

Passive sign convention: current enter at \oplus

$$\Delta q = i \Delta t \quad p(t) = \frac{dw}{dt} = \frac{dw}{dq} \frac{dq}{dt} = v(t) i(t)$$

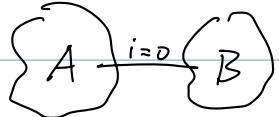
Assumptions 1. Lumped action is instantaneous

2. No ext. fields (E/M)

3. Electrical neutrality

KCL Two parts connected via only 1 wire, $i=0$

☒ antenna, not lumped (too long)



Node a region of circuit w/ connections

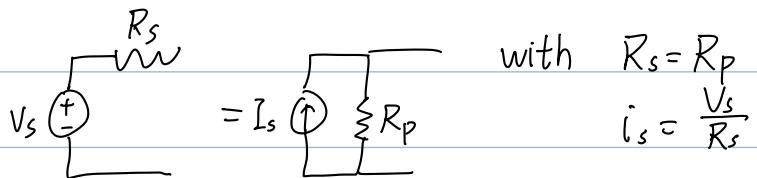
Circuit w/ n nodes \rightarrow $n-1$ indep. KCL

1. \sum_{enter} 2. \sum_{leave} 3. $\sum = \bar{\sum}$

KVL 1. $\sum = 0$ 2. $\sum_1 = \sum_2$

#KCL + #KVL = # elements

Ext eq. cir.



(X) - 2 v.s. in parallel (eg. short) (if $V_s = 0, \infty$ solns.)

- 2 i.s. in series (\sim open) ()

Wheatstone bridge: 2 v. dividers

Active circuits - Linear dependent

VCVS voltage controlled v. source

M V_C

v. gain

VCCS

$g_m V_C$

trans conductance

CCCS

B i_C

i. gain

CC VS

$R_m i_C$

trans resistance

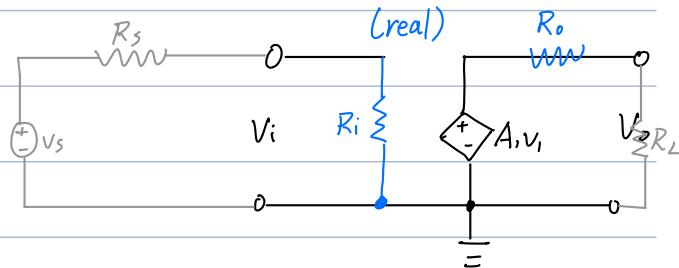
if indep. = 0 \rightarrow dep. = 0

Voltage amp., linear region & saturation

saturation \rightarrow distortion

$$V_I = \frac{R_i}{R_s + R_i} V_S \rightarrow \text{Need } R_i \gg R_s$$

$$V_O = \frac{R_L}{R_o + R_L} A_V V_I \rightarrow \sim R_o \ll R_L$$



OpAmp V_o come from $\pm V_{cc}$

$$V_o = A V_D = A (V_p - V_n) \quad \text{At linear}$$

ideal: input $i \rightarrow 0$

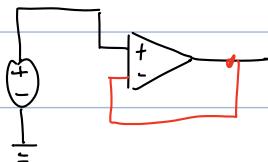
$$R_o \rightarrow 0$$

Negative feedback ensures $V_n = V_I$

$$A \rightarrow \infty$$

V_o, i_o no limit

buffer



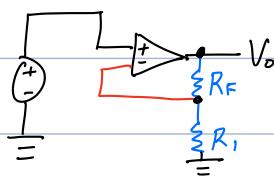
$$V_D = V_I - V_o = V_I - A V_D$$

neg. feedback

$$V_D \approx 0$$

$$V_o \approx V_I \quad \left(= \frac{A}{A+1} V_I \right)$$

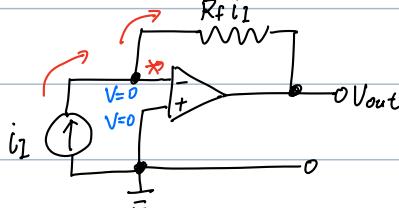
Ni. amp.



Voltage divider

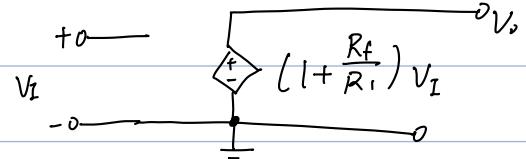
$$V_o = \left(1 + \frac{R_f}{R_i} \right) V_I \xrightarrow{\text{ratio, makes it (E. temp.) indep.}}$$

$i \rightarrow V$ converter

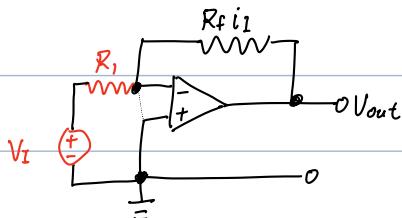


$$V_o = -R_f i_1$$

E. transducer, photodiode



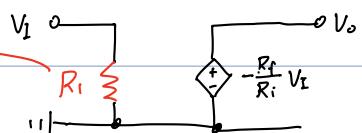
I. amp.



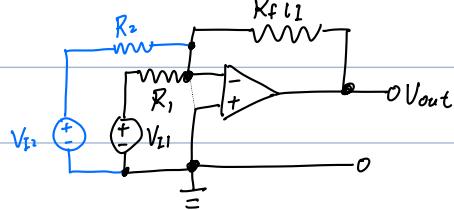
$$i_2 \rightarrow V_{I2}, R, V_o = -\frac{R_f}{R_1} V_I$$

problem!

Typically start with n1 (buffer)



Sum amp.

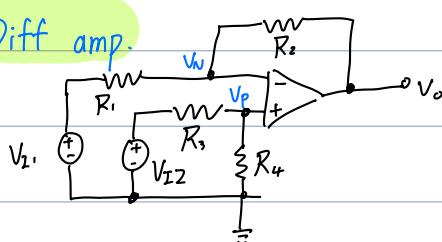


Combine currents

$$V_o = -\left(\frac{R_f}{R_1} V_{I1} + \frac{R_f}{R_2} V_{I2}\right)$$

Cascade gain X

Diff amp.

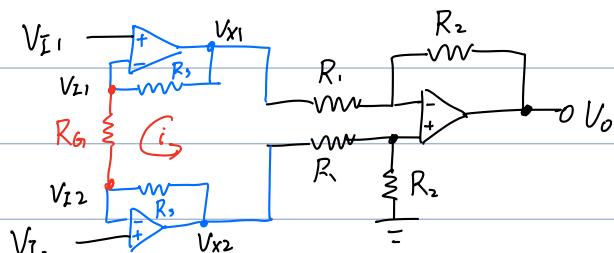
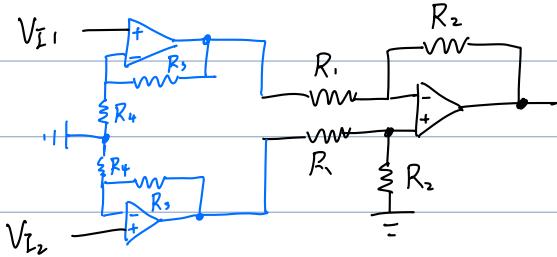


$$\begin{aligned} KCL \quad \frac{V_{I1} - V_N}{R_1} &= \frac{V_N - V_O}{R_2} \\ V_P &= V_{I2} \frac{R_4}{R_3 + R_4} \\ V_N &= V_P \end{aligned}$$

$$R_2 (V_{I1} - V_{I2} \frac{R_4}{R_3 + R_4}) = R_1 V_{I2} \frac{R_4}{R_3 + R_4} - V_O$$

If $\frac{R_3}{R_4} = \frac{R_1}{R_2}$, $V_O = \frac{R_2}{R_1} (V_{I2} - V_{I1})$

w/ buffer:



instrumentation amp.

$$i = \frac{V_{N1} - V_{N2}}{R_G} = \frac{V_{I1} - V_{I2}}{R_G}$$

$$V_{X1} - V_{X2} = (R_G + 2R_3) i$$

$$= \left(1 + \frac{2R_3}{R_G}\right) (V_{I1} - V_{I2})$$

$$V_O = -\frac{R_2}{R_1} \left(1 + \frac{2R_3}{R_G}\right) (V_{I1} - V_{I2})$$

Positive feedback

blow up! (until sat.)

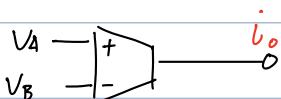


unstable eq.

Comparator no feedback, saturated, faster

Transconductor V_{CCS} , $i_o = G_m (V_A - V_B)$

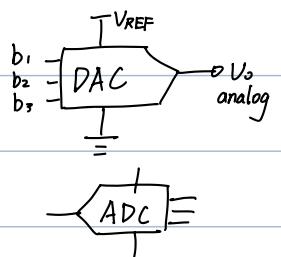
E. transistor, but idealized linear relation



D \rightarrow A converter (sum amp.), V_o is inverted!

A \rightarrow D converter compare V_I with diff. refs. (E. 3-bit: $0.5V \sim 6.5V$)

\Rightarrow logic converter \rightarrow output



Systematic tech.

Node voltage analysis E. circuit w/ \textcircled{D} , \textcircled{m} . Use KCL on every node

E. book p 228, $\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} i_{s1} \\ i_{s1} + i_{s2} \end{bmatrix}$

std. form $\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} i_{s1} \\ i_{s1} + i_{s2} \end{bmatrix}$

unknown node \checkmark indep. i. sources

$\underline{G} \underline{v} = \underline{i}$

$\underline{v} = \underline{G}^{-1} \underline{i}$

Cramer's rule

$$\Delta = G_1 G_2 + G_2 G_3 + G_3 G_1$$

To find V_2 , find Δ_2 by replacing 2nd col. of \underline{G} by \underline{i} and find det

$$\Delta_2 = \begin{vmatrix} G_1 + G_2 & i_{s1} \\ -G_2 & -i_{s1} + i_{s2} \end{vmatrix} = -G_1 i_{s1} + (G_1 + G_2) i_{s2}$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{-G_1}{G_1 G_2 + G_2 G_3 + G_3 G_1} i_{s1} + \frac{G_1 + G_2}{G_1 G_2 + G_2 G_3 + G_3 G_1} i_{s2}$$

+ dep. source $\textcircled{\text{P}}$ 1. VCCS (similar)

2. CCCS write i of interest in terms of node voltages

+ voltage source (don't know i thru $\textcircled{+}$)

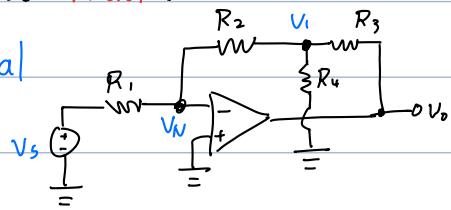
Already gives an eq. relating 2 node v.

Create supernode by finding all i crossing the $\textcircled{+}$ and any || R.

+ dep. voltage source $\textcircled{\text{P}}$ (same thing)

+ OpAmp Use model!

E. ideal



KCL

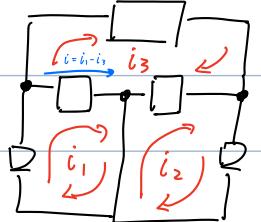
$$G_1(V_N - V_S) + G_2(V_N - V_1) = 0$$

$$G_2(V_1 - V_N) + G_4 V_1 + G_3(V_1 - V_o) = 0$$

$V_N = 0$ due to negative feedback

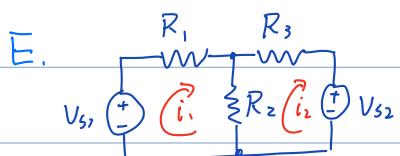
$$V_o = -V_S \left(\frac{R_2}{R_1} + \frac{R_3}{R_1} + \frac{R_2 R_3}{R_1 R_4} \right)$$

Mesh-current analysis - only valid for planar circuits (no $\frac{1}{T}$)



Mesh: loop w/o any inner loops

Unknown: mesh currents (usually same dir.)



$$\text{KVL} \quad -v_{s1} + R_1 i_1 + R_2 (i_1 - i_2) = 0 \Rightarrow (R_1 + R_2) i_1 - R_2 i_2 = v_{s1}$$

$$v_{s2} + R_2 (i_2 - i_1) + R_3 i_2 = 0 \Rightarrow -R_2 i_1 + (R_2 + R_3) i_2 = v_{s2}$$

$$B_i = v_s$$

* if see a $-$ in (\oplus) , put a $-$, when thru R , use $+$

+ ①, predefine an i branch, create **supermesh** skipping ①

Node v

Mesh i

planar only!

good for

parallel

series

solving for

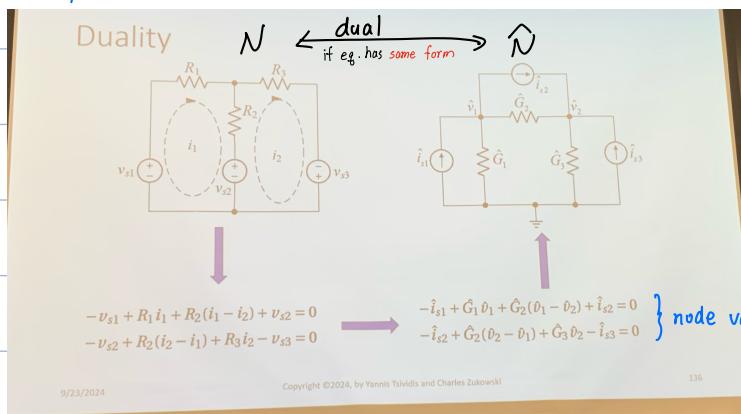
v

i

Always simplify!

Duality

node \leftrightarrow mesh



$V \rightarrow i$, $R \rightarrow G$, series \rightarrow parallel

exact dual if same numerical value

Pathological

$\Delta = 0$, or overconstrained

put small R in series w/ ①, large R in w/ ①

Linear circuit prop. & thm

$$y = C_1 X_{S1} + \dots + C_k X_{Sk}$$

↑↑↑ ↓↓↓
indep. sources (inputs)

Proportionality & superposition

$$y = \sum_{i=1}^k (output \text{ w/ only one source on})$$

Kill: Short ②, open ①

Eq.R only R and ◊

$$v = R_{eq} i$$

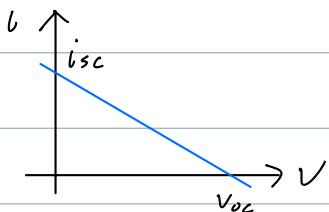
+ indep. sources ○

By conv., i is leaving the + term.

V_{oc} voltage when terminal of interest is open

i_{sc} current when terminal is shorted

Two-term $i-v$ char



Thévenin V_{oc} series w/ R_{Th} → R_{eq} , killing all indep. sources w/i

Pf. Superpos

Drive circuit by i

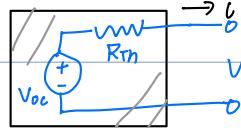
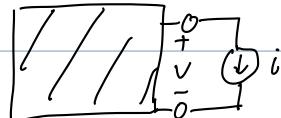
1. ext. source = 0, $v = V_{oc}$

indep!

2. int. source = 0, $v = R_{Th}(-i)$

1+2

$$v = V_{oc} - R_{Th} i$$



Norton i_{sc} parallel w/ R_{Th}

T vs N

$$R_{Th} = \frac{V_{oc}}{i_{sc}}$$

