Journal of International Money and Finance 37 (2013) 306–323

Contents lists available at SciVerse ScienceDirect

Journal of International Money and Finance

journal homepage: www.elsevier.com/locate/jimf

Optimal monetary policy in a small open economy with staggered wage and price contracts

Hyuk-jae Rhee*, Nurlan Turdaliev [1]

University of Windsor, Department of Economics, 401 Sunset Avenue, Windsor, Ontario N9B 3P4, Canada

JEL classification:
E31
E58
F41

Keywords:
Nominal wage rigidity
Sticky prices
Inflation targeting
Monetary policy
Small open economy

A B S T R A C T

We study optimal monetary policy for a small open economy in a model where both domestic prices and wages are sticky due to staggered contracts. The simultaneous presence of the two forms of nominal rigidities introduces an additional trade-off between domestic inflation and the output gap. We derive a second-order approximation to the average welfare losses that can be expressed in terms of the unconditional variances of the output gap, domestic price inflation, and wage inflation. As a consequence, the optimal policy seeks to minimize a weighted average of these variances. We analyze welfare implications of several alternative simple policy rules, and find that domestic price inflation targeting generates relatively large welfare losses, whereas CPI inflation targeting performs nearly as well as the optimal rule.

1. Introduction

The new Keynesian model has become a new theoretical consensus for studying various issues in monetary policy (Goodfriend, 2007). However, some features of the new Keynesian model are deemed to be unsatisfactory. For example, the monetary policy rule that stabilizes price inflation also stabilizes output gap variability. Therefore, a simple price inflation targeting monetary policy can achieve the (Pareto) optimal welfare level that would occur in the absence of nominal frictions. This property of the

* Corresponding author. Tel.: +1 519 253 3000x2385; fax: +1 519 973 7096.
E-mail addresses: jayrhee@uwindsor.ca (H.-j. Rhee), nurlan@uwindsor.ca (N. Turdaliev).
[1] Tel.: +1 519 253 3000x2391; fax: +1 519 973 7096.

0261-5606/$ – see front matter
http://dx.doi.org/10.1016/j.jimonfin.2013.06.007

new Keynesian framework is called the 'divine coincidence' (Blanchrad and Galí, 2007). In addition, recent empirical studies (e.g., Galí, 1992; Christiano et al., 1999; Neely and Rapach, 2008) reveal that a staggered price mechanism by itself is incapable of generating persistent real effects of monetary shocks. Instead, it has been argued that wage stickiness may be a more important force than price stickiness for generating output persistence (e.g., Ambler et al., 2012; Chari et al., 2000; Huang and Liu, 2002).

The small open economy version of the new Keynesian model with staggered price-setting (Galí and Monacelli, 2005: Clarida et al., 2002) also lead to a striking, but controversial result. In the model, domestic goods prices are sticky but foreign goods prices are flexible. This specification of pricing behavior leads to disappearance of CPI inflation from any of the structural equations needed to compute welfare of the households. Since the model's Phillips curve contains only domestic inflation, it is then not surprising that it is optimal for the central bank to target domestic rather than CPI inflation. According to the standard new Keynesian small open economy model, therefore, CPI inflation targeting in a small open economy is misguided. But all real world inflation targeting countries are open economies, and all of them target CPI inflation, not domestic inflation (Svensson, 2000). The description of the inflation dynamics and policy implications of the standard new Keynesian small open economy model, therefore, should be modified.

A number of studies have incorporated both wage and price stickiness into the new Keynesian model in a closed economy framework to help solve the issues mentioned above (Christiano et al., 2005; Erceg et al., 2000; Gali, 2011; Sbordone, 2002). In particular, Erceg et al. formulate a model in which both the labor and product markets exhibit monopolistic competition and staggered contracts. They find that the model (with both sticky prices and sticky wages) exhibits a tradeoff between stabilizing the output gap, price inflation, and wage inflation. They also find that price inflation targeting generates a relatively large welfare loss. Most of the studies involving both nominal wage and price rigidities, however, focus on the closed-economy framework. The recent papers by Adolfson et al. (2007, 2008) are among rare notable exceptions.[2]

In this paper, we employ the Calvo specification to incorporate both nominal wage and price rigidities into a small open economy new Keynesian framework. The goods market side of the model is similar in structure to the one developed in Galí and Monacelli (2005). Monopolistically competitive domestic producers set prices in staggered contracts as in Calvo (1983). Following Erceg et al. (2000), however, we modify the labor market where individual households supply differentiated labor services to domestic firms, and domestic firms combine this labor services to produce domestic goods. Monopolistically competitive households also set nominal wages in staggered fashion. These modifications lead to several distinctive features of our model. First, they necessitate the presence of the terms of trade, and thus CPI inflation, in the equation for wage inflation. Therefore, the behavior of wage inflation in this model is different from that in Erceg et al. (2000), which is a natural consequence of the small open economy framework. Second, there is a direct effect of CPI inflation on domestic price inflation, the dynamics of which, therefore, differs from that in the standard new Keynesian open economy Phillips curve of Galí and Monacelli (2005). This also has a very important policy implication (discussed below). Lastly, foreign output affects the dynamics of the real wage gap. It is through the real wage gap that shocks to foreign output affect both domestic price and wage inflation.

Within this model, we discuss how the economy responds to a contractionary monetary policy shock when both domestic prices and wages are sticky. In order to disentangle the role played by nominal wage rigidity, we examine four different assumptions on domestic price and wage stickiness. We consider an economy in which (i) both domestic prices and wages are flexible, (ii) only domestic prices are flexible, (iii) only wages are flexible, and (iv) both domestic prices and wages are sticky. We find that the existence of staggered wage setting influences the economy's equilibrium response to a monetary policy shock regardless of the presence of sticky domestic prices. This exercise also implies that wage stickiness is more important than price stickiness for generating persistent real effects of monetary shocks in a small open economy.

[2] Their work focuses mostly on estimating their model, whereas we derive some theoretical implications for several monetary policy rules for a small open economy that exhibits the simultaneous presence of the two forms of nominal rigidities.

In order to find the optimal monetary policy we derive a second-order approximation to the average welfare losses experienced by households in the economy with both wage and price stickiness around a steady state with zero inflation. The resulting welfare function can be expressed in terms of the unconditional variances of the output gap, domestic price inflation, and wage inflation, and the optimal policy seeks to minimize a weighted average of these variances. For the standard new Keynesian small open economy model as in Galí and Monacelli (2005), the optimal monetary policy requires full stabilization of domestic price inflation. Then the output gap is also stabilized under that optimal policy. As a result, a fully stabilizing domestic price inflation, which is optimal for the standard new Keynesian small open economy model, is no longer an optimal policy. Instead, the central bank should stabilize both domestic inflation and wage inflation in addition to the output gap.

We also employ our framework to analyze welfare implications of alternative policy rules. In addition to the optimal rule, we study four different simple policy rules whereby the domestic nominal interest rate responds to inflation and the output gap. The first rule requires that the domestic interest rate respond systematically to domestic inflation whereas the second assumes that the domestic interest rate responds to CPI inflation. They are referred to as the domestic inflation-based Taylor rule and the CPI inflation-based Taylor rule respectively. We consider an analogous rule for wage inflation (the wage inflation-based Taylor rule). The last rule considered seeks to stabilize a weighted average of domestic price and wage inflation and is referred to as the composite inflation-based Taylor rule. We use the derived approximation to the welfare function to evaluate the performance of alternative policy rules. The welfare level under the optimal monetary policy rules provides a benchmark. We rank these alternative policy rules in terms of their implied volatility for domestic inflation, wage inflation, and the output gap. From this exercise, we find that the domestic price inflation-based Taylor rule generates relatively large welfare losses due to excessive variation in wage inflation and the output gap. This finding is consistent with Erceg et al. (2000) where price inflation targeting induces substantial welfare costs. The CPI inflation-based Taylor rule, however, performs nearly as well as the optimal rule, leads to relatively small variation in inflation and output gap. There is a direct effect of CPI inflation on the dynamics of domestic price and wage inflation. Therefore, stabilizing CPI inflation is important for reducing volatility of domestic price and wage inflation. Less volatile domestic price inflation induces relatively small variance of the output gap under the CPI inflation-based Taylor rule. This result stands in sharp contrast with the policy implication of the standard new Keynesian model without staggered wage-settings (Clarida et al., 2002; Galí and Monacelli, 2005).

The plan of this paper is as follows. We describe the basic model in Section 2 and present the equilibrium conditions and dynamics of the model in Section 3. The implications and performance of each targeting regime are discusses in Section 4. And in Section 5 we draw the main conclusions.

2. The model

We consider a version of the dynamic new Keynesian model applied to a small open economy. The goods market side of the model is based on Galí and Monacelli (2005). To keep the analysis simple, we assume that there are two countries, home (H) and foreign (F). The two countries share the same preferences, technology, and market structure, but differ in size: it is assumed that the foreign country is a large economy, but the home country is small. Following Erceg et al. (2000), however, we modify the labor market. In particular, each household is assumed to supply a differentiated labor service. It is also assumed that households set nominal wages in a staggered fashion.

2.1. Firm

2.1.1. Technology

We first consider the production side of the economy. The market for domestic goods in the home country is populated by a continuum of domestic firms acting as monopolistic competitors indexed by $i \in [0,1]$, whose total is normalized to unity. Each domestic firm i produces a differentiated good with a linear technology represented by the production function $Y_t(i) = A_t N_t(i)$, where $a_t = \log A_t$ follows the AR (1) process $a_t = \rho_a a_{t-1} + \varepsilon_t$, and $N_t(i)$ is an index of labor input used by firm i and defined by

$$N_t(i) = \left[\int_0^1 N_t(i,j)^{1-\frac{1}{\varepsilon_W}} \mathrm{d}j\right]^{\frac{\varepsilon_W}{\varepsilon_W-1}}, \tag{1}$$

where $N_t(i,j)$ denotes the quantity of type j labor employed by firm i in period t, and parameter ε_W denotes the elasticity of substitution among labor varieties. We also assume a continuum of labor types indexed by $j \in [0,1]$.

Let $W_t(j)$ denote the nominal wage for type j labor effective in period t. As mentioned above, wages are set by workers of each type and taken as given by firms. Given the wages at any point in time, cost minimization yields a corresponding set of demand schedules for each firm i and labor type j: $N_t(i,j) = [W_t(j)/W_t]^{-\varepsilon_W} N_t(i)$, where $W_t = [\int_0^1 W_t(j)^{1-\varepsilon_W} \mathrm{d}j]^{1/1-\varepsilon_W}$ is an aggregate wage index.

Furthermore, it is assumed that each firm receives a subsidy of τ percent of its wage bill. Then the real marginal cost in terms of domestic goods prices (in log terms) will be the same across domestic firms and given by $mc_t = -\nu + w_t - p_{H,t} - a_t$, where $\nu = -\log(1-\tau)$.

For future reference, we derive an approximate aggregate production function in relation to aggregate employment. Notice that $N_t = \int_0^1 N_t(i)\mathrm{d}i = (Y_t/A_t)D_t$, where $D_t = \int_0^1 Y_t(i)/Y_t \mathrm{d}i$. Around the perfect foresight steady state, equilibrium variation in $d_t = \log D_t$ are of second order. Thus, up to a first-order approximation, we have an aggregate production relation $y_t = a_t + n_t$, where lower case letters denote the logs of the respective variables.

2.1.2. Price setting

Following Calvo (1983), we assume that a fraction $(1-\theta_{p_H})$ of (randomly selected) domestic firms set new prices each period, with an individual firm's probability of re-optimizing in any given period being independent of the time elapsed since it last reset its price. Then the optimal price-setting strategy for the typical firm resetting its price in period t can be approximated by the (log-linear) rule

$$\overline{p}_{H,t} = \mu^{p_H} + (1-\beta\theta_{p_H}) \sum_{k=0}^{\infty} (\beta\theta_{p_H})^k E_t\left\{mc_{t+k} + p_{H,t}\right\}, \tag{2}$$

where $\overline{p}_{H,t}$ denotes the newly set (log) domestic prices, β is the household's discount factor, and $\mu^{p_H} = \log(\varepsilon_p/\varepsilon_p - 1)$, which corresponds to the optimal (log) price mark-up in the flexible price equilibrium.[3]

2.2. Households

The home country is populated by infinitely lived households indexed by $j \in [0,1]$. A typical household seeks to maximize

$$E_0\left\{\sum_{t=0}^{\infty} \beta^t U(C_t(j), N_t(j))\right\}, \tag{3}$$

subject to a sequence of budget constraints (to be specified below), where $N_t(j)$ is the quantity of labor supplied, and C_t is a composite consumption index of domestic and imported goods defined as $C_t = [(1-\alpha)^{1/\eta} C_{H,t}^{\eta-1/\eta} + \alpha^{1/\eta} C_{F,t}^{\eta-1/\eta}]^{\eta/\eta-1}$ with $C_{H,t}$ being an index of consumption of domestic goods, $C_{H,t} = [\int_0^1 C_{H,t}(i)^{\varepsilon_p-1/\varepsilon_p} \mathrm{d}i]^{\varepsilon_p/\varepsilon_p-1}$, and $C_{F,t}$ an index of imported goods from the foreign country, $C_{F,t} = [\int_0^1 C_{F,t}(i^*)^{\varepsilon_p-1/\varepsilon_p} \mathrm{d}i^*]^{\varepsilon_p/\varepsilon_p-1}$. Notice that $i, i^* \in [0,1]$ denote the good varieties produced by monopolistically competitive firms at home and abroad, respectively, parameter $\varepsilon_p > 1$ denotes the elasticity of substitution between varieties of goods, and parameter $\alpha \in [0,1]$ is related to the share of imported goods in domestic consumption, which can also be interpreted as an index of openness. Parameter $\eta > 0$ measures the substitutability between domestic and foreign goods.

[3] For more on β and ε_p, see Eq. (3) and the discussion immediately following it.

Maximization of (3) is subject to a sequence of budget constraints

$$\int_0^1 P_{H,t}(i)C_{H,t}(i)\mathrm{d}i + \int_0^1 P_{F,t}(i^*)C_{F,t}(i^*)\mathrm{d}i^* + E_t\{\Psi_{t,t+1}B_{t+1}\} \le B_t + W_tN_t + T_t,$$

where B_{t+1} is the nominal payoff in period $t+1$ of a portfolio held at the end of period t, $\Psi_{t,t+1}$ the corresponding stochastic discount factor for nominal payoffs, N_t the household's supply of labor services, and W_t the corresponding nominal wage. Finally, T_t denotes the lump sum component of income (which includes transfers/taxes, and profits accruing from ownership of monopolistic firms). We assume that the household has access to a complete set of contingent claims traded internationally. The riskless short-term nominal interest rate, R_t, is found from $E_t\{\Psi_{t,t+1}\} = R_t^{-1}$.

The optimal allocation of any given expenditure within a category of goods yields the demand functions $C_{H,t}(i) = [P_{H,t}(i)/P_{H,t}]^{-\varepsilon_p}C_{H,t}$ and $C_{F,t}(i^*) = [P_{F,t}(i^*)/P_{F,t}]^{-\varepsilon_p}C_{F,t}$, where $P_{H,t}$ is the domestic price index defined as $P_{H,t} = [\int_0^1 P_{H,t}(i)^{1-\varepsilon_p}\mathrm{d}i]^{1/1-\varepsilon_p}$, and $P_{F,t} = [\int_0^1 P_{F,t}(i^*)^{1-\varepsilon_p}\mathrm{d}i^*]^{1/1-\varepsilon_p}$ is the price index for imported goods.

Similarly, the optimal allocation of expenditures between domestic and imported goods is given by $C_{H,t} = (1-\alpha)[P_{H,t}/P_t]^{-\eta}C_t$ and $C_{F,t} = \alpha[P_{F,t}/P_t]^{-\eta}C_t$, where $P_t = [(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}]^{1/1-\eta}$ is the consumer price index (CPI). Then the period budget constraint can be rewritten as $P_tC_t + E_t\{\Psi_{t,t+1}B_{t+1}\} \le B_t + W_tN_t + T_t$.

Unlike in Galí and Monacelli (2005), it is assumed that each household supplies a differentiated labor service indexed by $j\in[0,1]$. Furthermore, each household (with a monopoly power in the labor market) sets nominal wages in a staggered fashion with timing as in Calvo (1983): in each period, only a fraction $(1-\theta_w)$ of households, drawn randomly from the population, reoptimize their posted nominal wages.

2.2.1. Optimal wage setting

Consider a household resetting its wage in period t and let $\overline{W}_t$ denotes the newly set wage. Under the assumption of full consumption risk sharing across households, all households resetting their wages in any given period will choose the same wage. The household will choose $\overline{W}_t$ in order to maximize

$$E_t\left\{\sum_{k=0}^{\infty}(\beta\theta_w)^k U\left(C_{t+k|t}, N_{t+k|t}\right)\right\}, \tag{4}$$

where $C_{t+k|t}$ and $N_{t+k|t}$ respectively denote the composite consumption of domestic and imported goods and labor supply in period $t+k$ of a household that last reset its wage in period t.

Maximization of (4) is subject to the sequence of labor demand schedules and budget constraints that are effective while $\overline{W}_t$ remains in place:

$$\begin{aligned} N_{t+k|t} &= \left(\frac{\overline{W}_t}{W_{t+k}}\right)^{-\varepsilon_w} N_{t+k}, \\ P_{t+k}C_{t+k|t} + E_{t+k}&\left\{PSI_{t+k,t+k+1}B_{t+k+1|t}\right\} \le B_{t+k|t} + \overline{W}_tN_{t+k|t} + T_{t+k}, \end{aligned} \tag{5}$$

where $X_{t+k|t}$ denotes the value of X in period $t+k$ of a household that last reset its wage in period t. The remaining variables are defined as above.

The first-order condition is given by

$$\sum_{k=0}^{\infty}(\beta\theta_w)^k E_t\left\{N_{t+k|t}U_c\left(C_{t+k|t}, N_{t+k|t}\right)\left(\frac{\overline{W}_t}{P_{t+k}} - \frac{\varepsilon_w}{\varepsilon_w - 1}\mathrm{MRS}_{t+k|t}\right)\right\} = 0, \tag{6}$$

where $\mathrm{MRS}_{t+k|t} = -U_N(C_{t+k|t},N_{t+k|t})/U_C(C_{t+k|t},N_{t+k|t})$ denotes the marginal rate of substitution between consumption and labor in period $t+k$ for the household resetting the wage in period t.

Log-linearizing (6) around the zero-inflation steady state yields

$$\overline{w}_t = \mu^w + (1-\beta\theta_w)\sum_{k=0}^{\infty}(\beta\theta_w)^k E_t\left\{\text{mrs}_{t+k|t} + p_{t+k}\right\}, \tag{7}$$

where $\mu^w = \log(\varepsilon_w/\varepsilon_w - 1)$, which corresponds to the optimal (log) wage mark-up.

To obtain an explicit solution, we assume that the period utility function takes the specific functional form of $U(C_t, N_t) = C_t^{1-\sigma}/(1-\sigma) - N_t^{1+\varphi}/(1+\varphi)$, where $\sigma > 0$ and $\varphi > 0$. Let $\text{mrs}_{t+k} = \sigma c_{t+k} + \varphi n_{t+k}$. Then, the (log) marginal rate of substitution in period $t+k$ for a household that last reset its wage in period t can be written as

$$\text{mrs}_{t+k|t} = \text{mrs}_{t+k} + \varphi\left(n_{t+k|t} - n_{t+k}\right) = \text{mrs}_{t+k} - \varepsilon_w\varphi(\overline{w}_t - w_{t+k}),$$

where the last equality makes use of (5). Hence, we can rewrite (7) as

$$\overline{w}_t = \frac{1-\beta\theta_w}{1+\varepsilon_w\varphi}\sum_{k=0}^{\infty}(\beta\theta_w)^k E_t\{\mu^w + \text{mrs}_{t+k} + \varepsilon_w\varphi w_{t+k} + p_{t+k}\}. \tag{8}$$

2.2.2. *Other optimality conditions*

In addition to the optimal wage setting condition, the solution to the household's problem also yields

$$\beta\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{P_t}{P_{t+1}}\right] = \Psi_{t,t+1}, \tag{9}$$

which is a standard Euler equation for intertemporal consumption decision and represents the expectational IS curve.

Taking conditional expectations of both sides of (9) and rearranging with the riskless short-term nominal interest rate, we obtain a standard stochastic Euler equation:

$$\beta R_t E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{P_t}{P_{t+1}}\right\} = 1. \tag{10}$$

For future reference, we write (10) in log-linearized form as

$$c_t = E_t\{c_{t+1}\} - \sigma^{-1}\left[r_t - E_t\{\pi_{C,t+1}\} - \rho\right], \tag{11}$$

where $\pi_{C,t+1} = p_{t+1} - p_t$ is CPI inflation, c_t denotes total aggregate consumption, and $r_t = -\log\Psi_{t,t+1}$ is the nominal yield on the one-period bond.

3. Equilibrium

3.1. *Aggregate demand*

Goods market clearing in the home country requires

$$\begin{aligned} Y_t(i) &= C_{H,t}(i) + C^*_{H,t}(i) \\ &= \left(\tfrac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon_p}\left[(1-\alpha)\left(\tfrac{P_{H,t}}{P_t}\right)^{-\eta}C_t + \alpha\left(\tfrac{P^*_{H,t}}{P^*_t}\right)^{-\eta}C^*_t\right]. \end{aligned} \tag{12}$$

Substituting (12) into the definition of aggregate domestic output index together with the international risk-sharing condition, $C_t = Q_t^{1/\sigma}C^*_t$, yields

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t\left[(1-\alpha) + \alpha Q_t^{\eta-\frac{1}{\sigma}}\right], \tag{13}$$

where $Q_t = \varepsilon_p P^*_t/P_t$ is the real exchange rate.

The first-order log-linear approximation to (13) around the steady state:

$$y_t = c_t - \eta\left(p_{H,t} - p_t\right) + \alpha\left(\eta - \frac{1}{\sigma}\right)q_t = c_t + \frac{\alpha}{\sigma}\Omega s_t, \tag{14}$$

where $s_t = p_{F,t} - p_{H,t}$ is the terms of trade and $\Omega = \eta\sigma + (1-\alpha)(\eta\sigma - 1)$. The last equality in (14) follows from $q_t = (1-\alpha)s_t$.

For the foreign country, $y_t^* = c_t^*$. Hence, a condition similar to (14) for the foreign country can be written as

$$c_t^* = y_t^* + \frac{1-\alpha}{\sigma}s_t. \tag{15}$$

Combining (14) with (15), we obtain

$$y_t = y_t^* + \frac{1}{\sigma_\alpha}s_t, \tag{16}$$

where $\sigma_\alpha = \sigma/(1 - \alpha) + \alpha\Omega > 0$.

Finally, combining (14) with the Euler Equation (11) yields

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma_\alpha}\left[r_t - E_t\{\pi_{H,t+1}\} - \rho\right] + \alpha\Phi E_t\{\Delta y_{t+1}^*\},$$

where $\Phi = \sigma_\alpha(\Omega-1)/\sigma - \sigma_\alpha\alpha(\Omega-1)$.

3.2. The supply side

The (log-linearized) optimal price-setting condition (2) can be combined with the (log-linearized) difference equation describing the evolution of domestic prices to yield an equation determining domestic inflation as a function of deviations of marginal cost from its steady state value:

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} - \lambda_{p_H}\widehat{\mu}_t^{p_H}, \tag{17}$$

where $\widehat{\mu}_t^{p_H} = \mu_t^{p_H} - \mu^{p_H} = -\widehat{mc}_t$ and $\lambda_{p_H} = (1-\theta_{p_H})(1-\beta\theta_{p_H})/\theta_{p_H}$.

Let $\widehat{\mu}_t^w = \mu_t^w - \mu^w$ denote deviations of the economy's average (log) wage markup, defined as $\mu_t^w = (w_t - p_t) - \mathrm{mrs}_t$, from its steady state level μ^w. Then (8) can be rewritten as

$$\overline{w}_t = \beta\theta_w E_t\{\overline{w}_{t+1}\} + (1-\beta\theta_w)\left[w_t - (1-\varepsilon_w\varphi)^{-1}\widehat{\mu}_t^w\right]. \tag{18}$$

Since all households that are able to adjust their wages at time t will choose the same wages, the aggregate wage index will evolve according to

$$W_t = \left[\theta_w W_{t-1}^{1-\varepsilon_w} + (1-\theta_w)\left(\overline{W}_t\right)^{1-\varepsilon_w}\right]^{\frac{1}{1-\varepsilon_w}}. \tag{19}$$

A first-order Taylor expansion of (19) around the zero-inflation steady state yields

$$w_t = \theta_w w_{t-1} + (1-\theta_w)\overline{w}_t. \tag{20}$$

Combining (18) with (20) results in

$$\pi_{W,t} = \beta E_t\{\pi_{W,t+1}\} - \lambda_w\widehat{\mu}_t^w,$$

where $\pi_{W,t} = w_t - w_{t-1}$, and $\lambda_w = (1-\theta_w)(1-\beta\theta_w)/\theta_w(1+\varepsilon_w\varphi)$. Note that this wage inflation equation has a form analogous to Eq. (17) on the dynamics of domestic price inflation.

Next, we define the domestic natural level of output which is the equilibrium level in the absence of nominal rigidities. For that purpose, we, first, derive the real marginal cost as a function of domestic output. The real marginal cost is

$$mc_t = -\nu + (w_t - p_{H,t}) - a_t = -\nu + (w_t - p_t) + (p_t - p_{H,t}) - a_t$$
$$= -\nu + \sigma c_t + \varphi n_t + \alpha s_t - a_t,$$

where the last equality makes use of the fact that $p_t = p_{H,t} + \alpha s_t$. Finally, using (14) and (16), we can rewrite the real marginal cost in terms of domestic output and productivity as well as world output:

$$mc_t = -\nu + (\sigma_\alpha + \varphi)y_t - (\sigma - \sigma_\alpha)y_t^* - (1 - \varphi)a_t.$$

Note that under flexible prices, $mc_t = -\nu$ for all t. Thus, the natural level of output is given by

$$y_t^n = \Gamma_0 + \Gamma_a a_t + \Gamma_{y^*} y_t^*,$$

where $\Gamma_0 = \nu - \mu^{p_H}/\sigma_\alpha + \varphi, \Gamma_a = 1 + \varphi/\sigma_\alpha + \varphi$, and $\Gamma_{y^*} = \sigma - \sigma_\alpha/\sigma_\alpha + \varphi$. Note that the natural rate of output defined above is the equilibrium level of output in the absence of both price and wage rigidities.

3.3. *Equilibrium dynamics*

In this section, we derive equations for linearized equilibrium dynamics for the domestic price and wage inflation in terms of the output gap $\tilde{y}_t = y_t - y_t^n$.

First, we introduce a new variable, the real wage gap $\tilde{\omega}_t$, defined as

$$\tilde{\omega}_t = \omega_t - \omega_t^n,$$

where $\omega_t = w_t - p_t$ denotes the real wage, and where ω_t^n is the natural real wage, i.e., the real wage that would prevail in the flexible price and wage equilibrium. Then the natural real wage can be expressed as

$$\omega_t^n = \nu - \alpha s_t^n + a_t - \mu^p = \nu - \alpha\sigma_\alpha(y_t^n - y_t^*) + a_t - \mu^p,$$

where the last equality makes use of (16), and where s_t^n is the terms of trade at the natural level. Thus, we see that the small open economy's natural real wage is similar to that found in Erceg et al. (2000) and Galí (2008). However, two difference stand out. First, openness leads to dependence of the natural real wage on world output. Second, the degree of openness influences the sensitivity of the natural real wage to output.

Next, we relate the average price markup to the output and real wage gaps. Using the fact that $\widehat{\mu}_t^{p_H} = -(mc_t - mc_t^n)$, we obtain

$$\widehat{\mu}_t^{p_H} = -\left[(\omega_t - \omega_t^n) + \alpha(s_t - s_t^n)\right] = -\widetilde{\omega_t} - \alpha\widetilde{s_t} = -\widetilde{\omega_t} - \alpha\sigma_\alpha\widetilde{y}_t. \tag{21}$$

Hence, combining (17) and (21) yields

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \kappa_{p_H}\tilde{y}_t + \lambda_{p_H}\tilde{w}_t, \tag{22}$$

where $\kappa_{p_H} = \alpha\sigma_\alpha\lambda_{p_H}$.

Similarly, relate the average wage markup to the output and real wage gaps as

$$\widehat{\mu}_t^w = \omega_t - \mathrm{mrs}_t - \mu^w = -\widetilde{\omega_t} - \left[\sigma\widetilde{c_t} + \varphi\widetilde{n}_t\right] = -\widetilde{\omega_t} - [\sigma - \alpha\sigma_\alpha\Omega + \varphi]\widetilde{y}_t.$$

Therefore, we obtain

$$\pi_{W,t} = \beta E_t\{\pi_{W,t+1}\} + \kappa_W\tilde{y}_t - \lambda_W\tilde{w}_t, \tag{23}$$

where $\kappa_W = [\sigma - \alpha\sigma_\alpha\Omega + \varphi]\lambda_W$.

The major difference between the equations for domestic price inflation and wage inflation, (22) and (23), and those found in Erceg et al. (2000) is that the degree of openness α affects the dynamics of inflation through its influence on the slopes in (22) and (23).

Finally, the equation for the real wage gap is given by

$$\tilde{w}_t = \tilde{w}_{t-1} + \pi_{W,t} - \pi_{H,t} - \alpha\Delta s_t - \Delta w_t^n. \tag{24}$$

Eq. (24) differs from its closed economy counterpart in two ways. First, through a change in the natural real wage, foreign output has an effect on the real wage gap. Second, openness also makes the real wage gap depend on the terms of trade, and the degree of openness α affects the sensitivity of the real wage gap to the terms of trade.

The following important property of (22) and (23) is worth emphasizing at this point. Through the real wage gap, a change in the terms of trade affects both domestic price and wage inflation. The dependence of wage inflation on the terms of trade is a natural feature of an open economy. However, the presence of the terms of trade in the domestic inflation dynamics, which is called the direct exchange rate channel, is quite different from the standard new Keynesian open economy Phillips curve of Galí and Monacelli (2005). It has been argued that there exists a direct exchange rate channel to domestic price inflation (Guender, 2006; Svensson, 2000), that is not incorporated explicitly into the standard new Keynesian open economy Phillips curve. In this model, we incorporate the direct exchange rate channel into the domestic price inflation through staggered wage contracting. Using $\pi_{C,t} = \pi_{H,t} + \alpha \Delta s_t$, we can rewrite Eq. (24) as

$$\tilde{w}_t = \tilde{w}_{t-1} + \pi_{W,t} - \pi_{C,t} - \Delta w_t^n.$$

Thus, we see that CPI inflation has a direct effect on the dynamics of both domestic price inflation and wage inflation.

The dynamic IS equation for the small open economy can be obtained by rewriting (14) in terms of the output gap as

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma_\alpha}\left[r_t - E_t\{\pi_{H,t+1}\} - rr_t^n\right],$$

where $rr_t^n = \rho + \sigma_\alpha \Gamma_a(\rho_a - 1)a_t + \sigma_\alpha[\alpha\Phi - \Gamma_{y^*}]E_t\{\Delta y_{t+1}^*\}$ is the small open economy's natural rate of interest.

In order to close the model, we assume a Taylor-type interest rate rule of the form

$$r_t = \rho + \varphi_\pi \pi_{H,t} + \varphi_y \tilde{y}_t + \nu_t,$$

where ν_t is an exogenous monetary policy component, which is assumed to follow an AR (1) process:

$$\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_t^\nu,$$

where $\rho_\nu \in [0,1]$, and ε_t^ν is a white noise process with variance σ_ν^2.

3.4. Dynamic responses to a monetary policy shock

This section studies how the presence of staggered wage setting influences the economy's response to a monetary policy shock. In the baseline calibration of the model, one period corresponds to one quarter of a year.

It is assumed that $\beta = 0.99$, which generates to a real interest rate of around 4% per annum. It is also assumed that $\sigma = \eta = 1$ (which is consistent with the case considered in the next section). We set the value of α (degree of openness) to 0.3, which corresponds to the import/GDP ratio in Canada for the period 2003–2012. We set $\varphi = 3$, implying that the labor supply elasticity is taken as 1/3. The values of the steady state mark-up are set as $\varepsilon_p/\varepsilon_{p-1} = \varepsilon_W/\varepsilon_{W-1} = 1.2$. This implies that the elasticities of substitution between differentiated goods and labor services, ε_p and ε_W, equal 6. These parameters are taken from Galí and Monacelli (2005). The domestic price and wage contract duration parameters are set as $\theta_{p_H} = \theta_W = 0.75$ (implying the average contract duration of 4 quarters). The specification of the interest rate rules follows Taylor (1993): $\varphi_y = 0.125$ and $\varphi_\pi = 1.5$. Finally, the persistency parameter of monetary shocks is chosen as $\rho_\nu = 0.5$.

Fig. 1 illustrates the dynamic effects of a contractionary monetary policy shock on domestic inflation, wage inflation, CPI inflation, the rate of depreciation, real wage rate, and output gap. The shock consists of an increase of 0.25 percentage points in the exogenous component of the interest rate rule, which, in the absence of an endogenous change induced by the response of domestic inflation or the

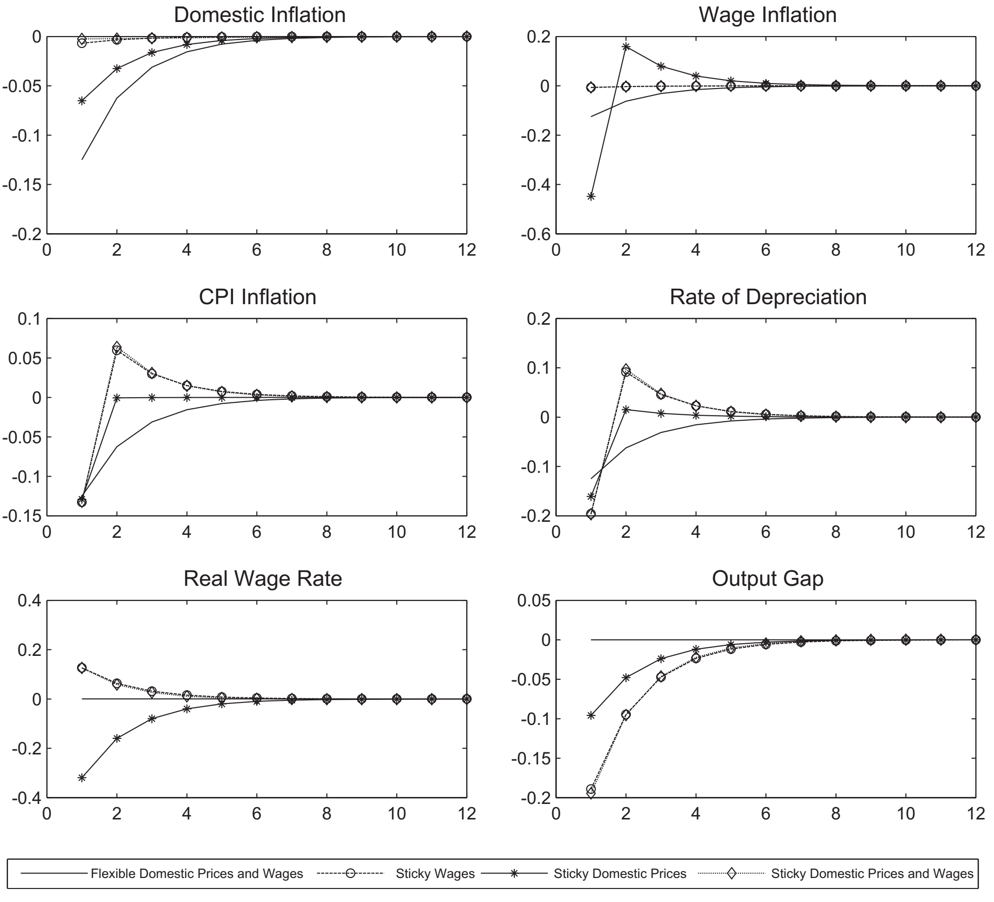

Fig. 1. Impulse responses to a monetary policy shock.

output gap, would lead to an impact increase of one percentage point in the annualized nominal interest rate. In order to disentangle the role played by each type of rigidity, the results are shown for four alternative cases for the values of θ_{p_H} and θ_W. The first case corresponds to an economy in which both domestic prices and wages are flexible ($\theta_{p_H} = \theta_W = 0$). The second case assumes flexible domestic prices and sticky wages ($\theta_{p_H} = 0, \theta_W = 0.75$). The third case corresponds to an economy with sticky domestic prices and flexible wages ($\theta_{p_H} = 075, \theta_W = 0$). Finally, in the fourth case we have an economy with both domestic price and wage rigidities (the wage and price contract duration parameters are set as $\theta_{p_H} = \theta_W = 0.75$ implying the average contract duration of 4 quarters).[4]

Our benchmark is the case when both domestic prices and wages are flexible (solid lines). In the flexible (domestic) price and wage economy, a contractionary monetary policy shock induces substantial decreases in domestic prices and nominal wages. Given the constancy of the foreign nominal interest rate, uncovered interest parity implies an initial nominal appreciation, which in turn, with a drop in domestic price inflation, reduces CPI inflation. The sizes of the CPI inflation drop and the reduction in wage inflation are almost the same. As a result, there is only a moderate response of the real wage, which in turn reduces the impact on the marginal rate of substitution and hence, a muted overall response of output. The monetary shock is absorbed by the price change, and the impact on the real economy is

[4] For computational reasons, when either θ_{p_H} or θ_W is assumed to be zero, we set their values equal to 0.000001.

limited. Notice that the nominal exchange rate does not overshoot. By contrast, the presence of both sticky domestic prices and wages (dotted lines with diamonds) generates, not surprisingly, more muted responses of domestic price and wage inflation, and a large and persistent reduction in output. Given the muted response of domestic price inflation, an initial large appreciation is followed by expectations of a future depreciation, which is reflected in the response of CPI inflation. The initial large decrease in CPI inflation with a muted response of wage inflation leads to a rise in the real wage.

Consider next the consequences of assuming sticky (domestic) prices and flexible wages (solid lines with stars). The presence of sticky domestic prices dampens the response of domestic inflation to a contractionary monetary policy shock. Wage inflation, however, falls considerably due to the absence of constraints on wage adjustment. The response of the output gap is more moderate than that under both flexible domestic prices and wages. A relatively large decrease in wage inflation (compared with CPI inflation) generates a persistent reduction in real wages. In the flexible domestic price/sticky wage economy the responses are similar to those under both sticky domestic prices and wages. The influence of sticky (domestic) prices on the economy's equilibrium response is very limited. It is evident that wage stickiness plays a more prominent role than price stickiness for generating persistent real effects of monetary shocks in our small open economy.

The panel for real wage rate in Fig. 1 indicates that real wages move countercyclically. This is a common criticism of models with nominal wage rigidity. The intuition for this in our model is as follows. When the central bank raises the nominal interest rate, CPI decreases through the exchange rate channel. Since nominal wages are sticky, real wages rise. We would like to point out here that cyclical behavior of real wages varies across time and countries. For example, real wages in the U.S. were mildly countercyclical between the two World Wars and mildly procyclical after World War II (Huang et al., 2004). Furthermore, Messina et al. (2009) find that open economies tend to show countercyclical wages; in the case of Spain, Bentolila et al. (2012) ascribe this countercyclical behavior of real wages to a deep insider-outsider divide in the labor market. Thus, the cyclical behavior of real wages in an open economy model should perhaps be modeled with a specific country in mind.

4. Monetary policy design in a small open economy

This section explores the implications of the existence of sticky wages in a small open economy, as modeled in Section 2, for the conduct of monetary policy. Let us first consider the efficient allocation, i.e., the equilibrium allocation in the absence of nominal rigidities. It corresponds to the solution of a sequence of static social planner problems of the form

$$\max \int_0^1 U(C_t(j), N_t(j))\mathrm{d}j,$$

subject to (i) the technological constraint $Y_t(i) = A_t N_t(i)$, (ii) the index of labor input (1) used by firm i, (iii) the market clearing condition (13), and (iv) a consumption/output possibilities set (the international risk-sharing condition $C_t = Q_t^{1/\sigma} C_t^*$). The efficient allocation must satisfy

$$\begin{aligned}
&C_t(j) = C_t, j \in [0,1], \\
&N_t(i,j) = N_t(j) = N_t(i) = N_t, \\
&-\tfrac{U_N(C_t,N_t)}{U_C(C_t,N_t)} = MPN_t,
\end{aligned}$$

where $MPN_t = A_t$. In order to keep the analysis as simple as possible, we restrict ourselves to the special case of $\sigma = \eta = 1$. Then, the efficient allocation can be characterized by $C_t N_t^{\varphi} = (1-\alpha)C_t/N_t$. Thus, at the efficient allocation the employment is constant: $N = (1-\alpha)^{1/1+\varphi}$.

Notice that the flexible price and wage equilibrium satisfies

$$\begin{aligned}
&\tfrac{\varepsilon_p - 1}{\varepsilon_p} = MC_t^n = -\tfrac{1-\tau}{A_t}\tfrac{W_t}{P_t}\left(S_t^n\right)^{\alpha} \\
&= \tfrac{(1-\tau)}{A_t}\tfrac{\varepsilon_w}{\varepsilon_w - 1}\left(N_t^n\right)^{\varphi} C_t^n \tfrac{Y_t^n}{C_t^n} = (1-\tau)\tfrac{\varepsilon_w}{\varepsilon_w - 1}\left(N_t^n\right)^{1+\varphi}.
\end{aligned}$$

Since at the efficient allocation $N = (1-\alpha)^{1/1+\varphi}$, by setting $(1-\tau)(1-\alpha) = (\varepsilon_p - 1/\varepsilon_p)(\varepsilon_w - 1/\varepsilon_w)$ the condition for the efficient allocation is also satisfied thus guaranteeing the efficiency of the flexible price equilibrium allocation. We assume that this property holds for the remainder of this study.

Following a well established tradition in macroeconomic literature (e.g., Woodford, 2003; chapter 6), we will next use a second-order approximation to expected welfare loss function, which, together with our linear structural equations allows one to use linear-quadratic optimization methods that are well understood and extensively studied. As is shown in the Appendix, the second-order approximation to the average welfare losses around the zero-inflation steady state, under the assumption of $\sigma = \eta = 1$, are given by

$$\mathbf{W} = \frac{1-\alpha}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ (1+\varphi)\tilde{y}_t^2 + \frac{\varepsilon_p}{\lambda_{p_H}} (\pi_{H,t})^2 + \frac{\varepsilon_W}{\lambda_W} (\pi_{W,t})^2 \right\} + t.i.p. , \tag{25}$$

where *t.i.p.* collects various terms that are independent of policy. Thus, the average period welfare loss is

$$\mathbf{L} = \frac{1-\alpha}{2} \left[(1+\varphi)\mathrm{var}(\tilde{y}_t) + \frac{\varepsilon_p}{\lambda_{p_H}} \mathrm{var}(\pi_{H,t}) + \frac{\varepsilon_W}{\lambda_W} \mathrm{var}(\pi_{W,t}) \right]. \tag{26}$$

Note that the relative weight of each of the variances is a function of the underlying parameter values. The period welfare loss (26) is similar to that derived in Erceg et al. (2000) except for the presence of degree of openness (α).

As discussed in Erceg et al. (2000) and Galí (2008), in the limiting case of flexible wages, $\lambda_W \to +\infty$, the term in the loss function associated with wage inflation volatility vanishes and the wage markup is constant. Hence,

$$\tilde{\omega}_t = \sigma\tilde{c}_t + \varphi\tilde{n}_t = \left[(1-\alpha)\sigma_\alpha + \frac{\varphi}{1-\alpha} \right] \tilde{y}_t,$$

which, substituted into (22), yields a standard new Keynesian open economy Phillips curve similar to that derived in Galí and Monacelli (2005):

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \lambda_{p_H}\left(\sigma_\alpha + \frac{\varphi}{1-\alpha}\right)\tilde{y}_t.$$

Therefore, there is no tradeoff between stabilization of domestic price inflation and stabilization of the output gap. In this case, the optimal monetary policy will attain the zero lower bound for welfare losses by fully stabilizing the output gap and domestic price inflation: $\pi_{H,t} = \tilde{y}_t = 0$ for all t. With both sticky prices and wages, however, monetary policy cannot attain the zero welfare losses outcome characterized by $\pi_{H,t} = \pi_{W,t} = \tilde{y}_t = 0$ for all t.

4.1. Optimal monetary policy

We are now ready to characterize optimal policy for our small open economy with sticky wages and prices. The optimal policy will strike a balance in stabilizing domestic price inflation, wage inflation, and the output gap. Hence, the central bank will seek to minimize (25) subject to the sequence of equilibrium constraints given by (22)–(24). The first-order conditions are:

$$(1-\alpha)(1+\varphi)\tilde{y}_t + \varsigma_{1t}\,\kappa_{p_H} + \varsigma_{2t}\,\kappa_W = 0, \tag{27}$$

$$(1-\alpha)\frac{\varepsilon_p}{\lambda_{p_H}}\pi_{H,t} - \varsigma_{1t} + \varsigma_{1t-1} - \varsigma_{3t} = 0, \tag{28}$$

$$(1-\alpha)\frac{\varepsilon_W}{\lambda_W}\pi_{W,t} - \varsigma_{2t} + \varsigma_{2t-1} + \varsigma_{3t} = 0, \tag{29}$$

$$-\varsigma_{3t} + \lambda_{p_H}\pi_{W,t} - \lambda_W\varsigma_{2t} + \beta E_t\{\varsigma_{3t+1}\} = 0, \tag{30}$$

where ς_{1t},ς_{2t}, and ς_{3t} are the Lagrange multipliers associated with the three period t constraints. The dynamical system describing the optimal monetary policy is thus composed of (27)–(30) together with constraints (22)–(24).

4.2. Evaluation of monetary policy rules

This section considers a number of simple monetary policy rules and provides a quantitative evaluation of their impact on welfare using the performance of the optimal monetary policy rule as a benchmark. The evaluation is based on the unconditional period losses implied by each simple rule, given by (26).

In addition to optimal rule, four different simple rules are studied. Each rule requires that the domestic interest rate respond systematically to the output gap and some rule-specific measure of inflation. The first rule, which is referred to as the domestic inflation-based Taylor rule, uses domestic inflation. The second employs CPI inflation and is referred to as the CPI inflation-based Taylor rule. We also consider an analogous rule for wage inflation (the wage inflation-based Taylor rule). And the last rule uses a weighted average of domestic price and wage inflation and is referred to as the composite inflation-based Taylor rule. Formally, the domestic inflation-based Taylor rule (DIT, for short) is assumed to take this form:

$$r_t = \rho + \varphi_\pi \pi_{H,t} + \varphi_y \tilde{y}_t. \tag{31}$$

The CPI inflation-based Taylor rule (CPIT), and the wage inflation-based Taylor rule (WIT) are specified respectively as follows:

$$r_t = \rho + \varphi_\pi \pi_{C,t} + \varphi_y \tilde{y}_t, \tag{32}$$

$$r_t = \rho + \varphi_\pi \pi_{W,t} + \varphi_y \tilde{y}_t. \tag{33}$$

Finally, the composite inflation-based Taylor rule (COMT) is given by

$$r_t = \rho + \varphi_\pi \left[\frac{\lambda_{p_H}}{\lambda_{p_H} + \lambda_W} \pi_{H,t} + \frac{\lambda_W}{\lambda_{p_H} + \lambda_W} \pi_{W,t} \right] + \varphi_y \tilde{y}_t. \tag{34}$$

We follow Galí and Monacelli (2005) to specify the exogenous processes as follows:

$$\begin{aligned} a_t &= 0.66 a_{t-1} + \varepsilon_t^a, \quad \sigma_a = 0.0071, \\ y_t^* &= 0.86 y_{t-1}^* + \varepsilon_t^{y^*}, \quad \sigma_{y^*} = 0.0078, \end{aligned}$$

where ε_t^a and $\varepsilon_t^{y^*}$ are white noises with variances σ_a and σ_{y^*} respectively. The remaining parameters are set at their baseline values. The wage and price contract duration parameters are chosen as follows: $\theta_{p_H} = \theta_W = 0.75$.

4.2.1. Impulse responses

The results of the quantitative analysis can be visualized through impulse response functions for a number of macroeconomic variables. In Fig. 2 we describe the dynamic effects of a domestic productivity shock on macroeconomic variables considered earlier under different policy regimes.

We start by describing impulse responses of main variables under the optimal policy. All variables remain stable to the shock under the optimal policy, and their responses are relatively small compared to those under the other four policies.

Let us contrast the dynamics of the variables under different policy regimes. Notice, at first, that the responses of domestic price inflation are qualitatively similar under the four different policy regimes except that the initial responses are more muted under the DIT and COMT rules since they explicitly target stabilization of domestic price inflation. The dynamics of the other key variables are also similar under the DIT and COMT rules. The two Taylor rules (DIT and COMT) lead to a large and persistent decline in wage inflation as well as the output gap. On impact, there is an appreciation of the nominal exchange rate, which in turn is reflected in an initial decrease in CPI inflation. The relatively large initial fall in CPI inflation leads to an initial increase in real wages under the DIT and COMT rules.

The CPI inflation-based Taylor rule, of course, generates a more muted response of CPI inflation than any other policy rule. We can also see a stable response of wage inflation under the wage-inflation

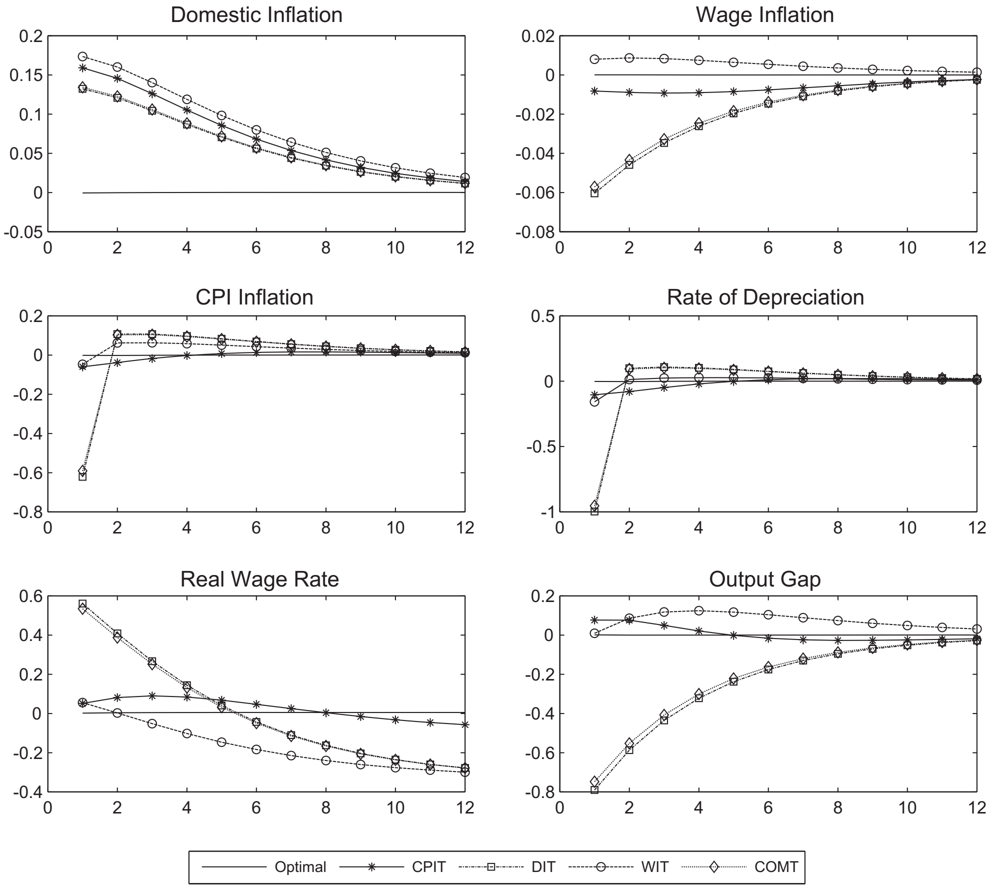

Fig. 2. Impulse responses to a technological shock: alternative policy rules.

based Taylor rule. However, it is interesting to notice that the response of wage inflation is smooth under both the CPI and WIT rules. This is because controlling CPI inflation reduces volatility of the real wage gap. The small volatility of the real wage gap leads to stable wage inflation. The response of the output gap is also more moderate under the CPIT and WIT rules. The intuition is as follows. The visual inspection of impulse responses suggests that the WIT rule performs nearly as well as CPIT rule. The major difference between the CPIT rule and WIT rule is that the CPI inflation-based Taylor rule generates, unlike the wage inflation-based rule, persistent decreases in both CPI and wage inflation. While under the WIT policy the output gap shows a hump-shaped pattern, under the CPIT rule the output gap increases on impact and then starts to revert to the steady state.

Let us comment on one of the main results in this paper: CPI inflation targeting policy yields higher welfare level than domestic inflation targeting policy. The second-order approximation to welfare losses can be expressed in terms of unconditional variances of the output gap, domestic price inflation, and wage inflation. The domestic price inflation-based Taylor rule generates relatively large losses due to excessive variation in wage inflation and the output gap. The latter can be explained by a large response of the real wage rate to technology shocks. On the other hand, as has been discussed in Subsection 3.3, there is a direct effect of CPI inflation on the dynamics of both domestic price inflation and wage inflation. Thus, the CPI inflation-based Taylor rule leads to relatively small variation in domestic inflation and wage inflation. The latter coupled with small variation in CPI inflation results in small variation in the real wage gap, which, in turn, yields small variation in output gap. As a result, CPI inflation targeting strikes a good balance and yields, compared to domestic inflation targeting, relatively low variation in both wage inflation and output gap, two of the three components of welfare measure. The third component of welfare, variation in domestic inflation, behaves quite similarly for both targeting regimes.

Our remaining main results and intuition behind them is presented in the following subsection.

4.2.2. *Second moments and welfare losses*

An alternative way to quantitatively analyze monetary policy is to calculate the standard deviations of macroeconomic variables and the loss incurred by the economy from the shocks in different cases. Table 1 reports the standard deviations of several key variables under alternative monetary policy rules with the optimal policy being a useful benchmark. The table confirms that the key variables are relatively less volatile under the optimal policy. In particular, we notice that the output gap is more stable under the optimal policy than under any other alternative policy regime.

We can also see that the DIT and COMT rules generate relatively less volatile domestic price inflation, but the CPIT and WIT rules result in relatively low variances of CPI inflation and wage inflation, respectively. This finding confirms what was already evident from the impulse response analysis. The critical feature of this exercise is that the CPI inflation-based Taylor rule leads to relatively small variation in domestic inflation and wage inflation as well as in the output gap among alternative policy regimes, but the domestic inflation-based Taylor rule induces relatively large volatility in wage inflation and the output gap. The DIT rule leads only to smoothness of domestic inflation.

Table 2 reports the variances of domestic price inflation, wage inflation, the output gap, and the welfare losses associated with the four alternative policy rules. We display the effects of changing the steady state domestic price and wage markups (i.e., changes in ε_{p_H} and ε_W). The top panel reports statistics corresponding to the benchmark calibration of the steady state markup, namely, the case when $\varepsilon_{p_H} = 6, \varepsilon_W = 6$. Relative to this benchmark, the second panel assumes a lower steady state wage markup ($\varepsilon_W = 11$), while the third panel reports results for a lower domestic price markup ($\varepsilon_{p_H} = 11$). And the last panel reports numbers for to the case of both lower steady state domestic price and wage markup ($\varepsilon_{p_H} = 11, \varepsilon_W = 11$). The main findings of this exercise are consistent with the quantitative evaluation of standard deviations conducted in Table 1: the domestic price inflation-based Taylor rule generates relatively large welfare losses due to excessive variation in wage inflation and the output gap, whereas the CPI inflation-based Taylor rule performs nearly as well as the optimal rule and leads to relatively small variation in wage inflation and the output gap. There is a direct effect of CPI inflation on the dynamics of domestic price and wage inflation. Therefore, stabilizing CPI inflation is important for reducing volatility of domestic price and wage inflation. Less volatile domestic price inflation induces small variance of the output gap under the CPI inflation-based Taylor rule. This result stands in sharp contrast with the policy implication of the standard new Keynesian model without staggered wage-settings (Clarida et al., 2002; Galí and Monacelli, 2005). The ranking among alternative policy rules does not change across the four mark-up cases considered.

We have also studied the behavior of our economy under a different version of the composite inflation-based Taylor rule which targets a weighted average of CPI and wage inflation (recall that COMT targets an average of domestic inflation and wage inflation). The responses of main variables are found to be more volatile than those under the CPIT and WIT rules but more muted than those under DIT and COMT.

Overall, the implication of the exercise in this section can be viewed as twofold. First, in the presence of both sticky wages and sticky domestic prices in a small open economy, the policy rule that

Table 1
Standard deviations in %.

	Optimal	DIT	CPIT	WIT	COMT
Domestic inflation	0.221	0.179	0.216	0.242	0.181
Wage inflation	0.010	0.065	0.017	0.013	0.062
CPI inflation	0.409	0.839	0.058	0.394	0.811
Output gap	0.023	0.836	0.102	0.206	0.787
Nominal depreciation rate	0.441	0.850	0.425	0.433	0.822

Table 2
Contribution to welfare losses.

	Optimal	CPIT	WIT	COMT	DIT
$\varepsilon_{p_H} = 6, \varepsilon_W = 6$					
$\text{Var}(\tilde{y}_t)$	0.0005	0.0104	0.0423	0.6190	0.6990
Var $(\pi_{H,t})$	0.0490	0.0466	0.0586	0.0329	0.0319
Var (w_t)	0.0001	0.0003	0.0002	0.0038	0.0043
Loss	1.0700	1.1100	1.3600	2.9600	3.2200
$\varepsilon_{p_H} = 6, \varepsilon_W = 11$					
$\text{Var}(\tilde{y}_t)$	0.0018	0.0106	0.0465	0.7260	0.7770
Var $(\pi_{H,t})$	0.0501	0.0508	0.0563	0.0399	0.0395
Var (w_t)	0.0000	0.0001	0.0000	0.0015	0.0016
Loss	1.0500	1.0800	1.2400	1.7100	1.7600
$\varepsilon_{p_H} = 11, \varepsilon_W = 6$					
$\text{Var}(\tilde{y}_t)$	0.0008	0.0104	0.0423	0.6190	0.6990
Var $(\pi_{H,t})$	0.0464	0.0466	0.0586	0.0329	0.0319
Var (w_t)	0.0003	0.0003	0.0001	0.0038	0.0043
Loss	1.9100	1.9300	2.3800	3.5400	3.7800
$\varepsilon_{p_H} = 11, \varepsilon_W = 11$					
$\text{Var}(\tilde{y}_t)$	0.0005	0.0106	0.0465	0.7260	0.7770
Var $(\pi_{H,t})$	0.0507	0.0490	0.0563	0.0399	0.0395
Var (w_t)	0.0000	0.0001	0.0000	0.0015	0.0016
Loss	2.0000	2.0500	2.3100	4.3400	4.5100

targets exclusively domestic price inflation is suboptimal. Second, the policy that responds to CPI inflation seems to outperform other alternative policy rules.

5. Conclusion

In this paper, we study several monetary policy rules for a small open economy within a framework where both domestic prices and wages are sticky due to staggered contracts. This study delivers three messages regarding the issue of monetary policy design in a small open economy. First, the optimal policy is to seek to minimize a weighted average of the variances of domestic price inflation, wage inflation, and the output gap. Second, the policy that exclusively targets domestic price inflation is suboptimal. Last, the policy that responds to CPI inflation is almost as good as the optimal policy. To bring our work in line with growing empirical evidence that fluctuations in exchange rates result in less than proportional changes in prices of traded goods and that prices respond with some delay (Engel, 1999; Parsley and Wei, 2001), we plan to extend this research to the case of incomplete and delayed exchange rate pass-through. We also intend to explore the relative performance of various composite inflation-based Taylor rules in these environments.

Appendix

In this appendix we derive a second-order approximation to the utility of the representative household around an efficient steady state. As has been discussed in the main text, we restrict our study to the special case of $\sigma = \eta = 1$. Frequent use is made of the following fact:

$$\frac{X_t - X}{X} = x_t + \frac{1}{2}x_t^2,$$

where x_t is the log deviation from steady state for the variable X_t. The second-order Taylor approximation of the household j's period t utility, $U_t(j)$, around a steady state combined with the goods market clearing condition and integrating across households yields

$$\int_0^1 U_t(j)\mathrm{d}j \simeq \log C_t^n + (1-\alpha)\tilde{y}_t - (1-\alpha)\tilde{n}_t - \frac{(1-\alpha)(1+\varphi)}{2}\int_0^1 \tilde{n}_t(j)^2 \mathrm{d}j,$$

where we have used $1-\alpha = (N_t^n)^{1+\varphi}$ and the market clearing condition $\tilde{c}_t = (1-\alpha)\tilde{y}_t + \alpha y_t^*$.

Define aggregate employment as $N_t = \int_0^1 N_t(j)\mathrm{d}j$, or, in terms of log deviations from the steady state and up to a second-order approximation,

$$\tilde{n}_t + \frac{1}{2}\tilde{n}_t^2 \simeq \int_0^1 \tilde{n}_t(j)\mathrm{d}j + \frac{1}{2}\int_0^1 \tilde{n}_t(j)^2\mathrm{d}j.$$

Note also that

$$\begin{aligned}\int_0^1 \tilde{n}_t(j)^2\mathrm{d}j &= \int_0^1 (\tilde{n}_t(j) - \tilde{n}_t + \tilde{n}_t)^2\mathrm{d}j = \tilde{n}_t^2 - 2\varepsilon_w\int_0^1 (\tilde{w}_t(j) - \tilde{w}_t)\mathrm{d}j + \varepsilon_w^2\int_0^1 (\tilde{w}_t(j) - \tilde{w}_t)^2\mathrm{d}j \\ &= \tilde{n}_t^2 + \varepsilon_w^2 \mathrm{var}_j\{w_t(j)\},\end{aligned}$$

where we have used the labor demand function $\tilde{n}_t(j) - \tilde{n}_t = \varepsilon_w(\widetilde{w_t}(j) - \tilde{w}_t)$, and the fact that $\int_0^1 (\tilde{w}_t(j) - \tilde{w}_t)\mathrm{d}j = 0$ and that $\int_0^1 (\tilde{w}_t(j) - \tilde{w}_t)^2\mathrm{d}j = \mathrm{var}_j\{w_t(j)\}$ is of second order.

The next step is to derive a relationship between aggregate employment and output:

$$\begin{aligned}N_t &= \int_0^1\int_0^1 N_t(i,j)\mathrm{d}j\mathrm{d}i = \int_0^1 N_t(i)\int_0^1 \frac{N_t(i,j)}{N_t(i)}\mathrm{d}j\mathrm{d}i = \Delta_{w,t}\int_0^1 N_t(i)\mathrm{d}i \\ &= \Delta_{w,t}\left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}}\int_0^1\left(\frac{Y_t(i)}{Y_t}\right)^{\frac{1}{1-\alpha}}\mathrm{d}i = \Delta_{w,t}\Delta_{p_H,t}\int_0^1\left(\frac{Y_t(i)}{Y_t}\right)^{\frac{1}{1-\alpha}}\mathrm{d}i,\end{aligned}$$

where $\Delta_{w,t} = \int_0^1 (w_t(j)/w_t)^{-\varepsilon_w}$ and $\Delta_{p_H,t} = \int_0^1 (p_{H,t}(i)/P_{H,t})^{-\varepsilon_p}$.

Thus, the following second-order approximation of the relation between (log) aggregate output and (log) aggregate employment holds:

$$\tilde{n}_t = \tilde{y}_t + d_{w,t} + d_{p_H,t},$$

where $d_{w,t} = \log\int_0^1 (w_t(j)/w_t)^{-\varepsilon_w}$ and $d_{p_H,t} = \log\int_0^1 (p_{H,t}(i)/P_{H,t})^{-\varepsilon_p}$.

Lemma 1.

$$d_{p_H,t} = \frac{\varepsilon_p}{2}\mathrm{var}_i\left\{p_{H,t}(i)\right\}.$$

Proof. See Galí and Monacelli (2005).

Lemma 2.

$$d_{w,t} = \frac{\varepsilon_w}{2}\mathrm{var}_j\{w_t(j)\}.$$

Proof. See Erceg et al. (2000).

Now, one-period aggregate welfare can be written as

$$\int_0^1 U_t(j)\mathrm{d}j = -\frac{1-\alpha}{2}\left[(1+\varphi)\tilde{y}_t^2 + \varepsilon_p\mathrm{var}_i\left\{p_{H,t}(i)\right\} + (\varepsilon_w + (1+\varphi)\varepsilon_w^2)\mathrm{var}_j\{w_t(j)\}\right] + t.i.p.\,,$$

where *t.i.p.* stands for terms independent of policy.

Lemma 3.

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t\mathrm{var}_i\left\{p_{H,t}(i)\right\} &= \frac{\theta_{p_H}}{(1-\beta\theta_{p_H})(1-\theta_{p_H})}\sum_{t=0}^{\infty}\beta^t\pi_{H,t}^2, \\ \sum_{t=0}^{\infty}\beta^t\mathrm{var}_j\{w_t(j)\} &= \frac{\theta_w}{(1-\beta\theta_w)(1-\theta_w)}\sum_{t=0}^{\infty}\beta^t\pi_{w,t}^2.\end{aligned}$$

Proof. See Woodford (2003, Chapter 6).

Collecting the previous results, we can write the second-order approximation to the small open economy's aggregate welfare function as follows:

$$\mathbf{W} = \frac{1-\alpha}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ (1+\varphi)\tilde{y}_t^2 + \frac{\varepsilon_p}{\lambda_{p_H}} \left(\pi_{H,t}\right)^2 + \frac{\varepsilon_w}{\lambda_w} \left(\pi_{w,t}\right)^2 \right\} + t.i.p.\,,$$

where $\lambda_{p_H} = (1-\beta\theta_{p_H})(1-\theta_{p_H})/\theta_{p_H}$ and $\lambda_w = (1-\beta\theta_w)(1-\theta_w)/\theta_w[1+\varepsilon_w(1+\varphi)]$.

References

Adolfson, M., Laseén, S., Linde, J., Villani, M., 2007. Bayesian estimation of an open economy DSGE model with incomplete pass-through. Journal of International Economics 72, 481–511.
Adolfson, M., Laseén, S., Linde, J., Villani, M., 2008. Evaluating an estimated New Keynesian small open economy model. Journal of Economic Dynamics and Control 32, 2690–2721.
Ambler, S., Guay, A., Phaneuf, L., 2012. Endogenous business cycle propagation and the persistence problem: the role of labor-market frictions. Journal of Economic Dynamics and Control 36, 47–62.
Bentolila, S., Dolado, J., Jimeno, J., 2012. Reforming an insider-outsider labor market: the Spanish experience. IZA Journal of European Labor Studies 1, 4.
Blanchrad, O., Galí, J., 2007. Real wage rigidities and the new Keynesian model. Journal of Money, Credit and Banking 39, 35–65.
Calvo, G., 1983. Staggered prices in a utility maximizing framework. Journal of Monetary Economics 12, 383–398.
Chari, V.V., Kehoe, P., McGrattan, E., 2000. Sticky price models of the business cycle: can the contract multiplier solve the persistence problem? Econometrica 68, 1151–1179.
Christiano, L., Eichenbaum, M., Evans, C., 1999. Monetary policy shocks: what have we learned and to what end? In: Taylor, J., Woodford, M. (Eds.), Handbook of Macroeconomics. Elsevier, New York, pp. 65–148.
Christiano, L., Eichenbaum, M., Evans, C.L., 2005. Nominal rigidities and the dynamic effects of a shock to monetary policy. Journal of Political Economy 113, 1–45.
Clarida, R., Galí, J., Gertler, M., 2002. A simple framework for international monetary policy analysis. Journal of Monetary Economics 49, 879–904.
Engel, C., 1999. Accounting for U.S. real exchange rate changes. Journal of Political Economy 107, 507–538.
Erceg, C., Henderson, D., Levin, A., 2000. Optimal monetary policy with staggered wage and price contracts. Journal of Monetary Economics 46, 281–384.
Galí, J., 1992. How well does the IS-LM model fit the postwar U.S. data? Quarterly Journal of Economics 107, 709–738.
Galí, J., 2008. Monetary Policy, Inflation, and the Business Cycle: an Introduction to the New Keynesian Framework. Princeton University Press, Princeton.
Gali, J., 2011. The return of the wage Phillips curve. Journal of the European Economic Association 9, 436–461.
Galí, J., Monacelli, T., 2005. Monetary policy and exchange rate volatility in a small open economy. Review of Economic Studies 75, 707–734.
Goodfriend, M., 2007. How the world achieved consensus on monetary policy. Journal of Economic Perspectives 21, 47–68.
Guender, A., 2006. Stabilizing properties of discretionary monetary policies in a small open economy. Economic Journal 116, 309–326.
Huang, K., Liu, Z., 2002. Staggered price-setting, staggered wage-setting, and business cycle persistence. Journal of Monetary Economics 49, 405–433.
Huang, K., Liu, Z., Phaneuf, L., 2004. Why does the cyclical behavior of real wages change over time? American Economic Review 94, 836–856.
Messina, J., Strozzi, C., Turunen, J., 2009. Real wages over the business cycle: OECD evidence from the time and frequency domains. Journal of Economic Dynamics and Control 33, 1183–1200.
Neely, C., Rapach, D., 2008. Real interest rate persistence: evidence and implications. Federal Reserve Bank of St. Louis Review 90, 609–641.
Parsley, D.C., Wei, S.-J., 2001. Explaining the border effect: the role of exchange rate variability, shipping costs and geography. Journal of International Economics 55, 87–105.
Sbordone, A.M., 2002. Prices and unit labor costs: testing models of pricing behavior. Journal of Monetary Economics 45, 265–292.
Svensson, L., 2000. Open-economy inflation targeting. Journal of International Economics 50, 155–183.
Taylor, J., 1993. Discretion versus policy rules in practice. Carnegie-Rochester Conference Series in Public Policy 15, 151–200.
Woodford, M., 2003. Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton University Press, Princeton.