# Black Hole Differential Cross Section — Michael Zhang

$$\Theta(b,v) = (A + Bb + Cb^2)(D + Ev + Fv^2)$$

let $\gamma \equiv \Theta / D + Ev + Fv^2$

$$\gamma = A + Bb + Cb^2 \rightarrow 0 = (A-\gamma) + Bb + Cb^2$$

$$b = \frac{-B + \sqrt{B^2 - 4(A-\gamma)C}}{2(A-\gamma)} \qquad b > 0$$

$$\frac{db}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

$$\frac{db}{d\theta} = \frac{-B\alpha\sqrt{-4AC + B^2 + \frac{4C}{\alpha}\Theta} + 2C(\Theta - A\alpha) + B^2\alpha}{2(\Theta - A\alpha)^2\sqrt{-4AC + B^2 + \frac{4C}{\alpha}\Theta}} \quad \text{where } \alpha \equiv D + Ev + Fv^2$$

$$\frac{db}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{2C(\Theta - A\alpha) + B^2\alpha - B\alpha\beta}{2(\Theta - A\alpha)^2\beta}\right| \quad \text{where } \alpha \equiv D + Ev + Fv^2, \; \beta \equiv \sqrt{\frac{4C}{\alpha}\Theta - B^2 - 4AC}$$