Algebra

$\sin n\pi = 0$
$1 - \cos n\pi = 2$ for odd n
$\arctan(\frac{1}{\sqrt{3}}) = \frac{\pi}{6}$, $\arctan(\sqrt{3}) = \frac{\pi}{3}$

$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

$\sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$
$\cos x \cos y = \frac{1}{2}[\cos(x-y) + \cos(x+y)]$
$\sin x \cos y = \frac{1}{2}[\sin(x-y) + \sin(x+y)]$

$\sin a \pm \sin b = 2 \sin \frac{a+b}{2} \cos \frac{a \mp b}{2}$
$\cos a + \cos b = 2 \cos \frac{a+b}{2} \cos \frac{a-b}{2}$
$\cos a - \cos b = -2 \sin \frac{a+b}{2} \sin \frac{a-b}{2}$

$C\mathrm{c}\,(\omega_0 t + \theta) = C\mathrm{c}\,(\theta)\mathrm{c}\,(\omega_0 t) - C\mathrm{s}\,(\theta)\mathrm{s}\,(\omega_0 t)$
$C\mathrm{s}\,(\omega_0 t + \theta) = C\mathrm{s}\,(\theta)\mathrm{c}\,(\omega_0 t) + C\mathrm{c}\,(\theta)\mathrm{s}\,(\omega_0 t)$

$\theta = \tan^{-1}(-\frac{b}{a})$, $\pm\pi$ when $a < 0$
$\sin t = \cos(t - \frac{\pi}{2})$
$-\cos t = \sin(t - \frac{\pi}{2})$

$\cos x = \frac{1}{2}\,[e^{jx} + e^{-jx}]$
$\sin x = \frac{1}{2j}\,[e^{jx} - e^{-jx}]$
$e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$

$z^* = a - jb = re^{-j\theta}$
$u^* v^* = (uv)^*$

$\angle z = \tan^{-1}(\frac{b}{a})$, $\pm\pi$ in Q2 and Q3

$z^{\frac{1}{n}} = r^{\frac{1}{n}} e^{j\frac{\theta + 2\pi m}{n}}$

Integrals

$\int \cos^2 at\, dt = \frac{t}{2} + \frac{\sin 2at}{4a}$

$\int t \cos at\, dt = \frac{1}{a^2}(\cos at + at \sin at)$
$\int t \sin at\, dt = \frac{1}{a^2}(\sin at - at \cos at)$
$\int t^2 \mathrm{c}\, at\, dt = \frac{1}{a^3}(2at\mathrm{c}\, at - 2\mathrm{s}\, at + a^2 t^2 \mathrm{s}\, at)$
$\int t^2 \mathrm{s}\, at\, dt = \frac{1}{a^3}(2at\mathrm{s}\, at + 2\mathrm{c}\, at - a^2 t^2 \mathrm{c}\, at)$

$\int te^{at}\, dt = \frac{1}{a^2}\, e^{at}(at - 1)$
$\int t^2 e^{at}\, dt = \frac{1}{a^3}\, e^{at}(a^2 t^2 - 2at + 2)$

$\int e^{at}\, \cos bt\, dt = \frac{1}{a^2+b^2}\, e^{at}(a \cos bt + b \sin bt)$
$\int e^{at}\, \sin bt\, dt = \frac{1}{a^2+b^2}\, e^{at}(a \sin bt - b \cos bt)$

$\int \frac{1}{x^2+a^2}\, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$

Signals

$\mathcal{E}_f = \int_{-\infty}^{\infty} |f(t)|^2 dt$ (complex);
$P_f = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |f(t)|^2 dt$;
rms power $= \sqrt{P_f}$

Cont; analog; periodic (extension); (non/anti)causal; energy/power (both); deterministic/stochastic (info)

$\int f(t) \cdot \delta(t - t_0) dt = f(t_0)$ (f cont at t_0)

$f(2x - 6)$: shift by 6, scale by 2;
$f(2(x - 6))$: scale by 2, shift by 6

$f_e(t) = \frac{1}{2}[f(t) + f(-t)]$
$f_o(t) = \frac{1}{2}[f(t) - f(-t)]$

Systems

$\mathcal{T}$: $\sum_{k=0} a_k D^k\, y(t) = \sum_{l=0} b_l D^l\, f(t)$

Linear $\mathcal{T}[kf_1(t) + f_2(t)] = ky_1(t) + y_2(t)$.
Lin if a_k, b_l are not func of $y(t), f(t)$
E. $\sin \dot{y}(t) + t^2 y(t) = (t+3) f(t)$

Time-inv $\mathcal{T}[f(t-\tau)] = y(t-\tau)$.
a_k, b_l indep of t (const coeff)
Let $g(t) \equiv f(t-\tau)$, find $z(t) = \mathcal{T}[g(t)]$, cmp $y(t-\tau)$

Causal $y(t)$ dep only on $f(\tau)$, $\tau \le t$. Compare t and τ.

Ins/dyn y only dep f at present (no $\int$, no memory)

Invertible given $y(t)$, we can know $f(t)$

Conv prop

$c(t) \equiv \int_{-\infty}^{\infty} f(\tau) g(t-\tau)\, d\tau$
$c[n] \equiv \sum_{m=-\infty}^{\infty} f[m] g[n-m]$

$f * g = g * f$
$f * (g + h) = f * h + g * h$
$f * (g * h) = (f * g) * h$
pf: $f * (g * h) = f * (h * g)$
$= \int f(\tau_1) \int h(\tau_2)\, g(t - \tau_2 - \tau_1)\, d\tau_2\, d\tau_1$
$= \int h(\tau_2) \int f(\tau_1)\, g(t - \tau_1 - \tau_2)\, d\tau_1\, d\tau_2$

$f(t - T_1) * g(t - T_2) = c(t - T_1 - T_2)$

$f(at) * g(at) = |\frac{1}{a}|\, c(at)$ (even/odd)

$f^{(m)}(t) * g^{(n)}(t) = c^{(m+n)}(t)$
pf: $\dot{f}(\tau) = \lim_{T\to 0} f(\tau) - f(\tau - T)$

Graph: shift left by $+t$, and reflect;
Every τ replaced by $t - \tau$; Reverted

Conv table

$f(t) * \delta(t - T) = f(t - T)$
$u(t) * u(t) = t\, u(t)$

$e^{at}\, u(t) *\, u(t) = \frac{1 - e^{at}}{-a}\, u(t)$
$e^{at}\, u(t) * e^{bt}\, u(t) = \frac{e^{at} - e^{bt}}{a - b}\, u(t)$
$a = b$, $te^{at}\, u(t)$
$e^{at}\, u(t) * e^{bt}\, u(-t) = \frac{e^{at}\, u(t) + e^{bt}\, u(-t)}{b - a}$

$te^{at}\, u(t) * e^{at}\, u(t) = \frac{1}{2}\, t^2 e^{at}\, u(t)$
$t^m\, u(t) * t^n\, u(t) = \frac{m!\, n!}{(m+n+1)!}\, t^{m+n+1}\, u(t)$

Don't forget $[u(t + T_1) - u(t - T_2)]$ term

LTI response

$Q(D) y(t) = P(D) f(t)$, typically integrating f
Assume causal input $f(t) u(t)$

$y_{zs}(t) = f(t) * h(t)$ from input
$y_{zs}(0^-) = 0$, $y_{zs}(0^+) \neq 0$

Let $h(t) = \mathcal{T}[\delta(t)]$ (impulse response)
$y_{zs}(t) = \mathcal{T}[f(t)] = \mathcal{T}[f(t) * \delta(t)]$
$= \mathcal{T}[\lim \sum f(n\Delta\tau) \delta(t - n\Delta\tau) \Delta\tau]$

$= \lim \sum f(n\Delta\tau) h(t - n\Delta\tau) \Delta\tau = f * h$

$y_{zi}(t)$ from ini, $f(t) = 0$, $Qy_{zi}(t) = 0$
$y_{zi}(0^-) = y_{zi}(0^+)$, $\dot{y_{zi}}(0^-) = \dot{y_{zi}}(0^+)$

Ortho set

$\mathcal{E}_e = \int_{t_1}^{t_2} [e(t)]^2 dt = \int_{t_1}^{t_2} f^2(t) dt$
$-2 \sum c_i \int f(t) x_i(t) dt + \int (\sum c_i x_i(t))^2 dt$
$= \mathcal{E}_f - 2 \sum \langle f, x_i \rangle + (\sum c_i^2 \int x_i(t)^2 dt +$
$\sum_{i \neq j} c_i c_j \int_{t1}^{t2} x_i(t) x_j(t) dt)$

$\frac{\partial \mathcal{E}_e}{\partial c_i} = 0 = -2 \langle f(t), x_i(t) \rangle + 2 \mathcal{E}_i c_i$
$\mathcal{E}_e^{\min} = \mathcal{E}_f - \sum_{i=1}^N c_i^2 \mathcal{E}_i$
$c_i = \frac{1}{\mathcal{E}_i} \langle f, x_i \rangle = \frac{\int f(t) x(t) dt}{\int x_i^2(t) dt}$

For ortho, $E_z = E_x + E_y$
$|u + v|^2 = |u|^2 + |v|^2 + u^* v + v^* u$
$\langle x(t), y(t) \rangle = \int_{t_1}^{t_2} x(t) y(t)^*\, dt$

FS

$a_0 = \frac{1}{T_0} \int_{T_0} f(t)\, dt$
$a_n = \frac{2}{T_0} \int_{T_0} f(t) \cos(n\omega_0 t)\, dt$
$b_n = \frac{2}{T_0} \int_{T_0} f(t) \sin(n\omega_0 t)\, dt$
Energy: T_0 for $n = 0$; $T_0/2$ else

Half wave sym $f(t - \frac{T_0}{2}) = -f(t)$
$a_{n_{\text{odd}}} = \frac{4}{T_0} \int_0^{T_0/2} f(t) \cos(n\omega_0 t)\, dt$

$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$
$F_n = \frac{1}{T_0} \int_{T_0} f(t) e^{-jn\omega_0 t}\, dt$

$C_n \cos(n\omega_0 t + \theta_n) =$
$\frac{C_n}{2}\, (e^{j(n\omega_0 t + \theta_n)} + e^{-j(n\omega_0 t + \theta_n)})$
$= (\frac{C_n}{2} e^{j\theta_n}) e^{jn\omega_0 t} + (\frac{C_n}{2} e^{-j\theta_n}) e^{-jn\omega_0 t}$

$F_n = \frac{C_n}{2}\, e^{j\theta_n} = \frac{1}{2}(a_n - jb_n) = |F_n| e^{j\angle F_n}$
$F_{-n} = \frac{C_n}{2}\, e^{-j\theta_n} = \frac{1}{2}(a_n + jb_n)$

Weak: finite $\int$, fin bounds a, b, fin power
Strong: fin m/m/disc over T_0, Converge

FS prop

Time shift $f(t - t_0) \to F_n e^{-jn(\omega_0 t_0)}$
$|F_n|$ same; $\angle F_n$ shifted by $-(\omega_0 t_0) n$

Reversal $f(-t) \to F_{-n}$
Scaling $T = \frac{T_0}{a}$, $\omega = a\omega_0$

Multiplication (same T_0):
$f(t) g(t) \to F_n * G_n$
$\frac{1}{T_0} \int_{T_0} f(t) g(t) e^{jn\omega_0 t}\, dt$
$=$
$\frac{1}{T_0} \int (\sum F_m e^{jm\omega_0 t})(\sum G_k e^{jk\omega_0 t}) e^{-jn\omega_0 t}\, dt$
$= \sum_m \sum_k F_m G_k \frac{1}{T_0} \int_{T_0} e^{j(m+k-n)\omega_0 t}\, dt$
$= \sum_m \sum_k F_m G_k \langle e^{j(m+k)\omega_0 t}, e^{jn\omega_0 t} \rangle$
$= \sum_{k=-\infty}^{\infty} G_k F_{n-k}$

Conjugation $f(t)^* = F_{-n}^*$

Parseval (power sig):
$P_f = \frac{1}{T_0} \int_{T_0} f(t) f(t)^* dt$
$= \frac{1}{T_0} \int_{T_0} (\sum_n F_n e^{jn\omega_0 t})(\sum_m F_m e^{jm\omega_0 t})^* dt$
$= \sum_n \sum_m F_n F_m^*\, \frac{1}{T_0} \int_{T_0} e^{j(n-m)\omega_0 t} dt$
$= \sum_n |F_n|^2 \cdot 1$

f real $\to |F|$ even, $\angle F$ odd
f real, even $\to F$ re, e; $F_{-n} = F_n = F_n^*$
f re, od $\to F$ im, o; $-F_{-n} = F_n = -F_n^*$

$f_e(t) \to \Re\{F_n\}$
$f_o(t) \to j\, \Im\{F_n\}$

FT

Let $F(\omega) \equiv \int f(t)e^{-j\omega t}\,dt$

$F_n = \frac{1}{T_0}\int_{T_0} f(t)e^{-jn\omega_0 t}\,dt$

Limit as $\omega_0 = \Delta\omega \to 0$,
$F_n = \frac{\Delta\omega}{2\pi}\int f(t)e^{-jn\Delta\omega t}\,dt \equiv \frac{\Delta\omega}{2\pi}F(n\Delta\omega)$

$f_{T_0}(t) = \sum F_n\, e^{jn\omega_0 t} =$
$\sum \frac{\Delta\omega}{2\pi}F(n\Delta\omega)\, e^{jn\Delta\omega t}$

$f(t) = \lim_{T_0\to\infty} f_{T_0}(t) = \frac{1}{2\pi}\int F(\omega)e^{jt\omega}d\omega$

$F(\omega) = |F(\omega)|\, e^{j\angle F(\omega)}$

Real signals: amp and phase symmetry

Existence: energy signal ($|e^{-j\omega t}| = 1$)
Strong: fin num max/min/discont

FT Table

$\delta(t) \to 1$
$1 \to 2\pi\delta(\omega)$

$e^{j\omega_0 t} \to 2\pi\delta(\omega-\omega_0)$
$\cos\omega_0 t \to \pi[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)]$
$\sin\omega_0 t \to j\pi[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)]$

$\sum\delta(t-nT_0) \to \omega_0\sum\delta(\omega-n\omega_0) \quad \omega_0 = \frac{2\pi}{T_0}$

$e^{-at}\,u(t) \to \frac{1}{a+j\omega} \qquad a>0$
$e^{-a|t|} \to \frac{2a}{a^2+\omega^2} \qquad a>0$
$u(t) = \lim_{a\to 0} e^{-at}u(t) \to \lim \frac{1}{a+j\omega}$
$\quad = \lim(\frac{a}{a^2+\omega^2} - j\frac{\omega}{a^2+\omega^2}) = \pi\delta(\omega) + \frac{1}{j\omega}$
$\mathrm{sgn}(t) \to \frac{2}{j\omega}$

$t^n e^{-at}u(t) \to \frac{n!}{(a+j\omega)^{n+1}} \qquad a>0$

$\mathrm{c}\,\omega_0 t\, u(t) \to \frac{\pi}{2}(\delta(\omega-\omega_0)+\delta(\omega+\omega_0)) + \frac{j\omega}{\omega_0^2-\omega^2}$
$\mathrm{s}\,\omega_0 t\, u(t) \to \frac{\pi}{2j}(\delta(-)-\delta(+)) + \frac{\omega_0}{\omega_0^2-\omega^2}$
$e^{-at}\cos\omega_0 t\, u(t) \to \frac{a+j\omega}{(a+j\omega)^2+\omega_0^2} \qquad a>0$
$e^{-at}\sin\omega_0 t\, u(t) \to \frac{\omega_0}{(a+j\omega)^2+\omega_0^2} \qquad a>0$

$\mathrm{rect}(\frac{t}{\tau}) \to \tau\,\mathrm{sinc}(\frac{\tau}{2}\omega)$
$\frac{W}{\pi}\,\mathrm{sinc}(Wt) \to \mathrm{rect}(\frac{\omega}{2W})$

$\triangle(\frac{t}{\tau}) \to \frac{\tau}{2}\,\mathrm{sinc}^2(\frac{\tau}{4}\omega)$
$\frac{W}{2\pi}\,\mathrm{sinc}^2(\frac{W}{2}t) \to \triangle(\frac{\omega}{2W})$

$[\omega^2 \mathrm{r}\,(\frac{\omega}{2\omega_0})] \leftarrow \frac{1}{2\pi}\frac{e^{j\omega t}}{(jt)^3}(-\omega^2t^2 - 2j\omega t + 2)_{-\omega_0}^{\omega_0}$
$\quad = \frac{(\omega_0^2t^2-2)\sin\omega_0 t + 2\omega_0 t\cos\omega_0 t}{\pi t^3}$

$[\frac{|\omega|}{\omega_0}\,\mathrm{rect}(\frac{\omega}{2\omega_0})] \leftarrow \frac{\cos\omega_0 t + \omega_0 t\sin\omega_0 t - 1}{\omega_0\pi t^2}$

Frequency domain prop

Linearity

Time shift $f(t-t_0) \to F(\omega)e^{-jt_0\omega}$
$\quad |F|$ unchanged; $\angle F = -t_0\omega$, lin shift

Freq shift $f(t)e^{j\omega_0 t} \to F(\omega-\omega_0)$

Duality $f(t) \to F(\omega)$, $F(t) \to 2\pi f(-\omega)$
$\quad$ pf. $f(t) = \frac{1}{2\pi}\int F(\lambda)e^{jt\lambda}\,d\lambda$
$\quad\quad 2\pi f(-t) = \int F(\lambda)e^{-tj\lambda}\,d\lambda = \mathcal{F}[F(\lambda)]$

Reversal $f(-t) \to F(-\omega)$
Scaling $f(at) \to \frac{1}{|a|}F(\frac{\omega}{a})$

Convolution $f*g \to FG$, $fg \to \frac{1}{2\pi}F*G$
$\quad \mathcal{F}[f*g] = \int e^{-j\omega t}\int f(\tau)g(t-\tau)d\tau dt$
$\quad\quad = \int f(\tau)\mathcal{F}[g(t-\tau)]d\tau$
$\quad\quad = \int f(\tau)G(\omega)e^{-j\omega\tau}d\tau$
$\quad \frac{1}{2\pi}\mathcal{F}^{-1}[F*G]$
$\quad\quad = (\frac{1}{2\pi})^2\int e^{j\omega t}\int F(\lambda)G(\omega-\lambda)d\lambda d\omega$

Diff $f^{(n)}(t) \to (j\omega)^n F(\omega)$ (diff $e^{j\omega t}$)

Int $\int_{-\infty}^t f(\tau)d\tau \to \frac{1}{j\omega}F(\omega) + \pi F(0)\delta(\omega)$
$\quad \int = f(t)*u(t) \to F(\omega)U(\omega)$
$\quad U(\omega) = \lim\frac{1}{a+j\omega} = \lim(\frac{a}{a^2+\omega^2} - j\frac{\omega}{a^2+\omega^2})$
$\quad\quad = \pi\delta(\omega) + \frac{1}{j\omega}$
$(\int\frac{a}{\omega^2+a^2}d\omega = \tan^{-1} = \pi)$

Conjugation $f(t)^* \to F(-\omega)^*$

Symmetry Re $\to$ mag even, phase odd
$\quad (F(-\omega) = F(\omega)^*)$
$\quad$ real, even $\to$ real, even;
$\quad$ real, odd $\to$ imaginary, odd

f even: $F(\omega) = 2\int_0^\infty f(t)\cos(\omega t)dt$
f odd: $F(\omega) = -2j\int_0^\infty f(t)\sin(\omega t)dt$

Parseval

$E_f = \int|f(t)|^2dt = \frac{1}{2\pi}\int|F(\omega)|^2d\omega$ for energy sig
$\quad = \int ff^*dt = \int f(t)\mathcal{F}^{-1}[F(-\omega)^*]dt$
$\quad = \int f(t)\frac{1}{2\pi}\int F(-\omega)^*e^{j\omega t}d\omega dt$
$\quad = \frac{1}{2\pi}\int f(t)\int F(\lambda)^*e^{-jt\lambda}d\lambda dt$
$\quad = \frac{1}{2\pi}\int F(\lambda)^*\int f(t)e^{-j\lambda t}dtd\lambda$

$\Delta E_f = \frac{2}{2\pi}\int_{\omega_1}^{\omega_2}|F(\omega)|^2d\omega$

Autocorrelation
$\psi_f(t) \equiv \int f(\tau)f(\tau-t)d\tau \to |F(\omega)|^2$

Modulation

$m(t)\cos(\omega_c t) \to \frac{1}{2}[M(\omega+\omega_c) + M(\omega-\omega_c)]$
$\quad e(t) = m(t)\cos^2\omega_c t$

$E(\omega) = \frac{1}{2}M + \frac{1}{4}[M(\omega+2\omega_c) + M(\omega-2\omega_c)]$

SSB 1/4 gain

$\phi_{\mathrm{AM}}(t) = [A+f(t)]\cos(\omega_0 t)$
$A \geq f(t)$ for all t

modulation index $\mu \equiv f_{\max}/A$
$\mu = \infty$, suppressed carrier; $\mu = 1$, marginal

LTIC sys trans, (marginally) stable

Let $e^{j\omega t} \Rightarrow H(\omega)e^{j\omega t}$

$\lim\sum\frac{F(n\Delta\omega)\Delta\omega}{2\pi}e^{jn\Delta\omega t} \Rightarrow$
$\lim\sum\frac{F(n\Delta\omega)H(n\Delta\omega)\Delta\omega}{2\pi}e^{jn\Delta\omega t}$
$y(t) = \frac{1}{2\pi}\int F(\omega)H(\omega)e^{j\omega t}d\omega$
$Y(\omega) = F(\omega)H(\omega)$

Distortionless $y(t) = kf(t-t_d)$, so
$H(\omega) = ke^{-j\omega t_d}$

Payley-Wiener H realizable, h causal iff
$\int\frac{|\ln|H(\omega)||}{1+\omega^2}d\omega < \infty$ (consecutive 0s)
$\quad$ Truncate $\hat{h}(t) = h(t)u(t)$

Periodic FT

$f(t) = \sum_n F_n e^{jn\omega_0 t}$
$\mathcal{F}[f(t)] = 2\pi\sum_n F_n\delta(\omega-n\omega_0)$

$Y = F(\omega)H(\omega) =$
$2\pi\sum F_nH(n\omega_0)\delta(\omega-n\omega_0)$
$Y_n \equiv F_nH(n\omega_0)$. Periodic with same ω_0

Eigen: $f(t) = e^{j\omega_0 t}$, $Y_1 = H(1\omega_0)$,
$y(t) = H(1\omega_0)e^{j\omega_0 t}$

$f(t) = \cos(\omega_0 t + \theta)$, assume $h(t)$ real

$y = \frac{1}{2}(e^{j(\theta+\omega_0 t)}H(\omega_0) + e^{-j(\theta+\omega_0 t)}H(-\omega_0))$
$\quad = |H(\omega_0)|\cos(\omega_0 t + \theta + \angle H(\omega_0))$

$\cos 2t * e^{-3t}\,u(t) \equiv f*h$
$\quad = |H(2)|\cos(2t + \angle H(2))$

Sampling

$\overline{f}(t) \equiv f(t)\delta_{T_s}(t) = \sum f(nT_s)\delta(t-nT_s)$
$\overline{F}(\omega) = \frac{1}{2\pi}F(\omega) * [\frac{2\pi}{T_s}\sum\delta(\omega-n\omega_s)]$
$= \frac{1}{T_s}\sum F(\omega-n\omega_s)$

$\omega_s \geq 4\pi B$, $F_s \geq F_N \equiv 2B$

Intrapolation when $F_s = 2B$
$F(\omega) = \overline{F}(\omega)T_s\,\mathrm{rect}(\frac{\omega}{4\pi B})$
If $F_s = 2B$, $f(t) = \overline{f}(t) * \frac{2B}{F_s}\,\mathrm{sinc}(2\pi Bt)$
$\quad = \sum_n f(nT_s)\delta(t-nT_s) * \mathrm{sinc}(2\pi Bt)$
$\quad = \sum_n f(nT_s)\,\mathrm{sinc}(2\pi Bt - n\pi)$
ana FS, with basis sinc, interpolation formula

If $F_s > 2B$, $f(t) = \sum f(nT_s)w(t-nT_s)$
not sinc weight
for some relaxed LP filter $w(t)$

Anti-alias before sampling: LPF of $F_s/2$

Practical sampling
$p_T(t) = \frac{\tau}{T_s} + \sum(\frac{2}{\pi n}\sin(n\pi\frac{\tau}{T_s}))\cos(n\omega_s t)$
$P_T(\omega) = 2\pi\frac{\tau}{T_s}\delta(\omega)$
$\quad + \sum\frac{\pi 2\sin(\ldots)}{\pi n}[\delta(\omega+n\omega_s) + \delta(\omega-n\omega_s)]$

Square $\frac{4}{\pi}(\cos t - \frac{1}{3}\cos 3t + \frac{1}{5}\cos 5t - \ldots)$
Triangle $\frac{8}{\pi^2}(\sin t - \frac{1}{9}\sin 3t + \frac{1}{25}\sin 5t - \ldots)$
Sawtooth $\frac{2}{\pi}(\sin t - \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t - \ldots)$
δ train $\delta_{T_0}(t) = \sum_{n=-\infty}^{\infty}\delta(t-nT_0)$