The Typer Piper: Automating Data Structure Transformations Through Type Chaining

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Abstract

We propose a system for automating multi-step type transformations in MIT Scheme. After registering a set of predicates and transformations between them, the programmer can request data be transformed into a given predicate, or set of predicates. The system then performs a backtracking search to find an appropriate type conversion flow, and can either perform the conversion or return generated conversion code. This eliminates much of the boilerplate and mental overhead involved in many software systems dealing with data of different types.

1 Introduction

Much of programming involves manipulation and transformation of data into different types. Internet connected systems serialize HTTP data into some format, then translate to a JSON type and finally to a normal data structure. An image processing system may convert images from a filename to a JPEG type to a RGB type to a gray-scale image. A command line program might parse a command line input, to a split list, to a data-structure with `ints` and `strings` and `flags`, and finally to commands to be executed.
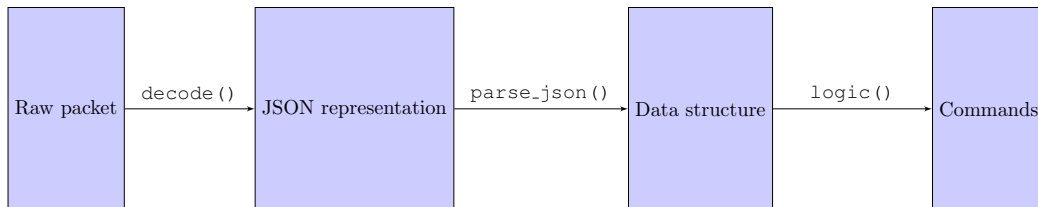

Figure 1: A type conversion diagram for a webserver.

Languages with type annotations or inference often have the information necessary to infer these flows, but instead force the programmer to explicitly notate the conversion process. This leads to more code and more thinking, which we believe to be generally worse than less code and less thinking.

To remedy this, we propose a system capable of automatically generating these conversion flows. To achieve this, we built the following functionality in MIT Scheme, which we'll elaborate on in later sections:

1. A method of registering predicates: functions that match a certain type.

2. A method of registering sub-types of predicates.

3. Support for "compound predicates," groups of element-wise predicates that match lists.

4. A method of registering conversions between predicates, building up a "predicate conversion graph" where nodes represent predicates, and edges represent transformations on predicates and an accompanying transformation on data that matches the start predicate.

5. A search engine that, given an input predicate, explores the predicate conversion graph to find paths to a given output predicate.

6. A programmer-facing API for explicitly converting data, exploring conversion paths, and generating type conversion code.

We'll demonstrate how this system provides an extremely flexible basis for automatic type conversion using multiple examples.

2 Type System

Trivially, any system capable of finding type transformations must have a type system, where the inputs and outputs of functions are labeled with a given type. We chose to use predicates to represent our types: functions that return true if the input matches the type. This provides us with a simple and flexible type system. We then provide functionality to *register* predicates by adding them as nodes to our predicate conversion graph, such that they can be used by the system.
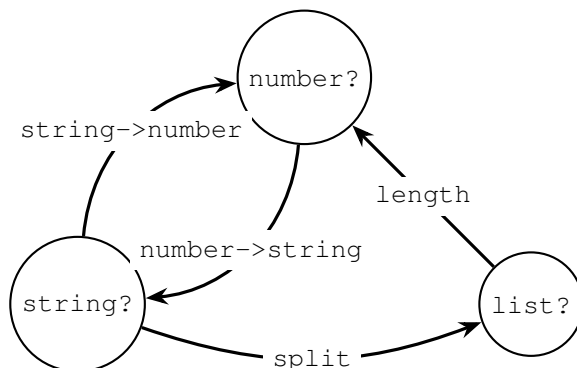
Figure 2: An example predicate conversion graph.

In this graph, edges represent transformations between predicates. However, unlike what the figure above may suggest, our system's type conversion graph is dynamic; rather than storing edges as having a source and a sink, edges have a source and a predicate transformation function, which given the source returns a sink. This allows fancy inferred conversions using dynamic types, such as doubling lists of arbitrary lengths.

In addition to the predicate transformation graph, we also store a graph of predicate subtypes and super-types. We chose to use a separate data structure for this (rather than just creating identity transformations in the predicate conversion graph) in order to allow using super-type transformations without losing information from the original predicate. For example, in the list doubling example (explained in greater detail below), imagine that a `list-len-2?` predicate were transformed into a `length-list?` predicate so that it could be doubled. Doing so would lose the information that the list is of length two, forcing the search to occur on the actual data rather than just on the

predicates. This would be far more computationally intensive and could lead to negative side effects from actually executing transformations during the search without the programmer's permission.

3 Search Engine

Given the previously defined predicate conversion graph, conversion paths between simple predicates such as `string?`, `number?`, and even `list-len-2?` can be found with a basic backtracking search. Introducing subtypes only slightly increases complexity by forcing the search to consider edges not only from the source node, but also from any supertype nodes.

However, if we want to allow even more complex type chaining, we should be able to operate not just on predicates, but on groups of predicates, or "compound predicates". For example, if we define a transformation from (`number?` `number?`) to `point?`, our search engine should be able to convert (`string?`, `string?`) to `point?` (by first separately converting each string to a number). We call these operations on compound predicates *compound transformations*.

Finally, in addition to being able to manipulate and transform compound predicates, we should also be able to branch transformations to form them. Using our `point?` example again, if we've defined `point:x` and `point:y` as transformations from `point?` to `number?`, we should be able to infer a path from `point?` to (`number?` `number?`). We call these operations of branching into compound predicates *joiner transformations*.

Our search engine supports both compound, joiner, and normal transformations.

3.1 Handling of compound transformations

Our standard search process works by querying the predicate transformation graph to find all possible transformations, filtering out predicates already visited in the current type flow, then recursing to try each transformation. We extend this to compound transformations by considering each possible transformation of each predicate (or none), and taking the crossproduct of the predicate transformations to get each possible transformation for the compound predicate.

$$(\texttt{string?} \quad \texttt{number?}) \rightarrow \begin{bmatrix} \texttt{string?} \\ \texttt{number?} \\ \texttt{list?} \end{bmatrix} \times \begin{bmatrix} \texttt{number?} \\ \texttt{string?} \end{bmatrix} \rightarrow \begin{matrix} (\texttt{string?} \quad \texttt{number?}) \\ (\texttt{string?} \quad \texttt{string?}) \\ (\texttt{number?} \quad \texttt{number?}) \\ \dots \\ (\texttt{list?} \quad \texttt{string?}) \end{matrix}$$

Figure 3: Transformations of (`string?` `number?`) using the Figure 2 graph.

We can now filter out already visited transformations, and recurse on the remaining transformations to explore paths they may be a part of.

3.2 Handling of joiner transformations

As discussed above, we'd like to be able to branch predicates into compound predicates. One motivating example for this is the `person?` record type, with attributes `first-name` and `last-name`, each with corresponding predicates. We'd like to be able to automatically convert a `person?` into a (`first-name?` `last-name?`) (and then automatically create a `full-name?` from the

`(first-name? last-name?))`. To do so, we would need two paths of execution, one generating the `(first-name?)` and the other the `(last-name?)`, and then a joiner that combines the output of these execution paths into a compound predicate.

Here's how we handle these joiner transformations:

1. Find all possible compound predicate targets or sub-targets by looking at the conversion target and at all compound predicates registered in the predicate conversion graph.

2. As we search, store all the predicates we've been able to create and the paths that we took to get there.

3. After every time we arrive at a new predicate, check if we can now create a new type of compound predicate. If so, create a joiner that transform that stores the sub-paths for creating each of the sub-prediactes in the compound-predicate, and start a new search from this intermediate compound predicate to the target predicate.

This method is less efficient than, for example, working backwards from the target predicate. However, it is an extensible modification to the original search process, and is likely asymptotically equivalent.

4 Path Visualization and Code Generation

Our search engine ouputs a list of structures that transform the input predicate into the desired output predicate. In most cases, this structure is a simple path where each transform function can be applied to the output of the previous transform. For example, requesting a transformation from `number?` to `symbol?` is a list of transforms `'(number->string string->symbol)`. (This example is simplified - our actual path data structure stores full transform datastructures that include the desired input-predicate and the predicate-transform function in addition to the data-transform function).

In cases with joiner transforms, our generated datastructure is a tree of paths. Each leaf of our tree is a transform that separately takes in the full input. Nodes with multiple inputs are joiner transforms that take the output of each path and combine it into a compound predicate.

Code generation proceeds by simply traversing over this datastructure to build up a LISP expression. Our engine knows the right type of code to generate for each type of transform. Normal transforms use simple function application. Compound transforms generate a map that separately applies each transform function to the input list (see the "compound transform" example below). Joiner transforms separately compute each sub-path and passing them into the transform function as a list of arguments.

5 API Reference

Programmers can interact with our system through the following method calls. More complete examples can be found in our codebase, linked to in Appendix A.

> `(register-predicate! predicate)`: Registers a given predicate by adding it to the predicate conversion graph.

   Example: `(register-predicate! number?)`

> (register-super! subpredicate superpredicate): Registers a predicate as a sub-predicate of another predicate.

Example:
```
(define (is-three? num) (eq? num 3))
(register-super! is-three? number?)
```

> (register-type-transform! input-type output-type transformation-function): Registers a function as a transformation between two registered predicates.

Example: (register-type-transform! number? string? number->string)

> (register-type-transform-f! input-type predicate-transform-function transformation-function): Registers a function as a transformation from the input predicate, into a new predicate generated by a predicate transform function. See example-list-types.scm for an example of where this could be useful.

Example:
```
(register-type-transform! number? (lambda (input-predicate) string?)
number->string)
```

> (get-transformations input-predicate output-predicate): Returns a list of transformations from the input predicate to the output predicate.

Example: (get-transformations string? number?)

> (create-compound-transformation transformation-path): Given a transformation path from get-transformations, return a method that transforms according to that path.

Example:
```
((create-compound-transformation (car
(get-transformations number? string?))) 5)
```

> (transform-with-first-path input-predicate output-predicate input-value): Find the first path between the input and output predicates, and transform the input accordingly.

Example: (transform-with-first-path number? string? 5)

> (debug-transform input-predicate output-predicate input-value): Return a list of all possible transformations of input-value, while printing to the console the transformation paths, intermediate values, and autogenerated code.

Example: (debug-transform number? string? 5)

> (debug-transform-to input-value output-predicate): Infer the input type, and return all possible transformations to the output-predicate while printing path information to the console.

Example: (debug-transform-to 5 string?)

5

6 Examples

Our attached code provides examples of our system in action. We highlight some examples here.

6.1 Compound Predicates

As a simple but illustrative example of compound predicates, imagine trying to convert (number?, string?) to (number?, number?). We presumably already have the following type conversion defined at a system level:

```
(register-type-transform! string? number? string->number)
```

Then, running the search automatically generates code for us:

```
(debug-transform (list number? string?) (list number? number?) (list 1 "2"))

;;(define (call f in) (f in))
;;(define (identity x) x)
;;(define (n:number?n:string?-to-n:number?n:number? input)
;;  (map
;;    call
;;    (list number->string identity)
;;    input))
```

6.2 Record Types

We explore an example where we have a "database" of records of people. We represent this in scheme with a record type for Person, and auxiliary record types for FullName and FormalTitleName (i.e. Professor Sussman).

```
(define (first-name? x) (printable-string? x))
(define (last-name? x) (printable-string? x))
(define (formal-title? x) (printable-string? x))
(define (person:age? x) (number? x))

(define-record-type Person
  (make-person first-name last-name age)
  person?
  (first-name person:first-name) ; first-name?
  (last-name person:last-name) ; last-name?
  (age person:age)) ; person:age?

(define-record-type FullName
  (make-full-name first-name last-name)
  full-name?
  (first-name full-name:first-name) ; first-name?
  (last-name full-name:last-name)) ; last-name?

(define-record-type FormalTitleName
  (make-formal-title-name formal-title last-name)
  formal-title-name?
```

```
    (formal-title formal-title-name:formal-title) ; formal-title?
    (last-name formal-title-name:last-name)) ; last-name?

;; ... boilerplate registering predicates for each of the above record types ...
;; (In a production-ready version of our systems, we would have a macro that
;; automatically registers these predicates for record types without
;; requiring any effort from the user other than specifying predicate names
;; for each field as we did in comments above.)

(define (first-name-to-formal-title fn)
  (if (is-female-first-name? fn) "Mrs." "Mr."))

(register-type-transform!
  first-name?
  formal-title?
  first-name-to-formal-title)

(register-type-transform!
  formal-title-name?
  printable-string?
  (lambda (ftn)
    (string-append
     (formal-title-name:formal-title ftn)
     " "
     (formal-title-name:last-name ftn)))  )
```

Now that we've defined our record types, we can automatically begin to search for transformations between them. For example, we can request a transformation from person? to full-name?.

```
(define gs (make-person "Gerald" "Sussman" 18))
(debug-transform person? full-name? gs)
;; One path gets returned, and the following code snippet is automatically generated.
;; Note that this transformation involves automatically identifying that the first-name?
;; and last-name? can be separately read out of the input and composed into
;; (first-name?, last-name?) which can then be transformed into a full-name?
;; (define (person?-to-full-name? input)
;;   (make-full-name (person:first-name input) (person:last-name input)))
```

For a more complicated example, we can automatically search for ways to print out a person:

```
(define gs (make-person "Gerald" "Sussman" 18))
(debug-transform person? full-name? gs)
;; Multiple possible transformations are found, resulting in the following values:
;; "Gerald Sussman", "Mr. Sussman", "Sussman", and "Gerald".
;; For example, the following code-snippet is generated to create "Mr. Sussman":
;;(define (person?-to-printable-string? input)
;;   (print-formal-title-name
;;       (make-formal-title-name
;;         (first-name-to-formal-title (person:first-name input))
;;         (person:last-name input))))
```

Note that generating this example required a few steps of translation:

1. Generate first-name? from person.

2. Generate last-name? from person.

3. Generate formal-title? from first-name?

4. Use a joiner transform to generate a compound predicate (formal-title?, last-name?) by combing the paths to formal-title? and last-name?

5. Generate formal-title-name? by passing in the (formal-title?, last-name?) compound predicate as a list of arguments to the formal-title-name constructor

6. Generate a printable-string? from the formal-title-name?

6.3 Dynamically Generated Predicates

Imagine that we want to add a transform that takes a list of a given length and outputs a list of twice the length by duplicating every item. We then want to be able to request generated code that takes a list of length two and returns a list of length 8. For our transform to be fully general, we need to be able to create list predicates that specify the length of our list. Then, we need to be able to compute the output type of our transform from the input type.

To support this functionality, our search engine allows the output-type of a transform to be computed from the input type. We then implement the above example as follows. Note how we dynamically generate a new list predicate from the input list predicate.

```
(define (duplicate-items-in-list lst)
  (apply append (list lst lst)))

(register-type-transform-f!
  length-list?
  (lambda (input_type)
  (generate-list-predicate
      (* 2 (get-list-predicate-length input_type)))))
duplicate-items-in-list)
```

To support this functionality, the programmer had to define some a generator for predicates of lists that specify their length:

```
;; a length-list is just a list whose type describes its length
(define (length-list? x) (list? x))

(register-predicate! length-list?)
(register-super! length-list? list?)

;; generate a new length-list? predicate with a specific length

(define generate-list-predicate
  (memoize
    (lambda (length)
      (define (list-predicate-with-length? item)
        (and
          (length-list? item)
          (= length item)))
```

```
        (register-list-predicate-length! list-predicate-with-length? length)
        (register-super! list-predicate-with-length? length-list?)

        list-predicate-with-length?)))

;; hash table for storing lengths associated with length-list? predicates
(define list-predicate-lengths (make-strong-eq-hash-table))

(define (register-list-predicate-length! list-predicate-with-length? length)
  (hash-table-set! list-predicate-lengths list-predicate-with-length? length))

(define (get-list-predicate-length list-predicate-with-length?)
  (hash-table/get list-predicate-lengths list-predicate-with-length? -1))
```

We can now request a transformation from a list of length 2 to a list of length 8. Note that the search engine realizes it can apply the this transformation to list-len-2? because each generated predicate is marked as a subtype of list-len?. Also note that search engine performs this transformation in a manner that doesn't lose information about the list's length (for example, we could not first convert list-len-2? to length-list? because we would lose the knowledge that len=2).

```
(define list-len-2? (generate-list-predicate 2))
(define list-len-8? (generate-list-predicate 8))

(debug-transform
  list-len-2?
  list-len-8?
  (list 2 3))

;; "Found 1 paths: "
;; "------"
;; "Code Gen:"
;; (define (list-predicate-with-length?-to-list-predicate-with-length? input)
;;   (duplicate-items-in-list (duplicate-items-in-list input)))
;; "Output value:"
;; (2 3 2 3 2 3 2 3)
```

7   Discussion

A common goal of programming languages is to minimize the amount of code that needs to be written. The programmer should be able to specify what they want as tersely as possible, and the computer should do the rest. We propose one such mechanism for allowing computers and languages to infer large components of programs, by focusing on types and conversion flows between them. Changing one's perspective to view more of computation as type conversion provides a framework from which more of the program can be easily inferred by a smart compiler.

Another interesting aspect of this project is the collaborative nature between our system and the programmer. By being less verbose in our programming, we force the system to ask us clarifying questions; in this case, which type flow to pick. We think this is a necessary trade-off for more advanced programming environments, and possibly a useful direction for future research.

9

We'd also like to note that many of the ideas in this paper are not wholly original. Our type inference is similar to Prolog's logic-based programming. Scala has "implicit conversions," which allows the definition of automatic conversions between types (but not chaining without significant extra work).

A   Github Repo Link

All of our code (including the LaTeX source for this writeup) can be found at the following link:

https://github.com/mfranzs/6905-final-project

We also provide a hard copy in the following pages.

B   Source Code

B.1   main.scm

```scheme
;; =============
;; =============
;; The Type Transformation Search Engine
;; =============
;; =============

;; =============
;; Type Transform Graph
;; =============

;; Association list of input-predicate -> '(
;;    (predicate-transformation . transformation)
;;    ...
;; )
(define %transform-graph)

(define (reset-transform-graph!)
  (set! %transform-graph (make-alist-store equal?)))

(reset-transform-graph!)

(define (add-to-transform-graph!
          input-predicate
          predicate-transformation
          transformation)
  (register-predicate! input-predicate)
  (let*
    ((existing-transforms (get-predicate-transforms input-predicate))
     (new-transform
       (make-transform input-predicate predicate-transformation transformation))
     (new-transforms (cons new-transform existing-transforms)))
    ((%transform-graph 'put!) input-predicate new-transforms)))

(define (register-predicate! predicate)
  (if (not (predicate? predicate))
      (begin
```

```scheme
        ((%transform-graph 'put!) predicate '())
        ((%supertype-graph 'put!) predicate '()))))

(define (predicate? function)
  ((%transform-graph 'has?) function))

(define (get-predicate-transforms predicate)
  (if (predicate? predicate)
      ((%transform-graph 'get) predicate)
      '()))

(define (register-type-transform! input-predicate output-predicate
                                  transformation)
  (assert (or (predicate? input-predicate) (list? input-predicate))
          input-predicate)
  (assert (or (predicate? output-predicate) (list? output-predicate)))
  (add-to-transform-graph! input-predicate
                           (lambda (x) output-predicate)
                           transformation))

(define (register-type-transform-f!
          input-predicate
          output-predicate
          transformation)
  (add-to-transform-graph! input-predicate
                           output-predicate
                           transformation))

(define (pred-to-string predicate)
  (if (list? predicate)
      (apply string-append (map pred-to-string predicate))
      (symbol->string (get-name predicate))))

(define (all-predicates)
  ((%transform-graph 'get-keys)))

(define (all-compound-predicates)
  (filter list? (all-predicates)))

;; ==============
;; Supertypes
;; ==============

;; Association list of input-predicate -> '(
;;   predicate-supertype?
;; )
(define %supertype-graph)

(define (reset-supertype-graph!)
  (set! %supertype-graph (make-alist-store equal?)))

(reset-supertype-graph!)

(define (register-super! predicate-sub predicate-super)
  (register-predicate! predicate-sub)
```

```
  (register-predicate! predicate-super)

  ((%supertype-graph 'put!) predicate-sub
   (cons predicate-super (get-predicate-supers predicate-sub))))

(define (get-predicate-supers predicate)
  ((%supertype-graph 'get-default) predicate '()))

;; ==============
;; Transforms
;; ==============

;; Transforms translate a value of type input-predicate to a new type

;; There are three types of transforms:
;; 1. Normal transforms
;; 2. Compound transforms (a list of transforms that transform a list
;; of values in parallel)
;; 3. Joiner transforms (a transform that takes a list of paths from
;; the input to intermediate predicates and joins them into a list)

;; Normal transforms are stored as (cons transform-input-predicate-to-output-fn
;; transform-data-fn). Note transforms can be a compound list of transforms to
;; apply to a list of predicates.

(define (make-transform input-predicate predicate-transformation transformation)
  (cons input-predicate (cons predicate-transformation transformation)))

;; Joiner transforms just store their compound predicate and their
;; list of paths

(define (make-joiner-transform compound-predicate paths-list)
  (cons 'joiner (cons compound-predicate paths-list)))

(define (joiner-transform-output-predicate transform)
  (cadr transform))

(define (joiner-transform-paths-list transform)
  (cddr transform))

(define (is-joiner-transform? transform)
  (equal? (car transform) 'joiner))

(define (is-compound-transform? transform)
  (and (not (is-joiner-transform? transform)) (list? transform)))

(define (transformation-input-predicate transformation)
  (if (is-compound-transform? transformation)
      (map transformation-input-predicate transformation)
      (car transformation)))

;; Returns a function that transforms the input-predicate to the output
(define (transformation-predicate-transform transformation)
  (lambda (in)
    (cond
```

```
      ((is-joiner-transform? transformation)
       (joiner-transform-output-predicate transformation))
      ((is-compound-transform? transformation)
       (assert (list? in) in)
       (assert (= (length in) (length transformation)))
       (map
         (lambda (value transform)
           ((transformation-predicate-transform transform) value))
         in
         transformation))
      (else
        ((cadr transformation) in)))))

;; Returns a function that transforms the input-value with the given
;; transformation
(define (transformation-data-transform transformation)
  (cond
    ((is-joiner-transform? transformation)
     (lambda (in)
       (map
         (lambda (path)
           ((create-compound-transformation path) in))
         (joiner-transform-paths-list transformation))))
    ((is-compound-transform? transformation)
     (lambda in
       (map
         (lambda (value transform)
           ((transformation-data-transform transform) value))
         in
         transformation)))
    (else
      (cddr transformation))))

;; Applies this transformation to the given in-value.
(define (apply-transformation-data-transform transformation in-value)
  (let ((dt-fn (transformation-data-transform transformation)))
    (if (list? (transformation-input-predicate transformation))
        (apply dt-fn in-value)
        (dt-fn in-value))))

(define identity-transform (make-transform always-true identity identity))

;; ==============
;; Paths
;; ==============

;; A path is a list (or a tree) of transforms that takes the input and
;; transforms it to the output.
;; (A path can be a tree if it has a joiner transform. In that case,
;; each of the leafs takes in the input).

(define (remove-from-path-before-joiner path)
  (let ((reversed-path (reverse path)))
    (define (recurse-build remaining-reversed-path built-path)
      (if (null? remaining-reversed-path)
```

```scheme
          built-path
          (let ((transform (car remaining-reversed-path)))
            (if (is-joiner-transform? transform)
                (cons transform built-path)
                (recurse-build
                  (cdr remaining-reversed-path)
                  (cons transform built-path))))))))
    (recurse-build reversed-path '()))))

(define (create-compound-transformation path)
  (if (null? path)
      identity
      (let ((transform-rest-of-path
              (create-compound-transformation (cdr path)))
            (transform (car path)))
        (if (is-joiner-transform? transform)
            (lambda (in)
              (transform-rest-of-path
                (map
                  (lambda (joiner-sub-path)
                    ((create-compound-transformation (reverse joiner-sub-path)) in))
                  (joiner-transform-paths-list transform))))
            (lambda (in)
              (transform-rest-of-path
                (apply-transformation-data-transform
                  transform in)))))))

(define (codegen path input-predicate output-predicate)
  (list
    'define
    (list (string->symbol (string-append
                            (pred-to-string input-predicate)
                            "-to-"
                            (pred-to-string output-predicate)))
          'input)
    (codegen-inner (reverse path))))

(define (codegen-inner path)
  (if (null? path)
      'input
      (let ((transform (car path)))
        (cond
        ((is-compound-transform? transform)
          ;; If it's a compound transform, we need to separately apply
          ;; each sub-transform.
          (list
          'map
          'call
          (cons
            'list
            (map
            (lambda (sub-transform)
              (get-name (transformation-data-transform sub-transform)))
            transform))
          (codegen-inner (cdr path)))))
```

```
        ((and (> (length path) 1) (is-joiner-transform? (cadr path)))
         ;; If our next thing is a joiner transform, we want to take the output of that transfor
         ;; and directly use them as the arguments to this function.
         ;; NOTE: We know this transform doesn't take a compound transform as input.
         (cons
           (get-name (transformation-data-transform transform))
           (map
             (lambda (joiner-sub-path)
               (codegen-inner joiner-sub-path))
             (joiner-transform-paths-list (cadr path)))))
        ((is-joiner-transform? transform)
         ;; If it's a joiner transform, we want to separately compute the output
         ;; of each subpath. Note that we only get here if we have a path o nothing but a
         ;; joiner trahsform, so we just want to output the answer as a list.
         (cons
           'list
           (map
             (lambda (joiner-sub-path)
               (codegen-inner joiner-sub-path))
             (joiner-transform-paths-list transform))))
        (else
         ;; If it's a normal transform, we just apply the transform function.
         (list
           (get-name (transformation-data-transform transform))
           (codegen-inner (cdr path))
           )))))

;; Visualizes the transforms involved in the path
(define (visualize-transformation-transforms path)
  (if (null? path)
      (lambda (x) '())
      (let ((transform-rest-of-path
             (visualize-transformation-transforms (cdr path)))
            (transform (car path)))
        (if (is-joiner-transform? transform)
            (lambda (in)
              (list
                (map
                  (lambda (joiner-sub-path)
                    ((visualize-transformation-transforms
                      (reverse joiner-sub-path)) in))
                  (joiner-transform-paths-list transform))
                (transform-rest-of-path
                  (map
                    (lambda (joiner-sub-path)
                      ((create-compound-transformation (reverse joiner-sub-path)) in))
                    (joiner-transform-paths-list transform)))))
            (lambda (in)
              (cons
                (get-name (transformation-data-transform transform))
                (transform-rest-of-path
                  (apply-transformation-data-transform
                    transform in))))))))))

;; Visualizes the intermediate predicates we reach in the path
```

```scheme
(define (visualize-transformation-predicates path)
  (if (null? path)
      (lambda (x) '())
      (let ((transform-rest-of-path
              (visualize-transformation-predicates (cdr path)))
            (transform (car path)))
        (if (is-joiner-transform? transform)
            (lambda (in)
              (list
                (map
                  (lambda (joiner-sub-path)
                    ((visualize-transformation-predicates
                      (reverse joiner-sub-path)) in))
                  (joiner-transform-paths-list transform))
                (transform-rest-of-path
                  (map
                    (lambda (joiner-sub-path)
                      ((create-compound-transformation (reverse joiner-sub-path)) in))
                    (joiner-transform-paths-list transform)))))
            (lambda (in)
              (cons
                (get-name ((transformation-predicate-transform transform)
                            (transformation-input-predicate transform)))
                (transform-rest-of-path
                  (apply-transformation-data-transform
                    transform in)))))))))

;; Visualizes the intermediate values we reach while performing a transformation along the path
(define (visualize-transformation-values path)
  (if (null? path)
      identity
      (let ((transform-rest-of-path
              (visualize-transformation-values (cdr path)))
            (transform (car path)))
        (if (is-joiner-transform? transform)
            (lambda (in)
              (list
                (map
                  (lambda (joiner-sub-path)
                    ((visualize-transformation-values
                        (reverse joiner-sub-path)) in))
                  (joiner-transform-paths-list transform))
                (transform-rest-of-path
                  (map
                    (lambda (joiner-sub-path)
                      ((create-compound-transformation (reverse joiner-sub-path)) in))
                    (joiner-transform-paths-list transform)))))
            (lambda (in)
              (cons
                in
                (transform-rest-of-path
                  (apply-transformation-data-transform
                    transform in)))))))))

;; ==============
```

```
;; Search Engine
;; =============
;; The core search engine.

(define (all-transforms-for-compound-predicate input-predicate)
  (assert (list? input-predicate))
  (crossproduct
    (map
      (lambda (pred) (cons
                       identity-transform
                       ;; Note we don't add the reached-predicates table
                       ;; here so it doesn't try to make nested compound
                       ;; predicates
                       (all-transforms-for-predicate pred
                                                     (make-equal-hash-table)
                                                     (list))))
      input-predicate)))

;; Find compound-predicates that we can make by using our
;; input-predicate at least once and filling the rest of the slots
;; with things from reached-predicates
(define (all-valid-compound-predicates input-predicate reached-predicates)
  (filter
    (lambda (compound-predicate)
      (and (member input-predicate compound-predicate)
           (every
             (lambda (sub-predicate)
               (or (equal? sub-predicate input-predicate)
                   (hash-table/get reached-predicates sub-predicate #f)))
             compound-predicate)))
    (all-compound-predicates)))

;; Find all compound predicates we could make using a our input-predicate and filling
;; the compound-predicates other slots from things in reached-predicates,
;; then generate transforms for each of those compound predicates.
(define (all-joiner-transforms input-predicate reached-predicates
                               path-so-far)
  (flatten-one-layer
    (map (lambda (compound-predicate)
           (map (lambda (paths-list)
                  (make-joiner-transform
                    compound-predicate
                    paths-list))
                ;; Find all possible paths we can combine to form this
                ;; compound-predicate
                (crossproduct
                  (map
                    (lambda (sub-predicate)
                      (if (equal? sub-predicate input-predicate)
                          (list path-so-far)
                          (hash-table-ref reached-predicates sub-predicate)))
                    compound-predicate))
                ))
         (all-valid-compound-predicates input-predicate reached-predicates))))
```

```scheme
(define (all-transforms-for-predicate input-predicate
                                       reached-predicates path-so-far)
  (append
    (if (list? input-predicate)
        (all-transforms-for-compound-predicate input-predicate)
        '())
    (get-predicate-transforms input-predicate)
    (all-joiner-transforms input-predicate reached-predicates path-so-far)
    (flatten-one-layer (map get-predicate-transforms
                            (get-predicate-supers input-predicate)))))

;; Computes all output predicates for these transforms.
(define (apply-all-transforms-to-predicate input-predicate transforms)
  (map
    (lambda (transform)
      ((transformation-predicate-transform transform) input-predicate))
    transforms))

(define (predicate-equal-or-supertype? pred target-pred)
  (or
    (equal? pred target-pred)
    ;; Or are any of the supertypes equal to target-pred?
    (any
      (lambda (super-pred)
        (predicate-equal-or-supertype? super-pred target-pred))
      (get-predicate-supers pred)
      )))

(define MAX_SEARCH_DEPTH 10)

(define (get-transformations-internal input-predicate output-predicate
                                       path-so-far reached-predicates
                                       seen-predicates)
  (if (> (length path-so-far) MAX_SEARCH_DEPTH)
      (list)
      (append
        ;; If we've hit the goal, add a "termination" to our path list, but
        ;; also keep search in case we're only actually at a subtype of our goal
        (if (predicate-equal-or-supertype? input-predicate
                                           output-predicate)
            (list (list)) ;; Valid path with no more transforms needed
            (list))
        (let*
          ((transforms (all-transforms-for-predicate
                         input-predicate reached-predicates path-so-far))
           (transform-intermediates
             (apply-all-transforms-to-predicate input-predicate
                                                transforms)))
          (hash-table-set!
            reached-predicates
            input-predicate
            (cons path-so-far (hash-table/get reached-predicates
                                              input-predicate '())))

          (write "Search reached" (get-name input-predicate)
```

```
                       "after␣#steps=␣" (length path-so-far))

               (map
                 remove-from-path-before-joiner
                 (flatten-one-layer
                   ;; Loop over each of the intermediate transforms and find all
                   ;; paths from there
                   (map
                     (lambda (intermediate-pred transformation)
                       ;; Check we haven't already been to this predicate
                       (if (member intermediate-pred seen-predicates)
                           '()
                           (let ((new-path-so-far
                                   (if (is-joiner-transform? transformation)
                                       (list transformation)
                                       (cons transformation path-so-far))))
                             ;; Recursively find all paths from this
                             ;; intermediate-predicate to the end-predicate
                             (map (lambda (path) (cons transformation path))
                                   (get-transformations-internal
                                     intermediate-pred
                                     output-predicate
                                     new-path-so-far
                                     reached-predicates
                                     (cons intermediate-pred seen-predicates))))))
                     transform-intermediates
                     transforms)))))))

(define (get-transformations input-predicate output-predicate)
  (get-transformations-internal input-predicate output-predicate '()
                                 (make-equal-hash-table) (list input-predicate)))

;; ==============
;; Visualizing Transformations
;; ==============

(define (debug-transform input-predicate
                                          output-predicate
                                          input-value)
  (write-line "")
  (write "********************")
  (write "********************")
  (write "Attempting␣to␣transform" (get-name input-predicate) "to"
         (get-name output-predicate) "and␣showing␣with␣value" input-value)
  (let ((paths (get-transformations input-predicate output-predicate)))
    (write "Found" (length paths) "paths:")
    (for-each (lambda (path)
                (visualize-path
                  path
                  input-predicate
                  output-predicate
                  input-value)) paths)))

(define (visualize-path path input-predicate output-predicate input-value)
  (write-line "------")
```

```scheme
  (write-line "Code Gen:")
  (pp (codegen path input-predicate output-predicate))

  (write-line "Output value:")
  (write-line ((create-compound-transformation path) input-value)))

  ; (write-line "Transforms:")
  ; (pp ((visualize-transformation-transforms path)
  ;      input-value))

  ; (write-line "Predicates:")
  ; (pp ((visualize-transformation-predicates path) input-value))

  ; (write-line "Values:")
  ; (pp ((visualize-transformation-values path) input-value)))


(define (transform-with-first-path input-predicate output-predicate input-value)
  ((create-compound-transformation
     (car (get-transformations input-predicate output-predicate)))
   input-value))

(define (debug-transform-to input-value output-predicate)
  (write-line "")
  (write "*********************")
  (write "********************")
  (write "Attempting to transform" input-value "to" (get-name output-predicate))
  (let*
    ((matching-predicates
       (filter (lambda (pred) (pred input-value)) (all-predicates)))
     (paths-by-predicate
       (map
         (lambda (input-predicate)
           (get-transformations input-predicate output-predicate))
         matching-predicates))
     (all-paths (flatten-one-layer paths-by-predicate)))

    (write "Found" (length all-paths) "paths:")
    (for-each
     (lambda (paths input-predicate)
       (for-each
         (lambda (path) (visualize-path
                          path
                          input-predicate
                          output-predicate
                          input-value))
         paths))
     paths-by-predicate
     matching-predicates)))

'loaded-type-search-engine-successfully
```

B.2   helpers.scm

```
(define (call f x) (f x))

(define (two-crossproduct a b)
  (flatten-one-layer
     (map
      (lambda (a-el)
         (map
            (lambda (b-el) (cons a-el b-el))
           b))
      a)))

(define (crossproduct lists)
  (fold-right two-crossproduct (list (list)) lists))

(define (identity x) x)

(define (flatten-one-layer list-of-lists) (apply append list-of-lists))

(define (get-name f)
  (if (equal? (get-name-proc f) '|#[unnamed-procedure]|)
      (get-name-search f)
      (get-name-proc f)))

(define (get-name-proc f)
  (if (list? f)
      (string->symbol (apply string-append (map (compose symbol->string get-name) f)))
      (car (lambda-components* (procedure-lambda f) list))))

(define (get-name-search f)
  (if (list? f)
    (map get-name f)
    (let ((matches (filter
        (lambda (el) (and (> (length el) 1) (eq? (car (cdr el)) f)))
        (environment-bindings user-initial-environment))))
      (if (>= (length matches) 1)
        (car (car matches))
        'missing-name-in-get-name-lookup))))


(define write
  (lambda args
    (write-line
     (apply string-append
            (map (lambda (x) (string-append (string x) "␣")) args)))))

(define (reverse items)
  (fold-right (lambda (x r) (append r (list x))) '() items))

(define (boolean->string val) (if val "#t" "#f"))

(define (alist:keys alist) (map car alist))

(define (always-true x) #t)
```

B.3   memoize.scm

```scheme
;; ===================
;; Helper for easily memoizing functions
;; ===================

(define (memoize function)

  ;; Hash table mapping arguments -> result
  (define stored-evaluations (make-equal-hash-table))

  (define memoize-inner
    (lambda args
      (hash-table/lookup
       stored-evaluations
       args
       (lambda (found-value) found-value)
       (lambda ()
         ;; Stored value not found
         (let ((result (apply function args)))
           (hash-table-set! stored-evaluations args result)
           result)))))

  memoize-inner)

;; Tests:

(define (test x)
  (write-line (list "Evaluate" x))
  x)

(define test-memoized (memoize test))

(test-memoized 1)                        ; -> Prints ("Evaluate" 1)
(test-memoized 1)                        ; -> Immediately returns 1
(test-memoized 2)                        ; -> Prints ("Evaluate 2)
```

B.4   example-basic.scm

```scheme
;; ===================
;; Basic examples
;; ===================

(load "load.scm")
(load "main.scm")

(register-predicate! list?)
(register-predicate! number?)
(register-predicate! string?)
(register-predicate! symbol?)

(define (is-three? num) (eq? num 3))

(register-predicate! is-three?)
(register-super! is-three? number?)
```

```scheme
(register-type-transform! list? number? length)
(register-type-transform! number? string? number->string)
(register-type-transform! string? number? string->number)
(register-type-transform! string? symbol? string->symbol)

; (debug-transform number? string? 1)

; (debug-transform list? string? '(1 2 3))

; (debug-transform is-three? string? 3)

; (debug-transform-to 3 string?)

(debug-transform number? symbol? 1)
```

B.5 example-compound-types.scm

```scheme
;; ===================
;; Examples with compound types
;; ===================

(load "load.scm")
(load "main.scm")

;; ===================
;; Example of applying transformations in parallel on a compound type.
;; ===================

(register-predicate! list?)
(register-predicate! number?)
(register-predicate! string?)

(register-type-transform! string? number? string->number)

(debug-transform (list number? string?) (list number? number?) (list 1 "2"))

;; ===================
;; Example of a transformation whose input is a compound predicate.
;; ===================

; (register-predicate! pair?)

; (register-type-transform! (list number? number?) pair? cons)

; (debug-transform (list number? number?) pair? (list 1 1))

;; ===================
;; Example of auto-broadcasting to a compound predicate.
;; ===================

;; (define (thing? x) (string? x))

;; (register-predicate! number?)
;; (register-predicate! thing?)
```

```scheme
;; (register-predicate! string?)
;; (register-predicate! pair?)

;; (define (add-three n) (number->string (+ 3 n)))

;; (register-type-transform! number? string? number->string)
;; (register-type-transform! number? thing? add-three)
;; (register-type-transform! (list thing? string?) pair? cons)

;; (debug-transform number? (list thing? string?) 1)
```

B.6   example-list-types.scm

```scheme
;; ===================
;; List predicates with certain lengths
;; ===================

(load "load.scm")
(load "main.scm")
(load "memoize.scm")

;; a length-list is just a list whose type describes its length
(define (length-list? x) (list? x))

(register-predicate! length-list?)
(register-super! length-list? list?)

;; generate a new length-list? predicate with a specific length

(define generate-list-predicate
  (memoize
   (lambda (length)
     (define (list-predicate-with-length? item)
       (and
        (length-list? item)
        (= length item)))

     (register-list-predicate-length! list-predicate-with-length? length)
     (register-super! list-predicate-with-length? length-list?)

     list-predicate-with-length?)))

;; hash table for storing lengths associated with length-list? predicates
(define list-predicate-lengths (make-strong-eq-hash-table))

(define (register-list-predicate-length! list-predicate-with-length? length)
  (hash-table-set! list-predicate-lengths list-predicate-with-length? length))

(define (get-list-predicate-length list-predicate-with-length?)
  (hash-table/get list-predicate-lengths list-predicate-with-length? -1))

;; ===================
;; Example: duplicating list length
;; ===================
```

```scheme
(define (duplicate-items-in-list lst)
  (apply append (list lst lst)))

(register-type-transform-f!
          length-list?
     (lambda (input_type)
             (generate-list-predicate
        (* 2 (get-list-predicate-length input_type))))
          duplicate-items-in-list)

(define list-len-2? (generate-list-predicate 2))
(define list-len-8? (generate-list-predicate 8))

(debug-transform
 list-len-2?
 list-len-8?
 (list 2 3))

;; ==================
;; Example: Points
;; ==================

; (define point? (generate-list-predicate 2))

; (define (add-elements-in-lists list-a list-b)
;    (map (lambda (a b) (+ a b)) list-a list-b))

; (duplicate-items-in-list (list 1 2))

; (register-type-transform!
;      length-list?
;          (lambda (input_type) input_type)
;          add-elements-in-listsx'xxl)

; (define list-len-2? (generate-list-predicate 2))
; (define list-len-4? (generate-list-predicate 4))

; (debug-transform
;  list-len-2?
;  list-len-4?
;   (list 2 3))
```

B.7 example-record-types.scm

```scheme
;; ==================
;; Example of Transforming Scheme Record-Types
;; ==================

(load "load.scm")
(load "main.scm")

(define (printable-string? x) (string? x))
(register-predicate! printable-string?)
(register-super! string? printable-string?)
```

```scheme
;; ====================
;; Example: Printing data about a person
;; ====================

(define (first-name? x) (printable-string? x))
(define (last-name? x) (printable-string? x))
(define (formal-title? x) (printable-string? x))
(define (person:age? x) (number? x))

;; Define the Person record type
(define-record-type Person
  (make-person first-name last-name age)
  person?
  (first-name person:first-name)
  (last-name person:last-name)
  (age person:age))

(register-predicate! person?)

(register-predicate! first-name?)
(register-super! first-name? printable-string?)
(register-predicate! last-name?)
(register-super! last-name? printable-string?)
(register-predicate! person:age?)
(register-super! person:age? integer?)

(register-type-transform! person? first-name?
              person:first-name)
(register-type-transform! person? last-name?
              person:last-name)
(register-type-transform! person? person:age?
              person:age)

;; Define the FullName record type
(define-record-type FullName
  (make-full-name first-name last-name)
  full-name?
  (first-name full-name:first-name)
  (last-name full-name:last-name))

(register-predicate! full-name?)

(register-type-transform! full-name? first-name? full-name:first-name)
(register-type-transform! full-name? last-name? full-name:last-name)
(register-type-transform! (list first-name? last-name?) full-name? make-full-name)
(define (print-full-name fn)
    (string-append
      (full-name:first-name fn)
      " "
      (full-name:last-name fn)))
(register-type-transform!
  full-name?
  printable-string?
  print-full-name )
```

```scheme
;; Define the FormalTitleName record type
(define-record-type FormalTitleName
  (make-formal-title-name formal-title last-name)
  formal-title-name?
  (formal-title formal-title-name:formal-title)
  (last-name formal-title-name:last-name))

(register-predicate! formal-title-name?)
(register-predicate! formal-title?)

(register-type-transform! formal-title-name? formal-title? formal-title-name:formal-title)
(register-type-transform! formal-title-name? last-name? formal-title-name:last-name)
(register-type-transform! (list formal-title? last-name?) formal-title-name? make-formal-title-na

;; Define a transformation from FirstName to FormalTitle
(define (is-female-first-name? x) #f) ;; In a real system, we would look this up in a database...

(define (first-name-to-formal-title fn)
  (if (is-female-first-name? fn) "Mrs." "Mr."))

(register-type-transform!
  first-name?
  formal-title?
  first-name-to-formal-title)

;; Define a transformation from FormalTitleName to PrintableString
(define (print-formal-title-name ftn)
  (string-append
    (formal-title-name:formal-title ftn)
    " "
    (formal-title-name:last-name ftn)))

(register-type-transform!
  formal-title-name?
  printable-string?
  print-formal-title-name )


;; Tests

(define gs (make-person "Gerald" "Sussman" 18))
(define fullname (make-full-name "Gerald" "Sussman"))

(debug-transform person? first-name? gs)
;; "Found 1 paths: "
;; "------"
;; "Code Gen:"
;; (define (person?-to-first-name? input)
;;    (person:first-name input))
;; "Output value:"
;; "Gerald"
;; "Transforms:"
;; (person:first-name)
;; "Predicates:"
```

```
;; (first-name?)
;; "Values:"
;; (#[person 12] . "Gerald")
;; ""

(debug-transform person? full-name? gs)
;; "Found 2 paths: "
;; "------"
;; "Code Gen:"
;; (define (person?-to-full-name? input)
;;   (make-full-name (person:first-name input) (formal-title-name:last-name (make-formal-title-na
;; "Output value:"
;; #[fullname 13]
;; "Transforms:"
;; (((person:first-name) (((person:first-name first-name-to-formal-title) (person:last-name)) (ma
;; "Predicates:"
;; (((first-name?) (((first-name? formal-title?) (last-name?)) (formal-title-name? last-name?)))
;; "Values:"
;; (((#[person 12] . "Gerald") (((#[person 12] "Gerald" . "Mr.") (#[person 12] . "Sussman")) (("M
;; "------"
;; "Code Gen:"
;; (define (person?-to-full-name? input)
;;   (make-full-name (person:first-name input) (person:last-name input)))
;; "Output value:"
;; #[fullname 16]
;; "Transforms:"
;; (((person:first-name) (person:last-name)) (make-full-name))
;; "Predicates:"
;; (((first-name?) (last-name?)) (full-name?))
;; "Values:"
;; (((#[person 12] . "Gerald") (#[person 12] . "Sussman")) (("Gerald" "Sussman") . #[fullname 17]
;; ""


(debug-transform person? printable-string? gs)
;; Outputs values including "Gerald", "Sussman", "Mr. Sussman", and "Gerald Sussman"
```