1 Theory

1.1 Numerical Integration Methods

1.1.1 Leap Frog

Leapfrog is a method that is used to numarically intergenerate that are in the form of

$$\ddot{x} = \frac{d^2x}{dt^2} = f(x)$$

It is also known as the Störmer-Verlet method, commonly used to numerically calculate the trejectory of particles. The name comes from the fact that calculation of updated **x** and **v** are done in some order and calculated for different time slices. The energy is approximately conserved during the calculation. It is stable in oscillatory motion as long as the time-step Δt is constant, and satisfies $\Delta t \leq 2/\omega$ [1]. The idea is straight forward; in the time interval Δt,

$$\begin{aligned} a(t_0) &= f(x_0) \\ x(t_0 + \Delta t) &= x(t_0) + v(t_0)\Delta t + a(t_0)\frac{\Delta t^2}{2} \qquad (1) \\ v(t_0 + \Delta t) &= v(t_0) + \{a(t_0) + a(t_0 + \Delta t)\}\frac{\Delta t}{2} \qquad (2) \end{aligned}$$

For more stability, this version can be rearranged to what is called 'kick-drift-kick' form,

$$\begin{aligned} v(t_0 + \Delta t/2) &= v(t_0) + a(t_0)\frac{\Delta t}{2} \\ x(t_0 + \Delta t) &= x(t_0) + v(t_0 + \Delta t/2)\Delta t \qquad (3) \\ v(t_0 + \Delta t) &= v(t_0 + \Delta t/2) + a(t_0 + \Delta t)\frac{\Delta t}{2} \end{aligned}$$

This version provides more time resolution to our calculation; however, it increases the number of calculations needed by about 50%.

1.1.2 Runge Kutta

Runge Kutta is another numerical integration method that is

References

[1] C. K. Birdsall and A. B. Langdon, *Plasma Physics via Computer Simulations*, McGraw-Hill Book Company, 1985, p. 56.