FinOpt-Workflow

October 19, 2025

1 FinOpt: Complete Workflow Validation

2 Configuration: Seasonality and Contribution

3 Initialize FinancialModel

```
[3]: # IncomeModel Instantiation
    income = IncomeModel(
        fixed=FixedIncome(
            base=1_480_000.0,
                                # CLP/month
            annual_growth=0.03, # 3% nominal annual growth
            salary_raises={
                date(2026, 4, 1): 400_000, # +400k in April 2026
                date(2027, 4, 1): 400_000 # +400k in April 2027
            },
            name="fixed"
        ),
        variable=VariableIncome(
            base=140_000.0,
                                    # Base variable income
            seasonality=seasonality_variable,
                                    # 10% monthly noise
            sigma=0.10,
```

```
floor=0.0,
                                # No negative income
                                # Maximum 400k/month
        cap=400 000.0.
        annual_growth=0.0,
                                # No growth in variable
        name="variable"
    ),
    monthly_contribution = monthly_contribution
)
# --- Account configuration ---
accounts = [
    Account.from annual(
        name="Cuenta Ahorro Vivienda (BE)",
        annual_return=0.025,
        annual_volatility=0.01,
        initial_wealth=0
    ),
    Account.from_annual(
        name="Conservative Clooney (Fintual)",
        annual_return=0.08,
        annual_volatility=0.09,
        initial_wealth=0
    ),
    Account.from_annual(
        name="Moderate Pitt (Fintual)",
        annual_return=0.12,
        annual_volatility=0.13,
        initial_wealth=0
    )
]
# --- Correlation matrix (2x2) ---
# UF portfolio have moderate positive correlation (rho = 0.2)
correlation_matrix = np.array([
    [1.0, 0.0, 0.0],
    [0.0, 1.0, 0.5],
    [0.0, 0.5, 1.0]
])
# --- Initialize FinancialModel ---
model = FinancialModel(income, accounts, default_correlation =_
⇔correlation matrix)
model
```

[3]: FinancialModel(M=3, accounts=['Cuenta Ahorro Vivienda (BE)', 'Conservative Clooney (Fintual)', 'Moderate Pitt (Fintual)'], cache=enabled)

4 Simulation parameters

```
[4]: # --- Simulation parameters ---
n_sims = 500
months = 48
start_date = date(2025, 10, 1)
```

5 Income Module

Total monthly income at time t is composed of fixed and variable parts:

$$Y_t = y_t^{\text{fixed}} + Y_t^{\text{variable}}$$

5.1 Fixed Income

The fixed component, y_t^{fixed} , reflects a baseline salary subject to compounded annual growth g and scheduled raises d_k, Δ_k (e.g., promotions or tenure milestones):

$$y_t^{\text{fixed}} = \text{current_salary}(t) \cdot (1+m)^{\Delta t}$$

where $m = (1+g)^{1/12} - 1$ is the **monthly compounded rate**, and Δt represents time since the last raise.

5.2 Variable Income

The variable component, Y_t^{variable} , models irregular income sources (e.g., freelance work or bonuses) with:

- Seasonality: $s \in \mathbb{R}^{12}$ (multiplicative monthly factors),
- Noise: $\epsilon_t \sim \mathcal{N}(0, \sigma^2)$ (Gaussian shocks),
- Growth: same compounded rate m applied to a base income level,
- Boundaries: optional floor and cap constraints.

The underlying stochastic projection is:

$$\tilde{Y}_t = \max(\text{floor}, \ \mu_t(1+\epsilon_t)), \quad \text{where } \mu_t = \text{base} \cdot (1+m)^t \cdot s_{(t \bmod 12)}$$

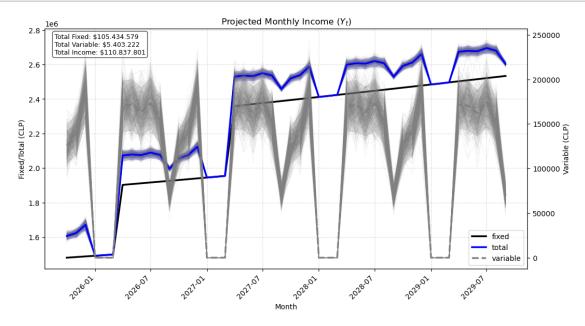
Then, guardrails are applied as:

$$Y_t^{\text{variable}} = \begin{cases} 0 & \text{if } \tilde{Y}_t < 0 \\ \tilde{Y}_t & \text{if } 0 \leq \tilde{Y}_t \leq \text{cap} \\ \text{cap} & \text{if } \tilde{Y}_t > \text{cap} \end{cases}$$

Note: In expectation (ignoring noise truncation), $\mathbb{E}[Y_t] = y_t^{\text{fixed}} + \mu_t$

5.3 Income Projection

Dual-axis plot with: - **Left axis**: Fixed income (deterministic) + Total income - **Right axis**: Variable income (stochastic with trajectories) - **Trajectories**: Individual noise realizations (n=300) - **Confidence band**: 95% CI for variable income

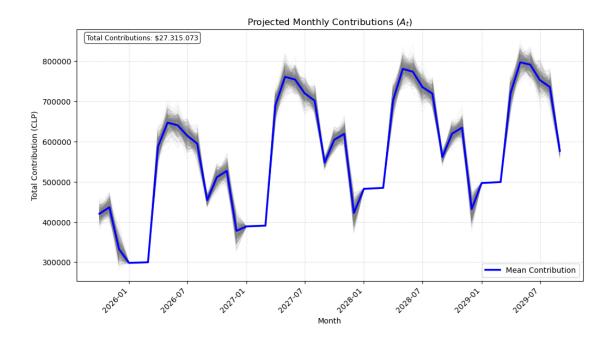


5.4 Contribution Projection

A fraction of income is allocated each month through calendar-rotating schedules:

$$A_t = \alpha_{(t \bmod 12)}^f \cdot y_t^{\text{fixed}} + \alpha_{(t \bmod 12)}^v \cdot Y_t^{\text{variable}}$$

where $\alpha^f, \alpha^v \in [0,1]^{12}$ control the fixed and variable contribution rates by applying the 12-month fractional arrays to projected incomes, rotated according to **start** date and repeated cyclically for horizons > 12 months.



6 Return Module

6.1 Multi-Account Return Model

For M accounts with correlated returns:

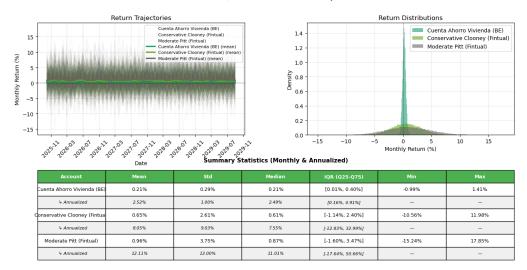
$$1 + R_t^m \sim \text{LogNormal}(\mu_{\text{log}}^m, \Sigma)$$

where $\Sigma = D \cdot \rho \cdot D$ is the covariance matrix: - $D = \operatorname{diag}(\sigma_{\log}^1, \dots, \sigma_{\log}^M)$ - $\rho \in \mathbb{R}^{M \times M}$ is the correlation matrix (symmetric, PSD)

6.2 Monthly Return Distribution (Marginal Analysis)

Visualizes IID monthly returns across both accounts with 4 panels: - **Trajectories**: Individual paths for each account - **Histograms**: Marginal distributions (overlaid) - **Statistics**: Mean, std, quantiles per account

Monthly Return Distributions (R_t^m)



T=48 months | n_sims=500 | seed=None

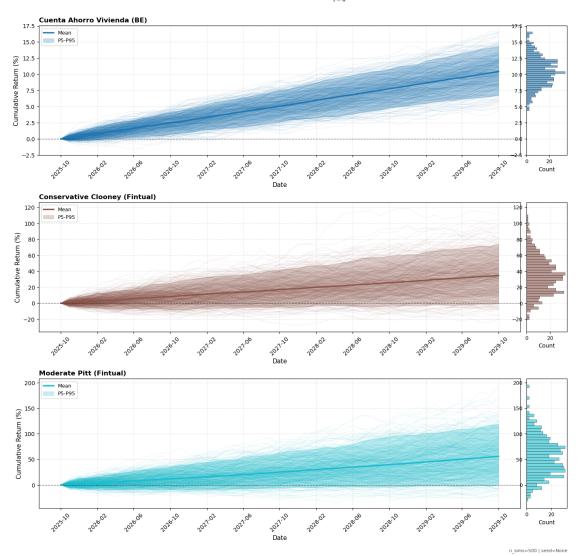
6.3 Cumulative Returns per Account

For M accounts with correlated returns:

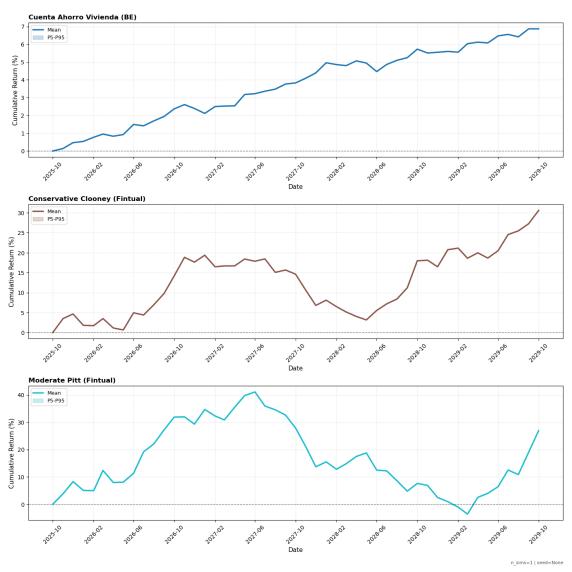
$$R_{\mathrm{cm}}^m(T) = \prod_{t=0}^{T-1} (1 + R_t^m) - 1$$

Cross-sectional correlation persists through time but does not compound.

Cumulative Returns per Account $\prod_{t=0}^{T-1} (1 + R_t^m) - 1$

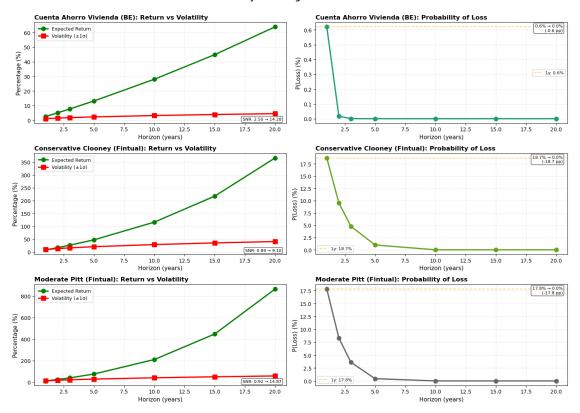


Random Example of Cumulative Returns per Account $(\frac{t=0}^{T-1}(1 + R_t^m) - 1)$



6.4 Horizon Analysis: Time Diversification by Account

Horizon Analysis: Average Portfolio Behavior



7 Portfolio Module

7.1 Wealth Projection Under Allocation Policy

Recursive dynamics: Wealth evolves as:

$$W_{t+1}^m = \left(W_t^m + A_t x_t^m\right) \left(1 + R_t^m\right)$$

We define $A_t^m = A_t \cdot x_t^m$ which is the contribution allocated to account m via policy $X = \{x_t^m\}_{t,m}$.

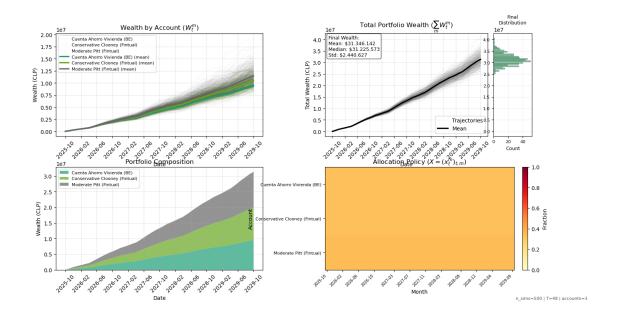
Closed-form representation:

$$W_t^m = W_0^m F_{0,t}^m + \sum_{s=0}^{t-1} A_s x_s^m F_{s,t}^m$$

with accumulation factor $F_{s,t}^m = \prod_{r=s}^{t-1} (1 + R_r^m)$.

Key insight: $W_t^m(X)$ is **linear affine in policy** $X \to$ analytical gradients enable convex optimization.

What to observe: - Top-left: wealth per account with Monte Carlo trajectories - Top-right: total wealth + final distribution histogram - Bottom-left: portfolio composition over time - Bottom-right: allocation policy heatmap



8 Goal-Driven Optimization

8.1 Problem Formulation

8.1.1 Non-Convex Chance Constraints

The **bilevel problem** seeks the minimum horizon T^* satisfying all goals:

$$\min_{T \in \mathbb{N}} \ T \max_{X \in \in \mathcal{F}_T} f(X)$$

where the **goal-feasible set** \mathcal{F}_T contains all policies $X \in \mathcal{X}_T$ satisfying probabilistic constraints: **Intermediate goals** (at fixed time t < T):

$$\mathbb{P}\big(W_t^m(X) \geq b_t^m\big) \geq 1 - \varepsilon_t^m, \quad \forall g \in \mathcal{G}_{\mathrm{int}}$$

Terminal goals (at horizon T):

$$\mathbb{P}\big(W^m_T(X) \geq b^m\big) \geq 1 - \varepsilon^m, \quad \forall g \in \mathcal{G}_{\mathrm{term}}$$

with decision space (simplex):

$$\mathcal{X}_T = \left\{ X \in \mathbb{R}^{T \times M} : x_t^m \geq 0, \ \sum_{m=1}^M x_t^m = 1, \ \forall t = 0, \dots, T-1 \right\}$$

Challenge: Chance constraints involve indicator functions $\mathbb{1}[\cdot]$, which are discontinuous and non-convex. Standard approaches (MILP, sigmoid smoothing) either scale poorly or find local optima.

8.2 Convex Reformulation via CVaR

8.2.1 CVaR Reformulation: Convex Upper Bound

We replace each chance constraint with a CVaR constraint (Rockafellar & Uryasev, 2000):

$$\boxed{\mathbb{P}(W \geq b) \geq 1 - \varepsilon \quad \Longleftrightarrow \quad \text{CVaR}_{\varepsilon}(b - W) \leq 0}$$

where the Conditional Value-at-Risk of shortfall L = b - W is:

$$\mathrm{CVaR}_\varepsilon(L) = \mathrm{VaR}_\varepsilon(L) + \frac{1}{\varepsilon} \mathbb{E} \big[(L - \mathrm{VaR}_\varepsilon(L))_+ \big]$$

Epigraphic formulation (convex, suitable for LP solvers):

$$\text{CVaR}_{\varepsilon}(L) = \min_{\gamma \in \mathbb{R}} \left\{ \gamma + \frac{1}{\varepsilon N} \sum_{i=1}^{N} [L^{i} - \gamma]_{+} \right\}$$

Introducing auxiliary variables $z^i \geq [L^i - \gamma]_+$:

$$\begin{split} \text{CVaR}_{\varepsilon}(L) &= \min_{\gamma,z} \left\{ \gamma + \frac{1}{\varepsilon N} \sum_{i=1}^{N} z^{i} \right\} \\ \text{s.t.} \quad z^{i} &\geq L^{i} - \gamma, \quad \forall i \in [N] \\ z^{i} &\geq 0, \quad \forall i \in [N] \end{split}$$

Key property: If W^i is affine in X (as in our wealth dynamics), then $\mathrm{CVaR}_{\varepsilon}(b-W)$ is convex in X.

8.2.2 Mathematical Relationship: Implication, Not Equivalence

Theorem (Rockafellar & Uryasev, 2000):

$$\text{CVaR}_{\varepsilon}(L) \leq 0 \implies \mathbb{P}(L \leq 0) \geq 1 - \varepsilon$$

Proof sketch: CVaR averages the worst ε -tail of the distribution. If this mean is non-positive, then at least $(1 - \varepsilon)$ of scenarios satisfy $L \le 0$.

The converse is NOT true: $\mathbb{P}(L \leq 0) \geq 1 - \varepsilon$ does NOT imply $\text{CVaR} \leq 0$ if the tail is heavy.

Interpretation: CVaR is a conservative approximation controlling both: 1. Frequency of violations (at most $\varepsilon \times 100\%$ scenarios fail) 2. Severity of violations (average loss in tail is non-positive)

The original chance constraint only controls frequency.

8.2.3 Convex Reformulated Problem - Multiple Objectives

Inner problem (for fixed T):

$$\begin{split} \max_{X,\gamma,z} & f(X) \\ \text{s.t.} & \sum_{m=1}^{M} x_t^m = 1, \quad \forall t = 0, \dots, T-1 \quad \text{(simplex)} \\ & x_t^m \geq 0, \quad \forall t, m \quad \text{(non-negativity)} \\ & z_g^i \geq (b_g - W_{t_g}^{i,m_g}(X)) - \gamma_g, \quad \forall g, i \quad \text{(epigraph)} \\ & z_g^i \geq 0, \quad \forall g, i \\ & \gamma_g + \frac{1}{\varepsilon_g N} \sum_{i=1}^N z_g^i \leq 0, \quad \forall g \quad \text{(CVaR constraint)} \end{split}$$

where: $W_t^{i,m}(X) = W_0^m F_{0,t,m}^i + \sum_{s=0}^{t-1} A_s^i \cdot x_s^m \cdot F_{s,t,m}^i$ (affine in X) - g indexes goals (both intermediate and terminal) - $t_g, m_g, b_g, \varepsilon_g$ are parameters of goal g - f(X) is a convex objective function (see supported objectives below)

Global optimality guaranteed via convex programming (interior-point methods).

8.2.4 Comparison: Original vs. CVaR

Observed conservativeness (empirical): - CVaR constraint: $\text{CVaR}_{\varepsilon}(L) \leq 0$ - Resulting violation rate: typically $(0.5-0.8) \times \varepsilon$ (better than required)

Example (from our results): - Goal: $\mathbb{P}(W_T \ge 1\text{M}) \ge 80\%$ (i.e., $\varepsilon = 20\%$) - CVaR solution: violation rate = 9% (margin of 11%)

The 11% buffer is the "price" of convexity, buying us certified global optimality and numerical stability.

References: - Rockafellar, R.T. & Uryasev, S. (2000). "Optimization of conditional value-atrisk." *Journal of Risk*, 2, 21-42. - Nemirovski, A. & Shapiro, A. (2006). "Convex approximations of chance constrained programs." *SIAM J. Optim.*, 17(4), 969-996.

8.3 Goal Specification

FINANCIAL GOALS SUMMARY

1.

Type: Terminal (horizon T to be optimized)

Account: Cuenta Ahorro Vivienda (BE)

Threshold: \$6,000,000 Confidence: 70% (=30%) 2.

Type: Terminal (horizon T to be optimized)
Account: Conservative Clooney (Fintual)

Threshold: \$5,000,000 Confidence: 75% (=25%)

3.

Type: Terminal (horizon T to be optimized)

Account: Moderate Pitt (Fintual)

Threshold: \$15,000,000 Confidence: 70% (=30%)

4.

Type: Intermediate (month 24)

Account: Cuenta Ahorro Vivienda (BE)

Threshold: \$2,200,000 Confidence: 80% (=20%)

8.4 Bilevel Optimization

8.4.1 Supported Convex Objectives

CVaROptimizer supports 7 convex objectives exploiting affine wealth structure: $W^{i,m}_t(X) = W^m_0 F^i_{0,t,m} + \sum_{s=0}^{t-1} A^i_s \cdot x^m_s \cdot F^i_{s,t,m}$

1. terminal_wealth — Growth maximization

$$\max_{X} \quad \mathbb{E}\left[\sum_{m=1}^{M} W_{T}^{m}\right] = \frac{1}{N} \sum_{i=1}^{N} \sum_{m=1}^{M} W_{T}^{i,m}(X)$$

2. min_cvar — Downside protection

$$\min_{X,\gamma,z} \quad \sum_{g \in \mathcal{G}} \left(\gamma_g + \frac{1}{\varepsilon_g N} \sum_{i=1}^N z_g^i \right)$$

3. low_turnover — Tax efficiency (lambda=0.1)

$$\max_{X} \quad \mathbb{E}\left[\sum_{m=1}^{M} W_T^m\right] - \lambda \sum_{t=1}^{T-1} \sum_{m=1}^{M} |x_t^m - x_{t-1}^m|$$

4. risk_adjusted — Markowitz mean-variance (lambda=0.5)

$$\max_{X} \quad \mathbb{E}\left[\sum_{m=1}^{M} W_{T}^{m}\right] - \lambda \cdot \operatorname{Var}\left(\sum_{m=1}^{M} W_{T}^{m}\right)$$

where
$$\text{Var}(W_T) = \frac{1}{N} \sum_{i=1}^{N} \left(W_T^i - \mathbb{E}[W_T]\right)^2$$

5. balanced — Multi-objective (lambda_risk=0.3, lambda_turnover=0.05)

$$\max_{X} \quad \mathbb{E}\left[\sum_{m=1}^{M} W_{T}^{m}\right] - \lambda_{r} \cdot \operatorname{Var}\left(\sum_{m=1}^{M} W_{T}^{m}\right) - \lambda_{t} \sum_{t=1}^{T-1} \sum_{m=1}^{M} |x_{t}^{m} - x_{t-1}^{m}|$$

6. min_variance — Capital preservation (target required)

$$\min_{X} \quad \operatorname{Var}\left(\sum_{m=1}^{M} W_{T}^{m}\right)$$
 s.t.
$$\mathbb{E}\left[\sum_{m=1}^{M} W_{T}^{m}\right] \geq W_{\operatorname{target}}$$

```
[13]: # --- Execute Optimization ---
      print("\n" + "=" * 70)
      print("STARTING BILEVEL OPTIMIZATION")
      optimizer = CVaROptimizer(n_accounts=model.M, objective='risk_adjusted')
      opt_result = model.optimize(
          goals=goals,
          optimizer=optimizer,
          T_{max}=120,
          n sims=300,
          seed=42,
          verbose=True,
          solver='ECOS', # 0 'SCS', 'CLARABEL'
          max iters=10000
      print(opt_result.summary())
      # Display optimization summary
      print("\n" + "=" * 70)
      print(opt_result.summary())
      print("=" * 70)
```

STARTING BILEVEL OPTIMIZATION

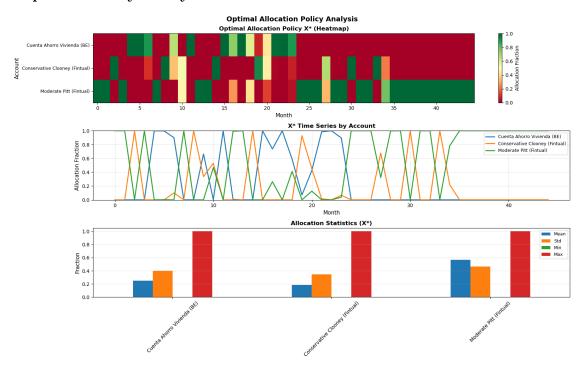
```
=== GoalSeeker: BINARY search T [24, 120] ===
[Iter 1] Binary search: testing T=72 (range=[24, 120])...

/home/mlioi/anaconda3/envs/finance/lib/python3.11/site-
packages/cvxpy/problems/problem.py:1539: UserWarning: Solution may be
inaccurate. Try another solver, adjusting the solver settings, or solve with
verbose=True for more information.
warnings.warn(
```

```
Feasible, obj=-2791158943700.42, time=2.752s
     [Iter 2] Binary search: testing T=48 (range=[24, 72])...
          Feasible, obj=-2459398695375.02, time=1.363s
     [Iter 3] Binary search: testing T=36 (range=[24, 48])...
          Infeasible, obj=0.00, time=0.136s
     [Iter 4] Binary search: testing T=42 (range=[37, 48])...
          Infeasible, obj=0.00, time=0.173s
     [Iter 5] Binary search: testing T=45 (range=[43, 48])...
          Feasible, obj=-2845265330067.62, time=0.529s
     [Iter 6] Binary search: testing T=44 (range=[43, 45])...
          Infeasible, obj=0.00, time=0.203s
     === Optimal: T*=45 (binary search converged) ===
     OptimizationResult(
       Status:
               Feasible
      Horizon: T=45 months
      Objective: -2845265330067.62
      Goals: 4 (1 intermediate, 3 terminal)
       Solve time: 0.529s
       Iterations: 0
     )
     ______
     OptimizationResult(
       Status:
               Feasible
       Horizon: T=45 months
       Objective: -2845265330067.62
       Goals: 4 (1 intermediate, 3 terminal)
       Solve time: 0.529s
       Iterations: 0
[14]: optimizer.objective
[14]: 'risk_adjusted'
```

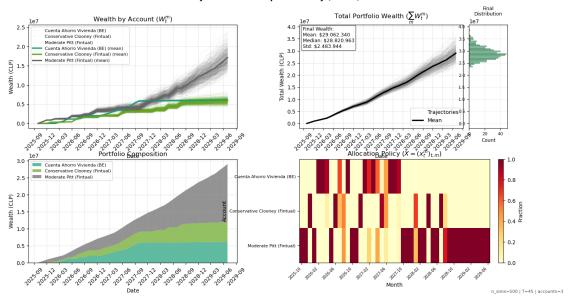
15

8.5 Optimal Policy Analysis



	======	======	=====	
OPTIMAL ALLOCATION SUMMARY				
	======	======	=====	
	Mean	Std	Min	Max
Cuenta Ahorro Vivienda (BE)	0.250	0.402	0.0	1.0
Conservative Clooney (Fintual)	0.185	0.347	0.0	1.0
Moderate Pitt (Fintual)	0.565	0.467	0.0	1.0
=======================================	======		=====	=======================================

Wealth Dynamics under Optimal Policy (T*=45)



8.6 Goal Verification

8.6.1 In-Sample Verification

GOAL VERIFICATION: IN-SAMPLE

Using optimization scenarios (n=300, seed=42)

Expected: All goals satisfied ()

Account: Cuenta Ahorro Vivienda (BE)

Threshold: \$6,000,000

Confidence: 70%

Status: SATISFIED

Violation rate: 12.33% (required 30.00%)

Margin: 17.67%

Median shortfall: \$29,467 N violations: 37 / 300

Account: Conservative Clooney (Fintual)

Threshold: \$5,000,000

Confidence: 75%

Status: SATISFIED

Violation rate: 10.67% (required 25.00%)

Margin: 14.33%

Median shortfall: \$233,420 N violations: 32 / 300

Account: Moderate Pitt (Fintual)

Threshold: \$15,000,000

Confidence: 70%

Status: SATISFIED

Violation rate: 14.33% (required 30.00%)

Margin: 15.67%

Median shortfall: \$575,870 N violations: 43 / 300

Account: Cuenta Ahorro Vivienda (BE)

Month: 24

Threshold: \$2,200,000

Confidence: 80%

Status: SATISFIED

Violation rate: 0.00% (required 20.00%)

Margin: 20.00% Median shortfall: \$0 N violations: 0 / 300

8.6.2 Out-of-Sample Verification

GOAL VERIFICATION: OUT-OF-SAMPLE

Using fresh scenarios (n=1000, seed=999)

Expected: All goals satisfied with safety margin

Account: Cuenta Ahorro Vivienda (BE)

Threshold: \$6,000,000

Confidence: 70%

Status: SATISFIED

Violation rate: 13.10% (required 30.00%)

Margin: 16.90%

Median shortfall: \$33,620 N violations: 131 / 1000

Account: Conservative Clooney (Fintual)

Threshold: \$5,000,000

Confidence: 75%

Status: SATISFIED

Violation rate: 10.90% (required 25.00%)

Margin: 14.10%

Median shortfall: \$194,851

N violations: 109 / 1000

Account: Moderate Pitt (Fintual)

Threshold: \$15,000,000

Confidence: 70%

Status: SATISFIED

Violation rate: 13.80% (required 30.00%)

Margin: 16.20%

Median shortfall: \$593,255 N violations: 138 / 1000

Account: Cuenta Ahorro Vivienda (BE)

Month: 24

Threshold: \$2,200,000

Confidence: 80%

Status: SATISFIED

Violation rate: 0.00% (required 20.00%)

Margin: 20.00%

Median shortfall: \$0 N violations: 0 / 1000

IN-SAMPLE vs OUT-OF-SAMPLE COMPARISON

In-Sample (n=300) Out-of-Sample (n=1000) Required ()

	In-Sample (n=300)	Out-of-Sample	(n=1000)	Required ()
0	0.1233		0.131	0.30
1	0.1067		0.109	0.25
2	0.1433		0.138	0.30
3	0.0000		0.000	0.20

 $[{\tt NbConvertApp}] \ \, {\tt Converting \ notebook \ FinOpt-Workflow.ipynb \ to \ pdf}$

[NbConvertApp] Support files will be in FinOpt-Workflow_files/

 $[{\tt NbConvertApp}] \ {\tt Making \ directory \ ./FinOpt-Workflow_files}$

[NbConvertApp] Writing 56166 bytes to notebook.tex

[NbConvertApp] Building PDF

[NbConvertApp] Running xelatex 3 times: ['xelatex', 'notebook.tex', '-quiet']

[NbConvertApp] Running bibtex 1 time: ['bibtex', 'notebook']

 $[{\tt NbConvertApp}] \ {\tt WARNING} \ | \ {\tt bibtex} \ {\tt had} \ {\tt problems}, \ {\tt most} \ {\tt likely} \ {\tt because} \ {\tt there} \ {\tt were} \ {\tt no}$

citations

[NbConvertApp] PDF successfully created

[NbConvertApp] Writing 2254492 bytes to FinOpt-Workflow.pdf

[NbConvertApp] Converting notebook FinOpt-Workflow.ipynb to html

[NbConvertApp] WARNING | Alternative text is missing on 9 image(s).

[NbConvertApp] Writing 3677141 bytes to FinOpt-Workflow.html

[]:[