Macroeconomics III - Project

Replication of New Keynesian Model for a Small Open Economy and Inclusion of Staggered Wages

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Abstract

This work first replicates the Galí and Monacelli (2005) new Keynesian model for a domestic small open economy that interacts with a continuum of identical foreign small economies that trade with each other and are not influenced by the policies and activity in the domestic economy. The authors compared three versions of this model based in different monetary rules: DITR (Domestic Inflation Taylor Rule), CITR (CPI Inflation Taylor Rule) and PEG (fixed exchange rate) with a benchmark optimal policy. Under this framework, similarly to the basic closed-economy New Keynesian model, the output gap and domestic inflation fall in response to a productivity shock, however the qualitative impact in the aggregate CPI depends on the monetary rule. The authors found that the DITR and CITR are the most efficient, while the PEG showed the largest losses. Our contribution was to add wage rigidity to the model. With rigidity in labor market the qualitative responses of the model are almost the same, but there is a quantitative impact change, mainly increasing the output gap volatility. We also found that, in this case, the ranking of policy alternatives is the same but the superiority of DITR and CITR over PEG narrows with more wage rigidity.

1 Introduction

Our work is based in Galí and Monacelli (2005) paper for a small open economy, that will be further referred as original paper. Its contribution figures in the intersection of two relevant fields in macroeconomics. First it extends the development of the basic version of the New Keynesian model that becomes the standard textbook model (as present in Galí (2015) and Woodford (2003)) and one of the main workhorses and benchmarks for monetary policy analysis, specially its impacts, objectives and implications [1]. On the other side this paper intersects with the open macroeconomics literature, using its contribution for modeling relevant parts of the economy as the exchange rate dynamics and interdependence between economies, as showed in Lane (2001) survey. Into this literature, Benigno and Benigno (2003) had already developed a new Keynesian model considering a two countries setup, however our basis paper considers a continuum of foreign countries, an approach that, at same time, drops out the strategic interaction between the domestic and foreign countries as the domestic one is small enough to prevent any impact on world activity and also allows the model to reach a log-linear representation very similar to the canonical new Keynesian model, with the foreign output only impacting the domestic economy's neutral rate.

Moreover, we extend the original model to include also wage rigidity similarly to Erceg et al. (2000) approach to a closed economy and Rhee and Turdaliev (2013) to an open economy. The last focus on analyze the optimal policy for this new framework (which can not achieve zero inflation and output gap) and create alternatives Taylor rules considering the wage inflation. Our approach differs from it, as we focus on the quantitative impact comparison between the model with wage rigidity and the original under the same monetary rules. We also study the impacts of productivity and world output shocks that are not present in the Rhee and Turdaliev's paper.

Finally, the main results are that under the original framework the domestic-inflation and CPI-inflation based Taylor rules are close to the optimal policy, while the PEG is the most distant. With wage rigidity there was a qualitative improvement, with most variables reaction now following a hump-shape, as also a quantitative increase in the output gap volatility. The welfare losses comparison indicates that the proportional superiority of the inflation based rules over PEG reduces with higher wage rigidity.

2 Description of the model

2.1 Consumers Problem

The model considers a standard representative consumer with separable preferences who maximizes his expected discounted payoff in an infinite time horizon:

$$\max_{C_t,N_t} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{\varphi+1}}{\varphi+1}\right] \quad \text{st.} \quad \forall t, \quad P_tC_t + \mathbb{E}_t[Q_{t,t+1}D_{t+1}] \leq D_t + W_tN_t + T_t \tag{1}$$

Where D_t is the nominal payoff in period t of the portfolio held at the end of period $t-1$. The consumer chooses between domestic goods ($C_{H,t}$) and foreign goods ($C_{F,t}$) with $\alpha \in [0,1]$ as the opening index of the economy and $\eta > 0$ as the elasticity of substitution between domestic and imported goods. The imported goods consists in a basket of goods imported from a continuum of different countries i with elasticity of substitution $\gamma > 0$. Finally, he can choose between goods j produced in the same country (domestic or foreign) with elasticity of substitution $\varepsilon > 0$.

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{2}$$

$$C_{F,t} \equiv \left(\int_0^1 C_{i,t}^{\frac{\gamma-1}{\gamma}} di\right)^{\frac{\gamma}{\gamma-1}} \quad (3a) \qquad C_{H,t} \equiv \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (3b) \qquad C_{i,t} \equiv \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (3c)$$

[1] See Clarida et al. (1999) for a survey

The solution of the consumer's problem is detailed in the appendix A and results are, as usual, the consumer Euler equation and the labor supply, with W_t^R as the real wage.

$$\beta R_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right] = 1 \tag{4}$$

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \equiv W_t^R \tag{5}$$

We get also the and the demand functions and the proportion of domestic and imported goods:

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t} \tag{6a}$$

$$C_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}} \right)^{-\varepsilon} C_{i,t} \tag{6b}$$

$$C_{i,t} = \left(\frac{P_{i,t}}{P_{F,t}} \right)^{-\gamma} C_{F,t} \tag{6c}$$

$$C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t \tag{7a}$$

$$C_{H,t} = (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \tag{7b}$$

2.2 Terms of trade

We define the bilateral terms of trade $S_{i,t}$ between domestic economy and the country i as $S_{i,t} \equiv \frac{P_{i,t}}{P_{H,t}}$. The effective terms of trade is given by:

$$S_t \equiv \left(\int_0^1 S_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} = \left(\int_0^1 \left(\frac{P_{i,t}}{P_{H,t}} \right)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} = \frac{1}{P_{H,t}} \left(\int_0^1 \left(P_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \right) = \frac{P_{F,t}}{P_{H,t}}$$

Using the fact that price level is given by $P_t^{1-\eta} = (1-\alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}$ we get:

$$P_t^{1-\eta} = (1-\alpha) P_{H,t}^{1-\eta} + \alpha \left(P_{H,t}^{1-\eta} S_t^{1-\eta} \right) = P_{H,t}^{1-\eta} \left[(1-\alpha) + \alpha S_t^{1-\eta} \right] \Rightarrow P_t = P_{H,t} \left[(1-\alpha) + \alpha S_t^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{8}$$

Dividing both sides by P_{t-1}:

$$\frac{P_t}{P_{t-1}} = \frac{P_{H,t} \left[(1-\alpha) + \alpha S_t^{1-\eta} \right]^{\frac{1}{1-\eta}}}{P_{H,t-1} \left[(1-\alpha) + \alpha S_{t-1}^{1-\eta} \right]^{\frac{1}{1-\eta}}} \Rightarrow \Pi_t = \Pi_{H,t} \left(\frac{(1-\alpha) + \alpha S_t^{1-\eta}}{(1-\alpha) + \alpha S_{t-1}^{1-\eta}} \right)^{\frac{1}{1-\eta}} \tag{9}$$

2.3 Exchange Rate

We assume that law of one price holds for all goods at all times, defining $\mathcal{E}_{i,t}$ as the bilateral exchange rate between domestic economy and country i and $P_{i,t}^i(j)$ as the price expressed in the producer's currency, then:

$$P_{i,t}(j) = \mathcal{E}_{i,t} P_{i,t}^i(j) \Rightarrow \left(\int_0^1 (P_{i,t}(j))^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}} = \left(\int_0^1 \left(\mathcal{E}_{i,t} P_{i,t}^i(j) \right)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}} \Rightarrow P_{i,t} = \mathcal{E}_{i,t} P_{i,t}^i$$

Aggregating for all countries i

$$\left(\int_0^1 (P_{i,t})^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} = \left(\int_0^1 \left(\mathcal{E}_{i,t} P_{i,t}^i \right)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \Rightarrow S_t P_{H,t} = \left(\int_0^1 \left(\mathcal{E}_{i,t} P_{i,t}^i \right)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}$$

Dividing $S_t P_{H,t}$ by $S_{t-1} P_{H,t-1}$ and using the same expression for the period $t-1$, we get:

$$\frac{S_t P_{H,t}}{S_{t-1} P_{H,t-1}} = \frac{\left(\int_0^1 \left(\mathcal{E}_{i,t} P_{i,t}^i \right)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}}{\left(\int_0^1 \left(\mathcal{E}_{i,t-1} P_{i,t-1}^i \right)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}} \Rightarrow \frac{S_t}{S_{t-1}} \Pi_{H,t} = \left(\frac{\int_0^1 \left(\mathcal{E}_{i,t} P_{i,t}^i \right)^{1-\gamma} di}{\int_0^1 \left(\mathcal{E}_{i,t-1} P_{i,t-1}^i \right)^{1-\gamma} di} \right)^{\frac{1}{1-\gamma}} \tag{10}$$

which defines implicitly $\mathcal{E}_t \equiv \left(\int_0^1 \mathcal{E}_{i,t}^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}$ as the nominal effective exchange rate [2].

The bilateral real exchange rate with country i is defined as $\mathcal{Q}_{i,t} \equiv \frac{\mathcal{E}_{i,t} P_{i,t}^i}{P_t}$ and using that $P_{i,t} = \mathcal{E}_{i,t} P_{i,t}^i$ then:

$$\begin{aligned} \mathcal{Q}_{i,t} &= \frac{\mathcal{E}_{i,t} P_{i,t}^i}{P_t} = \frac{P_{i,t}}{P_t} \Rightarrow \left(\int_0^1 \mathcal{Q}_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} = \left(\int_0^1 \left(\frac{P_{i,t}}{P_t} \right)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} = \frac{1}{P_t} \left(\int_0^1 P_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \\ \mathcal{Q}_t &= \frac{P_{F,t}}{P_t} = \frac{P_{H,t} S_t}{P_t} = \frac{P_{H,t} S_t}{P_{H,t} \left[(1-\alpha) + \alpha S_t^{1-\eta} \right]^{\frac{1}{1-\eta}}} = \left[(1-\alpha) S_t^{\eta-1} + \alpha \right]^{\frac{1}{\eta-1}} \end{aligned} \tag{11}$$

[2] Log-linearizing we get $\pi_{H,t+1} + s_{t+1} - s_t = \pi_{t+1}^* + \Delta e_{t+1}$, where $e_t \equiv \int_0^1 e_{i,t}$, $p_t^* \equiv \int_0^1 p_{i,t}^i$ and $\pi_t^* \equiv p_t^* - p_{t-1}^*$. The baseline model of the paper assumes $p_t^* = p^* = 0$, implying that $\pi_{H,t+1} + s_{t+1} - s_t = \Delta e_{t+1}$

2.4 Firms' Problem

A representative firm has a technology with constant returns to scale:

$$Y_t(j) = A_t N_t(j) \Rightarrow \int_0^1 Y_t(j)dj = \int_0^1 A_t N_t(j)dj \Rightarrow \int_0^1 Y_t(j)dj = A_t N_t \tag{12}$$

Where A_t is such that $a_t \equiv \ln(A_t)$ follows an AR(1) process and in steady state we normalize $A = 1$:

$$A_{t+1} = {A_t}^{\rho_a} \exp(\varepsilon_t^a) \tag{13}$$

The model assumes sticky prices following Calvo (1983): at each period a random selected fraction θ of firms have to keep their prices unchanged from the previous period, while the remaining $1-\theta$ firms can reset them. If a firm can adjust its prices at time t, it will fix it at $\overline{p}_{H,t}$ which maximizes the present value of its future profits:

$$\overline{p}_{H,t} = \max_{\overline{p}_{H,t}} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left[Q_{t,t+k}[Y_{t+k}(j)(\overline{p}_{H,t} - MC_{t+k}P_{H,t+k})] \right]$$

where MC_t is the real marginal cost. Combining the first order of this problem with the price dynamics (detailed solution is in appendix B) and defining $\widehat{MC}_t = \frac{MC_t}{1/\mathcal{M}}$ as the marginal cost deviation from steady state we get:

$$\Pi_{H,t} = \left[\theta + \frac{(1-\theta)}{P_{H,t-1}^{1-\varepsilon}} \left(\frac{\mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta)^k C_{t+k}^{-\sigma} \frac{1}{P_{t+k}} \widetilde{C}_{H,t+k} P_{H,t+k} \widehat{MC}_{t+k} \right]}{\mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta)^k C_{t+k}^{-\sigma} \frac{1}{P_{t+k}} \widetilde{C}_{H,t+k} \right]} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{14}$$

Finally, due to constant returns to scale, the real marginal cost does not depend on quantities. Using the consumers labor supply (5), the technology (12) and (8) we can derive the marginal cost deviation from its steady state:

$$\widehat{MC}_t = \mathcal{M} MC_t = \frac{W_t(1-\tau)\mathcal{M}}{P_{H,t}A_t} = \frac{W_t P_t (1-\tau)\mathcal{M}}{P_t P_{H,t} A_t} = \frac{W_t^R \left[(1-\alpha) + \alpha S_t^{1-\eta} \right]^{\frac{1}{1-\eta}} (1-\tau)\mathcal{M}}{A_t^{1+\varphi}} \tag{15}$$

where τ is an employment subsidy [3] and $\frac{W_t}{P_{H,t}}$ is the real wage in the firm's perspective[4].

2.5 Rest of the world

Given the small domestic economy, we assume that the rest of the world's output Y_t^* can be taken as exogenous and such that $y_t^* = \ln(Y_t^*)$ follows and AR(1) process:

$$Y_{t+1}^* = {Y_t^*}^{\rho_y} \exp(\varepsilon_t^*) \tag{16}$$

Consumer's Euler equations also holds for a representative consumer in any other country:

$$\beta \left(\frac{C_{t+1}^i}{C_t^i} \right) \left(\frac{P_t^i}{P_{t+1}^i} \right) \left(\frac{\mathcal{E}_t^i}{\mathcal{E}_{t+1}^i} \right) = Q_{t,t+1}^i$$

The model assumes complete markets, thus, there are perfect domestic and international risk sharing, implying that $Q_{t,t+1}^i = Q_{t,t+1}$:

$$\beta \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \left(\frac{P_t^i \mathcal{E}_t^i}{P_{t+1}^i \mathcal{E}_{t+1}^i} \right) = \beta \left(\frac{C_{t+1}}{C_t^i} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \Rightarrow \left(\frac{C_{t+1}}{C_t^i} \right) = \left(\frac{C_{t+1}^i}{C_t^i} \right) \left(\frac{\frac{\mathcal{E}_{i,t+1} P_{t+1}^i}{P_{t+1}}}{\frac{\mathcal{E}_{i,t} P_t^i}{P_t}} \right)^{\frac{1}{\sigma}}$$

$$\left(\frac{C_{t+1}}{C_t} \right) = \left(\frac{C_{t+1}^i}{C_t^i} \right) \left(\frac{\mathcal{Q}_{i,t+1}}{\mathcal{Q}_{i,t}} \right)^{\frac{1}{\sigma}} \Rightarrow \left[\frac{C_{t+1}}{C_{t+1}^i \mathcal{Q}_{i,t+1}^{\frac{1}{\sigma}}} \right] = \left[\frac{C_t}{C_t^i \mathcal{Q}_{i,t}^{\frac{1}{\sigma}}} \right]$$

Defining $t = 0$ and iterating we get that, for all period t

$$\left[\frac{C_t}{C_t^i \mathcal{Q}_{i,t}^{\frac{1}{\sigma}}} \right] = \left[\frac{C_0}{C_0^i \mathcal{Q}_{i,0}^{\frac{1}{\sigma}}} \right] \equiv v_{i,0} \Rightarrow C_t = v_{i,0} C_t^i \mathcal{Q}_{i,t}^{\frac{1}{\sigma}} \tag{17}$$

where $v_{i,0}$ depends on initial conditions, which the model assumes to be symmetric, then $v_{i,0} = 1$ for all i. Aggregating over all countries:

$$\left(\int_0^1 C_t^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \equiv C_t = \left(\int_0^1 \left(C_t^i \mathcal{Q}_{i,t}^{\frac{1}{\sigma}} \right)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \tag{18}$$

[3] See appendix B for discussion on optimal subsidy

[4] On consumer problem's the real wage $W_t^R \equiv W_t/P_t$ was defined considering the total price index, however when calculating the real marginal cost of the domestic firms the paper uses the real wage considering only the domestic price index $\left(\frac{W_t}{P_{H,t}} \right)$. This is consistent with no mobility of workers between countries.

2.6 Market clearing

For each domestic good, the total production is equal to domestic + external demands. Substituting the expressions obtained in the consumer problem and aggregating for all domestic goods (detailed steps are in appendix D) we obtain:

$$Y_t = C_t \left[(1-\alpha) + \alpha S_t^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\alpha) + \alpha \int_0^1 \left(\mathcal{S}_t^i \mathcal{S}_{i,t}\right)^{\gamma-\eta} \mathcal{Q}_{i,t}^{\eta-\frac{1}{\sigma}} di\right] \tag{19}$$

As we are considering the country as a small open economy: for the rest of the world, the domestic consumption and production are insignificant and can be ignored, so market clearing implies:

$$C_t^* = Y_t^* \tag{20}$$

3 Equilibrium conditions

The equilibrium conditions (without a monetary policy equation) are given by equations (4), (5), (9), (10), (11), (12), (13), (15), (14), (16), (18), (19) and (20) obtained above:

$$\beta R_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right)\right] = 1 \tag{21a}$$

$$C_t^\sigma N_t^\varphi = W_t^R \tag{21b}$$

$$\Pi_t = \Pi_{H,t} \left(\frac{(1-\alpha) + \alpha S_t^{1-\eta}}{(1-\alpha) + \alpha S_{t-1}^{1-\eta}}\right)^{\frac{1}{1-\eta}} \tag{21c}$$

$$\mathcal{Q}_t = \left[(1-\alpha) S_t^{\eta-1} + \alpha\right]^{\frac{1}{\eta-1}} \tag{21d}$$

$$\int_0^1 Y_t(j) dj = A_t N_t \tag{21e}$$

$$A_{t+1} = A_t{}^{\rho_a} \exp(\varepsilon_{t+1}^a) \tag{21f}$$

$$\frac{S_t}{S_{t-1}} \Pi_{H,t} = \left(\frac{\int_0^1 \left(\mathcal{E}_{i,t} P_{i,t}^i\right)^{1-\gamma} di}{\int_0^1 \left(\mathcal{E}_{i,t-1} P_{i,t-1}^i\right)^{1-\gamma} di}\right)^{\frac{1}{1-\gamma}} \tag{26g}$$

$$\widehat{MC}_t = \frac{W_t^R \left[(1-\alpha) + \alpha S_t^{1-\eta}\right]^{\frac{1}{1-\eta}} (1-\tau)\mathcal{M}}{A_t^{1+\varphi}} \tag{26h}$$

$$\Pi_{H,t} = \left[\theta + \frac{(1-\theta)}{P_{H,t-1}^{1-\varepsilon}} \left(\frac{\mathbb{E}_t \left[\sum_{k=0}^\infty (\beta\theta)^k C_{t+k}^{-\sigma} \frac{1}{P_{t+k}} \tilde{C}_{H,t+k} P_{H,t+k} \widehat{MC}_{t+k}\right]}{\mathbb{E}_t \left[\sum_{k=0}^\infty (\beta\theta)^k C_{t+k}^{-\sigma} \frac{1}{P_{t+k}} \tilde{C}_{H,t+k}\right]}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{26i}$$

$$Y_{t+1}^* = Y_t^{*\rho_y} \exp(\varepsilon_{t+1}^*) \tag{26j}$$

$$C_t = \left(\int_0^1 \left(C_t^i \mathcal{Q}_{i,t}^{\frac{1}{\sigma}}\right)^{1-\gamma} di\right)^{\frac{1}{1-\gamma}} \tag{26k}$$

$$Y_t = C_t \left[(1-\alpha) + \alpha S_t^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\alpha) + \alpha \int_0^1 \left(\mathcal{S}_t^i \mathcal{S}_{i,t}\right)^{\gamma-\eta} \mathcal{Q}_{i,t}^{\eta-\frac{1}{\sigma}} di\right] \tag{26l}$$

$$C_t^* = Y_t^* \tag{26m}$$

The equations resulting of the first-order linear approximation around the steady state [5] are:

$$c_t = \mathbb{E}_t[c_{t+1}] - \frac{1}{\sigma}(r_t - \mathbb{E}_t[\pi_{t+1}] - \rho) \tag{23a}$$

$$w_t^R = \sigma c_t + \varphi n_t \tag{23b}$$

$$\pi_t = \pi_{H,t} + \alpha s_t - \alpha s_{t-1} \tag{23c}$$

$$s_t - s_{t-1} + \pi_{H,t} = \Delta e_t \tag{23d}$$

$$q_t = (1-\alpha) s_t \tag{23e}$$

$$y_t = a_t + n_t \tag{23f}$$

$$a_{t+1} = \rho_a a_t + \varepsilon_{t+1}^a \tag{23g}$$

$$\widehat{mc}_t = -\nu + \mu + w_t^R + \alpha s_t - a_t \tag{23h}$$

$$c_t = c_t^* + \frac{1}{\sigma} q_t \tag{23i}$$

$$\pi_{H,t} = \beta \mathbb{E}_t[\pi_{H,t+1}] + \lambda \widehat{mc}_t \tag{23j}$$

$$y_{t+1}^* = \rho_y y_t^* + \varepsilon_{t+1}^* \tag{23k}$$

$$y_t = c_t + \alpha\gamma s_t + \alpha\left(\eta - \frac{1}{\sigma}\right) q_t \tag{23l}$$

$$c_t^* = y_t^* \tag{23m}$$

$$\overline{y}_t = \Omega + \Gamma a_t + \alpha \Psi y_t^* \tag{23n}$$

$$x_t = y_t - \overline{y}_t \tag{23o}$$

To the log-linearized system (equations 21) we also add (23n), a definition of potential output $\overline{y}_t$ obtained from (23h) when $\widehat{mc}_t = 0$ and manipulating to be a function only of exogenous variables; and (23o), a definition of (log) output gap given by output deviation from its potential.

The paper suggests 3 different monetary rules and an optimal monetary rule benchmark[6] and estimates the model in each case:

- Optimal Monetary Policy: $r_t = \overline{rr}_t + \phi_\pi \pi_{H,t}$
- Domestic Inflation Taylor Rule: $r_t = \rho + \phi_\pi \pi_{H,t}$
- CPI Inflation Taylor Rule: $r_t = \rho + \phi_\pi \pi_t$
- Exchange Rate Peg: $\Delta e_t = 0$.

Where $\overline{rr}_t = \rho + \sigma_\alpha \Gamma(1-\rho_a) a_t + \alpha\sigma_\alpha(\Theta + \Psi)(\mathbb{E}_t[y_{t+1}^*] - y_t^*)$ is the natural interest rate.

[5] Lower case letters represent the neperian logarithm of the original variables. We define $\rho \equiv \beta^{-1} - 1$, $\nu \equiv -ln(1-\tau)$, $\mu \equiv \ln(\mathcal{M})$, $\omega \equiv \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$, $\sigma_\alpha \equiv \frac{\sigma}{(1-\alpha)+\alpha\omega}$, $\Omega \equiv \frac{\nu-\mu}{\sigma_\alpha+\varphi}$, $\Gamma \equiv \frac{1+\varphi}{\sigma_\alpha+\varphi}$, $\Psi \equiv \frac{(1-\omega)\alpha}{\sigma_\alpha+\varphi}$, $\Theta \equiv \omega - 1$

[6] See appendix E for details

4 Calibration

The paper assumes the following calibration for the structural parameters:

Parameter	Description	Value
Common to Real Business Cycles Model		
β	Intertemporal discount factor	0.99
σ	Inverse elasticity of intertemporal subsititution	1
φ	Inverse Frisch elasticity of labour supply	3
ρ_a	Productivity shock smoothing	0.66
σ_a	Standard deviation of the productivity shocks	0.0071
Common to New Keynesian Model		
ε	Substitutability between varieties (from the same country)	6
θ	Calvo price stickiness	0.75
ϕ_π	Taylor rule response to inflation	1.5
Specific to Galí and Monacelli Model		
α	Opening index of the economy	0.4
η	Substitutability between domestic and imported goods	1
γ	Substitutability between goods from different foreign countries	1
ρ_{y^*}	World GDP shock smoothing	0.86
σ_{y^*}	Standard deviation of the world GDP shocks	0.0078
ρ_{ay^*}	Correlation between prod. and world GDP shocks	0.3

The authors did not justified exactly the source of all the values adopted in the calibration. The paper sets $\sigma = \eta = \gamma = 1$ to allow the use of the analytical result for the optimal labor subsidy that we showed in appendix B. The productivity and world GDP parameters were defined based in the authors estimation for Canada productivity and US GDP. There is no justification for the other parameters but they appear to close to the widely used values in the literature. In the paper's text and tables the parameter ρ_a is defined as 0.66, however the IRFs charts are compatible with value $\rho_a = 0.90$ (apparently authors reused the charts from an working paper version (Galí and Monacelli, 2002) without updating the figures and did not inform the different parameter in the text). We use $\rho_a = 0.90$ in the charts to be comparable with original ones in the paper. However, our other calculations, tables and text were done with the correct value $\rho_a = 0.66$.

5 Steady State Properties

In the steady state: (i) Purchasing Power Parity holds symmetrically for all other countries (then $\mathcal{Q}_i = \mathcal{Q}$, $S_i = S$, $S^i = 1$ and $C^i = C^*$), (ii) all the stationary variables are constant and (iii) there is not uncertainty ($\varepsilon^* = \varepsilon^a = 0$). Using the equilibrium conditions showed in (21) immediately by (g), (j) and (m) we obtain that $A = Y^* = C^* = 1$. The equation (a) defines $R = \beta^{-1}$. From (d) and (c) we have $\Pi = \Pi_H = 1$. Using (i) we find $\widehat{MC} = 1$. The remaining system (composed by equations (b), (e), (f), (l) and (l)) is:

$$C^\sigma N^\varphi = W^R \tag{24a}$$

$$\mathcal{Q} = \left[(1-\alpha)S^{\eta-1} + \alpha\right]^{\frac{1}{\eta-1}} \tag{24b}$$

$$Y = N \tag{24c}$$

$$1 = W^R \left[(1-\alpha) + \alpha S^{1-\eta}\right]^{\frac{1}{1-\eta}} (1-\tau)\mathcal{M} \tag{24d}$$

$$C = \mathcal{Q}^{\frac{1}{\sigma}} \tag{24e}$$

$$Y = C\left[(1-\alpha) + \alpha S^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\alpha) + \alpha \mathcal{S}^{\gamma-\eta}\mathcal{Q}^{\eta-\frac{1}{\sigma}}\right] \tag{24f}$$

This system cannot be solved analytically in the general case, however, using the values showed in calibration for $\alpha, \sigma, \eta, \gamma, \varepsilon$ and φ and defining the optimal subsidy ($1-\tau = \frac{1}{(1-\alpha)\mathcal{M}}$), as showed in appendix C, we obtain the following steady state values:

Variable	Description	Value
Y	Output	1.13622
C	Consumption	1.07964
W^R	Real wage	1.58367
C/Y	Consumption-to-GDP Ratio	0.95020
S	Terms of trade	1.13622
NX/Y	New exports in terms of domestic output	0.00000
(R^4-1)	Real annual interest rate	0.04102

In this model, the adopted optimal subsidy in the open economy is not the same of the closed one because there is an incentive to benefit the domestic economy with the better terms of trade as argued in appendix B. If we impose that the small economies had the same subsidy such that $1-\tau = \frac{1}{\mathcal{M}}$ the steady-state would be symmetric with values $Y = C = W^R = C/Y = S = 1$

Note that in the table the consumption-to-GDP ratio is lower than 1 while net exports is zero, showing an apparent contradiction. However it is justified as domestic output and domestic consumption uses different prices indexes (P_H and P, respectively), then net exports in terms of domestic output is given by $\frac{NX}{Y} = \frac{(Y - C\frac{P}{P_H})}{Y} = 1 - \frac{C}{Y}\frac{P}{P_H} = 1 - \frac{C}{Y}S^{0.4} = 0$.

6 Dynamic Properties

Simulating 1000 samples with 201 periods each we obtained the following dynamic properties:

	Optimal sd%	DITR sd%	CITR sd%	Peg sd%
Output	0.93	0.66	0.70	0.84
Domestic inflation	0.00	0.27	0.26	0.35
CPI inflation	0.38	0.40	0.26	0.21
Nominal int. rate	0.32	0.40	0.40	0.21
Terms of trade	1.52	1.43	1.33	1.08
Nominal depr. rate	0.95	0.85	0.52	0.00

Note: Sd denotes standard deviation in %

It's interesting to note that even with different measures of μ (markup) and φ, comparing results from CPI inflation and domestic inflation the conclusion is that the results are quite similar in both cases. On the other hand, when comparing the inflation target with peg, when the peg is chosen the output gap is three times as volatile and the domestic inflation is almost twice as volatile, what makes the households worse off. In the next section the difference between the policies will be more evident.

The contribution to welfare losses are:

	DI Taylor	CPI Taylor	Peg
Benchmark $\mu = 1.2$, $\varphi = 3$			
Var(Domestic infl.)	0.0150	0.0143	0.0259
Var(Output gap)	0.0009	0.0019	0.0052
Total	0.0159	0.0162	0.0311
Low steady state markup $\mu = 1.1$, $\varphi = 3$			
Var(Domestic infl.)	0.0277	0.0262	0.0472
Var(Output gap)	0.0009	0.0019	0.0052
Total	0.0287	0.0281	0.0524

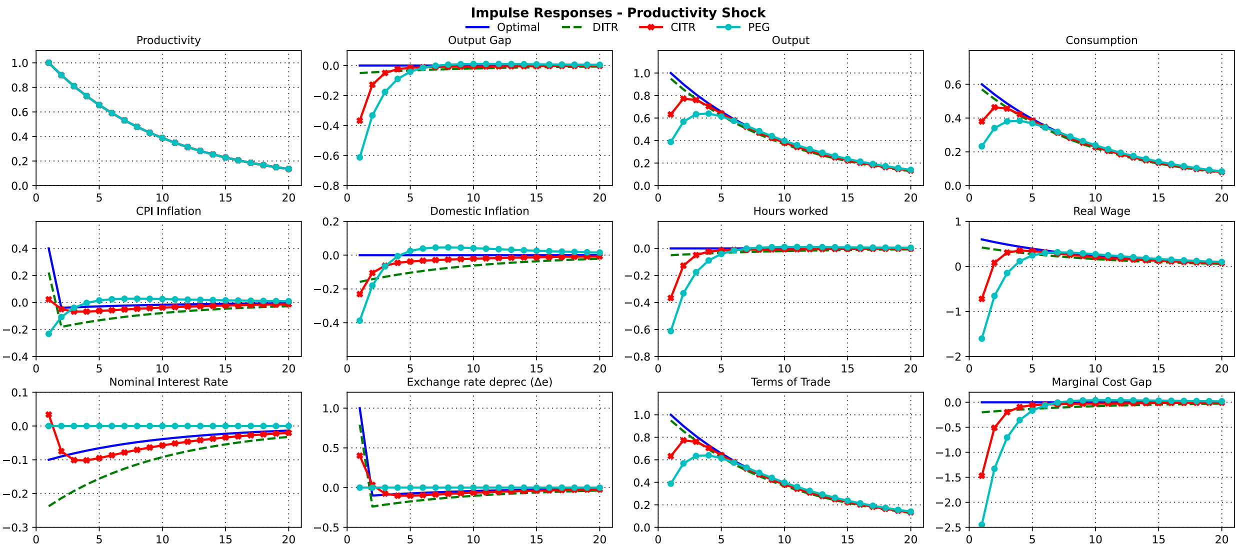
	DI Taylor	CPI Taylor	Peg
Low elasticity of labour supply $\mu = 1.2$, $\varphi = 10$			
Var(Domestic infl.)	0.0226	0.0230	0.0556
Var(Output gap)	0.0005	0.0020	0.0064
Total	0.0231	0.0250	0.0620
Low markup and elast. of labour supply $\mu = 1.1$, $\varphi = 10$			
Var(Domestic infl.)	0.0415	0.0421	0.1011
Var(Output gap)	0.0005	0.0020	0.0063
Total	0.0420	0.0440	0.1074

Note: Values are % units of steady state consumption

7 Impulse Response Functions

7.1 TFP shock

We show the IRFs relative to a productivity shock, as in the original paper. The shock hits only the domestic economy and is the same for all policy rules adopted.



First, let's analyze the responses under the optimal policy. A positive productivity shock impacts directly on potential output, as with higher productivity the economy can produce more with the same resources, which means that the marginal cost of firms lowers. It would generate an incentive to firms adjust prices, but the monetary authority reacts optimally cutting rates in a way that closes completely the output gap and offsets that incentive. However this rate cut depreciates the domestic currency, deteriorating the terms of trade. With higher productivity costs and same prices the profits increase, then the households feel richer and consume more. This increase in consumption is lower than the raise in output because the terms of trade imply an increase in exports. With domestic prices unchanged and a currency depreciation, the aggregate CPI, that includes foreign goods, increases.

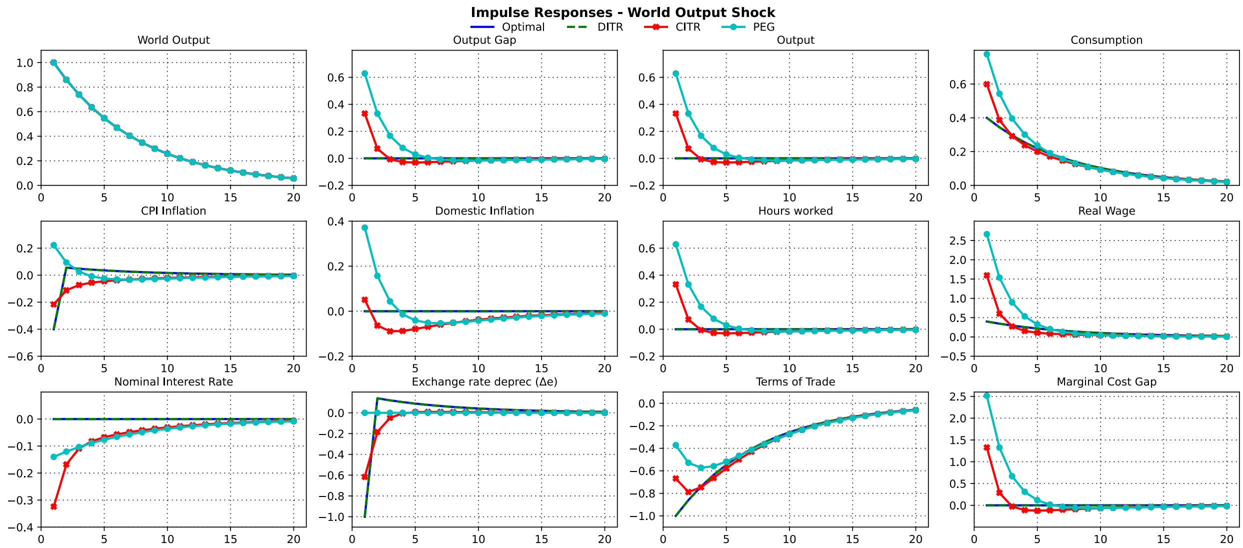
The dynamics under the Domestic Inflation Taylor Rule (DITR) is almost the same as the optimal policy. The difference is that, as monetary policy does not stabilize perfectly the domestic inflation it falls a bit (implying in small raise in aggregate CPI), but the response of the other variables are qualitative identical to those explained above.

Under the CPI Inflation Taylor Rule (CITR) there is a qualitative change in nominal rate impact. Like in the previous cases the increase in productivity raises the potential output more than real output (due to imperfect price adjustment) generating a negative output gap and a incentive to firms lower their domestic prices as also an increase in profits due to higher productivity. The domestic consumption increases due to higher income while the world consumption remain unchanged, which implies in a depreciation in real exchange rate due to PPP (see equation (23i). This leads to a deterioration in terms of trade and depreciation in nominal rate, raising the aggregate CPI, leading to a tightening response of monetary authority[7].

Finally, under the PEG there is no exchange rate depreciation, by hypothesis, and then no nominal interest rate change, as there were no changes on the external interest rates. In this case the aggregate CPI is directly affected by the domestic inflation drop in response to productivity shock. By the other side, there is no change in CPI's external component as foreign prices and nominal exchanged remain unchanged. Thus, the resulting impact in this case is a reduction in aggregate CPI.

7.2 World output shock

This shock can be seen as a productivity shock in the rest of the world and the corresponding monetary policy optimal response. Note that under the adopted calibration the composite parameters Ψ and Θ are 0, implying that y_t^* does not impact the neutral rate or the potential output (then, any impact on real output will affect proportionally the output gap). Moreover, when studying the responses to this shock we automatically set to zero the others ($a_t = 0$). As result the optimal rule becomes $r_t = \rho + \phi_\pi \pi_{H,t}$, identical to DITR, then (i) the DITR is the optimal rule in this case, fully stabilizing the domestic inflation and the output gap, and (ii) the IRFs are the same.



First, note that regardless of the monetary rule the higher foreign output impacts in foreign consumption, there's a greater demand of goods imported from the small economy, which would improve the terms of trade and appreciate the domestic currency. Due to better terms of trade[8] domestic consumption increases.

Under the Optimal Policy/DITR, the output gap remains at 0, implying that output and hours worked also remain at steady state level and that domestic inflation does not change. Finally, following the exchange rate appreciation, the aggregate CPI drops.

Under the CITR, the monetary policy does not stabilize output gap, implying in higher output and hours worked, and a small increase in domestic inflation in the first period. To work above potential, firms hire more workers increasing labor demand, causing higher wages that sustain the consumption boost. Anyway, this effect is small compared with the strong appreciation in exchange rate, resulting in a drop in the CPI. In response to lower inflation, monetary authority cuts the nominal interest rate.

Under the PEG, the central bank decreases the interest rate to prevent the exchange rate appreciation, but this cut incentives even more the consumption and then the production, generating stronger responses in the real variables. With positive output gap, the firms raise their prices, increasing domestic inflation. As external component of CPI does not change due to fixed exchange rate, the aggregate CPI also increases in this case.

8 Modification

We include wage stickiness following Erceg et al. (2000) and Galí (2015, Chap. 6) in the small open economy model. This framework adds imperfect competition in the labor market and considers unions that can define labor supply and set nominal wages aiming to maximize workers' utility and that decision is subject to nominal rigidities following Calvo, with the same mechanism of the price setting by the firms. This modification changes the consumer problem (as they do not decides individually their work hours) and the firms' (by way of technology), while the external sector relations are unaffected. In monetary policy we proceed our analysis with only DITR, CITR and PEG, as the previously obtained optimal policy rule is no longer optimal in that new framework due to change in the welfare costs function.

[7]This same channel also works in the other cases but as there the central bank responds only to domestic CPI it does not cause a monetary tightening.

[8]By convention, the terms of trade are defined as imports/exports prices, thus a negative (positive) ToT is good (bad) for domestic consumption and is called improvement (deterioration).

8.1 Firms

Each representative firm j now uses a continuum of different labor types ($x \in [0,1]$) as inputs:

$$Y_t(j) = A_t N_t(j) \quad N_t(j) \equiv \left(\int_0^1 N_t(j,\ell)^{\frac{\zeta-1}{\zeta}} d\ell\right)^{\frac{\zeta}{\zeta-1}} \tag{25}$$

Where ζ represents the elasticity of substitution among labor varieties and $W_t(\ell)$ is the nominal wage per unit of ℓ-type labor. Analogously to the consumer problem that solve for the optimal demand for each type of good given the individual aggregate consumption, for the firm cost minimization problem given $N_t(j)$ we have that the optimal demand for n-type labor is:

$$N_t(j,\ell) = \left(\frac{W_t(\ell)}{W_t}\right)^{-\zeta} N_t(j) \quad \text{where } W_t \equiv \left(\int_0^1 W_t(\ell)^{1-\zeta} d\ell\right)^{\frac{1}{1-\zeta}} \tag{26}$$

The W_t and $N_t(j)$ above are such that in aggregate terms $\int_0^1 W_t(\ell)N_t(j,\ell)d\ell = W_t N_t(j)$. It follows that the firms' price setting problem remains unchanged as marginal cost is constant due to constant returns to scale and W_t is calculated by the aggregator above.

8.2 Households

Moreover we assume that all consumers homogeneously supply all the labor types implying that his income is given by $\int_0^1 W_t(\ell)N_t(\ell)d\ell$, leading to the same problem discussed in the original model:

$$\max_{C_t} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{\varphi+1}}{\varphi+1}\right] \quad \text{st.} \quad \forall t, \quad P_t C_t + \mathbb{E}_t[Q_{t,t+1}D_{t+1}] \le D_t + \int_0^1 W_t(\ell)N_t(\ell)d\ell + T_t \tag{27}$$

The major difference is that now the consumers do not chose anymore the hours worked, thus labor income is now exogenous and the maximization problem solution consists only in the Euler equation showed in (4). The endogenous labor doesn't affect the consumption decision because the preferences are separable.

8.3 Wage setting

The model assumes that there exists a continuum of representative unions that can determine the nominal wages for each labor type. Analogously to the price setting problem we assume that each union is not completely free to adjust the wage at any period, but only a random selected fraction of them with measure ς can reset the wages in a given period, while the remaining fraction must keep the nominal wage unchanged. The union wage setting problem considers the utility maximization of the workers, taking as exogenous the other union decisions and the labor demand by the firms. Formally:

$$\max_{W_t^*} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\varsigma)^k \left(\frac{C_{t+k}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+k}} W_t^* N_{t+k|t} - \frac{N_{t+k|t}^{1+\varphi}}{1+\varphi}\right) Z_{t+k} \quad \text{such that} \quad N_{t+k|k} = \left(\frac{W_t^*}{W_{t+k}}\right)^{-\zeta} \left(\int_0^1 N_t(j)dj\right) \tag{28}$$

The first order condition is given by:

$$\sum_{k=0}^{\infty} (\beta\varsigma)^k \mathbb{E}_t \left[N_{t+k|t} Z_{t+k} \left(\frac{C_{t+k}^{-\sigma} W_t^*}{P_{t+k}} - \Xi N_{t+k|t}^{\varphi}\right)\right] = 0 \tag{29}$$

where $\Xi = \frac{\zeta}{\zeta-1}$. Log-linearizing and rearranging the expression above and defining $\xi = \ln(\Xi)$ we get:

$$w_t^* = (1-\beta\varsigma) \sum_{k=0}^{\infty} (\beta\rho)^k \mathbb{E}_t[\xi + \sigma c_{t+k} + \varphi n_{t+k|t} + p_{t+k}] \tag{30}$$

Log-linearizing the constraint in (28) we obtain that $n_{t+k|t} = -\zeta w_t^* + \zeta w_{t+k} + n_{t+k}$, thus:

$$w_t^* = (1-\beta\varsigma) \sum_{k=0}^{\infty} (\beta\rho)^k \mathbb{E}_t[\xi + \sigma c_{t+k} + \varphi(\zeta w_{t+k} - \zeta w_t^* + n_{t+k}) + p_{t+k}] = \frac{1-\beta\varsigma}{1+\zeta\varphi} \sum_{k=0}^{\infty} (\beta\varsigma)^k \mathbb{E}_t[\xi + \sigma c_{t+k} + \varphi n_{t+k} + \zeta\varphi w_{t+k} + p_{t+k}]$$

Writing in a recursive form:

$$w_t^* = \beta\varsigma \mathbb{E}_t[w_{t+1}^*] + (1-\beta\varsigma)\left[w_t - (1+\zeta\varphi)^{-1}(w_t^R - \sigma c_t - \varphi n_t - \xi)\right] \tag{31}$$

8.4 Wage inflation dynamics

The wage inflation is defined as $\Pi_{w,t} = \frac{W_t}{W_{t-1}}$ or, log-linearizing, $\pi_{w,t} = w_t - w_{t-1}$. As we defined real wage $W_t^R = \frac{W_t}{P_t}$ we can derive a relation between goods and wage inflation and its respective log-linearization:

$$\Pi_{w,t} = \frac{W_t}{W_{t-1}} = \frac{W_t^R P_t}{W_{t-1}^R P_{t-1}} = \frac{W_t^R}{W_{t-1}^R}\Pi_t \tag{32}$$

$$\pi_{w,t} = w_t^R - w_{t-1}^R + \pi_t \tag{33}$$

The aggregate wage dynamics is given by:

$$W_t = \left(\varsigma W_{t-1}^{1-\zeta} + (1-\varsigma)(W_t^*)^{1-\zeta}\right)^{\frac{1}{1-\zeta}} \tag{34}$$

$$w_t = \varsigma w_{t-1} + (1-\varsigma)w_t^* \Rightarrow \pi_{w,t} = (1-\varsigma)(w_t^* - w_{t-1}) \tag{35}$$

Substituting the expression for w_t^* found in (31), defining $\Lambda \equiv \frac{(1-\varsigma)(1-\beta\varsigma)}{\varsigma(1+\zeta\varphi)}$ and rearranging:

$$\pi_{w,t} = \beta\mathbb{E}_t[\pi_{w,t+1}] - \Lambda(w_t^R - \sigma c_t - \varphi n_t - \xi) \tag{36}$$

8.5 Equilibrium

The new log-linearized equilibrium is the same showed in (23) but removing the labor supply equation (23b) and adding (36) and (33). Note that this new model generalizes the original one in Galí and Monacelli (2005): assuming that workers have no market power to set wages due to perfect substitutability among labor types, that is $\zeta \to \infty$ (thus $\Xi \to 1$ and $\xi \to 0$), and there is not wage stickiness, $\varsigma \to 0$ (thus $\Lambda \to \infty$) we are back to the baseline model.

We assume the same calibration showed in section 4 and, for the new parameters, adopting $\varsigma = 0.75$ (consistent with 1 year as average time to change wages) and $\zeta = 4$ (following Erceg et al. (2000), that considered a wage markup of 1/3). Instead of the inconsistent use of $\phi_a = 0.66$ or $= 0.90$ in the original model related above, we will simply use 0.90 for all proposes. The tables below compares the models under this unified calibration thus some values for the original model differ from the section 6 due to recalculation.

9 New implications for the static and dynamics properties of the model

9.1 Steady state

Using the first order condition in the wage setting problem in (29) evaluated in the steady state the wage supply becomes $W^R = \Xi N^{\varphi} C^{\sigma}$. Thus, the exact same steady state found in (24) must hold. The only change is the substitution of the labor supply equation (24a) to the expression above. The numerical solution of the system with the adopted calibration is:

Variable	Description	Original Model	Model with Price Stickiness
Y	Output	1.13622	1.13622
C	Consumption	1.07964	1.07964
W^R	Real wage	1.58367	2.11156
C/Y	Consumption-to-GDP Ratio	0.95020	0.95020
S	Terms of trade	1.13622	1.13622
NX/Y	New exports in terms of domestic output	0.00000	0.00000
$(R^4 - 1)$	Real annual interest rate	0.04102	0.04102

9.2 Dynamic Properties

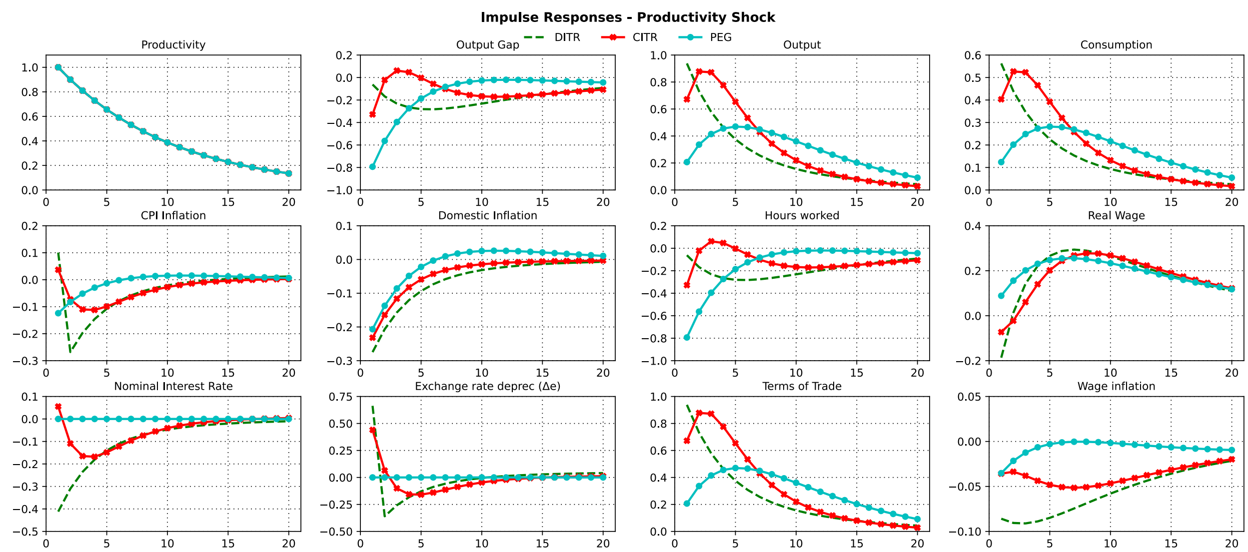
Again, simulating 1000 samples with 201 periods, we obtained the following dynamic properties.

	DI Taylor		CPI Taylor		Peg	
	Model sd%	Modif. sd%	Model sd%	Modif. sd%	Model sd%	Modif. sd%
Output	1.45	1.07	1.40	1.43	0.84	1.42
Output gap	0.08	0.59	0.33	0.75	0.64	1.44
Domestic inflation	0.24	0.30	0.26	0.28	0.39	0.25
CPI inflation	0.40	0.42	0.25	0.34	0.23	0.14
Nominal int. rate	0.36	0.44	0.38	0.51	0.21	0.28
Terms of trade	1.75	1.54	1.61	1.94	1.38	1.29
Nominal depr. rate	0.89	0.89	0.53	0.67	0.00	0.00

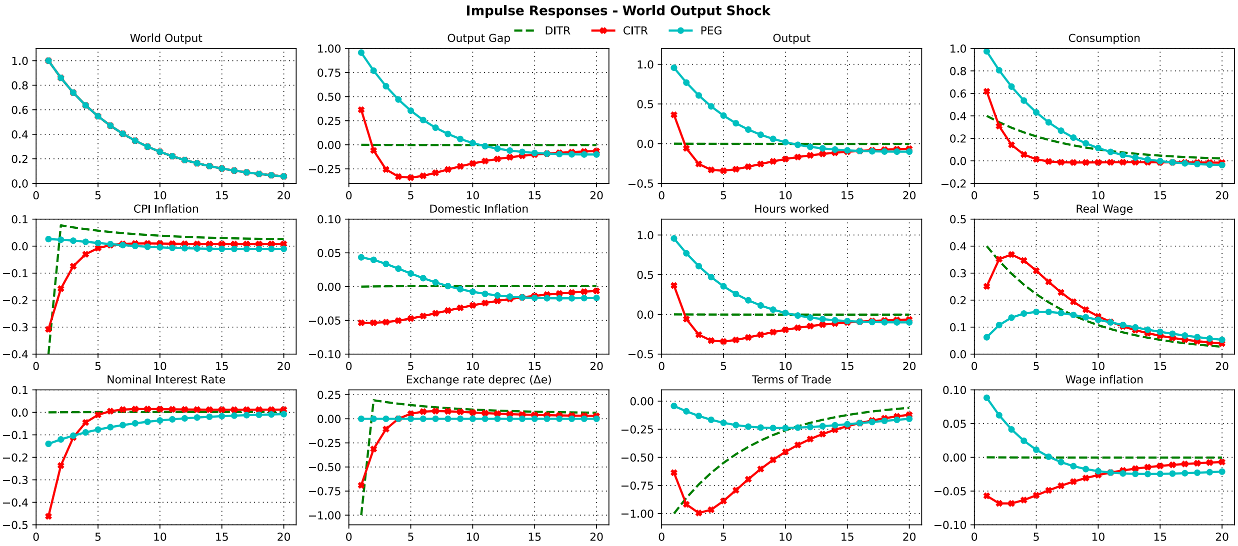
Note: Sd denotes standard deviation in %

What we can see here comparing these two models is that in this case the output is much more volatile, more than twice than the standard model. The domestic and CPI inflation turn to be more volatile with both Taylor rule specifications, but is less volatile when the peg is chosen. The terms of trade and the exchange rate have lower volatility for both Taylor rule policies, compared to the baseline.

9.3 Impulse Response Functions

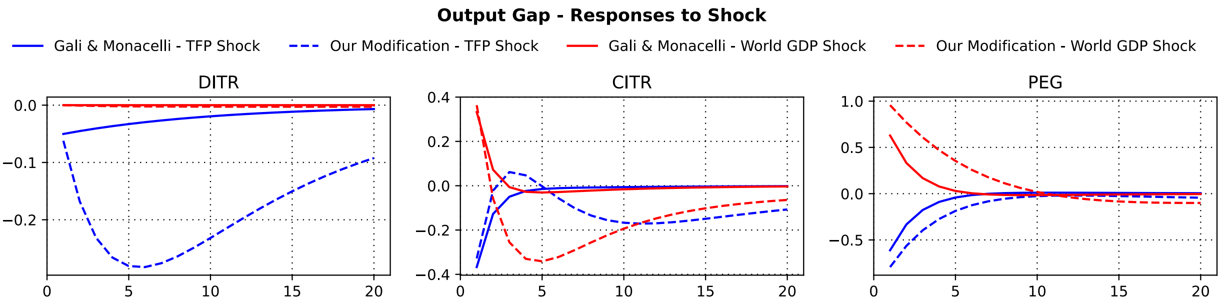


We can see that neither of the policies (DITR, CITR or peg) can push the economy very close to its potential in the first periods and all of them take more time to return to steady state than the original model. The reason is that now we have rigidity in two sides which impact the decision of firms: just some of them can re-optimize and lower its product's price, so the demand is lower than it would be with flexible prices; and because of wage rigidity just a fraction of households renegotiate and accept lower nominal wages, which causes the slow adjustment in real wages in the first periods (while in the original model without wage rigidity there was fast convergence of this variable). The result is that output volatility increases: under PEG there is a stronger initial decrease, while under CITR and DITR there is a hump-shape adjustment. Finally, we have that under DITR and CITR the negative effect on domestic inflation due to negative output gap is not strong enough to compensate the high exchange rate appreciation, increasing the aggregate CPI, while under PEG this affect does not occur and the aggregate CPI drops.



As the shock was external, the dynamics of the DITR rule are very close to the economy with flexible wages. That's because this shock only affects the households' (or unions') decision about salaries through the CPI inflation. It's interesting to note that, under the CITR, the output gap and hours worked decrease from the second period onward, before converging. This occurs because of exchange rate. Also, the domestic prices fall with rigid salaries, which leads to a deflation event higher in the CPI. Looking at the peg, similarly to original model, there are stronger impacts on real variables in comparison to DITR and CITR. As c_t^* increases with higher y_t^* the domestic economy faces higher demand, specially because the monetary authority prevents the expected exchange rate appreciation in this cases. Thus, the real output and hours worked increase, which also improve domestic households income, resulting in higher domestic consumption. Finally, the positive output gap creates a incentive to firms raise prices, increasing domestic inflation and, as consequence due to fixed exchange rate, the CPI.

Note that our modification causes a qualitative improvement in the model as the most variables reacts following hump-shapes. Moreover, there is also a quantitative change: for the both shocks as there is an additional nominal rigidity the impact over real variables is amplified regardless the monetary rule increasing the output gap variance, as we already seen in the dynamic properties table in section 9.2. These effects can be clearly seen in the chart below that compares the quantitative responses of the output gap in the original model (solid lines) and in our modification (dashed lines).



9.4 Welfare Losses

Following our derivation in appendix E, the second order approximation of the expected welfare losses as fraction of steady state consumption under after our modification is given by:

$$\mathbb{V} = -\frac{(1-\alpha)}{2}\left[(1+\varphi)var(x_t) + \frac{\varepsilon}{\lambda}var(\pi_t) + \frac{\zeta}{\Lambda}var(\pi_{w,t})\right] \tag{37}$$

The table below shows the contributions to welfare losses in the modified model, considering different levels of wage rigidity. We can see that, for both inflation target rules, the loss is bigger the higher is ς (the fraction with rigid salaries). We observe that the policy ranking are the same in comparison with the original model, with DITR and CITR close to each other and significantly better than PEG. However notice that the proportional difference between them reduces with more rigidity: while in the no wage rigidity version the PEG is about 3 times worse, under higher rigidity the policy alternatives became very close, with PEG losses only 15-20% higher.

	DI Taylor	CPI Taylor	Peg
No wage rigidity: $\varsigma = 0$			
Var(Home infl.)	0.0124	0.0157	0.0463
Var(Output gap)	0.0001	0.0016	0.0063
Var(Wage infl.)	0.0000	0.0000	0.0000
Total	0.0125	0.0173	0.0527
Low wage rigidity: $\varsigma = 0.25$			
Var(Home infl.)	0.0165	0.0143	0.0274
Var(Output gap)	0.0001	0.0026	0.0116
Var(Wage infl.)	0.0051	0.0083	0.0414
Total	0.0217	0.0252	0.0803

	DI Taylor	CPI Taylor	Peg
Intermediate wage rigidity: $\varsigma = 0.50$			
Var(Home infl.)	0.0202	0.0156	0.0181
Var(Output gap)	0.0005	0.0032	0.0173
Var(Wage infl.)	0.0163	0.0114	0.0468
Total	0.0370	0.0350	0.0822
Standard wage rigidity: $\varsigma = 0.75$			
Var(Home infl.)	0.0183	0.0162	0.0113
Var(Output gap)	0.0042	0.0074	0.0279
Var(Wage infl.)	0.0614	0.0620	0.0559
Total	0.0839	0.0855	0.0952

Note: Values are % units of steady state consumption

10 Conclusion

The paper that we replicated from Galí and Monacelli (2005) provided an important contribution to macroeconomics literature extending the basic new Keynesian model for a small open economy study, showing the dynamic properties of this new framework as also the responses to domestic and foreign shocks, comparing different monetary policy alternatives. Our contribution was include wage rigidity in the model to investigate potential qualitative and quantitative impacts.

The main findings were that in both models the welfare loss resulting from both the interest rate rules - targeting domestic inflation (DITR) and CPI inflation (CITR) - are very close to each other and significantly better than PEG policy. With wage rigidity we found that the variables responses have kept almost the same direction but with hump-shapes reactions and, mainly, now there is a considerable quantitative increase in output gap volatility. After our modification we conclude that, with increasing wage rigidity, the policy alternatives ranking does not change but the superiority of DITR and CITR over PEG reduces.

Appendix

A Consumer problem

The consumer problem is

$$\begin{aligned}\max_{C_t,N_t}\; E_0\sum_{t=0}^{\infty}\beta^t U(C_t,N_t) &= \max_{C_{H,t},C_{F,t},N_t}\; E_0\sum_{t=0}^{\infty}\beta^t U\left(\left[(1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}}+\alpha^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},N_t\right)\\ \text{s.t.}\int_0^1 P_{H,t}(j)C_{H,t}(j)dj &+ \int_0^1\int_0^1 P_{i,t}(j)C_{i,t}(j)dj\;di + E_t\{Q_{t,t+1}D_{t+1}\}\le D_t+W_tN_t+T_t\end{aligned} \tag{38}$$

where $C_{H_t} \equiv \left(\int_0^1 C_{H_t(j)}^{\frac{\varepsilon-1}{\varepsilon}}dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$, $C_{F,t} \equiv \left(\int_0^1 C_{i,t}^{\frac{\gamma-1}{\gamma}}dj\right)^{\frac{\gamma}{\gamma-1}}$, $C_{i,t} \equiv \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}}dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$. The restriction above is valid with equality, otherwise it would be possible to increase utility with the same resources.

Now we can calculate the MRS (marginal rate of substitution) between $C_{H,t}(j)$ and $C_{H,t}$, as by the optimal allocation (and well behaved utility functions), it has to be the rate of prices in every period of time (otherwise the consumer could by a little less of the product with relative higher price and buy another with relative lower price, increasing his utility).

$$\frac{\dfrac{\partial U(C_t,N_t)}{\partial C_{H,t}(j)}}{\dfrac{\partial U(C_t,N_t)}{\partial C_{H,t}}} = \frac{U_c(C_t,N_t)\left((1-\alpha)\dfrac{C_t}{C_{H,t}}\right)^{\frac{1}{\eta}}\displaystyle\int_0^1\left(\frac{C_{H,t}}{C_{H,t}(j)}\right)^{\frac{1}{\varepsilon}}dj}{U_c(C_t,N_t)\left((1-\alpha)\dfrac{C_t}{C_{H,t}}\right)^{\frac{1}{\eta}}} = \frac{\displaystyle\int_0^1 P_{H,t}(j)dj}{P_{H,t}} \tag{39}$$

After simplifying, we get the (6a) equation.

$$\int_0^1\left(\frac{C_{H,t}}{C_{H,t}(j)}\right)^{\frac{1}{\varepsilon}}dj = \int_0^1\frac{P_{H,t}(j)}{P_{H,t}}dj \;\Rightarrow\; C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}C_{H,t} \tag{40}$$

Similarly, with the MRS calculation we arrive at (6b) and (6c).

$$\int_0^1\int_0^1\left(\frac{C_{F,t}}{C_{i,t}}\right)^{\frac{1}{\gamma}}\left(\frac{C_{i,t}}{C_{i,t}(j)}\right)^{\frac{1}{\varepsilon}}dj\;di = \int_0^1\int_0^1\frac{P_{i,t}(j)}{P_{i,t}}\left(\frac{C_{F,t}}{C_{i,t}}\right)^{\frac{1}{\gamma}}dj\;di \;\Rightarrow\; C_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\varepsilon}C_{i,t} \tag{41}$$

$$\left(\frac{C_{F,t}}{C_{i,t}}\right)^{\frac{1}{\gamma}} = \frac{P_{i,t}}{P_{F,t}} \;\Rightarrow\; C_{i,t} = \left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\gamma}C_{F,t} \tag{42}$$

We can also calculate the optimal share of imported goods.

$$\alpha\frac{C_t}{C_{F,t}} = \left(\frac{P_{F,t}}{P_t}\right)^{\eta} \;\Rightarrow\; C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta}C_t;\;\text{ and }\; C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t \tag{43}$$

Considering the functional form for the utility function as $U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$ and deriving the first-order conditions with respect to C_t and N_t, we arrive at (5).

$$\begin{aligned} C_t^{-\sigma} &= \Phi_t P_t \\ N_t^{\varphi} &= \Phi_t W_t \qquad \Rightarrow \quad C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \end{aligned}$$

With the gross return equal to inverse of the stochastic discount factor expectation, we get the Euler equation (4).

$$\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} = \frac{\Phi_{t+1} P_{t+1}}{\lambda_t P_t} \quad \Rightarrow \quad \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+1}} = \frac{Q_{t,t+1}}{\beta} \quad \Rightarrow \quad \beta R_t E_t \left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+1}}\right] = 1$$

It remains to do the aggregation of the budget constraint of the representative consumer:

$$\int_0^1 P_{H,t}(j) C_{H,t}(j) dj = \int_0^1 P_{H,t}(j) \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} dj = \frac{C_{H,t}}{P_{H,t}^{-\varepsilon}} \int_0^1 P_{H,t}(j)^{1-\varepsilon} dj = \frac{C_{H,t}}{P_{H,t}^{-\varepsilon}} P_{H,t}^{1-\varepsilon} = P_{H,t} C_{H,t} \tag{44}$$

Doing twice the same steps as above (using the demand functions and price indices), we can aggregate the consumer's expenditure on imported goods.

$$\int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j) dj\ di = \int_0^1 \int_0^1 P_{H,t}(j) \left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\varepsilon} C_{i,t} dj\ di = \int_0^1 P_{i,t} C_{i,t} di = P_{F,t} C_{F,t} \tag{45}$$

As the total expenditure in goods is the sum of domestic goods (44) and imported goods (45), we achieve the consumer's problem described in (1): $P_t C_t + E_t[Q_{t,t+1} D_{t,t+1}] \leq D_t + W_t N_t + T_t$.

B Labor subsidy definition

As the model depends on imperfect competition assumption, the firms have market power implying that competitive equilibrium is not Pareto Optimal due to lower production and hiring. In this case, the domestic benevolent social planner maximizes the representative household discounted utility subject to technology (12), domestic/foreign consumption relation due to international risk sharing (18) and market clearing (19). For a analytically tractable solution we need to impose $\gamma = \sigma = \eta = 1$ (as adopted in the calibration). The problem becomes:

$$\max_{C_t, N_t} E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t) - \frac{N_t^{\varphi+1}}{\varphi+1}\right] \quad \text{s.t.} \begin{array}{l} Y_t = A_t N_t \\ C_t = C_t^* \mathcal{Q}_t \\ Y_t = C_t S_t^{\alpha} \end{array} \tag{46}$$

By (11) with $\gamma = \sigma = \eta = 1$ we have that $\mathcal{Q}_t = S_t^{1-\alpha}$, and using the global market clearing condition in (20): $Y_t^* = C_t^*$ we rewrite the social planner problem:

$$\max_{C_t, N_t} E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t) - \frac{N_t^{\varphi+1}}{\varphi+1}\right] \quad \text{s.t. } C_t = (A_t N_t)^{1-\alpha} (Y_t^*)^{\alpha} \tag{47}$$

As this problem is a static one we can solve separately for C_t and N_t at any period. The Lagrangean and the first order conditions are:

$$\mathcal{L} = \ln(C_t) - \frac{N_t^{\varphi+1}}{\varphi+1} - \lambda_t (C_t - (A_t N_t)^{1-\alpha} (Y_t^*)^{\alpha}) \tag{48a}$$

$$(C_t) \quad \frac{1}{C_t} - \lambda_t = 0 \tag{48b}$$

$$(N_t) \quad -N_t^{\varphi} + \lambda_t (Y_t^*)^{\alpha} (1-\alpha) A_t^{(1-\alpha)} N_t^{-\alpha} = 0 \tag{48c}$$

Manipulating we get that $N_t^{1+\varphi} = N^{1+\varphi} = (1-\alpha)$. In the competitive steady state in (24) again assuming $\gamma = \sigma = \eta = 1$ we obtain that $\frac{1}{\mathcal{M}} = (1-\tau) N^{1+\varphi}$. So, if the subsidy is such that $(1-\tau) = \frac{1}{(1-\alpha)\mathcal{M}}$ the steady state employment level coincides to Pareto optimal and, then, the efficiency is restored. Note that the solution above holds only for this specific parameter selection.

In the case of the closed economy the optimal subsidy would be $(1-\tau) = \frac{1}{\mathcal{M}}$ as present in Galí (2015). The intuition given by the authors for the difference is that with the open economy the benevolent planner have incentive to set a lower subsidy reducing the output a bit and distorting the terms of trade in a way beneficial to domestic consumers.

C Optimal price setting

The firms optimal price setting problem following the Calvo's model is given by:

$$\overline{P}_{H,t} = \max_{\overline{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left[Q_{t,t+k} [Y_{t+k}(j) (\overline{P}_{H,t} - MC_{t+k} P_{H,t+k})]\right] \tag{49}$$

The domestic demand for a specific variety is $C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}$ if the price remains unchanged at $\overline{P}_{H,t}$ until $t+k$ period then: $C_{t+k}(j) = \left(\frac{\overline{P}_{H,t}(j)}{P_{H,t+k}}\right)^{-\varepsilon} C_{H,t+k}$. Similarly the foreign consumption of this domestic good is $C_{t+k}^i(j) = \int_0^1 \left(\frac{\overline{P}_{H,t}(j)}{P_{H,t+k}}\right)^{-\varepsilon} C_{H,t+k}^i di$. Market clearing imposes that:

$$Y_{t+k}(j) = C_{H,t+k}(j) + \int_0^1 C_{H,t+k}^i(j) di = \left(\frac{\overline{p}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} \left(C_{H,t+k} + \int_0^1 C_{H,t+k}^i di\right) \equiv \left(\frac{\overline{p}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} \tilde{C}_{H,t+k} \tag{50}$$

Substituting $Q_{t,t+k}$ for the expression obtained in the consumer problem and $Y_{t+k}(j)$ for the expression above in the firms problem:

$$\overline{p}_{H,t} = \max_{\overline{p}_{H,t}} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left[\beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+k}} \right) \left(\frac{\overline{p}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \tilde{C}_{H,t+k} (\overline{p}_{H,t} - MC_{t+k} P_{H,t+k}) \right] \tag{51}$$

Calculating the first order condition with respect to $\overline{p}_{H,t}$ and rearranging we get:

$$\overline{p}_{H,t} = \frac{\mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta)^k C_{t+k}^{-\sigma} \frac{1}{P_{t+k}} \tilde{C}_{H,t+k} P_{H,t+k} MC_{t+k} \mathcal{M} \right]}{\mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta)^k C_{t+k}^{-\sigma} \frac{1}{P_{t+k}} \tilde{C}_{H,t+k} \right]} \tag{52}$$

In the zero inflation steady state $\overline{p}_{H,t} = P_{H,t} = P_t = P_H$, implying that, by the previous formula, $MC_t = \frac{1}{\mathcal{M}} \equiv \frac{\varepsilon-1}{\varepsilon}$. Thus we define $\widehat{MC}_t = \frac{MC_t}{1/\mathcal{M}}$ as the marginal cost deviation from steady state. Now using the price dynamics:

$$P_{H,t} = [\theta P_{H,t-1}^{1-\varepsilon} + (1-\theta)\overline{p}_{H,t}^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \Rightarrow \Pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}} = \frac{\left[\theta P_{H,t}^{1-\varepsilon} + (1-\theta)\overline{p}_{H,t}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}{P_{H,t-1}} = \left[\theta + (1-\theta) \frac{\overline{p}_{H,t}^{1-\varepsilon}}{P_{H,t-1}^{1-\varepsilon}} \right]^{\frac{1}{1-\varepsilon}} \tag{53}$$

Substituing $\overline{p}_{H,t}$ we reach:

$$\Pi_{H,t} = \left[\theta + \frac{(1-\theta)}{P_{H,t-1}^{1-\varepsilon}} \left(\frac{\mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta)^k C_{t+k}^{-\sigma} \frac{1}{P_{t+k}} \tilde{C}_{H,t+k} P_{H,t+k} \widehat{MC}_{t+k} \right]}{\mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta)^k C_{t+k}^{-\sigma} \frac{1}{P_{t+k}} \tilde{C}_{H,t+k} \right]} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{54}$$

Log-linearizing we get:

$$\frac{\pi_{H,t} + p_{H,t-1}}{(1-\theta)(1-\beta\theta)} = \sum_{k=0}^{\infty} (\beta\theta)^k \mathbb{E}_t[\widehat{mc}_{t+k} + p_{t+k}] \tag{55}$$

To obtain a recursive form, subtract for the same expression in $t+1$ multiplied by β, apply the Law of Iterated Expectations and rearrange:

$$\frac{\pi_{H,t} - \beta \mathbb{E}_t[\pi_{H,t+1}]}{(1-\theta)(1-\beta\theta)} = \sum_{k=0}^{\infty} (\beta\theta)^k \mathbb{E}_t[\widehat{mc}_t + p_{t+k}] - \beta \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k \mathbb{E}[\widehat{mc}_{t+1+k} + p_{t+k+1}]$$

Defining $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$ and manipulating we reach the New Keynesian Phillips Curve:

$$\pi_{H,t} = \beta \mathbb{E}_t[\pi_{H,t+1}] + \lambda \widehat{mc}_t \tag{56}$$

D Goods market clearing

Market clearing in goods market imposes that, for each domestic good, the total production is equal to domestic + external demands:

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t} + \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} \int_0^1 C_{H,t}^i di \tag{57}$$

Substituting $C_{H,t} = (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t$ and $C_{H,t}^i = \alpha \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P_{F,t}^i} \right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t^i} \right)^{-\eta} C_t^i$

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} \left((1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P_{F,t}^i} \right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t^i} \right)^{-\eta} C_t^i di \right) \tag{58}$$

As $Y_t = \left[\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$

$$\begin{aligned} Y_t &= \left\{ \int_0^1 \left[\left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} \left((1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P_{F,t}^i} \right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t^i} \right)^{-\eta} C_t^i di \right) \right]^{\frac{\varepsilon-1}{\varepsilon}} dj \right\}^{\frac{\varepsilon}{\varepsilon-1}} \\ Y_t &= \left\{ \int_0^1 \left(P_{H,t}(j)^{-\varepsilon} \right)^{\frac{\varepsilon-1}{\varepsilon}} dj \right\}^{\frac{\varepsilon}{\varepsilon-1}} \left(\frac{1}{P_{H,t}} \right)^{-\epsilon} \left((1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P_{F,t}^i} \right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t^i} \right)^{-\eta} C_t^i di \right) \end{aligned} \tag{59}$$

Considering that $\left[\int_0^1 \left(P_{H,t}(j)^{-\varepsilon} \right)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} = \left[\int_0^1 P_{H,t}^{1-\varepsilon}(j) dj \right]^{\frac{\varepsilon}{\varepsilon-1}} = P_{H,t}^{-\epsilon}$ we get:

$$\begin{aligned} Y_t &= (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P_{F,t}^i} \right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t^i} \right)^{-\eta} C_t^i di \\ Y_t &= \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} \left[(1-\alpha) C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P_{F,t}^i} \right)^{-\gamma} \left(\frac{\mathcal{E}_{i,t} P_{F,t}^i}{P_{H,t}} \right)^{-\eta} \left(\frac{P_t}{\mathcal{E}_{i,t} P_t^i} \right)^{-\eta} C_t^i di \right] \end{aligned} \tag{60}$$

$$Y_t = C_t \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha) + \alpha \int_0^1 \left(\mathcal{S}_t^i \mathcal{S}_{i,t}\right)^{\gamma-\eta} \mathcal{Q}_{i,t}^{\eta-\frac{1}{\sigma}} di\right] \tag{61}$$

And finally, using (8) we obtain:

$$Y_t = C_t \left[(1-\alpha) + \alpha S_t^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\alpha) + \alpha \int_0^1 \left(\mathcal{S}_t^i \mathcal{S}_{i,t}\right)^{\gamma-\eta} \mathcal{Q}_{i,t}^{\eta-\frac{1}{\sigma}} di\right] \tag{62}$$

E Optimal Monetary Policy and Welfare Losses

Here we'll derive the loss function in the general case with price rigidity, as we reached that the original paper model can be considered a special case when wage stickiness is set to zero ($\varsigma = 0$) and there is perfect competition in goods market ($\zeta \to \infty$). The welfare costs from deviations from the optimal monetary policy are derived from a second order Taylor expansion of the representative consumer's utility function around the steady-state.

Calculating the second order approximation from the product, we have

$$\frac{Y_t}{Y} = 1 + \ln\frac{Y_t}{Y} + \frac{1}{2}\left(\ln\frac{Y_t}{Y}\right)^2 + \frac{1}{3!}\left(\ln\frac{Y_t}{Y}\right)^3 + ... = 1 + y_t + \frac{1}{2}(y_t)^2 + o(||a||^n) \;\; \Rightarrow \;\; \frac{Y_t - Y}{Y} = 1 + y_t + \frac{y_t^2}{2} + o(||a||^n) \tag{63}$$

where a is the bound for the high order terms.

From equations (23i) and (23e), we have $c_t = c_t^* + \frac{1-\alpha}{\sigma} s_t = c_t^* + (1-\alpha)s_t$. Also, as $\gamma = \eta = \sigma = 1$, equation (23l) becomes

$$y_t = c_t + \alpha s_t \;\; \Rightarrow \;\; c_t = y_t - \alpha\frac{c_t - c_t^*}{1-\alpha} \;\; \Rightarrow \;\; c_t = (1-\alpha)y_t + \alpha y_t^*. \tag{64}$$

As $x_t \equiv y_t - \overline{y}_t$, in the stabilized economy, $x_t = 0$ and $y_t = \overline{y}_t$. Thus, $\overline{c}_t = \alpha y_t^* - (1-\alpha)\overline{y}_t$. Substituting, we get:

$$c_t = (1-\alpha)(\overline{y}_t + x_t) + \alpha y_t^* = (1-\alpha)\overline{y}_t + \alpha y_t^* + (1-\alpha)x_t \;\; \Rightarrow \;\; c_t = \overline{c}_t + (1-\alpha)x_t. \tag{65}$$

Expanding the log-deviation of the disutility of work, we have:

$$\left(\frac{N_t}{\overline{N}}\right)^{1+\varphi} = \exp[(1+\varphi)\widetilde{n}] = 1 + (1+\varphi)\widetilde{n}_t + \frac{1}{2}\widetilde{n}_t^2 + o(||a||^3) \;\; \Rightarrow \;\; \frac{N_t^{1+\varphi}}{1+\varphi} = \frac{\overline{N}^{1+\varphi}}{1+\varphi} + \overline{N}^{1+\varphi}\left[\widetilde{n}_t + \frac{1}{2}(1+\varphi)\widetilde{n}_t^2\right] + o(||a||^3) \tag{66}$$

Using the fact that

$$\int_0^1 N_t(j)dj = \int_0^1 \left(\frac{Y_t(j)}{A_t}\right) \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} di\; dj = \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\zeta} dj \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} di = \frac{N_t A_t}{Y_t} \tag{67}$$

$$\log\int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\zeta} dj + \log\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} di = \log\left(\frac{N_t A_t}{Y_t}\right) = n_t + a_t - y_t \tag{68}$$

If we define $z_t \equiv \log\int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\zeta} di + \log\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} di$, then $z_t = n_t + a_t - y_t = n_t + a_t - (\overline{y}_t + x_t)$. When prices are stabilized, $P_{H,t}(i) = P_{H,t}$, $W_t(j) = W_t$ and $\overline{z}_t = 0$. Also, there are no productivity shocks, so $a_t = \overline{y}_t - \overline{n}_t$. Thus,

$$z_t = \overline{n}_t + \widetilde{n}_t + \overline{y}_t - \overline{n}_t - (\overline{y}_t + x_t) \;\; \Rightarrow \;\; \widetilde{n}_t = z_t + x_t \tag{69}$$

Under the optimal subsidy assumption, from the consumer's FOC, we have that $\overline{N}_t^{1+\varphi} = (1-\alpha)$ (constant employment). Thus,

$$U(C_t, N_t) \equiv \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\overline{N}^{1+\varphi}}{1+\varphi} - \overline{N}^{1+\varphi}\left[x_t + z_t + \frac{1}{2}(1+\varphi)x_t^2\right] + o(||a||^3)$$

$$U(C_t, N_t) = -(1-\alpha)\left[x_t + z_t + \frac{1}{2}(1+\varphi)x_t^2\right] + \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{(1-\alpha)}{1+\varphi} o(||a||^3) = -(1-\alpha)\left[z_t + \frac{1}{2}(1+\varphi)x_t^2\right] + t.i.p + o(||a||^3) \tag{70}$$

there t.i.p. denotes terms independent of policy and under the optimal policy, $x_t = 0$.

We can sum the consumer's utility for the whole period to construct the loss function

$$\mathbb{W} = \sum_{i=0}^{\infty} \beta^t U(C_t, N_t) = \sum_{i=0}^{\infty} \beta^t \left[-(1-\alpha)\left(z_t + \frac{1}{2}(1+\varphi)x_t^2\right) + t.i.p + o(||a||^3)\right] \tag{71}$$

Using a lemma proved in Erceg et al. (2000) for the wage variance, $z_t = \frac{\varepsilon}{2} var_i[p_{H,t}(i)] + \frac{\zeta}{2} var_j[w_t(j)] + o(||a||^3)$, we have

$$\mathbb{W} = \sum_{i=0}^{\infty} \beta^t \left[-(1-\alpha)\left(\frac{\varepsilon}{2} var_i[p_{H,t}(i)] + \frac{\zeta}{2} var_j[w_t(j)] + o(||a||^3) + \frac{1}{2}(1+\varphi)x_t^2\right)\right] + t.i.p + o(||a||^3) \tag{72}$$

Now using that $\sum_{t=0}^{\infty} \beta^t var_i[p_{H,t}(i)] = \frac{1}{\lambda}\sum_{t=0}^{\infty} \beta^t \pi_{H,t}^2$ where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$ (proof in Woodford, 2003, Chapter 6), and the analogous $\sum_{t=0}^{\infty} \beta^t var_j[w_t(j)] = \frac{1}{\Lambda}\sum_{t=0}^{\infty} \beta^t \pi_{w,t}^2$ where $\Lambda = \frac{(1-\varsigma)(1-\beta\varsigma)}{\varsigma(1+\zeta\varphi)}$ we have:

$$\mathbb{W} = -\frac{(1-\alpha)}{2}\sum_{i=0}^{\infty}\beta^t\left[\frac{\varepsilon}{\lambda}\pi_{H,t}^2 + \frac{\zeta}{\Lambda}\pi_{w,t}^2 + (1+\varphi)x_t^2\right] + t.i.p + o(||a||^3) \tag{73}$$

Taking out the constant $(t.i.p)$, which are the terms independent of monetary policy and $o(||a||^3)$, which is a third order term, and letting $\beta \to 1$, we achieve the desired welfare loss equation expressed as function of the variances:

$$\mathbb{V} = -\frac{(1-\alpha)}{2}\left[\frac{\varepsilon}{\lambda}var(\pi_{H,t}) + \frac{\zeta}{\Lambda}var(\pi_{w,t}) + (1+\varphi)var(x_t)\right] \tag{74}$$

In the limit case when there is neither wage rigidity nor imperfect competition in labor market the second term of the equation above vanishes as $\frac{\zeta}{\Lambda} \to 0$, and we have the welfare function present in the original paper.

Finally, in order to derive the optimal rule without wage rigidity we first reduce the log-linear system in (23) to a 2 equations representation [9] (Dynamic IS and NKPC) obtaining:

$$\begin{aligned} x_t &= \mathbb{E}_t[x_{t+1}] + \frac{1}{\sigma_\alpha}(r_t - \mathbb{E}_t\pi_{H,t+1} - \rho + \sigma_\alpha\Gamma(1-\rho_a)a_t - \alpha\sigma_\alpha(\Theta+\Psi)(\mathbb{E}_t[y_{t+1}^*] - y_t^*)) \\ \pi_{H,t} &= \beta\mathbb{E}_t[\pi_{H,t+1}] + \kappa_\alpha x_t \end{aligned} \tag{75}$$

Defining $\overline{rr}_t \equiv \rho - \sigma_\alpha\Gamma(1-\rho_a)a_t + \alpha\sigma_\alpha(\Theta+\Psi)(\mathbb{E}_t[y_{t+1}^*] - y_t^*)$ as the neutral interest rate we can define an optimal monetary rule when $r_t = \overline{rr}_t$. In this case, the remaining system results exactly in $x_t = \pi_{H,t}\ \forall t$, implying in zero output and domestic inflation variance, thus no welfare losses. For implementation we define the optimal rule as: $r_t = \overline{rr}_t + \phi_\pi\pi_{H,t}$ with parameter $\phi_\pi > 1$ having no real impact but defined in order to satisfy the Taylor principle, necessary to attend Blanchard and Khan conditions to unique stationary equilibrium.

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[9] In the section 3 we opted for keep the 16 equations representation instead of this simplified one as this reduction drops out many variables that are useful to understand more clearly the model mechanisms.