Fast Rudin

Davi Sales Barreira

March 30, 2021

Abstract

This document are condensed notes on the book "Real and Complex Analysis" [Rudin, 1987]. The goal is to cover the first 9 chapters of the book.

1 Abstract Integration

Definition 1.1 (Topological Space)**.** (X, τ) is a topological space where τ is the collection of open sets in X, such that:

(i) $X \in \tau$;

(ii) Se $A_1, ..., A_n \in \tau \implies \cap_{i=1}^n A_i \in \tau$;

(iii) Se $A_\alpha \in \tau$ for any $\alpha \in \Lambda \implies \cup_{\alpha \in \Lambda} A_\alpha \in \tau$;

τ is the topology of X.

Definition 1.2 (Measurable Space)**.** $(X, \mathcal{F})$ is a measurable space where $\mathcal{F}$ is a σ-algebra in X. A σ-algebra is defined such that:

(i) $X \in \mathcal{F}$;

(ii) If $A \in \mathcal{F}$ then $A^c \in \mathcal{F}$;

(iii) If $A_n \in \mathcal{F}$ $\forall n \in \mathbb{N}$ then $\cup_{n \in \mathbb{N}} A_n \in \mathcal{F}$;

Definition 1.3 (Continuous Function)**.** For (X, τ) and (Y, τ') topological spaces, we say $f : X \to Y$ is continuous if for every open set $V \subset Y$ we have that $f^{-1}(V) \subset X$ is open.

Definition 1.4 (Measurable Function)**.** For $(X, \mathcal{F})$ a measurable space and (Y, τ') a topological space, we say $f : X \to Y$ is $\mathcal{F}$-measurable if for every open set $V \subset Y$ we have that $f^{-1}(V) \in \mathcal{F}$.

References

Walter Rudin. Real and complex analysis. 1987. *Cited on*, 156, 1987.