

Digital

Inverter (ideal) ideality based on suppression on noise/error

E. chain $V_{in} = \epsilon \xrightarrow{\rightarrow 0} \text{---} \text{---} \text{---} V_{out}$

E. extremely nonideal (slope = -1 curve) $\rightarrow V_{out} = V_{DD} - \epsilon \rightarrow$ not suppressed \therefore

slope = -1 $\rightarrow V_{ZH}, V_{ZL} \rightarrow$ H/L suppressed.

CMOS $V_{DD} - V_{ZH} = NMH, V_{ZL} - 0 = NML$

TTL $V_{max} \neq V_{DD}, V_{min} \neq 0$

Do 1. $i_d =$ p sat $i_{dp} = i_{dn} = k' \left(\frac{W}{L} \right)_p (V_{gs} - V_{Tp})^2 = k' \left(\frac{W}{L} \right)_n (V_{gs} - V_{Tn} - \frac{1}{2} \frac{V_{out}}{V_{ds}} V_{ds}) V_{ds}$

2. $\frac{\partial V_{out}}{\partial V_{in}} = -1$

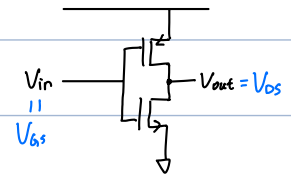
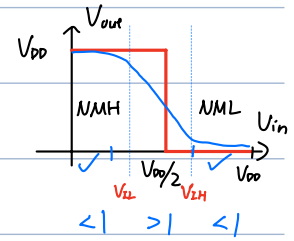
3. set $V_{in} = V_{ZH} \rightarrow$ solve for V_{out}

4. plug back to 1.

if PMOS used to pull down, can't continue beyond V_{Tp}

present drain side to output, if source side \rightarrow ~ src follower, X

E. if flip I/O, and P/N, won't work \rightarrow source follower, noninverting



N	P
cutoff	linear
$V_{ZL} \rightarrow$ sat	lin/tri
sat	sat
$V_{ZH} \rightarrow$ tri/lin	sat
lin	cut

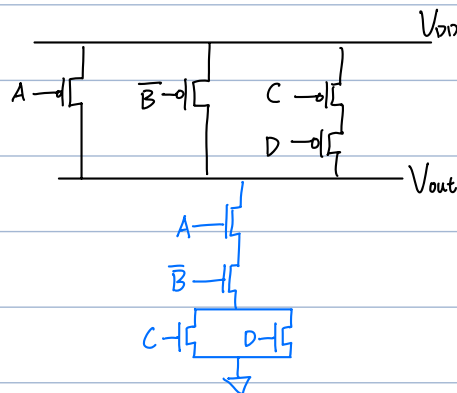
AND need pull down on 0 if any input is 0 \rightarrow PMOS only \rightarrow X

NAND up on 0 \rightarrow PMOS pullup \rightarrow \checkmark \rightarrow AND

down on both 1 \rightarrow NMOS pull down

NOR

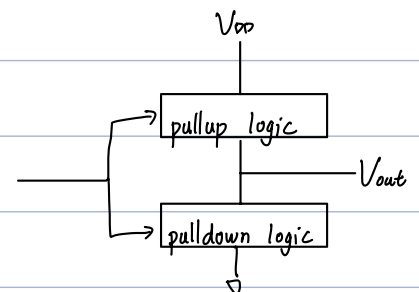
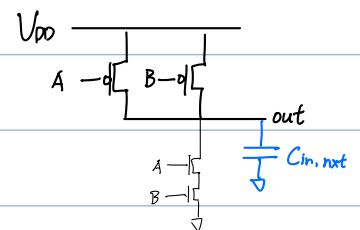
custom pullup (SOP)



pull down

$$Y = \overline{A \overline{B} (C + D)} = \overline{A} + B + \overline{C} \overline{D}$$

$$\overline{Y} = A \overline{B} (C + D)$$



Delay prop T_{pLH} T_{pHL} , 50%
 $T_p \propto \frac{C}{k' \frac{W}{L} V_{DD}}$ for long channel

$$k' \frac{W}{L} \propto I_D \text{ (all regions)}$$

$$\text{sat } I_D = \frac{1}{2} k' \frac{W}{L} (V_{DD} - V_T)^2$$

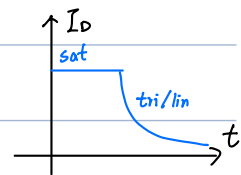
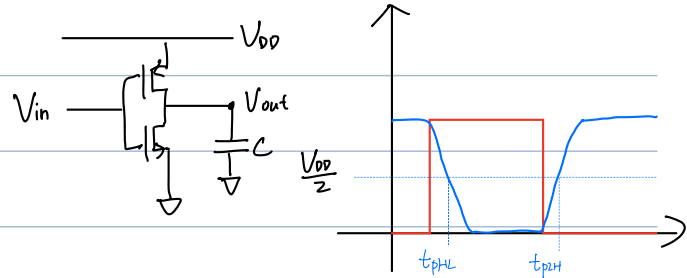
$$\text{tri} = k' \frac{W}{L} (V_{DD} - V_T - \frac{V_{sp}}{2})^2$$

$$\text{lin} = V k' \frac{W}{L} (V_{DD} - V_T)$$

V_{DD}

$$\text{lin. } V_{out} = V_{DD} (1 - e^{-t/R_C}) \quad e^{-t/R} \text{ outweighs } V_{DD}$$

$$C \propto WL \rightarrow \text{reduce } L$$



Short-channel \rightarrow sat \rightarrow assume resistive pull up/down

$$\text{E. pulldown} \rightarrow \text{PMOS off, NMOS} \rightarrow R_N \rightarrow V_{out}(t) = V_{DD} e^{-t/R_N C}$$

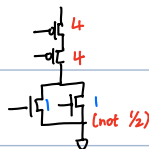
$$(50\%) \quad t_{pHL} \approx \ln 2 R_N C, \quad R_N \text{ is empirical}$$

symmetry $(k' \frac{W}{L})_n$ and $(k' \frac{W}{L})_p$ (typically $k'_n > k'_p$, need make p wider)

series, looks like $\frac{W}{2L} \rightarrow$ slower $\therefore L \rightarrow$ need double widths } worst case

parallel $\frac{2W}{L} \rightarrow$ faster, but worst case same

E. NOR



$$k'_n = 2k'_p$$

\rightarrow 10 units

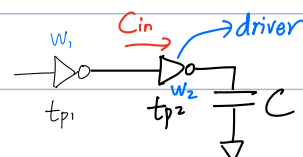
E. NAND

2 units for N, 2 for P \rightarrow 8 units \rightarrow smaller!

D_0 drives an off-chip C ($\gg C_{gs}, C_{gd}$)

$$t_{p2} = \frac{C}{k' \frac{W_2}{L} V_{DD}}$$

$$C_{in2} \propto W_2 L \text{ is large, since } (\frac{W}{L})_2 \text{ is large } t_{p1} = \frac{C_{in2}}{k' \frac{W_1}{L} V_{DD}}$$

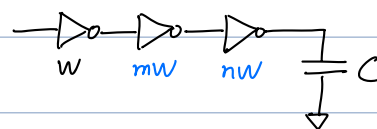


$(\frac{W}{L})_1$ need large

$\rightarrow L$ typ. fixed, $W \uparrow$

$$t_{pA} \propto \frac{mWL}{k' \frac{W}{L} V_{DD}} \quad t_{pB} \propto \frac{nWL}{k' \frac{W}{L} V_{DD}}$$

$$t_{pA} + t_{pB} \propto \frac{L^2}{k' V_{DD}} (m + \frac{n}{m}) \rightarrow m = \sqrt{n} \text{ at min delay}$$



Oscillator $T = 2.3 t_d = 6 t_d$

Phase lock loop (PLL) match TX/RX phase

limit (starve) gate current, set I mirror, so T match

Single

$$Z_{LC} \approx \frac{j\omega L}{1 - \omega^2 LC}, \infty \text{ if } \omega_0 = \frac{1}{\sqrt{LC}}$$

$$i_c = C \frac{dv}{dt} = C \omega_0 \cos(\omega_0 t) = \sqrt{\frac{C}{L}} \cos(\omega_0 t)$$

$$i_L = \frac{1}{L} \int v dt \approx -\sqrt{\frac{C}{L}} \cos(\omega_0 t)$$

XTAL
Crystal

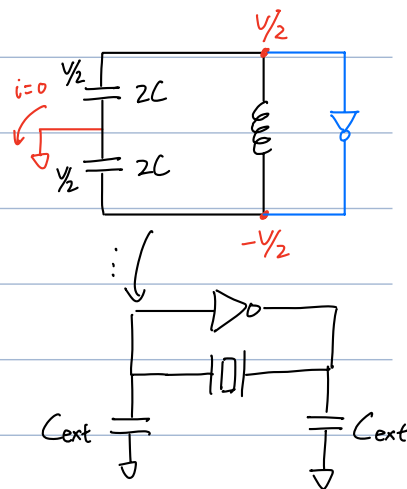
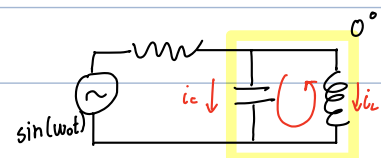
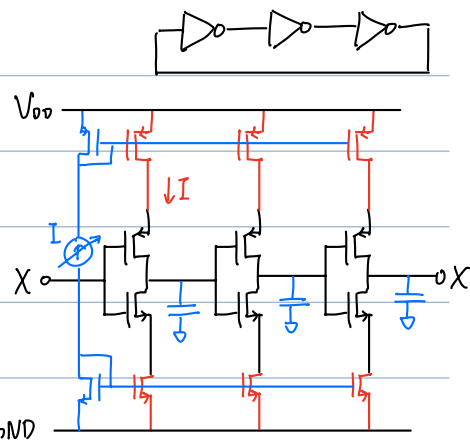
$$Z_{\text{crystal}} = \frac{C_0 \text{ (para)}}{1 + \frac{L_c R_c}{C_0}} \approx \frac{j}{\omega} \frac{\omega^2 L_c C_0 - 1}{(C_0 + C_c) - \omega^2 L_c C_c C_0}$$

can be made ∞ (parallel res.) or 0 (series res.)

parallel $f_{\text{parallel}} \approx \frac{1}{2\pi \sqrt{L_c C_c}} \sqrt{1 + \frac{C_c}{C_0}} \approx \frac{1}{2\pi \sqrt{L_c C_c}}$

if external caps added, $L = 1$ if $C_0 \gg C_c$

\Rightarrow is positive feedback supply



CMOS power no continuous power

Transition (driving a C) $1 \rightarrow 0$: C's energy $\frac{1}{2} C V_{DD}^2$ diss from Q

$$0 \rightarrow 1 \quad P_a = I_D V_{SD} = I_D (V_{DD} - V_c)$$

$$p_a(t) = C \frac{dv_c}{dt} (V_{DD} - V_c)$$

$$w = \int_0^\infty p(t) dt$$

$$= \int_0^{V_{DD}} C (V_{DD} - V_c) dV_c = \frac{1}{2} C V_{DD}^2$$

Total: $C V_{DD}^2 \rightarrow p = f C V_{DD}^2$

E. Apple M4, 4GHz ($\times \frac{1}{500}$), 28G transistors ($\times \frac{1}{4}$), $C = 1fF$, $V_{DD} = 0.9V$

$$\frac{4E9}{500} \times 1E-15 \times 0.9^2 \times \frac{28E9}{4} = 45W$$

if P/N both on $\rightarrow I_{\text{crowbar}} \rightarrow$ want low (sharp waveform)

(assume matching $(k' \frac{W}{L})_{p,n}$)

avoid staying near $V_{DD}/2$

typically ignored

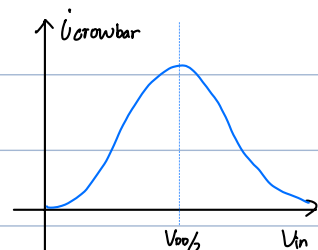
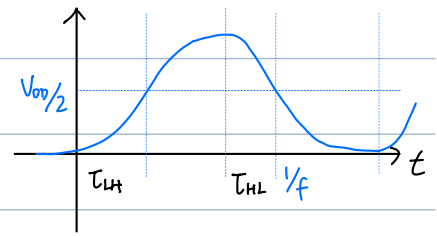
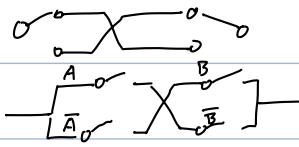


Figure of merit: power-delay product = $f C V_{DD}^2 t_{P,avg} = \frac{1}{2} C V_{DD}^2$

E. clk an DO to the fastest $t_p = \frac{1}{2}(\tau_{LH} + \tau_{HL}) \approx \frac{1}{2} \frac{1}{f}$



Pass-transistor Logic E. XOR switch

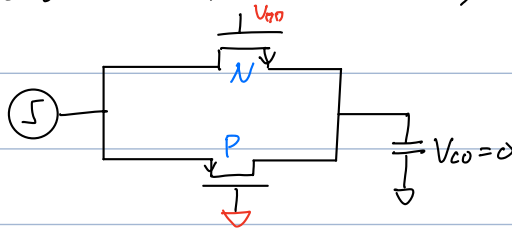


N \rightarrow source follower \rightarrow max: $V_{DD} - V_T$

pullup \checkmark if $V_p = 0$, $V_c = V_{DD}$ (symmetrical)

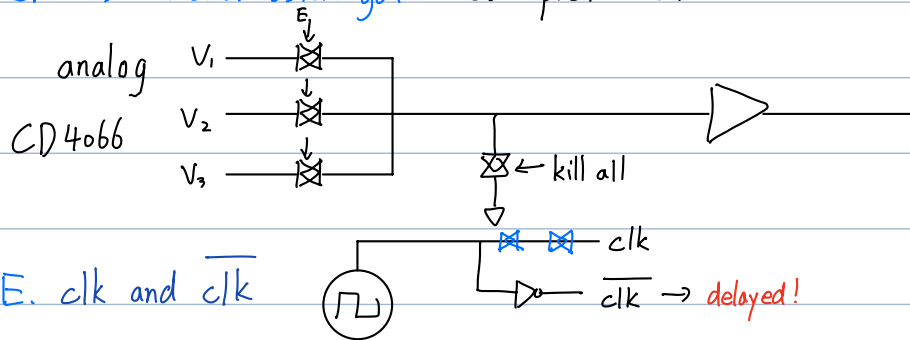
P oppo ($\checkmark 0 \rightarrow V_{DD}$, $\times V_{DD} \rightarrow 0$) \Rightarrow CMOS

CMOS sw



For rising, when N fails, P takes over

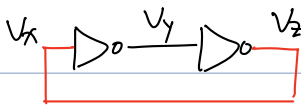
CMOS transmission gate (sample/hold)



E. clk and $\overline{\text{clk}}$

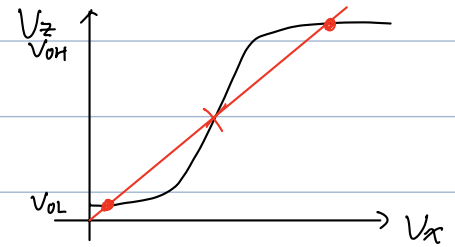
fix: buffer clk w/ trans. gates \rightarrow passive delay

Bistable



Delay: curve

Loadline: $V_x = V_z$



\rightarrow SRAM, fast, $4Q + 2SW = 6$ how to set?

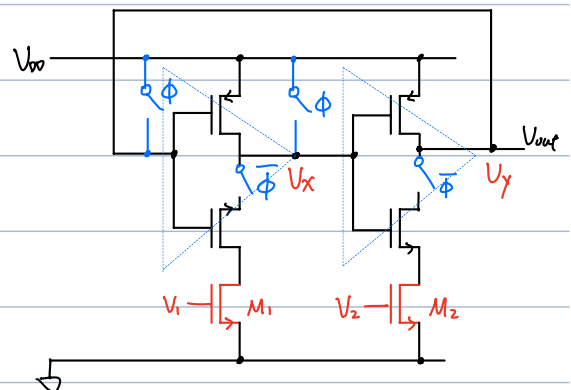
1. Brute force: switch V_x/V_y to 0

(crowbar current while switching)

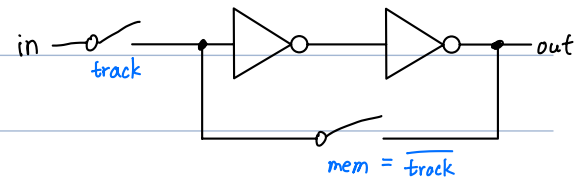
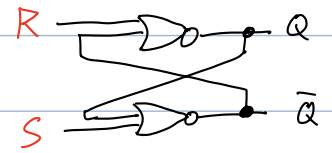
2. clocked comparator clk: ϕ

M_1, M_2 current-starve pulldown.

if $V_1 > V_2$, $V_x \rightarrow 0$ faster than $V_y \rightarrow$ pos. feedback



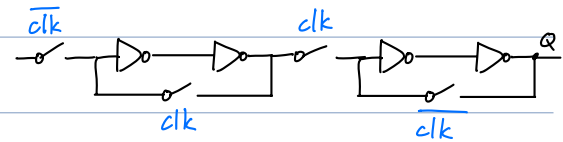
SR	S	R	$Q(t+1)$	$\bar{Q}(t+1)$
set	1	0	1	0
res	0	1	0	1
hold	0	0	$Q(t)$	$\bar{Q}(t)$
invalid	1	1	0	0



Dynamic gates track / hold

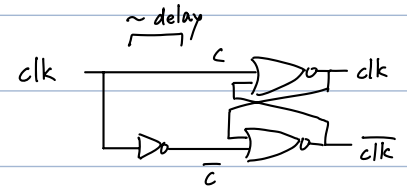
DFF updated at CLK rising edge

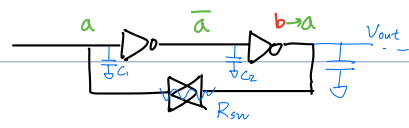
Master-slave Take turns track & hold

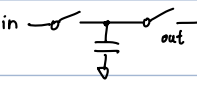


Non-overlapping $\text{clk}, \bar{\text{clk}} \rightarrow \sim \text{SR}$ latch ensures not both high

If inputs both high \rightarrow both outputs 0



Latch  wants to overwrite $b \rightarrow a$, b needs to drive $C_{in,next}$ and $R_{sw} C_1 \rightarrow$ slower than charging C_2 , a wins

dynamic DRAM (sample-hold) 

long readline \rightarrow huge $C_{line} \gg C_{mem}$

Store $Q = C_{mem} V_{DD}$

Recall Charge C_{line} starts w/ $\frac{V_{DD}}{2}$

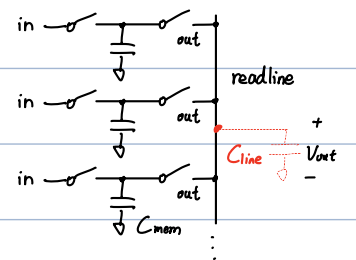
$$V_{out} = \frac{V_{DD}}{2} \frac{C_{line}}{C_{mem} + C_{line}} \text{ if } C_{mem} = 1; \frac{V_{DD}}{2} \left(\frac{C_{line} + 2C_{mem}}{C_{mem} + C_{line}} \right) \text{ if } C_{mem} = 0$$

Make a sensitive comparator (so DRAM slower than SRAM)

1. precharge readlines B and \bar{B} to $\frac{V_{DD}}{2}$

2. select cell. B/\bar{B} will shift slightly

3. Diff. amp when latch turned on $\xrightarrow{\text{pos feedback}}$ saturates



CLK recovery TX clk match & sync w/ RX clock, RX sample w/i a period!

1. Avoid consecutive 0/1s in encoding

E. alphabet: 26 \leq 5-bits (32) \rightarrow avoid 00000 11111 01111 10000 00001 11110

2. extract clk from data

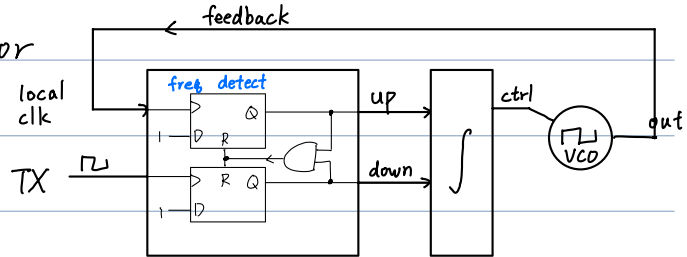
① rough tuning of local clk: TX sends oscillating 1/0 \rightarrow compare TX/local clk counter

Coarse adjust local clk (change $\frac{1}{T}$, D_0 , etc.)

② Fine tuning (lock f): voltage-controlled oscillator

TX still oscillates 1/0

first CLK sets DFF \rightarrow up/down \rightarrow reset DFF TX



3. Lock the phase TX sends data

sample a, b, c from local clk \rightarrow 2 dff for a, c, 1 for b

if $a=b \neq c \rightarrow$ early \rightarrow down pulse $up = \bar{a}bc + a\bar{b}\bar{c}$

$a \neq b = c \rightarrow$ late \rightarrow up pulse $down = ab\bar{c} + \bar{a}b\bar{c}$

$a = b = c \rightarrow$ no change

$a \neq b \neq c \rightarrow$ error \rightarrow nothing

