Abstract

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1 Introduction

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2 Description of the model

2.1 Consumers Problem

The model considers a standard representative consumer with separable preferences that maximizes his expected discounted payoff in a infinite time horizon:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{\varphi+1}}{\varphi+1} \quad \text{st.} \quad \forall t, \quad P_t C_t + \mathbb{E}_t[Q_{t,t+1} D_{t+1}] \leq D_t + W_t N_t + T_t \tag{1}$$

Where D_t is the nominal payoff in period t of the portfolio held at the end of period $t-1$. The consumer chooses between domestic goods ($C_{H,t}$) and foreign goods ($C_{F,t}$) with $\alpha \in [0,1]$ as the opening index of the economy and elasticity of substitution $\eta > 0$ between domestic and imported goods. The imported goods consumption is divided in goods imported from a continuum of different countries i with elasticity of substitution $\gamma > 0$. Finally, he chooses between goods j made in the same country (domestic or foreign) with elasticity of substitution $\varepsilon > 0$.

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{2}$$

$$C_{F,t} \equiv \left(\int_0^1 C_{i,t}^{\frac{\gamma-1}{\gamma}} \, di \right)^{\frac{\gamma}{\gamma-1}} \tag{3}$$

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{4}$$

$$C_{i,t} \equiv \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{5}$$

The detailed solution of the consumer problem is detailed in the appendix A and results are, as usual, the consumer Euler equation and the labour supply:

$$\beta R_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right] = 1 \tag{6}$$

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \tag{7}$$

2.2 Terms of trade

We define the bilateral terms of trade $S_{i,t}$ between domestic economy and the country i as $S_{i,t} \equiv \frac{P_{i,t}}{P_{H,t}}$. The effective terms of trade given by:

$$S_t \equiv \left(\int_0^1 S_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} = \left(\int_0^1 \left(\frac{P_{i,t}}{P_{H,t}} \right)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} = \frac{1}{P_{H,t}} \left(\int_0^1 \left(P_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \right) = \frac{P_{F,t}}{P_{H,t}} \tag{8}$$

Using the that price level are given by $P_t^{1-\eta} = (1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}$ we get:

$$P_t^{1-\eta} = (1-\alpha)P_{H,t}^{1-\eta} + \alpha \left(P_{H,t}^{1-\eta} S_t^{1-\eta} \right) = P_{H,t}^{1-\eta} \left[(1-\alpha) + \alpha S_t^{1-\eta} \right] \Rightarrow P_t = P_{H,t} \left[(1-\alpha) + \alpha S_t^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{9}$$

Dividing P_t by P_{t-1}:

$$\frac{P_t}{P_{t-1}} = \frac{P_{H,t} \left[(1-\alpha) + \alpha S_t^{1-\eta} \right]^{\frac{1}{1-\eta}}}{P_{H,t-1} \left[(1-\alpha) + \alpha S_{t-1}^{1-\eta} \right]^{\frac{1}{1-\eta}}} \Rightarrow \Pi_t = \Pi_{H,t} \left(\frac{(1-\alpha) + \alpha S_t^{1-\eta}}{(1-\alpha) + \alpha S_{t-1}^{1-\eta}} \right)^{\frac{1}{1-\eta}} \tag{10}$$

2.3 Exchange Rate

We assume that law of one price holds for all goods and all times, defining $\mathcal{E}_{i,t}$ as the bilateral exchange rate between domestic economy and country i and $P_{i,t}^i(j)$ as the price expressed in the producer's currency, then:

$$P_{i,t}(j) = \mathcal{E}_{i,t} P_{i,t}^i(j) \Rightarrow \left(\int_0^1 \left(P_{i,t}(j) \right)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}} = \left(\int_0^1 \left(\mathcal{E}_{i,t} P_{i,t}^i(j) \right)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}} \Rightarrow P_{i,t} = \mathcal{E}_{i,t} P_{i,t}^i \tag{11}$$

Aggregating for all countries i

$$\left(\int_0^1 \left(P_{i,t} \right)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} = \left(\int_0^1 \left(\mathcal{E}_{i,t} P_{i,t}^i \right)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \Rightarrow S_t P_{H,t} = \left(\int_0^1 \left(\mathcal{E}_{i,t} P_{i,t}^i \right)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \tag{12}$$

Dividing $S_t P_{H,t}$ by $S_{t-1} P_{H,t-1}$ we get :

$$\frac{S_t P_{H,t}}{S_{t-1} P_{H,t-1}} = \frac{\left(\int_0^1 \left(\mathcal{E}_{i,t} P_{i,t}^i \right)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}}{\left(\int_0^1 \left(\mathcal{E}_{i,t-1} P_{i,t-1}^i \right)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}} \Rightarrow \frac{S_t}{S_{t-1}} \Pi_{H,t} = \left(\frac{\int_0^1 \left(\mathcal{E}_{i,t} P_{i,t}^i \right)^{1-\gamma} di}{\int_0^1 \left(\mathcal{E}_{i,t-1} P_{i,t-1}^i \right)^{1-\gamma} di} \right)^{\frac{1}{1-\gamma}} \tag{13}$$

Which defines implicitly $\mathcal{E}_t \equiv \left(\int_0^1 \mathcal{E}_{i,t}^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}$ as the nominal effective exchange rate [1].

[1] Log-linearizing we get $\pi_{H,t} + \alpha s_{t+1} - \alpha s_t = \pi_t^* + \Delta e_t$, where $e_t \equiv \int_0^1 e_{i,t}$, $p_t^* \equiv \int_0^1 p_{i,t}^i$ and $\pi_t^* \equiv p_t^* - p_{t-1}^*$. The baseline model of the paper assumes $p_t^* = p^* = 0$, implying that $\pi_{H,t} + \alpha s_{t+1} - \alpha s_t = \Delta e_t$

The bilateral real exchange rate with country i is defined as $\mathcal{Q}_{i,t} \equiv \frac{\mathcal{E}_{i,t}P^i_{i,t}}{P_t}$ and using that $P_{i,t} = \mathcal{E}_{i,t}P^i_{i,t}$ then:

$$\begin{aligned}
\mathcal{Q}_{i,t} &= \frac{\mathcal{E}_{i,t}P^i_{i,t}}{P_t} = \frac{P_{i,t}}{P_t} \Rightarrow \left(\int_0^1 \mathcal{Q}_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}} = \left(\int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{1-\gamma} di\right)^{\frac{1}{1-\gamma}} = \frac{1}{P_t}\left(\int_0^1 P_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}} \\
\mathcal{Q}_t &= \frac{P_{F,t}}{P_t} = \frac{P_{H,t}S_t}{P_t} = \frac{P_{H,t}S_t}{P_{H,t}\left[(1-\alpha)+\alpha S_t^{1-\eta}\right]^{\frac{1}{1-\eta}}} = \left[(1-\alpha)S_t^{\eta-1}+\alpha\right]^{\frac{1}{\eta-1}}
\end{aligned} \tag{14}$$

2.4 Firms Problem

A representative firm has a technology with constant returns:

$$Y_t(j) = A_t N_t(j) \Rightarrow \int_0^1 Y_t(j)dj = \int_0^1 A_t N_t(j)dj \Rightarrow Y_t = A_t N_t \tag{15}$$

$$A_{t+1} = {A_t}^{\rho_a}\exp(\varepsilon^a_t) \tag{16}$$

Where A_t is such that $a_t \equiv \ln(A_t)$ follows an AR(1) process and in steady state we normalize $A = 1$.

The model assume sticky prices following Calvo (1983): at each period a random selected measure θ of firms have to keep their prices unchanged from the previous period, while the remaining $1-\theta$ firms can reset them. If a firm could adjust price at time t, we will set its price at $\bar{P}_{H,t}$ which maximizes the present value of its future profit:

$$\bar{P}_{H,t} = \max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t\left[Q_{t,t+k}[Y_{t+k}(j)(\bar{P}_{H,t} - MC_{t+k}P_{H,t+k})]\right] \tag{17}$$

Where MC_t is the real marginal cost. Combining the first order of this problem with the price dynamics (detailed solution is in appendix B) and defining $\widehat{MC}_t = \frac{MC_t}{1/\mathcal{M}}$ as the marginal cost deviation from steady state we get:

$$\Pi_{H,t} = \left[\theta + \frac{(1-\theta)}{P_{H,t-1}^{1-\varepsilon}}\left(\frac{\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\theta)^k C_{t+k}^{-\sigma}\frac{1}{P_{t+k}}\tilde{C}_{H,t+k}P_{H,t+k}\widehat{MC}_{t+k}\right]}{\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\theta)^k C_{t+k}^{-\sigma}\frac{1}{P_{t+k}}\tilde{C}_{H,t+k}\right]}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{18}$$

Finally, due to constant returns to scale, the real marginal cost does not depend on quantities. Using the consumers labor supply (7), the technology (15) and (9) we can derive the marginal cost deviation from its steady state:

$$\widehat{MC}_t = \mathcal{M}MC_t = \frac{W_t(1-\tau)\mathcal{M}}{P_{H,t}A_t} = \frac{C_t^{\sigma}N_t^{\varphi}P_t(1-\tau)\mathcal{M}}{P_{H,t}A_t} = \frac{C_t^{\sigma}Y_t^{\varphi}\left[(1-\alpha)+\alpha S_t^{1-\eta}\right]^{\frac{1}{1-\eta}}(1-\tau)\mathcal{M}}{A_t^{1+\varphi}} \tag{19}$$

Where τ is an employment subsidy [2].

[2] See appendix C for discussion on optimal subsidy

2.5 Rest of the world

Given the small domestic economy, we assume the rest of the world's output Y_t^* can be taken as exogenous and such that $y_t^* = \ln(Y_t^*)$ follows and AR(1) process:

$$Y_{t+1}^* = Y_t^{*\rho_y} \exp(\varepsilon_t^*) \tag{20}$$

Consumer's Euler equations also holds for a representative consumer in any other country:

$$\beta \left(\frac{C_{t+1}^i}{C_t^i} \right) \left(\frac{P_t^i}{P_{t+1}^i} \right) \left(\frac{\mathcal{E}_t^i}{\mathcal{E}_{t+1}^i} \right) = Q_{t,t+1}^i$$

The model assumes complete markets, then there are perfect domestic and international risk sharing, implying that $Q_{t,t+1}^i = Q_{t,t+1}$:

$$\begin{gathered}\beta \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \left(\frac{P_t^i \mathcal{E}_t^i}{P_{t+1}^i \mathcal{E}_{t+1}^i} \right) = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \Rightarrow \left(\frac{C_{t+1}}{C_t} \right) = \left(\frac{C_{t+1}^i}{C_t^i} \right) \left(\frac{\frac{\mathcal{E}_{i,t+1} P_{t+1}^i}{P_{t+1}}}{\frac{\mathcal{E}_{i,t} P_t^i}{P_t}} \right)^{\frac{1}{\sigma}} = \\ \left(\frac{C_{t+1}}{C_t} \right) = \left(\frac{C_{t+1}^i}{C_t^i} \right) \left(\frac{\mathcal{Q}_{i,t+1}}{\mathcal{Q}_{i,t}} \right)^{\frac{1}{\sigma}} \Rightarrow \left[\frac{C_{t+1}}{C_{t+1}^i \mathcal{Q}_{i,t+1}^{\frac{1}{\sigma}}} \right] = \left[\frac{C_t}{C_t^i \mathcal{Q}_{i,t}^{\frac{1}{\sigma}}} \right]\end{gathered} \tag{21}$$

Defining $t = 0$ and iterating we get that, for all period t

$$\left[\frac{C_t}{C_t^i \mathcal{Q}_{i,t}^{\frac{1}{\sigma}}} \right] = \left[\frac{C_0}{C_0^i \mathcal{Q}_{i,0}^{\frac{1}{\sigma}}} \right] \equiv v_{i,0} \Rightarrow C_t = v_{i,0} C_t^i \mathcal{Q}_{i,t}^{\frac{1}{\sigma}} \tag{22}$$

Where $v_{i,0}$ depends on initial conditions, which the model assumes be symmetric, then $v_{i,0} = 1$ for all i. Aggregating over all countries:

$$\left(\int_0^1 C_t^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} = C_t = \left(\int_0^1 \left(C_t^i \mathcal{Q}_{i,t}^{\frac{1}{\sigma}} \right)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \tag{23}$$

2.6 Market clearing

For each domestic good, the total production is equal to domestic + external demands. Substituting the expressions obtained in the consumer problem and aggregating for all domestic goods (detailed steps are in appendix D) we obtain:

$$Y_t = C_t \left[(1-\alpha) + \alpha S_t^{1-\eta} \right]^{\frac{\eta}{1-\eta}} \left[(1-\alpha) + \alpha \int_0^1 \left(\mathcal{S}_t^i \mathcal{S}_{i,t} \right)^{\gamma-\eta} \mathcal{Q}_{i,t}^{\eta - \frac{1}{\sigma}} di \right] \tag{24}$$

As we are considering the country as a small open economy, for the rest of the world the domestic consumption and production are insignificant and can be ignored, so market clearing implies:

$$C_t^* = Y_t^* \tag{25}$$

3 Equilibrium conditions

The equilibrium conditions (without a monetary policy equation) are given by equations (6), (10), (13), (14), (15), (16), (19), (18), (20), (23), (24) and (25) obtained above:

$$\beta R_t \mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\right]=1 \tag{26a}$$

$$\Pi_t=\Pi_{H,t}\left(\frac{(1-\alpha)+\alpha S_t^{1-\eta}}{(1-\alpha)+\alpha S_{t-1}^{1-\eta}}\right)^{\frac{1}{1-\eta}} \tag{26b}$$

$$\frac{S_t}{S_{t-1}}\Pi_{H,t}=\left(\frac{\int_0^1\left(\mathcal{E}_{i,t}P_{i,t}^i\right)^{1-\gamma}di}{\int_0^1\left(\mathcal{E}_{i,t-1}P_{i,t-1}^i\right)^{1-\gamma}di}\right)^{\frac{1}{1-\gamma}} \tag{26c}$$

$$\mathcal{Q}_t=\left[(1-\alpha)S_t^{\eta-1}+\alpha\right]^{\frac{1}{\eta-1}} \tag{26d}$$

$$Y_t=A_tN_t \tag{26e}$$

$$A_{t+1}={A_t}^{\rho_a}\exp(\varepsilon_t^a) \tag{26f}$$

$$\widehat{MC}_t=\frac{C_t^\sigma Y_t^\varphi\left[(1-\alpha)+\alpha S_t^{1-\eta}\right]^{\frac{1}{1-\eta}}(1-\tau)\mathcal{M}}{A_t^{1+\varphi}} \tag{26g}$$

$$\Pi_{H,t}=\left[\theta+\frac{(1-\theta)}{P_{H,t-1}^{1-\varepsilon}}\left(\frac{\mathbb{E}_t\left[\sum_{k=0}^\infty(\beta\theta)^kC_{t+k}^{-\sigma}\frac{1}{P_{t+k}}\tilde{C}_{H,t+k}P_{H,t+k}\widehat{MC}_{t+k}\right]}{\mathbb{E}_t\left[\sum_{k=0}^\infty(\beta\theta)^kC_{t+k}^{-\sigma}\frac{1}{P_{t+k}}\tilde{C}_{H,t+k}\right]}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{26h}$$

$$Y_{t+1}^*={Y_t^*}^{\rho_y}\exp(\varepsilon_t^*) \tag{26i}$$

$$C_t=\left(\int_0^1\left(C_t^i\mathcal{Q}_{i,t}^{\frac{1}{\sigma}}\right)^{1-\gamma}di\right)^{\frac{1}{1-\gamma}} \tag{26j}$$

$$Y_t=C_t\left[(1-\alpha)+\alpha S_t^{1-\eta}\right]^{\frac{\eta}{1-\eta}}\left[(1-\alpha)+\alpha\int_0^1\left(\mathcal{S}_t^i\mathcal{S}_{i,t}\right)^{\gamma-\eta}\mathcal{Q}_{i,t}^{\eta-\frac{1}{\sigma}}di\right] \tag{26k}$$

$$C_t^*=Y_t^* \tag{26l}$$

The log-linear approximation of these equations around the steady state is [3]:

[3] Lower case letters represent the neperian logarithm of the original variables. We define $\rho\equiv\beta^{-1}-1$, $\nu\equiv-ln(1-\tau)$, $\mu\equiv\ln(\mathcal{M})$, $\omega\equiv\sigma\gamma+(1-\alpha)(\sigma\eta-1)$, $\sigma_\alpha\equiv\frac{\sigma}{(1-\alpha)+\alpha\omega}$, $\Omega\equiv\frac{\nu-\mu}{\sigma_\alpha+\varphi}$, $\Gamma\equiv\frac{1+\varphi}{\sigma_\alpha+\varphi}$, $\Psi\equiv\frac{(1-\omega)\alpha}{\sigma_\alpha+\varphi}$

$$c_t = \mathbb{E}_t[c_{t+1}] - \frac{1}{\sigma}(r_t - \mathbb{E}_t[\pi_{t+1}] - \rho) \tag{27a}$$
$$\pi_t = \pi_{H,t} + \alpha s_t - \alpha s_{t-1} \tag{27b}$$
$$s_t - s_{t-1} + \pi_{H,t} = \Delta e_t \tag{27c}$$
$$q_t = (1-\alpha)s_t \tag{27d}$$
$$y_t = a_t + n_t \tag{27e}$$
$$a_{t+1} = \rho_a a_t + \varepsilon_t^a \tag{27f}$$
$$\hat{mc}_t = -\nu + \sigma c_t + \varphi y_t + \alpha s_t - (1+\varphi)a_t + \mu \tag{27g}$$
$$\pi_{H,t} = \beta \mathbb{E}_t[\pi_{H,t+1}] + \lambda \hat{mc}_t \tag{27h}$$
$$y_{t+1}^* = \rho_y y_t^* + \varepsilon_t^* \tag{27i}$$
$$c_t = c_t^* + \frac{1}{\sigma} q_t \tag{27j}$$
$$y_t = c_t + \alpha\gamma s_t + \alpha\left(\eta - \frac{1}{\sigma}\right) q_t \tag{27k}$$
$$c_t^* = y_t^* \tag{27l}$$

To these equations we also adds a definition of potential output $\bar{y}_t$ obtained from (27g) when $\hat{mc}_t = 0$ (and manipulating to be a function only of exogenous variables) and a definition of (log) output gap given by output deviation from its potential:

$$\bar{y}_t = \Omega + \Gamma a_t - \alpha \Psi y_t^* \tag{27m}$$
$$x_t = y_t - \bar{y}_t \tag{27n}$$

To complete the model its necessary to include a last equation, relative to monetary policy. The paper suggests 3 different monetary rules and a optimal monetary rule benchmark[4] and estimates the model in each case:

- Optimal Monetary Policy: $r_t = \bar{rr}_t + \phi_\pi \pi_{H,t}$
- Domestic Inflation Taylor Rule: $r_t = \rho + \phi_\pi \pi_{H,t}$
- CPI Inflation Taylor Rule: $r_t = \rho + \phi_\pi \pi_t$
- Exchange Rate Peg[5]: $\Delta e_t = 0$.

Where $\bar{rr}_t = \rho + \sigma_\alpha \Gamma(1-\rho_a)a_t + \alpha\sigma_\alpha(\Theta + \Psi)(\mathbb{E}_t[y_{t+1}^*] - y_t^*)$ is the natural interest rate.

4 Calibration

The paper assumes the following calibration for the structural parameters:

[4] See appendix E for details

[5] The paper defines Peg as $e_t = 0$ but as the nominal variables are not determined in equilibrium, depending on the initial conditions (only their variations like inflation rates and Δe_t are determined) we considered peg as $\Delta e_t = 0$, imposing an initial condition $e_0 = 0$ which implies in $e_t = 0$

Parameter	Description	Value
Common to Real Business Cycles Model		
β	Intertemporal discount factor	0.99
σ	Inverse elasticity of intertemporal subsititution	1
φ	Inverse Frisch elasticity of labour supply	3
ρ_a	Productivity shock smoothing	0.66
σ_a	Standard deviation of the productivity shocks	0.0071
Common to New Keynesian Model		
ε	Substitutability between varieties (from the same country)	6
θ	Calvo price stickiness	0.75
ϕ_π	Taylor rule response to inflation	1.5
Specific to Galí and Monacelli Model		
α	Opening index of the economy	0.4
η	Substitutability between domestic and imported goods	1
γ	Substitutability between goods from different foreign countries	1
ρ_{y^*}	World GDP shock smoothing	0.86
σ_{y^*}	Standard deviation of the world GDP shocks	0.0078
ρ_{ay^*}	Correlation between prod. and world GDP shocks	0.3

In the paper's text and tables the parameter ρ_a is defined as 0.66, however the IRFs charts are compatible with value 0.90 (authors reused the charts from an working paper version which $\rho_a = 0.90$ without update the figure or inform the different parameter in the text). We use this same value in the charts to be comparable with original ones in the paper.

5 Steady State Properties

In the steady state: (i) Purchasing Power Parity holds symmetrically for all other countries (then $\mathcal{Q}_i = \mathcal{Q}$, $S_i = S$, $S^i = 1$ and $C^i = C^*$), (ii) all the stationary variables are constant and (iii) there is not uncertainty ($\varepsilon^* = \varepsilon^a = 0$). Using the equilibrium conditions showed in (26) immediately by (f), (i) and (l) we obtain that $A = Y^* = C^* = 1$. The equation (a) defines $R = \beta^{-1}$. From (c) and (b) we have $\Pi = \Pi_H = 1$. Using (h) we find $\widehat{MC} = 1$. The remaining system (composed by equations (d), (e), (j) and (k)) is:

$$\mathcal{Q} = \left[(1-\alpha)S^{\eta-1} + \alpha\right]^{\frac{1}{\eta-1}} \tag{29a}$$

$$Y = N \tag{29b}$$

$$1 = C^\sigma Y^\varphi \left[(1-\alpha) + \alpha S^{1-\eta}\right]^{\frac{1}{1-\eta}} (1-\tau)\mathcal{M} \tag{29c}$$

$$C = \mathcal{Q}^{\frac{1}{\sigma}} \tag{29d}$$

$$Y = C\left[(1-\alpha) + \alpha S^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\alpha) + \alpha \mathcal{S}^{\gamma-\eta}\mathcal{Q}^{\eta-\frac{1}{\sigma}}\right] \tag{29e}$$

This system cannot be solved analytically in the general case, however, using the values showed in calibration for $\alpha, \sigma, \eta, \gamma, \epsilon$ and φ and defining the optimal subsidy ($1-\tau = \frac{1}{(1-\alpha)\mathcal{M}}$), as showed in appendix C, we obtain the following steady state values:

Variable	Description	Value
Y	Output	1.13622
C	Consumption	1.07964
C/Y	Consumption-to-GDP Ratio	0.95020
S	Terms of trade	1.13622
NX/Y	New exports in terms of domestic output	0
$(R^4 - 1)$	Real annual interest rate	0.04102

Note that consumption-to-GDP ratio is lower than 1 while net exports is zero, showing an apparent contradiction. However it is justified as domestic output and domestic consumption uses different prices indexes (P_H and P, respectively), then net exports in terms of domestic output is given by $\frac{NX}{Y} = \frac{(Y - C\frac{P}{P_H})}{Y} = 1 - \frac{C}{Y}\frac{P}{P_H} = 1 - \frac{C}{Y}S^{0.4} = 0$.

6 Dynamic Properties

Simulating 1000 samples with 201 periods each we obtained the following dynamic properties:

	Optimal sd%	DI Taylor sd%	CPI Taylor sd%	Peg sd%
Output	0.93	0.67	0.70	0.84
Domestic inflation	0.00	0.27	0.26	0.35
CPI inflation	0.38	0.41	0.27	0.21
Nominal int. rate	0.32	0.40	0.40	0.21
Terms of trade	1.50	1.42	1.33	1.08
Nominal depr. rate	0.95	0.85	0.52	0.00

Note: Sd denotes standard deviation in %

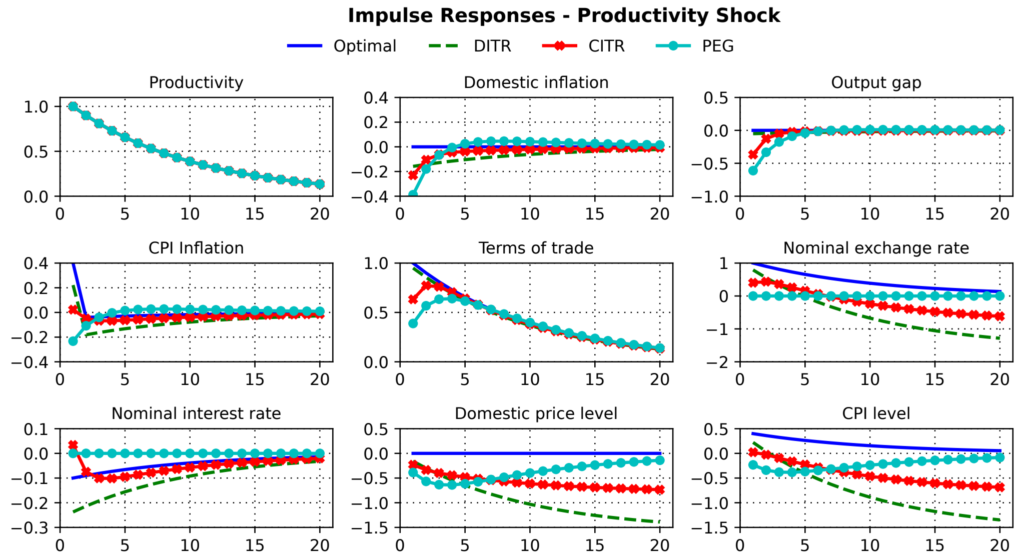
The contribution to welfare losses are:

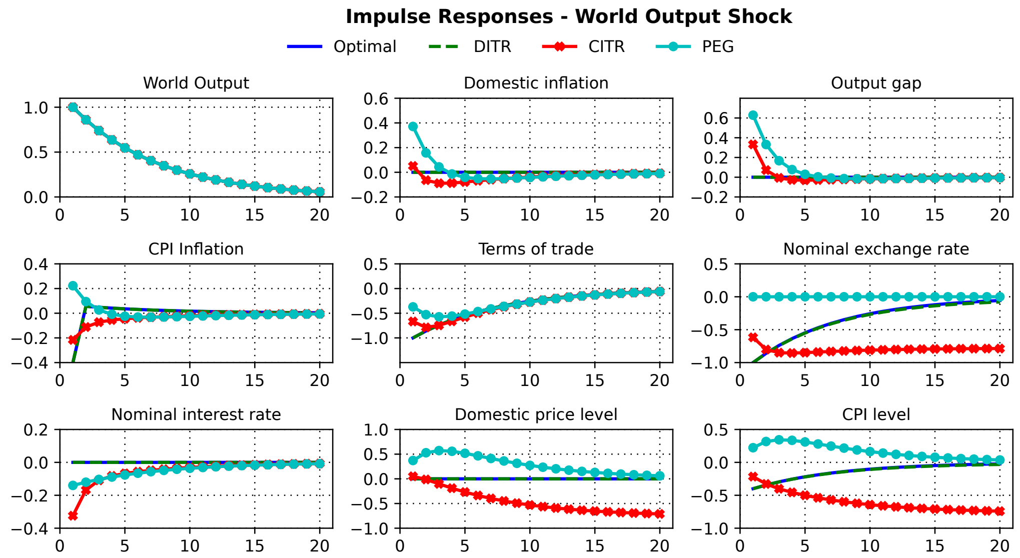
	DI Taylor	CPI Taylor	Peg
Benchmark $\mu = 1.2$, $\varphi = 3$			
Var(Domestic infl.)	0.0151	0.0142	0.0261
Var(Output gap)	0.0009	0.0019	0.0052
Total	0.0160	0.0161	0.0313
Low steady state markup $\mu = 1.1$, $\varphi = 10$			
Var(Domestic infl.)	0.0278	0.0262	0.0478
Var(Output gap)	0.0009	0.0019	0.0052
Total	0.0286	0.0281	0.0529
Low elasticity of labour supply $\mu = 1.2$, $\varphi = 3$			
Var(Domestic infl.)	0.0225	0.0230	0.0551
Var(Output gap)	0.0005	0.0005	0.0063
Total	0.0230	0.0250	0.0614
Low markup and elasticity of labour supply $\mu = 1.1$, $\varphi = 10$			
Var(Domestic infl.)	0.0414	0.0422	0.1013
Var(Output gap)	0.0005	0.0020	0.0063
Total	0.0419	0.0419	0.1076

Note: Values are percentage units of steady state consumption

7 Impulse Response Functions

We show the IRFs relative to both shocks present in the model (TFP and world output shocks) in output gap, domestic and total inflation, terms of trade, nominal exchange rate, nominal interest rate and dometic and total price levels (same charts showed in the original paper).





8 Modification

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9 The new implications for the static and dynamics properties of the model

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10 Conclusion

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A Consumer problem

Consumer problem solution.

B Labor subsidy definition

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C Optimal price setting

The firms optimal price setting problem is given by:

$$\bar{P}_{H,t} = \max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left[Q_{t,t+k} [Y_{t+k}(j)(\bar{P}_{H,t} - MC_{t+k} P_{H,t+k})] \right] \tag{30}$$

The domestic demand for a specific variety is $C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}$ if the price remains unchanged at $\bar{P}_{H,t}$ until $t+k$ period then: $C_{t+k}(j) = \left(\frac{\bar{P}_{H,t}(j)}{P_{H,t+k}}\right)^{-\varepsilon} C_{H,t+k}$. Similarly the foreign consumption of this domestic good is $C^i_{t+k}(j) = \int_0^1 \left(\frac{\bar{P}_{H,t}(j)}{P_{H,t+k}}\right)^{-\varepsilon} C^i_{H,t+k} di$. Market clearing imposes that:

$$Y_{t+k}(j) = C_{H,t+k}(j) + \int_0^1 C^i_{H,t+k}(j) di = \left(\frac{\bar{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} \left(C_{H,t+k} + \int_0^1 C^i_{H,t+k} di \right) \equiv \left(\frac{\bar{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} \tilde{C}_{H,t+k} \tag{31}$$

Substituting $Q_{t,t+k}$ for the expression obtained in the consumer problem and $Y_{t+k}(j)$ for the expression above in the firms problem:

$$\bar{P}_{H,t} = \max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left[\beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+k}}\right) \left(\frac{\bar{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} \tilde{C}_{H,t+k} (\bar{P}_{H,t} - MC_{t+k} P_{H,t+k}) \right] \tag{32}$$

Calculating the first order condition with respect to $\bar{P}_{H,t}$ and rearranging we get:

$$\bar{P}_{H,t} = \frac{\mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta)^k C_{t+k}^{-\sigma} \frac{1}{P_{t+k}} \tilde{C}_{H,t+k} P_{H,t+k} MC_{t+k} \mathcal{M} \right]}{\mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta)^k C_{t+k}^{-\sigma} \frac{1}{P_{t+k}} \tilde{C}_{H,t+k} \right]} \tag{33}$$

In the zero inflation steady state $\bar{P}_{H,t} = P_{H,t} = P_t = P_H$, implying that, by the previous formula, $MC_t = \frac{1}{\mathcal{M}} \equiv \frac{\varepsilon-1}{\varepsilon}$. Thus we define $\widehat{MC}_t = \frac{MC_t}{1/\mathcal{M}}$ as the marginal cost deviation from steady state. Now using the price dynamics:

$$P_{H,t} = [\theta P_{H,t-1}^{1-\varepsilon} + (1-\theta)\bar{P}_{H,t}^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \Rightarrow \Pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}} = \frac{\left[\theta P_{H,t}^{1-\varepsilon} + (1-\theta)\bar{P}_{H,t}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}{P_{H,t-1}} \tag{34}$$

$$P_{H,t} = [\theta P_{H,t-1}^{1-\varepsilon} + (1-\theta)\bar{P}_{H,t}^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \Rightarrow \Pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}} = \left[\theta + (1-\theta)\frac{\bar{P}_{H,t}^{1-\varepsilon}}{P_{H,t-1}^{1-\varepsilon}}\right]^{\frac{1}{1-\varepsilon}} \tag{35}$$

Substituing $\bar{P}_{H,t}$ we reach:

$$\Pi_{H,t} = \left[\theta + \frac{(1-\theta)}{P_{H,t-1}^{1-\varepsilon}}\left(\frac{\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\theta)^k C_{t+k}^{-\sigma}\frac{1}{P_{t+k}}\tilde{C}_{H,t+k}P_{H,t+k}\widehat{MC}_{t+k}\right]}{\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\theta)^k C_{t+k}^{-\sigma}\frac{1}{P_{t+k}}\tilde{C}_{H,t+k}\right]}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{36}$$

Log-linearizing we get:

$$\frac{\pi_{H,t} + p_{H,t-1}}{(1-\theta)(1-\beta\theta)} = \sum_{k=0}^{\infty}(\beta\theta)^k \mathbb{E}_t[\hat{mc}_{t+k} + p_{t+k}] \tag{37}$$

To obtain a recursive form, subtract for the same expression in $t+1$ multiplied by β, apply the Law of Iterated Expectations and rearrange:

$$\frac{\pi_{H,t} - \beta\mathbb{E}_t[\pi_{H,t+1}]}{(1-\theta)(1-\beta\theta)} = \sum_{k=0}^{\infty}(\beta\theta)^k \mathbb{E}_t[\hat{mc}_t + p_{t+k}] - \beta\mathbb{E}_t\sum_{k=0}^{\infty}(\beta\theta)^k\mathbb{E}[\hat{mc}_{t+1+k} + p_{t+k+1}] \tag{38}$$

$$\pi_{H,t} = \beta\mathbb{E}_t[\pi_{H,t+1}] + \lambda\hat{mc}_t \tag{39}$$

Where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$.

D Goods market clearing

Market clearing in goods market imposes that, for each domestic good, the total production is equal to domestic + external demands:

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} + \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\int_0^1 C_{H,t}^i di \tag{40}$$

Substituing $C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t$ and $C_{H,t}^i = \alpha\left(\frac{P_{H,t}}{\mathcal{E}_{i,t}P_{F,t}^i}\right)^{-\gamma}\left(\frac{P_{F,t}^i}{P_t^i}\right)^{-\eta} C_t^i$

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left((1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha\int_0^1\left(\frac{P_{H,t}}{\mathcal{E}_{i,t}P_{F,t}^i}\right)^{-\gamma}\left(\frac{P_{F,t}^i}{P_t^i}\right)^{-\eta} C_t^i di\right) \tag{41}$$

As $Y_t = \left[\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$

$$Y_t = \left\{\int_0^1 \left[\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left((1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t di\right)\right]^{\frac{\varepsilon-1}{\varepsilon}} dj\right\}^{\frac{\varepsilon}{\varepsilon-1}}$$

$$Y_t = \left\{\int_0^1 \left(P_{H,t}(j)^{-\varepsilon}\right)^{\frac{\varepsilon-1}{\varepsilon}} dj\right\}^{\frac{\varepsilon}{\varepsilon-1}} \left(\frac{1}{P_{H,t}}\right)^{-\epsilon} \left((1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t di\right) \tag{42}$$

Considering that $\left[\int_0^1 \left(P_{H,t}(j)^{-\varepsilon}\right)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}} = \left[\int_0^1 P_{H,t}^{1-\varepsilon}(j) dj\right]^{\frac{\varepsilon}{\varepsilon-1}} = P_{H,t}^{-\epsilon}$ we get:

$$\begin{aligned} Y_t &= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t di \\ Y_t &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha) C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{\mathcal{E}_{i,t} P^i_{F,t}}{P_{H,t}}\right)^{-\eta} \left(\frac{P_t}{\mathcal{E}_{i,t} P^i_t}\right)^{-\eta} C^i_t di\right] \end{aligned} \tag{43}$$

$$Y_t = C_t \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha) + \alpha \int_0^1 \left(\mathcal{S}^i_t \mathcal{S}_{i,t}\right)^{\gamma-\eta} \mathcal{Q}_{i,t}^{\eta-\frac{1}{\sigma}} di\right] \tag{44}$$

And finally, using (9) we obtain:

$$Y_t = C_t \left[(1-\alpha) + \alpha S_t^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\alpha) + \alpha \int_0^1 \left(\mathcal{S}^i_t \mathcal{S}_{i,t}\right)^{\gamma-\eta} \mathcal{Q}_{i,t}^{\eta-\frac{1}{\sigma}} di\right] \tag{45}$$

E Optimal Monetary Policy and Welfare Losses

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