Series and Their Convergence Tests

Math 100 Vantage College

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Annoucements

Where to find help?

- Email and talk to Prof. Bachmann.
- TA Office hours: Monday, 5-6 pm, Leonard S. Klinck Building (LSK).
- Math Learning Centre (MLC): Monday-Friday, 12-5 pm,LSK 301/302. https://www.math.ubc.ca/~MLC/

Slides are posted online:

- http://www.math.ubc.ca/~mqiu/m100vc.html

Series

Definition (Series)

Given a sequence $(a_n)_{n\in\mathbb{N}}$. We form the sequence of its partial sums:

$$S_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n.$$

For short, we call this sequence $(S_n)_{n\in\mathbb{N}}$ a series and write it as

$$\sum_{i=1}^{\infty} a_i.$$

If S_n converges, then we say that the series converges, otherwise we say that the series diverges.

Remark:

- A series represents the sequence of partial sums.

Absolute and conditional convergence

Definition

If the series $\sum_{n=1}^{\infty} |a_n|$ converges, then we say that the series $\sum_{n=1}^{\infty} a_n$ converges absolutely.

If the series $\sum_{n=1}^{\infty} |a_n|$ diververges, but the series $\sum_{n=1}^{\infty} a_n$ converges, then we say that the series $\sum_{n=1}^{\infty} a_n$ converges conditionally.

Two special series

Geometric series:

$$\sum_{n=1}^{\infty} r^n$$

- converges when $|r| < 1$;
- diverges when $|r| \geq 1$.

p-series:

$$\sum_{n=1}^{\infty} \frac{1}{n^p}, \ p > 0.$$

- converges when $p > 1$;
- diverges when $p \leq 1$.
- When $p = 1$, it is called the harmonic series.

Convergence tests for series (CLP2, p. 297)

1. Divergence test
   - works well when the terms of the series fail to converge to zero as n tends to infinity.
2. Alternating series test
   - successive terms alternate in sign;
   - the magnitude of the terms decays to zero.
3. Ratio test
   - works well when $\frac{a_{n+1}}{a_n}$ simplifies a lot so that you can easily verify $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = L$.
   - this often happens when a_n contains powers or factorials.
   - $L = 1$ is inconclusive.
4. Limit comparison test and comparison test
   - works well when, for very large n, a_n is approximately the same as a simple term b_n and it's easy to tell the convergence status of the series $\sum_{n=1}^{\infty} b_n$.
   - $b_n \geq 0$.
   - The limit comparison test is often easier to apply.