Small_Open_Economy_Model

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2022-09-06

R Markdown

Problem of the consumer:

$$\underset{C_t,N_t}{max}\ E_0 \sum_{t=0}^{\infty} \beta^t U(C_t,N_t) = \underset{C_{H,t},C_{F,t},N_t}{max}\ E_0 \sum_{t=0}^{\infty} \beta^t U\left(\left[(1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, N_t\right)$$

subject to the budget constraint (specified below), where

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}},\ C_{F,t} \equiv \left(\int_0^1 (C_{i,t})^{\frac{\gamma-1}{\gamma}} di\right)^{\frac{\gamma}{\gamma-1}} \text{ and } C_{i,t} \equiv \left(\int_0^1 C_{i,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$$

Substituting, we get

$$\underset{C_{H,t},C_{F,t},N_t}{max}\ E_0 \sum_{t=0}^{\infty} \beta^t U\left(\left[(1-\alpha)^{\frac{1}{\eta}}\left[\left(\int_0^1 C_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}\right]^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}\left[\left(\int_0^1 (C_{F,t})^{\frac{\gamma-1}{\gamma}} di\right)^{\frac{\gamma}{\gamma-1}}\right]^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, N_t\right) =$$

$$\underset{C_{H,t},C_{i,t},N_t}{max}\ E_0 \sum_{t=0}^{\infty} \beta^t U\left(\left[(1-\alpha)^{\frac{1}{\eta}}\left[\left(\int_0^1 C_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}\right]^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}\left[\left(\int_0^1 \left(\left(\int_0^1 C_{i,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}\right)^{\frac{\gamma-1}{\gamma}} di\right)^{\frac{\gamma}{\gamma-1}}\right]^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, N_t\right)$$

subject to the budget constraint:

$$\int_0^1 P_{H,t}(j)C_{H,t}(j)dj + \int_0^1\int_0^1 P_{i,t}(j)C_{i,t}(j)dj\ di + \mathbb{E}_t\{Q_{t,t+1}D_{t+1}\} \leq D_t + W_tN_t + Tt$$

As the budget constraint is binding, otherwise the consumer could spend more resources on more consumption and would not being optimizing his (or her consumption)

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U\left(\left[(1-\alpha)^{\frac{1}{\eta}}\left[\left(\int_0^1 C_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}\right]^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}\left[\left(\int_0^1 (C_{i,t})^{\frac{\gamma-1}{\gamma}} di\right)^{\frac{\gamma}{\gamma-1}}\right]^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, N_t\right) + \lambda_t \left(D_t + W_tN_t + Tt - \int_0\right.\right.$$

The first order condition (FOC) for $C_{H,t}(j)$ is:

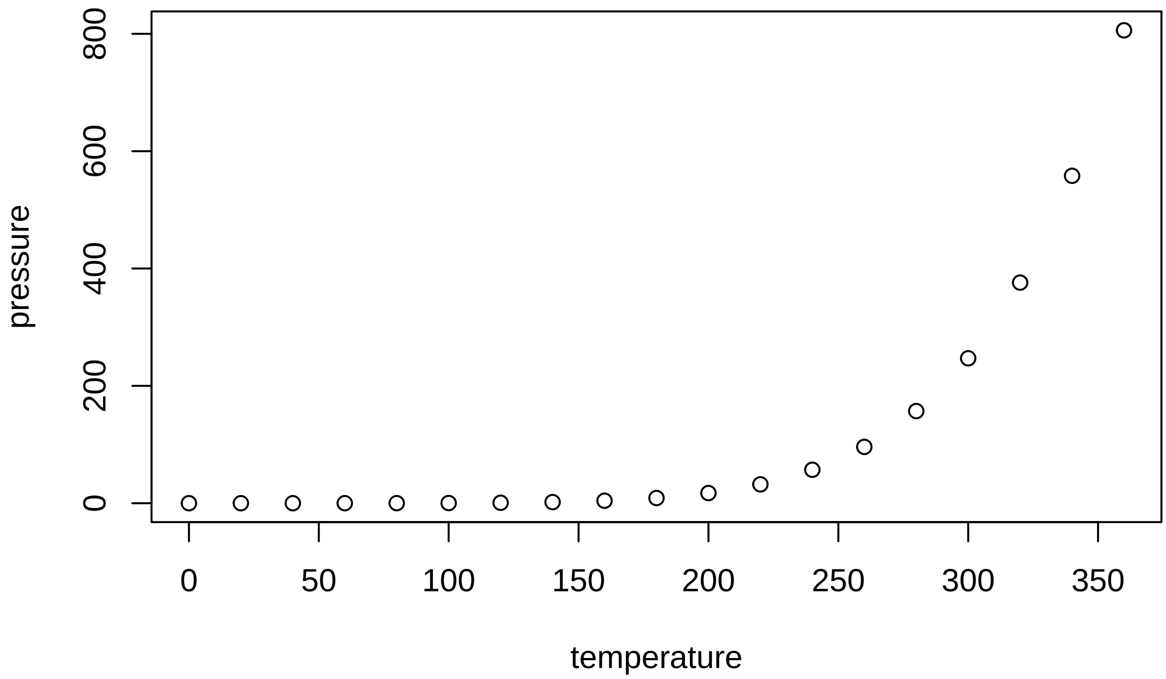
$$U_c(C_t,N_t)\frac{\eta}{1-\eta}\left(C_t^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}-1}(1-\alpha)^{\frac{1}{\eta}}\frac{\eta-1}{\eta}(C_{H,t})^{-\frac{1}{\eta}}\frac{\epsilon}{\epsilon-1}\left(C_{H,t}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}-1}\int_0^1\frac{\epsilon-1}{\epsilon}C_{H,t}(j)^{-\frac{1}{\epsilon}}dj = \lambda_t\int_0^1 P_{H,t}(j)dj$$

```
summary(cars)
```

```
##      speed           dist       
##  Min.   : 4.0   Min.   :  2.00  
##  1st Qu.:12.0   1st Qu.: 26.00  
##  Median :15.0   Median : 36.00  
##  Mean   :15.4   Mean   : 42.98  
##  3rd Qu.:19.0   3rd Qu.: 56.00  
##  Max.   :25.0   Max.   :120.00
```

Including Plots

You can also embed plots, for example:



Note that the `echo = FALSE` parameter was added to the code chunk to prevent printing of the R code that generated the plot.