Sector Calibration along a Transition to Lower Robot Prices

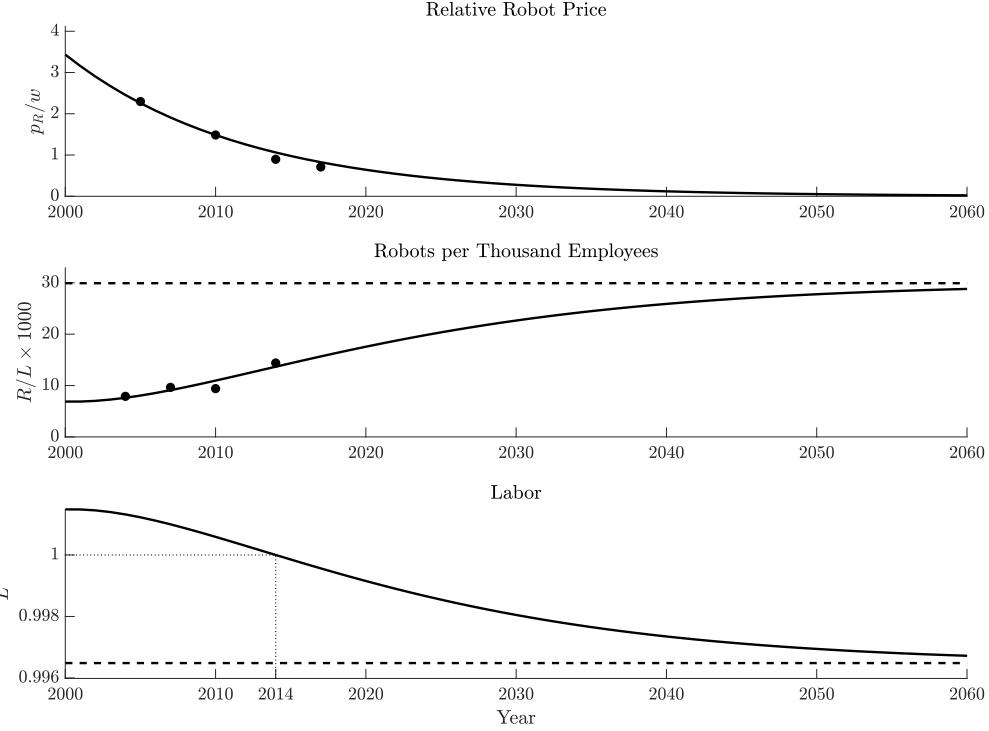
In our baseline setting, we considered 2010 and 2014 as steady states, which might result in too high a value for ψ_R , and lead to an underestimate of the employment effects of future developments in robotics. Indeed, the change in robot penetration over the period that we observe in the data could just reflect a partial adjustment to lower robot prices. In this section, we consider manufacturing as a single sector, and the increase in robot penetration between 2010 and 2014 as occurring along an equilibrium transition towards a future steady state with a robot price of 0. This alternative calibration delivers a lower value for the adjustment cost parameter, and it features larger employment responses to long-run changes in the parameters of the model.

We set the initial steady state to the year 2000, in order for it to be in the time interval where we have robot price data (1995-2017), and before the first year for which we have robot penetration data, 2004. The reason why we choose a single sector is that this structure allows us to compute the transition in labor market equilibrium, using an algorithm based on [Guerrieri and Lorenzoni \(2017\)](#).¹⁶ Since we wish to calibrate the model to represent the whole of manufacturing, we set the parameters, θ , λ , and σ through the same estimation procedures detailed in Section 3.3, lumping all the firms together and ignoring their affiliation with specific IFR sectors. This procedure results in a decreasing returns to scale parameter, $\theta = 0.298$, and EOU parameters, $\lambda = 0.879$, $\sigma = 0.141$, which fall in the range of parameters for the multi-sector model reported in Table 2. In order to characterize the transition, we need a path of relative robot prices, which we obtain by fitting an exponential function to the series of observed relative robot prices. The resulting path is depicted in the upper panel of Figure 6, where dots mark observed data points. We then seek values of Γ and ψ_R to minimize the distance between the model-implied path of robots per thousand employees

¹⁶The algorithm consists in guessing a wage path for the transition between two steady states, solving the HJB equation backwards and iterating the KFE forward using the guessed price path. This returns excess labor demands for each period, which are used to update the wage path. The procedure is repeated until excess labor demands are sufficiently close to 0 in each period.

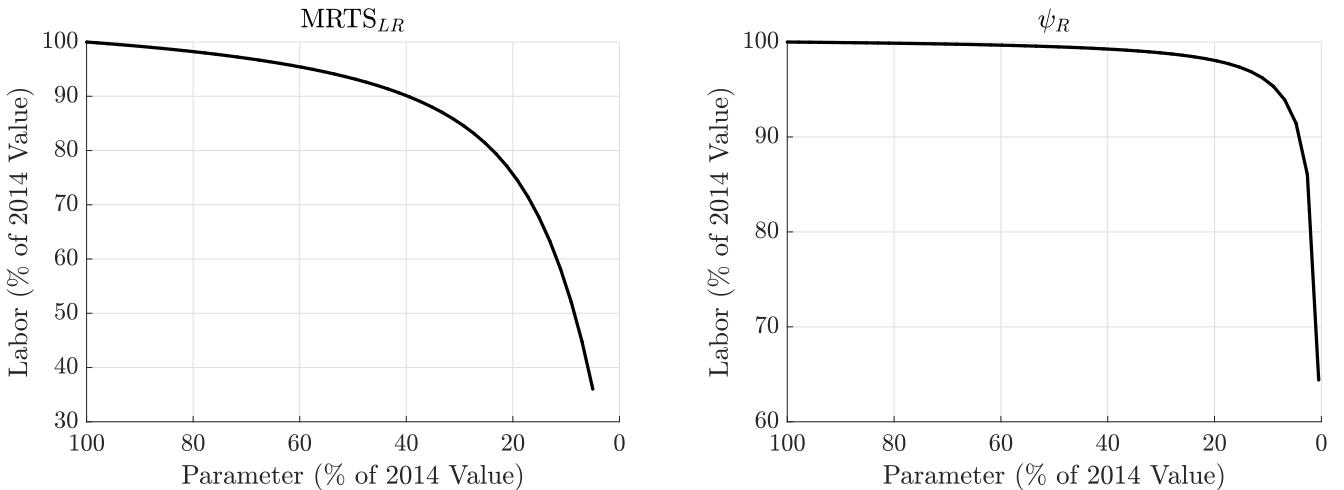
and the observed data points for the years 2004, 2007, 2010, 2014. This procedure can be thought of as an impulse-response matching exercise targeting the perfect-foresight response to a future path of robot prices revealed at the 2000 steady state. We obtain a value of $\Gamma = 0.653$, and $\psi_R = 499.79$, and report the calibrated path of robot penetration in the middle panel of Figure 6. The bottom panel of Figure 6 shows the model-implied employment path. The long-run employment losses generated by the fall in the robot price are about 0.35%, which compare with approximately 0.15% we obtained for the multi-sector model. Figure 7 displays the results of comparative statics exercises analogous to Figure 5a in the multi-sector model. Here, we fix the relative price of robots to the value in 2014 in the single-sector model and vary the other two parameters of interest, the MRTS between robots and workers, and the adjustment cost parameter, ψ_R . The results are closely aligned to the multi-sector model. Qualitatively, we still observe that, for a given percentage reduction, a fall in the MRTS causes larger employment losses than a fall in ψ_R . Due to a lower calibrated value of ψ_R , and a value of Γ that falls in the lower range of those estimated for the multi-sector model, the employment losses implied by percentage reductions in these parameters are larger than their multi-sector counterparts. In the one-sector calibration, an 80% fall in the MRTS causes about a 25% decrease in labor, while in the multi-sector calibration this value is about 15%. A similar reduction in ψ_R in the one-sector model causes about a 2.5% reduction in employment, compared with about 1% in the multi-sector model.

Figure 6: Transition to $p_R = 0$



Note: This figure presents the calibrated time paths of the relative price of robots (upper panel), robots per thousand employees (middle panel), and labor relative to the 2014 level, along a perfect-foresight transition to a 0 relative robot price in 2100, announced in year 2000. Solid lines depict the time paths of the three variables, with black dots denoting the data points that inform our calibration. The dashed lines denote the asymptotic values of the quantities of interest. Relative robot prices come from [Korus \(2019\)](#) and the OES series for production-line workers real wages in manufacturing. The data on robot counts per thousand employees are from [Acemoglu and Restrepo \(2020\)](#).

Figure 7: Comparative Statics for the Single-Sector Calibration Exercise



Note: This figure presents the general-equilibrium comparative statics results for the representative-sector model calibration. These results compare with the solid lines in the left panels in Figure 5a for the multi-sector model.

4.2 Calibration with Non-Stationary Shocks

In this section, we assess the robustness of our quantitative findings to the inclusion of non-stationary shocks. In previous sections, we focus on stationary shocks, because we wish to capture the volatility arising from short-run risk, and not uncertainty about the growth prospects of individual firms. Nevertheless, it is interesting to explore whether our mechanism extends to more persistent sources of risk. In order to obtain a stationary distribution, we need to model firm entry and exit as detailed below.

We assume that the stochastic process for the revenue-shock z for a surviving firm in sector s follows a Geometric Brownian Motion (GBM):

$$dz_{it} = \mu_s z_{it} dt + \sigma_s z_{it} dW.$$

We further assume that the process is killed at a constant exogenous exponential rate v_s . Therefore, firms' lifespans T in each sector s are distributed as follows:

$$T \sim f_{T,s}(x) = v_s \exp\{-v_s x\}.$$

We assume that there is a constant measure of firms. When a firm in sector s is eliminated, another immediately enters, drawing a value of the revenue shock z_0 from a log-normal distribution:

$$\log z_0 \sim \mathcal{N}(\mu_s^{\text{reset}}, \sigma_s^{\text{reset}}).$$

This process gives rise to a double Pareto-lognormal distribution for the revenue-shock (Reed and Jorgensen, 2004). This distribution is particularly suitable to fit size distributions, as both tails follow a power law.

To calibrate the stochastic process, we use the raw TFP residuals obtained in Section 3.3, since now we do not seek to make them stationary. We then choose the parameters of the process for each sector to match the empirical kernel density of residuals for firms with more than 10 observations, and the average overall firm lifespan.¹⁷ In particular, we set the reset parameter v_s to the reciprocal of the average lifespan in each sector, which produces a cross-sector average of 0.1, consistent with Clementi and Palazzo (2016). We then look for the parameters $\mu_s, \sigma_s, \mu_s^{\text{reset}}, \sigma_s^{\text{reset}}$ to match the estimated kernel density of TFP residuals. B.5 reports further details on the procedure, as well as raw and calibrated TFP distributions. The calibrated Γ_s , as well as the parameters of the shock process are listed in Table 4. We calibrate a value $\psi_R = 1894.16$, which is about 50% higher than the main calibration exercise. This is due to the fatter tails of the shock processes, which, by increasing the probability that firms get a very high shock, makes investment in robots more profitable. To match the observed increase in robot penetration, we then require a higher value for the adjustment cost

¹⁷We drop firms with less than ten observations to avoid excessive noise from imprecise TFP estimates.

Table 4: Calibrated Parameters in the Non-Stationary Shock Case

Sector	Γ_s	μ_s	σ_s	σ_s^{reset}	μ_s^{reset}	v_s
Automotive	0.2336	-0.0000	0.0000	-0.0193	0.1372	0.1028
Electronics	0.4677	0.0009	0.0015	-0.0415	0.1838	0.1173
Food and Beverages	0.6694	0.0039	0.0613	-0.0364	0.0461	0.1060
Wood and Furniture	0.8576	0.0010	0.0452	-0.0192	0.0802	0.1078
Miscellaneous Manufacturing	0.6416	0.0010	0.0010	-0.0460	0.1864	0.1248
Basic Metals	0.8120	0.0021	0.0469	-0.0225	0.0592	0.1102
Industrial Machinery	0.8264	0.0009	0.0533	-0.0176	0.0745	0.0992
Metal Products	0.6341	0.0038	0.0510	-0.0342	0.0600	0.1099
Clay Glass and Minerals	0.8586	0.0015	0.0387	-0.0165	0.0749	0.1013
Paper and Publishing	0.8592	0.0004	0.0000	-0.0083	0.1197	0.1043
Plastics Chemicals and Pharmaceuticals	0.4822	0.0265	0.1366	-0.2447	0.0374	0.1190
Apparel and Textiles	0.8693	0.0010	0.0366	-0.0160	0.0440	0.0875
Shipbuilding and Aerospace	0.8565	0.0005	0.0000	-0.0156	0.1434	0.0976

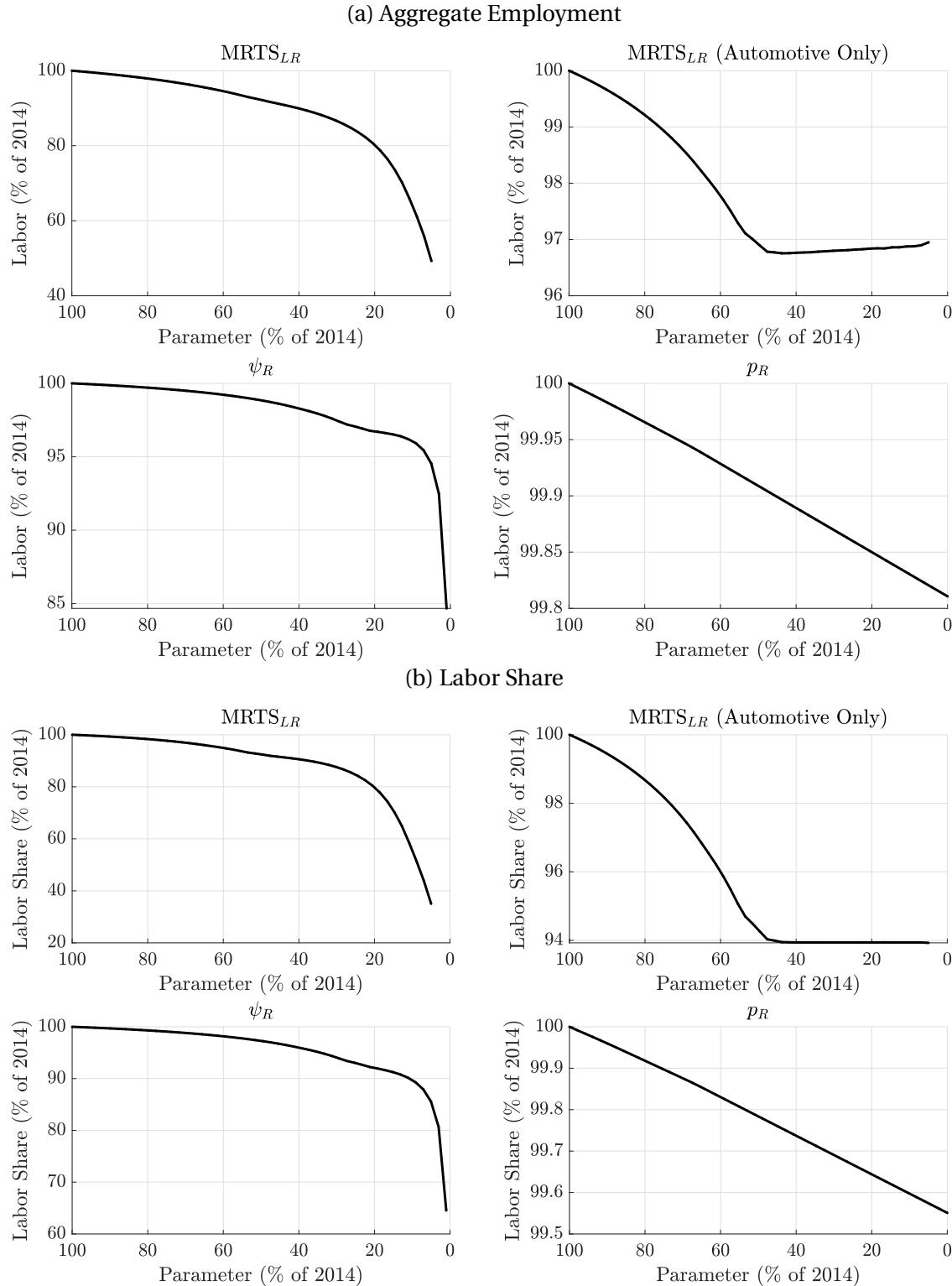
parameter.

The procedure to simulate the model is slightly modified relative to our baseline in two ways. First, the firms' discount factor is now augmented by the sector-specific death rate v_s . Second, we have to specify how new entrants choose their capital stocks. For simplicity, we make each of the new entrants select the value $R = R^*(z_0)$, depending on the productivity draw. Note that, for any given level of ψ_R , this value is higher than what entrants would choose if we let them maximize:

$$V(R, z_0) - p_R R - \frac{\psi_R}{2}(R^2).$$

This tends to increase average robot penetration in the model, representing a worst-case scenario for workers. In numerical simulations, we therefore first solve the survivors' problem, and obtain their transition matrix, to which we add the flows related to firm entry and exit. We present continuous comparative statics in Figure 8. Similarly to Section 4.1, we find that the employment losses are somewhat larger than our baseline calibration.

Figure 8: Comparative Statics for the Calibrated Model with Non-Stationary Shocks



Note: This figure presents comparative statics results for the percentage change in aggregate employment (panel (a)) and the labor share (panel (b)), relative to the 2014 steady state of the multi-sector non-stationary model with the parameters listed in Table 4.

4.3 Extension with Labor Adjustment Costs

Until now, we have assumed that labor can adjust instantly in response to shocks. In this section, we introduce quadratic costs on labor. We verify numerically that this model produces approximately the same results as our baseline model when labor adjustment costs are very close to zero. We show that the severity of labor frictions—as captured by the ratio of labor to capital adjustment costs—is a key variable in determining the adverse effects of automation on employment: the higher the relative adjustment costs, the lower the comparative advantage of labor in responding to shocks, and the higher the disemployment effects of improvements in automation technology.

In what follows, we choose quadratic costs on labor (Sargent, 1978) for two reasons. First, using the same type of adjustment costs for capital and labor allows us to assess the relative rigidity of the two factors in a direct way. Second, this type of cost can be easily managed by our solution method.¹⁸ The firm's problem yields the recursive formulation

$$\begin{aligned} \rho V(R, L, z) = & \max_{I_R, I_L} \Pi(R, L, z) - p_R I_R - \frac{\psi_R}{2} (I_R)^2 + (I_R - \delta R) V_R(R, L, z) - \frac{\psi_L}{2} (I_L)^2 + \\ & + (I_L - s L) V_L(R, L, z) + \mu(z) V_z(R, L, z) + \frac{\sigma^2(z)}{2} V_{zz}(R, L, z), \end{aligned}$$

where ψ_L parametrizes labor adjustment costs and s is an exogenous separation rate. We augment the profit function to allow for under-utilization of labor:

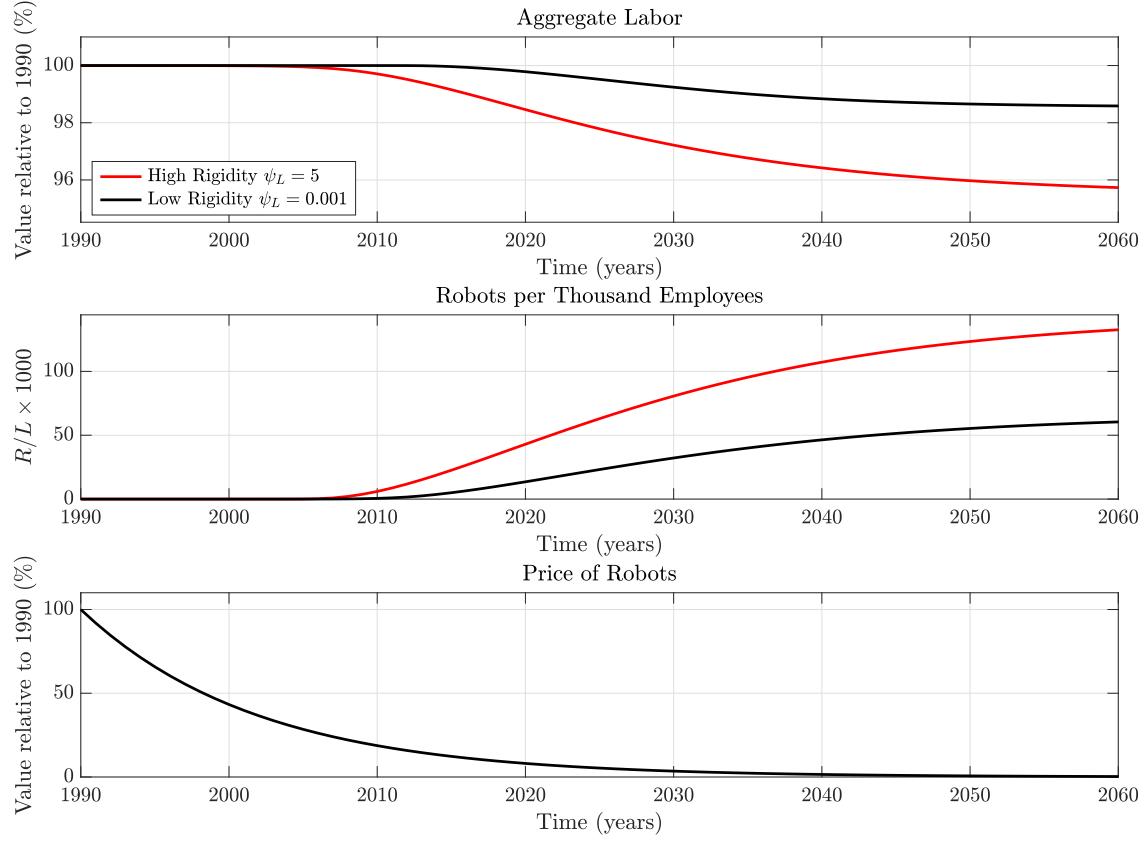
$$\Pi(R, L, z) = \max_{0 \leq u_R, u_L \leq 1} p z (\Gamma u_L L + (1 - \Gamma) u_R R)^\theta - m u_R R - w L.$$

Note that we do not allow the firm to pay workers in proportion to the utilization rate, in order to better capture labor market rigidities. Accordingly, the only way to avoid paying workers is to fire them. The solution entails full utilization of labor, regardless of the level of robots. By contrast, robots are turned off whenever full utilization would bring the firm above the desired scale $\hat{R}(z)$. We perform a simple exercise where we fix the wage rate—a worst-case scenario for workers—and simulate a perfect-foresight transition to a 0 price of robots. This setting is analogous to Section 4.1, except that we do not allow wages to adjust. We therefore fix all common parameters at the value of the initial steady state in that scenario, and simulate the model with the same calibrated price path. We choose a value of the separation rate $s = 0.313$, obtained from JOLTS data for 2019.¹⁹ In Figure 9, we consider two alternative scenarios for the labor adjustment cost parameter ψ_L . In a first scenario, we set $\psi_L = 0.001$, and obtain quantitative results that are in line with the flexible-labor case in Section 4.1. Clearly, since wages are fixed, employment effects are larger. Second, we

¹⁸Note that the additional state variable imposes a non-negligible computational burden on the algorithm, as now there are two endogenous state variables and the exogenous forcing term. This means that we need to be more parsimonious with the choice of approximating grids for the variables in our numerical scheme. We present this model with illustrative intent, as calibrating it to match the data points as in Section 3 would be infeasible.

¹⁹See <https://www.bls.gov/news.release/jolts.t16.htm>.

Figure 9: Transition to Steady State with $p_R = 0$



Note: This figure presents the transition paths of aggregate labor, robot penetration and the price of robots in the model with rigid labor and the calibration described in Section 4.1.

consider a higher value of $\psi_L = 5$, which is about 1% of the robot adjustment cost parameter.²⁰

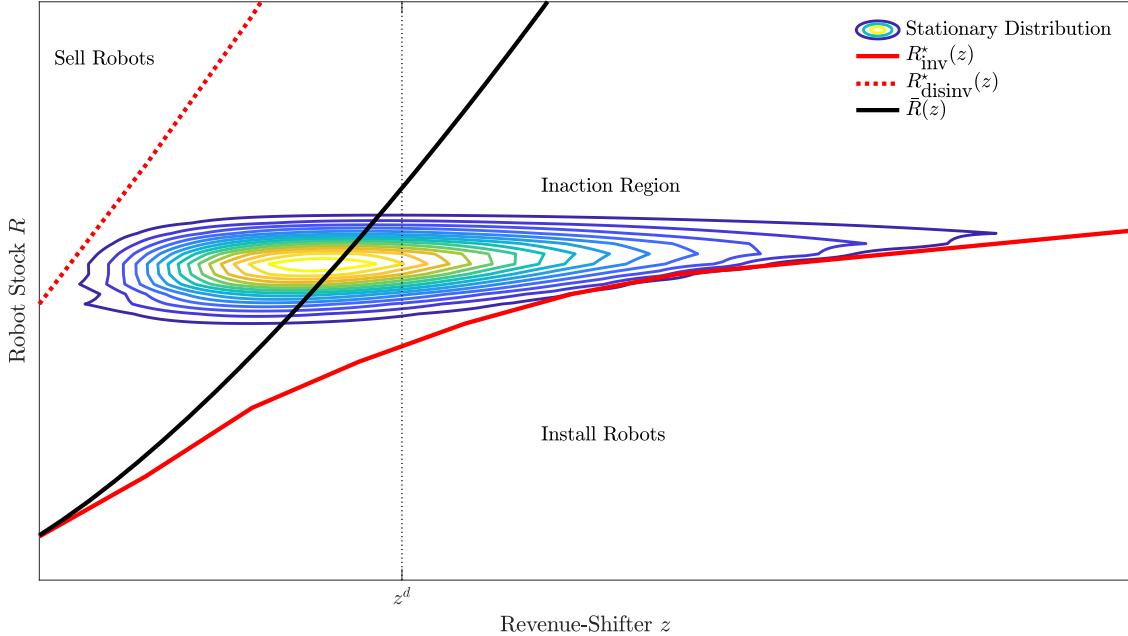
The economy with more sizeable labor market frictions starts adopting robots before its low-friction counterpart. This can speak to the evidence in [Acemoglu and Restrepo \(2018a\)](#) that countries with higher unionization rates also feature higher robot adoption. In the long run of this simple model, labor protections eliminate workers' distinctive comparative advantage. This effect could be magnified in the real world, as labor-substituting innovations respond to the incentives provided by a rigid labor market.

4.4 Robustness to Alternative Adjustment Costs Specifications

In this section, we explore the robustness of our findings to alternative adjustment costs specifications. We focus on linear adjustment costs, following [Bentolila and Bertola \(1990\)](#). Linear costs are interesting because they generate an inaction region, similar to fixed costs of adjustment ([Stokey, 2009](#)). However, linear costs are closer to the evidence provided in [Conway \(2014\)](#), which highlights unit installation costs. We describe the solution algorithm we use for our numerical results in detail

²⁰Note that steady-state aggregate labor tends to 0 as the ratio ψ_L/ψ_R increases.

Figure 10: Stationary Distribution for the Linear Adjustment Costs Model



Note: This figure presents the stationary distribution for an illustrative calibration the linear-cost model. Contour lines denote the stationary distribution. The red lines depict the investment (solid line) and disinvestment (dotted) thresholds. The inaction region is the area between these two thresholds. The black line denotes the full automation cutoff. All firms to the south-east of this locus are partially automated.

in D. The firm's problem is now

$$\rho V(R, z) = \max_{I \in \mathbb{R}} \quad \Pi(R, z) - \mathbf{1}\{I > 0\}(\psi_+ + p_R)I - \mathbf{1}\{I < 0\}(-\psi_- + p_R)I + \\ + (I - \delta R)V_R(R, z) + \mu(z)V_z(R, z) + \frac{\sigma^2(z)}{2}V_{zz}(R, z),$$

where ψ_+ and ψ_- denote linear adjustment costs for positive and negative investment. The solution implies that, for each z , the firm will be inactive for $R \in [R_{\text{inv}}^*(z), R_{\text{disinv}}^*(z)]$, where

$$V_R(R_{\text{inv}}^*(z), z) = p_R + \psi_+ \quad \text{and} \quad V_R(R_{\text{disinv}}^*(z), z) = p_R - \psi_-,$$

and the firm jumps to the investment/disinvestment cutoffs, $R_{\text{inv}}^*(z)$ and $R_{\text{disinv}}^*(z)$, if the revenue shock is such that the installed capital stock is outside of the inaction region.

This solution immediately implies that the stationary distribution features positive density only inside the inaction region. Therefore, we can numerically verify a result that is analogous to Proposition 1. Given the other parameters and for any p_R , there exists a $\psi_+ > 0$ such that the stationary distribution does not feature full automation. The increase in ψ_+ shifts down the investment barrier $R_{\text{inv}}^*(z)$ for all z , while $\bar{R}(z)$, the full automation threshold, is invariant to adjustment costs. As a result, it is possible to find a ψ_+ large enough that the investment barrier falls below $\bar{R}(z)$ for all z in a non-zero measure set of the support of $F(z)$. In Figure 10, we show an example of this instance,

where part of the stationary distribution falls below the $\bar{R}(z)$ schedule.

We sketch the solution of a fixed-cost variant of the model to show that it produces results that are similar to the linear specification we have analyzed thus far. Namely, a fixed cost model would also entail a desired level of capital to which firms want to adjust. Conditional on adjusting, a firm with a revenue shock z would like to set the robot stock to $R_{\text{adj}}^*(z)$, given by

$$V_R(R_{\text{adj}}^*(z), z) = p_R.$$

Given (R, z) , a firm that does not adjust gets the value $V(R, z)$, defined recursively as

$$\rho V(R, z) = \Pi(R, z) - \delta R V_R(R, z) + \mu(z) V_z(R, z) + \frac{\sigma^2(z)}{2} V_{zz}(R, z).$$

It follows that, given a fixed cost of adjustment F , a firm chooses to adjust if and only if

$$V(R_{\text{adj}}^*(z), z) \geq V(R, z) + p_R(R_{\text{adj}}^*(z) - R) + F.$$

Given the properties of the value function, the right hand side of the above expression is concave in R , so in general there exist two robot stock cutoffs, defined as functions of z . Analogous to the case of linear adjustment costs, these functions provide the bounds for an inaction region where firms let installed robot stocks depreciate. The condition above clearly shows that a sufficiently high F will push the lower bound of the inaction region towards $R = 0$ —and therefore below $\bar{R}(z)$ —for any z . As a result, a positive mass of firms will inevitably end up partially automated.

5 Conclusions and Future Work

We have shown that when robot capital involves substantial rigidity, and firms operate in a risky environment, labor can survive substantial innovations in automation if it is relatively more flexible. This occurs even if there is perfect substitution between factors, and even if robots have a flow cost advantage. However, this comes at a cost: job stability. As the simulated time paths in Figure 3 show, labor is mostly hired in short-lived bursts. Moreover, our calibrated model has shown that the main threat to labor comes from significant improvements in robot productivity or reductions in robot reallocation frictions. By contrast, we do not envision an important role for falling list prices in generating employment losses.

Our main insight on human labor flexibility has important policy consequences. Labor can survive in the absence of barriers to human-robot substitution, and it can do so by being more flexible than robots. If this source of advantage is taken away—for example by overly rigid regulations—human labor may be eliminated even in the presence of revenue risk and adjustment costs for robots.

While we do not develop this point formally, our framework can speak to the supply-side consequences of a host of robot taxes that have been recently proposed. Our calibration suggests that a tax on robot purchases, or one tied to ownership or utilization of robots, might not be enough to significantly alter the incentives for automation if robots become more productive and less costly to install. However, our results suggest that reducing the wage bill for firms, for instance by limiting social security contributions or income taxes on low-income individuals, might go a long way towards safeguarding unskilled and routine jobs.

Flexibility in performing tasks might be an inherently human ability that machines may never be able to reproduce. Our model can be easily extended to incorporate an *intensive* dimension of volatility, given by the stochasticity arising from the complexity of the environment in which robots have to operate. The simplest way to study this dimension in our framework is to focus on a robot-specific productivity shock, and move the scope of our analysis from tasks to jobs. In this setting, each job is an aggregate of tasks, which workers are asked to carry out stochastically. This could shed light on which jobs and sectors are inherently human, and which types of technical innovations could be most threatening for labor.

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A Theory Appendix

A.1 Omitted Proofs for Propositions Reported in the Main Text

Proposition 1 (Conditions for Long-Run Partial Automation). Consider a non-degenerate diffusion process or Continuous Time Markov Chain for z admitting a stationary distribution on \mathcal{Z} such that $z_1 \equiv \inf \mathcal{Z}$ and $z_N \equiv \sup \mathcal{Z}$, $z_1 \geq 0$. Define $G(\tilde{R}, \tilde{z}, t) \equiv \mathbb{P}\{R(t) \leq \tilde{R}, z(t) \leq \tilde{z}\}$, the CDF of the distribution of firms at time t , and $g(\tilde{R}, \tilde{z}, t)$ the associated PDF. For any purchase price of robots p_R , as long as $\delta > 0$, there exist a finite value of the adjustment cost parameter ψ_R such that the stationary distribution $G(R, z, \infty)$ does not feature full automation. If $z_N = \infty$, any $\psi_R > 0$ implies that the stationary distribution does not feature full automation.

Proof. If $\delta > 0$, the stationary distribution has support bounded from above by R_{\max}^* (see Proposition 5). To prove the statement, we need to show that there exists a finite value of ψ_R such that $\bar{R}(z_N) > R_{\max}^*$. Since $\bar{R}(z)$ is increasing in z , this amounts to setting ψ_R such that $\check{z} < z_N$. If the process has unbounded support, the statement is trivial as $\check{z} < \infty$ for any $\psi_R > 0$. Otherwise, using the definition of \check{z} , we can set

$$\psi_R > \left(\frac{w}{p\theta\Gamma z_N} \right)^{\frac{1}{1-\theta}} \left[\frac{1-\Gamma}{\delta} \left(\frac{\Omega}{\rho+\delta} - p_R \right) \right].$$

□

Proposition 2 (Comparative Statics in General Equilibrium). Consider the general equilibrium model with elasticity of substitution across industries $\eta = 1$, $(1 - F_s(\check{z}_s)) \rightarrow 1$, and $\theta_s = \theta$ for all s . In the neighborhood of an equilibrium supported by prices (w, \mathbf{p}) , aggregate equilibrium labor is increasing in m , p_R and ψ_R , and aggregate robot penetration—defined as the ratio of the aggregate robot stock R to aggregate labor L —is decreasing in m , p_R and ψ_R .

Proof. We define an equilibrium as in C.3. Under the assumption $\eta = 1$, the system of equations pinning down the equilibrium prices, (w, \mathbf{p}) reads,

$$LM(w, \mathbf{p}; \boldsymbol{\theta}) \equiv \sum_{s=1}^N \left\{ E_s \left(\frac{p_s \theta_s \Gamma_s^{\theta_s}}{w} \right)^{\frac{1}{1-\theta_s}} - \frac{1-\Gamma_s}{\Gamma_s} R_{\max,s}^* \right\} - \left(\frac{w}{\chi} \right)^\varphi = 0,$$

$$IM_s(w, \mathbf{p}; \boldsymbol{\theta}) \equiv \left[E_s \left(\frac{p_s \theta_s \Gamma_s}{w} \right)^{\frac{\theta_s}{1-\theta_s}} \right] - \xi_s \frac{\sum_j p_j^{\frac{1}{1-\theta_s}} E_j \left(\frac{\theta_j \Gamma_j}{w} \right)^{\frac{\theta_j}{1-\theta_j}}}{p_s} = 0 \quad \forall s = 1, \dots, N,$$

where the symbol $\boldsymbol{\theta}$ represents a generic parameter affecting the equilibrium on which we wish to perform the comparative statics. For ease of notation, we have defined

$$E_s \equiv \int_0^\infty z^{\frac{1}{1-\theta_s}} dF_s(z).$$

Here, the function $LM(\cdot)$ expresses excess demand for labor, while the functions $IM_s(\cdot)$'s give the excess demands for each intermediate good market. By Walras' law, we can omit the final good market. By the implicit function theorem, we can characterize the effect that a change in any of these parameters has on the equilibrium price system. To this end, pre-multiply the vector of direct effects of the parameters on the system of equation by the inverse of the Jacobian of the system with respect to prices. This yields,

$$\begin{bmatrix} \frac{dw}{d\theta} \\ \frac{dp_1}{d\theta} \\ \vdots \\ \frac{dp_N}{d\theta} \end{bmatrix} = - \begin{bmatrix} \frac{dLM(w, \mathbf{p}; \theta)}{dw} & \frac{dLM(w, \mathbf{p}; \theta)}{dp_1} & \dots & \frac{dLM(w, \mathbf{p}; \theta)}{dp_N} \\ \frac{dIM_1(w, \mathbf{p}; \theta)}{dw} & \frac{dIM_1(w, \mathbf{p}; \theta)}{dp_1} & \dots & \frac{dIM_1(w, \mathbf{p}; \theta)}{dp_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{dIM_N(w, \mathbf{p}; \theta)}{dw} & \frac{dIM_N(w, \mathbf{p}; \theta)}{dp_1} & \dots & \frac{dIM_N(w, \mathbf{p}; \theta)}{dp_N} \end{bmatrix}^{-1} \begin{bmatrix} \frac{dLM(w, \mathbf{p}; \theta)}{d\theta} \\ \frac{dIM_1(w, \mathbf{p}; \theta)}{d\theta} \\ \vdots \\ \frac{dIM_N(w, \mathbf{p}; \theta)}{d\theta} \end{bmatrix}.$$

Starting from the first row of the Jacobian, we can see that,

$$\begin{aligned} \frac{dLM(w, \mathbf{p}; \theta)}{dw} &= - \sum_{s=1}^N \left\{ \frac{1}{1-\theta_s} E_s \left(\frac{p_s \theta_s \Gamma_s^{\theta_s}}{w^{\theta_s}} \right)^{\frac{1}{1-\theta_s}} + \frac{1-\Gamma_s}{\Gamma_s} \underbrace{\frac{dR_{\max,s}^*}{dw}}_{>0} \right\} - \varphi \chi \left(\frac{w}{\chi} \right)^{\varphi-1} < 0, \\ \frac{dLM(w, \mathbf{p}; \theta)}{dp_s} &= \frac{1}{1-\theta_s} E_s \left(\frac{p_s^{\theta_s} \theta_s \Gamma_s^{\theta_s}}{w} \right)^{\frac{1}{1-\theta_s}} > 0 \quad \forall s = 1, \dots, N. \end{aligned}$$

Next, considering the effect of the wage on the market clearing condition in each sector,

$$\frac{dIM_s(w, \mathbf{p}; \theta)}{dw} = \frac{dY_s^S}{dw} - \frac{\xi_s}{p_s} \left[\sum_j \left\{ p_j \frac{dY_j^S}{dw} \right\} \right] \leq 0.$$

However, note that the above is monotone in ξ_s . Plugging in the expressions for dY_s^S/dw , yields

$$\frac{dIM_s(w, \mathbf{p}; \theta)}{dw} = \frac{1}{p_s} \left\{ -\frac{\theta_s}{1-\theta_s} w p_s \underbrace{\left(\frac{p_s \theta_s \Gamma_s}{w} \right)^{\frac{\theta_s}{1-\theta_s}}}_{=Y_s^S} E_s + \xi_s \sum_j p_j Y_j^S \frac{\theta_j}{1-\theta_j} w \right\}.$$

Setting the above to 0 and rearranging delivers a threshold value of the shares of value added:²¹

$$\tilde{\xi}_s = \frac{\frac{\theta_s}{1-\theta_s}}{\sum_j \xi_j \frac{\theta_j}{1-\theta_j}} \xi_s.$$

Note that if $\theta_s = \theta_j$ for all $s \neq j$, then $\tilde{\xi}_s = \xi_s \forall s = 1, \dots, N$. That is, under the assumption that

²¹Recall that, by definition, the parameters ξ_s satisfy $\xi_s = \frac{p_s Y_s^S}{\sum_j p_j Y_j^S}$.

decreasing returns to scale are equal across sectors, there is no direct effect of the wage on excess demand. Finally,

$$\frac{dIM_s(w, \mathbf{p}; \boldsymbol{\theta})}{dp_s} = (1 - \xi_s) \frac{dY_s^S}{dp_s} + \frac{\xi_s}{p_s^2} \sum_{j \neq s} p_j Y_j^S > 0,$$

$$\frac{dIM_s(w, \mathbf{p}; \boldsymbol{\theta})}{dp_j} = -\frac{\xi_s}{p_s} p_j \frac{dY_j^S}{dp_j} < 0.$$

Under the assumption that $\theta_s = \theta$ for all s , the sign pattern of the Jacobian is,

$$\text{sign}(J) = \begin{bmatrix} - & + & \dots & \dots & + \\ 0 & + & - & \dots & - \\ \vdots & - & + & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & - \\ 0 & - & \dots & - & + \end{bmatrix} = \begin{bmatrix} - & + & \dots & \dots & + \\ 0 & \underbrace{\begin{bmatrix} + & - & \dots & - \end{bmatrix}}_{\equiv S} \\ \vdots & \begin{bmatrix} - & + & \ddots & \vdots \end{bmatrix} \\ \vdots & \begin{bmatrix} \ddots & \ddots & \ddots & - \end{bmatrix} \\ 0 & \underbrace{\begin{bmatrix} - & \dots & - & + \end{bmatrix}}_{\equiv S} \end{bmatrix}.$$

Note that m , p_R and ψ_R do not affect the excess demand for any of the intermediate goods, and they only affect the excess demand for labor. Thus,

$$\text{sign} \left(\begin{bmatrix} \frac{dLM(w, \mathbf{p}; \boldsymbol{\theta})}{dp_R} \\ \frac{dIM_1(w, \mathbf{p}; \boldsymbol{\theta})}{dp_R} \\ \vdots \\ \frac{dIM_N(w, \mathbf{p}; \boldsymbol{\theta})}{dp_R} \end{bmatrix} \right) = \begin{bmatrix} + \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

Therefore,

$$\frac{dw}{dp_R} = -(J^{-1})_{11} \frac{dLM(w, \mathbf{p}; \boldsymbol{\theta})}{dp_R}.$$

Due to the structure of the matrix J ,

$$(J^{-1})_{11} = \frac{1}{\det|J|} \det|S| = \frac{1}{J_{11} \det|S|} \det|S| = \frac{1}{J_{11}}.$$

We therefore have that,

$$\frac{dw}{dp_R} = -\frac{\frac{dLM(w, \mathbf{p}; \boldsymbol{\theta})}{dp_R}}{\frac{dLM(w, \mathbf{p}; \boldsymbol{\theta})}{dw}} > 0,$$

since,

$$\frac{dLM(w, \mathbf{p}; \boldsymbol{\theta})}{dp_R} = \frac{1}{\psi_R \delta} \sum_{s=1}^N \left\{ \frac{1 - \Gamma_s}{\Gamma_s} \right\} > 0.$$

The proof follows the same steps for ψ_R and m . Indeed,

$$\text{sign} \left(\begin{bmatrix} \frac{dLM(w, \mathbf{p}; \boldsymbol{\theta})}{d\psi_R} \\ \frac{dIM_1(w, \mathbf{p}; \boldsymbol{\theta})}{d\psi_R} \\ \vdots \\ \frac{dIM_N(w, \mathbf{p}; \boldsymbol{\theta})}{d\psi_R} \end{bmatrix} \right) = \text{sign} \left(\begin{bmatrix} \frac{dLM(w, \mathbf{p}; \boldsymbol{\theta})}{dm} \\ \frac{dIM_1(w, \mathbf{p}; \boldsymbol{\theta})}{dm} \\ \vdots \\ \frac{dIM_N(w, \mathbf{p}; \boldsymbol{\theta})}{dm} \end{bmatrix} \right) = \begin{bmatrix} + \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

and,

$$\frac{dLM(w, \mathbf{p}; \boldsymbol{\theta})}{d\psi_R} = \frac{1}{\psi_R} \sum_{s=1}^N \left\{ \frac{1-\Gamma_s}{\Gamma_s} \right\} R_{\max,s}^* > 0$$

$$\frac{dLM(w, \mathbf{p}; \boldsymbol{\theta})}{dm} = \frac{1}{\psi_R \delta} \frac{1}{\rho + \delta} \sum_{s=1}^N \left\{ \frac{1-\Gamma_s}{\Gamma_s} \right\} > 0.$$

Since there is no wealth effect on the labor supply, the comparative statics on aggregate employment can be carried out trivially by noting that the supply of labor is only an increasing function of the equilibrium wage. \square

A.2 Solution of the Static Revenue Maximization Problem

We assume that $\Omega > 0$. The static problem reads

$$\Pi(R, z) \equiv \max_{L \geq 0, 0 \leq u \leq 1} z p (\Gamma L + (1 - \Gamma) u R)^\theta - w L - m u R.$$

Solving for labor yields,

$$L^*(R, z) = \begin{cases} \frac{1}{\Gamma} \left(\left(\frac{w}{pz\Gamma\theta} \right)^{\frac{1}{\theta-1}} - (1 - \Gamma) u^* R \right) & \text{if } uR \leq \left(\frac{w}{pz\Gamma\theta} \right)^{\frac{1}{\theta-1}} \frac{1}{1-\Gamma} \equiv \bar{R}(z) \\ 0 & \text{else} \end{cases}.$$

Thus,

$$\Pi(R, z) \equiv \begin{cases} (1 - \theta) pz(1 - \Gamma)^\theta \bar{R}(z)^\theta + \left(\frac{(1 - \Gamma)w}{\Gamma} - m \right) uR & uR \leq \bar{R}(z) \\ pz(1 - \Gamma)^\theta u^\theta R^\theta - muR & uR > \bar{R}(z) \end{cases}.$$

Now, by the F.O.C. for u , we obtain u^* in the case of an interior solution,

$$\theta p A (\Gamma L^* + (1 - \Gamma) u^* R)^{\theta-1} (1 - \Gamma) R - m R = 0.$$

Plugging in optimal labor from above yields,

$$u^*(R, z) = \frac{1}{(1 - \Gamma)R} \left(\frac{m}{(1 - \Gamma)\theta p A} \right)^{\frac{1}{\theta-1}} = \frac{\hat{R}(z)}{R}.$$

In particular, $u^* = 1$ if

$$\bar{R}(z) \leq R \leq \hat{R}(z) = \left(\frac{pz\theta(1 - \Gamma)^\theta}{m} \right)^{\frac{1}{1-\theta}},$$

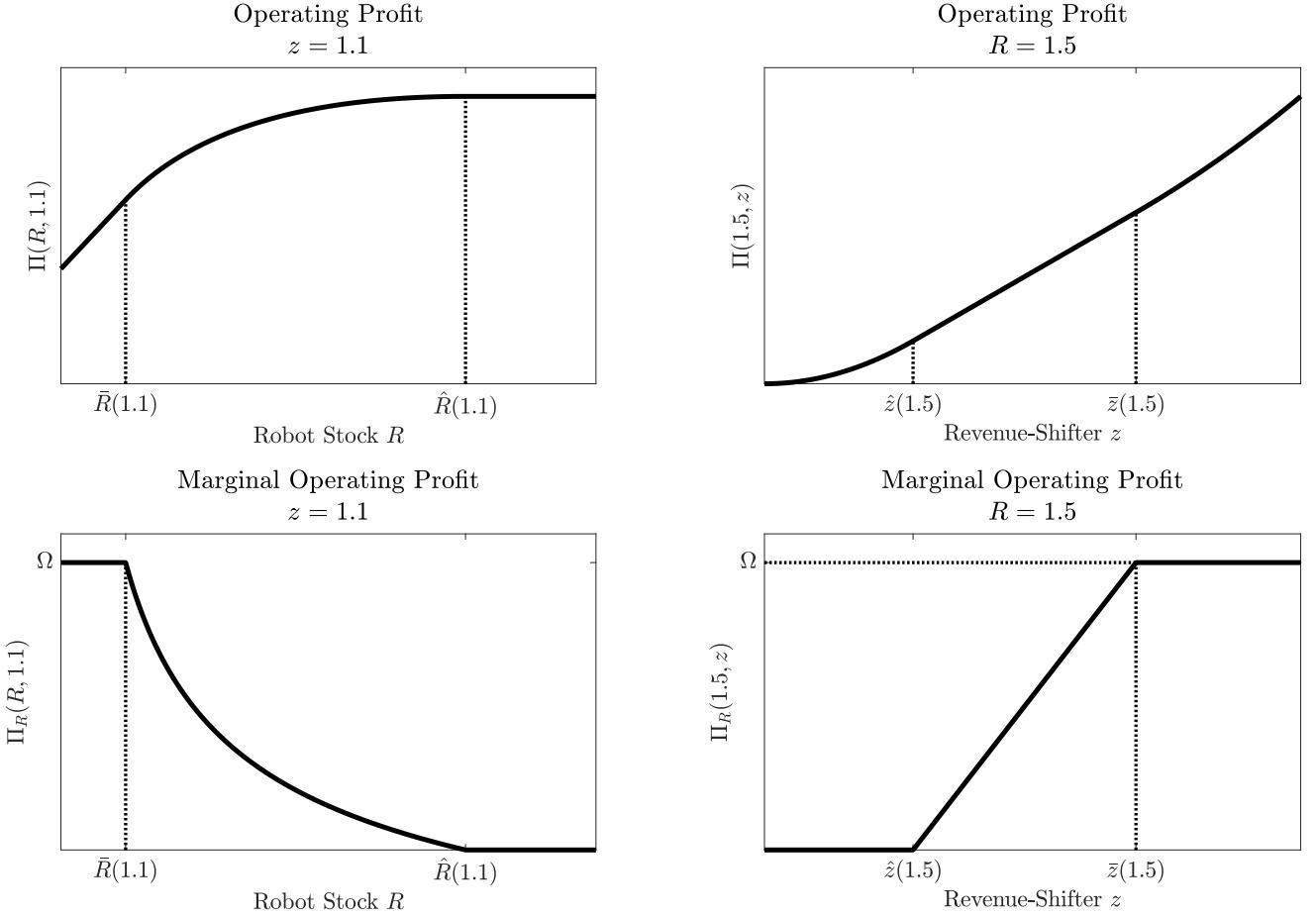
the optimal static scale of the firm. Under our assumption that labor savings are strictly positive, i.e. $\Omega > 0$, it is straightforward to see that $u^*(R, z) > 0$. We now show that in the case where both robots and labor are used, $u^* = 1$. Indeed, for an internal solution in the first region, the F.O.C. requires

$$\frac{w}{\Gamma} (1 - \Gamma) R - m R = 0,$$

i.e. no labor savings from having a (fully utilized) robot quantity R . Therefore, in the first region $u^* = 1$ if labor savings are positive. Otherwise, the firm sets $u^* = 0$. This static solution allows us to write the operating profit function as reported in the main text.

Figure 1 displays the properties of the profit function. The two upper panels show that operating profit function, $\Pi(R, z)$, is weakly increasing and concave in R for given z (upper left panel), and weakly increasing and convex in z for given R (upper right panel). The bottom panels highlight that the marginal profit from an additional robot, $\Pi_R(R, z)$, is bounded from above by Ω , the unit labor

Figure 1: Operating Profit and Marginal Operating Profit



savings achieved when robots substitute workers, and from below by 0, as redundant robots can be turned off. Moreover, the bottom left panel shows that the marginal profit is weakly decreasing in R for given z , and weakly increasing in z for a given R .

Lemma 1 (Properties of $\Pi(R, z)$). *If labor savings, $\Omega > 0$, the operating profit function, $\Pi(R, z)$, is weakly increasing in R for all z and in z for all R , weakly concave in R for all z , and weakly convex in z for all R . Moreover, the marginal operating profit, $\Pi_R(R, z)$, is bounded from above by Ω and from below by 0, weakly decreasing in R , and weakly increasing in z .*

Lemma 1. The operating profit function reads:

$$\Pi(R, z) = \begin{cases} (1-\theta)pz(1-\Gamma)^\theta \bar{R}(z)^\theta + \left(\frac{(1-\Gamma)w}{\Gamma} - m\right)R & R \leq \bar{R}(z) \\ pz(1-\Gamma)^\theta R^\theta - mR & \hat{R}(z) \geq R > \bar{R}(z) \\ pz(1-\Gamma)^\theta (\hat{R}(z))^\theta - m\hat{R}(z) & \hat{R}(z) < R \end{cases}$$

where:

$$\bar{R}(z) \equiv \frac{1}{1-\Gamma} \left(\frac{pz\theta\Gamma}{w} \right)^{\frac{1}{1-\theta}} \quad \hat{R}(z) \equiv \frac{1}{1-\Gamma} \left(\frac{pz\theta(1-\Gamma)}{m} \right)^{\frac{1}{1-\theta}}$$

Define $\bar{z}(R), \hat{z}(R)$, as the inverse functions of $\bar{R}(z), \hat{R}(z)$, respectively. The derivative of the profit function with respect to R is given by,

$$\Pi_R(R, z) \equiv \begin{cases} \Omega \equiv \left(\frac{(1-\Gamma)w}{\Gamma} - m \right) & z > \bar{z}(R) \\ \theta z p(1-\Gamma)^\theta R^{\theta-1} - m & \bar{z}(R) \geq z > \hat{z}(R) \\ 0 & z \leq \hat{z}(R) \end{cases}$$

Under the assumption of positive labor savings Ω , the above is non-negative $\forall z$. Furthermore, this derivative is decreasing in R . Indeed, both $\bar{z}(R)$ and $\hat{z}(R)$ are increasing in R , since they are the inverses of the functions $\bar{R}(z), \hat{R}(z)$, which are increasing in z . Moreover, its maximum is given by Ω (which in particular is also the value of the derivative when R is 0). Now, for a given (\tilde{R}, \tilde{z}) , consider an increase in z . If the initial \tilde{z} is not at any of thresholds, taking derivatives shows that $\Pi_R(\tilde{R}, \tilde{z})$ is strictly increasing in z if the (\tilde{R}, \tilde{z}) are such that $\bar{z}(\tilde{R}) > \tilde{z} > \hat{z}(\tilde{R})$. Otherwise, $\Pi_{R,z}(\tilde{R}, \tilde{z}) = 0$. Finally, this function is increasing in z if $\tilde{z} = \bar{z}(\tilde{R})$, or $\tilde{z} = \hat{z}(\tilde{R})$, since $\Omega \geq \theta z p(1-\Gamma)^\theta R^{\theta-1} - m \geq 0$ for the values of R that imply full robot-stock utilization. It follows that $\Pi_R(R, z)$ is increasing in z . Following similar steps for R shows that $\Pi_R(R, z)$ is decreasing in R .

The derivative of the operating profit function with respect to z reads

$$\Pi_z(R, z) \equiv \begin{cases} (1-\theta)p(1-\Gamma)^\theta \bar{R}(z)^\theta + (1-\theta)pz(1-\Gamma)^\theta \theta \bar{R}(z)^{\theta-1} \bar{R}_z(z) & z > \bar{z}(R) \\ p(1-\Gamma)^\theta R^\theta & \bar{z}(R) \geq z > \hat{z}(R) \\ p(1-\theta)(\hat{R}(z))^\theta + pz(1-\theta)\theta (\hat{R}(z))^{\theta-1} \hat{R}_z(z) & z \leq \hat{z}(R) \end{cases}$$

Where the last line follows from replacing the definition of $\hat{R}(z)$ into $\Pi(R, z)$ for $\hat{R}(z) < R$, which yields,

$$\begin{aligned} pz(1-\Gamma)^\theta (\hat{R}(z))^\theta - m \hat{R}(z) &= pz(1-\Gamma)^\theta \left(\frac{1}{1-\Gamma} \left(\frac{pz\theta(1-\Gamma)}{m} \right)^{\frac{1}{1-\theta}} \right)^\theta - \frac{m}{1-\Gamma} \left(\frac{pz\theta(1-\Gamma)}{m} \right)^{\frac{1}{1-\theta}} \\ &= \left(\frac{pz\theta(1-\Gamma)}{m} \right)^{\frac{\theta}{1-\theta}} (pz - \theta pz) \\ &= pz(1-\theta) \hat{R}(z)^\theta. \end{aligned}$$

Further note that

$$\hat{R}_z(z) \equiv \frac{\partial \hat{R}(z)}{z} = \frac{1}{1-\theta} z^{\frac{\theta}{1-\theta}} \frac{1}{1-\Gamma} \left(\frac{p\theta(1-\Gamma)}{m} \right)^{\frac{1}{1-\theta}} = \frac{1}{(1-\theta)z} \hat{R}(z) \geq 0,$$

and similarly,

$$\bar{R}_z(z) \equiv \frac{\partial \bar{R}(z)}{z} = \frac{1}{(1-\theta)z} \bar{R}(z) \geq 0.$$

We can therefore rewrite

$$\Pi_z(R, z) \equiv \begin{cases} p(1-\Gamma)^\theta \bar{R}(z)^\theta & z > \bar{z}(R) \\ p(1-\Gamma)^\theta R^\theta & \bar{z}(R) \geq z > \hat{z}(R) \\ p(1-\Gamma)^\theta (\hat{R}(z))^\theta & z \leq \hat{z}(R) \end{cases}$$

It immediately follows that the derivative $\Pi_z(R, z) \geq 0$ for all (R, z) . Moreover,

$$\Pi_{zz}(R, z) \equiv \begin{cases} p(1-\Gamma)^\theta \theta \bar{R}(z)^{\theta-1} \bar{R}_z(z) & z > \bar{z}(R) \\ 0 & \bar{z}(R) \geq z > \hat{z}(R), \\ p(1-\Gamma)^\theta \theta (\hat{R}(z))^{\theta-1} \hat{R}_z(z) & z \leq \hat{z}(R) \end{cases}$$

is nonnegative, given the derivatives reported above. \square

A.3 Dynamic Problem

We assume that the stochastic process has the FOSD property as defined below.

Definition 1. Denoting as $F(z_{t+s}|z_t)$ the conditional distribution of the revenue shifter at horizon $t+s$, given a starting point $z(t)=z$, we say that the stochastic process for z has the FOSD property if

$$z'_t > z_t \Rightarrow F(z_{t+s}|z'_t) \succeq_{FOSD} F(z_{t+s}|z_t).$$

Following Dixit and Pindyck (1994), we can write the problem of the firm recursively as

$$\rho V(R, z) = \max_{I \in \mathbb{R}} \Pi(R, z) - p_R I - \frac{\psi_R}{2} I^2 + (I - \delta R) V_R(R, z) + \mu(z) V_z(R, z) + \frac{\sigma^2(z)}{2} V_{zz}(R, z).$$

Lemma 2 (Properties of the Value Function). The value function $V(R, z)$ is increasing in R for all z , and concave in R for all z . $V_R(R, z)$ is bounded from above by $\frac{\Omega}{\rho+\delta}$, and weakly decreasing in w . Moreover, if the stochastic process has the FOSD property as in Definition 1, then $V(R, z)$ is increasing in z , and $V_R(R, z)$ is nondecreasing in z .

Lemma 2. The properties cited are inherited by the instantaneous operating profit function $\Pi(R, z)$ (see e.g. Stokey (2009), p.233). Following Stokey (2009) (p.229), and considering a small time interval $dt \rightarrow 0$, we can write an approximation for the value function as follows:

$$V(R_0, z_0) \approx \Pi(R_0, z_0) dt - p_R(I^*(R_0, z_0)) - \frac{\psi_R}{2}(I^*(R_0, z_0))^2 + \frac{1}{1+\rho dt} \mathbb{E}[V(R_0 + dR, z_0 + dz)] \quad (1)$$

Where:

$$dR \equiv [I^*(R_0, z_0) - \delta R_0] dt,$$

is the drift associated to optimal investment $I^*(R_0, z_0)$. Note that the above Equation 1 defines a contraction mapping $T_{dt}(V)$ for all $dt > 0$. We can therefore apply the results in Stokey et al. (1989) Corollary 1 (p.52). In particular, it is straightforward to verify the following properties of V that derive directly from the properties of $\Pi(R, z)$ summarized in Lemma 1:²²

- $\Pi(R, z)$ increasing in R for all z implies that $V(R, z)$ is increasing in R for all z , $\Pi(R, z)$ weakly concave in R for all z implies that $V(R, z)$ is weakly concave in R for all z .²³

²²These properties follow directly from Π since the Envelope Theorem ensures that the effects of changes in z_0 through the optimal investment I^* and the associated drift dR on the maximized value function V are second order.

²³An alternative proof of concavity follows Dixit and Pindyck (1994), 1993, p.360. Consider two initial values of R , R_1, R_2 with associated optimal paths $\{R_{1,t}\}, \{R_{2,t}\}$ and investment policies $\{\Delta R_{1,t}\}, \{\Delta R_{2,t}\}$. Now consider the firm having initial capital stock:

$$\alpha R_1 + (1-\alpha) R_2, \quad \alpha \in [0, 1]$$

Consider now the path $\{\alpha R_{1,t} + (1-\alpha) R_{2,t}\}$. This is clearly feasible, so V will have a value at least as large as the one

- If the FOSD property is assumed, $\Pi(R, z)$ increasing in z for all R implies that $V(R, z)$ is increasing in z for all R , as it is immediate to see that T_{dt} will map a V increasing in z for all R into a value function $V' = T(V)$ that has the same property. This is ensured by the definition of FOSD. For any $z''_0 > z'_0$, $\mathbb{E}[V(R_0 + dR, z''_0 + dz)] \geq \mathbb{E}[V(R_0 + dR, z'_0 + dz)]$. The statement follows combining this fact with the fact that $\Pi(R, z)$ is increasing in z for all R .

To prove the properties of V_R , Consider the Envelope condition that can be obtained by differentiating the HJB in R side by side. Doing so yields:

$$(\rho + \delta) V_R(R, z) - \Pi_R(R, z) - \mu(z) V_{Rp}(R, z) - \frac{1}{2} \sigma^2(z) V_{Rpp}(R, z) - \dot{R} V_{RR}(R, z) = 0,$$

which can be rewritten using Itô's formula as:

$$(\rho + \delta) V_R(R, z) - \Pi_R(R, z) - \mathbb{E}\left[\frac{dV_R(R, z)}{dt}\right] = 0,$$

As for Equation (1), we can use an approximation for the derivative of the value function along the optimal path as:

$$V_R(R_0, z_0) \approx \Pi_R(R_0, z_0) dt + \frac{1}{1 + (\rho + \delta) dt} \mathbb{E}[V_R(R_0 + dR, z_0 + dz)]. \quad (2)$$

The RHS of Equation (2) also defines a contraction $T_{R,dt}$ for any $dt > 0$, as it satisfies the hypotheses of Blackwell's theorem.²⁴ To prove that $V_{RR} \leq 0$, consider a function V'_R that satisfies the property. Then it is immediate to see that $V''_R = T_{R,dt}(V'_R)$ satisfies it as well. Indeed, the returns function $\Pi_R(R, z)$ has weakly negative derivative in R as well. It follows that the operator maps weakly concave value functions into weakly concave value functions. Since $T_{R,dt}$ is a contraction mapping, we conclude

obtained following this path. Following such path the firm obtains in each instant:

$$u_t(\{\alpha R_{1,t} + (1-\alpha) R_{2,t}\}, z_t) \equiv \Pi(\alpha R_{1,t} + (1-\alpha) R_{2,t}, z_t) - p_R(\alpha \Delta R_{1,t} + (1-\alpha) \Delta R_{2,t}) - \Psi(\alpha \Delta R_{1,t} + (1-\alpha) \Delta R_{2,t})$$

By concavity of Π and convexity of $\Psi(I) \equiv \frac{\psi_R}{2} I^2$:

$$u_t(\{\alpha R_{1,t} + (1-\alpha) R_{2,t}\}, z_t) \geq \alpha u_t(R_{1,t}, z_t) + (1-\alpha) u_t(R_{2,t}, z_t)$$

This implies:

$$\begin{aligned} E_0 \int_0^\infty e^{-\rho t} u_t(\{\alpha R_{1,t} + (1-\alpha) R_{2,t}\}, z_t) dt &\geq \alpha E_0 \int_0^\infty e^{-\rho t} u_t(R_{1,t}, z_t) dt + (1-\alpha) E_0 \int_0^\infty e^{-\rho t} u_t(R_{2,t}, z_t) dt \\ E_0 \int_0^\infty e^{-\rho t} u_t(\{\alpha R_{1,t} + (1-\alpha) R_{2,t}\}, z_t) dt &\geq \alpha V(R_1, z_0) + (1-\alpha) V(R_2, z_0) \\ V(\alpha R_1 + (1-\alpha) R_2, P) &\geq \alpha V(R_1, z_0) + (1-\alpha) V(R_2, z_0) \end{aligned}$$

Proving the statement.

²⁴Monotonicity follows immediately from the fact that the operator is linear in V_R , while discounting is ensured by $(\rho + \delta) dt > 0$ for all $dt > 0$.

that $V_{RR} \leq 0$. A similar reasoning shows that $V_{Rw} > 0$, and, if the FOSD property holds, that $V_{Rz} \geq 0$.

To prove the upper bound of $V_R(R, z)$, we rewrite the envelope condition using Lemma 1 in Appendix B of Abel and Eberly (1993, p.22) to solve for $V_R(R, z)$ as²⁵

$$V_R(R_0, z_0) = \mathbb{E} \left[\int_0^\infty e^{-(\rho+\delta)t} \Pi_R(R_t, z_t) dt \right].$$

Using the bounds for $\Pi(R, z)$ in Lemma 1,

$$V_R(R_0, z_0) \leq \mathbb{E} \left[\int_0^\infty e^{-(\rho+\delta)t} \Omega dt \right] = \frac{\Omega}{\rho + \delta}.$$

□

Proposition 3 (Properties of the Optimal Investment Policy). *Suppose that the stochastic process for z satisfies the FOSD property in Definition 1. Then, the optimal investment policy $I^*(R, z)$ is non-increasing in R for all z , non-decreasing in z for all R , non-decreasing in w for all (R, z) , and bounded from above by*

$$\frac{1}{\psi_R} \left[\frac{\Omega}{\rho + \delta} - p_R \right].$$

Proof. Optimal investment satisfies the first order condition

$$I^*(R, z) = \frac{1}{\psi_R} [V_R(R, z) - p_R].$$

It follows that $I^*(R, z)$ inherits the properties of $V_R(R, z)$. By Lemma 2, the properties in the statement follow. □

Proposition 4 (Steady State of the Deterministic Model). *Consider the deterministic investment problem with $z_t = z$, $\forall t$. The model has a unique steady state level of robots $R^*(z)$. If either $\psi_R = 0$ or $\delta = 0$, the unique steady state features full automation. If $\delta > 0$, there exists a finite value $\tilde{\psi}_R > 0$ such that partial automation obtains for $\psi_R \geq \tilde{\psi}_R$ and full automation occurs if $\psi_R < \tilde{\psi}_R$.*

Proof. By the proof of Proposition 3, we can write,

$$I^*(R_t, z_t) = \frac{1}{\psi_R} \left(\int_t^\infty e^{-(\rho+\delta)(s-t)} \mathbb{E}_t \{ \Pi_R(R_s^*, z_s) \} ds - p_R \right).$$

Removing the expectation and evaluating at the steady state we obtain, for $R^* > 0$,

$$\delta R^*(z) = \frac{1}{\psi_R} \left(\frac{\Pi_R(R^*(z), z)}{\rho + \delta} - p_R \right).$$

²⁵This part shows the case of a diffusion, but the same reasoning can be applied to a CTMC, by replacing the stochastic terms appropriately. Indeed, while the statement of Lemma 1 in Abel and Eberly (1993) considers a diffusion, all their passages can be directly applied to a CTMC, and to the case where the function of interest is multivariate, as in our case.

First, note that, given z , we have

$$\Pi_R(R^*(z), z) = \begin{cases} \min \{\Omega, \theta z p(1-\Gamma)^\theta R^*(z)^{\theta-1} - m\} & R^*(z) < \hat{R}(z) \\ 0 & R^*(z) \geq \hat{R}(z). \end{cases}$$

Therefore, $R^*(z) = 0$ if $p_R > \frac{\Omega}{\rho+\delta}$, regardless of the value of ψ_R, δ . If either $\psi_R, \delta = 0$, the FOC for investment gives

$$\frac{\Pi_R(R^*(z), z)}{\rho + \delta} = p_R.$$

So if there are strictly positive labor savings, $0 < p_R < \frac{\Omega}{\rho+\delta}$ —our main assumption—the optimal solution entails full automation, $R^*(z) > \bar{R}(z)$.²⁶

Turning to the case $\delta, \psi_R > 0$ and rearranging,

$$\psi_R \delta R^*(z) + p_R = \frac{\Pi_R(R^*(z), z)}{\rho + \delta}$$

the above equation has a unique solution as $\Pi(R, z)$ is weakly concave in R , so the RHS is strictly increasing from p_R to ∞ and the LHS is weakly decreasing from $\frac{\Omega}{\rho+\delta}$ to 0. Since the revenue shock is fixed at z , this unique solution is the steady state of the model. Moreover, by Corollary 1 we have that $R^*(z)$ is strictly decreasing in ψ_R and goes to 0 as ψ_R tends to infinity. It follows that there exists a finite $\tilde{\psi}_R$ such that $R^*(z) < \bar{R}(z)$ for all $\psi_R > \tilde{\psi}_R$.

□

Definition 2. For any revenue shock z , we define the desired stochastic steady state as the stock of robots $R^*(z)$ such that optimal investment just covers depreciation,

$$I^*(R^*(z), z) = \delta R^*(z).$$

Corollary 1 (Properties of $R^*(z)$). Under the assumptions of Proposition 3, $R^*(z)$ is non-decreasing in z , and non-decreasing in w for all z . If $\delta > 0$, then $R^*(z)$ is bounded from above by

$$R_{\max}^* \equiv \frac{1}{\delta \psi_R} \left[\frac{\Omega}{\rho + \delta} - p_R \right], \forall z$$

and tends to 0 as $\psi_R \rightarrow \infty$.

Proof. Follows directly from Proposition 3, evaluating investment at R^* and using Definition 2. □

Proposition 5 (Bounds of the Stationary Distribution). Given a diffusion or a Continuous-Time Markov Chain (CTMC) for z that admits a stationary distribution with support \mathcal{Z} such that $\inf \mathcal{Z} = z_1$

²⁶When $\frac{\Omega}{\rho+\delta} = p_R$, the firm is indifferent between all values of the robot stock satisfying, $R^*(z) \leq \bar{R}(z)$, so there is a continuum of steady state distributions that depend on the initial distribution of robot stocks.

and $\sup \mathcal{Z} = z_N$, the stationary distribution $G(R, z)$ has support $[R^*(z_1), R^*(z_N)] \times \mathcal{Z}$. If $\delta > 0$, then the stationary distribution has support $[R^*(z_1), R_{\max}^*] \times \mathcal{Z}$.

Proof. For ease of notation, define $R_1^* = R^*(z_1)$, $R_N^* = R^*(z_N)$. The KFE for a Poisson process reads, defining $G(\bar{R}, z_i, t) = Pr(R_t \leq \bar{R}, z_t = z_i)$:

$$\frac{\partial}{\partial t} G(R_t, z_i, t) = -\frac{\partial R_t}{\partial t} \frac{\partial}{\partial R_t} G(R_t, z_i, t) - \sum_{j \neq i} \lambda_{ij} G(R_t, z_i, t) + \sum_{j \neq i} \lambda_{ji} G(R_t, z_j, t)$$

Integrating over z 's yields:

$$\begin{aligned} \sum_i \frac{\partial}{\partial t} G(R_t, z_i, t) &= -\sum_i \frac{\partial R_t}{\partial t} \frac{\partial}{\partial R_t} G(R_t, z_i, t) + \\ &\quad + \sum_i \left\{ -\sum_{j \neq i} \lambda_{ij} G(R_t, z_i, t) + \sum_{j \neq i} \lambda_{ji} G(R_t, z_j, t) \right\} \end{aligned}$$

At the stationary distribution it holds:

$$0 = -\sum_i \frac{\partial R}{\partial t} \frac{\partial}{\partial R} G(R, z_i, \infty)$$

since the last terms in parenthesis cancel out by definition of a stationary distribution. Note by definition of R_N^* : $\frac{\partial R_t(R_N^*, z_N)}{\partial t} = (I^*(R_N^*, z_N) - \delta R_N^*) = 0$. By Corollary 1, investment is increasing in z and decreasing in R . It follows that

$$\frac{\partial R_t(R, z)}{\partial t} < 0 \quad \forall R > R_N^* \wedge z \leq z_N$$

To avoid contradiction, in a stationary distribution we must have: $G(R, z, \infty) = 0 \quad \forall R > R_N^*$. A similar argument can be made for R_1^* by flipping all inequalities. Combining the two arguments, $G(R, z_i, \infty) = 0 \quad \forall R \notin [R_1^*, R_N^*]$.

Now, consider a diffusion with KFE:

$$0 = -\frac{\partial [I^*(R, z) - \delta R] g(R, z, t)}{\partial R} - \frac{\partial [\mu(z) g(R, z, t)]}{\partial z} + \frac{1}{2} \frac{\partial^2 [\sigma^2(z) g(R, z, t)]}{\partial z^2}$$

Suppose $g(R, z, t) > 0$ for some $R < R_1^*$ at all $z \in [z_1, z_N]$. Now, for all $R < R_1^*$, and for all $z \in [z_1, z_N]$, the investment drift is strictly positive, by definition of R_1^* . Therefore, the joint distribution over (R, z) will feature outflows in R for each revenue shock $z \in [z_1, z_N]$. Integrating over z , the marginal stationary distribution for R is positive below R_1^* . Since $I^*(R, z) - \delta R > 0, \forall z, R < R_1^*$,

$$-\mathrm{d}[I^*(R, z) - \delta R] g(R, z, t) < 0 \quad \forall z, R < R_1^*$$

directly contradicting the definition of stationary distribution. Assume now that $\delta > 0$, then we can reason as above by flipping all inequalities for $R > R_N^*$. \square

Proposition 6 (Aggregate Labor Demand). *The aggregate labor demand is non-increasing in w , with $\lim_{w \rightarrow \infty} L^d(w) = 0$ and $\lim_{w \rightarrow 0} L^d(w) = \infty$.*

Proof. By Proposition 3, investment is non-decreasing in w . As a result, the cutoffs $R^*(z)$ are non-decreasing in w as well. From the Kolmogorov Forward Equation in Proposition 5, the stationary distribution entails weakly higher robot stocks for each z . Moreover, the cutoffs $\bar{R}(z)$ are strictly decreasing in w , which implies that individual labor demand is non-increasing in w for all (R, z) (strictly decreasing for firms with positive labor demand). Finally, individual labor demand is non-increasing in R for all z . Combining all these facts, the integral,

$$\int_{\mathcal{S}} L^*(R, z) dG(R, z),$$

is non-increasing in w . Note that as $w \rightarrow \infty$, the individual labor demand falls to 0. As $w \rightarrow 0$, we instead have that $\bar{R}(z) \rightarrow \infty$, implying that the individual labor policy tends to ∞ as well. \square

B Calibration

B.1 Illustrative Figures Calibration

The following table reports the detailed parametrization of the model we adopt for the illustrative figures in the main text, Section 2. All parameters are taken from the robustness check which calibrates Γ and ψ_R with a single representative manufacturing sector. However, note that we choose a much lower value for ψ_R , as the calibrated value would imply that almost all firms choose $R = R_{\max}^*$.

Table 1: Illustrative Calibration Parameters

Description	Variable/Parameter	Value
Wage	w	0.27
Robot Purchase Price	p_R	$1.02 \times w$
Normalized MRTS	Γ	0.653
Flow robot cost	m	0
Robot depreciation	δ	$\log(1 + 1/12)$
Discount factor	ρ	$\log(1 + 0.04)$
DRS parameter	θ	0.298
Robot adjustment cost	ψ_R	5
EOU mean-reversion parameter	λ	0.879
EOU long-run standard deviation	σ_p	0.141
EOU log-run mean	z	1
Total factor productivity	p	1

B.2 Data Sources

The data sources that inform our calibration are the following:

- Compustat Daily Updates - Fundamentals Annual (1950-2019) for US firms accessed through the WRDS service. We use the series: “sale” (firm sales), “emp” (firm employment), “ppent” (net property, plant and equipment), “ppegt” (gross property, plant and equipment). We use “gvkey” identifiers and four-digit SIC codes to classify industries into the 19 sectors covered by the IFR statistics;
- Crosswalk between IFR and SIC sectors kindly provided by Daron Acemoglu and Pascual Restrepo;
- Data on robots per worker in 2010 and 2014 from the replication files for [Acemoglu and Restrepo \(2020\)](#), retrieved at: <https://economics.mit.edu/faculty/acemoglu/data>. which aggregates IFR data by sector;
- We construct factor shares using the historical SIC-level data from the NBER-CES dataset, retrieved at: <http://www.nber.org/nberces/>.

- Fixed investment deflator (BEA): Implicit price deflator for fixed gross private investment deflator, seasonally adjusted quarterly and averaged annually. <https://fred.stlouisfed.org/series/A007RD3Q086SBEA>;
- GDP deflator by the BEA, seasonally adjusted quarterly and averaged annually. Retrieved at: <https://fred.stlouisfed.org/series/GDPDEF>;
- We use data from ARK investment and BCG to obtain a ballpark of the relative robot cost. In particular, [Korus \(2019\)](#) and [Sirkin et al. \(2015\)](#) contain time series for robot unit costs up to 2014;
- Average annual wage for production workers from the Occupation Employment Statistics database retrieved at: <https://www.bls.gov/oes/tables.htm>;
- Value added by industry from the BEA GDP-by-industry dataset. Retrieved at: https://apps.bea.gov/iTable/index_industry_gdpIndy.cfm

B.3 Calibrating the Parameters of the EOU Stochastic Process

The first step towards calibrating the model is estimating the parameters of the EOU process for z used throughout the paper. In order to do so, we choose to use Compustat firm-level TFP. We use the SIC codes to classify firms into the 13 IFR sectors with the help of the crosswalk mentioned above. We employ the following specification to recover log-TFP,

$$\log(\text{Sales}_{sit}) = \gamma_i + f_{ts} + \alpha_s \log(\text{Emp}_{sit}) + \beta_s \log(\text{K}_{si,t-1}) + \varepsilon_{ist}. \quad (3)$$

We include firm fixed effects, time-by-IFR-sector fixed effects and assume that the technological parameters α and β are the same for all firms within the same IFR sector. We deflate sales by the GDP deflator and capital stocks by the fixed investment deflator. Our real capital stock measure is built by perpetual inventory method, using as a starting point the total gross property plant and equipment (“ppegt”) in the first year the firm appears in our panel, deflated by the fixed investment deflator for that year. We then add the net investment in property plant and equipment obtained using the differences of the series “ppent”, deflated by the corresponding fixed investment deflator.²⁷

The residuals of the above regression give an estimate of log firm-level TFP. We follow [Hamilton \(2018\)](#) to extract the stationary component at one-year frequency. For each firm, we estimate $\log z_{it}$ as the residual, v_{it} , from the regression:

$$\hat{\varepsilon}_{ist} = \gamma_{1,is} + \sum_{l=2}^5 \gamma_{l,is} \hat{\varepsilon}_{is,t-l} + v_{ist}.$$

²⁷We deal with the missing values for “ppent” by interpolating linearly using the nearest non-missing observations.

In this regression, $\hat{\varepsilon}_{ist}$ is the estimate for the firm-level residual ε_{ist} . We test the stationarity of the estimated residuals, \hat{v}_{ist} using an Augmented Dickey-Fuller test. We restrict our sample to firms with more than 20 observations for which we can reject the presence of a unit root at a 10% significance level. The rationale behind this procedure is that we want to keep stationary residuals, all the while ensuring that the ADF test has sufficient power.

We set $\log(z_{ist}) = \hat{v}_{ist}$. In the main text, we assume that z_{ist} follows a sector-specific exponential Ornstein-Uhlenbeck. Equivalently, its logarithm follows the Ornstein-Uhlenbeck process:

$$d\log(z_{ist}) = -\lambda_{\log,s}(\log(z_{ist}) - \mu_{\log,s})dt + \sigma_{s,\log}dW.$$

We estimate the parameters of this process by maximum likelihood, following the formulas in [Tang and Chen \(2009\)](#), amended to account for the fact that all firms within each sector share the same process.²⁸ We also winsorize the residual series at the first and 99th percentile, to avoid an excessive influence of extreme observations.²⁹ Applying Itô's lemma we have that z_{ist} follows the exponential Ornstein-Uhlenbeck process:

$$dz_{ist} = -\lambda_{a,s}z_{ist}(\log(z_{ist}) - \mu_{a,s})dt + \sigma_{a,s}z_{ist}dW,$$

with parameters:

$$\lambda_{a,s} = \lambda_{\log,s} \quad \mu_{a,s} = \left(\mu_{\log,s} - \frac{\sigma_{\log,s}^2}{2\lambda_{\log,s}} \right) \quad \sigma_{a,s} = \sigma_{\log,s}.$$

Recall that in our model, we normalize the mean of sector-level TFP to 1. To carry out this transformation, we look for a constant κ such that the transformed process $\tilde{z}_{ist} \equiv \kappa z_{ist}$ has a stationary log-normal distribution with mean 1. This process is also an exponential Ornstein Uhlenbeck with the same λ and σ parameter, but with with a different μ . This procedure delivers our sector-specific values for the EOU parameters, as a function of the MLE estimators (denoted with hats) for the log-process:

$$\lambda_{a,s} = \hat{\lambda}_{\log,s} \quad \mu_s = \left(\frac{\hat{\sigma}_{\log,s}^2}{4\hat{\lambda}_{\log,s}} \right) \quad \sigma_s = \hat{\sigma}_{\log,s}.$$

[Figure 1](#) displays the stationary shock distributions obtained through the procedure detailed above together with the kernel densities of Compustat TFP residuals used for their estimation.

²⁸This just requires replacing all sums over time, t , with double sums over both firms and time (i and t). Underlying this procedure is the assumption that all firms are sampled in an i.i.d. fashion in each sector, so that we can pool the data of all firms in each sector.

²⁹Recall that a small variance goes against the long-run preservation of labor, so our estimates represent a worst-case scenario for our labor survival mechanism.

B.4 Calibrating the Parameter θ

In order to calibrate θ in a model-consistent way, we target the share of income going to production-line employees in the sectors of interest. Our model implies that this quantity is exactly equal to θ when aggregate robot penetration, defined as R/L , is zero. To obtain the relevant labor share, we use the NBER-CES data for 1958-2011, and compute the share of income going to production-line employees as the wage bill of production-line employees over value added. Once again, this quantity is computed by IFR sector, to which we map the SIC sectors by using our crosswalk. In order to purge the estimated series by cyclical fluctuations, we apply a HP filter with smoothing parameter, $\lambda = 6.25$, to our annual-frequency data, and keep the trend component.

We do not have data for robot penetration before the year 2004, so we cannot establish exactly when the robot penetration is sufficiently close to 0. However, the data reported in [Acemoglu and Restrepo \(2020\)](#) suggests that most sectors had a reasonably low penetration of robots in 2004, with the exception of automotive. To calibrate θ we choose to take a mean of the HP-filtered series for the years 1956-1980. We choose 1980 as the final point for the time average as the 1980s saw a sharp increase in robot adoption in the US automotive industry.

B.5 Calibration with Non-Stationary Shocks

As described in Section 4.2, we assume that the revenue shock process for each firm in sector s is described by the following three equations:

$$dz_{it} = \mu_s z_{it} dt + \sigma_s z_{it} dW, \quad \forall t \in [0, T_i] \quad (4)$$

$$T_i \sim f_{T_s}(x) \equiv v_s \exp\{-v_s x\}. \quad (5)$$

$$\log z_{i0} \sim \mathcal{N}(\mu_s^{\text{reset}}, \sigma_s^{\text{reset}}), \quad (6)$$

where as usual dW denotes a Brownian increment, and v_s the exogenous sector-specific kill rate of the process. We set v_s to its sample analogue, the inverse of the average number of years for which we observe firms in our sample in sector s . We estimate TFP residuals as by taking the exponent of estimated log-TFP residuals from Equation (3). We then estimate their Kernel density on a grid of $N_p = 150$ equally-spaced points between 0.001 and the minimum between 6 and the 99.9 percentile of the empirical distribution.³⁰ We use the grid thus obtained to approximate the process described by Equations (4) - (6). We then choose the parameters $\mu_s, \sigma_s, \mu_s^{\text{reset}}, \sigma_s^{\text{reset}}$ for each sector to minimize

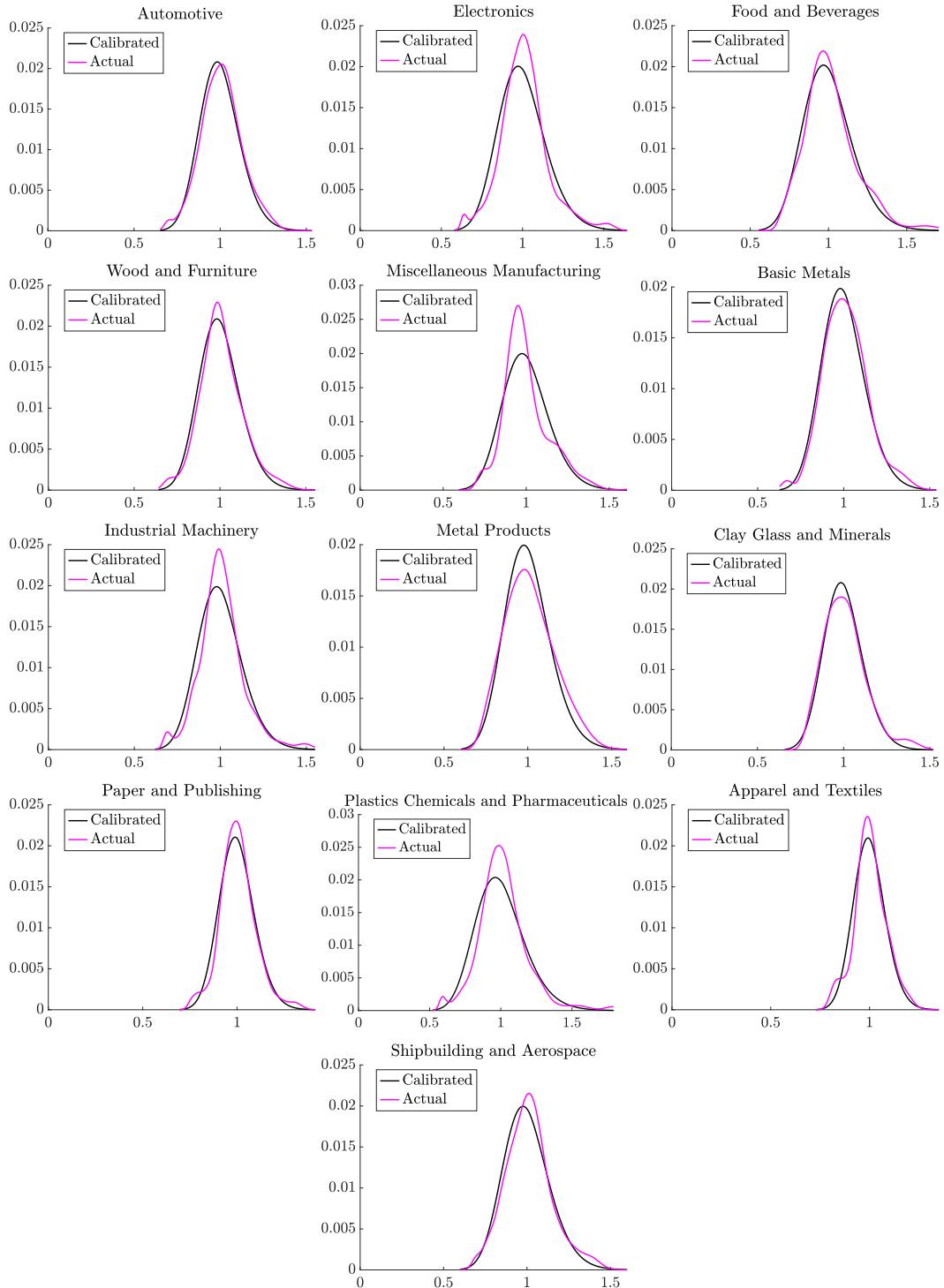
³⁰The upper bound of 6 is binding only in the case of four sectors: Electronics; Other Manufacturing; Metal Products; and Plastics, Chemicals and Pharmaceuticals. All these sectors have very few extreme outliers, that would worsen the distribution approximation by lowering the amount of points on our equally-spaced grid in the intervals where the density is higher.

the quadratic loss function:

$$\sum_{j=1}^{N_p} f_{j,s} \cdot (f_{j,s} - \tilde{f}_{j,s}(\mu_s, \sigma_s, \mu_s^{\text{reset}}, \sigma_s^{\text{reset}}; v_s))^2,$$

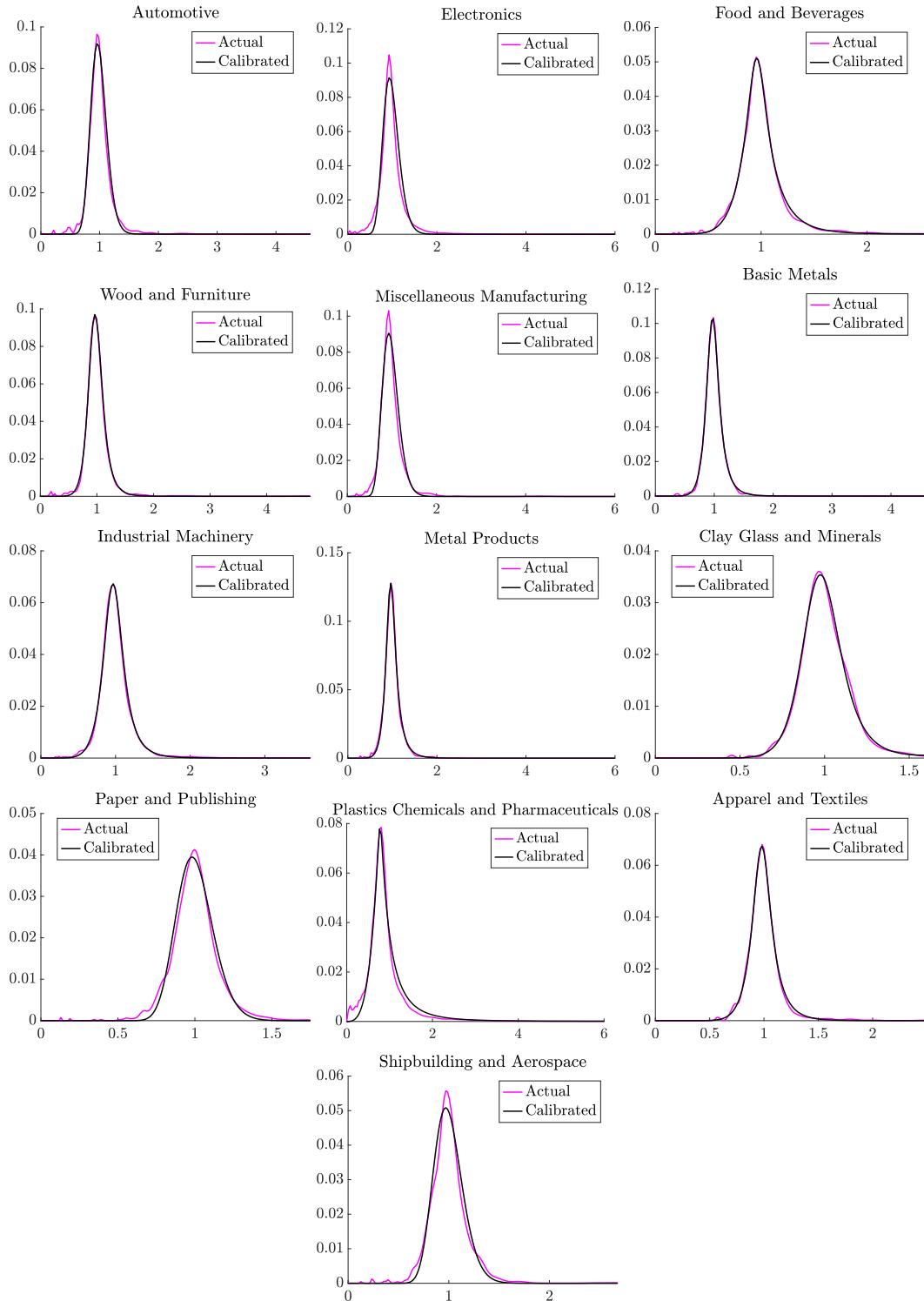
where $f_{j,s}$ denotes the normalized kernel density of residual TFP, evaluated at point $j \in \{1, \dots, N_p\}$ and $\tilde{f}_{j,s}(\cdot)$ is the stationary distribution of the shock process described by Equations (4) - (6), evaluated on the same grid. We weight the loss by the normalized density of the estimated TFP distribution at each point. Figure 2 displays the results of this procedure. For each sector, we report the distribution of estimated TFP from the data in magenta (“Actual”) and the stationary distribution resulting from the above minimization in black (“Calibrated”). For a majority of sectors, the fit of the calibrated distribution is almost perfect.

Figure 1: TFP Distributions (Actual and Calibrated) by Manufacturing Sector in the EOU Shock Case



Note: This figure presents the kernel density estimated on Compustat TFP residuals used to estimate the EOU parameters for each sector (denoted by “Actual”), together with the stationary distributions arising from the calibration of the stochastic shock (“Calibrated”) obtained from B.3.

Figure 2: TFP Distributions (Actual and Calibrated) by Manufacturing Sector in the Non-Stationary Case



Note: This figure presents the kernel density estimated on Compustat TFP residuals for each sector (denoted by “Actual”), together with the stationary distributions arising from the calibration of the stochastic (“Calibrated”) shock in the non-stationary model presented in Section 4.2.

C General Equilibrium

This Appendix provides additional details on the general equilibrium model described in Section 3.1. Throughout, time indexes are suppressed.

C.1 Goods Producers

We denote the final good consumed by agents in our economy by Y , that we take as the numéraire. The final good producer operates a CES production function that aggregates intermediate goods Y_s , at unit cost p_s . Under these assumptions, the static cost-minimization problem of the final good firm reads:

$$\begin{aligned} & \max_{\{Y_s\}} \sum_{s=1}^N p_s Y_s \\ \text{s.t. } & A_F \left(\sum_{s=1}^N \xi_s Y_s^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} = \bar{Y}. \end{aligned}$$

here η denotes the elasticity of substitution between goods from different intermediate sectors, and $\sum_{s=1}^N \xi_s = 1$. We then have that the demand for each intermediate good is given by:

$$Y_s^D = \left(\frac{\xi_s \tilde{P}}{p_s} \right)^\eta \frac{Y}{A^F} = \left(\frac{\xi_s}{p_s} \right)^\eta \frac{Y}{A^F}$$

where,

$$\tilde{P} = \left(\sum_{s=1}^N p_s^{1-\eta} \xi_s^\eta \right)^{\frac{1}{1-\eta}} = 1,$$

is the ideal price index, which equals one due to our choice of numéraire.

The intermediate good supplied by each sector is an aggregate of the net output of the firms described in the main section:

$$Y_s^S = \int \{Q_s(z, L, R, u)\} dG_s(R, z) \equiv \int \{z(\Gamma_s L + (1 - \Gamma_s) u R)^{\theta_s}\} dG_s(R, z).$$

In this context, we interpret the revenue-shifter shock as an idiosyncratic productivity shock faced by each firm. The solution of the intermediate firms' problem, as described in section 2, determines the labor demand coming from each sector s . The firm's problem also leads to an individual robot demand. The expenditures faced by the firm to purchase, maintain and adjust the robot stock are given by:

$$\Psi_s \equiv \int \left\{ mR + p_R I_s^*(R, z) + \frac{\psi_R}{2} (I_i^*(R, z))^2 \right\} dG_s(R, z).$$

C.2 Households

The economy is populated by a measure-one mass of hand-to-mouth agents. The representative household also receives all profits and adjustment costs in the economy.³¹ Accordingly, the household's problem reads,

$$\begin{aligned} & \max_{c_w, \ell} U(c_w, \ell) \\ & \text{s.t. } c_w = w\ell + \Pi + \Psi, \end{aligned}$$

where $\Psi = \sum_{s=1}^N \Psi_s$. Since we are not concerned with wealth effects on the labor supply, we shut down this channel assuming GHH preferences,

$$U(c_w, \ell) = \frac{\left(c_w - \chi \frac{\varphi}{1+\varphi} \ell^{1+\frac{1}{\varphi}} \right)^{1-\frac{1}{\eta}} - 1}{1 - \frac{1}{\eta}}.$$

A simple derivation gives the optimal labor supply of the household,

$$\ell^*(w) = \left(\frac{w}{\chi p_y} \right)^\varphi,$$

which, as usual for GHH preferences, only depends on the level of the real wage.

C.3 Equilibrium

The model is closed requiring equilibrium in the main markets. Labor market clearing requires,

$$\ell^*(w) = L^d(w, \mathbf{p}),$$

where labor demand is given by the sum of sectoral labor demands,

$$L^d(w, \mathbf{p}) = \sum_{s=1}^N \int_0^\infty L_s(w, p_s, R, z) dG_s(R, z).$$

Final goods' market clearing requires,

$$Y = c_w.$$

³¹We could equivalently have assumed that there is a sector producing all these goods/services which aggregates intermediaries in the same way as the final good. This equivalent formulation stresses that ultimately robots produced will generate income for somebody in the economy.

The remaining market clearing conditions are simply given by equating demand and supply for intermediate goods,

$$Y_s^S(p_s) = Y_s^D(p_s) \quad \forall s = 1, \dots, N.$$

We define a *stationary equilibrium* in a N -sector economy as follows. Given a Markovian stochastic process for the productivity shock z_t for each sector s that admits a stationary distribution with CDF $F_s(z)$, , exogenous productivity parameter A^F and robot prices p_R , a stationary equilibrium is given by a set of CDF's $G_s(R, z)$ prices $\{w, \{p_s\}\}$, allocations $\{L_s, R, I, Y_s, Y, \ell, c_w\}$, firms' values $V_s(R, z)$, utilization choices u , satisfying

1. Individual optimal labor supply:

$$\ell^*(w) = \left(\frac{w}{\chi} \right)^\varphi;$$

2. Optimal workers' consumption:

$$c_w = w\ell + \Pi + \Psi;$$

3. Final goods' production function:

$$Y = A^F \left(\sum_{i=1}^N \xi_s Y_s^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}};$$

4. Perfect competition in the final goods' sector (price of the final good equals unit cost):

$$1 = \left(\sum_{s=1}^N p_s^{1-\eta} \xi_s^\eta \right)^{\frac{1}{1-\eta}};$$

5. Cost minimization by final goods' producers:

$$Y_s^D = \left(\frac{\xi_s}{p_s} \right)^\eta \frac{Y}{A^F} \quad \forall s = 1, \dots, N;$$

6. Static profit optimization by firms for labor:

$$\begin{aligned} L_s^*(R, z; w, p_s) &= \mathbf{1} \left\{ u_s^*(R, z) R \leq \left(\frac{w}{p_s z \Gamma_s \theta_s} \right)^{\frac{1}{\theta_s - 1}} \frac{1}{1 - \Gamma_s} \right\} \times \\ &\quad \times \frac{1}{\Gamma_s} \left(\left(\frac{w}{p_s z \Gamma \theta} \right)^{\frac{1}{\theta_s - 1}} - (1 - \Gamma_s) u_s^*(R, z) R \right) \quad \forall s; \end{aligned}$$

7. Static profit optimization by firms for utilization (use only if positive labor savings, get as close

as possible to desired size):

$$u_s^*(R, z; w, p_s) = \mathbf{1} \left\{ R \leq \left(\frac{p_s z \theta (1 - \Gamma_s)^{\theta_s}}{m} \right)^{\frac{1}{1-\theta_s}} \right\} + \\ + \mathbf{1} \left\{ R > \left(\frac{p_s z \theta_s (1 - \Gamma_s)^{\theta_s}}{m} \right)^{\frac{1}{1-\theta_s}} \right\} \times \\ \times \frac{1}{(1 - \Gamma_s)R} \left(\frac{m}{(1 - \Gamma_s)\theta_s p_s z} \right)^{\frac{1}{\theta_s-1}} \quad \forall s;$$

8. Optimal individual investment by firms to solve the firms' problem given value $V(R, z)$:

$$I_s^*(R, z; w, p_s) = \arg \max_I \Pi_s(R, z) - p_R I - \frac{\psi}{2}(I)^2 + E_s \left[\frac{dV_s(R, z)}{dt} \right] \quad \forall s;$$

9. Value of an individual firm:

$$\rho V_s(R, z) = \Pi_s(R, z) - p_R I_s^*(R, z) - \frac{\psi}{2}(I_s^*(R, z))^2 + E_s \left[\frac{dV_s(R, z)}{dt} \right] \quad \forall s;$$

10. Law of motion of individual robot stocks:

$$dR_s = [I_s^*(R, z) - \delta R_s] dt \quad \forall s;$$

11. Labor market equilibrium:

$$\sum_{i=1}^N \left[\int L_i^*(R, z; w, p_s) dG_s(R, z) \right] - \ell^*(w) = 0;$$

12. Intermediate goods' market clearing:

$$\int \left\{ z (\Gamma L_s^*(R, z; w, p_s) + (1 - \Gamma) u_s^*(R, z) R)^\theta \right\} dG_s(R, z) - Y_s^D(p_s) = 0 \quad \forall i;$$

13. Final goods' market clearing:

$$Y = c_w;$$

14. Kolmogorov Forward Equation for the stationary CDF of firms:

$$\frac{dG_s(R, z)}{dt} = 0 \quad \forall s.$$

D Linear Costs Solution Algorithm

In this appendix, we describe the solution algorithm adopted to solve the model in Section 4.4. We follow a scheme similar to Achdou et al. (2017), making the adjustments that are required by the problem at hand.

Consider a discretization of the state space on a increasing grid for R, z with N_R and N_z points respectively. First, recall that the policy is to adjust immediately if outside of the inaction region, bringing the state variable all the way to the boundary, and to be inactive otherwise. This means that a conventional PDE defines the value function within the inaction region, while the value function outside of the inaction region is a simple linear function of the value function evaluated at the boundary. We seek the following matrix representation of the optimized HJB equation,

$$\rho \mathbf{v} = \mathbf{u} + A\mathbf{v}.$$

Where \mathbf{v} is a vector of length $N_z \times N_R$, and A is a (sparse) matrix that we describe below. Consider first a case with just one value of z . Given an initial guess for the value function $V^0(R)$, we iterate on the following steps until convergence.

Step 1. Find cutoffs R_{inv} and R_{disinv} . We compute the forward and backward differences to approximate the derivative of the value function, dV^F, dV^B . By the concavity of the value function, we know that these two objects are decreasing in R . Therefore, we divide the state space by finding the first occurrence (starting from R_0 , the smallest value on the grid) of an index i such that,

$$dV^F(R_i) < p_R + \psi_+.$$

This value for R_i gives us the first value of robots strictly inside the inaction region. Therefore we set $R_{\text{inv}}^* \equiv R_{i-1}$. We proceed analogously to find the cutoff $R_{\text{disinv}}^* \equiv R_j$, where j is defined as the index such that,

$$dV^B(R_j) < p_R - \psi_-.$$

Note that the above procedure imposes that the two cutoffs lie on the grid and that the inaction region contains at least one point. While this reduces the accuracy of the solution, the error in the computation of the cutoffs vanishes as the size of the grid for R increases. Moreover, this greatly improves the numerical stability of the algorithm. Given these cutoffs, we can define the inaction region consistently with the main text as: $\{R_{\text{inv}}, \dots, R_{\text{disinv}}\}$. Investment will then be positive for all indexes i such that $R_i < R_{\text{inv}}$ and negative for all indexes j such that $R_j > R_{\text{disinv}}$. Inside the inaction region, the robot stock will depreciate at rate δ . Now we note that the optimal solution for investment entails,

$$V_R(R_i) = p_R + \psi_+ \quad \forall i | R_i < R_{\text{inv}},$$

and,

$$V_R(R_j) = p_R - \psi_- \quad \forall j | R_j > R_{\text{disinv}}.$$

Integrating, we immediately get,

$$V(R_i) = V(R_{\text{inv}}) - (\psi_+ + p_R)(R_{\text{inv}} - R_i), \quad \forall i | R_i < R_{\text{inv}},$$

and,

$$V(R_j) = V(R_{\text{disinv}}) - (-\psi_- + p_R)(R_{\text{disinv}} - R_j), \quad \forall j | R_j > R_{\text{disinv}}.$$

Step 2. By the above results, we can rewrite the above matrix representation as follows, denoting by i_{inv} the index such that $R_{\text{inv}} = R_{i_{\text{inv}}}$, and similarly for i_{disinv} :

$$\begin{bmatrix} V(R_0) \\ V(R_1) \\ \vdots \\ V(R_{i_{\text{inv}}-1}) \\ \rho V(R_{i_{\text{inv}}}) \\ \vdots \\ \rho V(R_{i_{\text{disinv}}}) \\ V(R_{i_{\text{disinv}}+1}) \\ \vdots \\ V(R_{\text{end}}) \end{bmatrix} = \begin{bmatrix} -(\psi_+ + p_R)(R_{i_{\text{inv}}} - R_0) \\ -(\psi_+ + p_R)(R_{i_{\text{inv}}} - R_1) \\ \vdots \\ -(\psi_+ + p_R)(R_{i_{\text{inv}}} - R_{i_{\text{inv}}-1}) \\ \Pi(R_{i_{\text{inv}}}) \\ \vdots \\ \Pi(R_{i_{\text{disinv}}}) \\ -(\psi_- - p_R)(R_{i_{\text{disinv}}+1} - R_{i_{\text{disinv}}}) \\ \vdots \\ -(\psi_- - p_R)(R_{\text{end}} - R_{i_{\text{disinv}}}) \end{bmatrix} + \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 & 0 & \cdots & \cdots & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \cdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \cdots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & \cdots & \delta R_{i_{\text{inv}}} & -\delta R_{i_{\text{inv}}} & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \vdots & \cdots & \delta R_{i_{\text{disinv}}} & -\delta R_{i_{\text{disinv}}} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & \cdots & \cdots & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \cdots & \cdots & 0 & 1 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} V(R_0) \\ V(R_1) \\ \vdots \\ V(R_{i_{\text{inv}}-1}) \\ V(R_{i_{\text{inv}}}) \\ \vdots \\ V(R_{i_{\text{disinv}}}) \\ V(R_{i_{\text{disinv}}+1}) \\ \vdots \\ V(R_{\text{end}}) \end{bmatrix}$$

We can then update the value function using either an iterative or implicit scheme as described in the appendix to [Achdou et al. \(2017\)](#).

Extending the problem to multiple price levels is trivial. The only difference is that the indexes denoting the inaction region will vary for each z . Moreover, the matrix described above becomes only one of the diagonal blocks in a bigger sparse matrix. Differently from the standard case, outflows from each block into other blocks are only allowed for states inside the inaction region within each block. Indeed, outside of these blocks, the linearity of the value function is ensured as de-

scribed above, and there is no jump over z to be included in the matrix A , as control is instantaneous.

In order to compute the stationary distribution, we proceed as in [Achdou et al. \(2017\)](#), by using the adjoint of the matrix A , A^T , to iterate on an initial guess. The only difference is that now we have to ensure that any firm starting outside the inaction region will eventually abandon it. To do so, we just add -1 to the diagonal of all indexes outside the inaction region, ensuring that the rows of A sum to zero and that therefore A^T is indeed an infinitesimal generator. The resulting matrix will therefore eventually push all the mass outside the inaction region to relevant cutoffs.