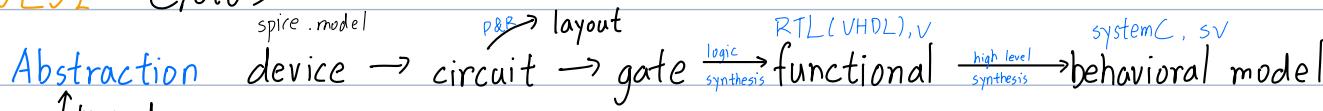


VLSI CMOS



↑ transcend
Problem: power distribution integrity (V_{DD}); reliability; noise; interference

A vs D NM

A more energy efficient, less reliable

Material Si vs 35 vs wide-bandgap (SiC, GaN)

MOS vs BJT power

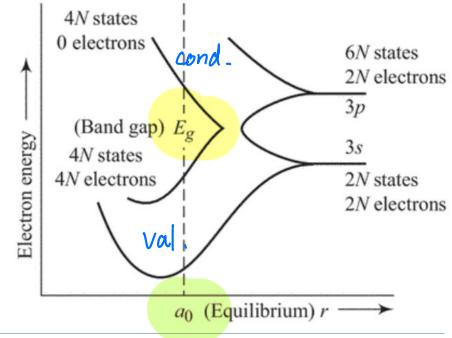
Semi $T \uparrow$, e^- jump to $E_c \rightarrow EHP$

$$n_p = n_i^2$$

$$\text{Fermi Dirac } f(E) = \frac{1}{1 + e^{\frac{E_f - E_i}{kT}}}$$

$$n = n_i e^{\frac{E_f - E_i}{kT}}$$

$$p = n_i e^{\frac{E_i - E_f}{kT}}$$



pnj band bend $\leftarrow E \rightarrow \Delta V$

diffuse $\rightarrow q^{+/-}$

$$\frac{dE}{dx} = \frac{P}{K_s \epsilon_0}$$

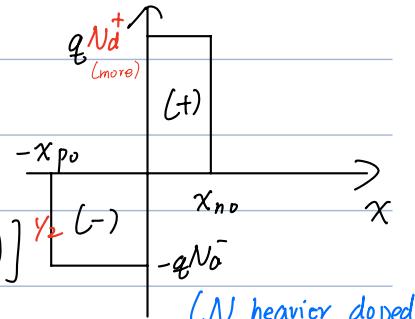
$$K_s = 11.7, \epsilon_0 = 8.85 \times 10^{-14}$$

$$\int_0^{E_{max}} dE = \int_{-x_p}^0 \frac{P}{K_s \epsilon_0} dx$$

$$\text{lin } E_{max} = \frac{q N_d}{K_s \epsilon_0} x_p = \frac{q N_d}{K_s \epsilon_0} x_n$$

$$V_{bi} = \frac{1}{q} (kT \ln \frac{N_d}{n_i} + kT \ln \frac{N_a}{n_i}) = \frac{kT}{q} \ln \left(\frac{N_a N_d}{n_i^2} \right) = \frac{1}{2} E_{max} W$$

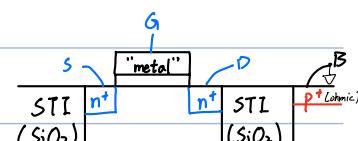
$$W = \left[\frac{2 K_s \epsilon_0}{q} \left(\frac{1}{N_a} + \frac{1}{N_d} \right) \right]^{1/2}$$



bias V_A : non-eq

$$I \sim e^{\frac{qV_A}{kT}}$$

$$C. C_o = \frac{K_s \epsilon_0}{W} \quad (\text{per area, ss, } \downarrow \text{ as RBT, parasitic})$$



MOS/FET SD sym., depends on how biased in circuit

\approx for ideal

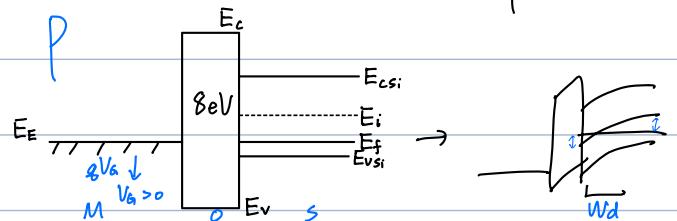
$V_G = 0$, flat band V_{FB} to ensure $E = 0$

$V_G > 0$, band bend \rightarrow depletion in S

$$E_i - E_f = kT \ln \left(\frac{N_d}{n_i} \right) \equiv \phi_f$$

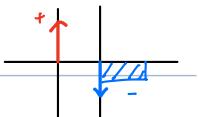
@ interface @ bulk

Once $E_f - E_i = E_i - E_f \rightarrow$ inversion



past V_T

dep. region no longer grows. e^- grows in inv. to balance



$V_{ov} = V_G - V_T$ decides current

$$W_d = \sqrt{\frac{2K_s \epsilon_0 \phi_s}{q N_A}}$$

ϕ_s : surface vs bulk;

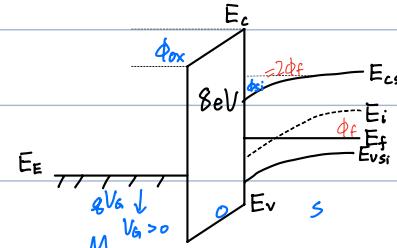
$$W_d^{\max} = \sqrt{\frac{2K_s \epsilon_0 2\phi_f}{q N_A}} \quad Q_{blk} = q N_A W_d^{\max}$$

$$V_T = \phi_{ox} + \phi_s$$

$$= \frac{Q_{bulk}}{C_{ox}} + 2|\phi_f| + V_{FB}$$

compensate for non-flat-band

$$C_{ox} = \frac{K_{ox} \epsilon_0}{t_{ox}}$$



Use nonuniform doping profile to control W_d and V_T

As size \downarrow , # dopants countable (random dopant fluctuation) \rightarrow :

sln. deplete Si \rightarrow no dopant \rightarrow engineer ϕ_m for a good V_{FB}

Mobility $v_{drift} = \mu E \rightarrow v_{sat} \approx 10^7 \text{ cm/s}$

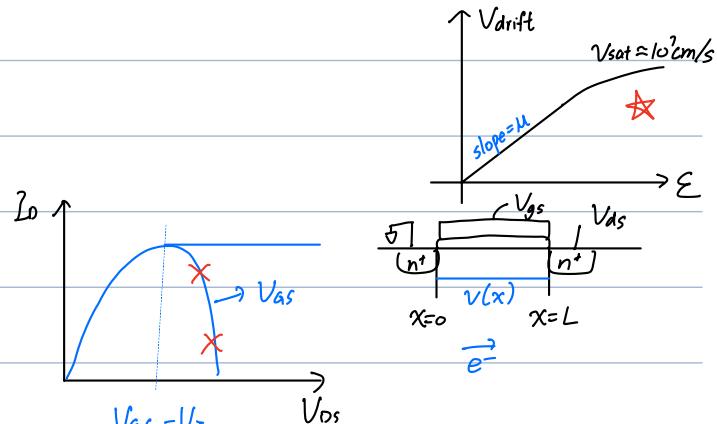
long-channel Shockley (assume \downarrow)

$$I_D = W Q_i v_{drift}$$

$$= W C_{ox} (V_{gs} - V(x) - V_T) (\mu \frac{dv}{dx})$$

$$\int_0^L I_D dx = \int_0^{V_{ds}} W C_{ox} (V_{gs} - V_T - V(x)) \mu dv$$

$$I_D = \frac{W}{L} \mu C_{ox} [(V_{gs} - V_T) V_{ds} - \frac{V_{ds}^2}{2}] \quad (\text{linear})$$



$V_{ds} > V_{ov}$ pinchoff (no free carriers, e^- swept to drain) $\rightarrow I_{sat} = \frac{W}{2L} \mu C_{ox} (V_{ds} - V_T)^2$

$I_{sat} \sim V_{ov}^2$, since $Q_i \cdot v_{drift}$

short channel $Q_i \cdot v_{sat} \rightarrow \sim V_{ov}$

$$v_{drift} = \frac{\mu E}{1 + \epsilon / \epsilon_{crit}} \sim 10^4 \text{ V/cm}$$

$$I_D = W C_{ox} (V_{gs} - V(x) - V_T) \frac{\mu E}{1 + \epsilon / \epsilon_{crit}}$$

$$I_D = \frac{W}{L} \frac{\mu C_{ox}}{1 + \frac{V_{ds}}{\epsilon_{crit} L}} \left[(V_{gs} - V_T) V_{ds} - \frac{V_{ds}^2}{2} \right]$$

$$\frac{\partial I_D}{\partial V_{ds}} = 0 \rightarrow V_{ds} = V_{dsat} = \frac{2(V_{gs} - V_T)}{1 + \sqrt{\frac{1 + 2\mu(V_{gs} - V_T)}{\epsilon_{crit} L}}}$$

$$I_{dsat} = C_{ox} W V_{sat} (V_{gs} - V_T) \sqrt{\frac{-1}{1 + \sqrt{\frac{1 + 2\mu(V_{gs} - V_T)}{\epsilon_{crit} L}}}} \quad \text{if } \frac{V_{gs} - V_T}{L} \gg \frac{\epsilon_{crit}}{2}, I_{dsat} \rightarrow C_{ox} W V_{sat} (V_{gs} - V_T)$$

lin

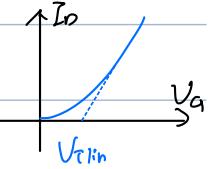
Enhancement : off when $V_G = 0$ ($V_T < 0$ for p, > 0 for n) TAOH?

Depletion on \sim (rare)

V_T 1. device physical: onset of strong inv.

def {

2. triode $I_D = \sim (V_{GS} - V_T) V_{DS}$ (extrapolate lin. part $\rightarrow V_{T\text{lin}}$)



3. sat. find V_{DS} s.t. I_D reaches a specific density when $V_{DS} = V_{DD}$ ($E \frac{1\mu\text{A}}{\mu\text{m}^2}$)

Scaling (Dennard) $w \rightarrow \frac{w}{K}$; $L \rightarrow \frac{L}{K}$; $t_{ox} \rightarrow \frac{t_{ox}}{K}$; $V \rightarrow \frac{V}{K}$ (const. E scaling)

1. Depletion region $W_{dep} \sim \sqrt{\frac{V}{N}} \sim \frac{1}{K}$ want $N \rightarrow K N$

but $V = V_{bias} + V_{built-in}$ want \sim bandgap \rightarrow can't scale \rightarrow short channel

2. C $C_{gate} = \frac{\epsilon_{WL}}{t_{ox}} \sim \frac{1}{K}$

inv. layer charge density

3. $Q_i \sim \frac{EV}{t_{ox}} \sim 1$, but $V:L$

4. $I_D \sim Q_i W \frac{1}{V_{DS}} \sim \frac{1}{K}$

5. delay $\approx \frac{CV}{I} \sim \frac{1}{K} \rightarrow f \sim K$

6. power $= IV \sim \frac{1}{K^2}$ (density ~ 1)

V_T inability $I_D \sim e^{\frac{q(V_{GS}-V_T)}{mKT}}$

\log_{10} slope $= \frac{mKT}{q} \frac{1}{\log_{10} e} \approx 60 \text{ mV/dec}$ of I_D

if $V_T \downarrow \rightarrow I_{leak} \uparrow$

if size \downarrow , but $W_{dep} \nabla \rightarrow D$ starts to control Q_{dep}

$$V_T = V_{FB} + 2|\phi_F| + \frac{Q_{bulk}}{C_{ox}} \downarrow \text{not much control}$$

If $V_{DS} \uparrow$, $V_{T,\text{eff}} \downarrow$, so $I_D \uparrow$ slightly \rightarrow short channel effect

(DIBL: drain Induced Barrier Lowering $\sim \frac{mV}{\nu}$)

V_T rolloff if L too short, $V_T \downarrow$, same reason

want minimal variation

Cap from device / wires ($0.1 \text{ fF}/\mu\text{m}$)

intrinsic: $C_{gate} \sim I_D$

extrinsic (parasitic) \leftarrow overlap from V_{GS}, V_{DD} ; also between M contact & gate stack

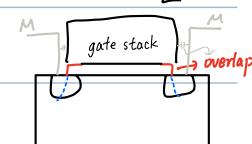
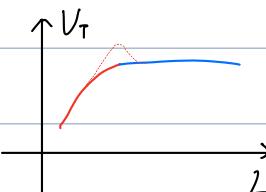
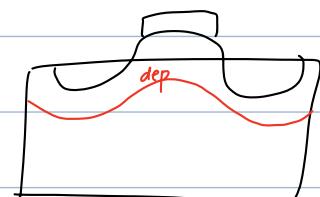
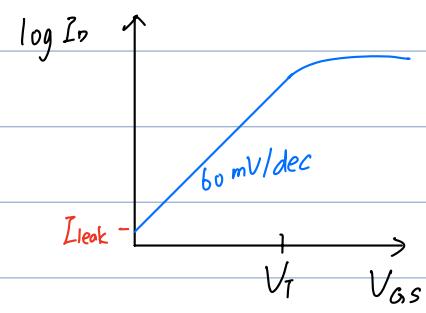
diffusion (junction): jct C across W_{dep} , controlled by layout

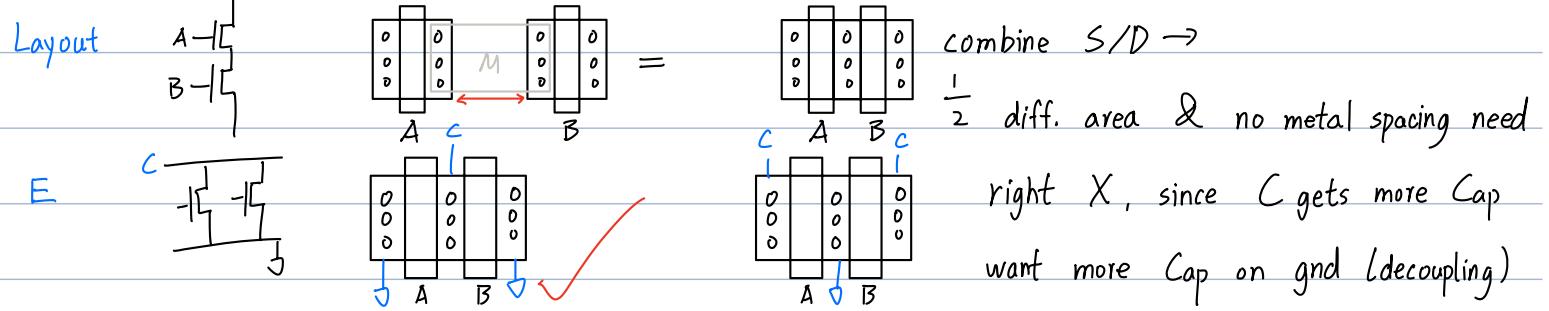
Intrinsic distribution $C_{gs} \ C_{gd} \ C_{gb}$

cutoff no inv. $\rightarrow C_{gs} = C_{gd} = 0$, $C_{gb} = C_{ox} w L \xrightarrow{\text{series w/}} C_{bulk}$

triode uniform inv. $\rightarrow C_{gs} = C_{gd} = \frac{1}{2} C_{ox} w L$; $C_{gb} = 0$

sat. most C not at drain $\rightarrow C_{gs} = \frac{2}{3} C_{ox} w L$; $C_{gd} = C_{gb} = 0$





Body effect if $V_{SB} \neq 0$

$$V_T = V_{FB} + 2|\phi_F| + \frac{Q_B}{C_{ox}} ; Q_B = \sqrt{2g_{es} N_A 2|\phi_F|}$$

$$2\phi_F + V_{SB} \rightarrow V_T \uparrow$$

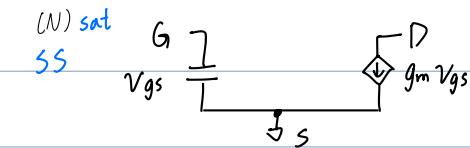
must apply larger V_S to create inv. May also use V_{SB} to control V_T

Transfer curve

f_T : unity current gain cutoff f

F04 } delay of D₀ driving 4 D₀s of same size

F01 |



$$g_m = \frac{\partial I_0}{\partial V_{gs}}$$

$$V_{gs} = i_g \cdot \frac{1}{sC_{as}}$$

$$i_d = i_g \frac{g_m}{sC_{as}}$$

$$|A_i(j\omega)| = \frac{g_m}{wC_{as}} = 1$$

$$f_T = \frac{1}{2\pi} \frac{g_m}{C_{as}} = \frac{1}{2\pi} \frac{wC_{ox}V_{sat}}{wL C_{ox}}$$

$$(transit time) = \frac{1}{2\pi} \frac{V_{sat}}{L} \equiv \frac{1}{2\pi} \cdot \frac{1}{t_{tr}}$$

F01 $V_{sat,p} \approx \frac{1}{2} V_{sat,n}$, so $w_p = 2w_n$ (but ~ same for modern)

$$N t \approx \frac{V_{DD}}{I_{sat}} (C_{gn} + C_{dn}) \quad \eta \equiv \frac{C_g}{C_d} \sim 1.5 \sim 2$$

$$= \frac{V_{DD}}{I_{sat}} C_{gn} \left(1 + \frac{1}{\eta}\right)$$

$$= \frac{V_{DD} w L C_{ox} (1 + \frac{1}{\eta})}{w C_{ox} (V_{DD} - V_T) V_{sat}}$$

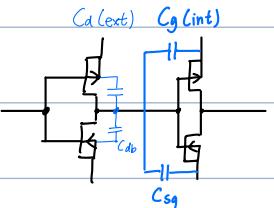
$$= \frac{L}{V_{sat}} \left(1 + \frac{1}{\eta}\right)$$

$$= T_{tr} \left(1 + \frac{1}{\eta}\right)$$

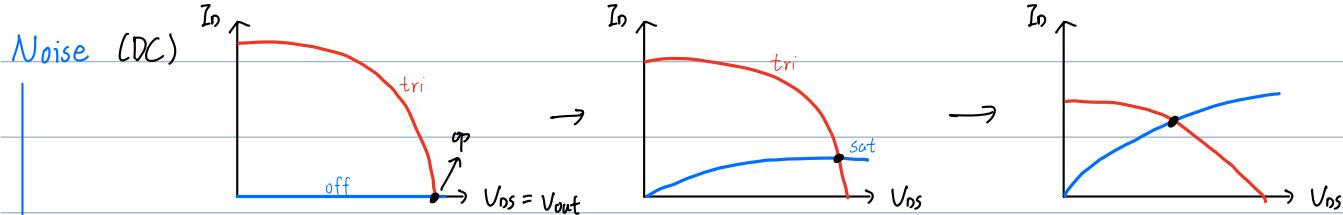
$$P+N: t(2w) = 3 T_{tr} \left(1 + \frac{1}{\eta}\right) \equiv T_{inv} \quad \eta = \frac{C_{gn} \times (2 + 1)}{C_{dn} \times (2 + 1)} \text{ (P+n)}$$

$$F04 = \frac{4C_g + C_d}{C_g + C_d} \cdot F01 \quad (= \times 4 \text{ if } \eta = \infty)$$

$$= (1 + 4\eta) \cdot \frac{3}{\eta} T_{tr} = 3 T_{tr} \left(4 + \frac{1}{\eta}\right)$$



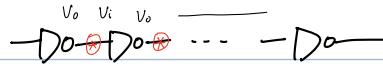
(S are not switching, irrelevant)



rapid change in middle \rightarrow D_o trans. curve

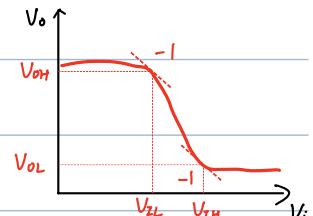
Noise at end will be restored

NM (DC, acting on every node)



+ worst-case DC noise (*)

if AC (NM_{AC}) , will be filtered by C_g (better)



$$NM_L = (V_{IL} - V_{OL}) ; NM_H = V_{OH} - V_{IH}$$

Switching energy output charges C_L

$$\begin{aligned} \xrightarrow{0 \rightarrow 1} E &= \int dt i_{DD}(t) V_{DD} \\ &= V_{DD} \int_0^\infty dt C_L \frac{dV_{out}}{dt} \\ &= C_L V_{DD}^2 \quad \text{but } E_c = \frac{1}{2} C_L V_{DD}^2 \rightarrow \text{rest heat} \end{aligned}$$

$\xrightarrow{1 \rightarrow 0}$ stored $\frac{1}{2}$ diss.

$$P = C_L V_{DD}^2 f$$

I_{corner}: while switching, a transient time where both Q on ($\sim 10\text{-}20\%$ p)

especially when V_{in} switch slow (low slew rate)

spice E. channel length E. in a wafer

Compact model: many params. Have a distribution (may be correlated)

corner model: set 3 σ from μ \rightarrow ensure still work

not too much σ , since some chip

TT, SS, FF, SF, FS \rightarrow corners

PVT: process, voltage, temperature

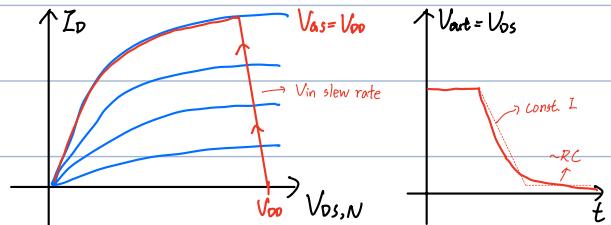
Delay $V_{in}: 0 \rightarrow 1$

\sim saturated ramp. delay T_{PHL}^{out} , $T_{PLH} = \Delta t$ for 50% V_{DD}

slew \sim ramp slope of 10% - 90%

or extrapolate to 0-100 ($\div 0.8$), up to ur def

$$C_g \left[\frac{fF}{\mu m} \right] @ \text{min length}$$



$$R_{nFET} = W_n R_{pulldown} \quad [k\Omega \cdot \mu m]$$

$$R_{pFET} = W_p R_{pullup}$$

$$C_g = C_{gn} = C_{gp} = \frac{C_{gate}}{W} \approx \frac{K\epsilon_0 L}{t_{ox}}$$

$$\left[\frac{fF}{\mu m} \right]$$

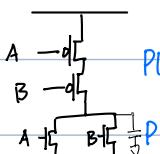
@ min width

For p: $2W$; n: W inverter: logical effort $g_{inv} \equiv 1$

$$\text{electrical effort } h_{inv} \equiv \frac{C_{out}}{C_{in}} = 1$$

$$\text{normalized delay } F \equiv \frac{\tau_{logic\ gate}}{\tau_{inv}} = g_{gate} \cdot h_{gate}$$

E. nor

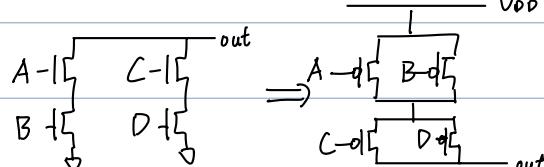


exactly 1 on at a time



want input w/ more delay to be closer to output

Ser. in PDN \leftrightarrow || in PDN



want $\leq 2 Q$ high per network

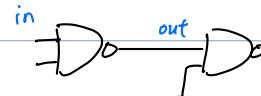
$$\tau_{inv} = FO|_{inv} = \frac{R_{pFET}}{W} \cdot 3WC_g = 3C_g R_{nFET}$$

E. NAND size wrt to inv. $\rightarrow p: 2W; n: 2W$

$$T_{nand} = FO|_{nand} = (4WC_g) \times \left(\frac{R_{nFET}}{2W} \right) \cdot 2 = 4C_g R_{nFET}$$

$$f \equiv \frac{T_{nand}}{\tau_{inv}} = \frac{4}{3}$$

$$h = \frac{4WC_g}{4WC_g} = 1 \rightarrow g_{nand} = \frac{f}{h} = \frac{4}{3}$$



E2. NOR $\rightarrow p: 4W, n: 1W$

$$T_{nor} = FO|_{nor} = (5WC_g) \times \left(\frac{R_{nFET}}{W} \right)^{4+1} = 5C_g R_{nFET}$$

$$d = \frac{5}{3}, \quad g_{nor} = \frac{5}{3}$$

E. if NOR driving 2 NORs w/ same $4W - W$

$$FO2_{nor} = (10WC_g) \times \left(\frac{R_{nFET}}{W} \right) = 10C_g R_{nFET}$$

$$d = \frac{10}{3}, \quad h = 2, \quad g = \frac{5}{3} \quad (\text{same logical, more electrical})$$

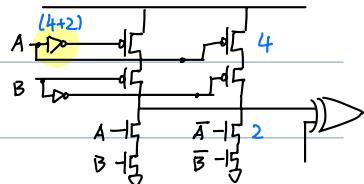
E3. XOR Need to size \triangleright

$$FO|xor = FO|_{main} \times FO|_{inv}$$

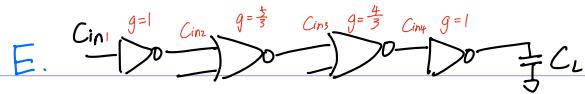
multiply stage effort

$$= (12WC_g) \left(\frac{R_{nFET}}{2W} \cdot 2 \right) \times \left(6WC_g \right) \left(\frac{R_{nFET}}{2W} \right)$$

$$= 12C_g R_{nFET} \times 3C_g R_{nFET} = 4\tau_{inv} \cdot 1\tau_{inv} \rightarrow F = 4$$



$$E. \text{ if inv is } 2W/W, \quad FO|xor = (9WC_g) \dots \times (6WC_g) \left(\frac{R_{nFET}}{W} \right) = 3\tau_{inv} \cdot 2\tau_{inv} \rightarrow F = 6 > 4 \therefore$$



Assume C_{in}, C_L fixed (if not, make $C_{in} \rightarrow \infty$)

$$F = \prod_{i=1}^n F_i = \prod_{i=1}^n (g_i h_i) = \prod_{i=1}^n g_i \cdot \prod_{i=1}^n h_i = \left(\frac{20}{9}\right) \left(\frac{C_{in2}}{C_{in1}} \times \dots \times \frac{C_L}{C_{in4}}\right) = \frac{20}{9} \frac{C_L}{C_{in1}} \rightarrow \text{fixed}$$

$$T_{path} = \sum_{i=1}^n (g_i h_i) T_{inv} \rightarrow \text{minimizable}$$

$$\text{Let norm. delay } y = f_{nor} + f_{nand} = \frac{F}{f_{nand}} + f_{nand} = f_{nand} \left(1 + \frac{F}{f_{nand}}\right)$$

$$\frac{\partial y}{\partial f_{nand}} = 1 - \frac{F}{f_{nand}^2} = 0 \rightarrow f_{nand} = \sqrt[2]{F}$$

General for n stages: $f_i = \sqrt[2]{F}$ for min delay (if no branching (const. H))

+ stages N-stage inv. chain, $y = \infty$, $f_i = \alpha$ ignore C_d

$$F = \prod_{i=1}^N f_i = \alpha^N = \frac{C_{out}}{C_{in}}$$

$$N = \frac{\ln(C_{out}/C_{in})}{\ln \alpha}$$

$$T_d = T_{inv} \sum_{i=0}^{N-1} F = T_{inv} \alpha N$$

$$= T_{inv} \alpha \frac{\ln(C_{out}/C_{in})}{\ln \alpha}$$

$$0 = \frac{\partial T_d}{\partial \alpha} =$$

$$\alpha = e \quad (\text{closest even})$$

if include C_d , $\alpha = 4$

E. $C_g = \frac{2F}{1.2\mu m}$, $R_{pf} = 16 k\Omega \cdot \mu m$, $R_{nf} = 8 k\Omega \cdot \mu m$, $T_{inv} = 3R_{nf}C_g = 48 ps$

① find eff C_{out} . $f_3 = \frac{500\mu m}{3 \cdot 1.2\mu m} = \frac{5}{3} \cdot \frac{1pF}{2F \cdot 6\mu m}$

inv. size to match gate
actual size
 $f_2 = \frac{6\mu m}{3 \cdot 0.6\mu m}$

$$f_1 = \frac{2.4\mu m}{3 \cdot 1.2\mu m}$$

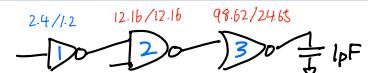
② $T_{delay} = T_{inv}(f_3 + f_2 + f_1) = 6858 ps$, too long

③ $F = f_3 f_2 f_1 = 308.6 \rightarrow f_{op} = \sqrt[3]{F} \approx 6.76 \rightarrow T_{delay} = 973 ps$

④ $W_{inv,eff,3} = \frac{500\mu m}{3 \cdot 6.76} = 24.65 \mu m \rightarrow 4W/W = 98.62/24.65$

$W_{inv,eff,2} = \frac{5.2465\mu m}{3 \cdot 6.76} = 6.08 \mu m \rightarrow 2W/2W = 12.16/12.16$

$W_{inv,eff,1} = \frac{4 \cdot 6.08 \mu m}{3 \cdot 6.76} = 1.2 \mu m \rightarrow$



E': add 2 Do $\rightarrow f_{op} = \sqrt[5]{F} \approx 3.15$

(if other gate added, $g \uparrow \rightarrow f \uparrow$)

$$T_{delay} = 756 ps$$



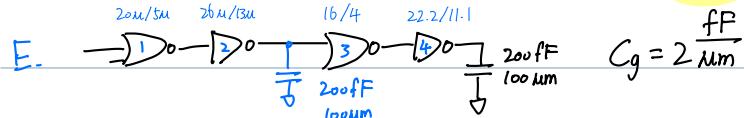


Add $C_x \rightarrow h$ no longer const. $H = \frac{C_L}{C_{in1}} \cdot \frac{C_{in4}}{C_{in3}} \cdot \frac{C_{in3} + C_x}{C_{in2}} \cdot \frac{C_{in2}}{C_{in1}}$

 $= \frac{C_L}{C_{in1}} \cdot \frac{C_{in3} + C_x}{C_{in3}}$ (if $C_{in3} \gg C_x$, ideal)

logical

G unaffected



size up the following gate

$f_1 = 2.6, f_2 = 3, f_3 = 3, f_4 = 3$ w/o C_x

11.7 T_{inv}

if $1.5 \times f_3 \rightarrow f'_2 = 3.25, f'_3 = 1.83$, better! (want $C_{in3} \uparrow$)

10.78 T_{inv}

↳ 4: 30/15 $\rightarrow f'_{2:4} = 3.25, 2.5, 2.27, \checkmark$

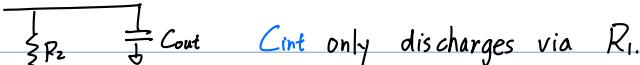
10.63 T_{inv}

Stacks 

if $A=1, B=0 \rightarrow 1$

C_{int} needs to be discharged first before M_2 can be on

B slower, delay $\propto (\text{stack level})^2$



C_{int} only discharges via R_1 .

$T_{elmore} = R_1 C_{int} + (R_1 + R_2) C_{out}$

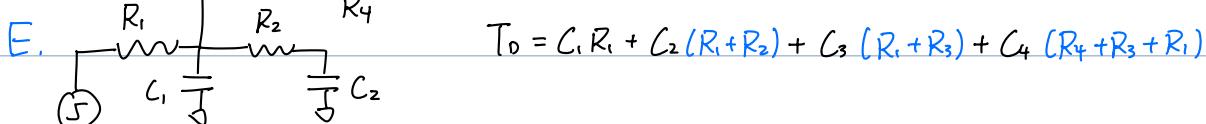
(\oplus)

(add $R_i C_i$ for each dev. in stack)

$T_D = \sum_{\text{node } i} R_{i \rightarrow \text{src}} C_i$ only valid for 1. single driving node &

2. all caps are gnded (path to R gnd)

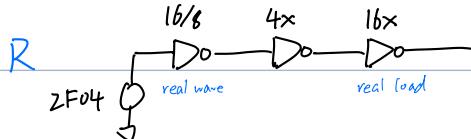
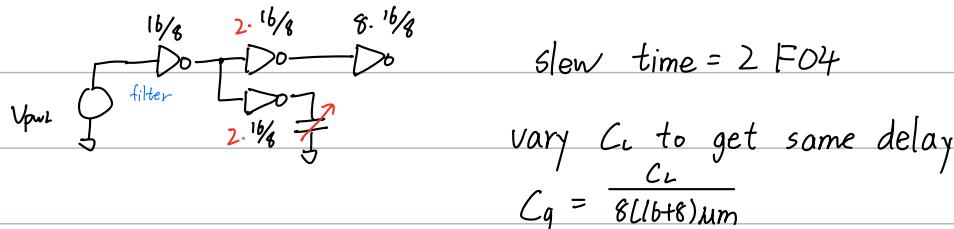
3. no resistive loops



$T_D = C_1 R_1 + C_2 (R_1 + R_2) + C_3 (R_1 + R_2 + R_3) + C_4 (R_1 + R_2 + R_3 + R_4)$

P53 C

want F04



$R_{nfet} = \frac{t_{pnl}}{C_L W_L} \times W_{nfet}$

$R_{pfet} = \frac{t_{pnr}}{C_L W_L} \times W_{pfet}$