Abstract

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1 Introduction

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2 Description of the model

2.1 Consumers Problem

The model considers a standard representative consumer with separable preferences that maximizes his expected discounted payoff in a infinite time horizon:

$$\max_{C_t,N_t} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{\varphi+1}}{\varphi+1} \right] \quad \text{st.} \quad \forall t, \quad P_t C_t + \mathbb{E}_t[Q_{t,t+1} D_{t+1}] \leq D_t + W_t N_t + T_t \tag{1}$$

Where D_t is the nominal payoff in period t of the portfolio held at the end of period $t-1$. The consumer chooses between domestic goods ($C_{H,t}$) and foreign goods ($C_{F,t}$) with $\alpha \in [0,1]$ as the opening index of the economy and elasticity of substitution $\eta > 0$ between domestic and imported goods. The imported goods consumption is divided in goods imported from a continuum of different countries i with elasticity of substitution $\gamma > 0$. Finally, he chooses between goods j made in the same country (domestic or foreign) with elasticity of substitution $\varepsilon > 0$.

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{2}$$

$$C_{F,t} \equiv \left(\int_0^1 C_{i,t}^{\frac{\gamma-1}{\gamma}} \, di \right)^{\frac{\gamma}{\gamma-1}} \qquad C_{H,t} \equiv \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \qquad C_{i,t} \equiv \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

The detailed solution of the consumer problem is detailed in the appendix A and results are, as usual, the consumer Euler equation and the labor supply:

$$\beta R_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right] = 1 \tag{3}$$

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \equiv W_t^R \tag{4}$$

Where W_t^R is the real wage.

2.2 Terms of trade

We define the bilateral terms of trade $S_{i,t}$ between domestic economy and the country i as $S_{i,t} \equiv \frac{P_{i,t}}{P_{H,t}}$. The effective terms of trade given by:

$$S_t \equiv \left(\int_0^1 S_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}} = \left(\int_0^1 \left(\frac{P_{i,t}}{P_{H,t}}\right)^{1-\gamma} di\right)^{\frac{1}{1-\gamma}} = \frac{1}{P_{H,t}}\left(\int_0^1 \left(P_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}}\right) = \frac{P_{F,t}}{P_{H,t}} \tag{5}$$

Using the that price level are given by $P_t^{1-\eta} = (1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}$ we get:

$$P_t^{1-\eta} = (1-\alpha)P_{H,t}^{1-\eta} + \alpha\left(P_{H,t}^{1-\eta}S_t^{1-\eta}\right) = P_{H,t}^{1-\eta}\left[(1-\alpha) + \alpha S_t^{1-\eta}\right] \Rightarrow P_t = P_{H,t}\left[(1-\alpha) + \alpha S_t^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{6}$$

Dividing P_t by P_{t-1}:

$$\frac{P_t}{P_{t-1}} = \frac{P_{H,t}\left[(1-\alpha) + \alpha S_t^{1-\eta}\right]^{\frac{1}{1-\eta}}}{P_{H,t-1}\left[(1-\alpha) + \alpha S_{t-1}^{1-\eta}\right]^{\frac{1}{1-\eta}}} \Rightarrow \Pi_t = \Pi_{H,t}\left(\frac{(1-\alpha) + \alpha S_t^{1-\eta}}{(1-\alpha) + \alpha S_{t-1}^{1-\eta}}\right)^{\frac{1}{1-\eta}} \tag{7}$$

2.3 Exchange Rate

We assume that law of one price holds for all goods and all times, defining $\mathcal{E}_{i,t}$ as the bilateral exchange rate between domestic economy and country i and $P_{i,t}^i(j)$ as the price expressed in the producer's currency, then:

$$P_{i,t}(j) = \mathcal{E}_{i,t}P_{i,t}^i(j) \Rightarrow \left(\int_0^1 (P_{i,t}(j))^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}} = \left(\int_0^1 \left(\mathcal{E}_{i,t}P_{i,t}^i(j)\right)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}} \Rightarrow P_{i,t} = \mathcal{E}_{i,t}P_{i,t}^i \tag{8}$$

Aggregating for all countries i

$$\left(\int_0^1 (P_{i,t})^{1-\gamma} di\right)^{\frac{1}{1-\gamma}} = \left(\int_0^1 \left(\mathcal{E}_{i,t}P_{i,t}^i\right)^{1-\gamma} di\right)^{\frac{1}{1-\gamma}} \Rightarrow S_tP_{H,t} = \left(\int_0^1 \left(\mathcal{E}_{i,t}P_{i,t}^i\right)^{1-\gamma} di\right)^{\frac{1}{1-\gamma}} \tag{9}$$

Dividing $S_tP_{H,t}$ by $S_{t-1}P_{H,t-1}$ we get:

$$\frac{S_tP_{H,t}}{S_{t-1}P_{H,t-1}} = \frac{\left(\int_0^1 \left(\mathcal{E}_{i,t}P_{i,t}^i\right)^{1-\gamma} di\right)^{\frac{1}{1-\gamma}}}{\left(\int_0^1 \left(\mathcal{E}_{i,t-1}P_{i,t-1}^i\right)^{1-\gamma} di\right)^{\frac{1}{1-\gamma}}} \Rightarrow \frac{S_t}{S_{t-1}}\Pi_{H,t} = \left(\frac{\int_0^1 \left(\mathcal{E}_{i,t}P_{i,t}^i\right)^{1-\gamma} di}{\int_0^1 \left(\mathcal{E}_{i,t-1}P_{i,t-1}^i\right)^{1-\gamma} di}\right)^{\frac{1}{1-\gamma}} \tag{10}$$

Which defines implicitly $\mathcal{E}_t \equiv \left(\int_0^1 \mathcal{E}_{i,t}^{\frac{\gamma-1}{\gamma}} di\right)^{\frac{\gamma}{\gamma-1}}$ as the nominal effective exchange rate [1].

[1]Log-linearizing we get $\pi_{H,t+1} + s_{t+1} - s_t = \pi_{t+1}^* + \Delta e_{t+1}$, where $e_t \equiv \int_0^1 e_{i,t}$, $p_t^* \equiv \int_0^1 p_{i,t}^i$ and $\pi_t^* \equiv p_t^* - p_{t-1}^*$. The baseline model of the paper assumes $p_t^* = p^* = 0$, implying that $\pi_{H,t+1} + s_{t+1} - s_t = \Delta e_{t+1}$

The bilateral real exchange rate with country i is defined as $\mathcal{Q}_{i,t} \equiv \frac{\mathcal{E}_{i,t} P^i_{i,t}}{P_t}$ and using that $P_{i,t} = \mathcal{E}_{i,t} P^i_{i,t}$ then:

$$
\begin{aligned}
\mathcal{Q}_{i,t} &= \frac{\mathcal{E}_{i,t} P^i_{i,t}}{P_t} = \frac{P_{i,t}}{P_t} \Rightarrow \left(\int_0^1 \mathcal{Q}_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} = \left(\int_0^1 \left(\frac{P_{i,t}}{P_t} \right)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} = \frac{1}{P_t} \left(\int_0^1 P_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \\
\mathcal{Q}_t &= \frac{P_{F,t}}{P_t} = \frac{P_{H,t} S_t}{P_t} = \frac{P_{H,t} S_t}{P_{H,t} \left[(1-\alpha) + \alpha S_t^{1-\eta} \right]^{\frac{1}{1-\eta}}} = \left[(1-\alpha) S_t^{\eta - 1} + \alpha \right]^{\frac{1}{\eta - 1}}
\end{aligned}
\tag{11}
$$

2.4 Firms Problem

A representative firm has a technology with constant returns:

$$
Y_t(j) = A_t N_t(j) \Rightarrow \int_0^1 Y_t(j) dj = \int_0^1 A_t N_t(j) dj \Rightarrow \int_0^1 Y_t(j) dj = A_t N_t \tag{12}
$$

$$
A_{t+1} = {A_t}^{\rho_a} \exp(\varepsilon_t^a) \tag{13}
$$

Where A_t is such that $a_t \equiv \ln(A_t)$ follows an AR(1) process and in steady state we normalize $A = 1$.

The model assume sticky prices following Calvo (1983): at each period a random selected measure θ of firms have to keep their prices unchanged from the previous period, while the remaining $1 - \theta$ firms can reset them. If a firm could adjust price at time t, we will set its price at $\bar{P}_{H,t}$ which maximizes the present value of its future profit:

$$
\bar{P}_{H,t} = \max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left[Q_{t,t+k} [Y_{t+k}(j)(\bar{P}_{H,t} - MC_{t+k} P_{H,t+k})] \right] \tag{14}
$$

Where MC_t is the real marginal cost. Combining the first order of this problem with the price dynamics (detailed solution is in appendix B) and defining $\widehat{MC}_t = \frac{MC_t}{1/\mathcal{M}}$ as the marginal cost deviation from steady state we get:

$$
\Pi_{H,t} = \left[\theta + \frac{(1-\theta)}{P_{H,t-1}^{1-\varepsilon}} \left(\frac{\mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta)^k C_{t+k}^{-\sigma} \frac{1}{P_{t+k}} \tilde{C}_{H,t+k} P_{H,t+k} \widehat{MC}_{t+k} \right]}{\mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta)^k C_{t+k}^{-\sigma} \frac{1}{P_{t+k}} \tilde{C}_{H,t+k} \right]} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{15}
$$

Finally, due to constant returns to scale, the real marginal cost does not depend on quantities. Using the consumers labor supply (4), the technology (12) and (6) we can derive the marginal cost deviation from its steady state:

$$
\widehat{MC}_t = \mathcal{M} MC_t = \frac{W_t (1-\tau) \mathcal{M}}{P_{H,t} A_t} = \frac{W_t P_t (1-\tau) \mathcal{M}}{P_t P_{H,t} A_t} = \frac{W_t^R \left[(1-\alpha) + \alpha S_t^{1-\eta} \right]^{\frac{1}{1-\eta}} (1-\tau) \mathcal{M}}{A_t^{1+\varphi}} \tag{16}
$$

Where τ is an employment subsidy [2] and $\frac{W_t}{P_{H,t}}$ is the real wage in the firm's perspective[3].

2.5 Rest of the world

Given the small domestic economy, we assume the rest of the world's output Y_t^* can be taken as exogenous and such that $y_t^* = \ln(Y_t^*)$ follows and AR(1) process:

$$Y_{t+1}^* = Y_t^{*\rho_y} \exp(\varepsilon_t^*) \tag{17}$$

Consumer's Euler equations also holds for a representative consumer in any other country:

$$\beta \left(\frac{C_{t+1}^i}{C_t^i} \right) \left(\frac{P_t^i}{P_{t+1}^i} \right) \left(\frac{\mathcal{E}_t^i}{\mathcal{E}_{t+1}^i} \right) = Q_{t,t+1}^i$$

The model assumes complete markets, then there are perfect domestic and international risk sharing, implying that $Q_{t,t+1}^i = Q_{t,t+1}$:

$$\begin{gathered}\beta \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \left(\frac{P_t^i \mathcal{E}_t^i}{P_{t+1}^i \mathcal{E}_{t+1}^i} \right) = \beta \left(\frac{C_{t+1}}{C_t^i} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \Rightarrow \left(\frac{C_{t+1}}{C_t^i} \right) = \left(\frac{C_{t+1}^i}{C_t^i} \right) \left(\frac{\frac{\mathcal{E}_{i,t+1} P_{t+1}^i}{P_{t+1}}}{\frac{\mathcal{E}_{i,t} P_t^i}{P_t}} \right)^{\frac{1}{\sigma}} = \\ \left(\frac{C_{t+1}}{C_t} \right) = \left(\frac{C_{t+1}^i}{C_t^i} \right) \left(\frac{\mathcal{Q}_{i,t+1}}{\mathcal{Q}_{i,t}} \right)^{\frac{1}{\sigma}} \Rightarrow \left[\frac{C_{t+1}}{C_{t+1}^i \mathcal{Q}_{i,t+1}^{\frac{1}{\sigma}}} \right] = \left[\frac{C_t}{C_t^i \mathcal{Q}_{i,t}^{\frac{1}{\sigma}}} \right]\end{gathered} \tag{18}$$

Defining $t = 0$ and iterating we get that, for all period t

$$\left[\frac{C_t}{C_t^i \mathcal{Q}_{i,t}^{\frac{1}{\sigma}}} \right] = \left[\frac{C_0}{C_0^i \mathcal{Q}_{i,0}^{\frac{1}{\sigma}}} \right] \equiv v_{i,0} \Rightarrow C_t = v_{i,0} C_t^i \mathcal{Q}_{i,t}^{\frac{1}{\sigma}} \tag{19}$$

Where $v_{i,0}$ depends on initial conditions, which the model assumes be symmetric, then $v_{i,0} = 1$ for all i. Aggregating over all countries:

$$\left(\int_0^1 C_t^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \equiv C_t = \left(\int_0^1 \left(C_t^i \mathcal{Q}_{i,t}^{\frac{1}{\sigma}} \right)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \tag{20}$$

2.6 Market clearing

For each domestic good, the total production is equal to domestic + external demands. Substituting the expressions obtained in the consumer problem and aggregating for all domestic goods (detailed steps are in appendix D) we obtain:

$$Y_t = C_t \left[(1-\alpha) + \alpha S_t^{1-\eta} \right]^{\frac{\eta}{1-\eta}} \left[(1-\alpha) + \alpha \int_0^1 \left(\mathcal{S}_t^i \mathcal{S}_{i,t} \right)^{\gamma-\eta} \mathcal{Q}_{i,t}^{\eta - \frac{1}{\sigma}} di \right] \tag{21}$$

[2] See appendix C for discussion on optimal subsidy

[3] On consumer problem's the real wage $W_t^R \equiv \frac{W_t}{P_t}$ was defined considering the total price index, however when calculating the real marginal cost of the domestic firms the paper uses the real wage considering only the domestic price index ($\frac{W_t}{P_{H,t}}$).

As we are considering the country as a small open economy, for the rest of the world the domestic consumption and production are insignificant and can be ignored, so market clearing implies:

$$C_t^* = Y_t^* \tag{22}$$

3 Equilibrium conditions

The equilibrium conditions (without a monetary policy equation) are given by equations (3), (4), (7), (10), (11), (12), (13), (16), (15), (17), (20), (21) and (22) obtained above:

$$\beta R_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right] = 1 \tag{23a}$$

$$C_t^\sigma N_t^\varphi = W_t^R \tag{23b}$$

$$\Pi_t = \Pi_{H,t} \left(\frac{(1-\alpha) + \alpha S_t^{1-\eta}}{(1-\alpha) + \alpha S_{t-1}^{1-\eta}} \right)^{\frac{1}{1-\eta}} \tag{23c}$$

$$\frac{S_t}{S_{t-1}} \Pi_{H,t} = \left(\frac{\int_0^1 \left(\mathcal{E}_{i,t} P_{i,t}^i \right)^{1-\gamma} di}{\int_0^1 \left(\mathcal{E}_{i,t-1} P_{i,t-1}^i \right)^{1-\gamma} di} \right)^{\frac{1}{1-\gamma}} \tag{23d}$$

$$\mathcal{Q}_t = \left[(1-\alpha) S_t^{\eta-1} + \alpha \right]^{\frac{1}{\eta-1}} \tag{23e}$$

$$\int_0^1 Y_t(j) dj = A_t N_t \tag{23f}$$

$$A_{t+1} = A_t{}^{\rho_a} \exp(\varepsilon_{t+1}^a) \tag{23g}$$

$$\widehat{MC}_t = \frac{W_t^R \left[(1-\alpha) + \alpha S_t^{1-\eta} \right]^{\frac{1}{1-\eta}} (1-\tau) \mathcal{M}}{A_t^{1+\varphi}} \tag{23h}$$

$$\Pi_{H,t} = \left[\theta + \frac{(1-\theta)}{P_{H,t-1}^{1-\varepsilon}} \left(\frac{\mathbb{E}_t \left[\sum_{k=0}^\infty (\beta\theta)^k C_{t+k}^{-\sigma} \frac{1}{P_{t+k}} \tilde{C}_{H,t+k} P_{H,t+k} \widehat{MC}_{t+k} \right]}{\mathbb{E}_t \left[\sum_{k=0}^\infty (\beta\theta)^k C_{t+k}^{-\sigma} \frac{1}{P_{t+k}} \tilde{C}_{H,t+k} \right]} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{23i}$$

$$Y_{t+1}^* = Y_t^{*\rho_y} \exp(\varepsilon_{t+1}^*) \tag{23j}$$

$$C_t = \left(\int_0^1 \left(C_t^i \mathcal{Q}_{i,t}^{\frac{1}{\sigma}} \right)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \tag{23k}$$

$$Y_t = C_t \left[(1-\alpha) + \alpha S_t^{1-\eta} \right]^{\frac{\eta}{1-\eta}} \left[(1-\alpha) + \alpha \int_0^1 \left(\mathcal{S}_t^i \mathcal{S}_{i,t} \right)^{\gamma-\eta} \mathcal{Q}_{i,t}^{\eta-\frac{1}{\sigma}} di \right] \tag{23l}$$

$$C_t^* = Y_t^* \tag{23m}$$

The log-linear approximation of these equations around the steady state is [4]:

[4] Lower case letters represent the neperian logarithm of the original variables. We define $\rho \equiv \beta^{-1} - 1$, $\nu \equiv -ln(1-\tau)$, $\mu \equiv \ln(\mathcal{M})$, $\omega \equiv \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$, $\sigma_\alpha \equiv \frac{\sigma}{(1-\alpha)+\alpha\omega}$, $\Omega \equiv \frac{\nu-\mu}{\sigma_\alpha+\varphi}$, $\Gamma \equiv \frac{1+\varphi}{\sigma_\alpha+\varphi}$, $\Psi \equiv \frac{(1-\omega)\alpha}{\sigma_\alpha+\varphi}$

$$c_t = \mathbb{E}_t[c_{t+1}] - \frac{1}{\sigma}(r_t - \mathbb{E}_t[\pi_{t+1}] - \rho) \tag{24a}$$

$$w_t^R = \sigma c_t + \varphi n_t \tag{24b}$$

$$\pi_t = \pi_{H,t} + \alpha s_t - \alpha s_{t-1} \tag{24c}$$

$$s_t - s_{t-1} + \pi_{H,t} = \Delta e_t \tag{24d}$$

$$q_t = (1-\alpha) s_t \tag{24e}$$

$$y_t = a_t + n_t \tag{24f}$$

$$a_{t+1} = \rho_a a_t + \varepsilon_{t+1}^a \tag{24g}$$

$$\hat{mc}_t = -\nu + w_t^R + \alpha s_t - (1+\varphi) a_t + \mu \tag{24h}$$

$$\pi_{H,t} = \beta \mathbb{E}_t[\pi_{H,t+1}] + \lambda \hat{mc}_t \tag{24i}$$

$$y_{t+1}^* = \rho_y y_t^* + \varepsilon_{t+1}^* \tag{24j}$$

$$c_t = c_t^* + \frac{1}{\sigma} q_t \tag{24k}$$

$$y_t = c_t + \alpha\gamma s_t + \alpha\left(\eta - \frac{1}{\sigma}\right) q_t \tag{24l}$$

$$c_t^* = y_t^* \tag{24m}$$

To these equations we also adds a definition of potential output $\bar{y}_t$ obtained from (24h) when $\hat{mc}_t = 0$ (and manipulating to be a function only of exogenous variables) and a definition of (log) output gap given by output deviation from its potential:

$$\bar{y}_t = \Omega + \Gamma a_t - \alpha \Psi y_t^* \tag{24n}$$

$$x_t = y_t - \bar{y}_t \tag{24o}$$

To complete the model its necessary to include a last equation, relative to monetary policy. The paper suggests 3 different monetary rules and a optimal monetary rule benchmark[5] and estimates the model in each case:

- Optimal Monetary Policy: $r_t = \bar{rr}_t + \phi_\pi \pi_{H,t}$
- Domestic Inflation Taylor Rule: $r_t = \rho + \phi_\pi \pi_{H,t}$
- CPI Inflation Taylor Rule: $r_t = \rho + \phi_\pi \pi_t$
- Exchange Rate Peg[6]: $\Delta e_t = 0$.

Where $\bar{rr}_t = \rho + \sigma_\alpha \Gamma(1-\rho_a) a_t + \alpha\sigma_\alpha(\Theta + \Psi)(\mathbb{E}_t[y_{t+1}^*] - y_t^*)$ is the natural interest rate.

[5] See appendix E for details

[6] The paper defines Peg as $e_t = 0$ but as the nominal variables are not determined in equilibrium, depending on the initial conditions (only their variations like inflation rates and Δe_t are determined) we considered peg as $\Delta e_t = 0$, imposing an initial condition $e_0 = 0$ which implies in $e_t = 0$

4 Calibration

The paper assumes the following calibration for the structural parameters:

Parameter	Description	Value
Common to Real Business Cycles Model		
β	Intertemporal discount factor	0.99
σ	Inverse elasticity of intertemporal subsititution	1
φ	Inverse Frisch elasticity of labour supply	3
ρ_a	Productivity shock smoothing	0.66
σ_a	Standard deviation of the productivity shocks	0.0071
Common to New Keynesian Model		
ε	Substitutability between varieties (from the same country)	6
θ	Calvo price stickiness	0.75
ϕ_π	Taylor rule response to inflation	1.5
Specific to Galí and Monacelli Model		
α	Opening index of the economy	0.4
η	Substitutability between domestic and imported goods	1
γ	Substitutability between goods from different foreign countries	1
ρ_{y^*}	World GDP shock smoothing	0.86
σ_{y^*}	Standard deviation of the world GDP shocks	0.0078
ρ_{ay^*}	Correlation between prod. and world GDP shocks	0.3

In the paper's text and tables the parameter ρ_a is defined as 0.66, however the IRFs charts are compatible with value 0.90 (authors reused the charts from an working paper version which $\rho_a = 0.90$ without update the figure or inform the different parameter in the text). We use this same value in the charts to be comparable with original ones in the paper.

5 Steady State Properties

In the steady state: (i) Purchasing Power Parity holds symmetrically for all other countries (then $\mathcal{Q}_i = \mathcal{Q}$, $S_i = S$, $S^i = 1$ and $C^i = C^*$), (ii) all the stationary variables are constant and (iii) there is not uncertainty ($\varepsilon^* = \varepsilon^a = 0$). Using the equilibrium conditions showed in (23) immediately by (g), (j) and (m) we obtain that $A = Y^* = C^* = 1$. The equation (a) defines $R = \beta^{-1}$. From (d) and (c) we have $\Pi = \Pi_H = 1$. Using (i) we find $\widehat{MC} = 1$. The remaining system (composed by equations (b), (e), (f), (l) and (l)) is:

$$C^\sigma N^\varphi = W^R \tag{26a}$$

$$\mathcal{Q} = \left[(1-\alpha)S^{\eta-1} + \alpha\right]^{\frac{1}{\eta-1}} \tag{26b}$$

$$Y = N \tag{26c}$$

$$1 = W^R \left[(1-\alpha) + \alpha S^{1-\eta}\right]^{\frac{1}{1-\eta}} (1-\tau)\mathcal{M} \tag{26d}$$

$$C = \mathcal{Q}^{\frac{1}{\sigma}} \tag{26e}$$

$$Y = C\left[(1-\alpha) + \alpha S^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\alpha) + \alpha \mathcal{S}^{\gamma-\eta}\mathcal{Q}^{\eta-\frac{1}{\sigma}}\right] \tag{26f}$$

This system cannot be solved analytically in the general case, however, using the values showed in calibration for $\alpha, \sigma, \eta, \gamma, \varepsilon$ and φ and defining the optimal subsidy ($1 - \tau = \frac{1}{(1-\alpha)\mathcal{M}}$), as showed in appendix C, we obtain the following steady state values:

Variable	**Description**	**Value**
Y	Output	1.13622
C	Consumption	1.07964
W^R	Real wage	1.58367
C/Y	Consumption-to-GDP Ratio	0.95020
S	Terms of trade	1.13622
NX/Y	New exports in terms of domestic output	0.00000
$(R^4 - 1)$	Real annual interest rate	0.04102

Note that consumption-to-GDP ratio is lower than 1 while net exports is zero, showing an apparent contradiction. However it is justified as domestic output and domestic consumption uses different prices indexes (P_H and P, respectively), then net exports in terms of domestic output is given by $\frac{NX}{Y} = \frac{(Y - C\frac{P}{P_H})}{Y} = 1 - \frac{C}{Y}\frac{P}{P_H} = 1 - \frac{C}{Y}S^{0.4} = 0$.

6 Dynamic Properties

Simulating 1000 samples with 201 periods each we obtained the following dynamic properties:

	Optimal sd%	DI Taylor sd%	CPI Taylor sd%	Peg sd%
Output	0.93	0.67	0.70	0.84
Domestic inflation	0.00	0.27	0.26	0.35
CPI inflation	0.38	0.41	0.27	0.21
Nominal int. rate	0.32	0.40	0.40	0.21
Terms of trade	1.50	1.42	1.33	1.08
Nominal depr. rate	0.95	0.85	0.52	0.00

Note: Sd denotes standard deviation in %

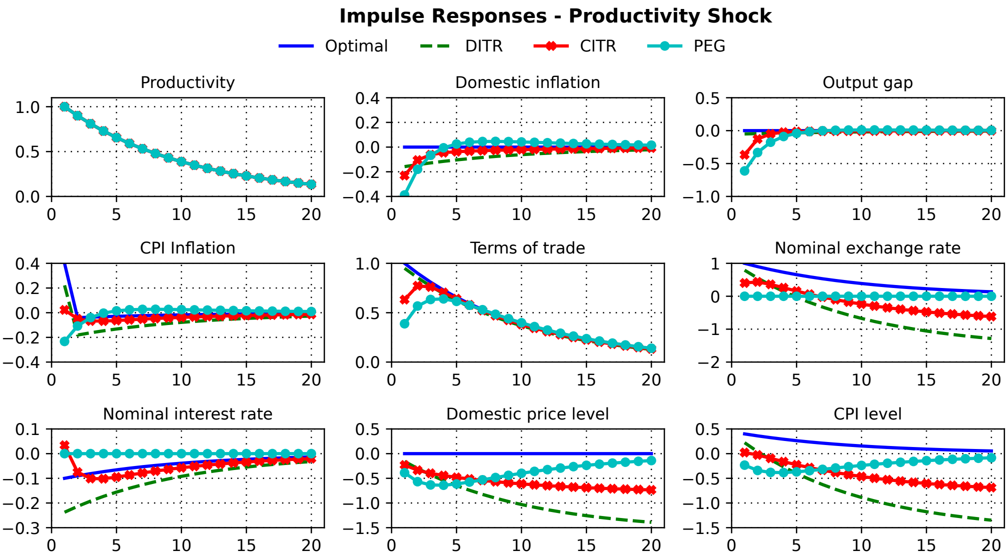
The contribution to welfare losses are:

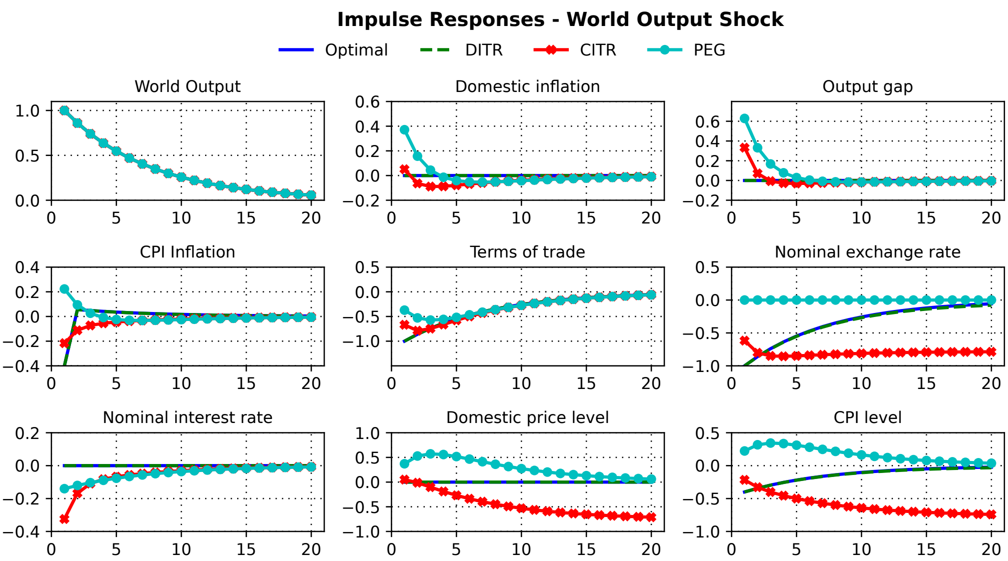
	DI Taylor	CPI Taylor	Peg
Benchmark $\mu = 1.2$, $\varphi = 3$			
Var(Domestic infl.)	0.0151	0.0142	0.0261
Var(Output gap)	0.0009	0.0019	0.0052
Total	0.0160	0.0161	0.0313
Low steady state markup $\mu = 1.1$, $\varphi = 10$			
Var(Domestic infl.)	0.0278	0.0262	0.0478
Var(Output gap)	0.0009	0.0019	0.0052
Total	0.0286	0.0281	0.0529
Low elasticity of labour supply $\mu = 1.2$, $\varphi = 3$			
Var(Domestic infl.)	0.0225	0.0230	0.0551
Var(Output gap)	0.0005	0.0005	0.0063
Total	0.0230	0.0250	0.0614
Low markup and elasticity of labour supply $\mu = 1.1$, $\varphi = 10$			
Var(Domestic infl.)	0.0414	0.0422	0.1013
Var(Output gap)	0.0005	0.0020	0.0063
Total	0.0419	0.0419	0.1076

Note: Values are percentage units of steady state consumption

7 Impulse Response Functions

We show the IRFs relative to both shocks present in the model (TFP and world output shocks) in output gap, domestic and total inflation, terms of trade, nominal exchange rate, nominal interest rate and dometic and total price levels (same charts showed in the original paper).





8 Modification

We include wage stickiness following Ergec, Hendenson and Levin (ref, 2000) and Gali (livro). Explicar melhor

8.1 Firms

Each represent firm j now uses a continuum of different labor types ($x \in [0, 1]$) as inputs:

$$Y_t(j) = A_t N_t(j) \quad N_t(j) \equiv \left(\int_0^1 N_t(j,x)^{\frac{\zeta-1}{\zeta}} dx \right)^{\frac{\zeta}{\zeta-1}} \tag{27}$$

Where ζ represents the elasticity of substitution among labor varieties and $W_t(x)$ is the nominal wage per unit of x-type labor. Analogously to the consumer problem that solve for the optimal demand for each type of good given the individual aggregate consumption, for the firm cost minimization problem given $N_t(j)$ we have that the optimal demand for n-type labor is:

$$N_t(j,x) = \left(\frac{W_t(x)}{W_t} \right)^{\zeta} N_t(j) \quad \text{where} \quad W_t \equiv \left(\int_0^1 W_t(x)^{1-\zeta} dx \right)^{\frac{1}{1-\zeta}} \tag{28}$$

The W_t and $N_t(j)$ above are such that in aggregate terms $\int_0^1 W_t(x) N_t(j,x)dx = W_t N_t(j)$. Follows that the firms price setting problem remains unchanged as marginal cost is constant due to constant returns to scale and W_t is calculated by aggregator above.

8.2 Households

Moreover we assume that all consumers homogeneously supply all the labor types implying that his income is given by $\int_0^1 W_t(x) N_t(x) dx$, leading to same problem discussed in original model:

$$\max_{C_t} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{\varphi+1}}{\varphi+1} \right] \quad \text{st.} \quad \forall t, \quad P_t C_t + \mathbb{E}_t[Q_{t,t+1} D_{t+1}] \leq D_t + \int_0^1 W_t(x) N_t(x) dx + T_t \tag{29}$$

The major difference is that now the consumers do not chose anymore the hours worked, thus labor income is now exogenous and the maximization problem solution consists only in the Euler equation showed in (3).

8.3 Wage setting

The model assumes that there exists a continuum of representative unions that can determine the nominal wages for each labor type. Analouslgy to price setting problem we assume that each union is not completely free to adjust their wage at any period, but only a random selected fraction of them with measure ς can reset the wages in a given period, while the remaining fraction must keep the nominal wage unchanged. The union wage setting problem consider the utility maximization of the workers, taking as exogenous the other union decisions and the labor demand by the firms. Formally:

$$\max_{W_t^*} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\varsigma)^k \left(\frac{C_{t+k}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+k}} W_t^* N_{t+k|t} - \frac{N_{t+k|t}^{1+\varphi}}{1+\varphi} \right) Z_{t+k} \quad \text{such that} \quad N_{t+k|k} = \left(\frac{W_t^*}{W_{t+k}} \right)^{-\zeta} \left(\int_0^1 N_t(j) dj \right) \tag{30}$$

The first order condition is given by:

$$\sum_{k=0}^{\infty} (\beta\varsigma)^k \mathbb{E}_t \left[N_{t+k|t} Z_{t+k} \left(\frac{C_{t+k}^{-\sigma} W_t^*}{P_{t+k}} - \Xi N_{t+k|t}^{\varphi} \right) \right] = 0 \tag{31}$$

Where $\Xi = \frac{\zeta}{\zeta - 1}$. Log-linearizing and rearranging the expression above and defining $\xi = \ln(\Xi)$ we get:

$$w_t^* = (1 - \beta\varsigma) \sum_{k=0}^{\infty} (\beta\rho)^k \mathbb{E}_t[\xi + \sigma c_{t+k} + \varphi n_{t+k|t} + p_{t+k}] \tag{32}$$

Log-linearizing the constraint in (30) we obtain that $n_{t+k|t} = -\zeta w_t^* + \zeta w_{t+k} + n_{t+k}$, thus:

$$w_t^* = (1 - \beta\varsigma) \sum_{k=0}^{\infty} (\beta\rho)^k \mathbb{E}_t[\xi + \sigma c_{t+k} + \varphi(\zeta w_{t+k} - \zeta w_t^* + n_{t+k}) + p_{t+k}] \tag{33}$$

$$w_t^* = \frac{1 - \beta\varsigma}{1 + \zeta\varphi} \sum_{k=0}^{\infty} (\beta\varsigma)^k \mathbb{E}_t[\xi + \sigma c_{t+k} + \varphi n_{t+k} + \zeta\varphi w_{t+k} + p_{t+k}] \tag{34}$$

Writing in a recursive form:

$$w_t^* = \beta\varsigma \mathbb{E}_t[w_{t+1}^*] + (1 - \beta\varsigma) \left[w_t - (1 + \zeta\varphi)^{-1} (w_t^R - \sigma c_t - \varphi n_t - \xi) \right] \tag{35}$$

8.4 Wage inflation dynamics

The wage inflation is defined as $\Pi_{w,t} = \frac{W_t}{W_{t-1}}$ or, log-linearizing, $\pi_{w,t} = w_t - w_{t-1}$. As we defined real wage $W_t^R = \frac{W_t}{P_t}$ we can derive a relation between goods and wage inflation:

$$\Pi_{w,t} = \frac{W_t}{W_{t-1}} = \frac{W_t^R P_t}{W_{t-1}^R P_{t-1}} = \frac{W_t^R \Pi_t}{W_{t-1}^R} \tag{36}$$

Log-linearizing:

$$\pi_{w,t} = w_t^R - w_{t-1}^R + \pi_t \tag{37}$$

The aggregate wage dynamics is given by:

$$W_t = \left(\varsigma W_{t-1}^{1-\zeta} + (1 - \varsigma)(W_t^*)^{1-\zeta} \right)^{\frac{1}{1-\zeta}} \tag{38}$$

Log-linearizing and calculating $\pi_{w,t}$:

$$\Rightarrow w_t = \varsigma w_{t-1} + (1 - \varsigma) w_t^* \Rightarrow \pi_{w,t} = (1 - \varsigma)(w_t^* - w_{t-1}) \tag{39}$$

Substituting the expression for w_t^* found in (35) and rearranging:

$$\pi_{w,t} = \beta \mathbb{E}_t[\pi_{w,t+1}] - \Lambda(w_t^R - \sigma c_t - \varphi n_t - \xi) \tag{40}$$

Where $\Lambda \equiv \frac{(1-\varsigma)(1-\beta\varsigma)}{\varsigma(1+\zeta\varphi)}$.

8.5 Equilibrium

The new log-linearized equilibrium is the same showed in (24) but removing the labor supply equation (b) and adding (37) and (40).

9 The new implications for the static and dynamics properties of the model

9.1 Steady state

Using the first order condition in wage setting problem in (31) evaluated in steady state it becomes $W^R = \Xi N^{\varphi} C^{\sigma}$. Thus, the exact same steady state found in (26) must hold, with only change the substitution of labor supply equation (26)a) to the expression above:

$$\Xi C^{\sigma} N^{\varphi} = W^R \tag{41a}$$

$$\mathcal{Q} = \left[(1-\alpha)S^{\eta-1} + \alpha\right]^{\frac{1}{\eta-1}} \tag{41b}$$

$$Y = N \tag{41c}$$

$$1 = W^R \left[(1-\alpha) + \alpha S^{1-\eta}\right]^{\frac{1}{1-\eta}} (1-\tau)\mathcal{M} \tag{41d}$$

$$C = \mathcal{Q}^{\frac{1}{\sigma}} \tag{41e}$$

$$Y = C\left[(1-\alpha) + \alpha S^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\alpha) + \alpha \mathcal{S}^{\gamma-\eta} \mathcal{Q}^{\eta-\frac{1}{\sigma}}\right] \tag{41f}$$

It is immediate that if $\zeta \to \infty$ then $\Xi \to 1$ and the system above results in the same of the original model, as it means that workers has not market power to set wages due to perfect substitutability among labor types, which is equivalent to a model without wage stickiness. Assuming the same calibration showed in section 4 and, for the new parameters, adopting $\varsigma = 0.75$ (consistent 1 year as average time to change wages) and $\zeta = 6$ (similarly to elasticity of substitution ε between goods) we obtain that steady state values are identical to original model except the real wage:

Variable	**Description**	**Original Model**	**Model with Price Stickiness**
Y	Output	1.13622	1.13622
C	Consumption	1.07964	1.07964
W^R	Real wage	1.58367	1.90040
C/Y	Consumption-to-GDP Ratio	0.95020	0.95020
S	Terms of trade	1.13622	1.13622
NX/Y	New exports in terms of domestic output	0.00000	0.00000
$(R^4 - 1)$	Real annual interest rate	0.04102	0.04102

9.2 Dynamic Properties

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9.3 Impulse Response Functions

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10 Conclusion

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A Consumer problem

Consumer problem solution.

B Labor subsidy definition

As the model depends on imperfect competition assumption the firms have market power implying that competitive equilibrium is not Pareto Optimal due to lower production and hiring. In this case, the domestic benevolent social planner maximizes the representative household discounted utility subject to technology (12), domestic/foreign consumption relation due to international risk sharing (20) and market clearing (21). For a analytically tractable solution we need to impose $\gamma = \sigma = \eta = 1$ (as adopted in the calibration). The problem becomes:

$$\max_{C_t,N_t} E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t) - \frac{N_t^{\varphi+1}}{\varphi+1}\right] \quad \text{s.t.} \quad \begin{aligned} Y_t &= A_t N_t \\ C_t &= C_t^* \mathcal{Q}_t \\ Y_t &= C_t S_t^{\alpha} \end{aligned}$$

By (11) with $\gamma = \sigma = \eta = 1$ we have that $\mathcal{Q}_t = S_t^{1-\alpha}$, and using the global market clearing condition in (22): $Y_t^* = C_t^*$ we rewrite the social planner problem:

$$\max_{C_t,N_t} E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t) - \frac{N_t^{\varphi+1}}{\varphi+1}\right] \quad \text{s.t.} \;\; C_t = (A_t N_t)^{1-\alpha}(Y_t^*)^{\alpha}$$

As this problem is a static one we can solve separately for C_t and N_t at any period. The Lagrangean and the first order conditions are:

$$\begin{aligned} \mathcal{L} &= \ln(C_t) - \frac{N_t^{\varphi+1}}{\varphi+1} - \lambda_t(C_t - (A_t N_t)^{1-\alpha}(Y_t^*)^{\alpha}) \\ (C_t) &\;\; \frac{1}{C_t} - \lambda_t = 0 \\ (N_t) &\;\; -N_t^{\varphi} + \lambda_t (Y_t^*)^{\alpha}(1-\alpha)A_t^{(1-\alpha)} N_t^{-\alpha} = 0 \end{aligned}$$

Manipulating we get that $N_t^{1+\varphi} = N^{1+\varphi} = (1-\alpha)$. In the competitive steady state in (26) again assuming $\gamma = \sigma = \eta = 1$ we obtain that $\frac{1}{\mathcal{M}} = (1-\tau)N^{1+\varphi}$. So, if the subsidy is such that $(1-\tau) = \frac{1}{(1-\alpha)\mathcal{M}}$ the steady state employment level coincides to Pareto optimal and, then, the efficiency is restored.

C Optimal price setting

The firms optimal price setting problem following the Calvo's model is given by:

$$\bar{P}_{H,t} = \max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left[Q_{t,t+k}[Y_{t+k}(j)(\bar{P}_{H,t} - MC_{t+k}P_{H,t+k})]\right] \tag{42}$$

The domestic demand for a specific variety is $C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}$ if the price remains unchanged at $\bar{P}_{H,t}$ until $t+k$ period then: $C_{t+k}(j) = \left(\frac{\bar{P}_{H,t}(j)}{P_{H,t+k}}\right)^{-\varepsilon} C_{H,t+k}$. Similarly the foreign consumption of this domestic good is $C^i_{t+k}(j) = \int_0^1 \left(\frac{\bar{P}_{H,t}(j)}{P_{H,t+k}}\right)^{-\varepsilon} C^i_{H,t+k} di$. Market clearing imposes that:

$$Y_{t+k}(j) = C_{H,t+k}(j) + \int_0^1 C^i_{H,t+k}(j)di = \left(\frac{\bar{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} \left(C_{H,t+k} + \int_0^1 C^i_{H,t+k}di\right) \equiv \left(\frac{\bar{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} \tilde{C}_{H,t+k} \tag{43}$$

Substituting $Q_{t,t+k}$ for the expression obtained in the consumer problem and $Y_{t+k}(j)$ for the expression above in the firms problem:

$$\bar{P}_{H,t} = \max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left[\beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+k}}\right) \left(\frac{\bar{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} \tilde{C}_{H,t+k}(\bar{P}_{H,t} - MC_{t+k}P_{H,t+k})\right] \tag{44}$$

Calculating the first order condition with respect to $\bar{P}_{H,t}$ and rearranging we get:

$$\bar{P}_{H,t} = \frac{\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\theta)^k C^{-\sigma}_{t+k}\frac{1}{P_{t+k}}\tilde{C}_{H,t+k}P_{H,t+k}MC_{t+k}\mathcal{M}\right]}{\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\theta)^k C^{-\sigma}_{t+k}\frac{1}{P_{t+k}}\tilde{C}_{H,t+k}\right]} \tag{45}$$

In the zero inflation steady state $\bar{P}_{H,t} = P_{H,t} = P_t = P_H$, implying that, by the previous formula, $MC_t = \frac{1}{\mathcal{M}} \equiv \frac{\varepsilon-1}{\varepsilon}$. Thus we define $\widehat{MC}_t = \frac{MC_t}{1/\mathcal{M}}$ as the marginal cost deviation from steady state. Now using the price dynamics:

$$P_{H,t} = [\theta P^{1-\varepsilon}_{H,t-1} + (1-\theta)\bar{P}^{1-\varepsilon}_{H,t}]^{\frac{1}{1-\varepsilon}} \Rightarrow \Pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}} = \frac{\left[\theta P^{1-\varepsilon}_{H,t} + (1-\theta)\bar{P}^{1-\varepsilon}_{H,t}\right]^{\frac{1}{1-\varepsilon}}}{P_{H,t-1}} \tag{46}$$

$$P_{H,t} = [\theta P^{1-\varepsilon}_{H,t-1} + (1-\theta)\bar{P}^{1-\varepsilon}_{H,t}]^{\frac{1}{1-\varepsilon}} \Rightarrow \Pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}} = \left[\theta + (1-\theta)\frac{\bar{P}^{1-\varepsilon}_{H,t}}{P^{1-\varepsilon}_{H,t-1}}\right]^{\frac{1}{1-\varepsilon}} \tag{47}$$

Substituing $\bar{P}_{H,t}$ we reach:

$$\Pi_{H,t} = \left[\theta + \frac{(1-\theta)}{P^{1-\varepsilon}_{H,t-1}}\left(\frac{\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\theta)^k C^{-\sigma}_{t+k}\frac{1}{P_{t+k}}\tilde{C}_{H,t+k}P_{H,t+k}\widehat{MC}_{t+k}\right]}{\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\theta)^k C^{-\sigma}_{t+k}\frac{1}{P_{t+k}}\tilde{C}_{H,t+k}\right]}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{48}$$

Log-linearizing we get:

$$\frac{\pi_{H,t} + p_{H,t-1}}{(1-\theta)(1-\beta\theta)} = \sum_{k=0}^{\infty}(\beta\theta)^k \mathbb{E}_t[\hat{mc}_{t+k} + p_{t+k}] \tag{49}$$

To obtain a recursive form, subtract for the same expression in $t+1$ multiplied by β, apply the Law of Iterated Expectations and rearrange:

$$\frac{\pi_{H,t} - \beta\mathbb{E}_t[\pi_{H,t+1}]}{(1-\theta)(1-\beta\theta)} = \sum_{k=0}^{\infty}(\beta\theta)^k\mathbb{E}_t[\hat{mc}_t + p_{t+k}] - \beta\mathbb{E}_t\sum_{k=0}^{\infty}(\beta\theta)^k\mathbb{E}[\hat{mc}_{t+1+k} + p_{t+k+1}] \tag{50}$$

$$\pi_{H,t} = \beta\mathbb{E}_t[\pi_{H,t+1}] + \lambda\hat{mc}_t \tag{51}$$

Where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$.

D Goods market clearing

Market clearing in goods market imposes that, for each domestic good, the total production is equal to domestic + external demands:

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} + \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \int_0^1 C^i_{H,t} di \tag{52}$$

Substituing $C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t$ and $C^i_{H,t} = \alpha \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t$

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left((1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t di\right) \tag{53}$$

As $Y_t = \left[\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$

$$\begin{aligned} Y_t &= \left\{\int_0^1 \left[\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left((1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t di\right)\right]^{\frac{\varepsilon-1}{\varepsilon}} dj\right\}^{\frac{\varepsilon}{\varepsilon-1}} \\ Y_t &= \left\{\int_0^1 \left(P_{H,t}(j)^{-\varepsilon}\right)^{\frac{\varepsilon-1}{\varepsilon}} dj\right\}^{\frac{\varepsilon}{\varepsilon-1}} \left(\frac{1}{P_{H,t}}\right)^{-\epsilon} \left((1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t di\right) \end{aligned} \tag{54}$$

Considering that $\left[\int_0^1 \left(P_{H,t}(j)^{-\varepsilon}\right)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}} = \left[\int_0^1 P_{H,t}^{1-\varepsilon}(j) dj\right]^{\frac{\varepsilon}{\varepsilon-1}} = P_{H,t}^{-\epsilon}$ we get:

$$\begin{aligned} Y_t &= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t di \\ Y_t &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha) C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{\mathcal{E}_{i,t} P^i_{F,t}}{P_{H,t}}\right)^{-\eta} \left(\frac{P_t}{\mathcal{E}_{i,t} P^i_t}\right)^{-\eta} C^i_t di\right] \end{aligned} \tag{55}$$

$$Y_t = C_t \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha) + \alpha \int_0^1 \left(\mathcal{S}^i_t \mathcal{S}_{i,t}\right)^{\gamma-\eta} \mathcal{Q}_{i,t}^{\eta-\frac{1}{\sigma}} di\right] \tag{56}$$

And finally, using (6) we obtain:

$$Y_t = C_t \left[(1-\alpha) + \alpha S_t^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\alpha) + \alpha \int_0^1 \left(\mathcal{S}^i_t \mathcal{S}_{i,t}\right)^{\gamma-\eta} \mathcal{Q}_{i,t}^{\eta-\frac{1}{\sigma}} di\right] \tag{57}$$

E Optimal Monetary Policy and Welfare Losses