

$$\arctan(\frac{1}{\sqrt{3}}) = \frac{\pi}{6}, \arctan(\sqrt{3}) = \frac{\pi}{3}$$

$$\begin{aligned}\sin n\pi &= 0 \\ 1 - \cos n\pi &= 2 \text{ for odd } n\end{aligned}$$

$$\begin{aligned}\sin(x \pm y) &= \sin x \cos y \pm \cos x \sin y \\ \cos(x \pm y) &= \cos x \cos y \mp \sin x \sin y\end{aligned}$$

$$\begin{aligned}\sin x \sin y &= 1/2[\cos(x - y) - \cos(x + y)] \\ \cos x \cos y &= 1/2[\cos(x - y) + \cos(x + y)] \\ \sin x \cos y &= 1/2[\sin(x - y) + \sin(x + y)]\end{aligned}$$

$$Cc(\omega_0t + \theta) = C\,c\,(\theta)\,c\,(\omega_0t) - Cs\,(\theta)s\,(\omega_0t)$$

$$\begin{aligned}\theta &= \tan^{-1}(-b/a), \pm\pi \text{ when } a < 0 \\ \sin t &= \cos(t - \pi/2)\end{aligned}$$

$$\begin{aligned}\cos x &= \frac{1}{2}[e^{jx} + e^{-jx}] \\ \sin x &= \frac{1}{2j}[e^{jx} - e^{-jx}] \\ e^{j\omega t} &= \cos(\omega t) + j\sin(\omega t)\end{aligned}$$

$$\begin{aligned}z^* &= a - jb = re^{-j\theta} \\ uv^* &= (uv)^*\end{aligned}$$

$$\angle z = \tan^{-1}(b/a), \pm\pi \text{ in Q2 and Q3}$$

$$z^{\frac{1}{n}} = r^{\frac{1}{n}}e^{j\frac{\theta+2\pi m}{n}}$$

$$\begin{aligned}\int \cos^2 at \, dt &= \frac{t}{2} + \frac{\sin 2at}{4a} \\ \int t \cos at \, dt &= \frac{1}{a^2}(\cos at + at \sin at) \\ \int t^2 \cos at \, dt &= \frac{1}{a^3}(2atc at - 2s at + a^2t^2s at)\end{aligned}$$

$$\begin{aligned}\int te^{at} \, dt &= \frac{1}{a^2}e^{at}(at - 1) \\ \int t^2e^{at} \, dt &= \frac{1}{a^3}e^{at}(a^2t^2 - 2at + 2)\end{aligned}$$

$$\int e^{at} \cos bt \, dt = \frac{1}{a^2+b^2}e^{at}(a \cos bt + b \sin bt)$$

$$\int \frac{1}{x^2+a^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$\begin{aligned}\mathcal{E}_f &= \int_{-\infty}^{\infty} |f(t)|^2 dt \text{ (complex);} \\ P_f &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |f(t)|^2 dt; \\ \text{rms power} &= \sqrt{P_f}\end{aligned}$$

$$\begin{aligned}\text{Cont; analog; periodic (extension);} \\ \text{(non/anti)causal; energy/power (both);} \\ \text{deterministic/stochastic (carries info)}\end{aligned}$$

$$\int f(t) \cdot \delta(t - t_0) dt = f(t_0) \text{ (} f \text{ cont at } t_0)$$

$$\begin{aligned}\text{out-in } f(2x - 6): &\text{ shift by 6, scale by 2;} \\ f(2(x - 6)): &\text{ scale by 2, shift by 6}\end{aligned}$$

$$\begin{aligned}f_e(t) &= 1/2[f(t) + f(-t)]; \\ f_o(t) &= 1/2[f(t) - f(-t)]\end{aligned}$$

$$\begin{aligned}\text{L: } \mathcal{T}[kf_1(t) + f_2(t)] &= ky_1(t) + y_2(t). \\ \mathcal{T}: \sum_{k=0}^{\infty} a_k D^k y(t) &= \sum_{l=0}^{\infty} b_l D^l f(t), \\ \text{L if } a_k, b_l &\text{ are not functions of } y(t), f(t) \\ \text{E. } \sin \dot{y}(t) + t^2 y(t) &= (t + 3)f(t)\end{aligned}$$

$$\begin{aligned}\text{TI: } \mathcal{T}[f(t - \tau)] &= y(t - \tau). \\ a_k, b_l \text{ indep of } t. &\text{ (const coeff)} \\ \text{Let } g(t) \equiv f(t - \tau), &\text{ find } z(t) = \mathcal{T}[g(t)]\end{aligned}$$

$$\begin{aligned}\text{Causal: } y(t) \text{ dep only on } &f(\tau), \tau < t. \text{ Just} \\ \text{compare } t \text{ and } \tau.\end{aligned}$$

$$\text{Ins/dyn: } y \text{ only dep } f \text{ at present (no } f)$$

$$\text{Invertible: given } y(t), \text{ we can know } f(t)$$

$$c(t) \equiv \int_{-\infty}^{\infty} f(\tau)g(t - \tau)d\tau$$

$$f * g = g * f$$

$$f * (g + h) = f * h + g * h$$

$$f * (g * h) = (f * g) * h$$

$$\begin{aligned}\text{Pf: } f * (g * h) &= f * (h * g) = \\ \int f(\tau_1) \int h(\tau_2) g(t - \tau_1 - \tau_2) &d\tau_2 \, d\tau_1\end{aligned}$$

$$f(t - T_1) * g(t - T_2) = c(t - T_1 - T_2)$$

$$f(at) * g(at) = |1/a| c(at) \text{ (even/odd)}$$

$$f^{(m)}(t) * g^{(n)}(t) = c^{(m+n)}(t)$$

$$\begin{aligned}\text{Graph: shift LEFT by } t, &\text{ and reflect.} \\ \text{Every } \tau \text{ replaced by } t - \tau, &\text{ reverted}\end{aligned}$$

$$f(t) * \delta(t - T) = f(t - T)$$

$$u(t) * u(t) = t \, u(t)$$

$$e^{at} u(t) * u(t) = \frac{1 - e^{at}}{-a} u(t)$$

$$e^{at} u(t) * e^{bt} u(t) = \frac{e^{at} - e^{bt}}{a - b} u(t) \text{ (} te^{at} u(t) \text{)}$$

$$e^{at} u(t) * e^{bt} u(-t) = \frac{e^{at} u(t) + e^{bt} u(-t)}{b - a}$$

$$te^{at} u(t) * e^{at} u(t) = 1/2 t^2 e^{at} u(t)$$

$$t^m u(t) * t^n u(t) = \frac{m!n!}{(m+n+1)!} t^{m+n+1} u(t)$$

$$\text{Don't forget } [u(t + T_1) - u(t - T_2)] \text{ term}$$

$$\begin{aligned}Q(D)y(t) &= P(D)f(t), \text{ typically } \int f \\ \text{Assume causal input } &f(t)u(t)\end{aligned}$$

$$\begin{aligned}y_{zs}(t) &= f(t) * h(t) \text{ from input} \\ y_{zs}(0^-) &= 0, y_{zs}(0^+) \neq 0\end{aligned}$$

$$\begin{aligned}\text{Let } h(t) &= \mathcal{T}[\delta(t)] \text{ (impulse response)} \\ y_{zs}(t) &= \mathcal{T}[f(t)] = \mathcal{T}[f(t) * \delta(t)] \\ &= \mathcal{T}[\lim \sum f(n\Delta\tau)\delta(t - n\Delta\tau)\Delta\tau] \\ &= \lim \sum f(n\Delta\tau)h(t - n\Delta\tau)\Delta\tau = f * h\end{aligned}$$

$$\begin{aligned}y_{zi}(t) \text{ from ini, } f(t) &= 0, Qy_{zi}(t) = 0; \\ y_{zi}(0^-) &= y_{zi}(0^+), y'_{zi}(0^-) = y'_{zi}(0^+)\end{aligned}$$

$$\begin{aligned}\mathcal{E}_e &= \int_{t_1}^{t_2} [e(t)]^2 dt = \int_{t_1}^{t_2} f^2(t) dt \\ -2 \sum c_i \int_{t_1}^{t_2} f(t)x_i(t)dt &+ \int_{t_1}^{t_2} (\sum c_i x_i(t))^2 dt \\ &= \mathcal{E}_f - 2 \sum \langle f, x_i \rangle + (\sum c_i^2 \int_{t_1}^{t_2} x_i(t)^2 dt + \\ &\sum_{i \neq j} c_i c_j \int_{t_1}^{t_2} x_i(t)x_j(t)dt)\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathcal{E}_e}{\partial c_i} &= 0 = -2 \langle f(t), x_i(t) \rangle + 2 \mathcal{E}_i c_i \\ \mathcal{E}_e^{\min} &= \mathcal{E}_f - \sum_{i=1}^N c_i^2 \mathcal{E}_i\end{aligned}$$

$$c_i = \frac{1}{\mathcal{E}_i} \langle f, x_i \rangle = \frac{\int f(t)x_i(t)dt}{\int f^2(t)dt}$$

$$\text{For ortho, } E_z = E_x + E_y$$

$$|u + v|^2 = |u|^2 + |v|^2 + u^*v + v^*u$$

$$\begin{aligned}\langle x(t), y(t) \rangle &= \int_{t_1}^{t_2} x(t)y(t)^* \, dt \\ &= \int_{t_1}^{t_2} x(t)y(t)dt \text{ if real} \\ \text{Use prod } \rightarrow &\text{ sum identities}\end{aligned}$$

$$a_0 = \frac{1}{T_0} \int_{T_0} f(t) \, dt$$

$$a_n = \frac{2}{T_0} \int_{T_0} f(t) \cos(n\omega_0t) \, dt$$

$$\begin{aligned}b_n &= \frac{2}{T_0} \int_{T_0} f(t) \sin(n\omega_0t) \, dt \\ \text{Energy: } T_0 \text{ for } n = 0; &T_0/2 \text{ else}\end{aligned}$$

$$\begin{aligned}\text{Half-w sym: } f(t - T_0/2) &= -f(t) \\ a_{n\text{odd}} &= \frac{4}{T_0} \int_0^{T_0/2} f(t) \cos(n\omega_0t) \, dt\end{aligned}$$

$$\begin{aligned}f(t) &= \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0t} \\ F_n &= \frac{1}{T_0} \int_{T_0} f(t) e^{-jn\omega_0t} \, dt\end{aligned}$$

$$\begin{aligned}C_n \cos(n\omega_0t + \theta_n) &= \\ C_n/2 (e^{j(n\omega_0t + \theta_n)} + e^{-j(n\omega_0t + \theta_n)}) &= \\ (\frac{C_n}{2} e^{j\theta_n}) e^{jn\omega_0t} + (\frac{C_n}{2} e^{-j\theta_n}) e^{-jn\omega_0t}\end{aligned}$$

$$\begin{aligned}F_n &= \frac{C_n}{2} e^{j\theta_n} = \frac{1}{2}(a_n - jb_n) = |F_n|e^{j\angle F_n} \\ F_{-n} &= \frac{C_n}{2} e^{-j\theta_n}\end{aligned}$$

$$\text{W: finite } \int, \text{ fin } a, b, \text{ fin power}$$

$$\text{S: fin m/m/dcont over } T_0, \rightarrow \frac{f(t_0^+) + f(t_0^-)}{2}$$

$$\begin{aligned}\text{Time shift: } f(t - t_0) &\leftrightarrow F_n e^{-jn(\omega_0t_0)}, |F_n| \\ \text{same, } \angle F_n &\text{ shifted by } -(\omega_0t_0)n\end{aligned}$$

$$\text{Reversal: } f(-t) \leftrightarrow F_{-n}$$

$$\text{Scale: } T = T_0/a, \omega = a\omega_0$$

$$\begin{aligned}\text{Multip (same } T_0): f(t)g(t) &\leftrightarrow F_n * G_n \\ \frac{1}{T_0} \int_{T_0} f(t)g(t)e^{jn\omega_0t} \, dt &= \\ \frac{1}{T_0} \int (\sum F_m e^{jm\omega_0t})(\sum G_k e^{jk\omega_0t})e^{-jn\omega_0t} \, dt &= \\ = \sum_m \sum_k F_m G_k \frac{1}{T_0} \int_{T_0} e^{j(m+k-n)\omega_0t} \, dt &= \\ = \sum_m \sum_k F_m G_k \langle e^{j(m+k)\omega_0t}, e^{jn\omega_0t} \rangle &= \\ = \sum_{k=-\infty}^{\infty} G_k F_{n-k}\end{aligned}$$

$$\text{Conjugation: } f(t)^* = F_{-n}^*$$

$$\begin{aligned}\text{Parseval (power): } P_f &= \frac{1}{T_0} \int_{T_0} f(t)f(t)^* dt \\ &= \frac{1}{T_0} \int_{T_0} (\sum_n F_n e^{jn\omega_0t})(\sum_m F_m e^{jm\omega_0t})^* dt \\ &= \sum_n \sum_m F_n F_m^* \frac{1}{T_0} \int_{T_0} e^{j(n-m)\omega_0t} dt \\ &= \sum_n |F_n|^2 \cdot 1\end{aligned}$$

$$\begin{aligned}f \text{ real } \rightarrow |F| \text{ even, } \angle F \text{ odd} \\ f \text{ real, even } \rightarrow F \text{ re, e; } F_{-n} &= F_n = F_n^* \\ f \text{ re, od } \rightarrow F \text{ im, o; } -F_{-n} &= F_n = -F_n^*\end{aligned}$$

$$\begin{aligned}f_e(t) &\leftrightarrow \text{Re}\{F_n\} \\ f_o(t) &\leftrightarrow j \text{Im}\{F_n\}\end{aligned}$$

$$\begin{aligned}\text{Square (} A = 1, T = 2\pi, \omega = 1) \\ \frac{4}{\pi}(\cos t - \frac{1}{3} \cos 3t + \frac{1}{5} \cos 5t - \dots) \\ \frac{4}{\pi}(\sin t + \frac{1}{3} \sin 3t + \frac{1}{5} \sin 5t + \dots)\end{aligned}$$

$$\begin{aligned}\text{Triangle: } \frac{8}{\pi^2}(\sin t - \frac{1}{9} \sin 3t + \frac{1}{25} \sin 5t - \dots) \\ \frac{8}{\pi^2}(\cos t + \frac{1}{9} \cos 3t + \frac{1}{25} \cos 5t + \dots)\end{aligned}$$

$$\begin{aligned}\text{Sawtooth: } \frac{2}{\pi}(\sin t - \frac{1}{2} \sin 2t + \frac{1}{3} \sin 3t - \dots) \\ \frac{2}{\pi}(-\sin t - \frac{1}{2} \sin 2t - \frac{1}{3} \sin 3t - \dots)\end{aligned}$$

$$\begin{aligned}\delta \text{ train: } \delta_{T_0}(t) &= \sum_{n=-\infty}^{\infty} \delta(t - nT_0) \\ \frac{1}{T_0} \sum_{n=-\infty}^{\infty} e^{jn\omega_0t}\end{aligned}$$

0.0.1 Fourier transform

$$\text{Let } F(\omega) \equiv \int f(t)e^{-j\omega t} dt$$

$$F_n = \frac{1}{T_0} \int_{T_0} f(t)e^{-jn\omega_0 t} dt$$

$$\text{Limit as } \omega_0 = \Delta\omega \rightarrow 0, \\ F_n = \frac{\Delta\omega}{2\pi} \int f(t)e^{-jn\Delta\omega t} dt \equiv \frac{\Delta\omega}{2\pi} F(n\Delta\omega)$$

$$f_{T_0}(t) = \sum F_n e^{jn\omega_0 t} = \\ \sum \frac{\Delta\omega}{2\pi} F(n\Delta\omega) e^{jn\Delta\omega t}$$

$$f(t) = \lim f_{T_0}(t) = \frac{1}{2\pi} \int F(\omega)e^{j\omega t} d\omega$$

$$F(\omega) = |F(\omega)| e^{j\angle F(\omega)}$$

Re signals: sym of || and \angle

Existence: energy signal ($|e^{-j\omega t}| = 1$)

Strong: fin num max/min/discont

0.0.2 FT Table

$$\delta(t) \leftrightarrow 1 \\ 1 \leftrightarrow 2\pi\delta(\omega)$$

$$e^{j\omega_0 t} \leftrightarrow 2\pi\delta(\omega - \omega_0) \\ \cos \omega_0 t \leftrightarrow \pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)] \\ \sin \omega_0 t \leftrightarrow j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$$

$$\sum \delta(t - nT_0) \leftrightarrow \omega_0 \sum \delta(\omega - n\omega_0)$$

$$e^{-at} u(t) \leftrightarrow \frac{1}{a+j\omega} \\ e^{-a|t|} \leftrightarrow \frac{2a}{a^2+\omega^2} \\ u(t) = \lim_{a \rightarrow 0} e^{-at} u(t) \leftrightarrow \lim_{a+j\omega} \frac{1}{a+j\omega} \\ = \lim(\frac{a}{a^2+\omega^2} - j\frac{\omega}{a^2+\omega^2}) = \pi\delta(\omega) + \frac{1}{j\omega} \\ \text{sgn}(t) \leftrightarrow \frac{2}{j\omega}$$

$$t^n e^{-at} u(t) \leftrightarrow \frac{n!}{(a+j\omega)^{n+1}}$$

$$c\omega_0 t u(t) \leftrightarrow \frac{\pi}{2}(\delta(-) + \delta(+)) + \frac{j\omega}{\omega_0^2 - \omega^2} \\ \sin \omega_0 t u(t) \leftrightarrow \frac{\pi}{2j}(\delta(-) - \delta(+)) + \frac{\omega_0}{\omega_0^2 - \omega^2} \\ e^{-at} \cos \omega_0 t u(t) \leftrightarrow \frac{a+j\omega}{(a+j\omega)^2 + \omega_0^2} \\ e^{-at} \sin \omega_0 t u(t) \leftrightarrow \frac{\omega_0}{(a+j\omega)^2 + \omega_0^2}$$

$$\text{rect}(\frac{t}{\tau}) \leftrightarrow \tau \text{sinc}(\frac{\tau}{2}\omega) \\ \frac{W}{\pi} \text{sinc}(Wt) \leftrightarrow \text{rect}(\frac{\omega}{2W})$$

$$\Delta(\frac{t}{\tau}) \leftrightarrow \frac{\tau}{2} \text{sinc}^2(\frac{\tau}{4}\omega) \\ \frac{W}{2\pi} \text{sinc}^2(\frac{W}{2}t) \leftrightarrow \Delta(\frac{\omega}{2W})$$

$$[\omega^2 \text{r}(\frac{\omega}{2\omega_0})] \leftarrow \frac{1}{2\pi} \frac{e^{j\omega t}}{(jt)^3} (-\omega^2 t^2 - 2j\omega t + 2) \omega_0 \\ = \frac{(\omega_0^2 t^2 - 2) \sin \omega_0 t + 2\omega_0 t \cos \omega_0 t}{\pi t^3}$$

$$[\frac{|\omega|}{\omega_0} \text{rect}(\frac{\omega}{2\omega_0})] \leftarrow \frac{\cos \omega_0 t + \omega_0 t \sin \omega_0 t - 1}{\omega_0 \pi t^2}$$

0.0.3 Frequency domain properties

Linearity

$$\text{Time shift: } f(t - t_0) \leftrightarrow F(\omega)e^{-jt_0\omega} \\ |F| \text{ unchanged; } \angle F = -t_0\omega, \text{ lin shift}$$

$$\text{Freq shift: } f(t)e^{j\omega_0 t} \leftrightarrow F(\omega - \omega_0)$$

$$\text{t-f dual } f(t) \leftrightarrow F(\omega), F(t) \leftrightarrow 2\pi f(-\omega)$$

$$\text{Pf. } f(t) = \frac{1}{2\pi} \int F(\lambda)e^{j\lambda t} d\lambda \\ 2\pi f(-t) = \int F(\lambda)e^{-tj\lambda} d\lambda = \mathcal{F}[F(\lambda)]$$

$$\text{Reversal: } f(-t) \leftrightarrow F(-\omega)$$

$$\text{Scaling: } f(at) \leftrightarrow \frac{1}{|a|} F(\frac{\omega}{a})$$

$$\text{Convolution: } f * g \leftrightarrow FG, fg \leftrightarrow \frac{1}{2\pi} F * G \\ \mathcal{F}[f * g] = \int e^{-j\omega t} \int f(\tau)g(t - \tau)d\tau dt = \\ \int f(\tau)\mathcal{F}[g(t - \tau)]d\tau = \int f(\tau)G(\omega)e^{-j\omega\tau}d\tau \\ = \frac{1}{2\pi} \mathcal{F}^{-1}[F * G] = \\ (\frac{1}{2\pi})^2 \int e^{j\omega t} \int F(\lambda)G(\omega - \lambda)d\lambda d\omega$$

$$\text{Diff: } f^{(n)}(t) \leftrightarrow (j\omega)^n F(\omega) \text{ (diff } e^{j\omega t})$$

$$\text{Int: } \int_{-\infty}^t f(\tau)d\tau \leftrightarrow \frac{1}{j\omega} F(\omega) + \pi F(0)\delta(\omega) \\ U(\omega) = \lim \frac{1}{a+j\omega} = \lim(\frac{a}{a^2+\omega^2} - j\frac{\omega}{a^2+\omega^2}) \\ = \pi\delta(\omega) + \frac{1}{j\omega} (\int \frac{a}{\omega^2+a^2} d\omega = \tan^{-1} = \pi) \\ \int = f(t) * u(t) \leftrightarrow F(\omega)U(\omega)$$

$$\text{Conjugation: } f(t)^* \leftrightarrow F(-\omega)^*$$

$$\text{Sym: Re } \leftrightarrow || \text{ e, } \angle \text{ o } (F(-\omega) = F(\omega)^*); \\ \text{re, e } \leftrightarrow \text{re, e; re, o } \leftrightarrow \text{im, o}$$

$$f \text{ even: } F(\omega) = 2 \int_0^\infty f(t) \cos(\omega t) dt \\ f \text{ odd: } F(\omega) = -2j \int_0^\infty f(t) \sin(\omega t) dt$$

0.0.4 Parseval

$$\text{Psval: } E_f = \int |f(t)|^2 dt = \frac{1}{2\pi} \int |F(\omega)|^2 d\omega \\ \text{for energy signal}$$

$$\text{Pf: } = \int f f^* dt = \int f(t) \mathcal{F}^{-1}[F(-\omega)^*] dt \\ = \int f(t) \frac{1}{2\pi} \int F(-\omega)^* e^{j\omega t} d\omega dt \\ = \frac{1}{2\pi} \int f(t) \int F(\lambda)^* e^{-j\lambda t} d\lambda dt = \int dt d\lambda \\ \Delta E_f = \frac{2}{2\pi} \int_{\omega_1}^{\omega_2} |F(\omega)|^2 d\omega$$

Autocorrelation

$$\psi_f(t) \equiv \int f(\tau)f(\tau - t)d\tau \leftrightarrow |F(\omega)|^2$$

0.0.5 AM

$$m(t) \cos(\omega_c t) \leftrightarrow \frac{1}{2}[M(\omega + \omega_c) + M(\omega - \omega_c)] \\ e(t) = m(t) \cos^2 \omega_c t \\ E(\omega) = \frac{1}{2}M + \frac{1}{4}[M(\omega + 2\omega_c) + M(\omega - 2\omega_c)]$$

SSB: 1/4 gain

$$\phi_{AM}(t) = [A + f(t)] \cos(\omega_0 t) \\ A \geq f(t) \text{ for all } t$$

$$\text{modulation index } \mu \equiv f_{\max}/A$$

$$\mu = \infty, \text{ SC, } \mu = 1, \text{ marginal}$$

0.0.6 LTIC system transmission

$$\text{Let } e^{j\omega t} \rightarrow H(\omega)e^{j\omega t}$$

$$\lim \sum \frac{F(n\Delta\omega)\Delta\omega}{2\pi} e^{jn\Delta\omega t} \\ \rightarrow \lim \sum \frac{F(n\Delta\omega)H(n\Delta\omega)\Delta\omega}{2\pi} e^{jn\Delta\omega t} \\ = \frac{1}{2\pi} \int F(\omega)H(\omega)e^{j\omega t} d\omega$$

$$Y(\omega) = F(\omega)H(\omega)$$

$$\text{Distortionless: } y(t) = kf(t - t_d), \\ \text{so } H(\omega) = ke^{-j\omega t_d}$$

$$\text{Payley-Wiener: } H \text{ realizable, } h \text{ causal iff} \\ \int \frac{|\ln|H(\omega)||}{1+\omega^2} d\omega < \infty \text{ (consecutive 0s)} \\ \hat{h}(t) = h(t)u(t)$$

0.0.7 Periodic FT

$$f(t) = \sum F_n e^{jn\omega_0 t}, \\ \mathcal{F}[f(t)] = 2\pi \sum F_n \delta(\omega - n\omega_0)$$

$$Y = F(\omega)H(\omega) = \\ 2\pi \sum F_n H(n\omega_0) \delta(\omega - n\omega_0) \\ Y_n \equiv F_n H(n\omega_0). \text{ Periodic with same } \omega_0$$

$$\text{Eigen: } f(t) = e^{j\omega_0 t}, Y_1 = H(1\omega_0), \\ y(t) = H(1\omega_0)e^{j\omega_0 t}$$

$$f(t) = \cos(\omega_0 t + \theta), \text{ assume } h(t) \text{ real} \\ y = \frac{1}{2}(e^{j(\theta+\omega_0 t)} H(\omega_0) + e^{-j(\theta+\omega_0 t)} H(-\omega_0)) \\ = |H(\omega_0)| \cos(\omega_0 t + \theta + \angle H(\omega_0))$$

$$\cos 2t * e^{-3t} u(t) \equiv f * h \\ = |H(2)| \cos(2t + \angle H(2))$$

0.0.8 Sampling

$$\bar{f}(t) \equiv f(t)\delta_{T_s}(t) = \sum f(nT_s)\delta(t - nT_s) \\ \bar{F}(\omega) = \frac{1}{2\pi} F(\omega) * [\frac{2\pi}{T_s} \sum \delta(\omega - n\omega_s)] \\ = \frac{1}{T_s} \sum F(\omega - n\omega_s)$$

$$\omega_s \geq 4\pi B, F_s \geq 2B$$

$$F(\omega) = \bar{F}(\omega)T_s \text{rect}(\frac{\omega}{4\pi B}) \\ \text{If } F_s = 2B, f(t) = \bar{f}(t) * \frac{2B}{F_s} \text{sinc}(2\pi Bt) \\ = \sum f(nT_s)\delta(t - nT_s) * \text{sinc}(2\pi Bt) \\ = \sum f(nT_s) \text{sinc}(2\pi Bt - n\pi)$$

ana FS, basis: sinc, interpolation formula

$$\text{If } F_s > 2B, f(t) = \sum f(nT_s)w(t - nT_s) \\ \text{for some relaxed filter } w(t)$$

Anti-alias before sampling: LPF of $F_s/2$

Practical sampling:

$$p_T(t) = \frac{\tau}{T_s} + \sum (\frac{2}{\pi n} \sin(n\pi \frac{\tau}{T_s})) \cos(n\omega_s t) \\ P_T(\omega) = 2\pi \frac{\tau}{T_s} \delta(\omega) \\ + \sum \frac{2 \sin(\dots)}{n} [\delta(\omega + n\omega_s) + \delta(\omega - n\omega_s)]$$