BLG453E COMPUTER VISION

Fall 2017 Term

Istanbul Technical University
Computer Engineering Department

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Week 1-2 Topics and Learning Objectives:
Introduction to Computer Vision
Pointwise Image Processing
Image Intensity Transformations, Image Histograms,
Image Enhancement

At the end of Week 1-2: Students will be able to:

1. Discuss the main problems of computer (artificial) vision, its uses and applications
2. Design and implement various image transforms: point-wise transforms, neighborhood operation-based spatial filters, and geometric transforms over images

What is an image?

An image is multi-dimensional signal that measures a physical quantity.

Multi-dimensional

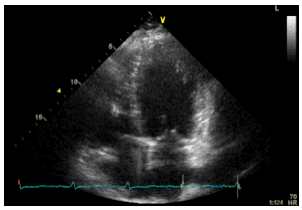
- 2D: Image (as a function of *x* and *y*)
- 3D: Video (as a function of *x*, *y*, and *t*)
- (others, e.g. CT, MRI)

Physical quantity

- Typically brightness for standard photographs
- Many other possibilities (temperature, depth, acoustic properties, etc.)

Digital photo

Thermal image



Ultrasound video

Multi-dimensional function

Mathematically, an image is a multi-dimensional function. For example, a grayscale image can be expressed a mapping from the set of real numbers in 2D space to the set of real numbers (brightness).

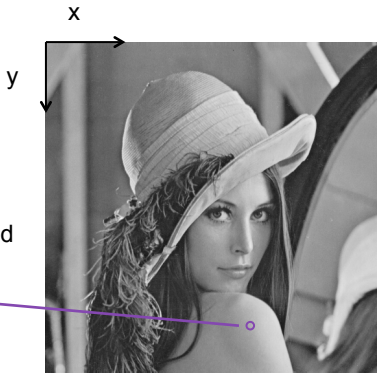
We could write

$$I : \mathbb{R}^2 \rightarrow \mathbb{R}$$

Or simply

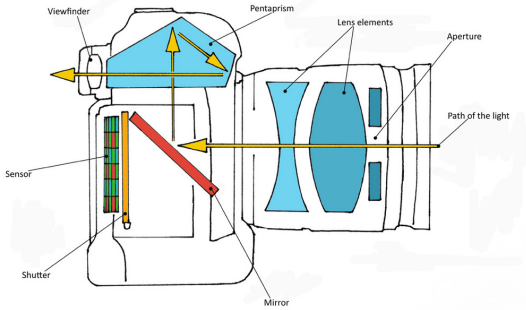
$$I(x, y)$$

Where *x* and *y* are the spatial variables, and *I* is the intensity



Camera

Typical components
- Aperture to let light in
- Lens
- Photosensitive image plane
- Housing to keep stray light out



http://murrayparkphotography.weebly.com/inside-a-dslr.html

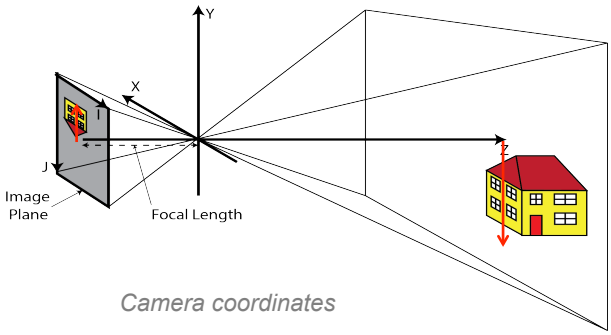
Pinhole camera

- All light passes through a single point, known as the *centre of projection*.
- On the other side of the camera is film that captures striking light.
- This creates a *perspective projection*.

http://petapixel.com/2013/03/26/how-to-create-a-homemade-large-format-pinhole-camera-using-a-shoebox/

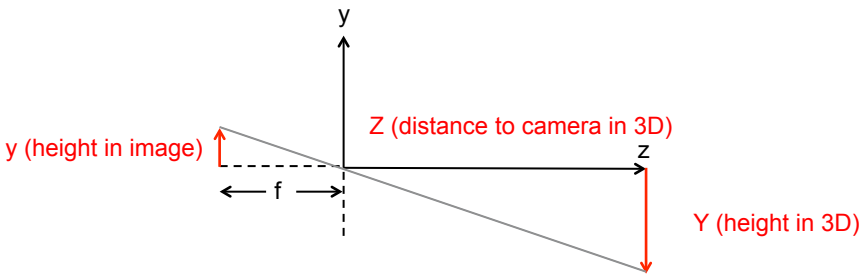
Pinhole camera model

This can be modelled mathematically
- Whilst simple, the model is reasonably realistic
- Deviations occur due to lenses that create distortions



Camera coordinates

Pinhole camera model

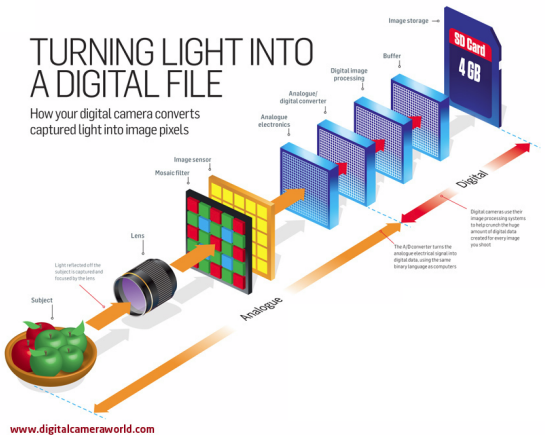


By similar triangles, $\frac{y}{f} = \frac{Y}{Z}$ or $y = \frac{fY}{Z}$

⇨ As Z increases, y decreases (i.e., farther away objects appear smaller!)

From light to pixels

The light impinging on the image sensor is a continuous signal. This is converted to a *digital* signal through *sampling* and *quantisation*, to form a set of *pixels* comprising the image.



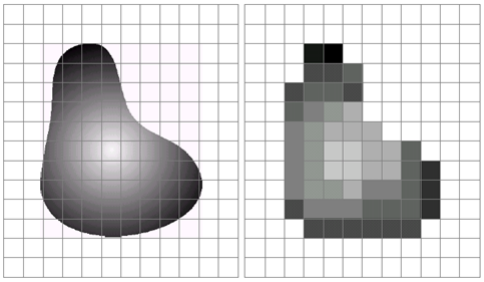
Digital image

A digital image is a collection of pixels, arranged on a 2D grid. Each pixel stores colour information comprising the image. The image *resolution* is the size of the image, in pixels (in width and height).

http://www.g2cs.org/media/interactives/image-compression/00.html

Sampling

Sampling converts a continuous signal into a discrete one. The light impinging the image plane is a continuous signal in *x* and *y*. In a digital camera, this signal is sampled on a regular grid. The sampling rate determines the resolution.



Well-known devices

iPhone 6

Rear **camera**
3264 x 2448 ~ 8 MP

Samsung Galaxy S6

Rear **camera**
5312 x 1988 ~ 16 MP

HDTV

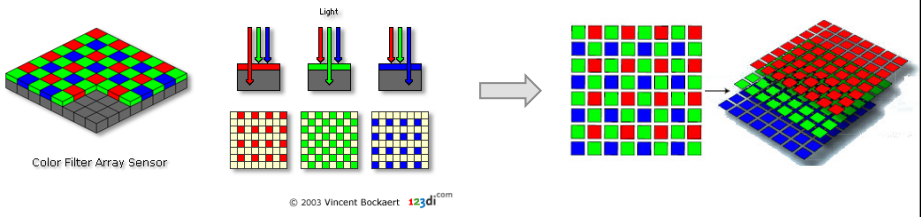
display
1920x1080 ~ 2 MP

Colour filter array and demosaicing

Light captured by digital cameras is usually binned into separate red, green, and blue values using a *colour filter array*, often made of a GRBG pattern called the Bayer pattern. A *demosaicing* algorithm interpolates the data so that each pixel has a red, green, and blue (RGB) value.

Mosaic

Demosaiced image



https://www.youtube.com/watch?v=2-stCNB8jT8

Quantisation

Quantisation limits the values a pixel can have to a finite set. For example, in a greyscale image, one normally uses 8 bits per pixel, so there are $2^8 = 256$ different intensities, normally represented in the range [0, 255].

8 bits per pixel
$2^8 = 256$ shades

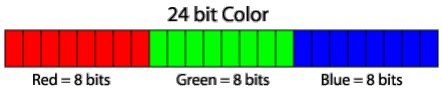
4 bits per pixel
$2^4 = 16$ shades

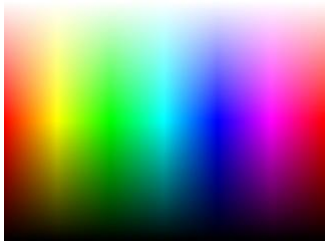
1 bit per pixel
$2^1 = 2$ shades
binary image

https://songbirdsings13.wordpress.com/year-12/new-media/unit-16/pmd1-unit-19/

Quantisation

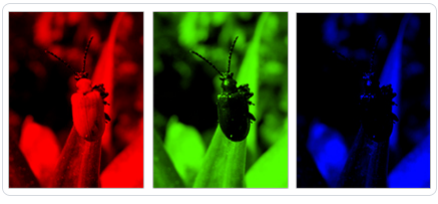
In standard colour photographic images, one uses 8 bits for *each* colour channel (red, green, blue), or 24 bits per pixel. That means there are 2^{24} possible colours for a pixel. This is roughly 16.7 million colours!





Colour

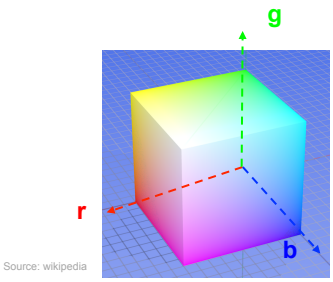
Colour images typically have three colour *channels* corresponding to the amount of red, green, and blue present at each pixel. Combining them together produces the final colour.



http://digital.pho.to/

RGB colour space

- Has its origin in colour television, now used in displays (flat panels, phones)
- *Additive* mixing or red, green, and blue light form the final colour
- RGB is a colour space
  - Red axis, green axis, blue axis
  - Values typically in the range from 0 (none) to 255 (full colour) along each axis
  - A colour is a point in this space, represented as a vector $[r, g, b]^T$

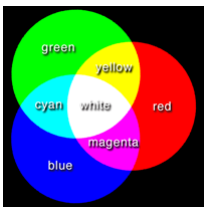


Source: wikipedia

RGB colour

RGB uses additive colour mixing, because it describes what kind of *light* needs to be *emitted* to produce a given colour. Light is added together to create form from out of the darkness.

Colour Component			Colour Common Name
R	G	B	
0	0	0	Black
0	0	255	Blue
0	255	0	Green
0	255	255	Cyan
255	0	0	Red
255	0	255	Magenta
255	255	0	Yellow
255	255	255	White

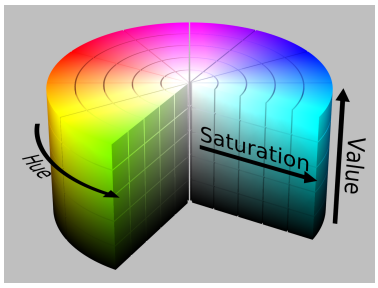


⇨ Of course, one can have values, like [255, 127, 0] -- orange

HSV

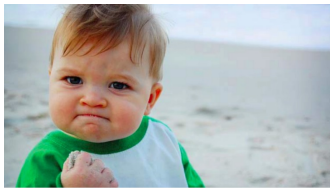
- Transforms the colour space to be more intuitive and perceptually relevant than RGB, using a cylindrical coordinate system.
  - H (hue) corresponds to the discernible colour based on the dominant wavelength
  - S (saturation) corresponds to the vividness of the colour. Low saturation means a grayscale colour in the centre of the cylinder.
  - V (value) corresponds to brightness.

⇨ For example, a bright red colour has a red hue, full saturation, and high value.

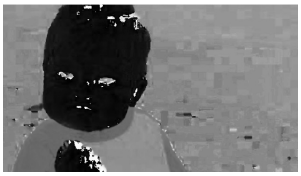


Other color spaces exist such as CIE LAB. In this course, we'll work in RGB color space

HSV



Original image



H

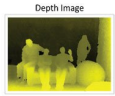
S

V

Other common image types

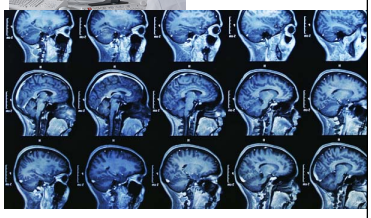
RGBD

- In addition to a colour image (RGB), acquires a depth image (D), which represents the distance from each pixel from the camera



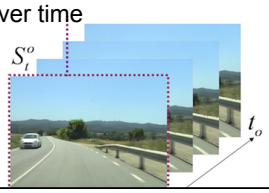
Volumetric images

- Image data stored as voxels (small cubes, or 3D pixels). Examples include computed tomography (CT) or magnetic resonance (MRI)



Video

- A sequence of images over time

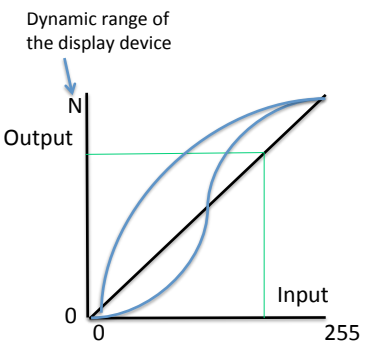


Point Processing of Images

- gamma | - brightness | original | + brightness | + gamma

- In a digital image, point = pixel.
- Point processing transforms a pixel's intensity value as function of its value alone;
- it does not depend on the values of the pixel's neighbors.

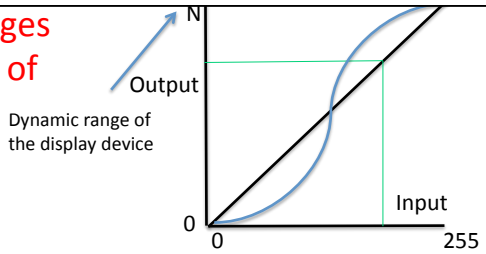
Image: a 2D pattern of intensity values

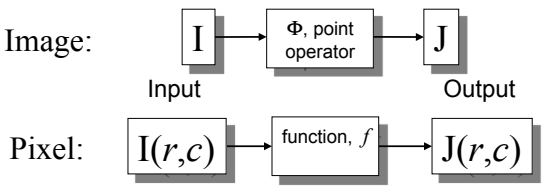


Point Processing of Images

- Brightness and contrast adjustment
- Gamma correction
- Histogram equalization
- Histogram matching

Point Operations on Images via Functional Mappings of intensities





$J = \Phi[I]$

The transformation of image I into image J is accomplished by replacing each input intensity, g, with a specific output intensity, k, at every location (r,c) where $I(r,c) = g$.

If $I(r,c) = g$ and $f(g) = k$ then $J(r,c) = k$.

The rule that associates k with g is usually specified with a function, f, so that $f(g) = k$.

Point Operations on Images via Functional Mappings

One-band Image

$$J(r,c) = f\left(I(r,c)\right),$$
for all pixels locations (r,c).

Three-band Image

$$J(r,c,b) = f\left(I(r,c,b)\right), \text{ or}$$
$$J(r,c,b) = f_b\left(I(r,c,b)\right),$$
for $b = 1,2,3$ and all (r,c).

Point Operations on Images via Functional Mappings

One-band Image

$$J(r,c) = f\left(I(r,c)\right),$$
for all pixels locations (r,c).

Either all 3 bands are mapped through the same function, f, or …

Three-band Image

$$J(r,c,b) = f\left(I(r,c,b)\right), \text{ or}$$
$$J(r,c,b) = f_b\left(I(r,c,b)\right),$$
for $b = 1,2,3$ and all $(r,c$

… each band is mapped through a separate function, f_b.

Point Operations Using Look-up Tables

A look-up table (LUT) implements a functional mapping.

If $k = f(g)$, for $g = 0,\ldots,255$, and if k takes on values in $\{0,\ldots,255\}, \ldots$

… then the LUT that implements f is a 256x1 array whose $(g+1)^{th}$ value is $k = f(g)$.

To remap a 1-band image, I, to J:

$$J = \mathrm{LUT}(I)$$

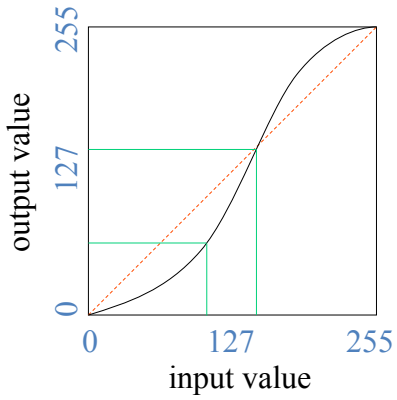
Point Operations Using Look-up Tables

If I is 3-band, then
a) each band is mapped separately using the same LUT for each band *or*
b) each band is mapped using different LUTs – one for each band.

a) $J = \mathrm{LUT}(I)$, *or*

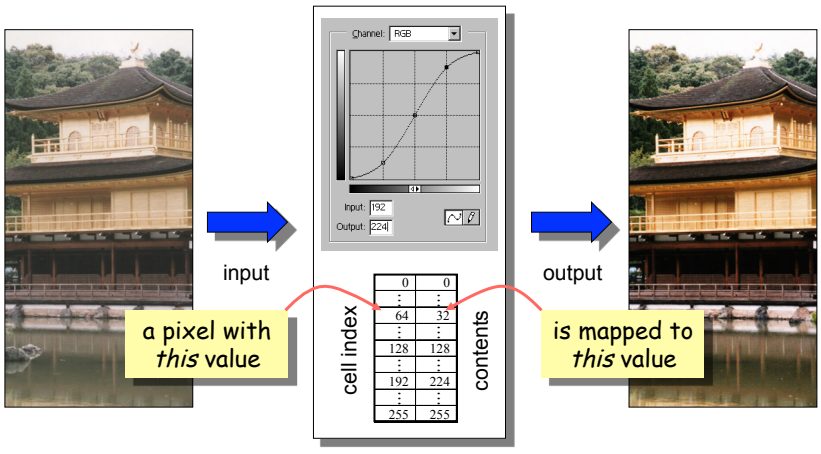
b) $J(:,:,b) = \mathrm{LUT}_b\left(I(:,:,b)\right)$ for $b = 1,2,3.$

Point Operations = Look-up Table Ops



E.g.:	index	value
	...	...
	101	64
	102	68
	103	69
	104	70
	105	70
	106	71
	...	...
	input	output

Look-Up Tables



by Richard Alan Peters II

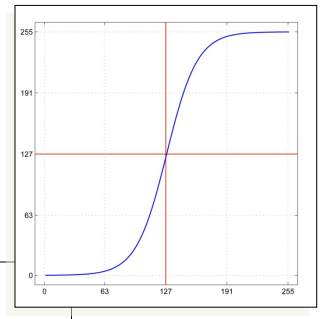
How to Generate a Look-Up Table

For example:

Let $a = 2.$

Let $x \in \{0,\ldots,255\}$

$$\sigma(x;a) = \frac{255}{1+e^{-a(x-127)/32}}$$



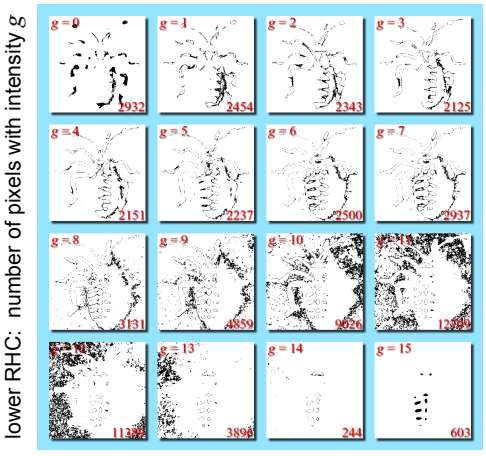
Or in Matlab:

```
a = 2;
x = 0:255;
LUT = 255 ./ (1+exp(-a*(x-127)/32));
```

This is just one example.

The Histogram of a Grayscale Image

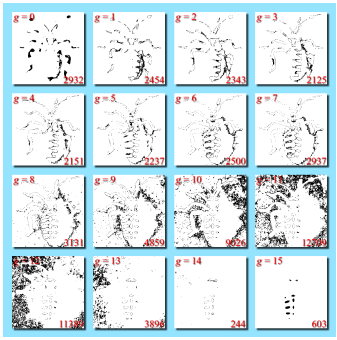
16-level (4-bit) image



The Histogram of a Grayscale Image

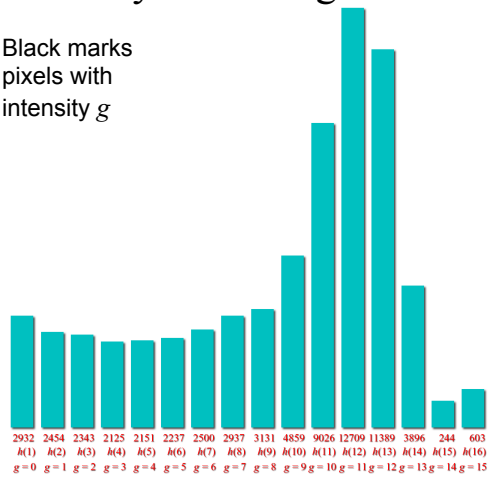
- Let *I* be a 1-band (grayscale) image.
- *I*(*r*,*c*) is an 8-bit integer between 0 and 255.
- Histogram, h_I, of *I:*
  - a 256-element array, h_I
  - $h_I(g)$, for g = 0, 2, 3, …, 255, is an integer
  - $h_I(g)$ = number of pixels in *I* that have value g.

The Histogram of a Grayscale Image



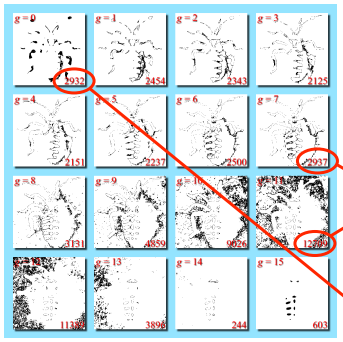
Plot of histogram:
number of pixels with intensity g

Black marks pixels with intensity g



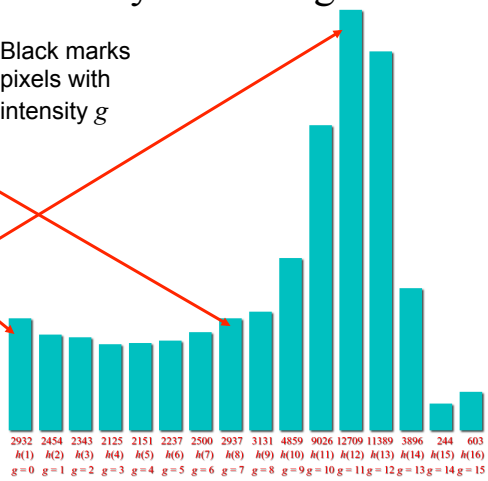
by Richard Alan Peters II

The Histogram of a Grayscale Image



Black marks pixels with intensity g

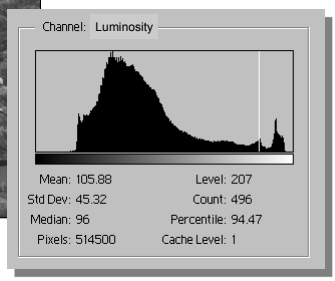
Plot of histogram: number of pixels with intensity g



by Richard Alan Peters II

The Histogram of a Grayscale Image

$h_I(g)$ = the number of pixels in I with graylevel g.



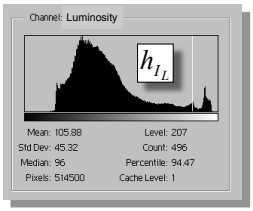
by Richard Alan Peters II

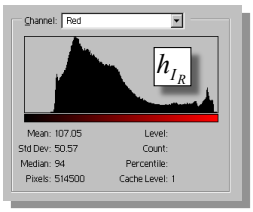
The Histogram of a Color Image

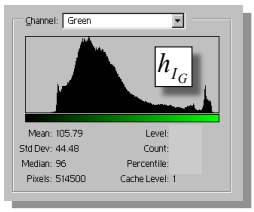
- If I is a 3-band image (truecolor, 24-bit)
- then I(r,c,b) is an integer between 0 and 255.
- Either I has 3 histograms:
  - $h_R(g)$ = # of pixels in I(:,:,1) with intensity value g
  - $h_G(g)$ = # of pixels in I(:,:,2) with intensity value g
  - $h_B(g)$ = # of pixels in I(:,:,3) with intensity value g
- or 1 vector-valued histogram, $h(g,1,b)$ where
  - $h(g,1,1)$ = # of pixels in I with red intensity value g
  - $h(g,1,2)$ = # of pixels in I with green intensity value g
  - $h(g,1,3)$ = # of pixels in I with blue intensity value g

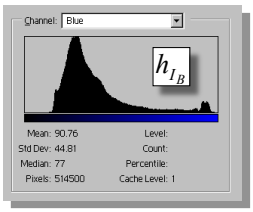
The Histogram of a Color Image

There is one histo-gram per color band R, G, & B. Luminosity histogram is from 1 band = (R+G+B)/3

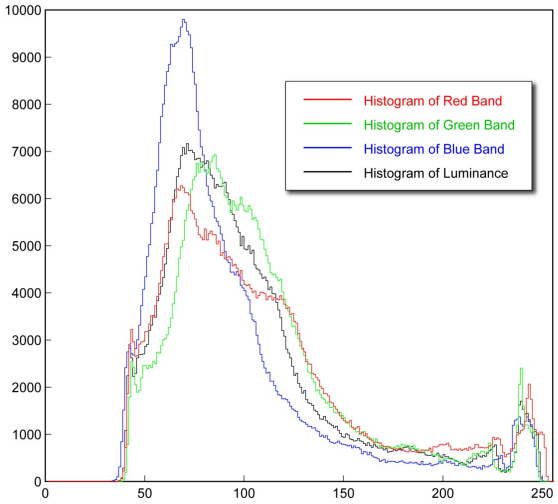








The Histogram of a Color Image



Value or Luminance Histograms

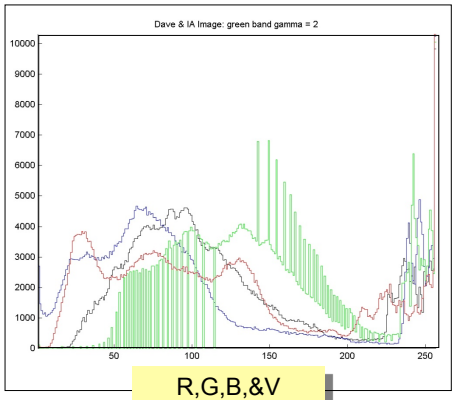
The value histogram of a 3-band (truecolor) image, I, is the histogram of the value image,

$$V(r,c) = \frac{1}{3}\left[R(r,c) + G(r,c) + B(r,c)\right]$$

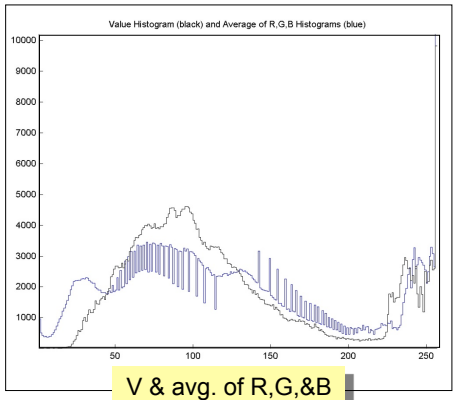
Where R, G, and B are the red, green, and blue bands of I.
The luminance histogram of I is the histogram of the luminance image,

$$L(r,c) = 0.299 \cdot R(r,c) + 0.587 \cdot G(r,c) + 0.114 \cdot B(r,c)$$

Value Histogram vs. Average of R,G,&B Histograms



R,G,B,&V histograms



V & avg. of R,G,&B histograms

Multi-Band Histogram Calculator in Matlab

```
% Multi-band histogram calculator
function h=histogram(I)

[R C B]=size(I);

% allocate the histogram
h=zeros(256,1,B);

% range through the intensity values
for g=0:255
   h(g+1,1,:) = sum(sum(I==g)); % accumulate
end

return;
```

Multi-Band Histogram Calculator in Matlab

```
% Multi-band histogram calc
function h=histogram(I)

[R C B]=size(I);

% allocate the histogram
h=zeros(256,1,B);

% range through the intensi
for g=0:255
   h(g+1,1,:) = sum(sum(I==g)); % accumulate
end
```

Loop through all intensity levels (0-255)
Tag the elements that have value g.
The result is an RxCxB *logical* array that has a 1 wherever $I(r,c,b) = g$ and 0's everywhere else.
Compute the number of ones in each band of the image for intensity g.
Store that value in the 256x1xB histogram at $h(g+1,1,b)$.

sum(sum(I==g)) computes one number for each band in the image.

If B==3, then h(g+1,1,:) contains 3 numbers: the number of pixels in bands 1, 2, & 3 that have intensity g.

The Probability Density Function of an Image

- p_I is the normalized histogram of the image

  A: total number of pixels in the image.

  $$p_I(g) = \frac{1}{A} h_I(g)$$

- $p_I(g)$ is the fraction of pixels in (a specific band of) an image that have intensity value g.
- $p_I(g)$ is the probability that a pixel randomly selected from the given image band has intensity value g.
- Sum of $p_I(g)$ over all g is 1 (whereas the sum of the histogram $h_I(g)$ over all g from 0 to 255 is equal to the number of pixels in the image)

Images and Probability

Parametric pdf's:

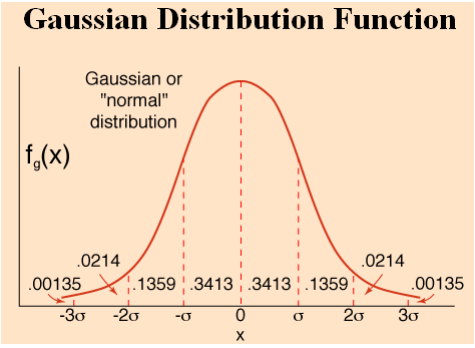
Question: Which is the probability density function that can be described by just 2 parameters, mean and variance?

Images and Probability

The distribution that can be described by just 2 parameters, mean and variance:

Important family of probability distributions: Gaussian

$$\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$



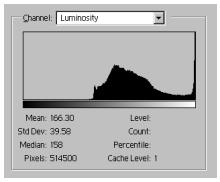
A.k.a. Cumulative Distribution Function.

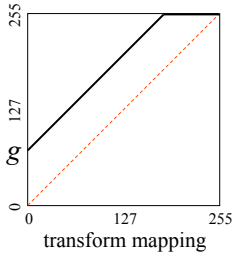
The Probability Distribution Function of an Image

- $P_I(g)$ is the fraction of pixels in (a specific band of) an image that have intensity values less than or equal to g.
- $P_I(g)$ is the probability that a pixel randomly selected from the given band has an intensity value less than or equal to g.
- $P_I(g)$ is the cumulative (or running) sum of $p_I(g)$from 0 through g inclusive.
- $P_I(0) = p_I(0)$ and $P_I(255) = 1$; $P_I(g)$ is nondecreasing.

Note: the Probability Distribution Function (PDF, capital letters) and the Cumulative Distribution Function (CDF) are exactly the same things. Both PDF and CDF will refer to it. However, pdf (small letters) is the *density* function.

Increase Brightness





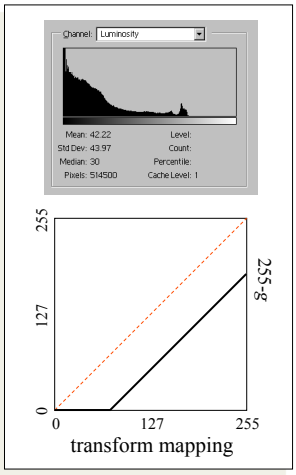
$$J_k(r,c) = \begin{cases} I_k(r,c) + g, & \text{if } I_k(r,c) + g < 256 \\ 255, & \text{if } I_k(r,c) + g > 255 \end{cases}$$

$g \geq 0$ and $k \in \{1,2,3\}$ is the band index.

Point Processing: Decrease Brightness

$$J_k(r,c) = \begin{cases} 0, & \text{if } I_k(r,c) - g < 0 \\ I_k(r,c) - g, & \text{if } I_k(r,c) \end{cases}$$

$g \geq 0$ and $k \in \{1,2,3\}$ is the band index.

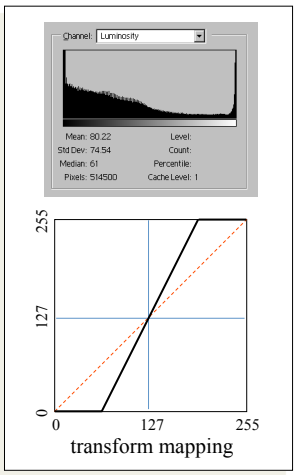


by Richard Alan Peters II

Point Processing: Increase Contrast

Let $T_k(r,c) = a[I_k(r,c) - 127] + 127$, where $a > 1.0$

$$J_k(r,c) = \begin{cases} 0, & \text{if } T_k(r,c) < 0, \\ T_k(r,c), & \text{if } 0 \leq T_k(r,c) \leq 255, \\ 255, & \text{if } T_k(r,c) > 255. \end{cases} \quad k \in \{1,2,3\}$$

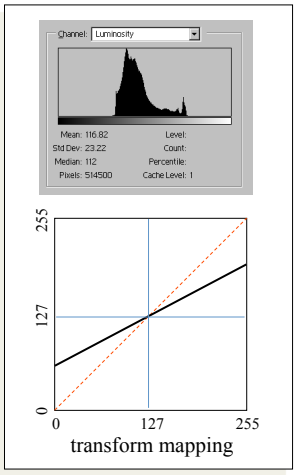


by Richard Alan Peters II

Point Processing: Decrease Contrast

$$T_k(r,c) = a\left[I_k(r,c) - 127\right] + 127,$$

where $0 \le a < 1.0$ and $k \in \{1,2,3\}$.



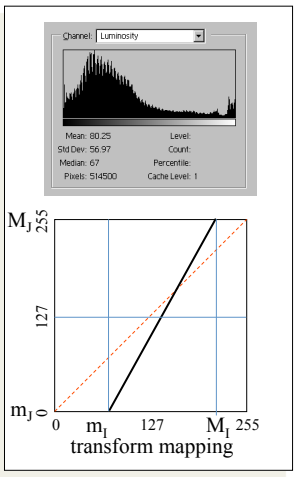
by Richard Alan Peters II

Point Processing: Contrast Stretch

Let $m_I = \min\left[I(r,c)\right]$, $M_I = \max\left[I(r,c)\right]$,
$m_J = \min\left[J(r,c)\right]$, $M_J = \max\left[J(r,c)\right]$.

Then,

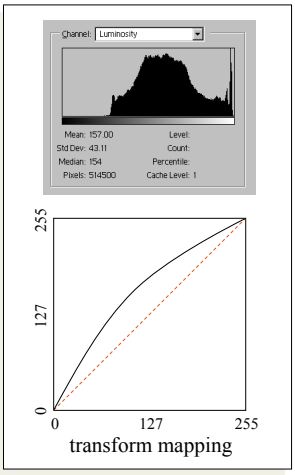
$$J(r,c) = (M_J - m_J)\frac{I(r,c) - m_I}{M_I - m_I} + m_J.$$



by Richard Alan Peters II

Point Processing: Increased Gamma

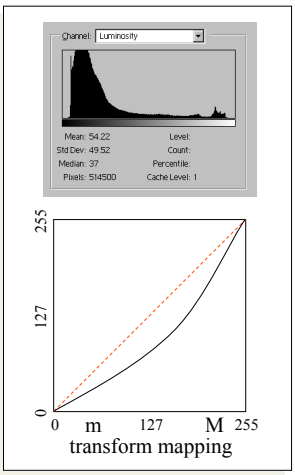
$$J(r,c) = 255 \cdot \left[\frac{I(r,c)}{255}\right]^{1/\gamma} \quad \text{for} \quad \gamma > 1.0$$



by Richard Alan Peters II

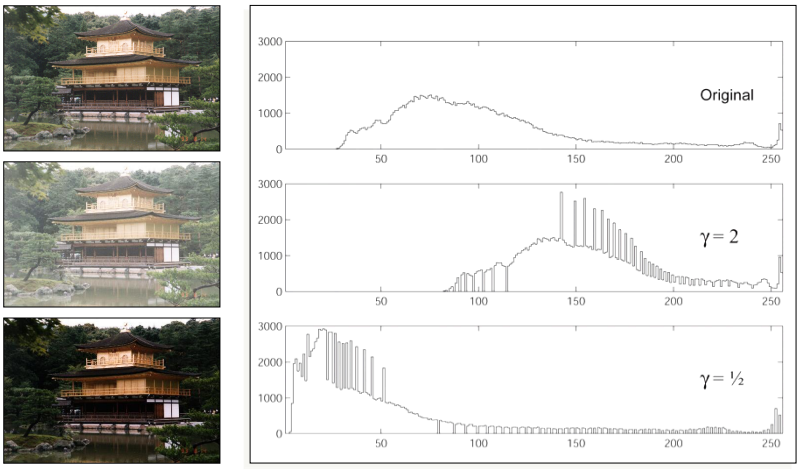
Point Processing: Decreased Gamma

$$J(r,c) = 255 \cdot \left[\frac{I(r,c)}{255}\right]^{1/\gamma} \quad \text{for} \quad \gamma < 1.0$$



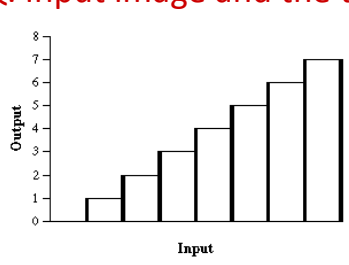
by Richard Alan Peters II

Gamma Correction: Effect on Histogram

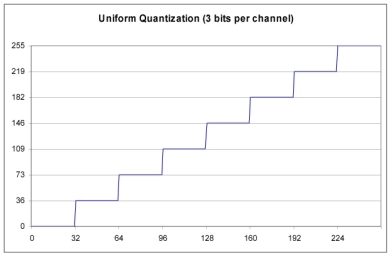


by Richard Alan Peters II

Q: Input image and the transform below is given



?



Uniform Quantization of Intensities is a Pointwise image transform too

Point Processing: Histogram Equalization

Recall from Probability Theory the theorem: If you plug in a random variable into its own CDF, you get a Uniform distribution: P(x) ~ U

Task: remap image *I* so that its histogram is as close to uniform as possible

Let $P_I(\gamma)$ be the cumulative (probability) distribution function of *I*.

Then *J* has, as closely as possible, the correct histogram if

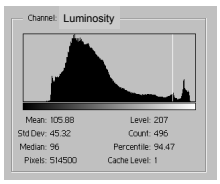
$$J(r,c) = 255 \cdot P_I\left[I(r,c)\right].$$

all bands processed similarly

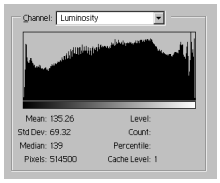
The CDF itself is used as the LUT.

Point Processing: Histogram Equalization

$$J(r,c) = 255 \cdot P_I(g)$$



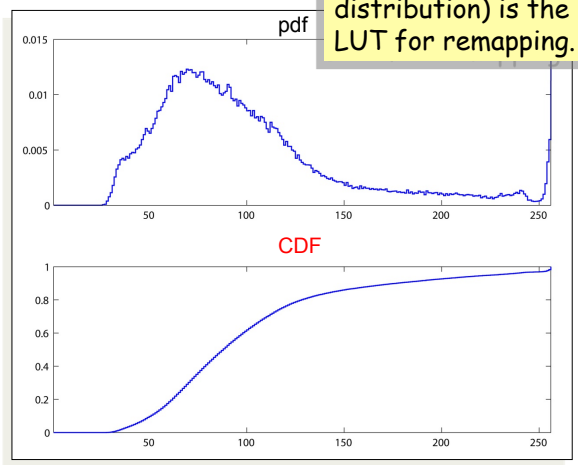
before



after

Histogram EQ

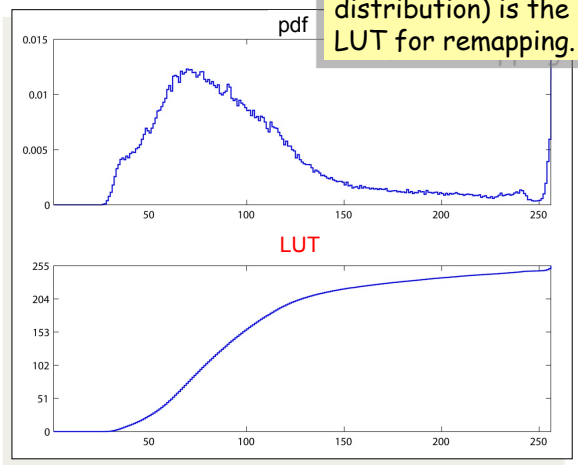
The CDF (cummulative distribution) is the LUT for remapping.



by Richard Alan Peters II

Histogram EQ

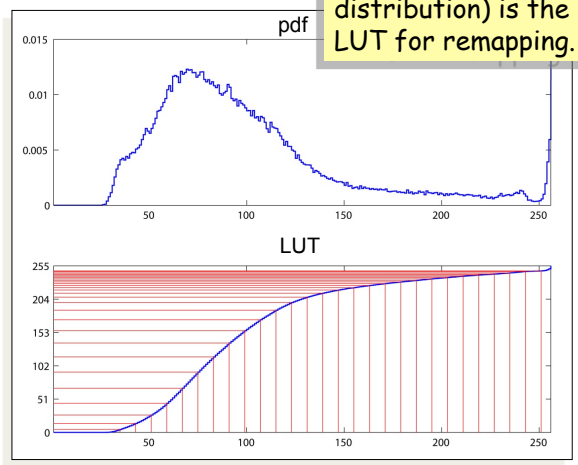
The CDF (cummulative distribution) is the LUT for remapping.



by Richard Alan Peters II

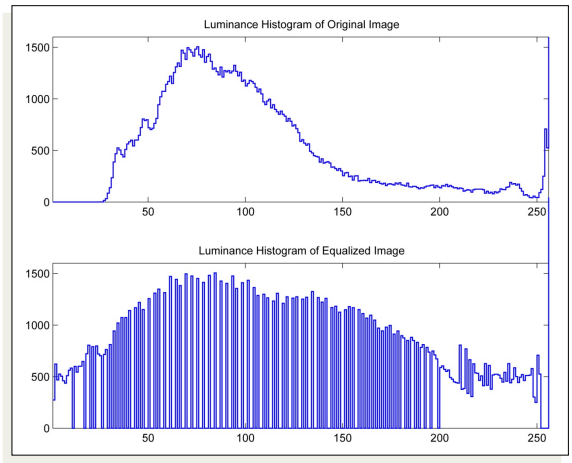
Histogram EQ

The CDF (cummulative distribution) is the LUT for remapping.



by Richard Alan Peters II

Histogram EQ



by Richard Alan Peters II

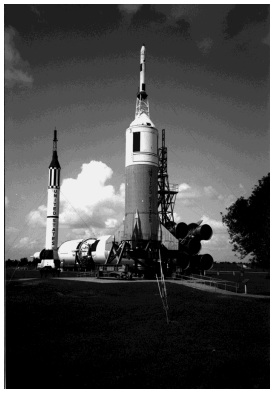
Point Processing: Histogram Matching

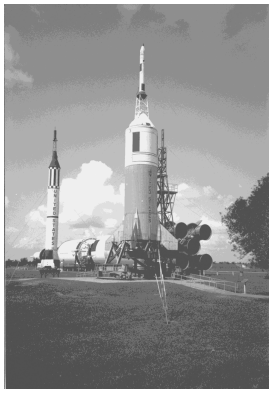
Task: remap image I so that it has, as closely as possible, the same histogram as image J.

Because the images are digital it is not, in general, possible to make $h_I \equiv h_J$. Therefore, $p_I \not\equiv p_J$.

Q: How, then, can the matching be done?
A: By matching percentiles.

Example: Histogram Matching





original target remapped

Matching Percentiles

... assuming a 1-band image or a single band of a color image.

Recall:

- The CDF of image I is such that $0 \le P_I(g_I) \le 1$.
- $P_I(g_I) = c$ means that c is the fraction of pixels in I that have a value less than or equal to g_I.
- $100c$ is the *percentile* of pixels in I that are less than or equal to g_I.

To match percentiles, replace all occurrences of value g_I in image I with the value, g_J, from image J whose percentile in J most closely matches the percentile of g_I in image I.

Matching Percentiles

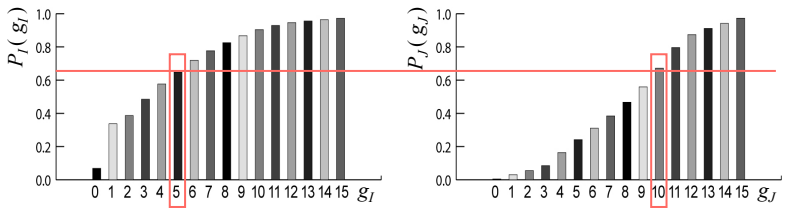
... assuming a 1-band image or a single band of a color image.

So, to create an image, K, from image I such that K has nearly the same CDF as image J do the following:

If $I(r,c) = g_I$ then let $K(r,c) = g_J$ where g_J is such that

$P_I(g_I) > P_J(g_J\text{-}1)$ AND $P_I(g_I) \le P_J(g_J)$.

Example:
$I(r,c) = 5$
$P_I(5) = 0.65$
$P_J(9) = 0.56$
$P_J(10) = 0.67$
$K(r,c) = 10$



Histogram Matching Algorithm

... assuming a 1-band image or a single band of a color image.

```
[R,C] = size(I);
K = zeros(R,C);
g_J = m_J;
for g_I = m_I to M_I
    while g_J < 255 AND P_I(g_I + 1) < 1 AND
                P_J(g_J + 1) < P_I(g_I + 1)
        g_J = g_J + 1;
    end
    K = K + [g_J × (I == g_I)]
end
```

This directly matches image I to image J.

$P_I(g_I + 1)$: CDF of I

$P_J(g_J + 1)$: CDF of J.

$m_J = \min J,$

$M_J = \max J,$

$m_I = \min I,$

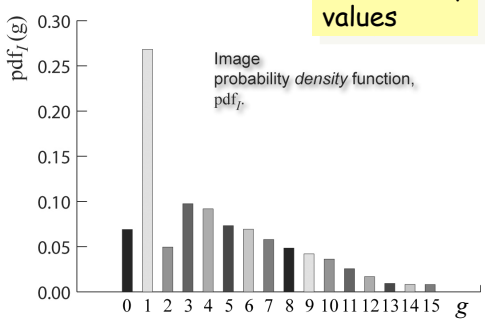
$M_I = \max I.$

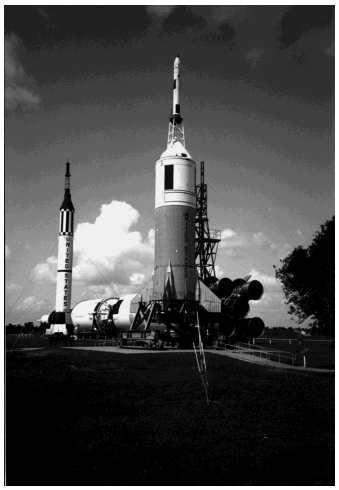
Better to use a LUT. See the slide (6 next)

Example: Histogram Matching

Image pdf

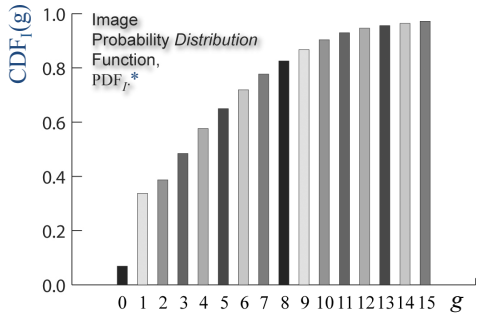
Image with 16 intensity values



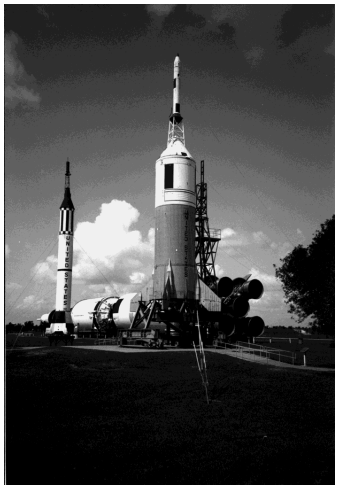


Example: Histogram Matching

Image CDF



*a.k.a Cumulative Distribution Function, CDF_I.

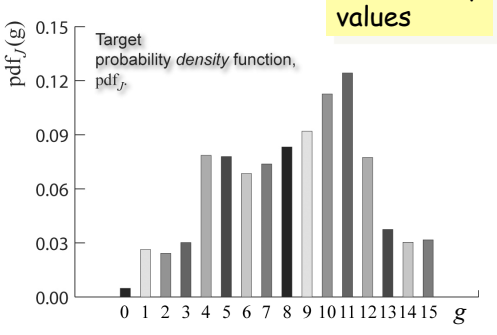


by Richard Alan Peters II

Example: Histogram Matching

Target pdf

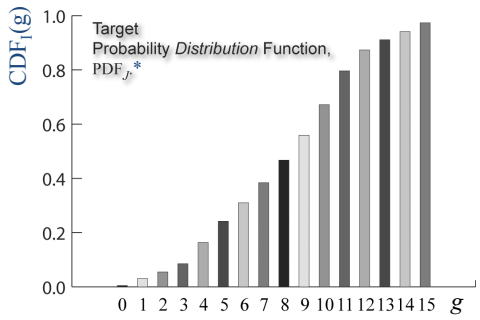
Target with 16 intensity values



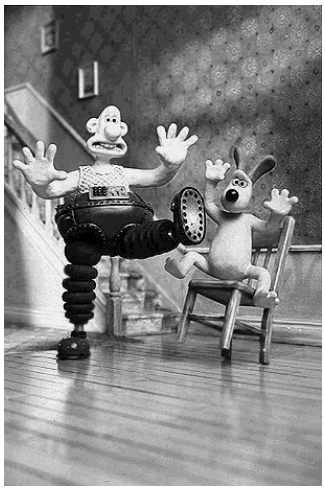
by Richard Alan Peters II

Example: Histogram Matching

Target CDF



*a.k.a Cumulative Distribution Function, CDF_J.



by Richard Alan Peters II

Histogram Matching with a Lookup Table

The given algorithm (5 slides previously) matches one image to another directly. Often it is faster or more versatile to use a lookup table (LUT). Rather than remapping each pixel in the image separately, one can create a table that indicates to which target value each input value should be mapped. Then

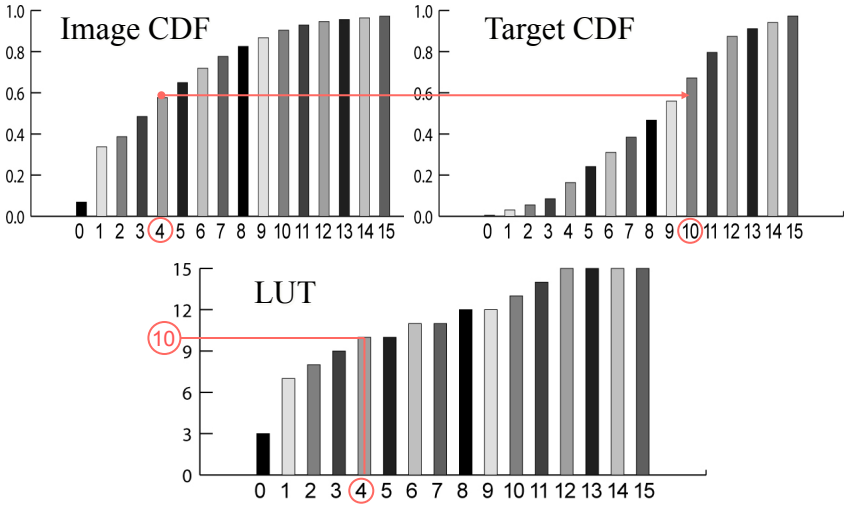
K = LUT[I] (see the graphics on the next slide)

In *Matlab* if the LUT is a 256 × 1 matrix with values from 0 to 255 and if image I is of type **`uint8`**, it can be remapped with the following code:

```
K = uint8(LUT(double(I)+1));
```


by Richard Alan Peters II

LUT Creation



by Richard Alan Peters II

Look Up Table for Histogram Matching

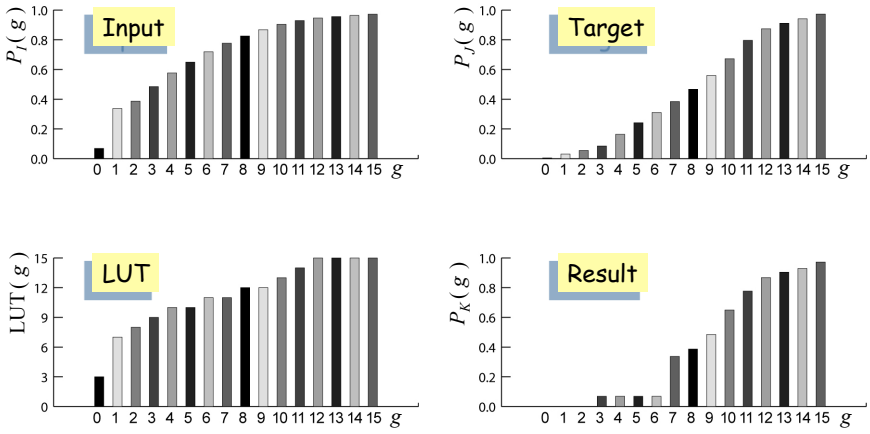
```
LUT = zeros(256,1);
g_J = 0;
for g_I = 0 to 255
    while P_J(g_J + 1) < P_I(g_I + 1) AND g_J < 255
        g_J = g_J + 1;
    end
    LUT(g_I + 1) = g_J;
end
```

This creates a look-up table which can then be used to remap the image.

$P_I(g_I+1)$: CDF of I,
$P_J(g_J+1)$: CDF of J,
$\mathrm{LUT}(g_I+1)$: Look- Up Table

by Richard Alan Peters II

Input & Target CDFs, LUT and Resultant CDF



Example: Histogram Matching

original target remapped

Remap a color Image:

To Have Two of its Color pdfs Match the Third

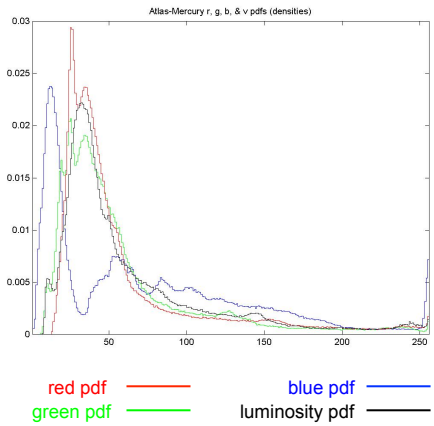
original

G & B ←R

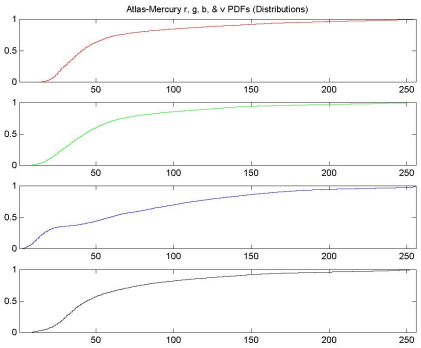
B & R ←G

R & G ←B

Probability Density Functions of a Color Image

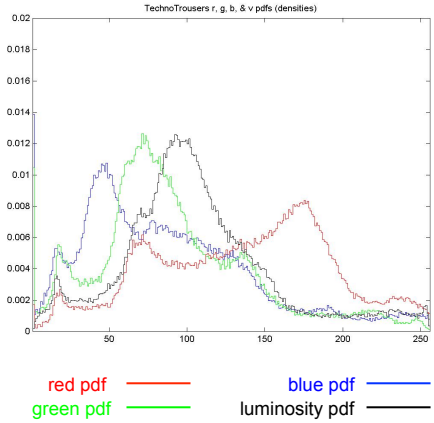


Cumulative Distribution Functions (CDF)



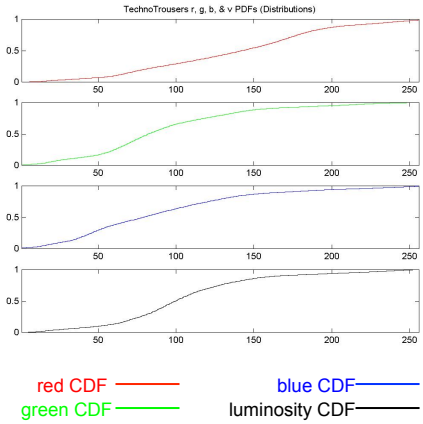
red CDF
green CDF
blue CDF
luminosity CDF

Probability Density Functions of a Color Image



red pdf
green pdf
blue pdf
luminosity pdf

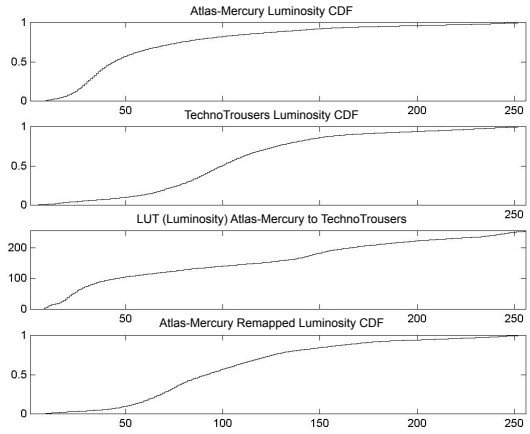
Cumulative Distribution Functions (CDF)



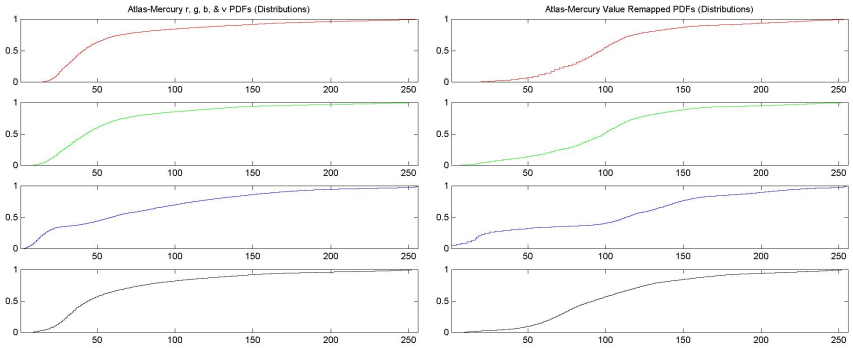
Remap an Image to have the Lum. CDF of Another

original target luminosity remapped

CDFs and the LUT



Effects of Luminance Remapping on CDFs



Before

After

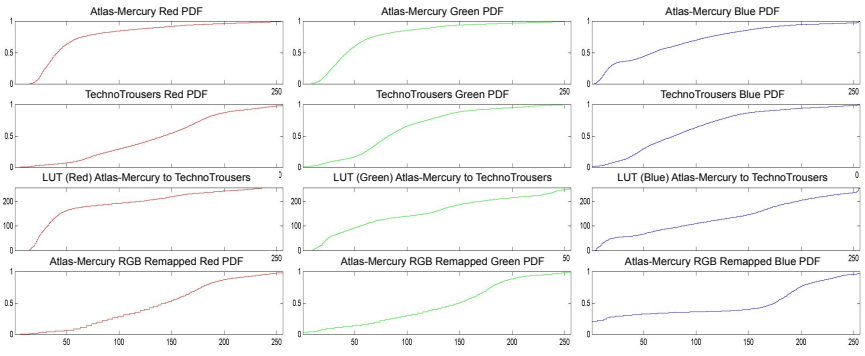
Remap an Image to have the rgb CDF of Another

original

target

R, G, & B remapped

CDFs and the LUTs



END OF LECTURE

Recall Learning objectives of Week 2: Students will be able to:

1. Discuss the main problems of computer (artificial) vision, its uses and applications

2. Design and implement various image transforms: point-wise transforms

Reading Assignments:
[Klette Book] 1.1, 1.3, 2.1.1, 2.1.2

[Gonzalez and Woods]: Chap 3.1 – 3.3

HW Assignment: you get your hands dirty by starting with your first Image Processing implementations