

Cryptography
Using Python

by

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Course Outline

- ✔ First Day—Python Crash Course
 - Programming Overview
 - Python Installations
 - Python Programming Concepts and Hands-on

Course Outline (cont.)

- Second Day—Cryptography using Python
 - Cryptography Overview
 - Caesar Ciphering
 - Modular Arithmetic
 - Modular Inverse
 - Greatest Common Divisor
 - Affine Ciphering
 - Attacking Encrypted Messages

Course Outline (cont.)

- Third Day—Advanced Topics
 - Vigenère and one-time pad ciphers
 - Base64 CODEC
 - Hashing and password verification
 - DES and AES Cryptography
 - Public Key and Digital Signature
- References

Review

• Solve the Google Assessment you have received in your email.

Cryptography

- Cryptography—it is the process of converting a readable document to a non-readable document, to hide the content of the original document while transferring it over shared communication channel(s).
- A secrete key—it is the key that the cryptography algorithm use to encrypt or decrypt a message.
- Cryptography algorithm—it is the procedure followed to apply the secrete key to a plain-text (or encryptedtext) to transform it into an encrypted-text (or decrypted plaint-ext).

Cryptography Algorithm Categories

- Symmetric—same key for encryption and decryption processes.
 - Stream cipher (Ceasar, Affine, ...)
 - Block cipher (DES, 3DES, AES, ...)
- Asymmetric—different keys for encryption and decryption processes. (Public key cryptography)
- Protocol-Based Cryptography

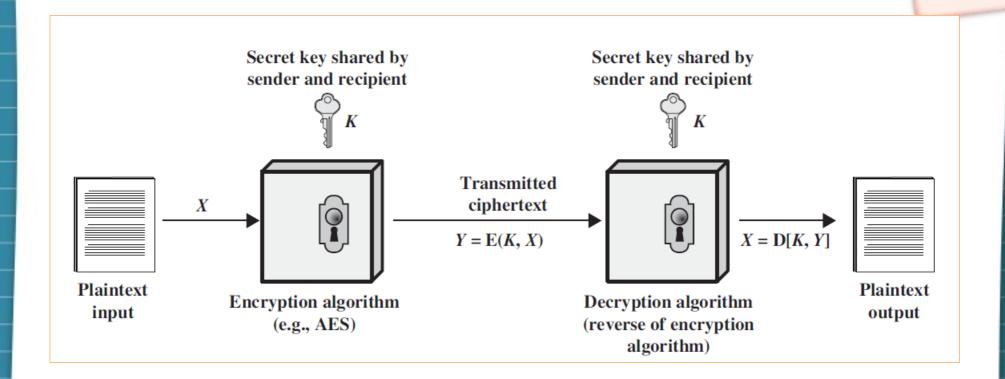
Cryptography (cont.)

- Attacks strategies—there are two strategies for attacking encrypted messages.
 - The first is to try all possible keys (Brute-Force attack).
 - The second is to analyze the encrypted message and detect patterns to be the guide for penetration (Cryptanalysis).

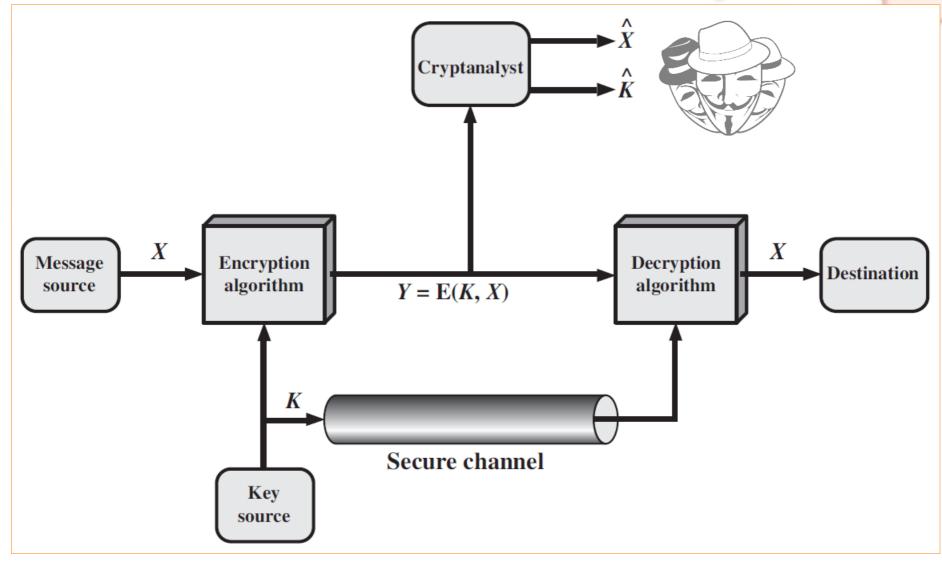
Steganography

- It is another form of cryptography
- Its philosophy depend on hiding a secret message within a larger one in such a way that others cannot discern the presence of the contents of the hidden message.
- Example of steganogrphy—hiding secrete messages into images (or video!!)

Symmetric Cryptography



Symmetric Cryptography Scenario



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Ciphering

- Ciphering—is a simple symmetric cryptography technique that depends on substituting each character of a plain-text by another one to encrypt the original message.
- The reverse cipher—is the simplest ciphering way to encrypt/decrypt a text message by just reversing it...
- Caesar Cipher (additive cipher)—works by substituting each letter of a message with a new letter after shifting the alphabet over.
 - The same key is used for both encryption and decryption processes...so it is symmetric/asymmetric cryptography?
 - The key is limited: 0 > key < length of the symbol set

Lab—Saesar Ciphering

Additive Cipher

- see "caesar.ipynb"
 - Notice the use of the following:
 - import statement
 - constants and variables
 - for loops
 - if , else , and elif statements
 - "in" and "not in" operators
 - find() string method
 - wraparound the symbol set

Pyperclip is a cross-platform Python module for copy and paste clipboard functions. It works with Python 2 and 3.

Assignments

- Using caesarCipher.py, encrypt the following sentences with the given keys:
 - "You can show black as white by argument," said Filby, "but you will never convince me." with keys 8 and 23, -15
 - Try encrypting the message with key 77
 Why ?????

Suggest a solution ...

Multiplicative Cipher

- It is similar to the Caesar cipher but encrypts using multiplication rather than addition.
- For multiplicative cipher, if the key is equal 11, here's the mapping you would end up with:

'ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz12 34567890 !?.'

'ALWhs4ALWhs

 Notice that the symbols A, G, and M all encrypt to the same letter, A.

Multiplicative Cipher (cont.)

- The multiplicative cipher has two concepts to be concerned: the Modular inverse and the Greatest Common Divisor (GCD) of the proper key.
- Each of the two concepts uses the modular operator %
- For the multiplicative cipher, the key and the size of the symbol set must be *relatively prime* to each other.
- Two numbers are relatively prime (or co-prime) if their Greatest Common Divisor is 1.

А	В	С	D	Е	F	G	Н	1	J	K	L	М	N	0	Р	Q	R	S	Т	U	٧	W	Х	Υ	Z	AA
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
	Α	В	С	D	Е	F	G	Н	1	J	K	L	M	N	0	Р	Q	R	S	Т	U	V	W	Х	Υ	Z
Vov. – 2	2	4	6	8	10	12	14	16	18	20	22	24	26	2	4	6	8	10	12	14	16	18	20	22	24	26
Key = 2	В	D	F	Н	J	L	Ν	Р	R	Т	V	Х	Z	В	D	F	Н	J	L	N	Р	R	Т	V	Х	Z
Vov. – 2	3	6	9	12	15	18	21	24	1	4	7	10	13	16	19	22	25	2	5	8	11	14	17	20	23	26
Key = 3	С	F	1	L	0	R	U	X	Α	D	G	J	M	Р	S	٧	Υ	В	Ε	Н	K	N	Q	Т	W	Z
Gcd(3,26)	1																									

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Greatest Common Divisor (GCD)

- GCD—is the greatest common factor among positive numbers.
- Factorization is used to find the GCD of two numbers.
- Example:
 - $-\gcd(24,30)=6$
 - Factors of 24 are: 1, 2, 3, 4, 6, 8, 12, 24
 - Factors of 30 are: 1, 2, 3, 5, **6**, 10, 15, 30

Greatest Common Divisor (cont.)

- Euclid's Algorithm for Finding the GCD
 - Given two integers 0 < b < a, make a repeated division that eventually terminate with a zero remainder: $\frac{a}{b} = q + \frac{r}{b}$
 - Where q is the quotient, and the remainder r satisfies $0 \le r < b$. If we clear fractions, this is the equation a = bq + r.

$$30 = 24 (1) + 6$$

 $24 = 6 (4) + 0$
 $gcd(24, 30) = 6$

$$a = bq_1 + r_1, 0 < r_1 < b,$$

$$b = r_1q_2 + r_2, 0 < r_2 < r_1,$$

$$r_1 = r_2q_3 + r_3, 0 < r_3 < r_2,$$

$$...$$

$$r_{j-2} = r_{j-1}q_j + r_j, 0 < r_j < r_{j-1}$$

$$r_{j-1} = r_jq_{j+1}.$$

• The greatest common divisor gcd(a,b) of a and b is r_j , the last nonzero remainder in the division process.

Greatest Common Divisor (cont.)

Example 1. Find the gcd of 42823 and 6409.

```
42823 = 6409(6) + 4369
6409 = 4369(1) + 2040
4369 = 2040(2) + 289
2040 = 289(7) + 17
289 = 17(17) + 0
Therefore gcd(42823, 6409) = 17.
```

See the "gcd.py"

```
def gcd(a, b):
    while a != 0:
        a, b = b % a, a
    return b
```

Notice that: 0 < a < b

Quiz

- What is the GCD of 10 and 15? using factorization
- The GCD of 17 and 31 is 1. Are 17 and 31 relatively prime?
- Why aren't 6 and 8 relatively prime?
- Exercise: Find the gcd's of the following pairs of numbers using Euclid's Algorithm.
 - 7469 and 2464
 - 2689 and 4001

Modulo Operator

- Mathematical operation based on modulo operator is called modular arithmetic or clock arithmetic.
- The modulo operator % is used in cryptography to wraparound the symbol set.
 - Example: if the clock is 15, where is the arrow on the clock?

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Modular Inverse

- If modular operator is used for encryption, modular inverse should be used for decryption.
- Modular inverse i for two numbers a, m is

$$(a * i) % m = 1$$

- Modular inverse is a brute-force process...
- The brute-force approach starts by testing the integer 1, and then 2, and then 3, and so on.

Modular Inverse (cont.)

- Example—The modular inverse of 5 mod 7 would be some number i where (5 * i) % 7 is equal to 1.
- You can brute-force this calculation like this:
 - 1 isn't the modular inverse of 5 mod 7, because (5 * 1) % 7 = 5.
 - 2 isn't the modular inverse of 5 mod 7, because (5 * 2) % 7 = 3.
 - 3 is the modular inverse of 5 mod 7, because (5 * 3) % 7 = 1.
- This process is time-consuming for large keys.
- Instead, use Euclid's extended algorithm to find the GCD of two numbers.

Euclid's Extended Algorithm

• Given two integers 0 < b < a, gcd(a, b) = r_j

$$a = bq + r$$

 $r = a - bq$

$$r_{1} = a - bq_{1},$$

$$r_{2} = b - r_{1}q_{2},$$

$$r_{3} = r_{1} - r_{2}q_{3},$$

$$...$$

$$r_{j-1} = r_{j-3} - r_{j-2}q_{j-1}$$

$$r_{j} = r_{j-2} - r_{j-1}q_{j}.$$

- Then, in the last of these equations , replace r_{j-1} with its expression in terms of r_{j-3} and r_{j-2} from the equation immediately above it. Continue this process successively, replacing r_{j-2} , r_{j-3} , ... until you obtain the final equation $r_j = ax + by$,
- In the special case that gcd(a, b) = 1, the integer equation reads 1 = ax + by. Therefore we deduce 1 ≡ by mod a, so that (the residue of) y is the multiplicative inverse of b, mod a.

 Example: Find the multiplicative inverse of 8 mod 11, using the Euclid's extended algorithm.

$$(8 * ?) \% 11 = 1$$

• Solution. We'll organize our work carefully. We'll do the Euclidean Algorithm in the left column It will verify that gcd(11, 8) = 1. Then we'll solve for the remainders in the right column, before back solving:

$$11 = 8(1) + 3 | 3 = 11 - 8(1)$$

 $8 = 3(2) + 2 | 2 = 8 - 3(2)$
 $3 = 2(1) + 1 | 1 = 3 - 2(1)$
 $2 = 1(2)$

Now reverse the process using the equations on the right.

$$1 = 3 - 2(1)$$

$$1 = 3 - (8 - 3(2))(1) = 3 - (8 - (3(2)) = 3(3) - 8$$

$$1 = (11 - 8(1))(3) - 8 = 11(3) - 8(4) = 11(3) + 8(-4)$$

• Therefore $1 \equiv 8(-4) \mod 11$, or if we prefer a residue value for the multiplicative inverse, $1 \equiv 8(7) \mod 11$.

Let us examine the previous example. Find the modular inverse of 5 mod 7.

Find the gcd(7,5)	Modular inverse							
7 = 5(1) + 2	2 = 7 - 5(1)							
5 = 2(2) + 1	1 = 5 - 2(2)							
2 = 1(2) + 0	Therefore:							
gsd(5,7) = 1	1 = 5 - 2 (7 - 5(1)) $1 = 5 + 7(2) + 5(2)$ $1 = 5(3) + 7(2)$							
5, 7 are co-prime	1 – 3(3) + 7(2)							
	Then 3 is the modular inverse of 5%7							

- As long as the two numbers a and m you pass to Euclid's function are relatively prime, you will receive the modular inverse of a: (a*i)%m=1.
- see "findmodinverse.ipynb".
 - "gcd.py" should be in the same folder
- Examine the program by yourself.
- For further explanation of the Euclid's algorithm, see "Euclid.pdf".

```
In [201]: import gcd
          def findModInverse(a, m):
              if gcd.gcd(a, m) != 1:
                  return None # No mod inverse if a & m aren't relatively prime.
              u1, u2, u3 = 1, 0, a
             v1, v2, v3 = 0, 1, m
             while v3 != 0:
                 q = u3 // v3 # Note that // is the integer division operator.
                 \#print(u3, ' = ', v3, '(', q, ')', ' + ', u3%v3)
                  print(u3%v3, ' = ', u3, ' - ', v3, '(', q, ')')
                 v1, v2, v3, u1, u2, u3 = (u1 - q * v1), (u2 - q * v2),
                                         (u3 - a * v3), v1, v2, v3
              return u1%m
In [202]: print(findModInverse(7, 5))
          2 = 7 - 5(1)
          1 = 5 - 2(2)
          0 = 2 - 1(2)
```

Assignment

Apply findModInverse() to display the following

```
The steps to find the Modular inverse of 56 , 15 are:

11 = 56 - 15 ( 3 )

4 = 15 - 11 ( 1 )

3 = 11 - 4 ( 2 )

1 = 4 - 3 ( 1 )

0 = 3 - 1 ( 3 )

The Modular inverse is: 11
```

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Summary and Simple Explanation

- if *n* is the length of the character set.
- and **e** is the encryption key (a random integer).
- e, n should be co-prime numbers satisfying the rule that gcd(e,n) = 1 following the Euclid's Algorithm.
- The decryption key is a brute-force process based on the rule (e * d) % n = 1
- Instead Euclid's extended Algorithm is used to estimate the modular inverse.

Quiz

Is gcd(a,m) equivalent to gcd(m,a)?

How?

Why?

 Based on the previous question, is there a difference between the gcd() implementation based on Euclid's algorithm and the traditional factorization method?

Affine Cipher

- Affine cipher is actually the multiplicative cipher combined with the Caesar cipher.
- It needs two keys: one for the multiplicative cipher and another for the Caesar cipher.
- How many available keys can Affine cipher has?

Affine Cipher (cont.)

- How Many Keys Can the Affine Cipher Have?
 - Each of the additive and multiplicative key is limited to the size of the symbol set.
 - When you multiply 66 possible Key A keys by 66 possible Key B keys, the result is 4356 possible combinations.
 - Then when you subtract the integers that can't be used for Key A because they're not relatively prime with 66, the total number of possible key combinations for the Affine cipher drops to 1320.

Lab—Affine Cipher

- see "affinecipher.ipynb"
- simple run

```
Key: 2894
Key A and Key B: 43 56
Encrypted text:
"5QG9ol3La6QI93!xQxaia6faQL9QdaQG1!!axQARLa!!A-5!1RQP36ARu
Full encrypted text copied to clipboard.
```

- There is an unused function. Which one?
- Assignment: randomize the selection of the additive and the multiplicative keys. Loop the selection until the keys are valid.

Quizzes

- The affine cipher is the combination of which two other ciphers? _____ and ____
- What is a tuple? How is a tuple different from a list?
- If Key A is 1, why does it make the affine cipher weak?
- If Key B is 0, why does it make the affine cipher weak?
- Does the main() function of affineCipher.py get called if another program runs import affineCipher?

Hacking Encrypted Messages

- Hacking the Caesar cipher using brute-force technique.
 - See the "CaesarHacker.ipynb"
 - the brute-force technique
 - the range() function
 - string formatting (string interpolation)

Detect English

- Hacking encrypted messages using the CryptAnalysis strategy by detecting English phrase
- It is also used to validate the decrypted "English" message.
 - See the "detectEnglish.ipynb"
 - use of dictionary { }
 - Evaluate the program

Lab—Hacking the Affine Algorithm

- Because Affine cipher is limited to only a few thousand keys. It is easily possible to perform a brute-force attack against it.
- See "affineHacker.ipynb"
- Sample Run

```
III LEU NEY ZOOB... ( KNILO47: VBN.O: DUNUVUVB: VN ONSVN
Tried Kev 2890... ("okCOMPUTERkWOULDkDESERVEKTOkBEKCALLEDkI)
Tried Key 2891... ("!8Zljmrqbo8tlria8abpbosb8ql8Yb8ZXiiba8f)
Tried Key 2892... ("UQw970A.y!QC9A6xQxy?y!ByQ.9QvyQwu66yxQ3)
Tried Key 2893... ("rnFRPSXWHUnZRXOGnGHVHUYHnWRnEHnFD00HGnL)
Tried Key 2894... ("A computer would deserve to be called i)
Possible encryption hack:
Kev: 2894
Decrypted message: "A computer would deserve to be called intellige
that it was human." -Alan Turing
Enter D if done, anything else to continue hacking:
> d
key is: 2894
Hacking time is: 8.77 seconds
Copying hacked message to clipboard:
"A computer would deserve to be called intelligent if it could dece
n." -Alan Turing
```

Lab—Hacking the Affine Algorithm (cont.)

- Hacking process could be stopped at any time: hackAffine(message)
- The Total Number of Possible Keys is equal to (length of the symbol set) ** 2
- run the program by yourself...

Programming Tasks

Wrap-around the developed Caesar and Affine ciphering programs (encoding/decoding) and do perform your touch to them...

