

# FinOpt-Workflow

November 3, 2025

## 1 FinOpt: Complete Workflow Validation

## 2 Configuration: Seasonality and Contribution

```
[2]: # --- Seasonality pattern for variable income (12 months, Jan-Dec) ---

# Structure: months = ["Jan", "Feb", "Mar", "Apr", "May", "Jun",
#                      "Jul", "Aug", "Sep", "Oct", "Nov", "Dec"]

seasonality_variable = [0.00, 0.00, 0.00, 1.22, 1.22, 1.16,
                        1.24, 1.10, 0.50, 0.90, 1.00, 1.31]

monthly_contrib_fixed = [0.20, 0.20, 0.20, 0.22, 0.25, 0.25,
                         0.23, 0.23, 0.20, 0.20, 0.20, 0.10]

monthly_contrib_variable = [1.0] * 12

monthly_contribution = {"fixed": monthly_contrib_fixed, "variable": monthly_contrib_variable}
```

## 3 Initialize FinancialModel

```
[3]: # IncomeModel Instantiation

income = IncomeModel(
    fixed=FixedIncome(
        base=1_480_000.0,           # CLP/month
        annual_growth=0.03,         # 3% nominal annual growth
        salary_raises={
            date(2026, 4, 1): 400_000, # +400k in April 2026
            date(2027, 4, 1): 400_000  # +400k in April 2027
        },
        name="fixed"
    ),
    variable=VariableIncome(
        base=140_000.0,             # Base variable income
        seasonality=seasonality_variable,
        sigma=0.10,                 # 10% monthly noise
    )
)
```

```

        floor=0.0,                      # No negative income
        cap=400_000.0,                   # Maximum 400k/month
        annual_growth=0.0,               # No growth in variable
        name="variable"
    ),
    monthly_contribution = monthly_contribution
)

# --- Account configuration ---
accounts = [
    Account.from_annual(
        name="Cuenta Ahorro Vivienda (BE)",
        annual_return=0.025,
        annual_volatility=0.01,
        initial_wealth=1600000
    ),
    Account.from_annual(
        name="Conservative Clooney (Fintual)",
        annual_return=0.08,
        annual_volatility=0.09,
        initial_wealth=744747
    ),
    Account.from_annual(
        name="Moderate Pitt (Fintual)",
        annual_return=0.12,
        annual_volatility=0.13,
        initial_wealth=496879
    )
]

# --- Correlation matrix (2x2) ---
# UF portfolio have moderate positive correlation (rho = 0.2)
correlation_matrix = np.array([
    [1.0, 0.0, 0.0],
    [0.0, 1.0, 0.5],
    [0.0, 0.5, 1.0]
])

# --- Initialize FinancialModel ---
model = FinancialModel(income, accounts, default_correlation = correlation_matrix)
model

```

[3]: FinancialModel(M=3, accounts=['Cuenta Ahorro Vivienda (BE)', 'Conservative Clooney (Fintual)', 'Moderate Pitt (Fintual)'], cache=enabled)

## 4 Simulation parameters

```
[4]: # --- Simulation parameters ---
n_sims = 500
months = 48
start_date = date(2025, 11, 1)
```

## 5 Income Module

Total monthly income at time  $t$  is composed of fixed and variable parts:

$$Y_t = y_t^{\text{fixed}} + Y_t^{\text{variable}}$$

### 5.1 Fixed Income

The fixed component,  $y_t^{\text{fixed}}$ , reflects a baseline salary subject to compounded annual growth  $g$  and scheduled raises  $d_k, \Delta_k$  (e.g., promotions or tenure milestones):

$$y_t^{\text{fixed}} = \text{current\_salary}(t) \cdot (1 + m)^{\Delta t}$$

where  $m = (1 + g)^{1/12} - 1$  is the **monthly compounded rate**, and  $\Delta t$  represents time since the last raise.

### 5.2 Variable Income

The variable component,  $Y_t^{\text{variable}}$ , models irregular income sources (e.g., freelance work or bonuses) with:

- **Seasonality:**  $s \in \mathbb{R}^{12}$  (multiplicative monthly factors),
- **Noise:**  $\epsilon_t \sim \mathcal{N}(0, \sigma^2)$  (Gaussian shocks),
- **Growth:** same compounded rate  $m$  applied to a base income level,
- **Boundaries:** optional floor and cap constraints.

The underlying stochastic projection is:

$$\tilde{Y}_t = \max(\text{floor}, \mu_t(1 + \epsilon_t)), \quad \text{where } \mu_t = \text{base} \cdot (1 + m)^t \cdot s_{(t \bmod 12)}$$

Then, guardrails are applied as:

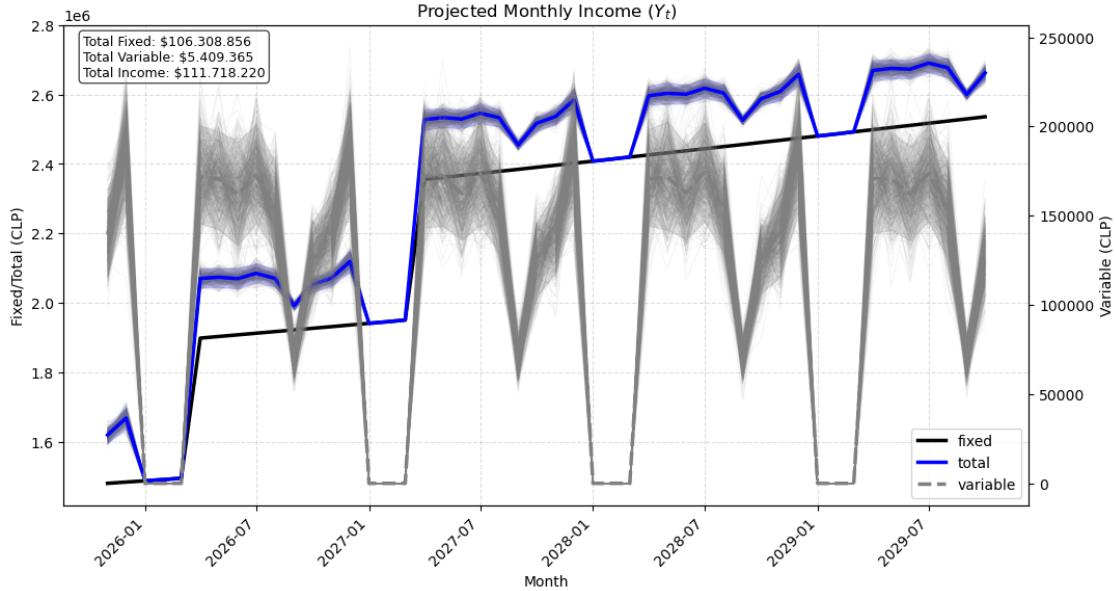
$$Y_t^{\text{variable}} = \begin{cases} 0 & \text{if } \tilde{Y}_t < 0 \\ \tilde{Y}_t & \text{if } 0 \leq \tilde{Y}_t \leq \text{cap} \\ \text{cap} & \text{if } \tilde{Y}_t > \text{cap} \end{cases}$$

**Note:** In expectation (ignoring noise truncation),  $\mathbb{E}[Y_t] = y_t^{\text{fixed}} + \mu_t$

### 5.3 Income Projection

Dual-axis plot with:  
- **Left axis:** Fixed income (deterministic) + Total income  
- **Right axis:** Variable income (stochastic with trajectories)  
- **Trajectories:** Individual noise realizations (n=300)  
- **Confidence band:** 95% CI for variable income

```
[5]: # Income Projection Simulation
model.plot(mode='income', T=months, start=start_date)
```



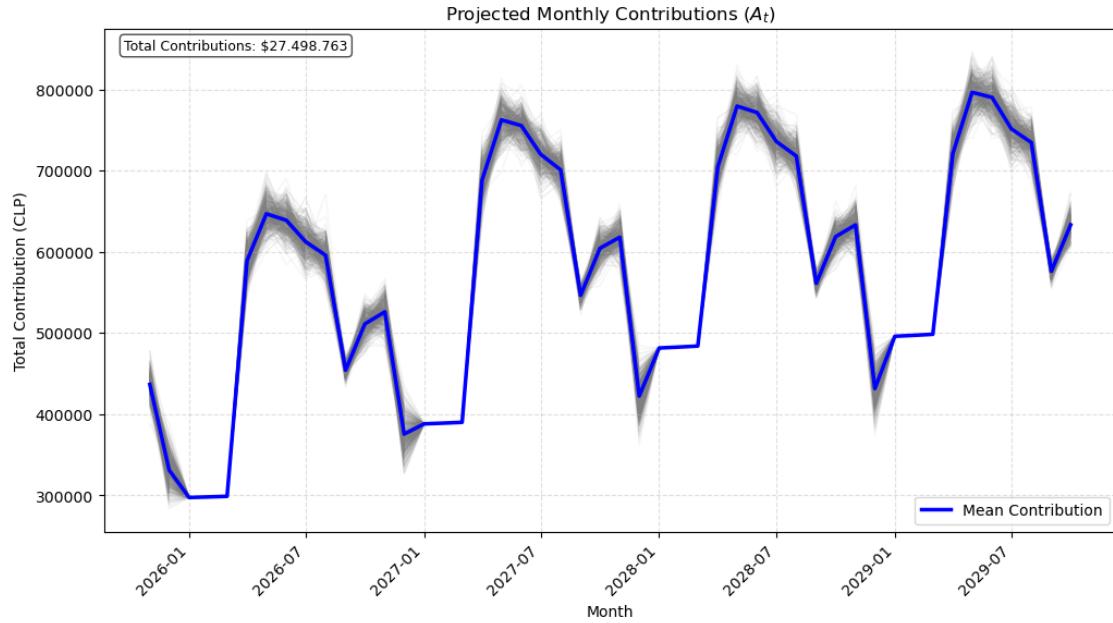
### 5.4 Contribution Projection

A fraction of income is allocated each month through calendar-rotating schedules:

$$A_t = \alpha_{(t \bmod 12)}^f \cdot y_t^{\text{fixed}} + \alpha_{(t \bmod 12)}^v \cdot Y_t^{\text{variable}}$$

where  $\alpha^f, \alpha^v \in [0, 1]^{12}$  control the fixed and variable contribution rates by applying the 12-month fractional arrays to projected incomes, rotated according to `start` date and repeated cyclically for horizons > 12 months.

```
[6]: # Contribution Projection Simulation
model.plot(mode='contributions', T=months, start=start_date)
```



## 6 Return Module

### 6.1 Multi-Account Return Model

For  $M$  accounts with correlated returns:

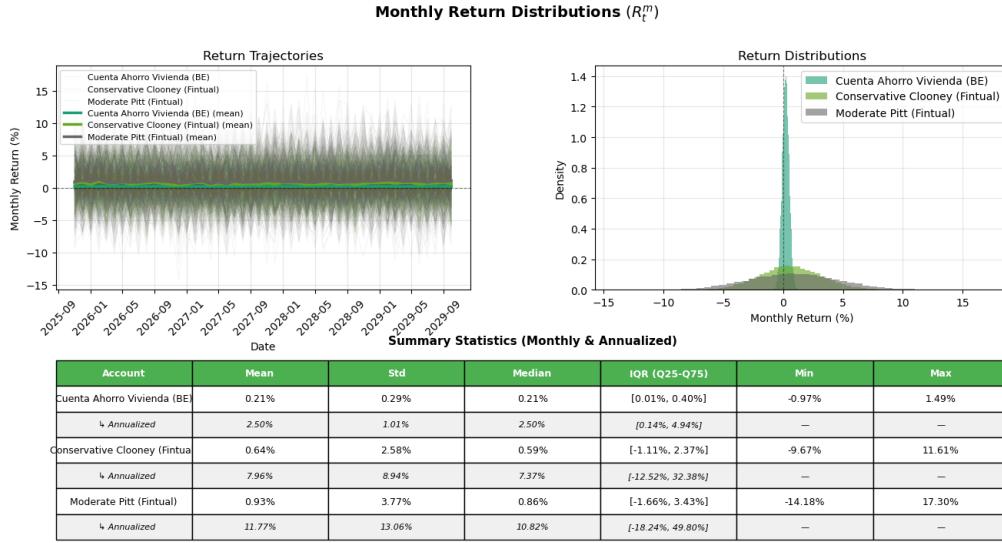
$$1 + R_t^m \sim \text{LogNormal}(\mu_{\log}^m, \Sigma)$$

where  $\Sigma = D \cdot \rho \cdot D$  is the covariance matrix: -  $D = \text{diag}(\sigma_{\log}^1, \dots, \sigma_{\log}^M)$  -  $\rho \in \mathbb{R}^{M \times M}$  is the correlation matrix (symmetric, PSD)

### 6.2 Monthly Return Distribution (Marginal Analysis)

Visualizes IID monthly returns across both accounts with 4 panels: - **Trajectories**: Individual paths for each account - **Histograms**: Marginal distributions (overlaid) - **Statistics**: Mean, std, quantiles per account

```
[7]: model.plot(mode = 'returns', T = months, start=start_date)
```



T=48 months | n\_sims=500 | seed=None

### 6.3 Cumulative Returns per Account

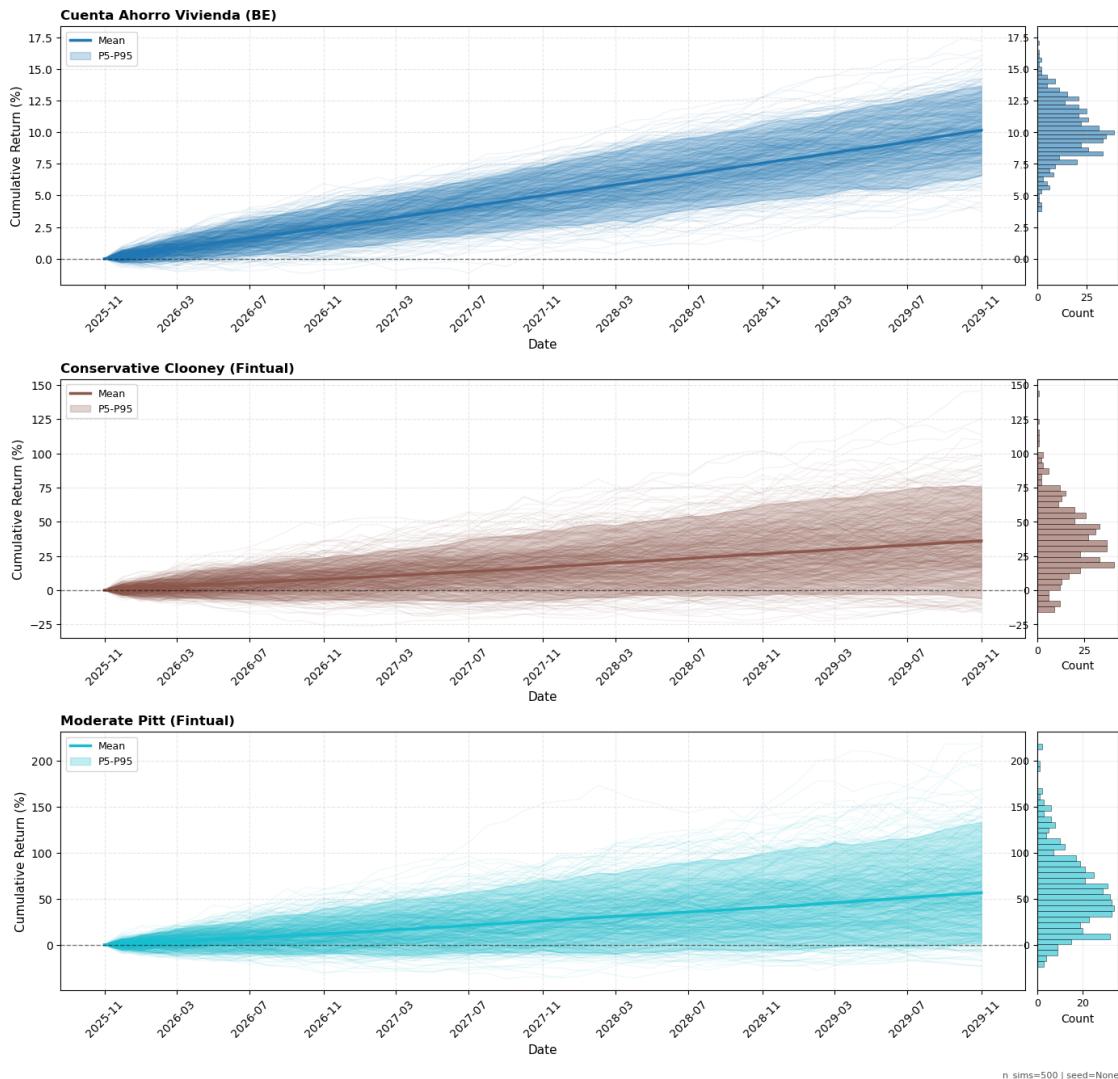
For  $M$  accounts with correlated returns:

$$R_{cm}^m(T) = \prod_{t=0}^{T-1} (1 + R_t^m) - 1$$

**Cross-sectional correlation** persists through time but does not compound.

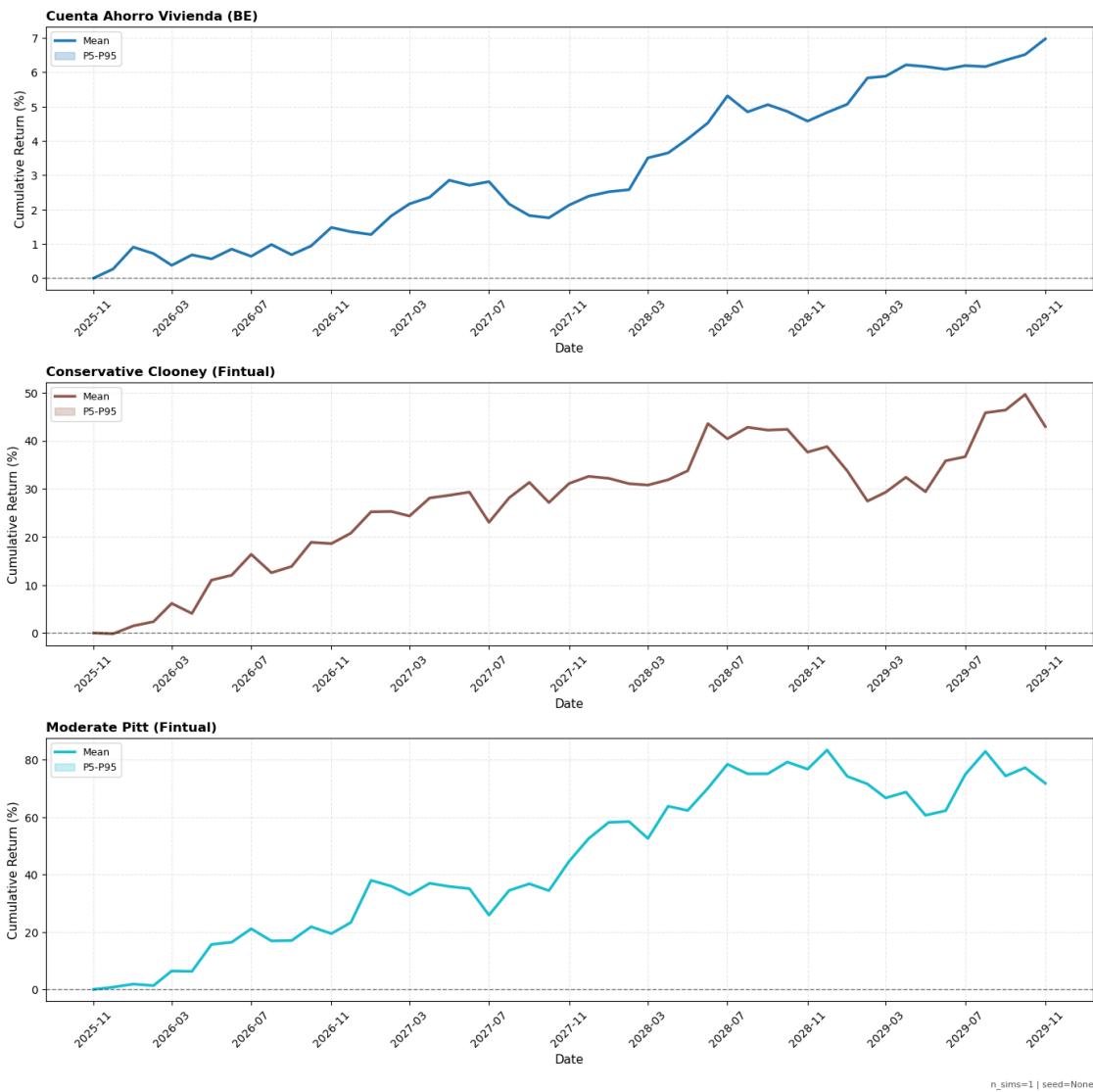
```
[8]: model.plot(mode = 'returns_cumulative', T = months, start=start_date)
```

$$\text{Cumulative Returns per Account} \left( \prod_{t=0}^{T-1} (1 + R_t^m) - 1 \right)$$



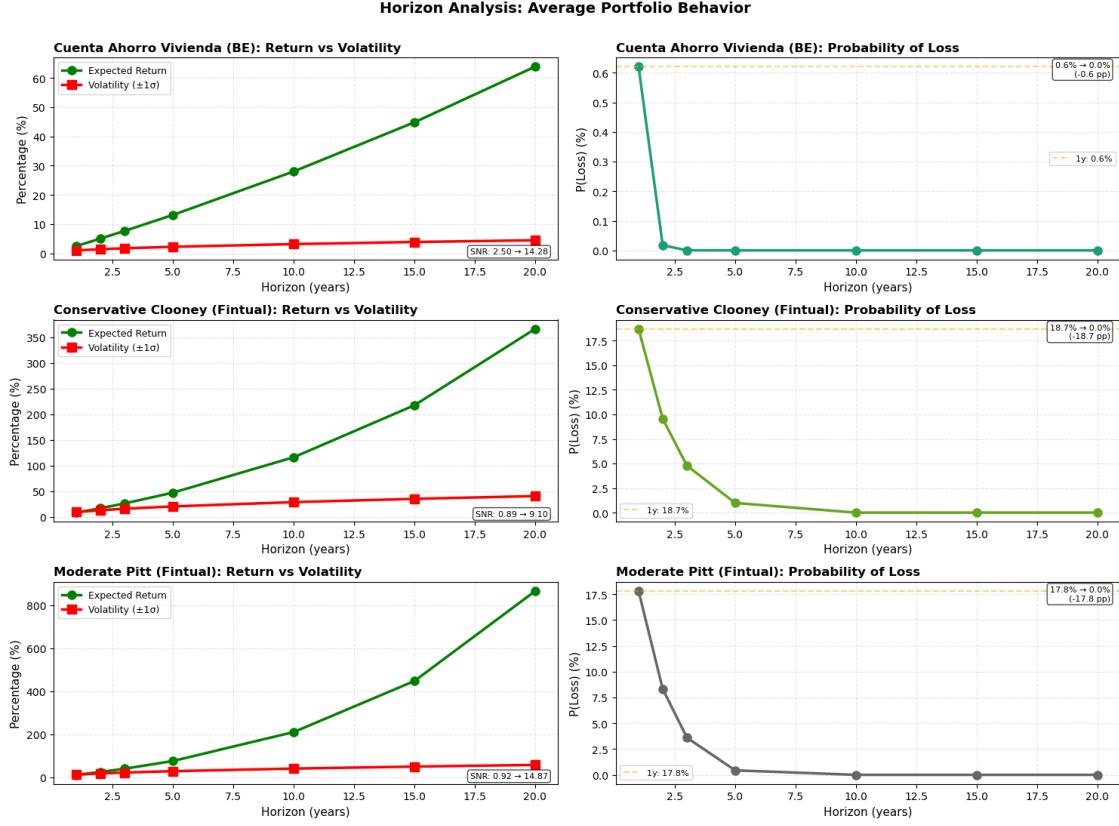
```
[9]: model.plot(mode = 'returns_cumulative', T = months, start=start_date, n_sims=1, title= r'Random Example of Cumulative Returns per Account' + $(\prod_{t=0}^{T-1}(1 + R_t^m) - 1)')
```

**Random Example of Cumulative Returns per Account  $\prod_{t=0}^{T-1} (1 + R_t)^m - 1$**



n\_sims=1 | seed=None

## 6.4 Horizon Analysis: Time Diversification by Account



## 7 Portfolio Module

### 7.1 Wealth Projection Under Allocation Policy

**Recursive dynamics:** Wealth evolves as:

$$W_{t+1}^m = (W_t^m + A_t x_t^m) (1 + R_t^m)$$

We define  $A_t^m = A_t \cdot x_t^m$  which is the contribution allocated to account  $m$  via policy  $X = \{x_t^m\}_{t,m}$ .

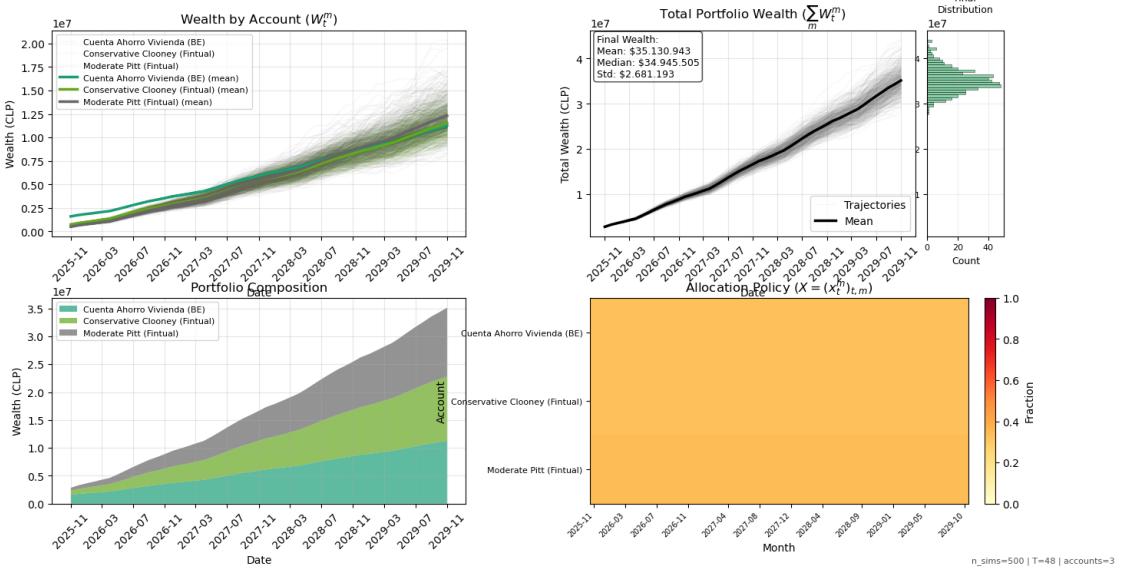
**Closed-form representation:**

$$W_t^m = W_0^m F_{0,t}^m + \sum_{s=0}^{t-1} A_s x_s^m F_{s,t}^m$$

with accumulation factor  $F_{s,t}^m = \prod_{r=s}^{t-1} (1 + R_r^m)$ .

**Key insight:**  $W_t^m(X)$  is **linear affine in policy  $X$**  → analytical gradients enable convex optimization.

**What to observe:** - Top-left: wealth per account with Monte Carlo trajectories - Top-right: total wealth + final distribution histogram - Bottom-left: portfolio composition over time - Bottom-right: allocation policy heatmap



## 8 Goal-Driven Optimization

### 8.1 Problem Formulation

#### 8.1.1 Non-Convex Chance Constraints

The **bilevel problem** seeks the minimum horizon  $T^*$  satisfying all goals:

$$\min_{T \in \mathbb{N}} T \max_{X \in \mathcal{F}_T} f(X)$$

where the **goal-feasible set**  $\mathcal{F}_T$  contains all policies  $X \in \mathcal{X}_T$  satisfying probabilistic constraints:

**Intermediate goals** (at fixed time  $t < T$ ):

$$\mathbb{P}(W_t^m(X) \geq b_t^m) \geq 1 - \varepsilon_t^m, \quad \forall g \in \mathcal{G}_{\text{int}}$$

**Terminal goals** (at horizon  $T$ ):

$$\mathbb{P}(W_T^m(X) \geq b^m) \geq 1 - \varepsilon^m, \quad \forall g \in \mathcal{G}_{\text{term}}$$

with decision space (simplex):

$$\mathcal{X}_T = \left\{ X \in \mathbb{R}^{T \times M} : x_t^m \geq 0, \sum_{m=1}^M x_t^m = 1, \forall t = 0, \dots, T-1 \right\}$$

**Challenge:** Chance constraints involve indicator functions  $\mathbb{1}[\cdot]$ , which are discontinuous and non-convex. Standard approaches (MILP, sigmoid smoothing) either scale poorly or find local optima.

## 8.2 Convex Reformulation via CVaR

### 8.2.1 CVaR Reformulation: Convex Upper Bound

We replace each chance constraint with a **CVaR constraint** (Rockafellar & Uryasev, 2000):

$$\boxed{\mathbb{P}(W \geq b) \geq 1 - \varepsilon \iff \text{CVaR}_\varepsilon(b - W) \leq 0}$$

where the **Conditional Value-at-Risk** of shortfall  $L = b - W$  is:

$$\text{CVaR}_\varepsilon(L) = \text{VaR}_\varepsilon(L) + \frac{1}{\varepsilon} \mathbb{E}[(L - \text{VaR}_\varepsilon(L))_+]$$

**Epigraphic formulation** (convex, suitable for LP solvers):

$$\text{CVaR}_\varepsilon(L) = \min_{\gamma \in \mathbb{R}} \left\{ \gamma + \frac{1}{\varepsilon N} \sum_{i=1}^N [L^i - \gamma]_+ \right\}$$

Introducing auxiliary variables  $z^i \geq [L^i - \gamma]_+$ :

$$\begin{aligned} \text{CVaR}_\varepsilon(L) &= \min_{\gamma, z} \left\{ \gamma + \frac{1}{\varepsilon N} \sum_{i=1}^N z^i \right\} \\ \text{s.t. } &z^i \geq L^i - \gamma, \quad \forall i \in [N] \\ &z^i \geq 0, \quad \forall i \in [N] \end{aligned}$$

**Key property:** If  $W^i$  is **affine in  $X$**  (as in our wealth dynamics), then  $\text{CVaR}_\varepsilon(b - W)$  is **convex in  $X$** .

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### 8.2.2 Mathematical Relationship: Implication, Not Equivalence

**Theorem** (Rockafellar & Uryasev, 2000):

$$\text{CVaR}_\varepsilon(L) \leq 0 \implies \mathbb{P}(L \leq 0) \geq 1 - \varepsilon$$

**Proof sketch:** CVaR averages the worst  $\varepsilon$ -tail of the distribution. If this mean is non-positive, then at least  $(1 - \varepsilon)$  of scenarios satisfy  $L \leq 0$ .

**The converse is NOT true:**  $\mathbb{P}(L \leq 0) \geq 1 - \varepsilon$  does NOT imply  $\text{CVaR} \leq 0$  if the tail is heavy.

**Interpretation:** CVaR is a **conservative approximation** controlling both: 1. **Frequency** of violations (at most  $\varepsilon \times 100\%$  scenarios fail) 2. **Severity** of violations (average loss in tail is non-positive)

The original chance constraint only controls frequency.

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### 8.2.3 Convex Reformulated Problem - Multiple Objectives

**Inner problem** (for fixed  $T$ ):

$$\begin{aligned}
 & \max_{X, \gamma, z} f(X) \\
 \text{s.t. } & \sum_{m=1}^M x_t^m = 1, \quad \forall t = 0, \dots, T-1 \quad (\text{simplex}) \\
 & x_t^m \geq 0, \quad \forall t, m \quad (\text{non-negativity}) \\
 & z_g^i \geq (b_g - W_{t_g}^{i,m_g}(X)) - \gamma_g, \quad \forall g, i \quad (\text{epigraph}) \\
 & z_g^i \geq 0, \quad \forall g, i \\
 & \gamma_g + \frac{1}{\varepsilon_g N} \sum_{i=1}^N z_g^i \leq 0, \quad \forall g \quad (\text{CVaR constraint})
 \end{aligned}$$

where: -  $W_t^{i,m}(X) = W_0^m F_{0,t,m}^i + \sum_{s=0}^{t-1} A_s^i \cdot x_s^m \cdot F_{s,t,m}^i$  (**affine in  $X$** ) -  $g$  indexes goals (both intermediate and terminal) -  $t_g, m_g, b_g, \varepsilon_g$  are parameters of goal  $g$  -  $f(X)$  is a convex objective function (see supported objectives below)

**Global optimality guaranteed** via convex programming (interior-point methods).

### 8.2.4 Comparison: Original vs. CVaR

**Observed conservativeness** (empirical): - CVaR constraint:  $\text{CVaR}_\varepsilon(L) \leq 0$  - Resulting violation rate: typically  $(0.5 - 0.8) \times \varepsilon$  (better than required)

**Example** (from our results): - Goal:  $\mathbb{P}(W_T \geq 1M) \geq 80\%$  (i.e.,  $\varepsilon = 20\%$ ) - CVaR solution: violation rate = 9% (margin of 11%)

The 11% buffer is the “price” of convexity, buying us certified global optimality and numerical stability.

**References:** - Rockafellar, R.T. & Uryasev, S. (2000). “Optimization of conditional value-at-risk.” *Journal of Risk*, 2, 21-42. - Nemirovski, A. & Shapiro, A. (2006). “Convex approximations of chance constrained programs.” *SIAM J. Optim.*, 17(4), 969-996.

## 8.3 Goal Specification

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FINANCIAL GOALS SUMMARY

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1.

Type: Terminal (horizon T to be optimized)  
 Account: Cuenta Ahorro Vivienda (BE)  
 Threshold: \$4,100,000  
 Confidence: 20% (=80%)

2.  
Type: Terminal (horizon T to be optimized)  
Account: Conservative Clooney (Fintual)  
Threshold: \$5,000,000  
Confidence: 20% (=80%)
3.  
Type: Terminal (horizon T to be optimized)  
Account: Moderate Pitt (Fintual)  
Threshold: \$15,000,000  
Confidence: 20% (=80%)
4.  
Type: Intermediate (month 6)  
Account: Cuenta Ahorro Vivienda (BE)  
Threshold: \$2,200,000  
Confidence: 20% (=80%)
- 

## 8.4 Bilevel Optimization

### 8.4.1 Supported Convex Objectives

CVaROptimizer supports 7 convex objectives exploiting affine wealth structure:  $W_t^{i,m}(X) = W_0^m F_{0,t,m}^i + \sum_{s=0}^{t-1} A_s^i \cdot x_s^m \cdot F_{s,t,m}^i$

1. **terminal\_wealth** — Growth maximization

$$\max_X \mathbb{E} \left[ \sum_{m=1}^M W_T^m \right] = \frac{1}{N} \sum_{i=1}^N \sum_{m=1}^M W_T^{i,m}(X)$$

2. **min\_cvar** — Downside protection

$$\min_{X, \gamma, z} \sum_{g \in \mathcal{G}} \left( \gamma_g + \frac{1}{\varepsilon_g N} \sum_{i=1}^N z_g^i \right)$$

3. **low\_turnover** — Tax efficiency (`lambda=0.1`)

$$\max_X \mathbb{E} \left[ \sum_{m=1}^M W_T^m \right] - \lambda \sum_{t=1}^{T-1} \sum_{m=1}^M |x_t^m - x_{t-1}^m|$$

4. **risk\_adjusted** — Markowitz mean-variance (`lambda=0.5`)

$$\max_X \mathbb{E} \left[ \sum_{m=1}^M W_T^m \right] - \lambda \cdot \text{Var} \left( \sum_{m=1}^M W_T^m \right)$$

where  $\text{Var}(W_T) = \frac{1}{N} \sum_{i=1}^N (W_T^i - \mathbb{E}[W_T])^2$

5. **balanced** — Multi-objective (`lambda_risk=0.3, lambda_turnover=0.05`)

$$\max_X \quad \mathbb{E} \left[ \sum_{m=1}^M W_T^m \right] - \lambda_r \cdot \text{Var} \left( \sum_{m=1}^M W_T^m \right) - \lambda_t \sum_{t=1}^{T-1} \sum_{m=1}^M |x_t^m - x_{t-1}^m|$$

6. **min\_variance** — Capital preservation (**target required**)

$$\begin{aligned} \min_X \quad & \text{Var} \left( \sum_{m=1}^M W_T^m \right) \\ \text{s.t.} \quad & \mathbb{E} \left[ \sum_{m=1}^M W_T^m \right] \geq W_{\text{target}} \end{aligned}$$

```
[56]: # --- Execute Optimization ---
```

```
print("\n" + "=" * 70)
print("STARTING BILEVEL OPTIMIZATION")

optimizer = CVaROptimizer(n_accounts=model.M, objective='risk_adjusted',
                           ↪objective_params= {'lambda': 0.25})
opt_result = model.optimize(
    goals=goals,
    optimizer=optimizer,
    T_max=120,
    n_sims=300,
    seed=42,
    verbose=True,
    solver='ECOS',  # O 'SCS', 'CLARABEL'
    max_iters=100000)

print(opt_result.summary())

# Display optimization summary
print("\n" + "=" * 70)
print(opt_result.summary())
print("=" * 70)
```

```
=====
```

```
STARTING BILEVEL OPTIMIZATION
```

```
== GoalSeeker: BINARY search T [6, 120] ==
[Iter 1] Binary search: testing T=63 (range=[6, 120])...
/home/mlioi/anaconda3/envs/finance/lib/python3.11/site-
packages/cvxpy/problems/problem.py:1539: UserWarning: Solution may be
inaccurate. Try another solver, adjusting the solver settings, or solve with
```

```

verbose=True for more information.

warnings.warn(
    Feasible, obj=-1357043388685.99, time=1.704s

[Iter 2] Binary search: testing T=34 (range=[6, 63])...
Infeasible, obj=0.00, time=0.304s

[Iter 3] Binary search: testing T=49 (range=[35, 63])...
Feasible, obj=-1833271314493.87, time=0.836s

[Iter 4] Binary search: testing T=42 (range=[35, 49])...
Feasible, obj=-1658307540107.95, time=2.294s

[Iter 5] Binary search: testing T=38 (range=[35, 42])...
Feasible, obj=-1406821156021.59, time=2.257s

[Iter 6] Binary search: testing T=36 (range=[35, 38])...
Feasible, obj=-1020865440607.72, time=1.528s

[Iter 7] Binary search: testing T=35 (range=[35, 36])...
Feasible, obj=-1070347915893.94, time=1.193s

==== Optimal: T*=35 (binary search converged) ===

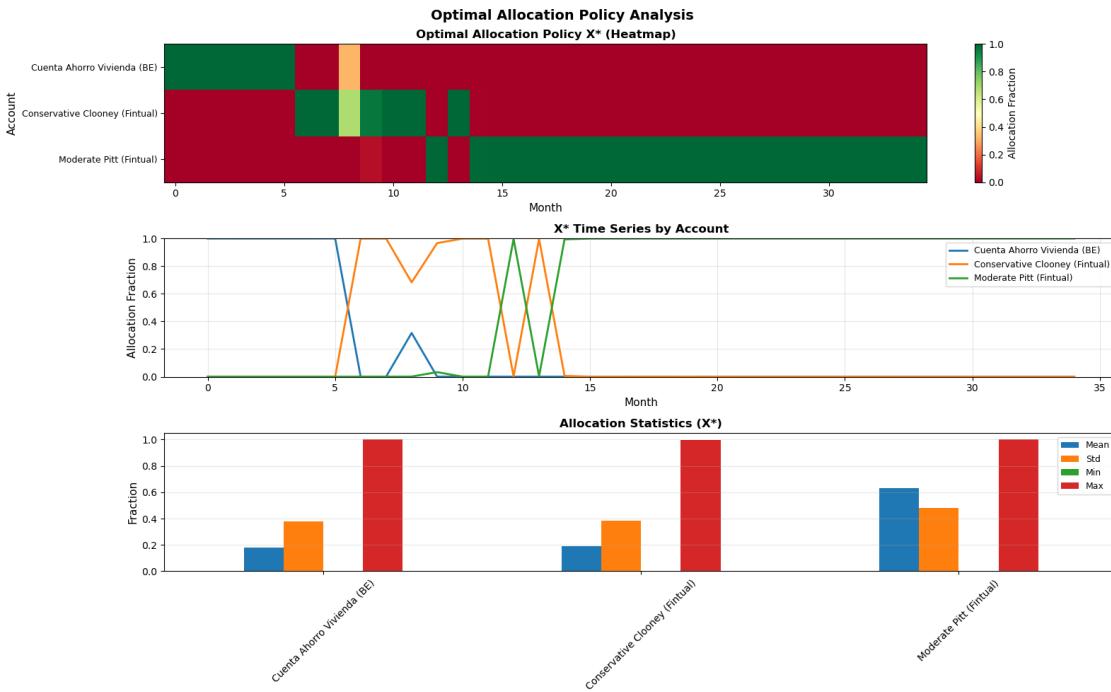
OptimizationResult(
    Status: Feasible
    Horizon: T=35 months
    Objective: -1070347915893.94
    Goals: 4 (1 intermediate, 3 terminal)
    Solve time: 1.193s
    Iterations: 0
)

=====
OptimizationResult(
    Status: Feasible
    Horizon: T=35 months
    Objective: -1070347915893.94
    Goals: 4 (1 intermediate, 3 terminal)
    Solve time: 1.193s
    Iterations: 0
)
=====

[57]: optimizer.objective
[57]: 'risk_adjusted'

```

## 8.5 Optimal Policy Analysis



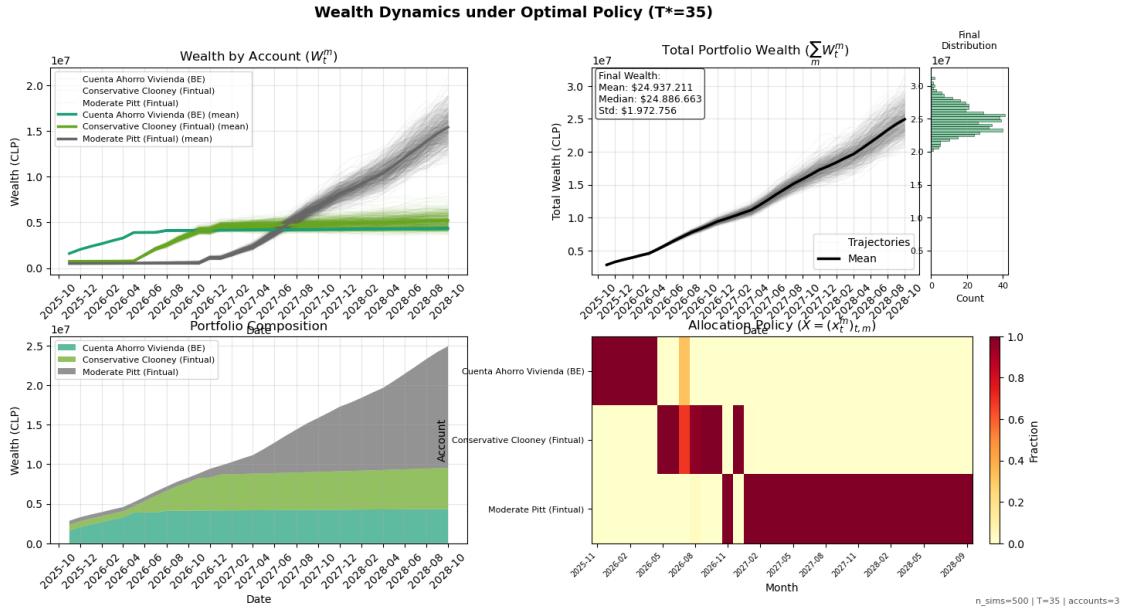

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### OPTIMAL ALLOCATION SUMMARY

---

	Mean	Std	Min	Max
Cuenta Ahorro Vivienda (BE)	0.181	0.376	0.0	1.000
Conservative Clooney (Fintual)	0.190	0.383	0.0	0.999
Moderate Pitt (Fintual)	0.629	0.482	0.0	1.000

---



## 8.6 Goal Verification

### 8.6.1 In-Sample Verification

---

GOAL VERIFICATION: IN-SAMPLE

---

Using optimization scenarios (n=300, seed=42)

Expected: All goals satisfied ()

Account: Cuenta Ahorro Vivienda (BE)

Threshold: \$4,100,000

Confidence: 20%

---

Status: SATISFIED

Violation rate: 0.33% (required 80.00%)

Margin: 79.67%

Median shortfall: \$32,374

N violations: 1 / 300

Account: Conservative Clooney (Fintual)

Threshold: \$5,000,000

Confidence: 20%

---

Status: SATISFIED

Violation rate: 35.67% (required 80.00%)

Margin: 44.33%

Median shortfall: \$427,871

N violations: 107 / 300

```
Account: Moderate Pitt (Fintual)
Threshold: $15,000,000
Confidence: 20%
---
Status: SATISFIED
Violation rate: 37.33% (required 80.00%)
Margin: 42.67%
Median shortfall: $1,010,871
N violations: 112 / 300
Account: Cuenta Ahorro Vivienda (BE)
Month: 6
Threshold: $2,200,000
Confidence: 20%
---
Status: SATISFIED
Violation rate: 0.00% (required 80.00%)
Margin: 80.00%
Median shortfall: $0
N violations: 0 / 300
```

---

### 8.6.2 Out-of-Sample Verification

---

```
=====
GOAL VERIFICATION: OUT-OF-SAMPLE
=====
Using fresh scenarios (n=1000, seed=999)
Expected: All goals satisfied with safety margin
```

```
Account: Cuenta Ahorro Vivienda (BE)
Threshold: $4,100,000
Confidence: 20%
---
Status: SATISFIED
Violation rate: 0.20% (required 80.00%)
Margin: 79.80%
Median shortfall: $3,087
N violations: 2 / 1000
Account: Conservative Clooney (Fintual)
Threshold: $5,000,000
Confidence: 20%
---
Status: SATISFIED
Violation rate: 39.50% (required 80.00%)
Margin: 40.50%
Median shortfall: $382,732
```

```
N violations: 395 / 1000
Account: Moderate Pitt (Fintual)
Threshold: $15,000,000
Confidence: 20%
---
Status: SATISFIED
Violation rate: 41.50% (required 80.00%)
Margin: 38.50%
Median shortfall: $928,055
N violations: 415 / 1000
Account: Cuenta Ahorro Vivienda (BE)
Month: 6
Threshold: $2,200,000
Confidence: 20%
---
Status: SATISFIED
Violation rate: 0.00% (required 80.00%)
Margin: 80.00%
Median shortfall: $0
N violations: 0 / 1000
```

```
=====
```

```
=====
```

#### IN-SAMPLE vs OUT-OF-SAMPLE COMPARISON

	In-Sample (n=300)	Out-of-Sample (n=1000)	Required ( )
0	0.0033	0.002	0.8
1	0.3567	0.395	0.8
2	0.3733	0.415	0.8
3	0.0000	0.000	0.8

```
=====
```

```
[NbConvertApp] Converting notebook FinOpt-Workflow.ipynb to html
[NbConvertApp] WARNING | Alternative text is missing on 9 image(s).
[NbConvertApp] Writing 3675874 bytes to FinOpt-Workflow.html
```

```
[ ]:
```